QMC calculations of electron-nucleus scattering in the Short-Time Approximation

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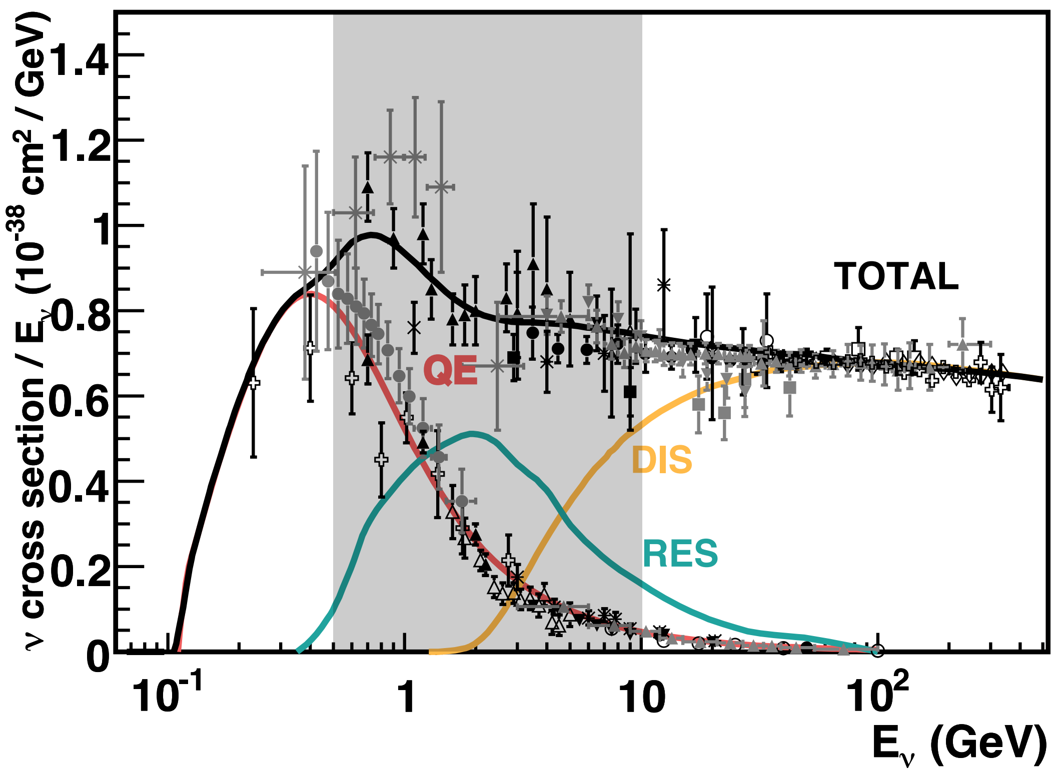
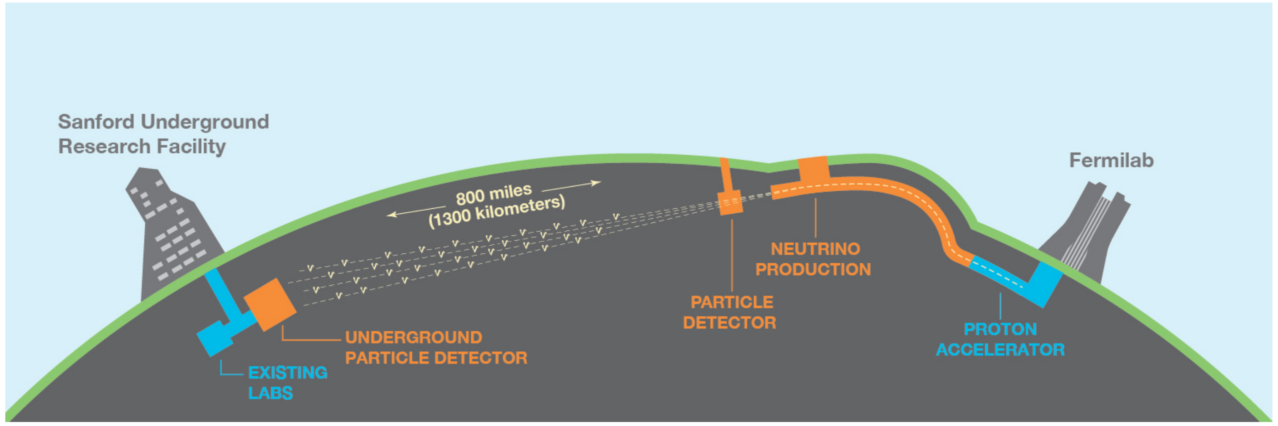
OLD DOMINION UNIVERSITY

Jefferson Lab

Lepton-nucleus scattering

Theoretical understanding of **nuclear effects** is extremely important for **electron** and **neutrino** experimental programs: oscillation experiments require accurate calculations of cross sections





Electron scattering can be used to test our nuclear model (**e4nu**):

- **same nuclear effects**
- **no need to reconstruct energies**
- **abundant experimental data**

Lepton-nucleus scattering

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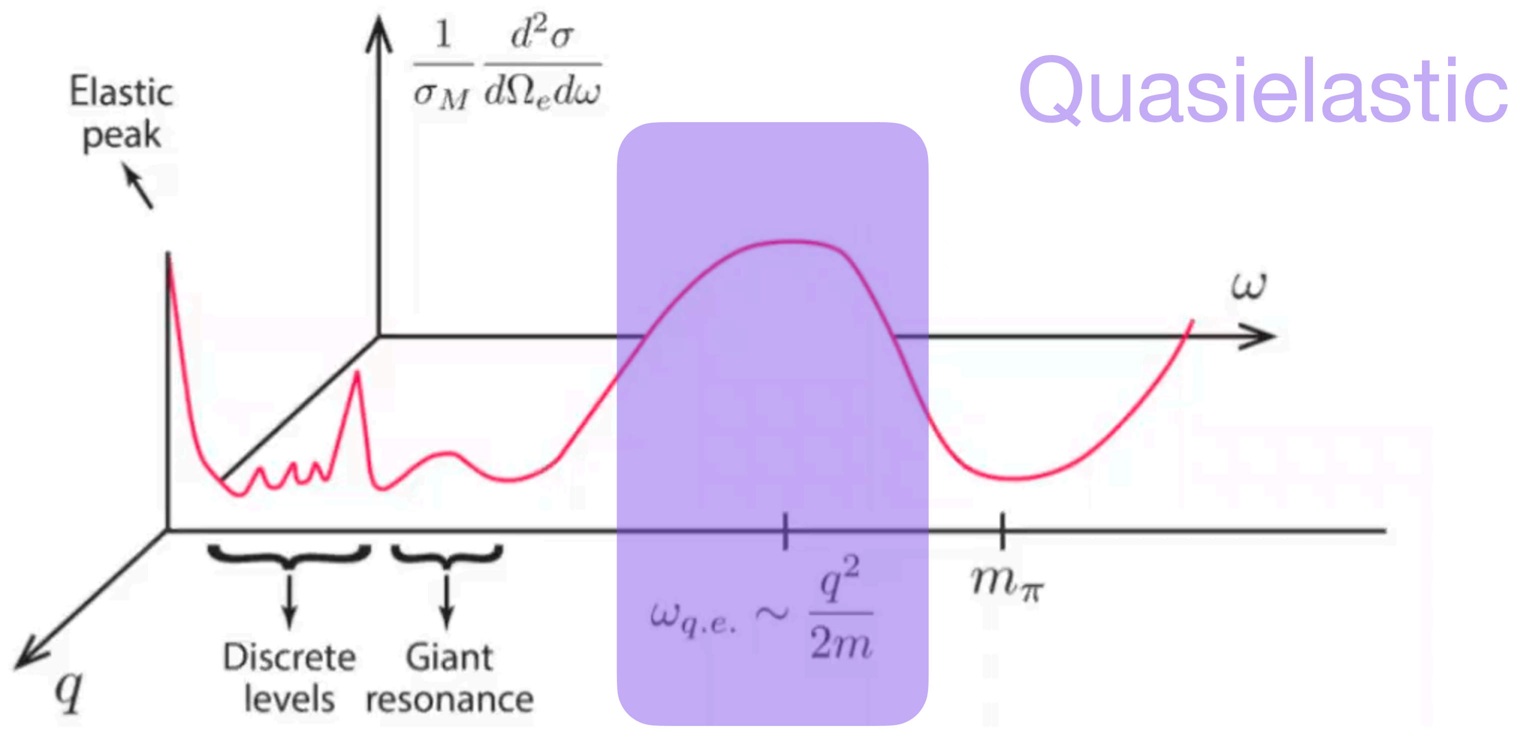

Electron scattering can be used to test our nuclear model:

- **same nuclear effects**
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- **abundant experimental data**

Outline

- **Ab initio description of nuclei:**
  - Nuclear interaction and ground state wave functions
  - Electromagnetic interaction of leptons with nucleons and clusters of correlated nucleons
- **Electron-nucleus scattering:**
  - Inclusive processes
  - Short-Time Approximation
- **Results**
- **Conclusions and outlook**

Many-body nuclear problem

Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

ψ are complex spin-isospin vectors in 3A dimensions with components $2^A \times \frac{A!}{Z!(A-Z)!}$

http://exascaleage.org/np/

^{4}He: 96

^{6}Li: 1280

^{8}Li: 14336

^{12}C: 540572

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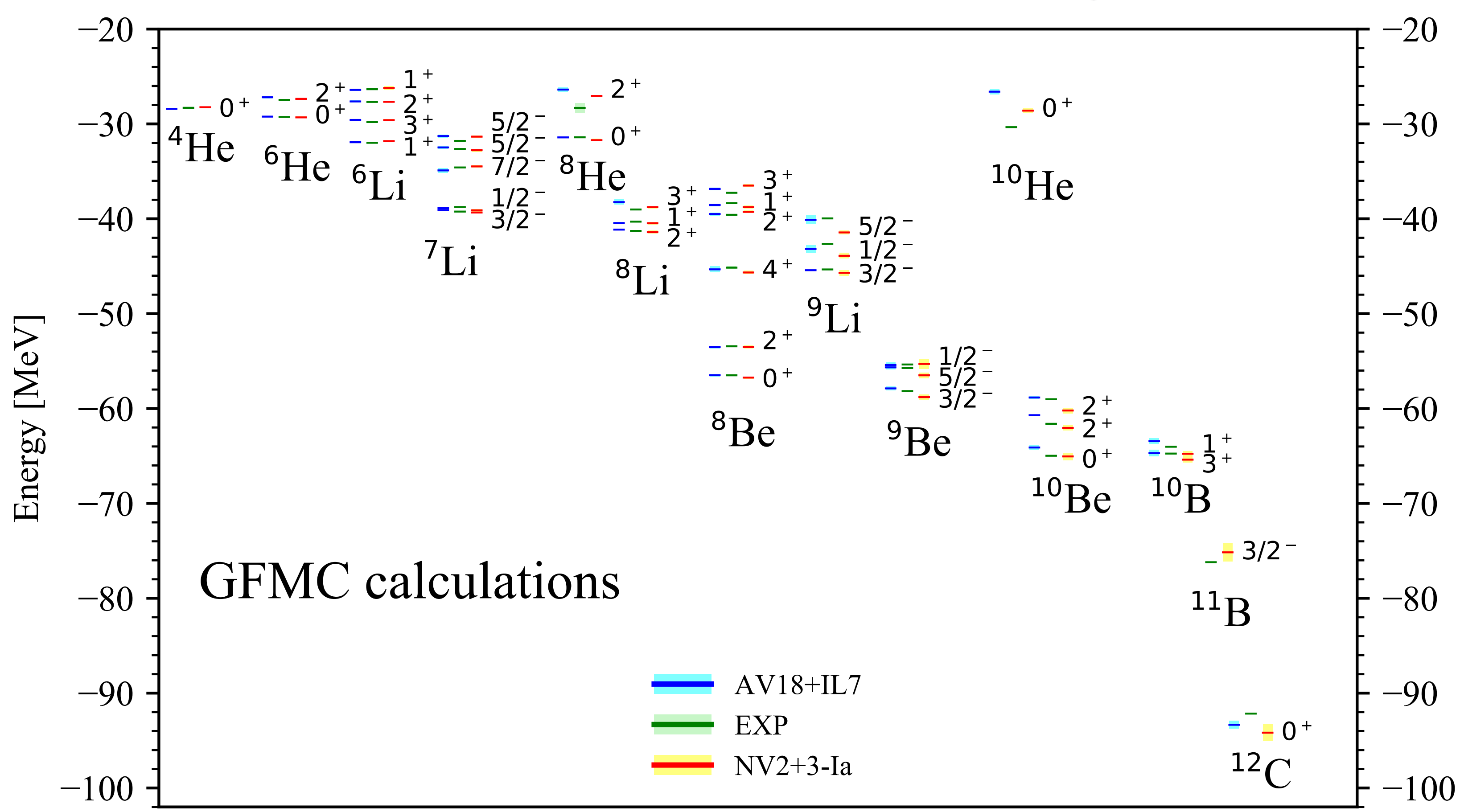


Spectra of light nuclei
Piarulli et al. PRL120(2018)052503

Many-body nuclear problem

Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

Quantum Monte Carlo method:
Use nuclear wave functions that minimize the expectation value of E

$$E_V = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \geq E_0$$

The evaluation is performed using Metropolis sampling

Nuclear Wave Functions

Variational wave function for nucleus in J state

$$|\psi\rangle = \mathcal{S}\prod_{i<j}^{A}\left[1 + U_{ij} + \sum_{k\neq i,j}^{A} U_{ijk}\right]\left[\prod_{i<j} f_c(r_{ij})\right]|\Phi(JMTT_3)\rangle$$

Two-body spin- and isospin-dependent correlations

$$U_{ij} = \sum_p f^p(r_{ij})\, O_{ij}^p$$

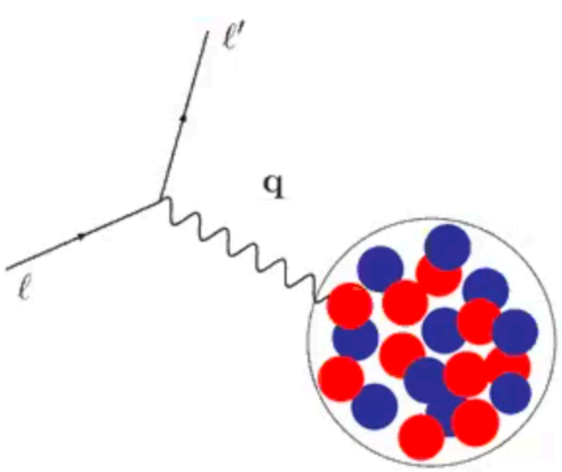
$$O_{ij}^p = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

$$U_{ijk} = \epsilon v_{ijk}(\bar{r}_{ij}, \bar{r}_{jk}, \bar{r}_{ki})$$

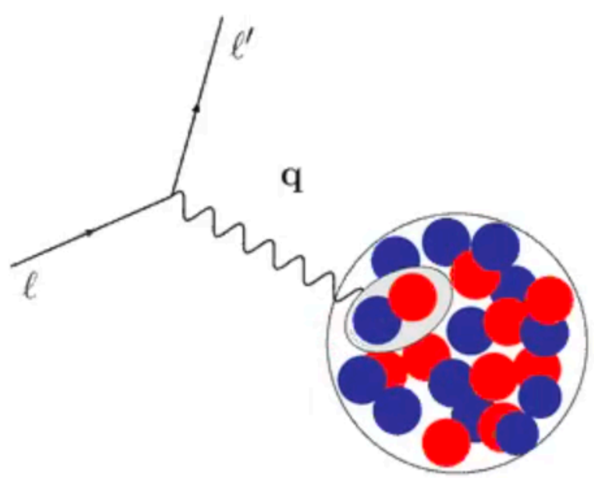
Electromagnetic interactions

Phenomenological Hamiltonian for NN and NNN

The interaction with external probes is described in terms on one- and two-body charge and current operators



one-body



two-body

Charge operators

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

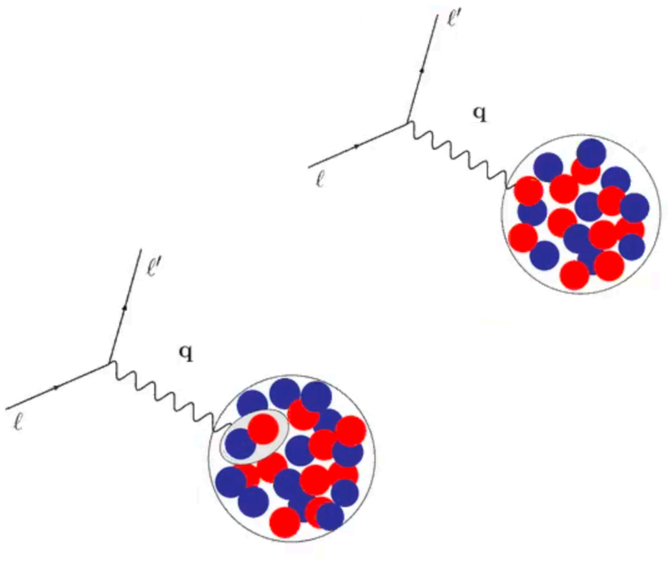
Current operators

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

Two-body currents are a manifestation of two-nucleon correlations

Electromagnetic interactions

- One body-currents: non-relativistic reduction of covariant nucleons' isoscalar and isovector currents
- Two-body currents: modeled on MEC currents constrained by commutation relation with the nuclear Hamiltonian
- Argonne v18 two-nucleon and Urbana potentials, together with these currents, provide a quantitatively successful description of many nuclear electroweak observables, including charge radii, electromagnetic moments and transition rates, charge and magnetic form factors of nuclei with up to A = 12 nucleons



Carlson, Schiavilla 1992. Marcucci et al. 2005

Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$
Transverse response induced by the current operator $O_T = \mathbf{j}$

5 responses in neutrino-nucleus scattering

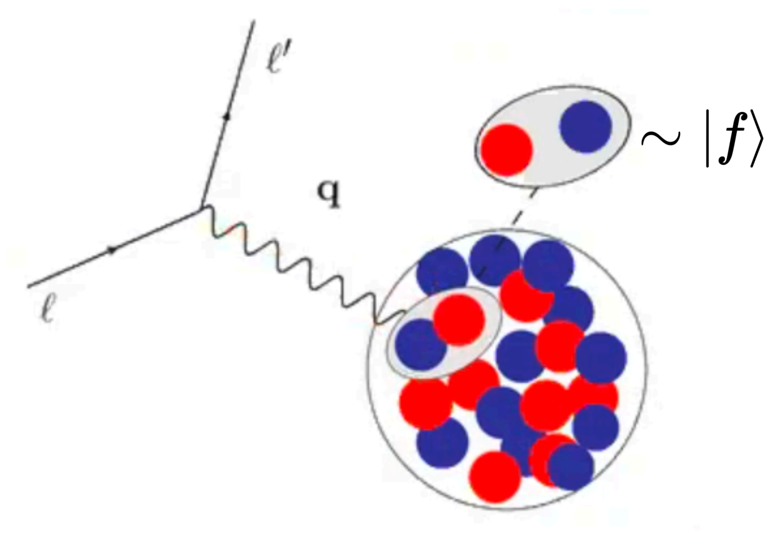
$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)]$$

One can exploit integral properties of the response functions to avoid explicit calculation of the final states: CC + Lorentz Integral Transfor, GFMC + Euclidean

Short-time approximation

S. Pastore, J. Carlson, S. Gandolfi, R. Schiavilla, and R. B. Wiringa PRC101(2020)044612

Factorization scheme: describe electroweak scattering from $A \geq 12$ without losing **two-body physics**, account for **exclusive processes**, incorporate **relativistic effects**



The sum over all final states is replaced by a two nucleon propagator

Response functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

$$O^\dagger e^{-iHt} O = \left(\sum_i O_i^\dagger + \sum_{i<j} O_{ij}^\dagger \right) e^{-iHt} \left(\sum_{i'} O_{i'} + \sum_{i'<j'} O_{i'j'} \right)$$

$$= \sum_i O_i^\dagger e^{-iHt} O_i + \sum_{i \neq j} O_i^\dagger e^{-iHt} O_j$$

$$+ \sum_{i \neq j} \Big(O_i^\dagger e^{-iHt} O_{ij} + O_{ij}^\dagger e^{-iHt} O_i$$

$$+ O_{ij}^\dagger e^{-iHt} O_{ij} \Big) + \ldots$$

Interference

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Short-time approximation

S. Pastore, J. Carlson, S. Gandolfi, R. Schiavilla, and R. B. Wiringa PRC101(2020)044612

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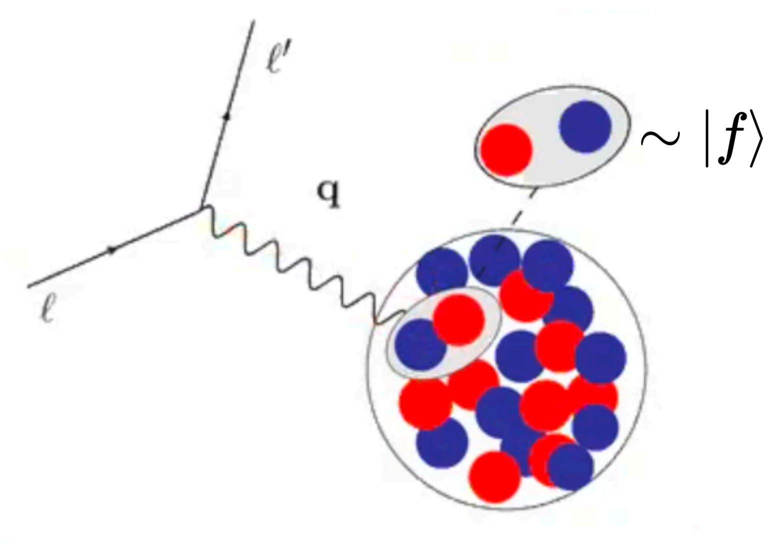


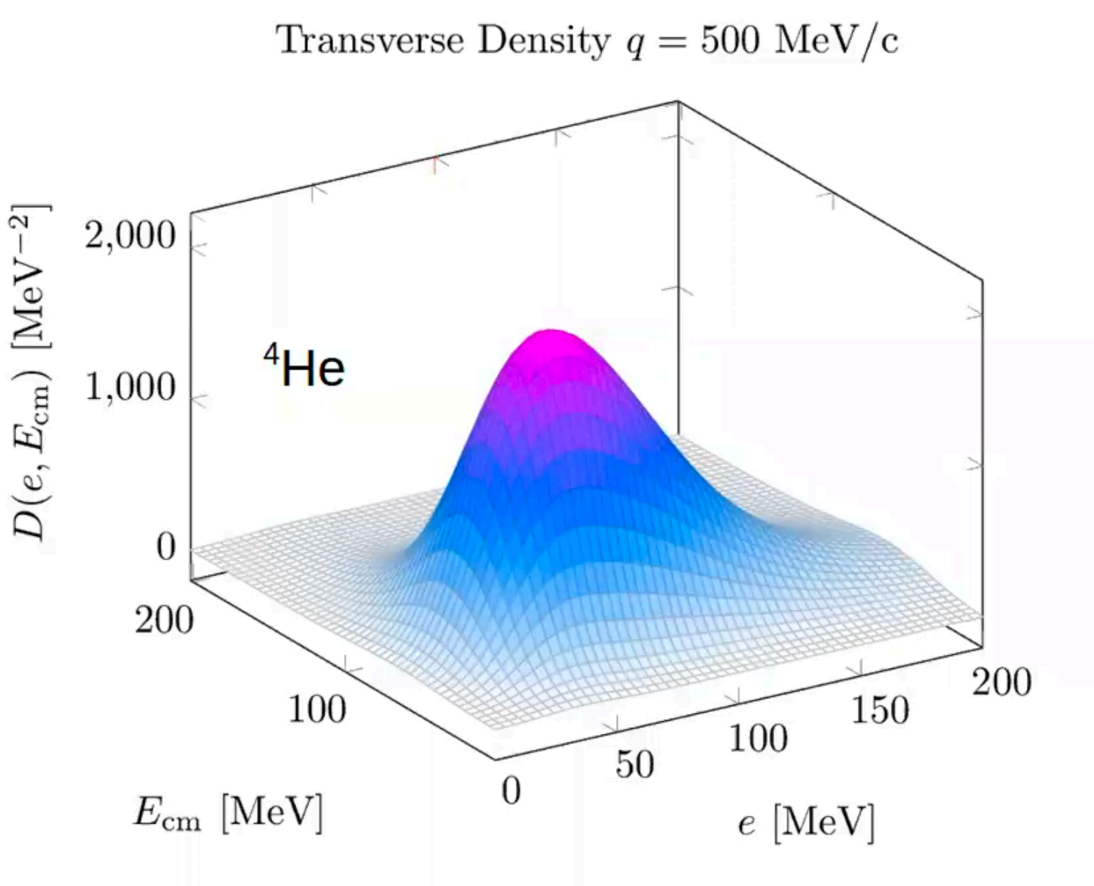
Response functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f)|\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Response densities

$$R^{\mathrm{STA}}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Transverse response density

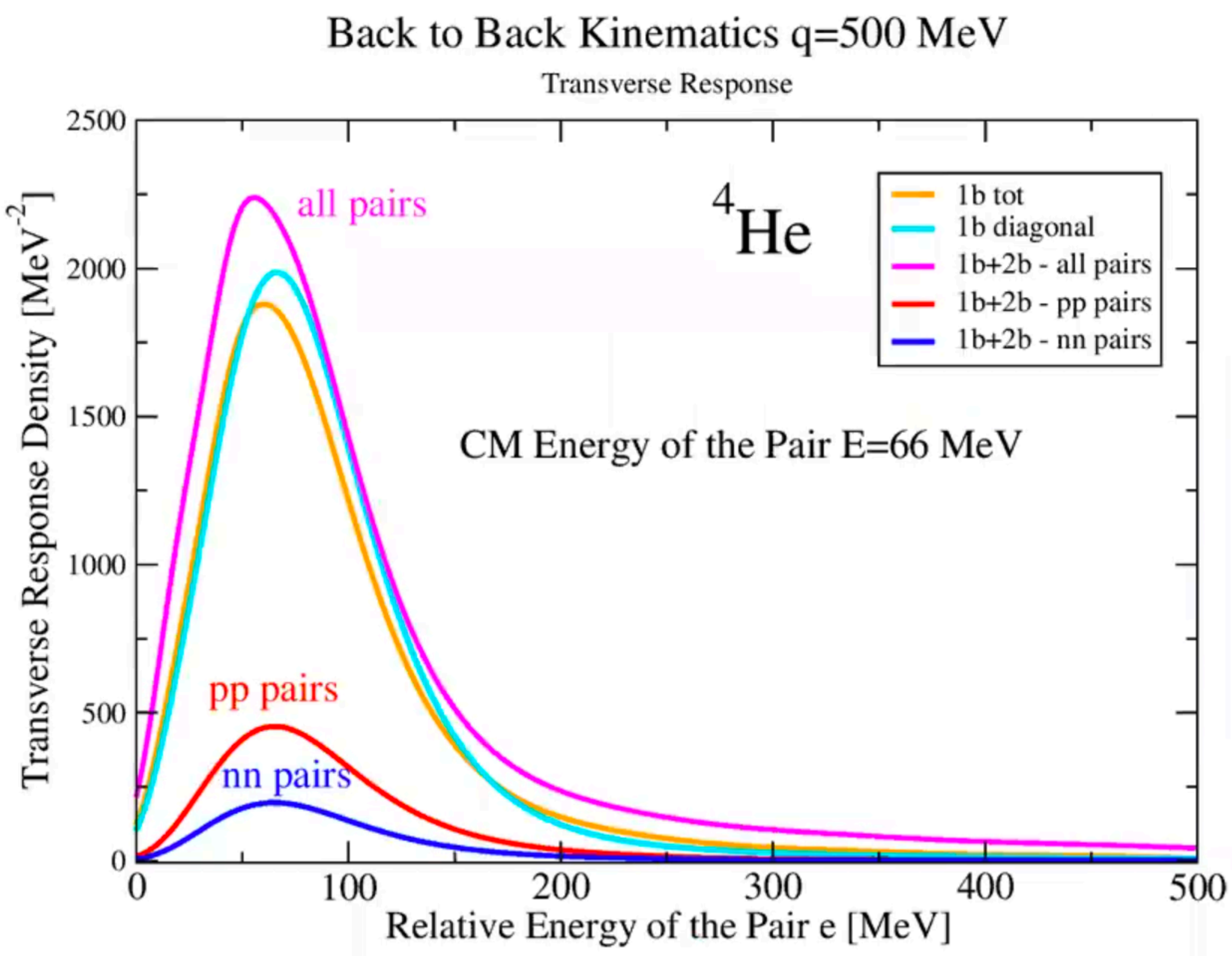


Electron scattering from ^4He in the STA:

- Provides “more” exclusive information in terms of nucleon-pair kinematics via the Response Densities as functions of (E,e)
- Give access to particular kinematics for the struck nucleon pair

PRC101(2020)044612

Back-to-back kinematic



We can select a particular kinematic, and assess the contributions from different particle identities

np dominance at high relative energy

PRC101(2020)044612

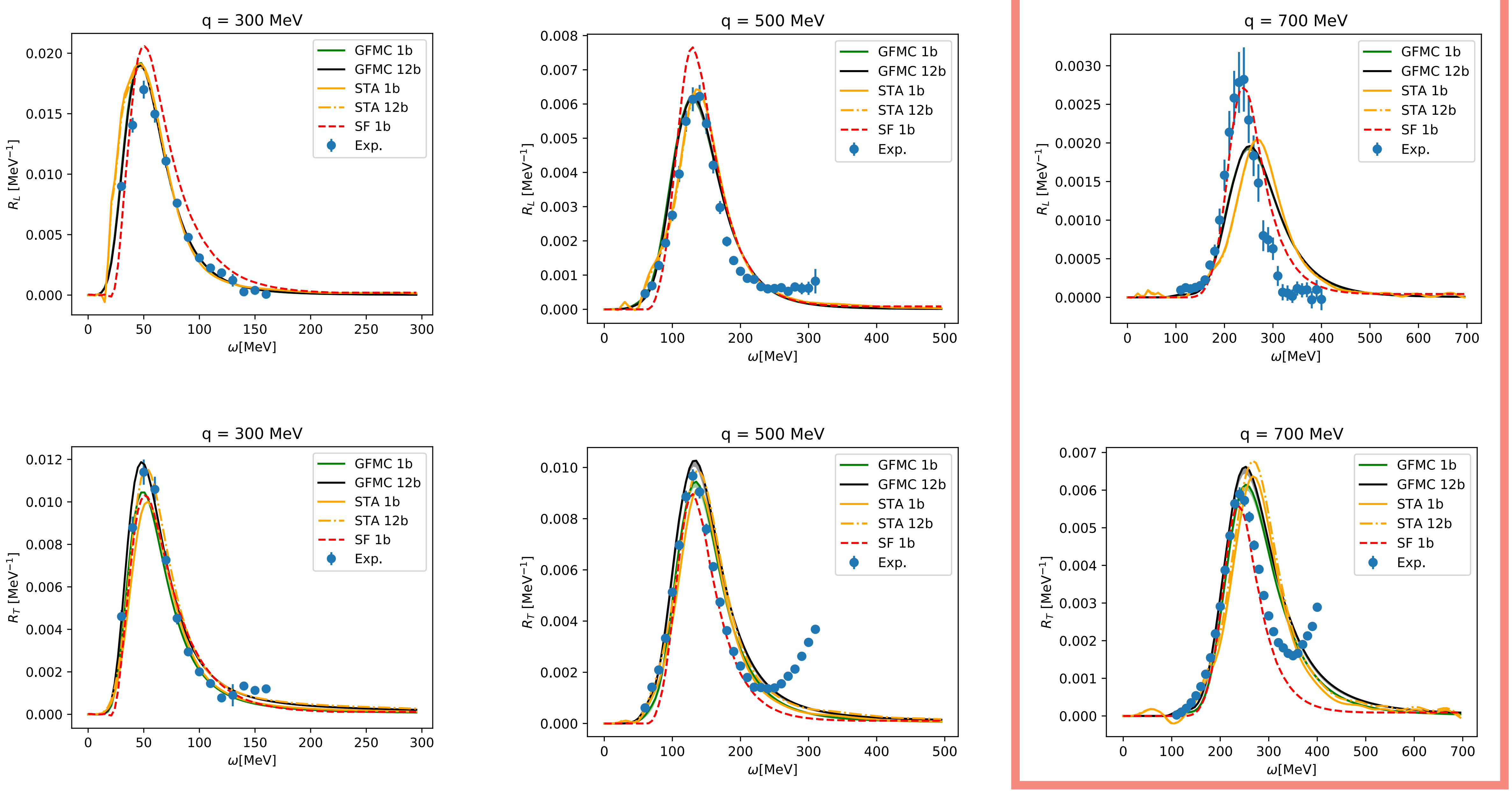
Benchmark

L.A, J. Carlson, A. Lovato, S. Pastore, N. Rocco, RB Wiringa PRC105(2022)014002

- We benchmarked three different methods based on the **same description of nuclear dynamics of the initial target state**
- Compared to the experimental data for the longitudinal and transverse electromagnetic response functions of ^{3}He, and the inclusive cross sections of both ^{3}He and ^{3}H
- Comparing the results allows for a precise quantification of the uncertainties inherent to factorization schemes

Benchmark

Longitudinal and transverse response function in ^{3}He



PRC105(2022)014002

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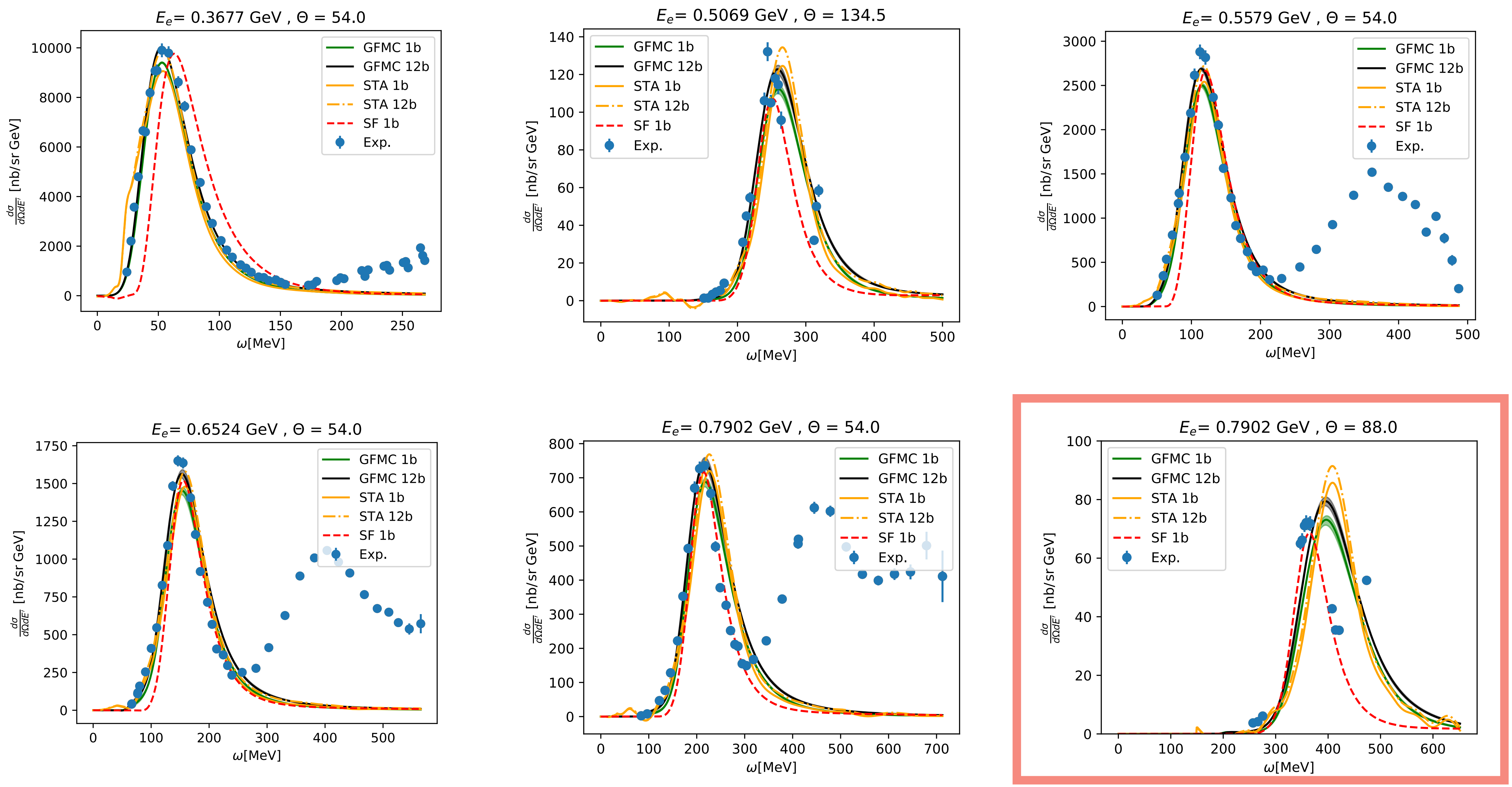
Benchmark

^3H



PRC105(2022)014002

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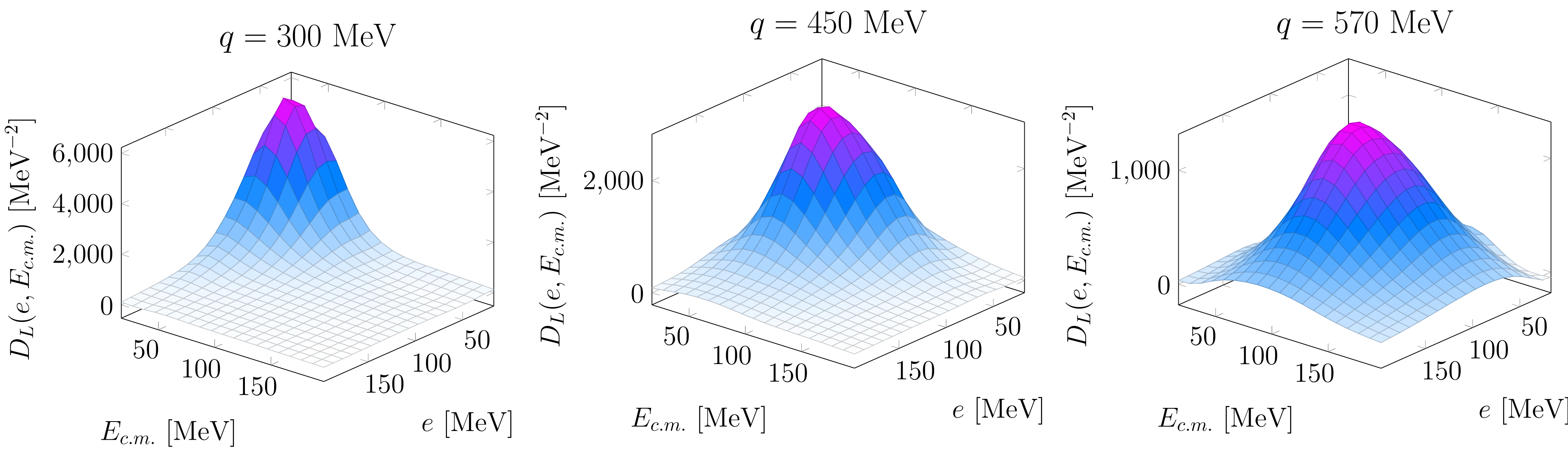
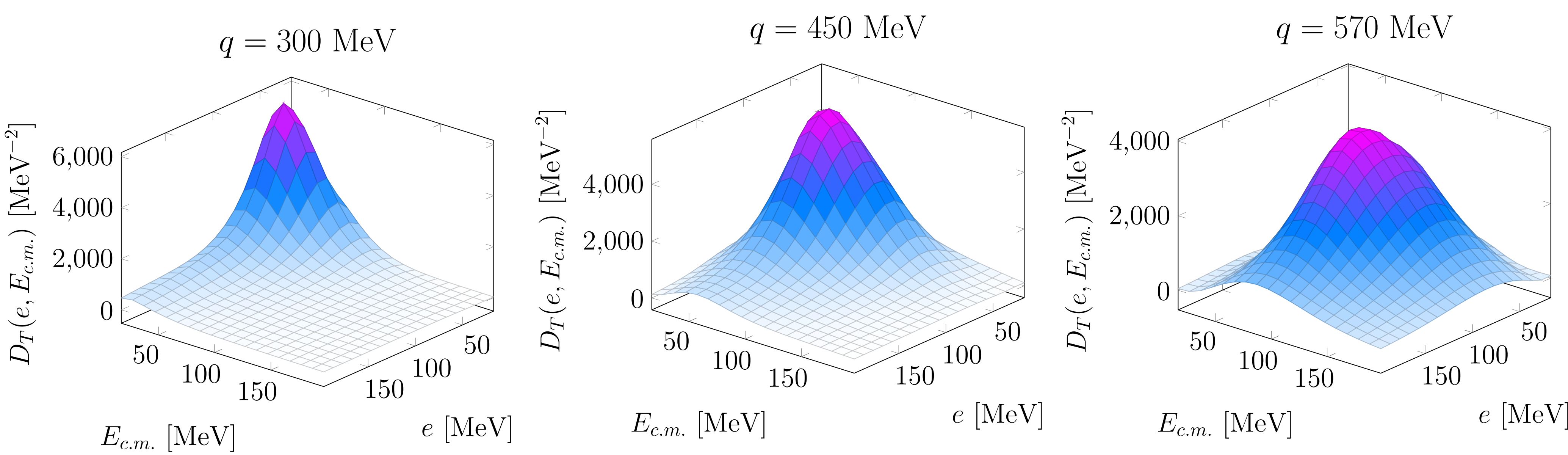
Responses for ^{12}C

Response densities are calculated for different values of momenta in the range **300 < q < 800 MeV**:

Longitudinal



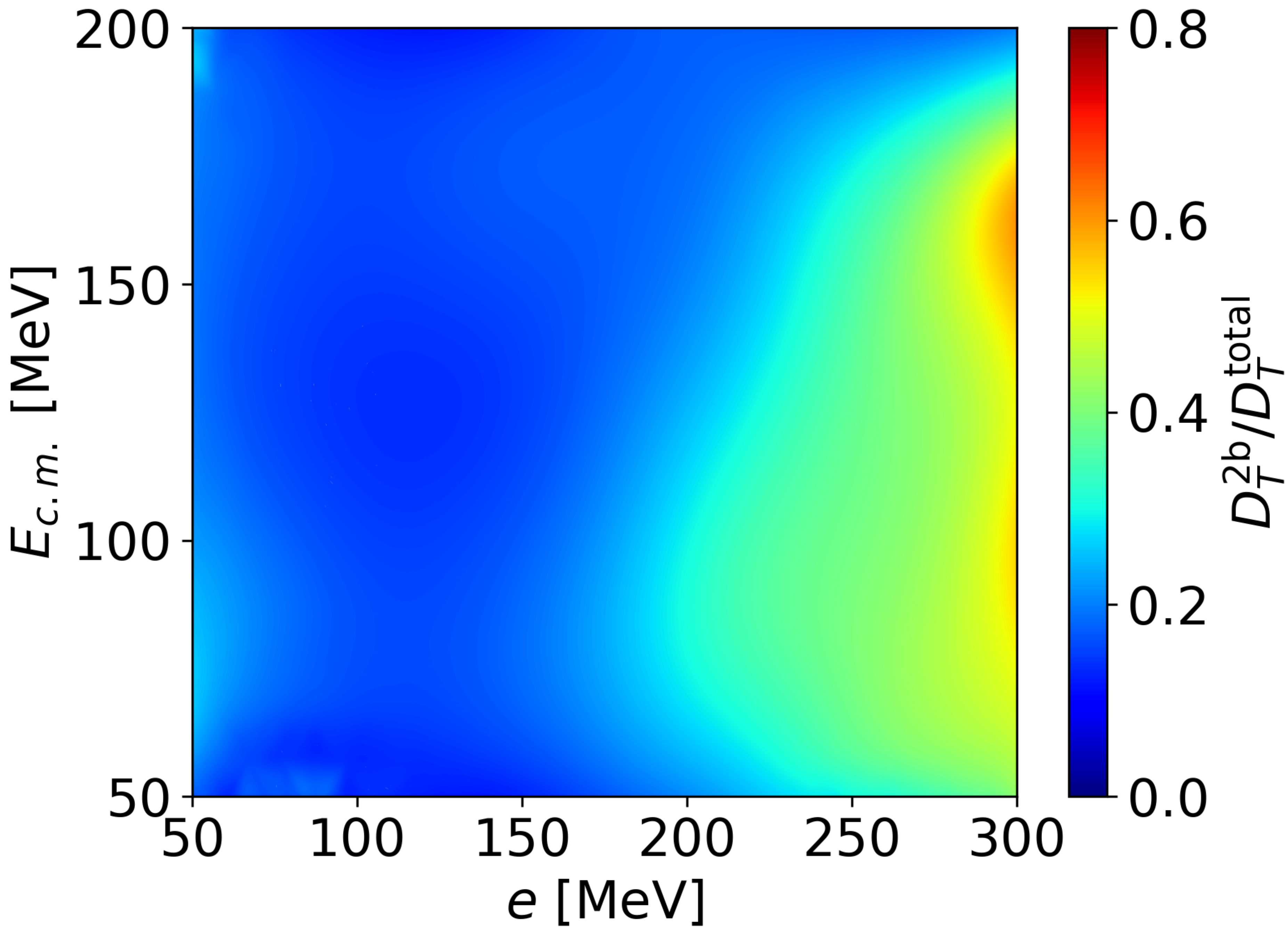
Transverse



https://arxiv.org/abs/2407.06986

Two-body contributions

Transverse response density at q=570 MeV:



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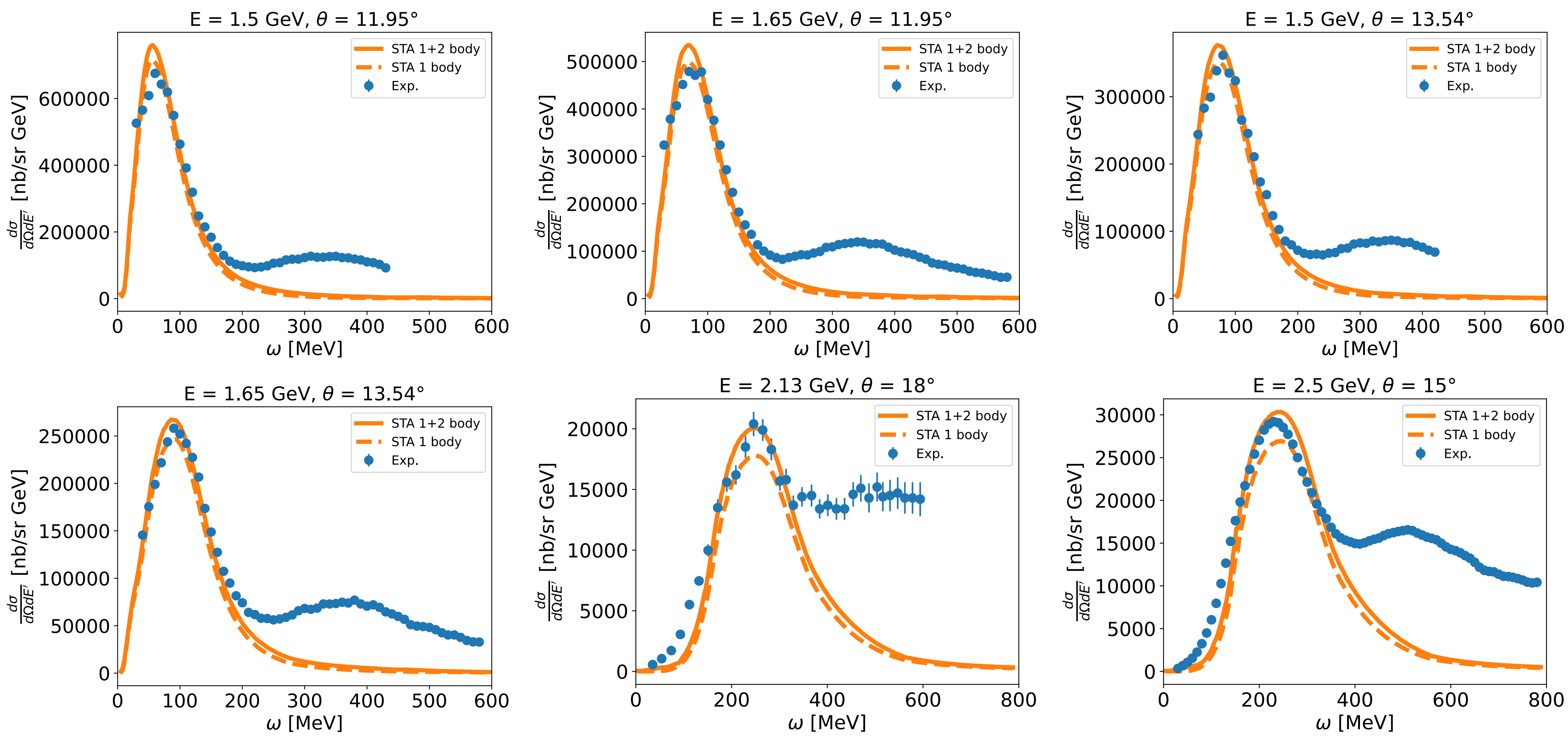
Cross sections results for ^{12}C



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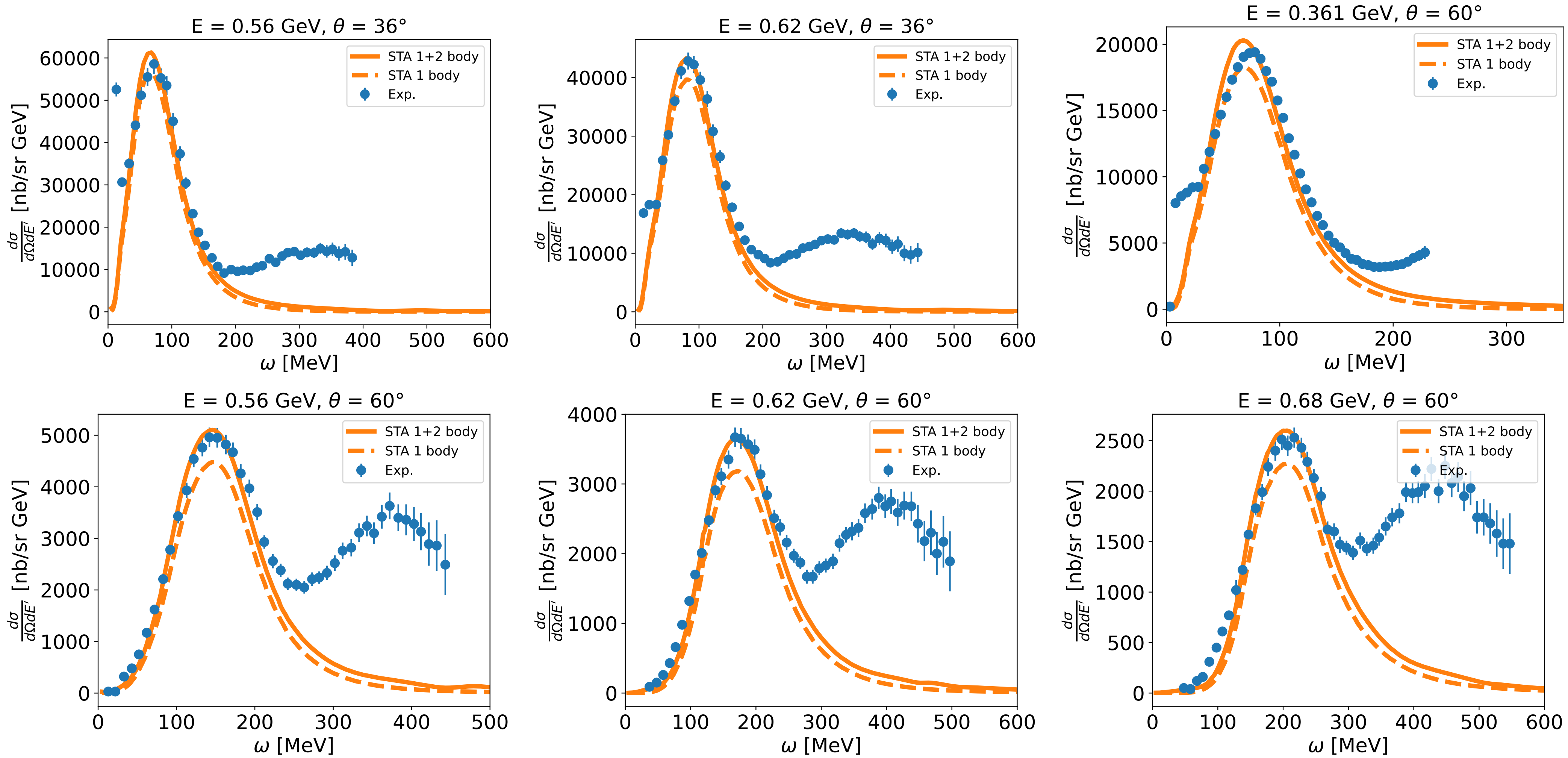

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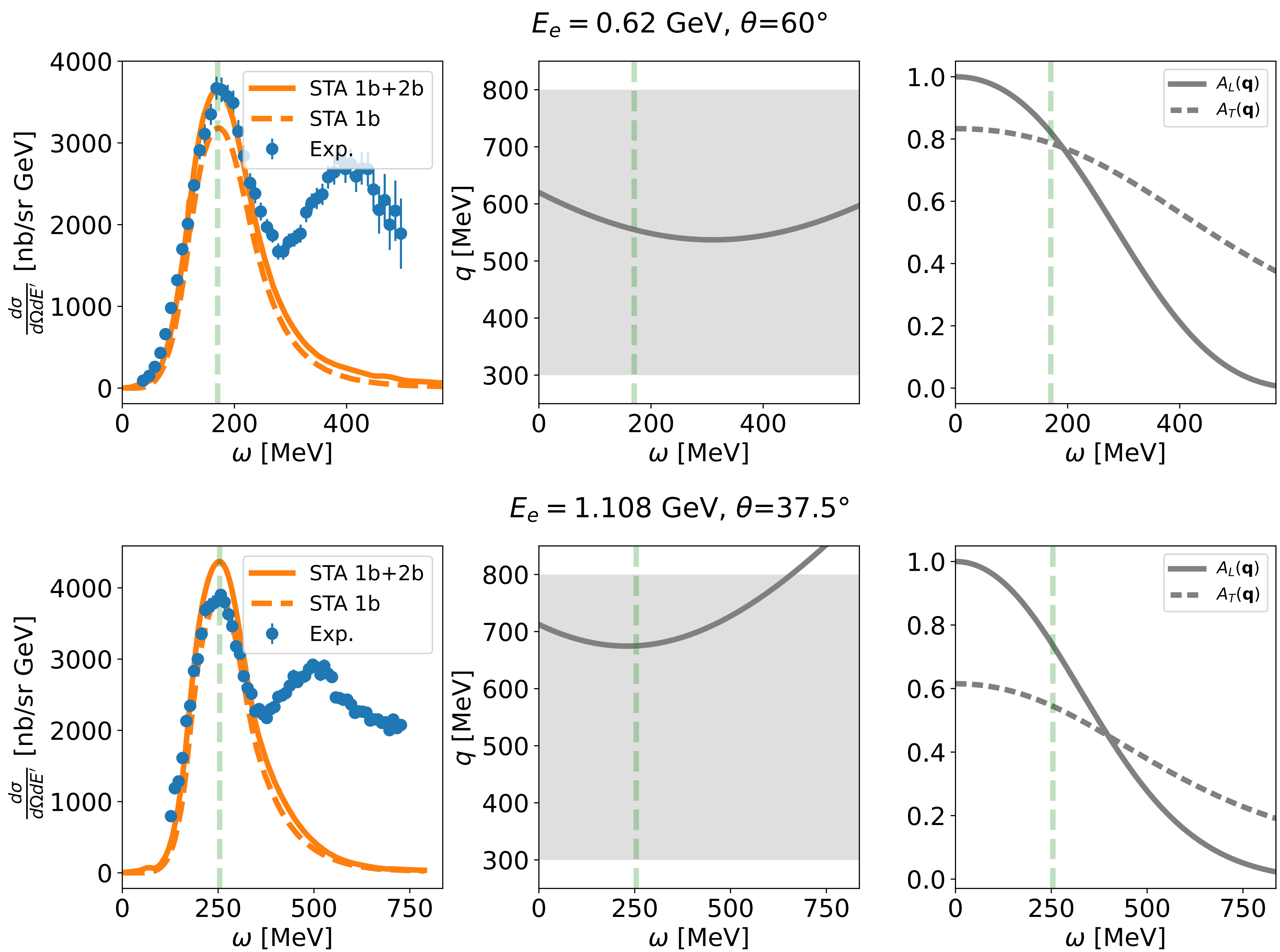


https://arxiv.org/abs/2407.06986

Relativistic corrections

Necessary to include relativistic correction at higher momentum q.

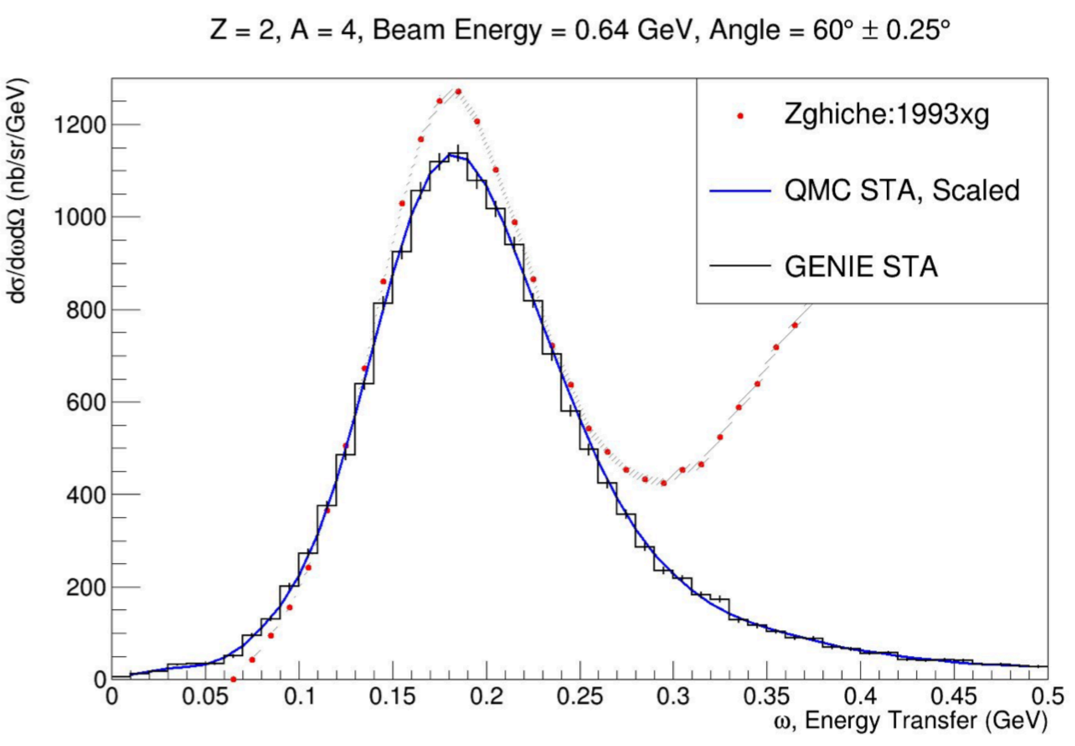
We are currently working on including **relativistic corrections within the STA formalism**:

R. Weiss, J. Carlson (LANL), G. Chambers-Wall, S. Pastore (WashU)

- **Relativistic kinematic**: allowed by STA factorization scheme
- **Relativistic currents**: expansion for a large value of the momentum transfer **q**

GENIE validation using e-scattering

- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE
- Electromagnetic processes (for which data are available) are used to validate the generator
- Next step: use the information contained in the **response densities** to generate events



Barrow, Gardiner, Pastore, Betancourt et al. PRD 103 (2021) 5, 052001

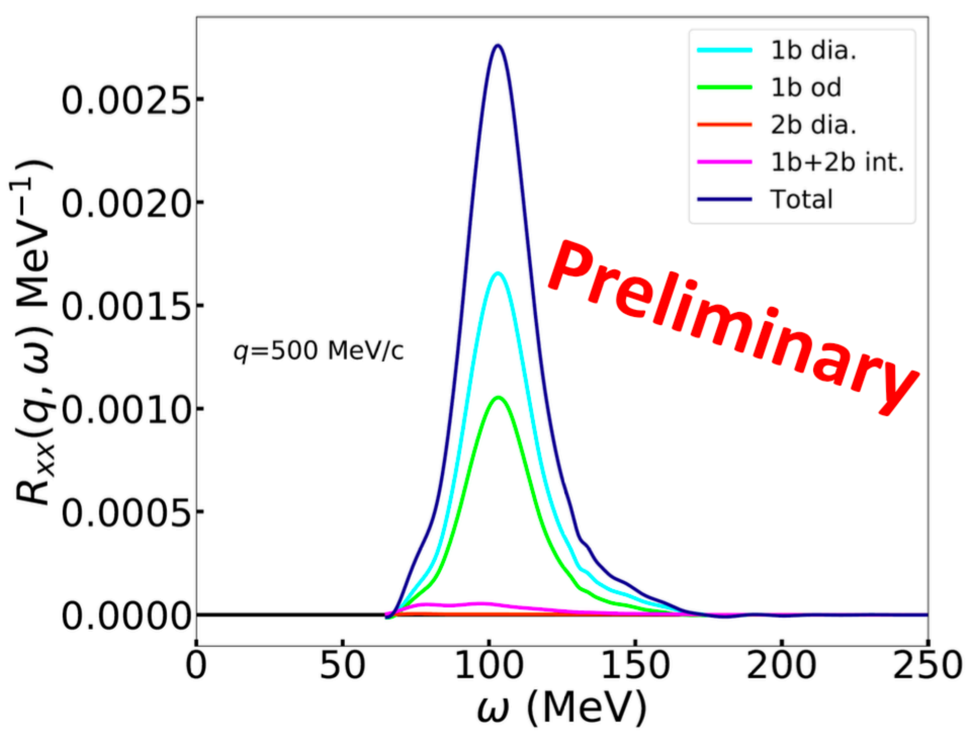
GENIE HadronTensorModelI Class: https://internal.dunescience.org/doxygen/classgenie_1_1HadronTensorModelI.html

EW interactions:

- The current work on EM interactions allows for a thorough evaluation of the method, and a comparison with the abundant experimental data for electron-nucleus scattering

The same STA formalism can be applied to neutrino-nucleus scattering:

- **G. King**: neutral weak currents quasi-elastic responses evaluated for ^2H



Conclusion:

- The STA responses for ^{12}C are in good agreement with the data, and are accurate up to moderate values of q (and consequently to moderate values of incoming electron beam for cross sections calculations)
- It can describe electromagnetic scattering from $A \geq 12$ accounting for two-body physics (**currents** and **correlations**), and is exportable to other QMC methods to address larger nuclei, e.g. AFDMC

Next:

- Incorporate **relativistic effects**, **pion production**, **heavier nuclei**
- Use of information from **response densities** in event generators: collaboration with GENIE Monte Carlo event generator (S. Gardiner, J. Barrow)

Thank you!

Collaborators:

ODU: A. Gnech

WashU: G. Chambers-Wall, G. King, S. Pastore and M. Piarulli

LANL: R. Weiss, S. Gandolfi, J. Carlson

ANL: R. B. Wiringa

FNAL: J. Barrow, M. Betancourt, S. Gardiner

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