Exclusive processes and generalized parton distributions

**C. Weiss (JLab), Lectures at International School of Hadron Femtography,
JLab, 16-25 Sep 2024** [Webpage]

Jefferson Lab

I) Exclusive processes in lepton-hadron scattering — "kinematics"

Kinematic variables
Collinear frames and light-cone components
L/T currents
Cross section
Meson production and Virtual Compton Scattering VCS

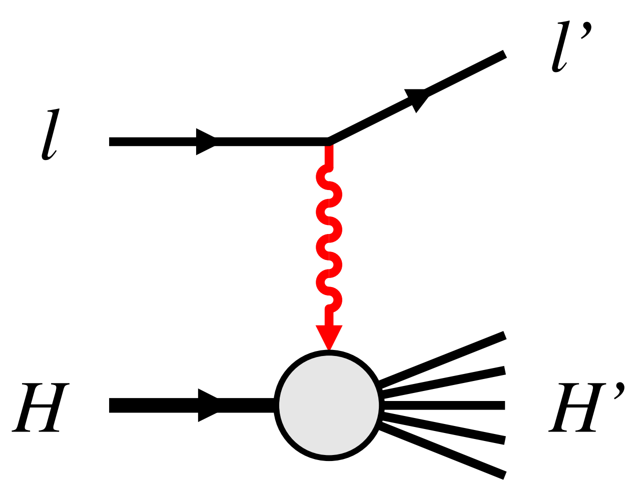
II) Asymptotic behavior and factorization — "dynamics"

Bjorken regime
QCD factorization
Quantum correlation functions
DVCS factorization from collinear expansion
Generalized parton distributions

III) Asymptotic behavior and factorization II

Meson production factorization and GPDs
GPD evolution
Characteristics of large and small x
Heavy quarkonium production near threshold
TBD

Lepton-hadron scattering



Lepton-hadron scattering $l + H \to l' + H'$

$l = e^{\pm}, \mu^{\pm}$ $H =$ nucleon p/n, nucleus A, meson

Mediated by electromagnetic (EM) interaction

Lowest order in EM coupling $\mathcal{O}(e^2)$

Amplitude = lepton current x hadron current

Also: QED radiative corrections, two-photon exchange

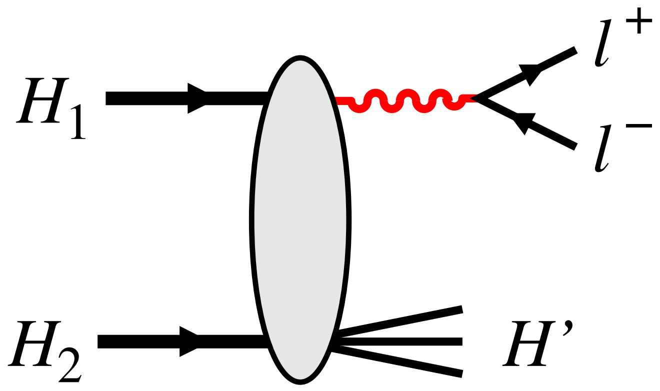
Final states

$H' = H$ only momentum changed — elastic scattering

$H' = H$ + meson, H + photon γ — inelastic scattering — exclusive

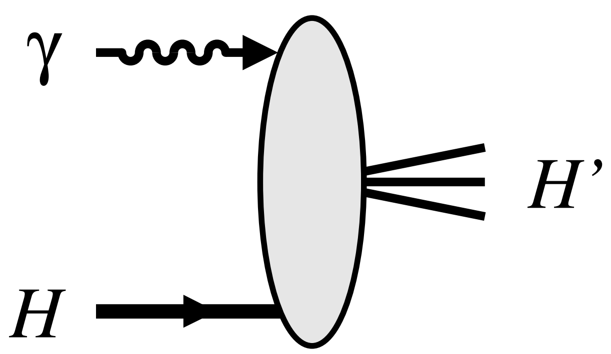
$H' = X$ anything — inelastic scattering — inclusive

Related: Hadron/photon-hadron scattering



Hadron-hadron scattering with dilepton production

$H_1 + H_2 \to l^+ l^- + H'$



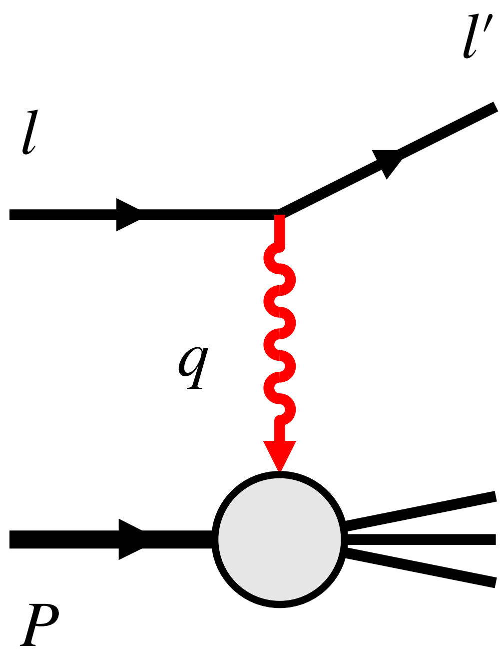
Photon-hadron scattering

$\gamma + H \to H'$

$H' = H$ + meson, H + heavy quarkonium $\bar{Q}Q$

Exclusive final states possible

Kinematic variables



Relativistically covariant formulation

Particles described by 4-momenta $p^\mu = (p^0, \mathbf{p})$

Kinematic variables defined as 4-vector products = invariants

Can be interpreted in particular frame, e.g. target rest frame

$q \equiv l - l'$ 4-momentum transfer, contains energy and momentum transfer (independent variables)

Invariant variables describing hadronic transition

$-q^2 \equiv Q^2 > 0$ invariant 4-momentum transfer

$(P + q)^2 \equiv W^2 > 0$ invariant CM energy of $\gamma^* H$ initial state
= invariant mass of total hadronic final state

Kinematic variables

$$x_B \equiv \frac{-q^2}{2Pq}$$

Bjorken scaling variable,
ratio of 4-momentum and energy transfer

Uses alt. invariant energy variable Pq, related to W^2 by

$$W^2 = (P+q)^2 = P^2 + q^2 - 2Pq = M^2 - Q^2 + 2Pq$$

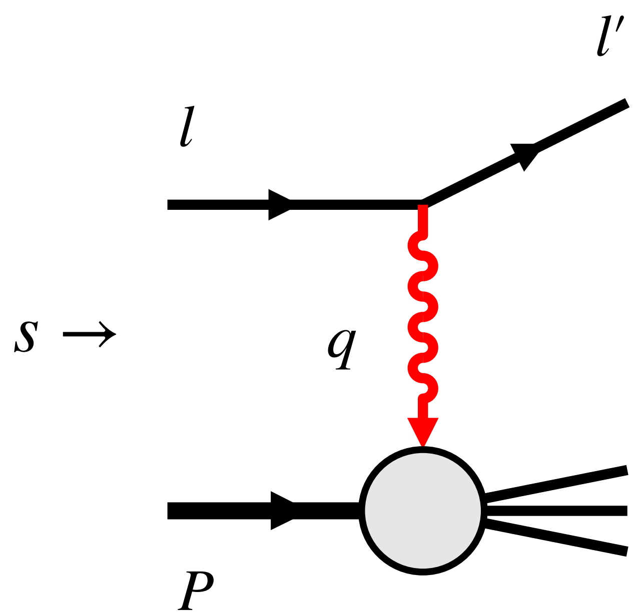
$M^2 = P^2$ target mass

In target rest frame: $P = (M, \mathbf{0}),\ \ q = (\nu, \mathbf{q}) \ \rightarrow\ Pq = \nu M$

energy transfer

Variables describing hadronic final state in exclusive processes:
Depend on specific final state, to be defined later

Kinematic range

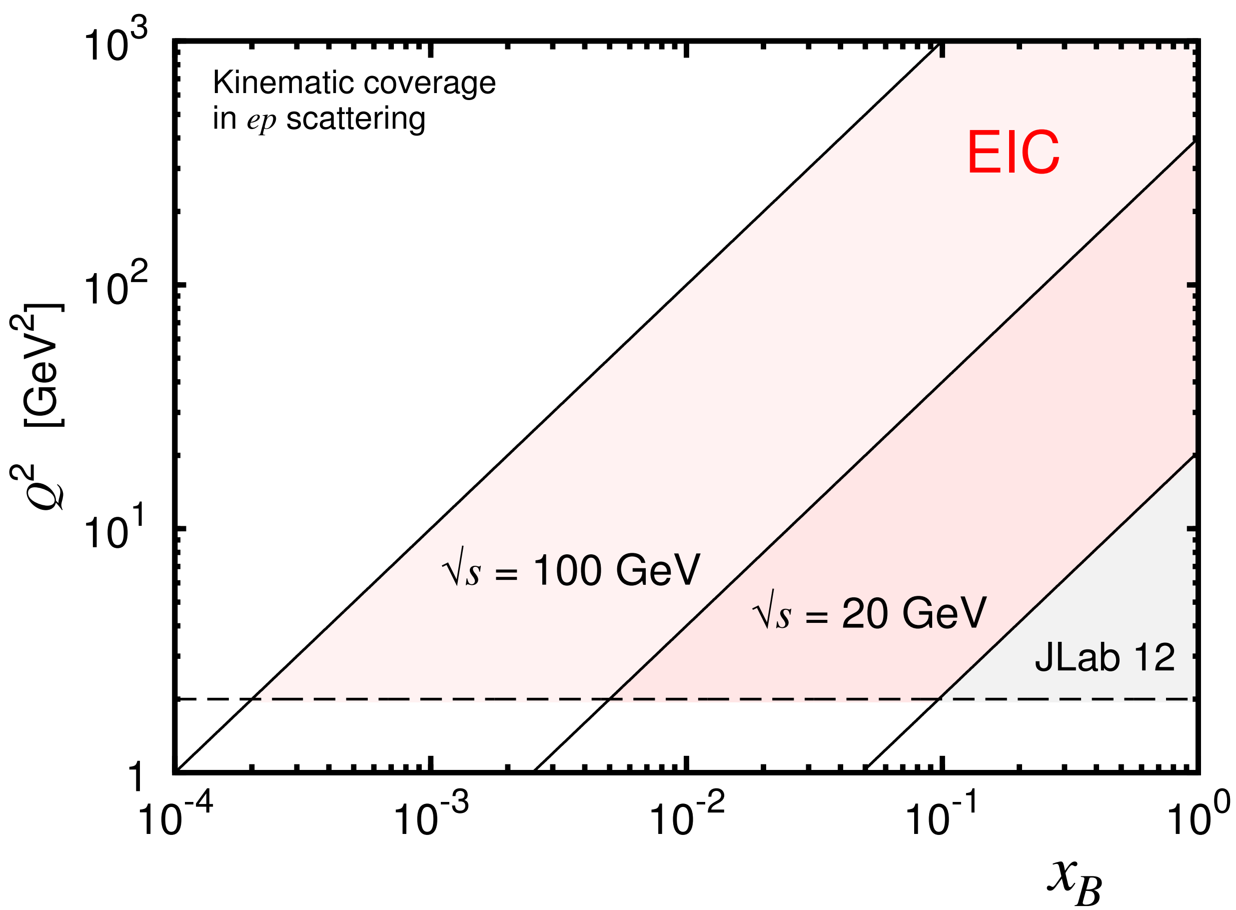


$s = (l + P)^2$ lepton-hadron invariant

$= (E_l + E_H)^2_{\text{CM}}$ energies of particles in CM frame

Kinematic range

$Q^2 < x_B(s - M^2)$ kinematic limit [E1]

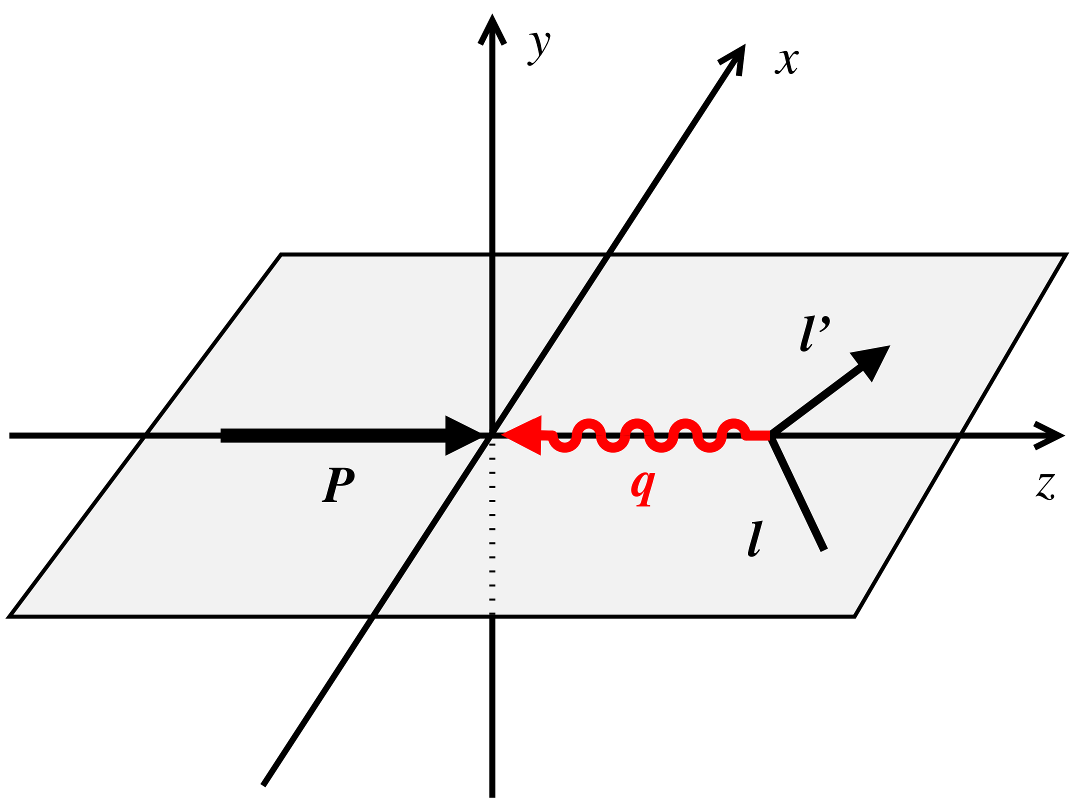


Large CM energies s needed to access small x_B, high Q^2

Theoretical coverage, experimental limitations at low Q^2 and large x_B

Additional limitations due to kinematic dependence of cross sections and rates, esp. in exclusive processes

Collinear frames



Scattering processes can be analyzed in any frame:
Invariant variables, cross section

Collinear frames $\mathbf{P} \parallel \mathbf{q}$ *)

Rotational symmetry around $\gamma^* H$ collision axis

Initial state: L/T currents, target spin L/T

Final-state: Particles described by azimuthal angle

High-energy limit

Longitudinal momentum components grow

Transverse components remain finite

Qualitative distinction, used in factorization

Coordinate system

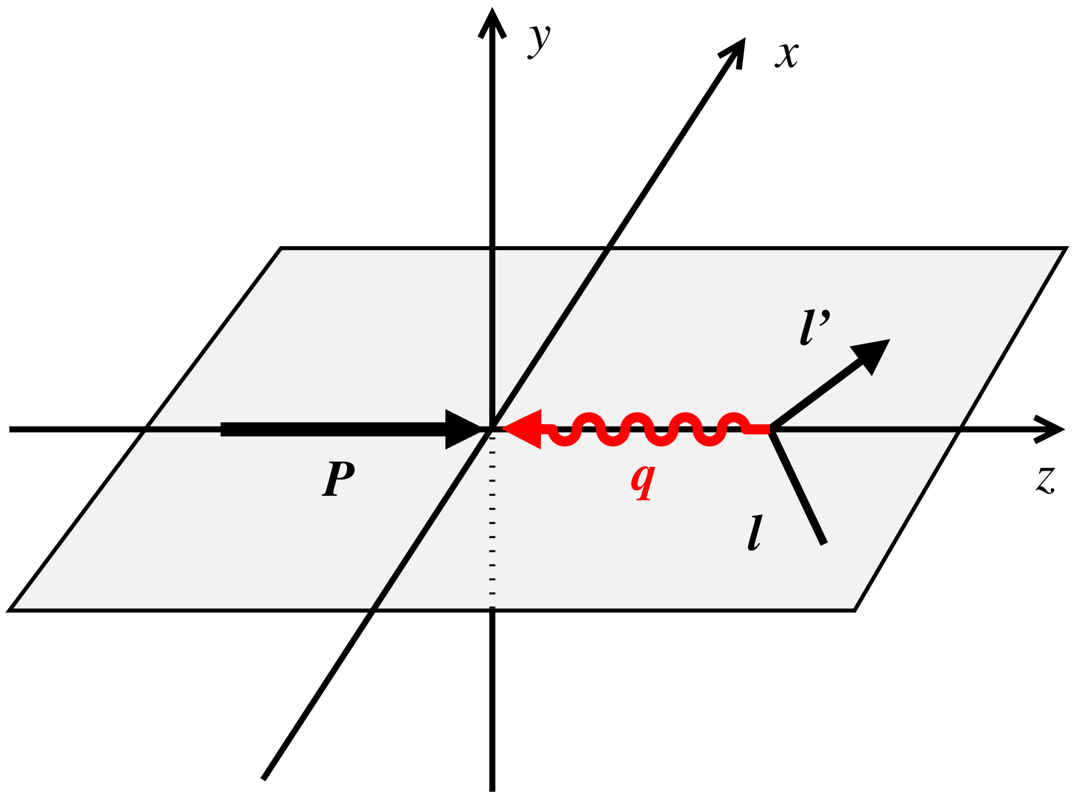
$\mathbf{P}, \mathbf{q}$ along z-axis, $\mathbf{q}$ in -z direction

$\mathbf{l}, \mathbf{l}'$ in xz-plane, with $\mathbf{l} + \mathbf{l}'$ in +x direction

Other conventions in use — verify!

*) Here the collinear frame is defined using the initial hadron momentum. In applications to exclusive processes one also defines frames using a combination of initial and final hadron momenta.

Collinear frames



Class of collinear frames

Collinear condition $\mathbf{P} \parallel \mathbf{q}$ defines not single frame, but equivalence class of frames related by longitudinal boosts

Members include:

$\mathbf{P} = 0$	Target rest frame
$\mathbf{P} + \mathbf{q} = 0$	γ^*-target CM frame
$\mathbf{P} = -\mathbf{q}/2 = 0$	Breit frame (only 3-momentum transfer, no energy transfer)

Light-cone components

$v^{\pm} \equiv v^0 \pm v^3$ $\quad$ $\mathbf{v}_T \equiv (v^1, v^2)$ $\quad$ light-cone components of 4-vector a^{μ}

$vw = \dfrac{v^+w^- + v^-w^+}{2} - \mathbf{v}_T \cdot \mathbf{w}_T$ $\quad$ $v^2 = v^+v^- - |\mathbf{v}_T|^2$ $\quad$ scalar product and square

Boost along 3-axis

$v^+ \to \Lambda v^+$ $\quad$ $v^- \to \Lambda^{-1} v^-$ $\quad$ transform homogeneously - simple! $\Lambda > 0$, number

$v^{\pm} \to e^{\pm y} v^{\pm}$ $\quad$ $y = \log \Lambda$ rapidity of boost

Particle momenta in collinear frame

$k^{\mu} = [k^+, k^-, \mathbf{k}_T]$ $\quad$ $k^- = \dfrac{k^2 + |\mathbf{k}_T|^2}{k^+}$ $\quad$ $k^+ = \alpha P^+$ $\quad$ fraction of P^+ momentum in collinear frame

Particular collinear frame specified by value of P^+.
No explicit boost needed to change between frames, just change value of P^+

E2

Calculations can be performed in general collinear frame, without specifying value of P^+

Longitudinal and transverse subspaces

4D spacetime can be decomposed into subspace spanned by $\{P, q\}$ and transverse subspace

4-vectors, currents, tensors have components in the subspaces, can be projected

Important for high-energy limit, scaling behavior → later

Basis 4-vectors

$$L \equiv P - \frac{(Pq)}{q^2} \qquad Lq = 0$$

orthogonal 4-vectors

$$e_L \equiv \frac{L}{\sqrt{L^2}} \qquad e_q \equiv \frac{q}{\sqrt{-q^2}}$$

unit 4-vectors, span space $\{P, q\}$

Projectors

$$g_L^{\mu\nu} \equiv e_L^\mu e_L^\nu \qquad g_T^{\mu\nu} \equiv g^{\mu\nu} + e_L^\mu e_L^\nu + e_q^\mu e_q^\nu$$

projectors on L and T subspaces (orthogonal to q)

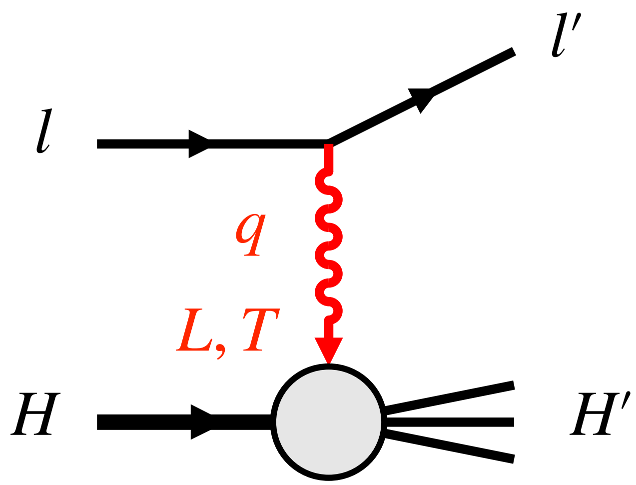
$$g^{\mu\nu} = g_L^{\mu\nu} + g_T^{\mu\nu} + e_q^\mu e_q^\nu$$

completeness relation

In collinear frame, vectors e_L, e_q have spatial components are along the 3-axis

Calculations with 4-vectors/tensors can be performed covariantly using the basis 4-vectors

Scattering amplitude



$$\mathscr{M}(lH \to l'H') \;=\; \frac{e^2}{q^2} \langle l' | j^\mu | l \rangle \, \langle H' | J_\mu | H \rangle$$

amplitude lepton hadron current

Longitudinal and transverse currents

$q^\mu \langle l' | j_\mu | l \rangle = 0 \qquad q^\mu \langle H' | J_\mu | H \rangle = 0$

lepton and hadron current matrix elements orthogonal to q, can be projected on L and T

$j^\mu_{L,T} \equiv g^{\mu\nu}_{L,T} \, j_\nu \qquad J^\mu_{L,T} \equiv g^{\mu\nu}_{L,T} \, J_\nu$

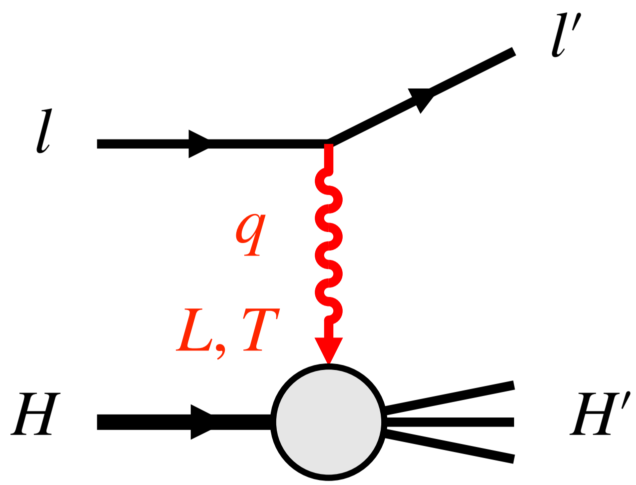
L and T projections of lepton and hadron currents
(→ observables, high-energy limit, QCD factorization)

Current matrix elements depend on momentum and spin variables

Lepton current: Point particle, current matrix elements computed explicitly

Hadron current: Composite system, current computed using approximate methods

Longitudinal-transverse ratio



Ratio of L and T projections of lepton current

$$\epsilon \equiv \frac{j_L^* j_L}{-j_T^* j_T} = \frac{\langle l' | j_L^\mu | l \rangle \langle l | j_{L\mu} | l' \rangle}{-\langle l' | j_T^\mu | l \rangle \langle l | j_{T\mu} | l' \rangle}$$

$$\epsilon = \frac{1 - y - y^2 x_B^2 M^2/Q^2}{1 - y + y^2/2 + y^2 x_B^2 M^2/Q^2}$$

$$y \equiv \frac{Pq}{Pl} = \frac{Q^2}{x_B(s - M^2)}$$

Explicit expression in terms of invariants

Depends on variables x_B, Q^2 and lepton-hadron CM energy s E3

Interpretation: Ratio of L and T polarization in virtual photon flux (probability)

Plays important role in cross section and observables

Cross section

$$d\sigma\,(lH \to l'H') \;=\; \text{Flux} \;\times |\,\mathscr{M}\,|^2 \;\times d\Gamma_{l'} \;\times \prod_{h \text{ in } H'} d\Gamma_h$$
$$\times\, (2\pi)^4 \delta^{(4)}(l + P - l' - \sum_{h \text{ in } H'} P_h)$$

general expression
of differential cross section

$$d\Gamma \;\equiv\; \frac{d^4 p}{(2\pi)^4}\, 2\pi\delta(p^2 - m^2)_{p^0 > 0} \;=\; \frac{d^3 p}{(2\pi)^3 2E(\mathbf{p})}$$

invariant phase space element
of particle (mass m)

$d\Gamma_{l'} \to$ Jacobian $\times\, dx_B\, dQ^2$ final lepton phase space element expressed in variables x_B, Q^2

$d\Gamma_h$ for h in H' final hadron phase space elements, specifics depend on channel

Spin degrees of freedom

Lepton helicity conserved in scattering process (up to terms $\propto m_l$ lepton mass)

Average over spins in initial hadron state H with spin density matrix describing polarization

Sum over spins of hadrons in final state H' (not observed)

Cross section

Represent lepton-hadron cross section as “virtual photon flux” and “virtual photon-hadron cross section”

$$d\sigma\,(lH \to l'H') \;=\; \Phi\;\, dx_B\, dQ^2 \;\times\; d\sigma\,(\gamma^* H \to H')$$

$$\Phi \;\equiv\; \frac{\alpha_{\text{em}}}{2\pi}\frac{y^2}{1-\epsilon}\frac{1-x_B}{x_B}\frac{1}{Q^2}$$

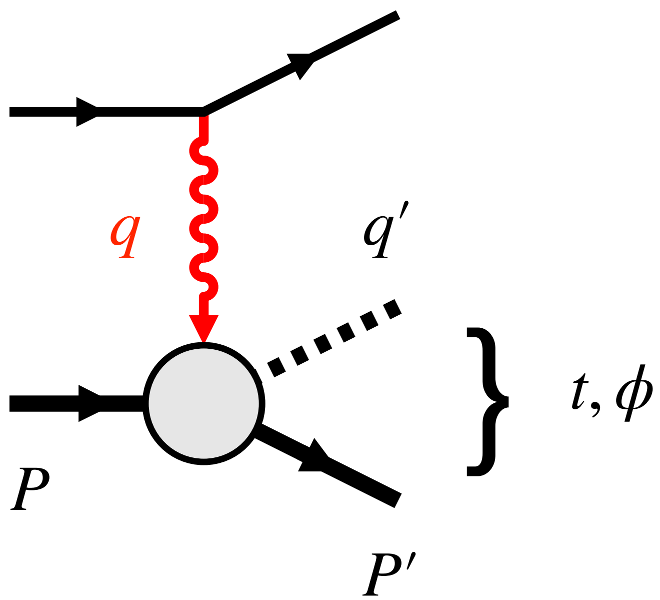
virtual photon flux factor, depends on convention

$$\epsilon \;= \text{function}(x_B, Q^2, s)$$

L/T ratio of virtual photon flux

Virtual photon-hadron cross section contains terms arising from L and T virtual photon amplitudes and their interference, can be decomposed accordingly.
Details depend on specific final state.

Exclusive meson production



$l + N \to l' + \pi + N$ exclusive pion production

Variables describing hadronic final state

$t = (P' - P)^2 = (q - q')^2$ invariant momentum transfer between initial and final nucleon, or virtual photon and pion

ϕ pion azimuthal angle around collinear axis

Differential cross section

$$d\sigma \; = \; \Phi \, dx_B \, dQ^2 \Big[F_T \; + \; \epsilon \, F_L \; + \; \sqrt{2\epsilon(1+\epsilon)} \cos\phi \, F_{LT} \; + \; \epsilon \cos(2\phi) \, F_{TT}$$

$$+ \; (2\lambda_e)\sqrt{2\epsilon(1-\epsilon)} \sin\phi \, F_{LT'} \; + \; \text{spin-dep. terms} \Big] \, dt \frac{d\phi}{2\pi}$$

$F_T, F_L, \ldots \; = \;$ functions(x_B, Q^2, t) structure functions, parametrize virtual photon-hadron cross section

Exclusive meson production

- Terms in cross section arise as bilinear forms in L and T hadron currents, including interference

$$F_T \propto J_T^{\,*} J_T \qquad F_L \propto J_L^{\,*} J_L \qquad F_{LT} \propto J_L^{\,*} J_T + J_T^{\,*} J_L$$

- Expression of cross section separates “kinematics” and “dynamics”

  Dependence on $s \to \epsilon,\ \phi$: Kinematics, exhibited explicitly

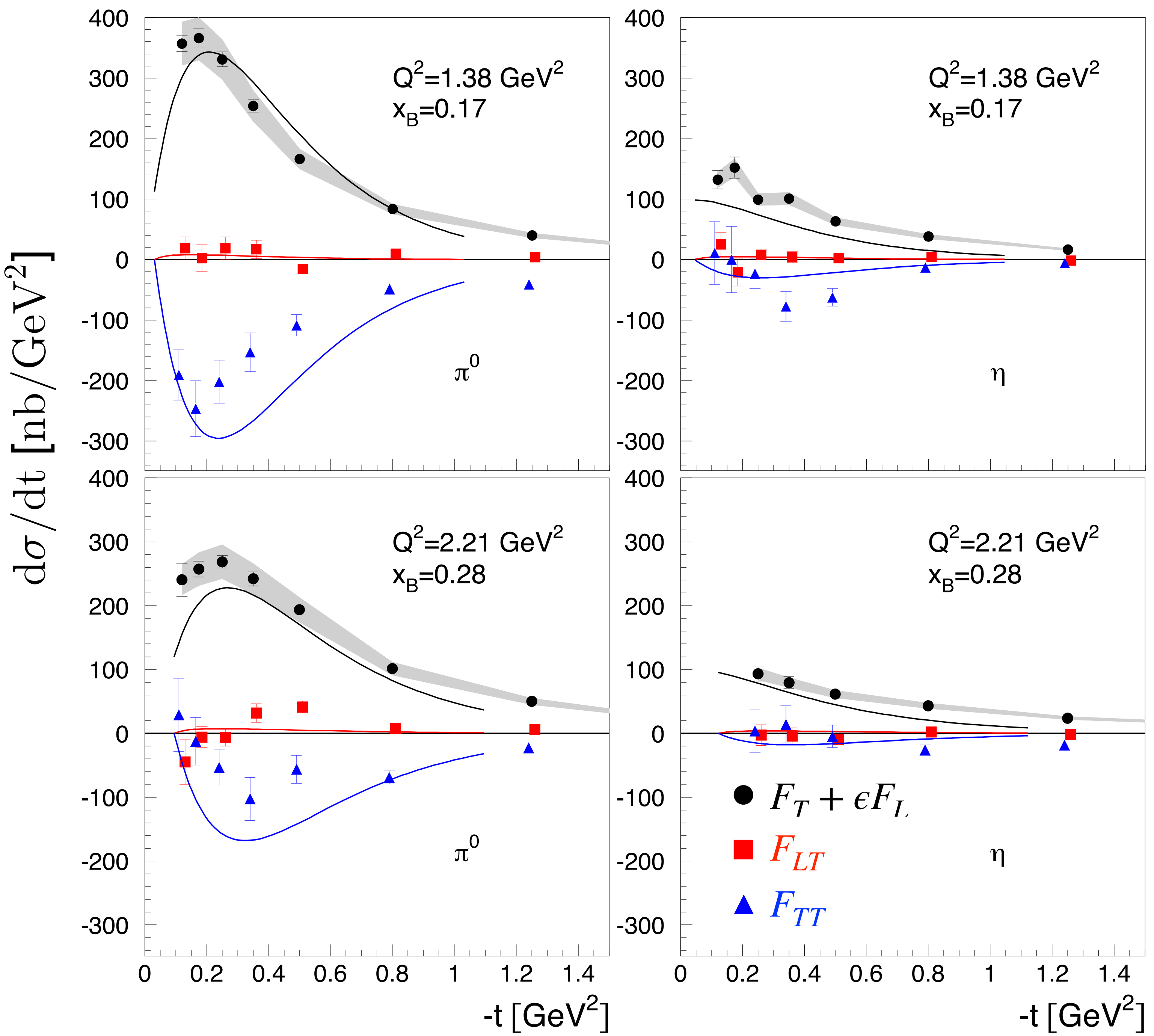
  Dependence on x_B, Q^2, t: Dynamics, included in structure functions

- Connection with real photoproduction in limit $Q^2 \to 0$, W fixed

$$F_T(Q^2 \to 0) = \frac{d\sigma}{dt}(\gamma N \to \pi N) \qquad F_L(Q^2 \to 0) = 0$$

- More structures arise with nucleon polarization (target or recoil)

Exclusive meson production



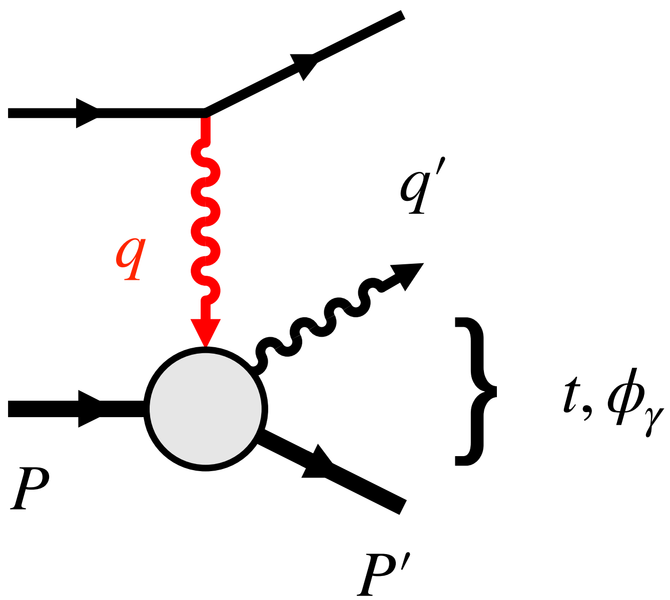
Example

Exclusive π^0 and η production cross sections (t-dependent structure functions) measured by JLab CLAS with 6 GeV beam energy

I. Bedlinskiy et al. [CLAS], Phys. Rev. C 95, 035202 (2017) [INSPIRE]

Curves: GPD-based theoretical model

Virtual Compton Scattering



$l + N \to l' + \gamma + N'$ exclusive production of real photon

Variables describing final state: t, ϕ_γ

Hadronic transition matrix element

$$\langle N' \gamma | J^\mu | N \rangle \;=\; T^{\mu\nu} \epsilon_\nu^*$$

$\epsilon^\nu \equiv \epsilon^\nu(q', \lambda')$ polarization vector of outgoing photon

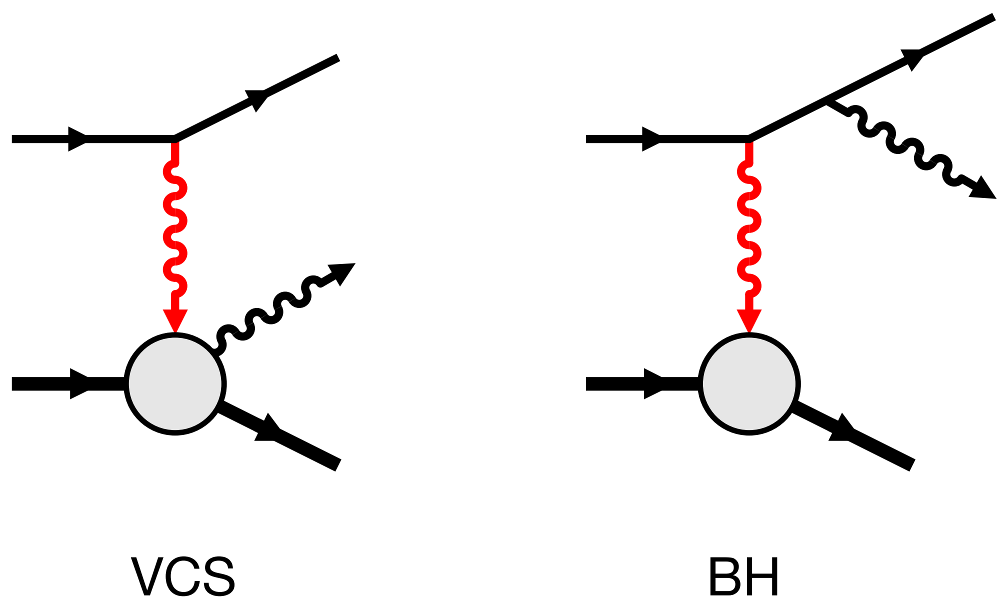
$$T^{\mu\nu} \;\equiv\; i \int d^4x \, e^{i(q'+q)x/2} \, \langle N' | \, T J^\mu(-x/2) \, J^\nu(x/2) \, | N \rangle$$

Compton tensor: Correlation function of EM currents in nucleon state

$$= \; \text{function}(P, q, q'; \sigma', \sigma)$$

Depends on momenta and nucleon spin quantum numbers σ, σ'

Can be derived from general arguments: Reduction theorem, EM current as interpolating operator



Cross section

Same final state produced by real photon emission from lepton: Bethe-Heitler process (BH)

BH and VCS amplitudes interfere in $lN \to l'\gamma N'$ cross section

$$d\sigma(lN \to l'\gamma N') \;\propto\; |\mathscr{M}(\text{BH}) + \mathscr{M}(\text{VCS})|^2$$

Rich structure of cross section and observables

BH amplitude given in terms of nucleon elastic form factors, real function (Im = 0)

Analysis methods use unique features of BH-VCS interference

→ Lectures Marija Cuic, Herve Dutrieux

E1: Kinematic range in lepton-hadron scattering

1) Derive the kinematic limit $Q^2 < x_B(s - M^2)$!

Hint: Introduce the variable $y \equiv \frac{Pq}{Pl} = \frac{P(l - l')}{Pl}$ = relative energy loss of lepton in target rest frame

and show that $0 < y < 1$

2) Evaluate lepton-hadron CM energy s for fixed-target setup (JLab) and colliding-beam setup (EIC)

E2: 4-vector components in collinear frame

1) Determine the light-cone components of the 4-vector q in Pq-collinear frames in terms of x_B and P^+ !

$$P = \left[P^+, \frac{M^2}{P^+}, 0\right], \quad q = [q^+, q^-, 0],$$

what are the values of q^+, q^- ?

E3: L/T ratio

1) Derive explicit expression of ϵ as function of x_B, Q^2 and y (or s) !

Hint: Use explicit expression of lepton current $j^\mu = \bar{u}(l', \sigma')\, \gamma^\mu\, u(l, \sigma)$, where $\sigma, \sigma' =$ helicity

Compute spin average $\frac{1}{2} \sum_{\sigma,\sigma'} j^{\nu *} j^\mu$ using spin projector $\sum_\sigma u(l, \sigma) \bar{u}(l, \sigma) = l \cdot \gamma + m$

(put lepton mass $m = 0$

Contract $\frac{1}{2} \sum_{\sigma,\sigma'} j^{\nu *} j^\mu$ with projectors formed from basis 4-vectors e_L, e_q

Express result in terms of invariant variables x_B, Q^2 and y

2) Plot ϵ as function of x_B and Q^2 for a given value of s (e.g. JLab12, EIC),
or as function of s for a given x and Q^2 !