# GPD analysis

## 1st International School of Hadron Femtography

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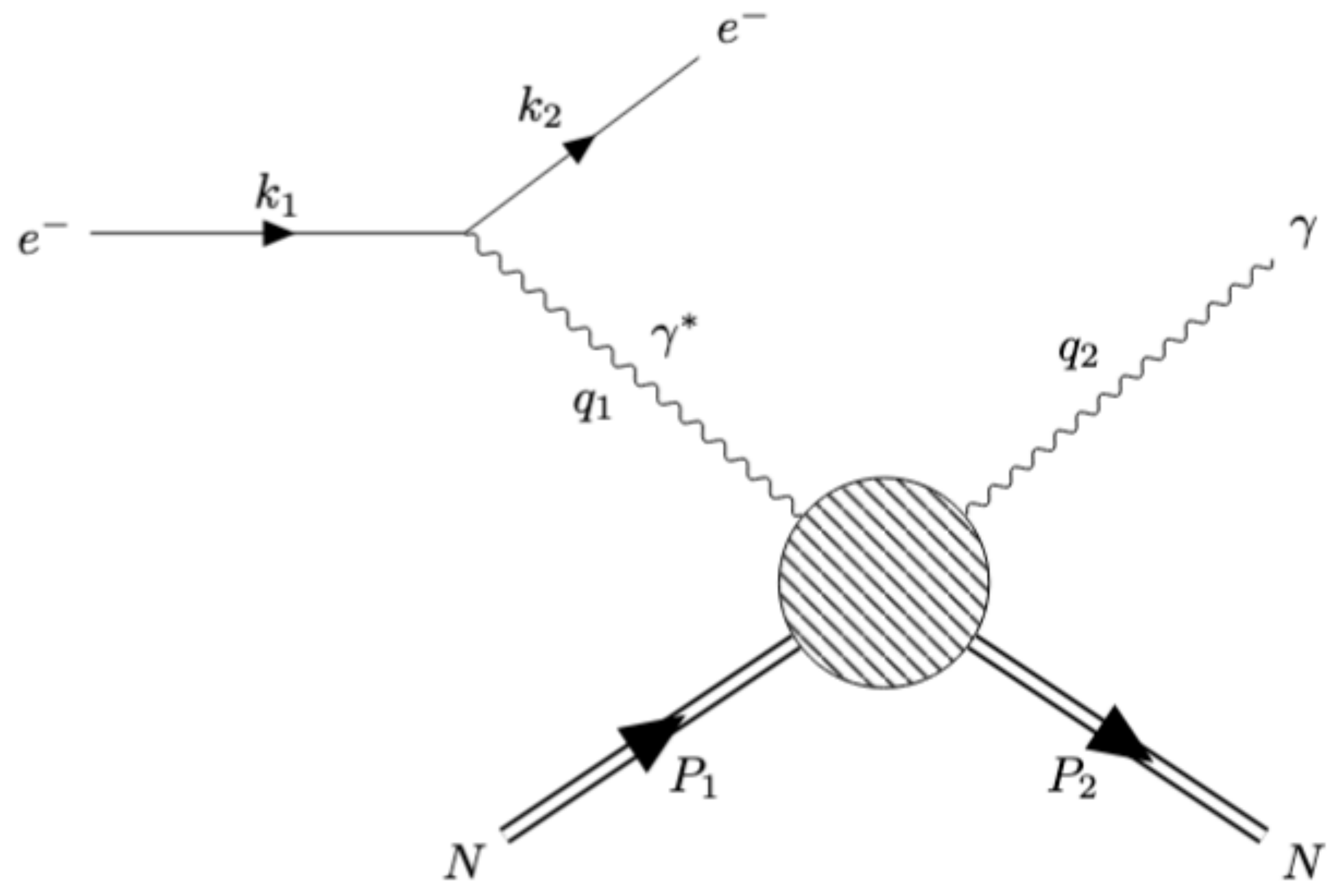
# Part I: Exclusive processes and Compton form factors

# Exclusive processes

## Deeply virtual Compton scattering ($\ell(k_1) + N(P_1) \to \ell(k_2) + N(P_2) + \gamma(q_2)$)




**Gold standard channel** due to a simple final state, the 3D structure of the proton is uncovered because the photon is emitted from the nucleon and because the initial and final nucleon states are not the same

Kinematic variables:

- Photon virtuality: $q_1^2 = -Q^2$
- Bjorken x: $x_\mathrm{B} = \dfrac{Q^2}{2P_1 \cdot q_1}$
- Four-momentum transfer to the hadron: $t = \Delta^2 = (P_2 - P_1)^2$
- Momenta combinations: $q = \dfrac{1}{2}(q_1 + q_2), \quad \mathcal{Q}^2 = -q^2, \quad P = P_1 + P_2$
- Skewness: $\xi = -\dfrac{\Delta \cdot q}{P \cdot q}$
- Generalized Bjorken variable: $\xi_\mathrm{B} = \dfrac{\mathcal{Q}^2}{P \cdot q}$

Generalized Bjorken limit:

$s = (P_1 + q_1)^2 \sim q_1^2 \to \infty, \quad -\Delta^2 \ll s, \quad x_\mathrm{B} = \text{fixed}$

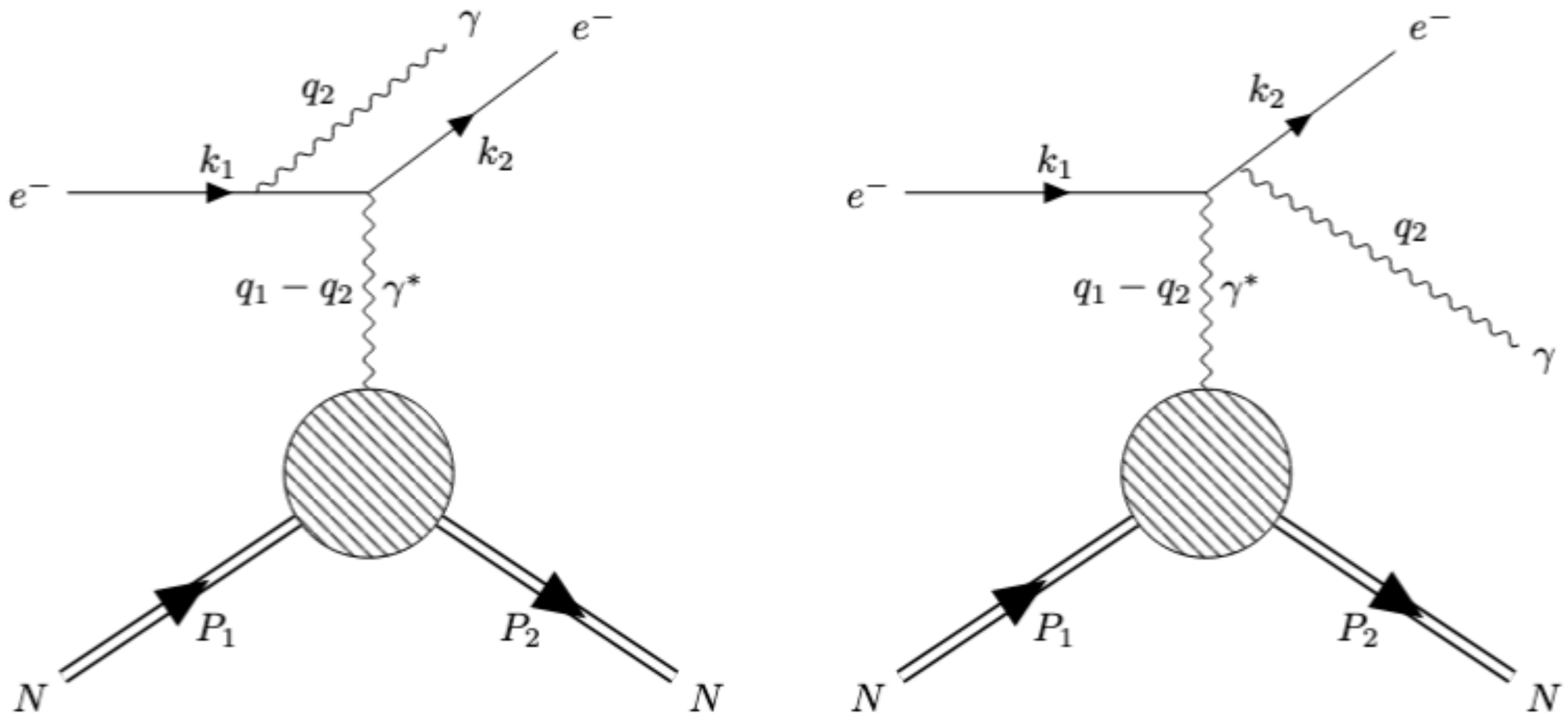
→ the virtual photon has high energy to take a good look into the proton, but the momentum transfer is small enough to not excite or break apart the hadron

→ in this limit we have

$\xi_\mathrm{B} \simeq \xi, \quad Q^2 \simeq 2\mathcal{Q}^2, \quad \xi \simeq \dfrac{x_\mathrm{B}}{2 - x_\mathrm{B}}, \quad s \simeq 2P \cdot q$

# **Bethe-Heitler ($\ell(k_1) + N(P_1) \to \ell(k_2) + N(P_2) + \gamma(q_2)$)**

The same initial and final state as DVCS, but the real photon is emitted as Bremsstrahlung from the initial or final lepton.




Detectors cannot differentiate between DVCS or BH, so we need to **add** their amplitudes coherently in order to calculate the total cross section for the transition between $\ell + N$ to $\ell + N + \gamma$:

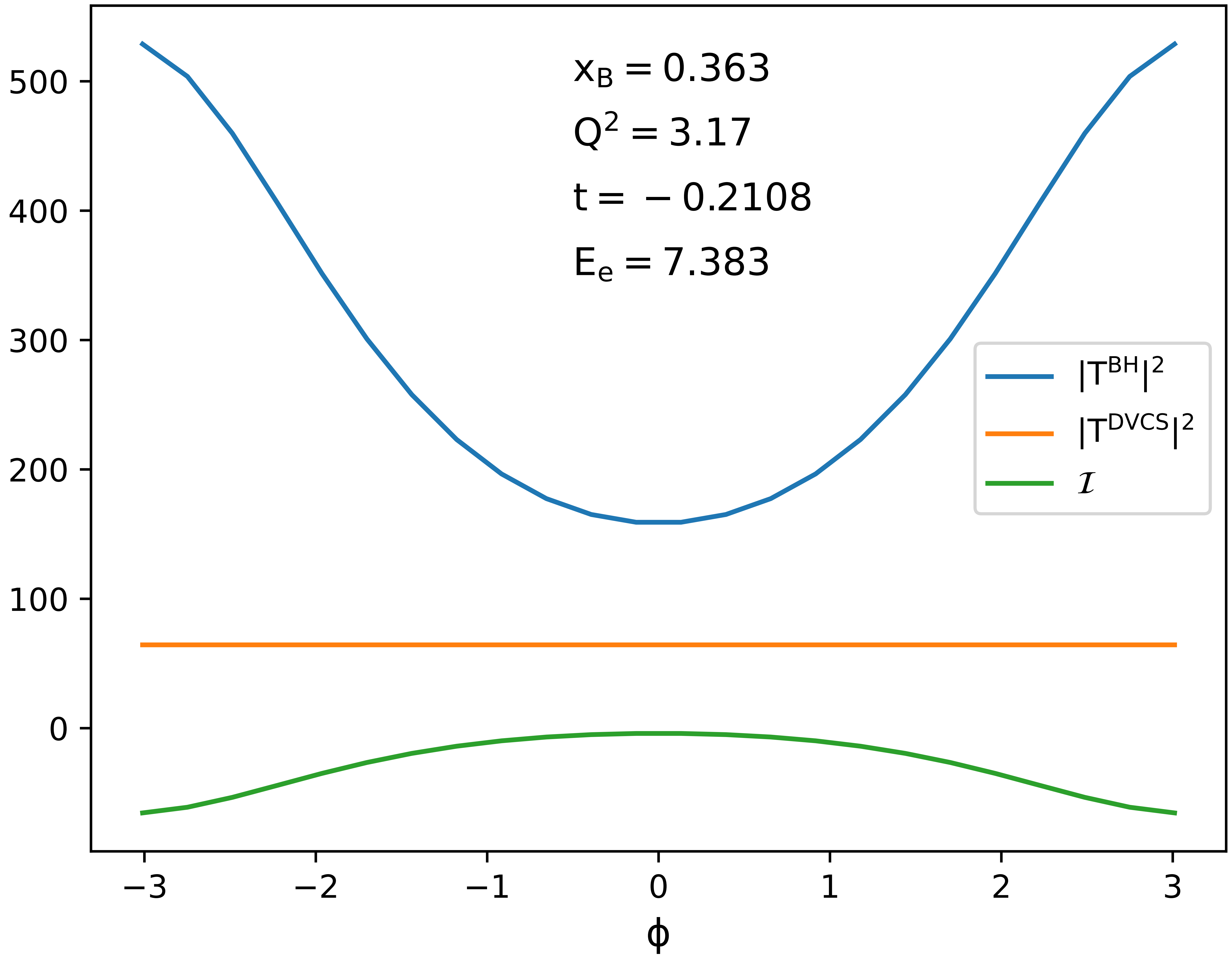
cross section: $\dfrac{d\sigma}{dx_B dy d\left|\Delta^2\right| d\phi d\varphi} = \dfrac{\alpha^3 x_B y}{16\pi^2 Q^2 \sqrt{1+\epsilon^2}} \left|\dfrac{\mathcal{T}}{e^3}\right|^2$

amplitude: $|\mathcal{T}|^2 = \left|\mathcal{T}_{\mathrm{DVCS}}\right|^2 + \left|\mathcal{T}_{\mathrm{BH}}\right|^2 + \mathcal{I}$

interference term: $\mathcal{I} = \mathcal{T}^*_{\mathrm{DVCS}}\mathcal{T}_{\mathrm{BH}} + \mathcal{T}_{\mathrm{DVCS}}\mathcal{T}^*_{\mathrm{BH}}$

This process gives access to elastic form factors, it does not see the 3D structure of hadrons.

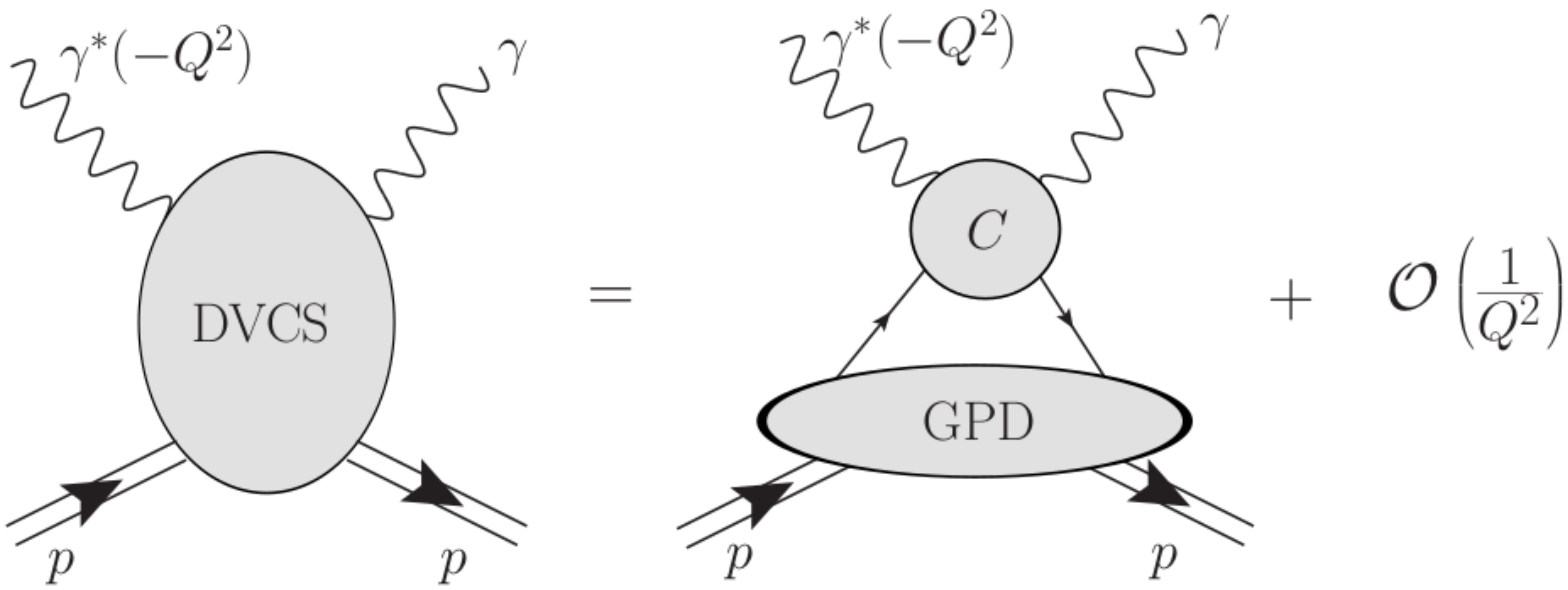
BH usually dominates over the DVCS and the interference term. This strong background may pose experimental challenges. A comparison between the squared DVCS amplitude, squared BH amplitude and the interference term for an unpolarized initial electron and proton at typical JLab kinematics is depicted bellow:




This plot was produced using the KM15 model, which is an LO model fitted to low-x HERA and JLab kinematics. We can see that all of the terms are symmetric with respect to $\phi$.

# Factorization and the structure of the proton

How does the structure of hadrons come into play?




**Factorization theorems** tell us that the process can be separated into a hard (perturbative) part, which is the upper blob denoted by C, and a soft (unperturbative )part, which contains functions that describe the 3D structure of hadrons, the **generalized parton distributions** (GPDs). GPDs describe the transition between the initial hadron state into the final hadron state.

# Compton form factors (CFFs)

Owing to the factorization theorem, we can depict DVCS using hand-bag diagrams, where the upper part is perturbatively calculated, and the lower part, which contains GPDs, is not.

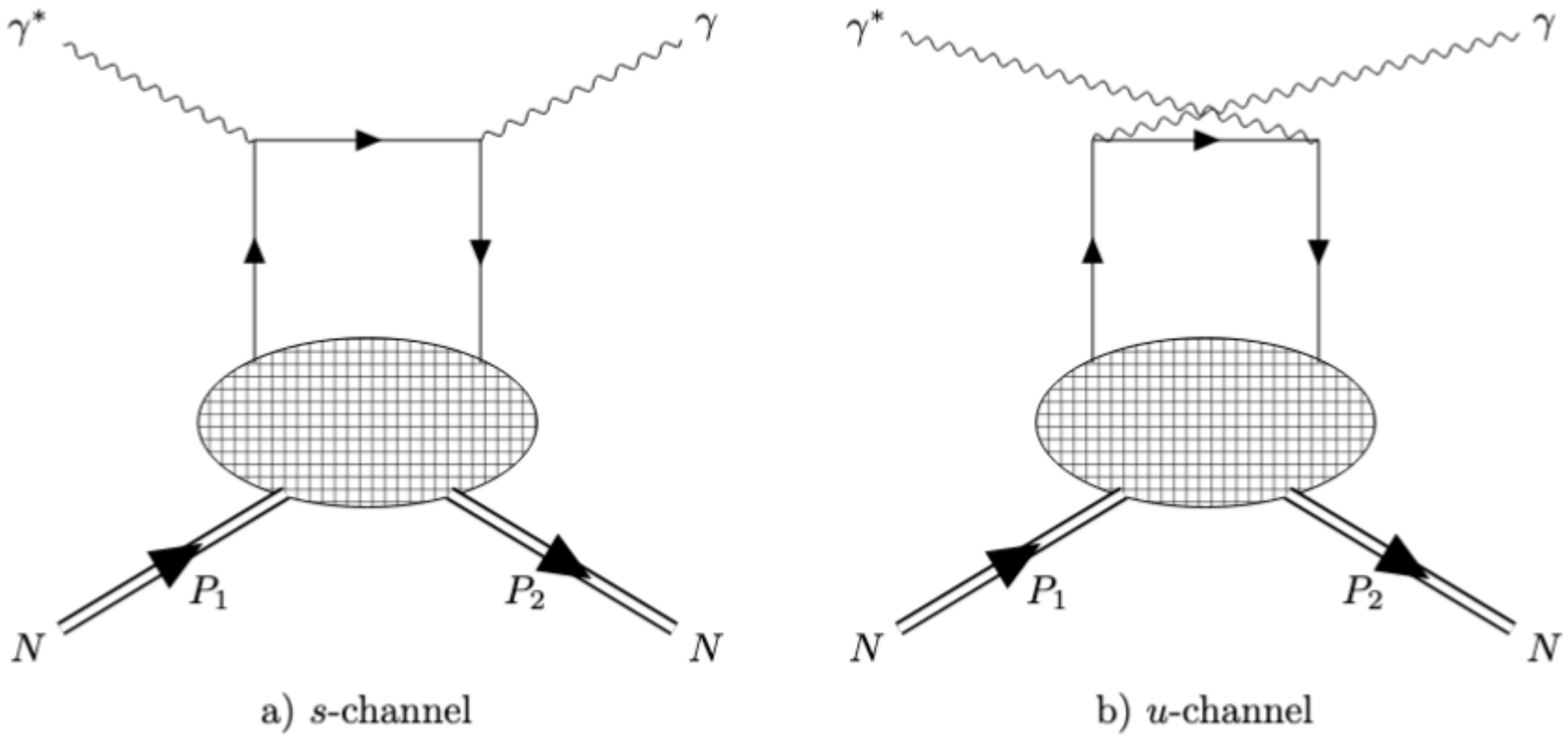
Unfortunately, we cannot access GPDs directly. The cross section is parametrized in terms of **Compton form factors** (CFFs), which are **convolutions** of the hard and soft parts in the process. We can write them in general as:

hard scattering amplitude

$$\mathscr{F}^A\left(\xi, \Delta^2, Q^2\right) = \int_{-1}^{1} \frac{\mathrm{d}x}{2\xi}\, {}^A T\left(x, \xi \,\middle|\, \alpha_s\left(\mu_R\right), \frac{Q^2}{\mu_F^2}\right) F^A\left(x, \xi, \Delta^2, \mu_F^2\right)$$

twist 2 GPDs: $F \in \{H, E, \tilde{H}, \tilde{E}\}$

$$A \in \{u, d, s, \ldots, g\}, \quad \mathscr{F} = \sum_A Q_A^2 \mathscr{F}^A, \quad Q_G^2 = \frac{1}{N_f}\sum_q Q_q^2$$




Hand-bag diagram for LO DVCS

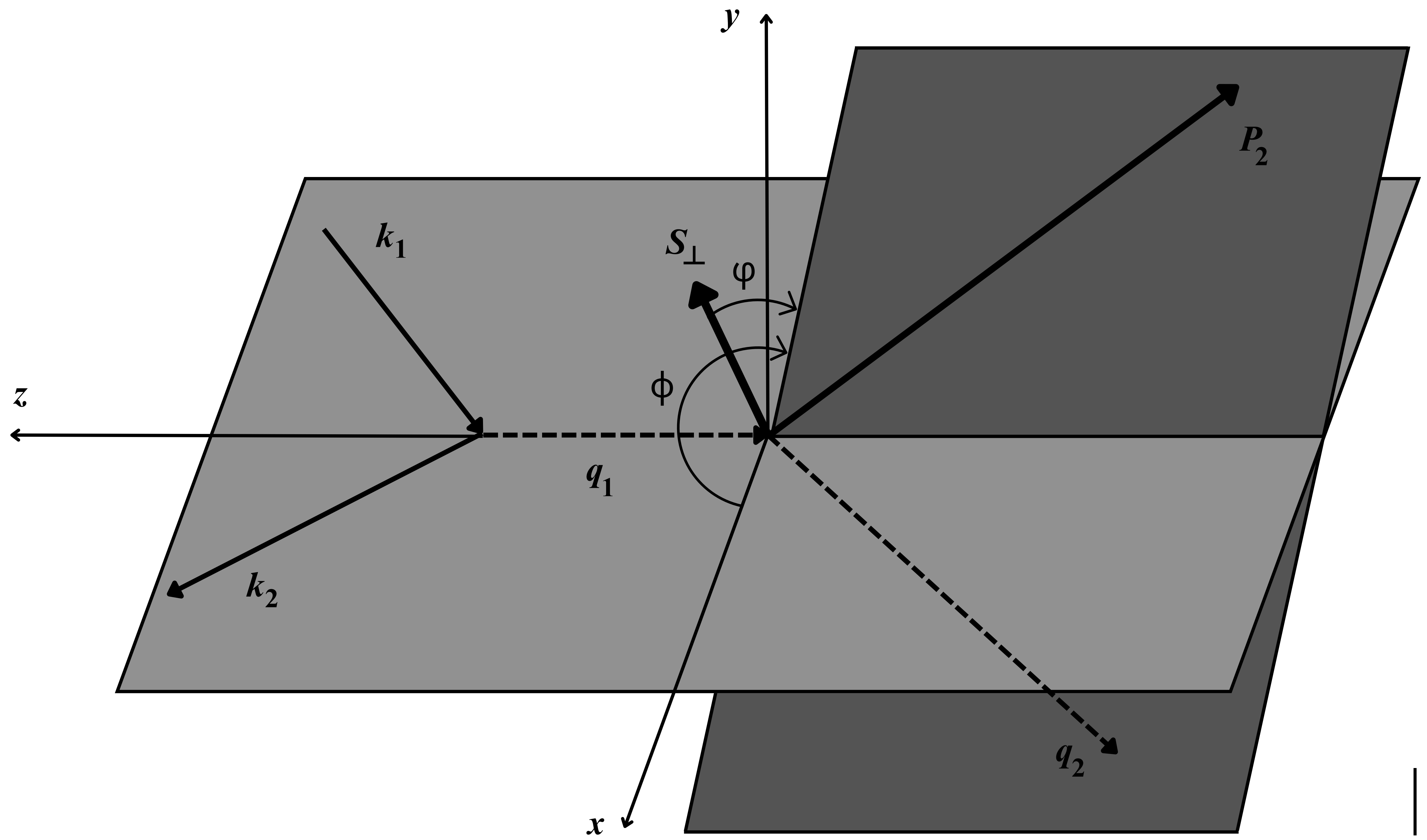
LO CFFs:

$$\mathscr{F}^A = \int_{-1}^{+1} dx \left(\frac{1}{x - \xi - i\epsilon} - \frac{1}{x + \xi - i\epsilon}\right) F^A(x, \xi, t)$$

$$= \underbrace{\mathscr{P}\int_{-1}^{+1} dx \left(\frac{1}{x - \xi} - \frac{1}{x + \xi}\right) F^A(x, \xi, t)}_{\text{real part}} - i\,\underbrace{\pi(F^A(\xi, \xi, t) - F^A(-\xi, \xi, t))}_{\text{imaginary part}}$$

# Cross section

In order to calculate the cross section, we need to choose a frame. A common choice is depicted bellow:




Belitsky-Kirchner-Mueller frame

Because we have a $2 \to 3$ process, it cannot happen in one plane. We define the leptonic plane, where the initial and final lepton are contained, as well as the virtual photon. At an angle $\phi$ to this plane, we have the hadronic plane, which contains the initial and final hadron, as well as the real photon. The virtual photon is contained in the intersection of these two planes, and we define the $z$-axis so that the virtual photon moves along it in the negative direction. In the BKM formalism, the cross section is written as:

$$\left|\mathcal{T}_{\mathrm{BH}}\right|^2 = \frac{e^6}{x_{\mathrm{B}}^2 y^2 \left(1+\epsilon^2\right)^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin(\phi) \right\}$$

$$\left|\mathcal{T}_{\mathrm{DVCS}}\right|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left[ c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right] \right\}$$

$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{B}} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^I + \sum_{n=1}^{3} \left[ c_n^I \cos(n\phi) + s_n^I \sin(n\phi) \right] \right\}, \quad \varepsilon = 2x_{\mathrm{B}} \frac{M}{Q^2}$$

The fact that the BH process, which we can describe sufficiently well at the level of precision we have for DVCS, comes into the full $\ell N \to \ell N \gamma$ cross section provides a unique opportunity to access CFFs linearly and quadratically!

$$\left|\mathcal{T}_{\mathrm{DVCS}}\right|^2 = \frac{2\left(2-2y+y^2\right)}{y^2 Q^2 \left(2-x_{\mathrm{B}}\right)^2} \left[ 4\left(1-x_{\mathrm{B}}\right)\left( |\mathcal{H}|^2 + |\widetilde{\mathcal{H}}|^2 \right) - \left( x_{\mathrm{B}}^2 + \left(2-x_{\mathrm{B}}\right)^2 \frac{\Delta^2}{4M^2} \right) |\mathcal{E}|^2 - x_{\mathrm{B}}^2 \left( \mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^* \right) - x_{\mathrm{B}}^2 \frac{\Delta^2}{4M^2} |\widetilde{\mathcal{E}}|^2 \right], \quad y = \frac{Q^2}{xs}$$

We can also write the cross section in terms of **helicity amplitudes**:

$$\sigma_{h\Lambda} = \Gamma \sum_{\Lambda'_\gamma,\Lambda'} \left(T^{h\Lambda'_\gamma}_{\text{DVCS},\Lambda\Lambda'}\right)^* T^{h\Lambda'_\gamma}_{\text{DVCS},\Lambda\Lambda'} = \Gamma \sum_{\Lambda^{(1)}_{\gamma^*},\Lambda^{(2)}_{\gamma^*}} \mathscr{L}_h^{\Lambda^{(1)}_{\gamma^*},\Lambda^{(2)}_{\gamma^*}} H_\Lambda^{\Lambda^{(1)}_{\gamma^*},\Lambda^{(2)}_{\gamma^*}}$$

lepton tensor: $\mathscr{L}_h^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}} = \sum_{\Lambda^{(1)}_{\gamma^*},\Lambda^{(2)}_{\gamma^*}} A_h^{\Lambda^{(1)}_{\gamma^*}} A_h^{\Lambda^{(2)}_{\gamma^*}}$

lepton helicity amplitude: $A_h^{\Lambda_{\gamma^*}} = \dfrac{1}{Q^2}\overline{u}(k',h)\gamma^\mu u(k,h)\left(\varepsilon_\mu^{\Lambda_{\gamma^*}}(q)\right)^*$

hadron tensor: $H_\Lambda^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}} = \sum_{\Lambda'} F_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}}$

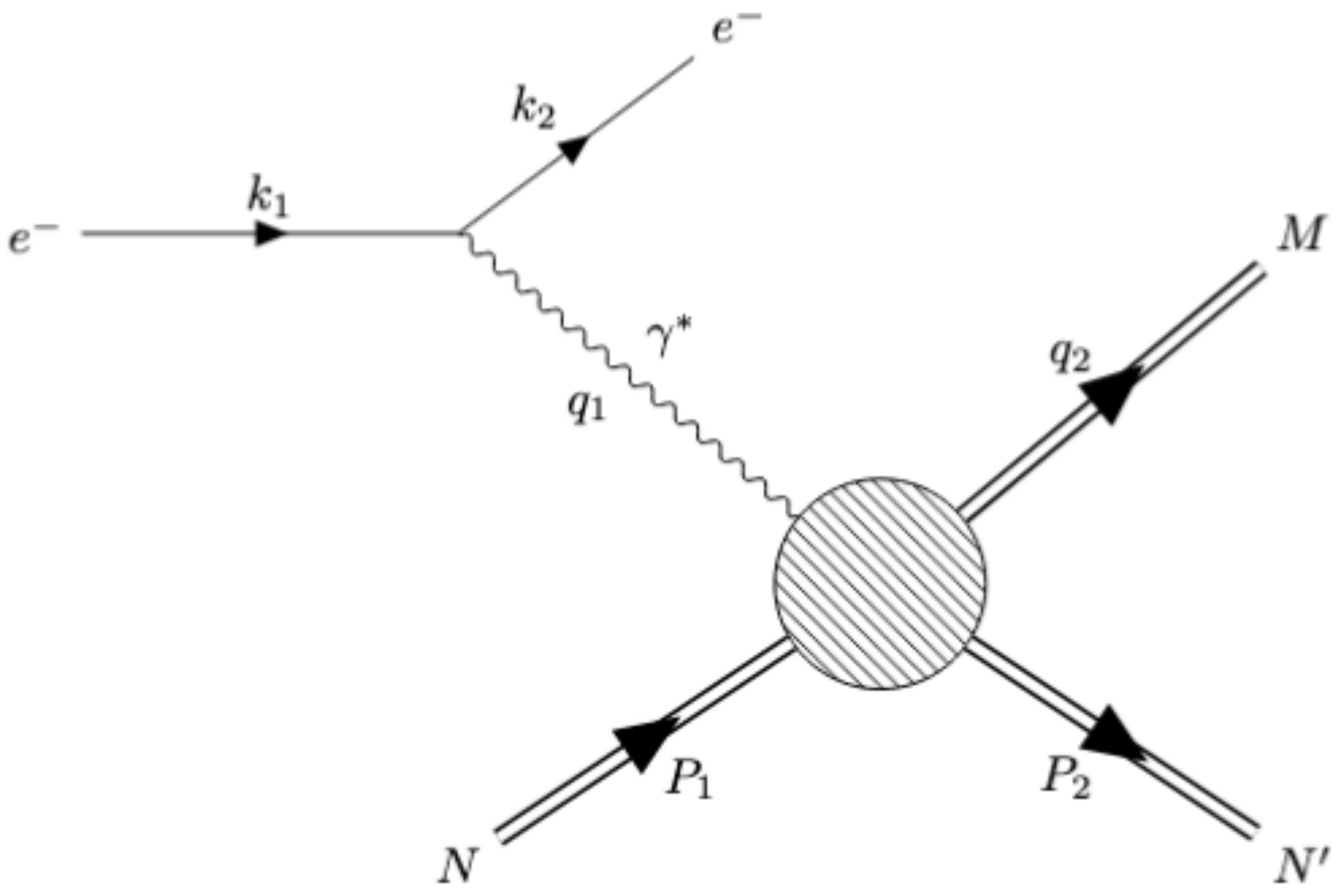
hadron helicity amplitudes: $F_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}} = \sum_{\Lambda'_\gamma} \left[f_{\Lambda,\Lambda'}^{\Lambda^{(1)}_{\gamma^*},\Lambda'_\gamma}\right]^* f_{\Lambda,\Lambda'}^{\Lambda^{(2)}_{\gamma^*},\Lambda'_\gamma}$

$\Rightarrow$

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \Gamma\left|T_{DVCS}\right|^2$$

$$\begin{aligned}
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} \\
&+\sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} \\
&+(2\Lambda)\Big[\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} \\
&+(2h)\Big(\sqrt{1-\epsilon^2}F_{LL} + 2\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}\Big)\Big] \\
&+\left(2\Lambda_T\right)\Big[\sin\left(\phi-\phi_S\right)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
&+\epsilon\sin\left(\phi+\phi_S\right)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin\left(3\phi-\phi_S\right)F_{UT}^{\sin(3\phi-\phi_S)} \\
&+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin\left(2\phi-\phi_S\right)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+(2h)\left(2\Lambda_T\right)\Big[\sqrt{1-\epsilon^2}\cos\left(\phi-\phi_S\right)F_{LT}^{\cos(\phi-\phi_S)} \\
&+\sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&+\sqrt{2\epsilon(1-\epsilon)}\cos\left(2\phi-\phi_S\right)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}$$

# Deeply virtual meson production $\ell(k_1) + N(P_1) \to \ell(k_2) + N'(P_2) + M(q_2)$

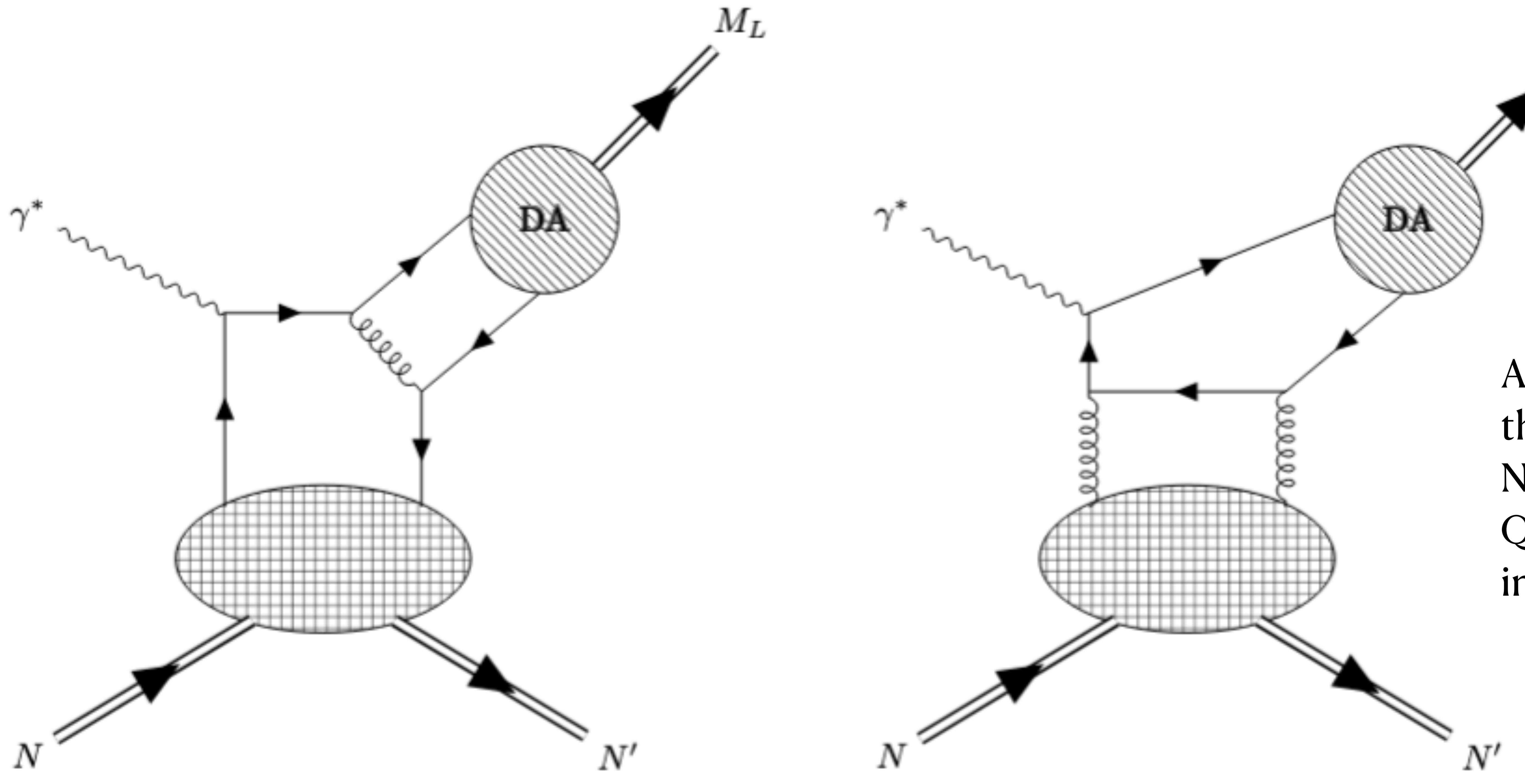



Factorization was proven for light pseudoscalar mesons and longitudinally polarized vector mesons. The structure of the hadron is contained inside **transition form factors**:

$$\mathscr{F}^A\left(\xi, \Delta^2, Q^2\right) = \frac{fC_\mathrm{F}}{QN_\mathrm{c}} \int_{-1}^{1} \frac{\mathrm{d}x}{2\xi} \int_0^1 \mathrm{d}v \underbrace{\varphi(v)^\mathrm{A}}_{\text{Distribution amplitude}} \overbrace{T\left(x, v, \xi \,\middle|\, \alpha_\mathrm{s}\left(\mu_\mathrm{R}\right), \frac{Q^2}{\mu_\mathrm{F}^2}, \frac{Q^2}{\mu_\varphi^2}, \frac{Q^2}{\mu_\mathrm{R}^2}\right)}^{\text{hard scattering amplitude}} \underbrace{F^A\left(x, \xi, \Delta^2, \mu_\mathrm{F}^2\right)}_{\text{GPDs}}$$

DAs are also unperturbative soft scale functions. They describe the transition between vacuum and the final state meson, and $v$ describes the fraction of the longitudinal momentum of a parton inside the meson.

Depending on the charge of the final meson, the hadron in the final state can differ to the initial hadron. Due to an intricate flavor structure of the final-state mesons, we can access several flavor combinations of GPDs through various DVMP measurements. We expect these GPDs to be universal for all DVCS and DVMP variations, as well as other processes that probe GPDs (or more precisely, their convolutions).




a) quark subprocess

b) gluon subprocess

At LO we already probe gluons, because we need gluons to produce the final meson. Gluons only appear in DVCS through evolution or NLO hard scattering coefficients. Because of gluons, we only have one QED vertex, so the cross section should be larger. Due to two hadrons in the final state, factorization is messier to prove.

LO DVMP

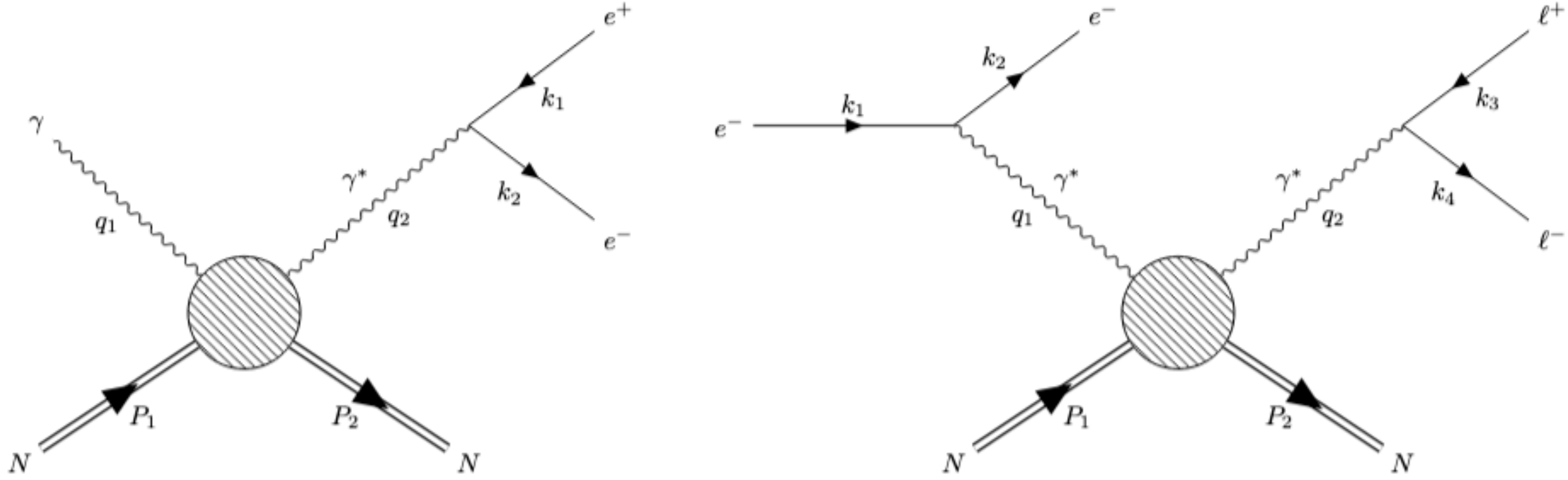
The cross section is given as:

$$\frac{\mathrm{d}^2\sigma^{\gamma_L^* \longmapsto MN'}}{\mathrm{d}\Delta^2\ \mathrm{d}\phi} = \frac{\alpha_{\mathrm{em}} x_{\mathrm{B}}^2 y^2}{32\pi Q^2\sqrt{1+\epsilon^2}} \frac{1}{1-y} \left|\mathcal{T}^{\mathrm{DVMP}}\right|^2,$$

$$\left|\mathcal{T}^{\mathrm{V_L}}\right|^2 = 16\frac{1-y}{y^2\left(2-x_{\mathrm{B}}\right)^2}\left[4\left(1-x_{\mathrm{B}}\right)|\mathcal{H}|^2 - x_{\mathrm{B}}^2\left(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^*\right) - \left(x_{\mathrm{B}}^2 + \left(2-x_{\mathrm{B}}\right)^2\frac{\Delta^2}{4M^2}\right)|\mathcal{E}|^2\right],$$

$$\left|\mathcal{T}^{\mathrm{PS}}\right|^2 = 16\frac{1-y}{y^2\left(2-x_{\mathrm{B}}\right)^2}\left[4\left(1-x_{\mathrm{B}}\right)|\widetilde{\mathcal{H}}|^2 - x_{\mathrm{B}}^2\left(\widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^*\right) - x_{\mathrm{B}}^2\frac{\Delta^2}{4M^2}|\widetilde{\mathcal{E}}|^2\right]$$

# Other exclusive processes



a) TCS b) DDVCS

Timelike Compton scattering and double DVCS contain the same GPDs, but probe them at different kinematics. TCS and DVCS are limiting cases of DDVCS.

+ processes with more particles in the final state, like two photons or two mesons

# Dispersion relations

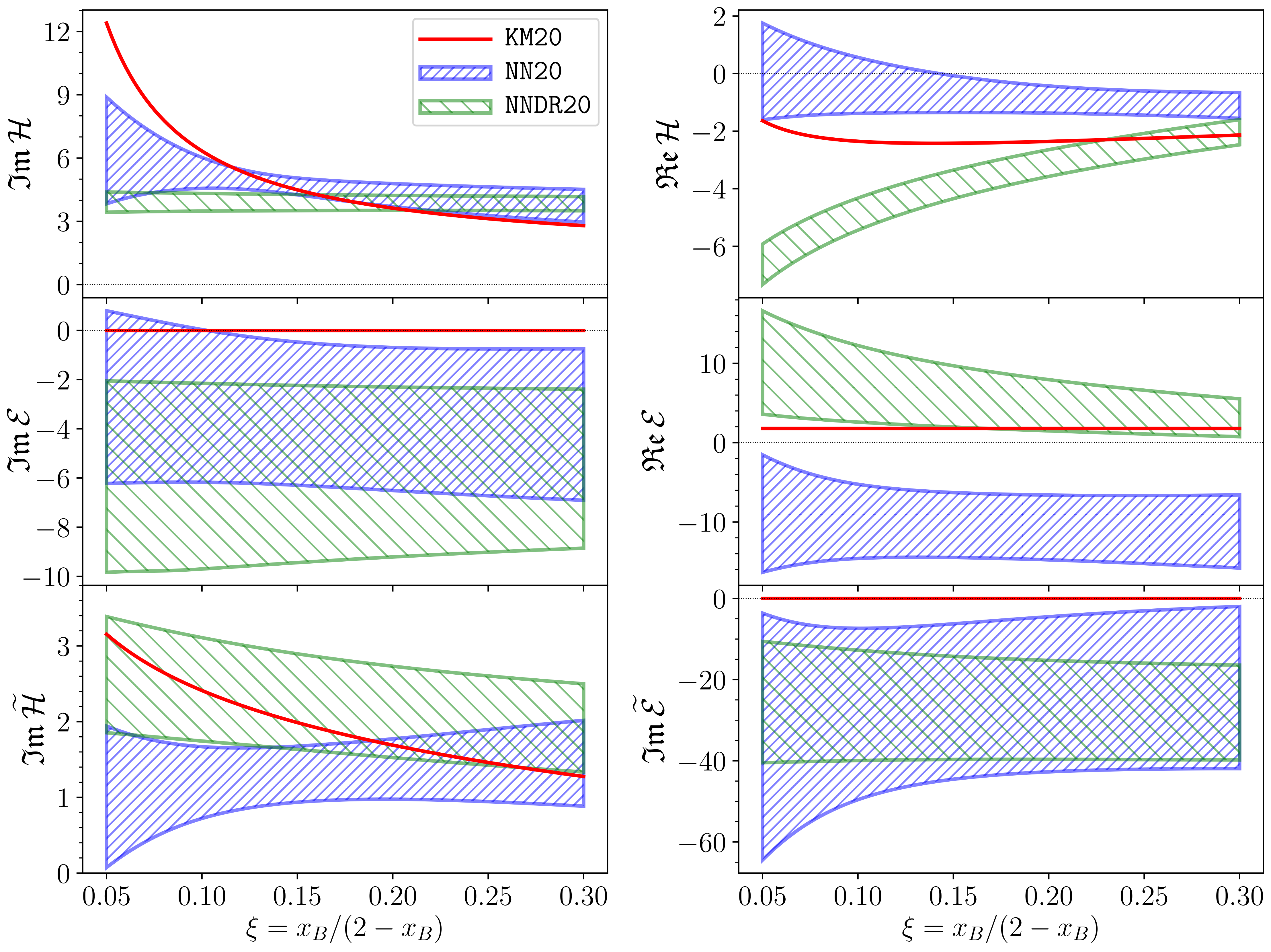
Analysis of DVCS (and other processes) can be done at the amplitude level, i.e. at the CFF level. One property that comes from analyticity and causality considerations of CFFs are (once subtracted) dispersion relations, which relate their real and imaginary parts as:

$$\mathfrak{Re}\mathscr{H}\left(\xi, t, Q^2\right) = \frac{1}{\pi}\,\text{P.V.}\int_0^1 d\xi' \mathfrak{Im}\mathscr{H}\left(\xi', t, Q^2\right)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) + \underbrace{\Delta_{\mathscr{H}}\left(t, Q^2\right)}_{\Downarrow\ \text{Subtraction constant}}$$

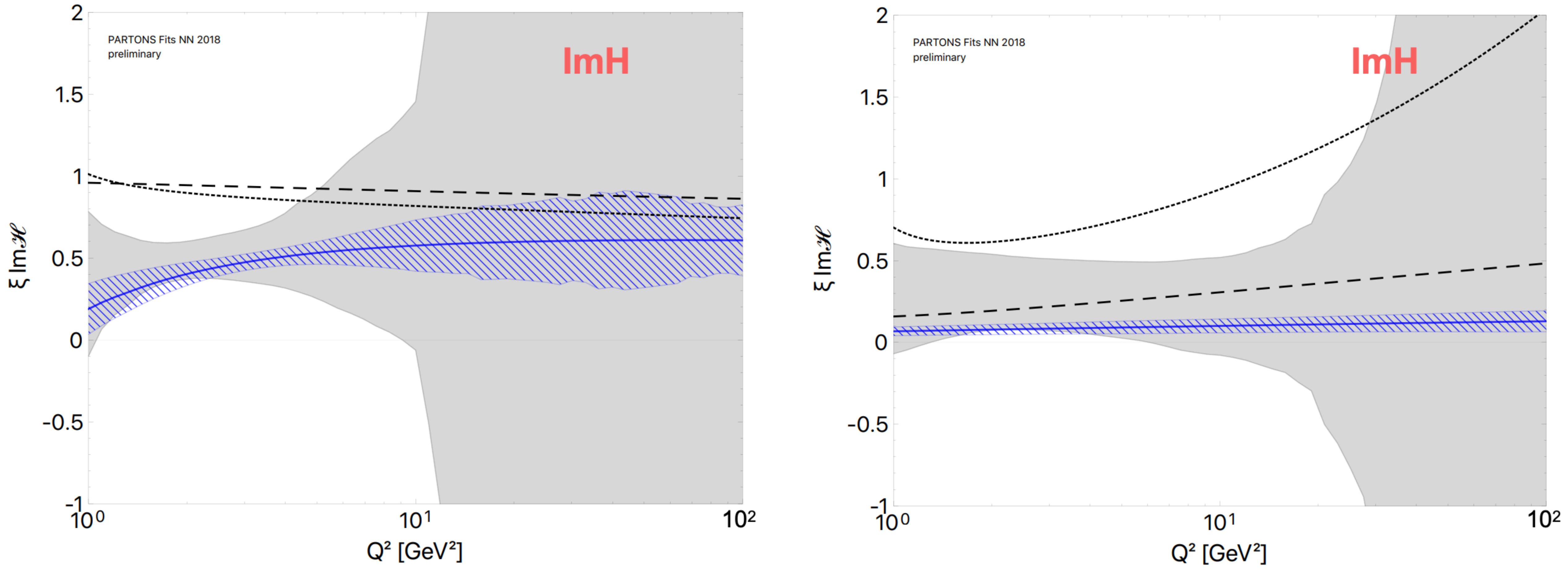
The subtraction constant is necessary to cancel out divergences at the point $\xi = 0$ and $x = 0$ that occur in CFFs. They rely on analytically continuing CFFs outside of the physical domain of $\xi$, which is defined as:

$$|\xi| \leq \frac{\sqrt{-t}}{\sqrt{-t+4M^2}}$$

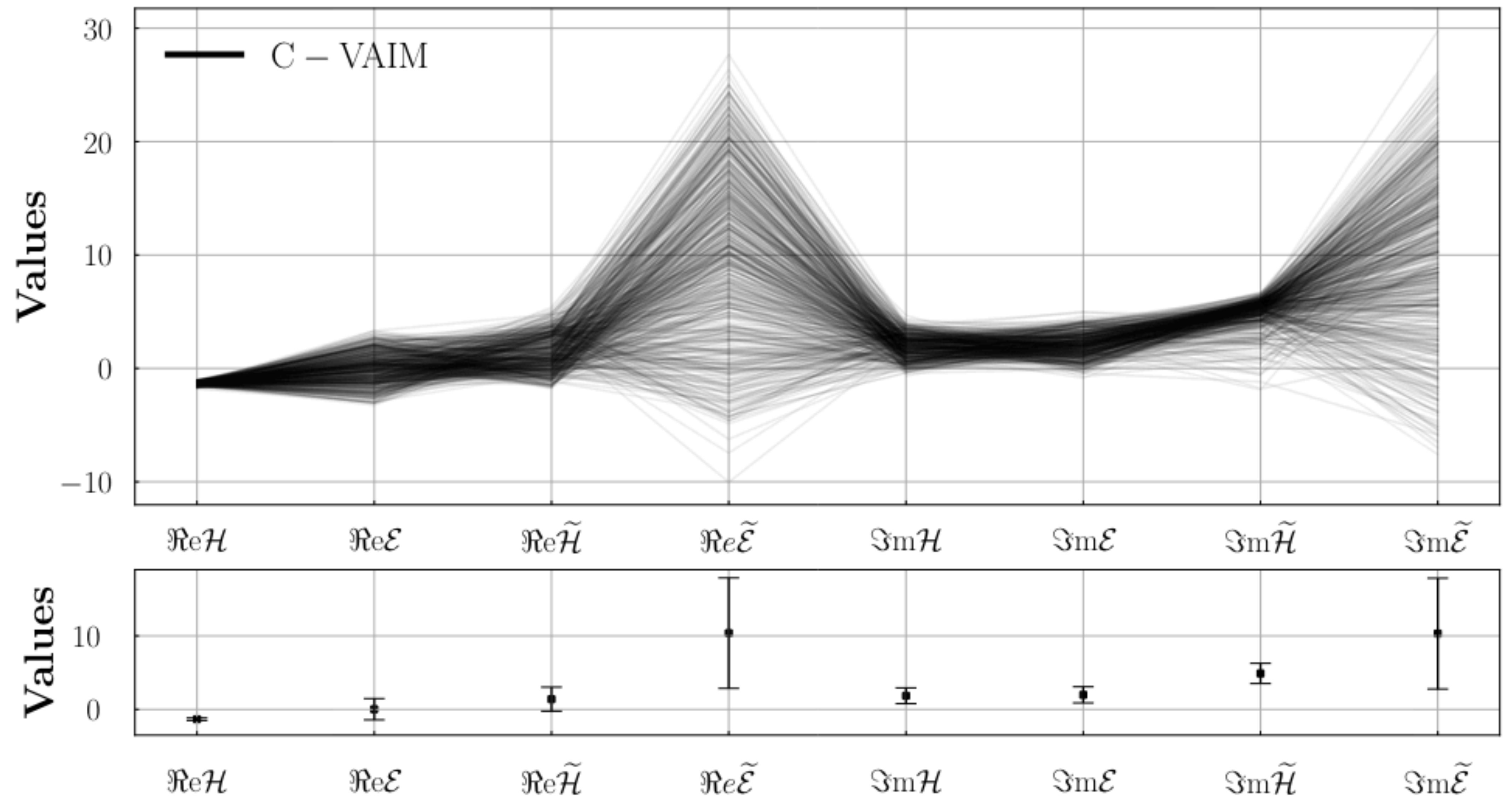
# CFF extraction




Čuić, Kumerički, Schäfer, 2020




Partons, 2019




Exclaim, 2024

# Observables

beam-spin asymmetry: $A_{\mathrm{LU}}(\phi) = \dfrac{\mathrm{d}\sigma^{\uparrow}(\phi) - \mathrm{d}\sigma^{\downarrow}(\phi)}{\mathrm{d}\sigma^{\uparrow}(\phi) + \mathrm{d}\sigma^{\downarrow}(\phi)} \propto \mathfrak{Im}\left\{F_1\mathscr{H} + \xi\left(F_1 + F_2\right)\widetilde{\mathscr{H}} - \dfrac{\Delta^2}{4M^2}F_2\mathscr{E}\right\}\sin(\phi)$

target-spin asymmetry: $A_{\mathrm{UL}}(\phi) = \dfrac{\mathrm{d}\sigma^{\Rightarrow}(\phi) - \mathrm{d}\sigma^{\Leftarrow}(\phi)}{\mathrm{d}\sigma^{\Rightarrow}(\phi) + \mathrm{d}\sigma^{\Leftarrow}(\phi)} \propto \mathfrak{Im}\left[F_1\widetilde{\mathscr{H}} + \xi\left(F_1 + F_2\right)\left(\mathscr{H} + \dfrac{x_{\mathrm{B}}}{2}\mathscr{E}\right) - \xi\left(\dfrac{x_{\mathrm{B}}}{2}F_1 + \dfrac{t}{4M^2}F_2\right)\widetilde{\mathscr{E}}\right]\sin(\phi)$

double spin asymmetry: $A_{\mathrm{LL}}(\phi) = \dfrac{\mathrm{d}\sigma^{\uparrow\Uparrow}(\phi) - \mathrm{d}\sigma^{\downarrow\Uparrow}(\phi) - \mathrm{d}\sigma^{\uparrow\Downarrow}(\phi) + \mathrm{d}\sigma^{\downarrow\Downarrow}(\phi)}{\mathrm{d}\sigma^{\uparrow\Uparrow}(\phi) + \mathrm{d}\sigma^{\downarrow\Uparrow}(\phi) + \mathrm{d}\sigma^{\uparrow\Downarrow}(\phi) + \mathrm{d}\sigma^{\downarrow\Downarrow}(\phi)} \propto \mathfrak{Re}\left[F_1\widetilde{\mathscr{H}} + \xi\left(F_1 + F_2\right)\left(\mathscr{H} + \dfrac{x_{\mathrm{B}}}{2}\mathscr{E}\right) - \xi\left(\dfrac{x_{\mathrm{B}}}{2}F_1 + \dfrac{t}{4M^2}F_2\right)\widetilde{\mathscr{E}}\right]\cos\phi + BH$

beam-charge asymmetry: $A_{\mathrm{C}} = \dfrac{\sigma^{+}_{\mathrm{UU}} - \sigma^{-}_{\mathrm{UU}}}{\sigma^{+}_{\mathrm{UU}} + \sigma^{-}_{\mathrm{UU}}} \propto \mathfrak{Re}\left[F_1\mathscr{H} + \xi\left(F_1 + F_2\right)\widetilde{\mathscr{H}} - \dfrac{\Delta^2}{4M^2}F_2\mathscr{E}\right]\cos\phi$

beam-spin sum: $\mathrm{d}^4\sigma = \dfrac{1}{2}\left[\dfrac{\mathrm{d}^4\sigma(\lambda = +1)}{\mathrm{d}Q^2\mathrm{d}x_{\mathrm{B}}\mathrm{d}t\mathrm{d}\phi} + \dfrac{\mathrm{d}^4\sigma(\lambda = -1)}{\mathrm{d}Q^2\mathrm{d}x_{\mathrm{B}}\mathrm{d}t\mathrm{d}\phi}\right]$

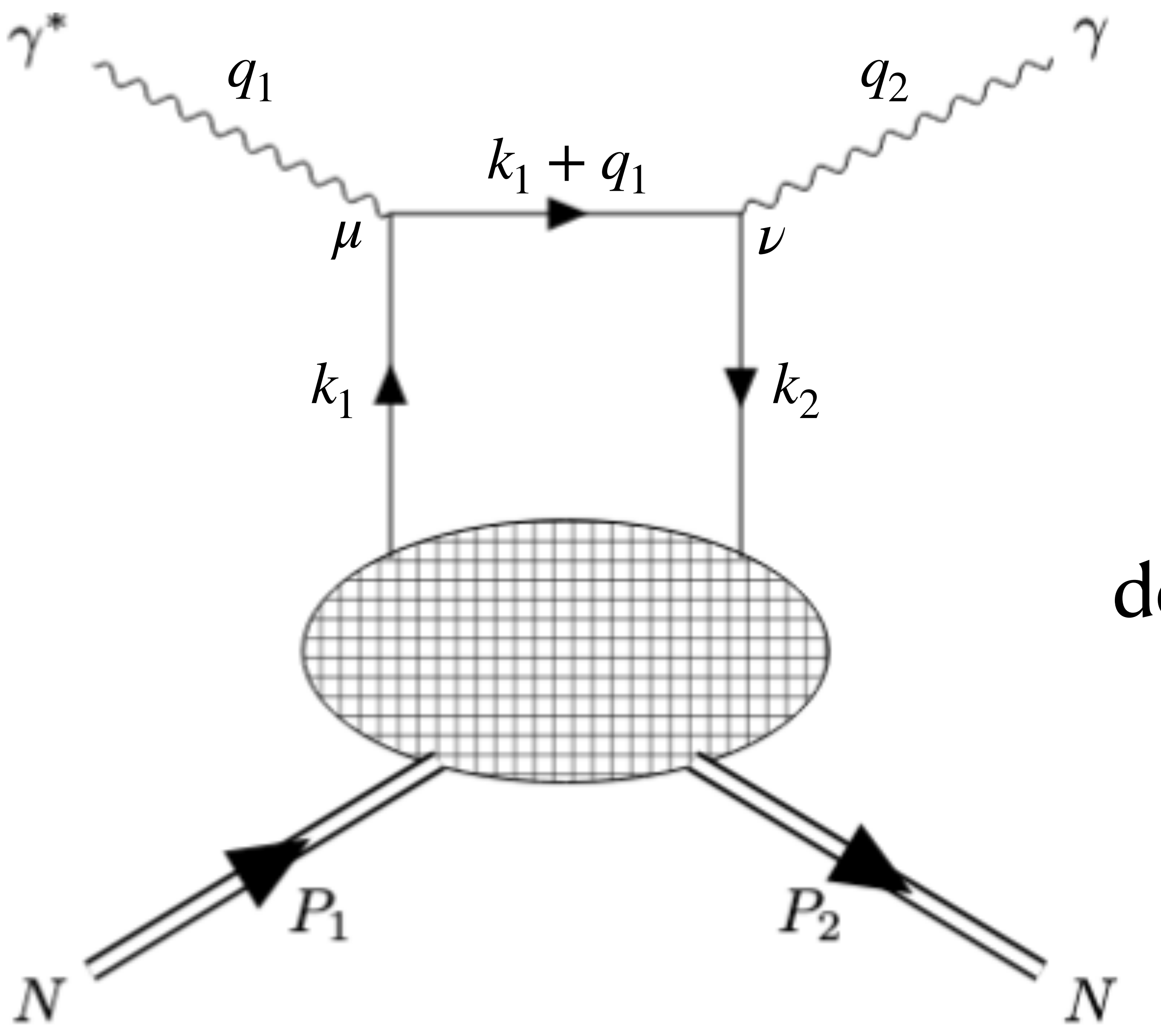
beam-spin difference: $\Delta^4\sigma = \dfrac{1}{2}\left[\dfrac{\mathrm{d}^4\sigma(\lambda = +1)}{\mathrm{d}Q^2\mathrm{d}x_{\mathrm{B}}\mathrm{d}t\mathrm{d}\phi} - \dfrac{\mathrm{d}^4\sigma(\lambda = -1)}{\mathrm{d}Q^2\mathrm{d}x_{\mathrm{B}}\mathrm{d}t\mathrm{d}\phi}\right]$

| Experiment | Kinematics | | |
|---|---|---|---|
| | $x_{\mathrm{B}}$ | $Q^2\ [\mathrm{GeV}^2]$ | $t\ [\mathrm{GeV}^2]$ |
| HERMES | 0.09 | 2.50 | -0.12 |
| CLAS | 0.19 | 1.25 | -0.19 |
| HALL A | 0.36 | 2.30 | -0.23 |
| HERA | 0.001 | 8.00 | -0.30 |

Typical experiment kinematics

| Experiment | Observable | Normalized CFF dependence |
|---|---|---|
| HERMES | $A_{\mathrm{C}}^{\cos 0\phi}$ | $\mathfrak{Re}\mathcal{H} + 0.06\mathfrak{Re}\mathcal{E} + 0.24\mathfrak{Re}\widetilde{\mathcal{H}}$ |
| | $A_{\mathrm{C}}^{\cos \phi}$ | $\mathfrak{Re}\mathcal{H} + 0.05\mathfrak{Re}\mathcal{E} + 0.15\mathfrak{Re}\widetilde{\mathcal{H}}$ |
| | $A_{\mathrm{LU,I}}^{\sin \phi}$ | $\mathfrak{Im}\mathcal{H} + 0.05\mathfrak{Im}\mathcal{E} + 0.12\mathfrak{Im}\widetilde{\mathcal{H}}$ |
| | $A_{\mathrm{UL}}^{+,\sin \phi}$ | $\mathfrak{Im}\widetilde{\mathcal{H}} + 0.10\mathfrak{Im}\mathcal{H} + 0.01\mathfrak{Im}\mathcal{E}$ |
| | $A_{\mathrm{UL}}^{+,\sin 2\phi}$ | $\mathfrak{Im}\widetilde{\mathcal{H}} - 0.97\mathfrak{Im}\mathcal{H} + 0.49\mathfrak{Im}\mathcal{E} - 0.03\mathfrak{Im}\widetilde{\mathcal{E}}$ |
| | $A_{\mathrm{LL}}^{+,\cos 0\phi}$ | $1 + 0.05\mathfrak{Re}\widetilde{\mathcal{H}} + 0.01\mathfrak{Re}\mathcal{H}$ |
| | $A_{\mathrm{LL}}^{+,\cos \phi}$ | $1 + 0.79\mathfrak{Re}\widetilde{\mathcal{H}} + 0.11\mathfrak{Im}\mathcal{H}$ |
| | $A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)}$ | $\mathfrak{Im}\mathcal{H}\mathfrak{Re}\mathcal{E} - \mathfrak{Im}\mathcal{E}\mathfrak{Re}\mathcal{H}$ |
| | $A_{\mathrm{UT,I}}^{\sin(\phi-\phi_S)\cos\phi}$ | $\mathfrak{Im}\mathcal{H} - 0.56\mathfrak{Im}\mathcal{E} - 0.12\mathfrak{Im}\widetilde{\mathcal{H}}$ |
| CLAS | $A_{\mathrm{LU}}^{-,\sin \phi}$ | $\mathfrak{Im}\mathcal{H} + 0.06\mathfrak{Im}\mathcal{E} + 0.21\mathfrak{Im}\widetilde{\mathcal{H}}$ |
| | $A_{\mathrm{UL}}^{-,\sin \phi}$ | $\mathfrak{Im}\widetilde{\mathcal{H}} + 0.12\mathfrak{Im}\mathcal{H} + 0.04\mathfrak{Im}\mathcal{E}$ |
| | $A_{\mathrm{UL}}^{-,\sin 2\phi}$ | $\mathfrak{Im}\widetilde{\mathcal{H}} - 0.79\mathfrak{Im}\mathcal{H} + 0.30\mathfrak{Im}\mathcal{E} - 0.05\mathfrak{Im}\widetilde{\mathcal{E}}$ |
| HALL A | $\Delta\sigma^{\sin\phi}$ | $\mathfrak{Im}\mathcal{H} + 0.07\mathfrak{Im}\mathcal{E} + 0.47\mathfrak{Im}\widetilde{\mathcal{H}}$ |
| | $\sigma^{\cos 0\phi}$ | $1 + 0.05\mathfrak{Re}\mathcal{H} + 0.007\mathcal{H}\mathcal{H}^*$ |
| | $\sigma^{\cos\phi}$ | $1 + 0.12\mathfrak{Re}\mathcal{H} + 0.05\mathfrak{Re}\widetilde{\mathcal{H}}$ |
| HERA | $\sigma_{\mathrm{DVCS}}$ | $\mathcal{H}\mathcal{H}^* + 0.09\mathcal{E}\mathcal{E}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^*$ |

# Bonus: structure of the cross section




$$\mathscr{L} \propto \varepsilon_\mu^{\Lambda_{\gamma^*}}(q_1)\gamma^\mu \frac{1}{\not{k}_1 + \not{q}_1}\gamma^\nu(\varepsilon_\nu^{\Lambda'_\gamma}(q_2))^* = \gamma^\mu \frac{\not{k}_1 + \not{q}_1}{(k_1+q_1)^2}\gamma^\nu \varepsilon_\mu^{\Lambda_{\gamma^*}}(q_1)(\varepsilon_\nu^{\Lambda'_\gamma}(q_2))^*$$

denominator: $(k_1+q_1)^2 = k_1^2 + 2k_1\cdot q_1 + q_1^2 \approx 2k_1 \cdot q_1 - Q^2 = Q^2\left(-1 + \underbrace{\frac{2k_1\cdot q_1}{Q^2}}_{Q^2 \approx 2\mathcal{Q}^2}\right)$

$$\to \text{insert } \left\{ q_1 = q + \frac{\Delta}{2} \right\}$$

$$= Q^2\left(-1 + (x+\xi)\underbrace{\frac{P\cdot q}{2\mathcal{Q}^2}}_{1/2\xi_B}\right) = Q^2\left(-1 + (x+\xi)\underbrace{\frac{1}{2\xi_B}}_{\xi_B\approx\xi}\right)$$

$$\approx \frac{Q^2}{2\xi}(x-\xi)$$

(we ignore quark masses everywhere)

**Structure of the numerator:**

We ignore $k_1$ and set $q_1$ to move along the negative z-axis at high energy, which means $q_1^0 \approx -q_1^3 \Rightarrow q^+ \approx 0$ & $q^- \gg$

$$\gamma^\mu (k_1 + q_1)_\alpha \gamma^\alpha \gamma^\nu \propto \gamma^\mu \gamma^+ \gamma^\nu = g^{\mu+}\gamma^\nu - g^{\mu\nu}\gamma^+ + g^{\nu+}\gamma^\mu + \mathrm{i}\epsilon^{\sigma\mu+\nu}\gamma_5\gamma_\sigma$$

Given that the outgoing photon is real, its polarization can only be transverse, so the index ν can only take on the values 1 and 2, which we will write as i. In terms of the light-cone coordinates, this index lives in the transverse space, so it is orthogonal to the indices ±.

$$\gamma^\mu \gamma^+ \gamma^\nu = g^{\mu+}\gamma^i - g^{\mu i}\gamma^+ + g^{i+}\gamma^\mu + \mathrm{i}\epsilon^{\sigma\mu+i}\gamma_5\gamma_\sigma = \underbrace{g^{\mu+}\gamma^i}_{\text{twist-3}} - \underbrace{g^{\mu i}\gamma^+}_{\text{twist-2}} + \underbrace{i\epsilon^{\sigma\mu+i}\gamma_5\gamma_\sigma}_{\text{axial twist-2 and 3}}$$

$\mu$ has to be = +,
longitudinal photon

$\mu$ has to be = i,
transverse photon

Photon polarization in the hadronic plane: $\varepsilon^{\pm 1}(q)_\mu = \dfrac{e^{\pm i\phi}}{\sqrt{2}}(0, \mp 1, i, 0), \quad \varepsilon^0(q)_\mu = \dfrac{1}{Q}(q^0, 0, 0, |\boldsymbol{q}|)$

Hadronic tensor $\propto \displaystyle\sum_{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}\Lambda'_\gamma\Lambda'} \left[f^{\Lambda^{(1)}_{\gamma^*},\Lambda'_\gamma}_{\Lambda,\Lambda'}\right]^* f^{\Lambda^{(2)}_{\gamma^*},\Lambda'_\gamma}_{\Lambda,\Lambda'} \Rightarrow$

- if $\Lambda^{(1)}_{\gamma^*} = \Lambda^{(2)}_{\gamma^*} = i$ there is no $\phi$ dependence, twist-2
- if $\Lambda^{(1)}_{\gamma^*} = 0, \quad \Lambda^{(2)}_{\gamma^*} = i$, we have $e^{\pm i\phi}$, twist-3
- if $\Lambda^{(1)}_{\gamma^*} = \pm 1, \quad \Lambda^{(2)}_{\gamma^*} = \mp 1$, we have $e^{\pm i2\phi}$, twist-2 gluon transversity GPDs
- if $\Lambda^{(1)}_{\gamma^*} = \Lambda^{(2)}_{\gamma^*} = 0$ there is no $\phi$ dependence, twist-4

Angle dependence corresponds to twist expansion of GPDs