



# $F_2$ neutron data-driven extraction update from CJ collaboration

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

**Matteo Cerutti**

# CTEQ-JLab collaboration

**Main focus:** Investigate the internal structure of nucleons  
in their valence region

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**collinear factorization**

$$d\sigma_{\text{hadron}} = \sum_{f_1, f_2, i, j} \phi_{f_1} \otimes \hat{\sigma}_{\text{parton}}^{f_1 f_2 \rightarrow ij} \otimes \phi_{f_2}$$

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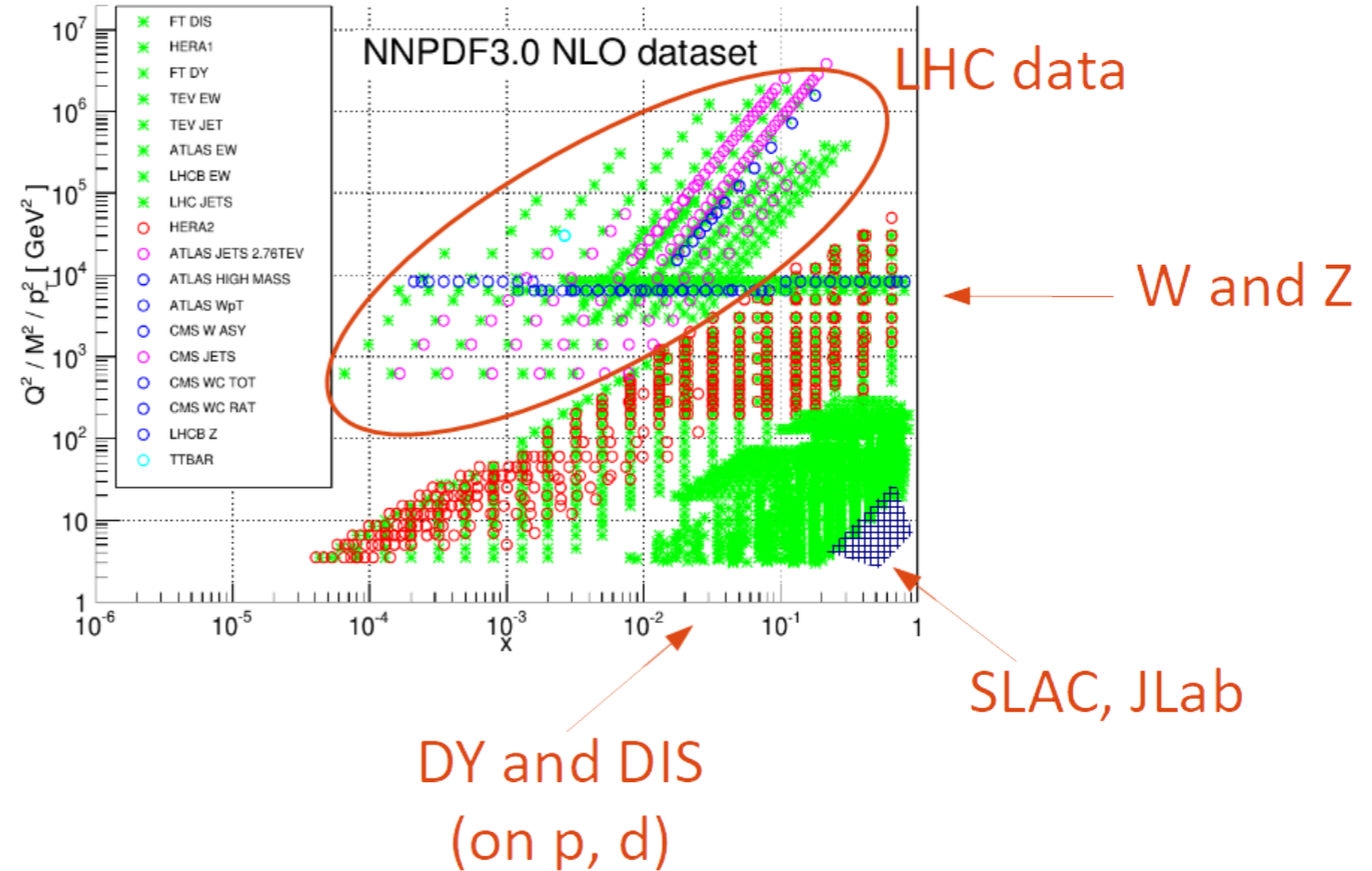
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## universality

- DIS  $p, d$  targets
- $pp$  collisions Drell-Yan
- W/Z boson production
- Jets



**40+ years of experience**

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## Coordinate **theory**+**experiment** effort within Jefferson Lab

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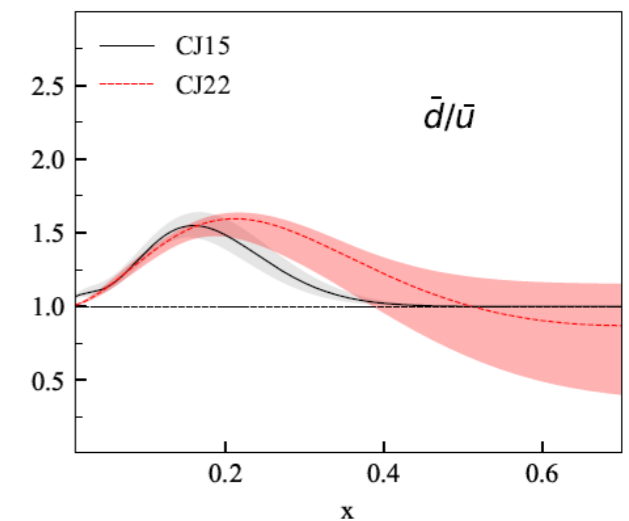
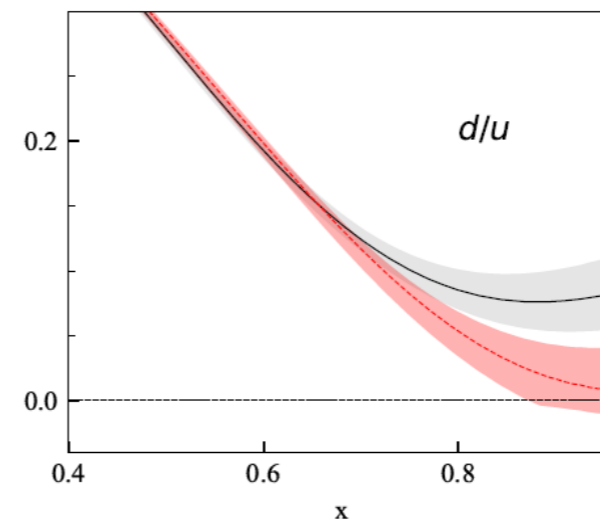
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## Recent works:

- Extraction of PDFs at large  $x$   
**CJ22** Accardi, Jing, Owens et al., PRD 107 (2023)



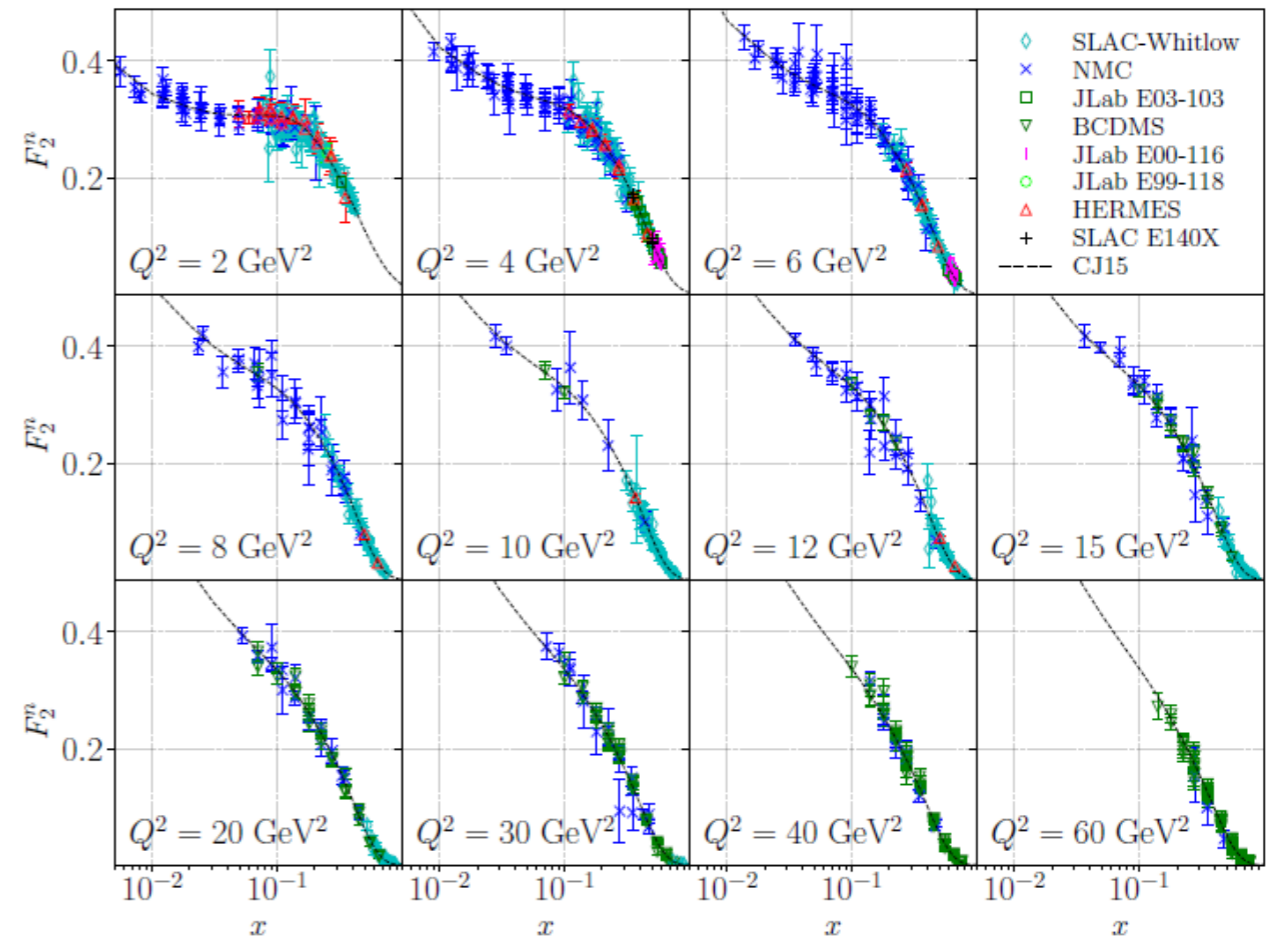
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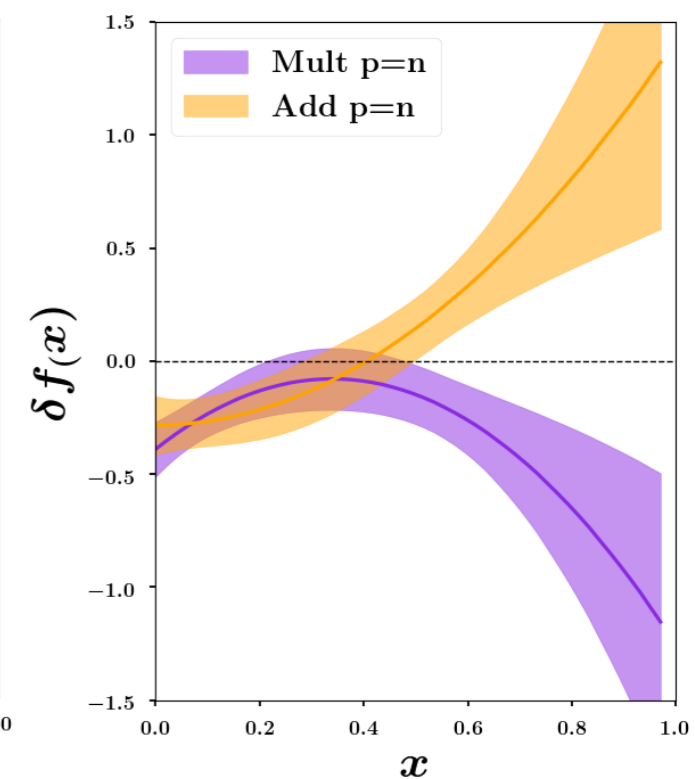
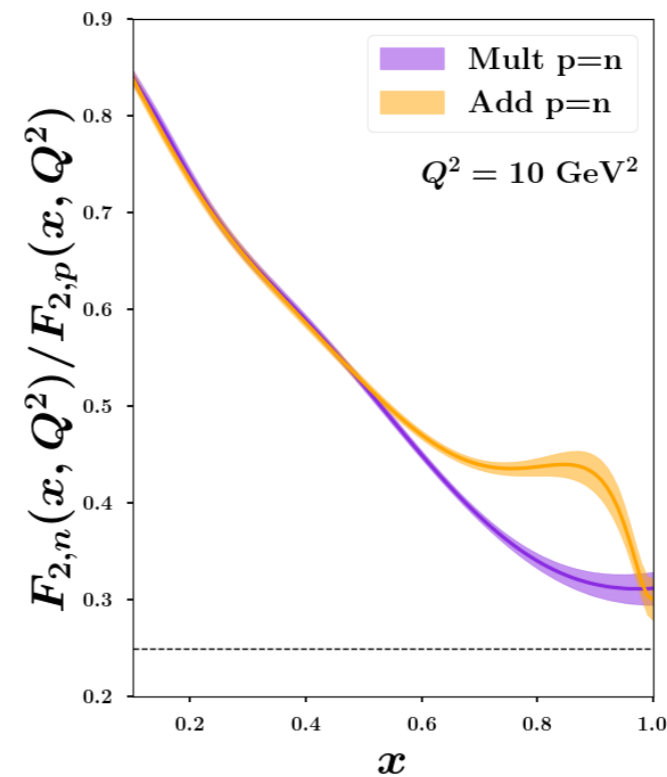
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- Systematic uncertainties from HT and off-shell corrections  
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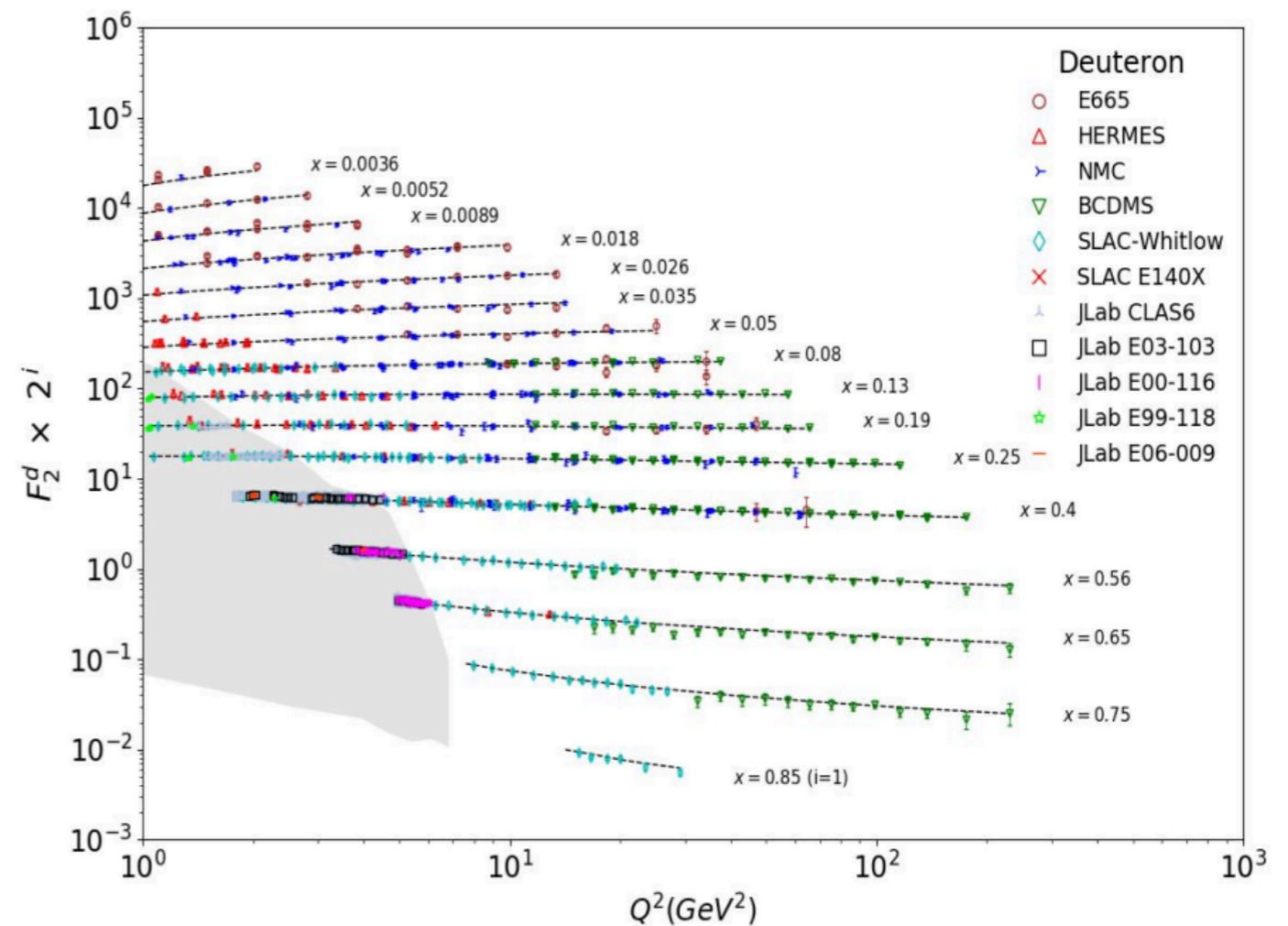
+ ...

# Extraction of neutron $F_2$ structure function

## DIS on deuteron target

CJ global data set:

- 1000+ data points
- high- $x$  and low- $Q^2$
- $W^2 > 3 \text{ GeV}^2, Q^2 > 1.69 \text{ GeV}^2$



# Extraction of neutron $F_2$ structure function

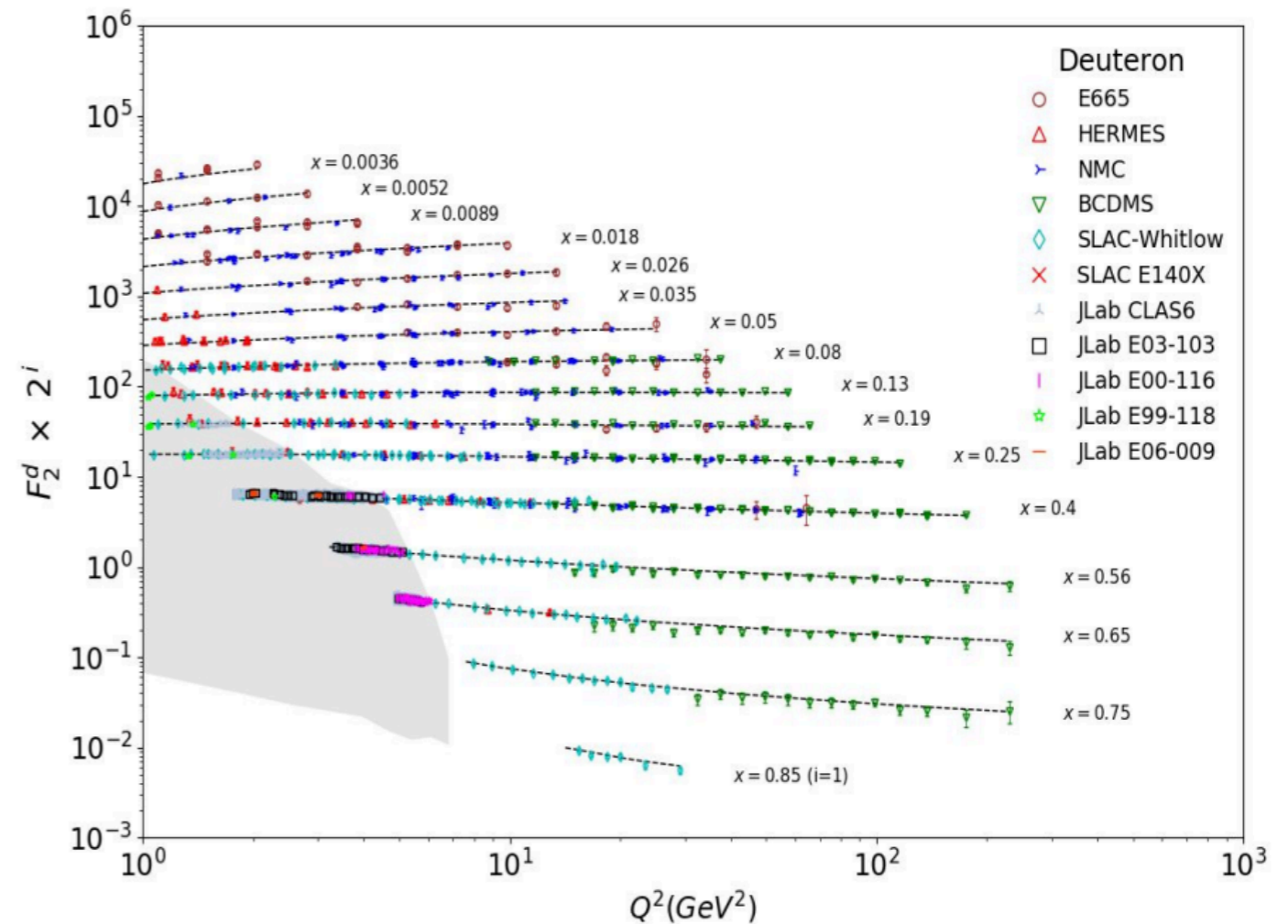
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Full treatment of nuclear corrections

Binding effects, Fermi motion, off-shell corrections, Higher Twist (HT), Target Mass Corrections (TMC)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

Smearing function

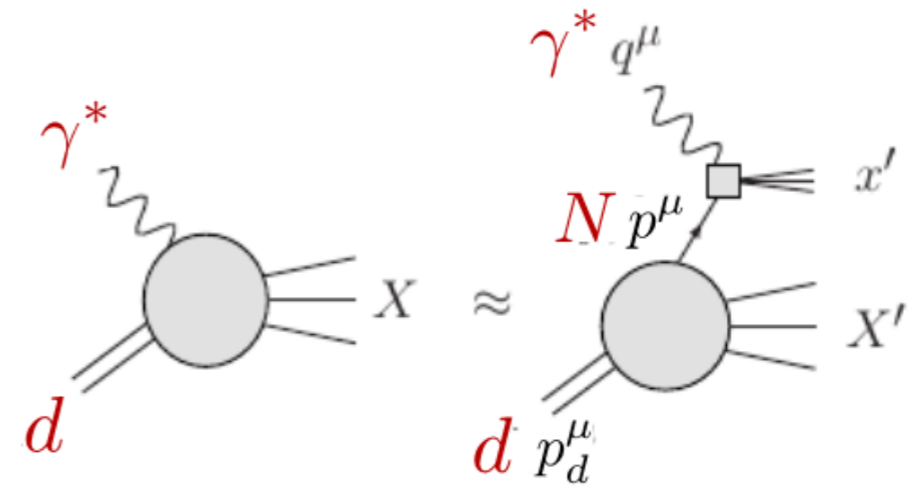
Structure function of a bound, off-shell nucleon

# Deuterium: off-shell corrections

Bound, off-shell nucleon inside the deuteron

$$p^2 < m_N^2$$

Structure functions are deformed at large  $x$

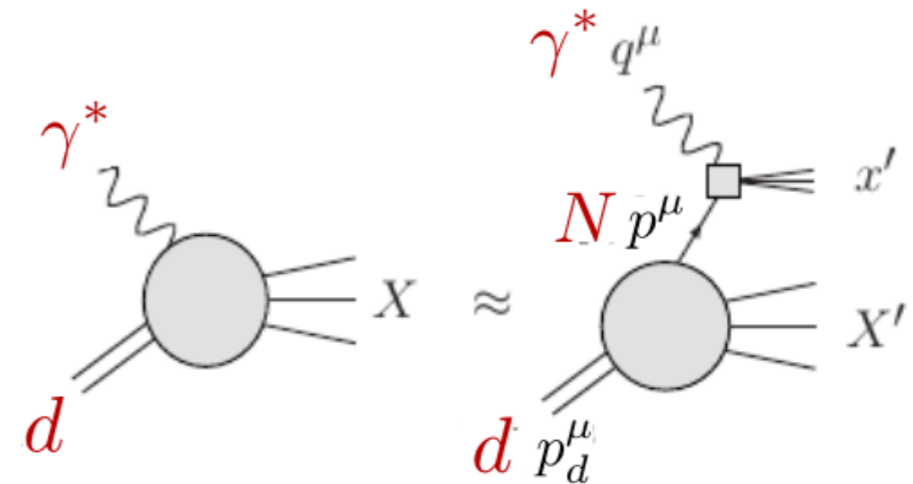


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**Off-shell expansion (in nucleon virtuality  $p^2$ )**

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2) \left[ 1 + \frac{p^2 - M^2}{M^2} \delta f(x) \right]$$

$$F_{2N}(x, Q^2, p^2) = F_{2N}^{\text{free}}(x, Q^2) \left[ 1 + \frac{p^2 - M^2}{M^2} \delta F(x) \right]$$

parton level

Kulagin, Piller, Weise, PRC 50 (1994)

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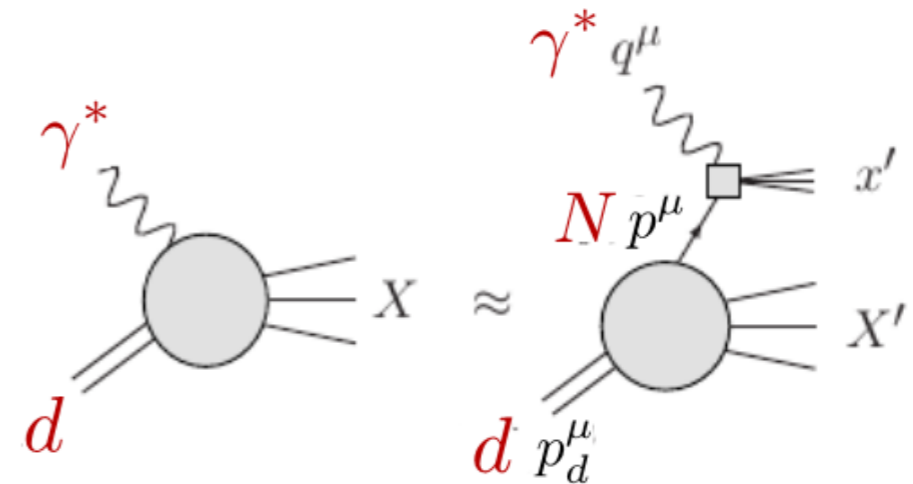
struct. func level

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struct. func level

Free nucleon pdfs/SFs

$$p^2 = m_N^2$$

Off-shell function

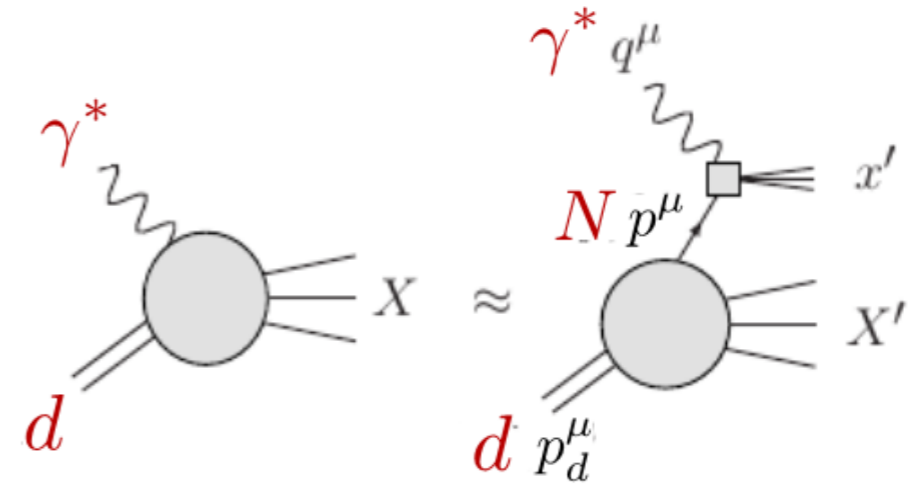
(To be fitted)

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Off-shell function

(To be fitted)

KP-like model

$$\delta f(x) = C(x - x_0)(x - x_1)(1 + x_0 - x)$$

$$\int_0^1 dx \delta f(x) [q(x) - \bar{q}(x)] = 0$$

# Higher-Twist function

## Higher Twist correction

Multiplicative

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{\mathbf{C}(x)}{Q^2} \right)$$

Additive

$$F_2 = F_2^{LT}(x, Q^2) + \frac{\mathbf{H}(x)}{Q^2}$$



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they are related

$$\begin{aligned} F_2^{LT}(x, Q^2) \left( 1 + \frac{\mathbf{C}(x)}{Q^2} \right) &= F_2^{LT}(x, Q^2) + F_2^{LT}(x, Q^2) \frac{\mathbf{C}(x)}{Q^2} \\ &= F_2^{LT}(x, Q^2) + \frac{\tilde{\mathbf{H}}(x, Q^2)}{Q^2} \end{aligned}$$

# Higher-Twist function

## Higher Twist correction

### Multiplicative

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{C(x)}{Q^2} \right)$$

$$C(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1 + a_{ht}^{(2)} x)$$

CJ fits

### Additive

$$F_2 = F_2^{LT}(x, Q^2) + \frac{H(x)}{Q^2}$$

### BIAS in isospin-symmetric case

MC, Accardi, Fernando, Li, arXiv:2407.03589

they are related

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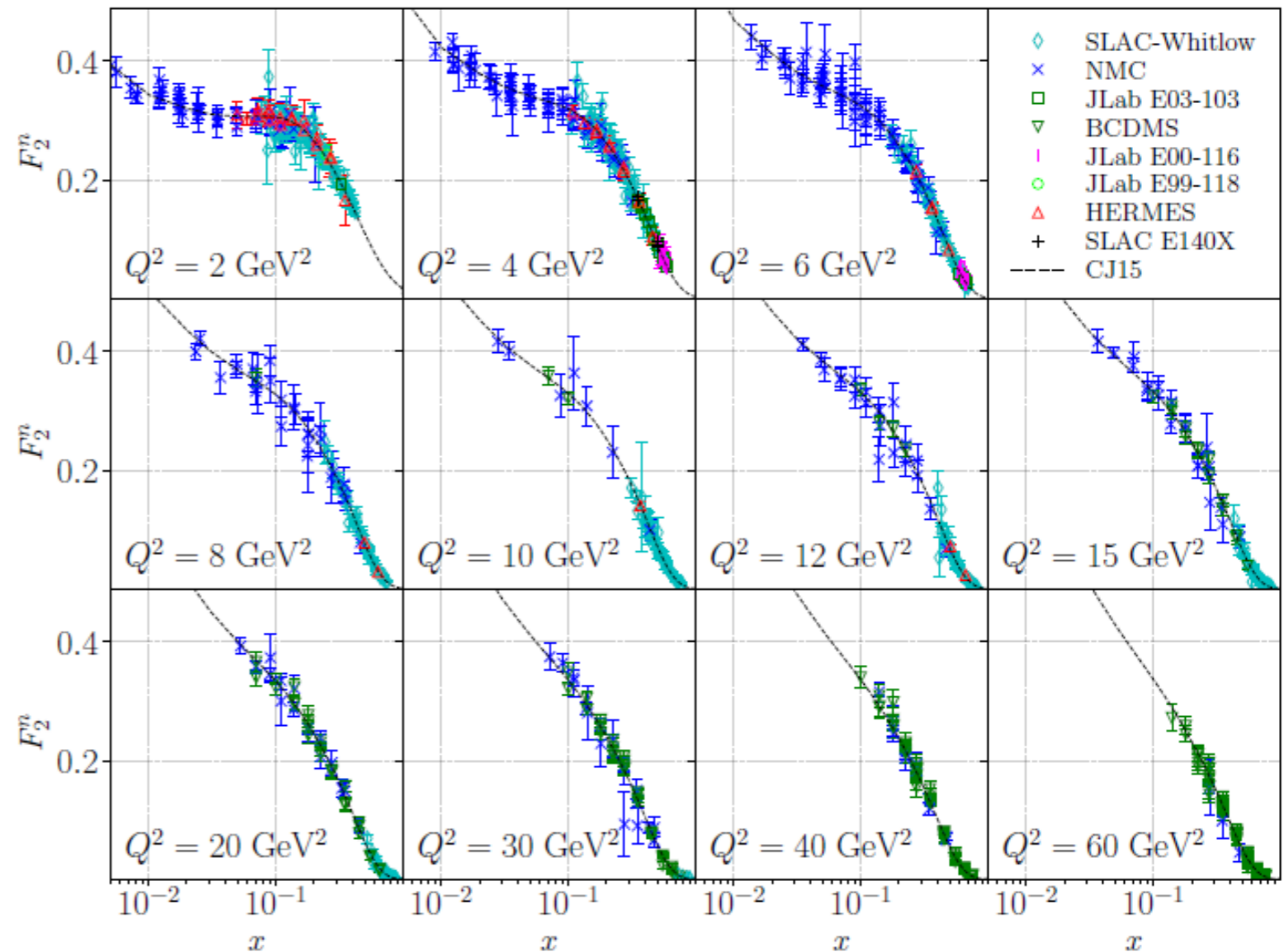
# Extraction of neutron $F_2$ structure function

## Basic idea

$F_{2,n}$

$$\widehat{F}_2^{n(0)}(x, Q^2) = \frac{2 \widehat{F}_2^{d(0)}(x, Q^2)_{\text{exp}}}{R_{d/N}^{CJ}(x, Q^2)} - \widehat{F}_2^{p(0)}(x, Q^2)_{\text{exp}}$$

- p, d data matching
- data cross normalization
- results based on CJ15 analysis
- extracted experimental bins centered for applications



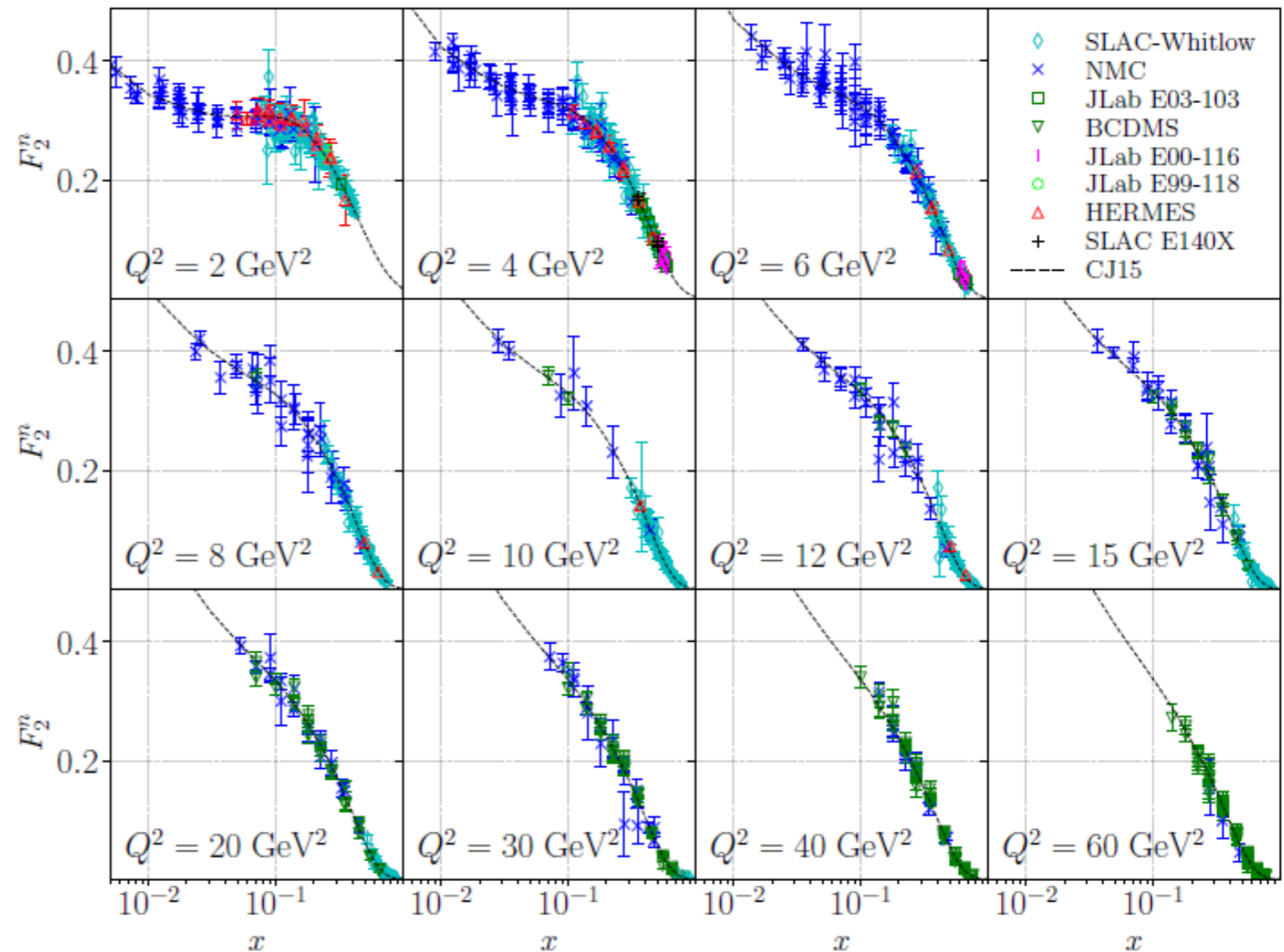
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**Neutron F2 data sets and grids available!!!**

<https://github.com/JeffersonLab/CJ-database/>

# Extraction of neutron $F_2$ structure function

## Experimental data sets

*extension of CJ15 DIS included data set*

Experiments	# of Proton F2 Data Points
SLAC-Whitlow <sup>[2]</sup>	564
BCDMS	351 <sup>[3]</sup>
HERMES <sup>[5]</sup>	45
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NMC <sup>[7]</sup>	275
+	
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# of Deuteron F2 Data Points
582
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## Matching conditions

- Same beam energy
- $|x_p - x_d| < 0.01$
- $|Q_p^2 - Q_d^2| < 1\%$

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# of Deuteron F2 Data Points	# of Constructed Neutron Points
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254 <sup>[4]</sup>	254
45	45
136	120
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13	9
69	37
1723 <sup>[11]</sup>	0
0	0
79	0
2	2

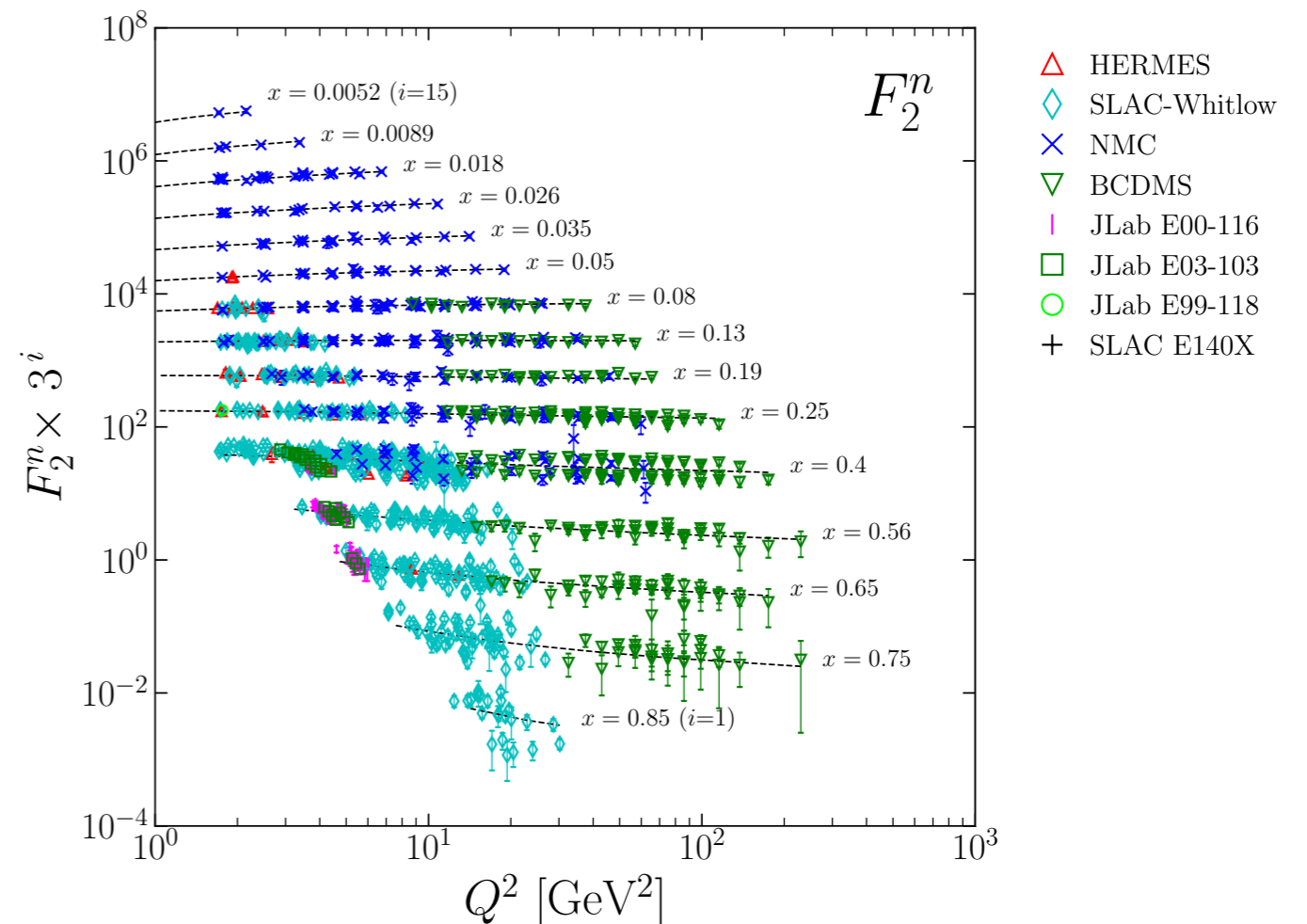
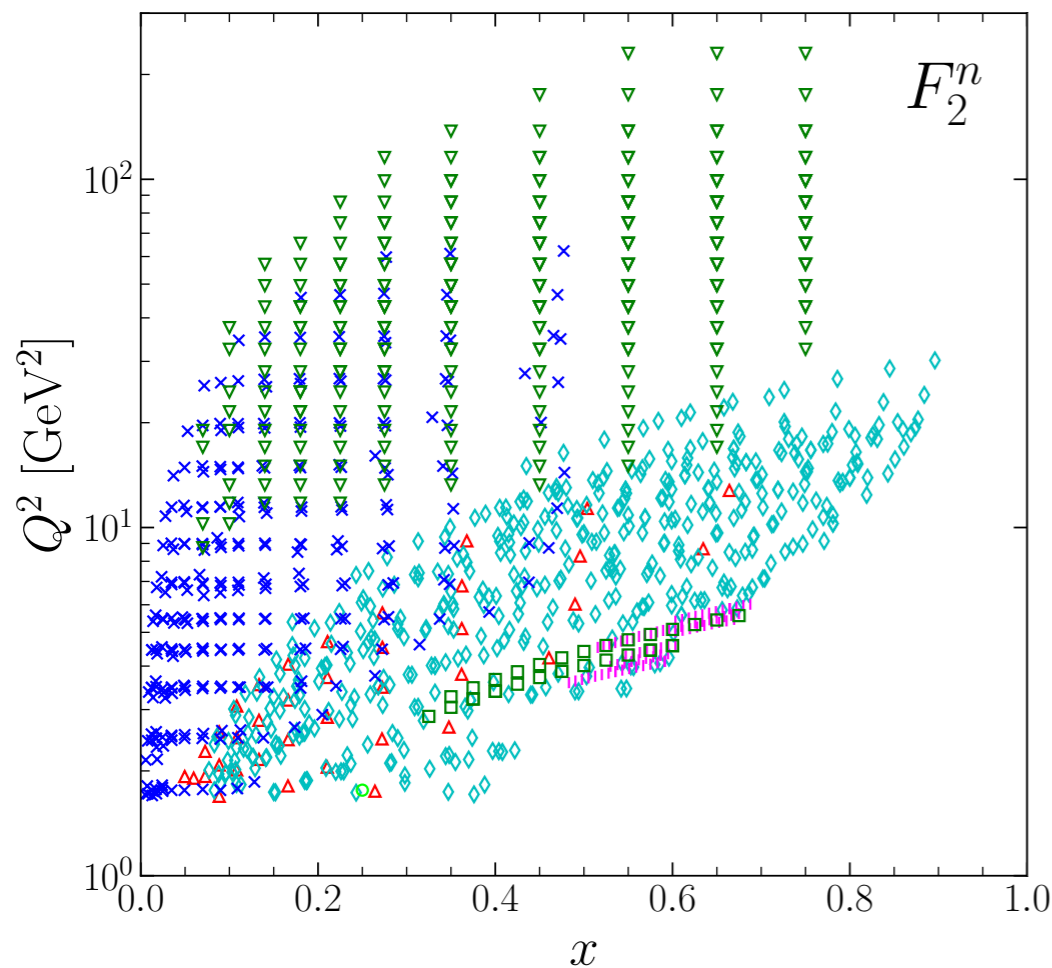
**1192** matched data points

# Extraction of neutron $F_2$ structure function

## Experimental data sets

*extension of CJ15 DIS included data set*

$W^2 > 3.5 \text{ GeV}^2$



**1192** matched data points



# Extraction of neutron $F_2$ structure function

## Cross normalization

$$\widehat{F}_2^{n(0)}(x, Q^2) = \frac{2 \widehat{F}_2^{d(0)}(x, Q^2)_{\text{exp}}}{R_{d/N}^{CJ}(x, Q^2)} - \widehat{F}_2^{p(0)}(x, Q^2)_{\text{exp}}$$

Avoid large fluctuations  
due to their relative  
systematic uncertainties

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Avoid large fluctuations due to their relative systematic uncertainties

## Through CJ framework

$$\chi^2 = \sum_{\text{exp}} \left[ \sum_{i=1}^{N_{\text{data}}} \left( \frac{D_i + \Delta_i - T_i/n}{\delta D_i} \right)^2 + (\lambda^{\text{norm}})^2 + \sum_{k=1}^K \lambda_k^2 \right]_{\text{exp}}$$

↙ ↘

Uncorrelated error

Normalization error

$$n = 1 + \lambda^{\text{norm}} \delta n$$

$$\Delta_i = \sum_k \lambda_k \beta_{k,i}$$

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Correlated error

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↙ ↘

Correlated error

**Fix** theoretical prediction

**Fit** simultaneously the nuisance parameters  $\lambda$

$$\widehat{D}_i^{(0)} = n^{(0)} (D_i + \Delta_i^{(0)})$$

# Extraction of neutron $F_2$ structure function

## Uncertainties

$$\hat{F}_2^n = \hat{F}_2^{n(0)} \pm \delta \hat{F}_2^n \pm \delta^{CJ} \hat{F}_2^n$$

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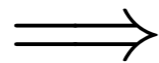
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*Uncorrelated uncertainties*



standard propagation from d, p data

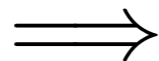
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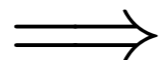
*Uncorrelated uncertainties*



standard propagation from d, p data

*Procedural uncertainties:*

Cross normalization



Hessian method with “error PDF sets”

Calculation of CJ ratio

CJ15n1o\_mod  $2 * N_{\text{parameters}} + 1 = 49$

(negligible)

$$\delta^{CJ} \lambda = \frac{1}{2} \sqrt{\sum_{j=1}^{24} [\lambda^{(2j-1)} - \lambda^{(2j)}]^2}$$

$$\delta^{CJ} \widehat{D}_i = \frac{1}{2} \sqrt{\sum_{j=1}^{24} [\widehat{D}^{(2j-1)} - \widehat{D}^{(2j)}]^2}$$

# Extraction of neutron $F_2$ structure function

## **Bin centering**

Better visualization of data points

Better evaluation of structure function moments - applications

# Extraction of neutron $F_2$ structure function

## Bin centering

Better visualization of data points

Better evaluation of structure function moments - applications

*Use CJ framework*

$$\tilde{F}_2(x, Q_0^2) \equiv R_{bc}(Q_0^2, Q^2) \hat{F}_2(x, Q^2)$$

$$R_{bc}(Q_0^2, Q^2) \equiv \left. \frac{F_2(x, Q_0^2)}{F_2(x, Q^2)} \right|_{CJ}$$

Procedural uncertainty: negligible



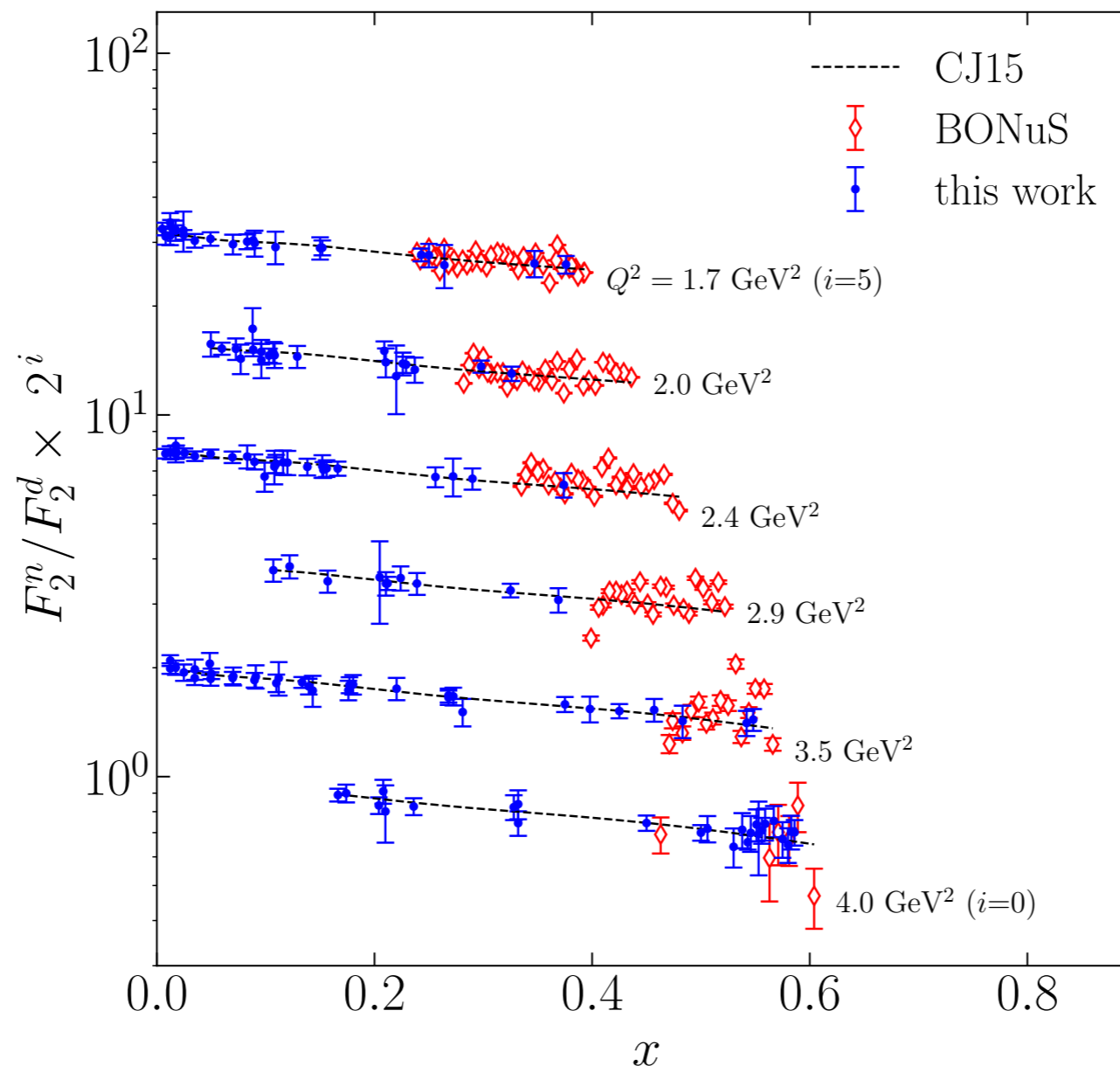
# Extraction of neutron $F_2$ structure function

## Comparison to BONuS data set

CLAS Collab., PRC 89 (2014)

$F_{2,n}/F_{2d}$   $\longrightarrow$  Experimental data

$\downarrow$   
This extraction



**GOOD  
AGREEMENT!**

# Extraction of neutron $F_2$ structure function

## Basic idea

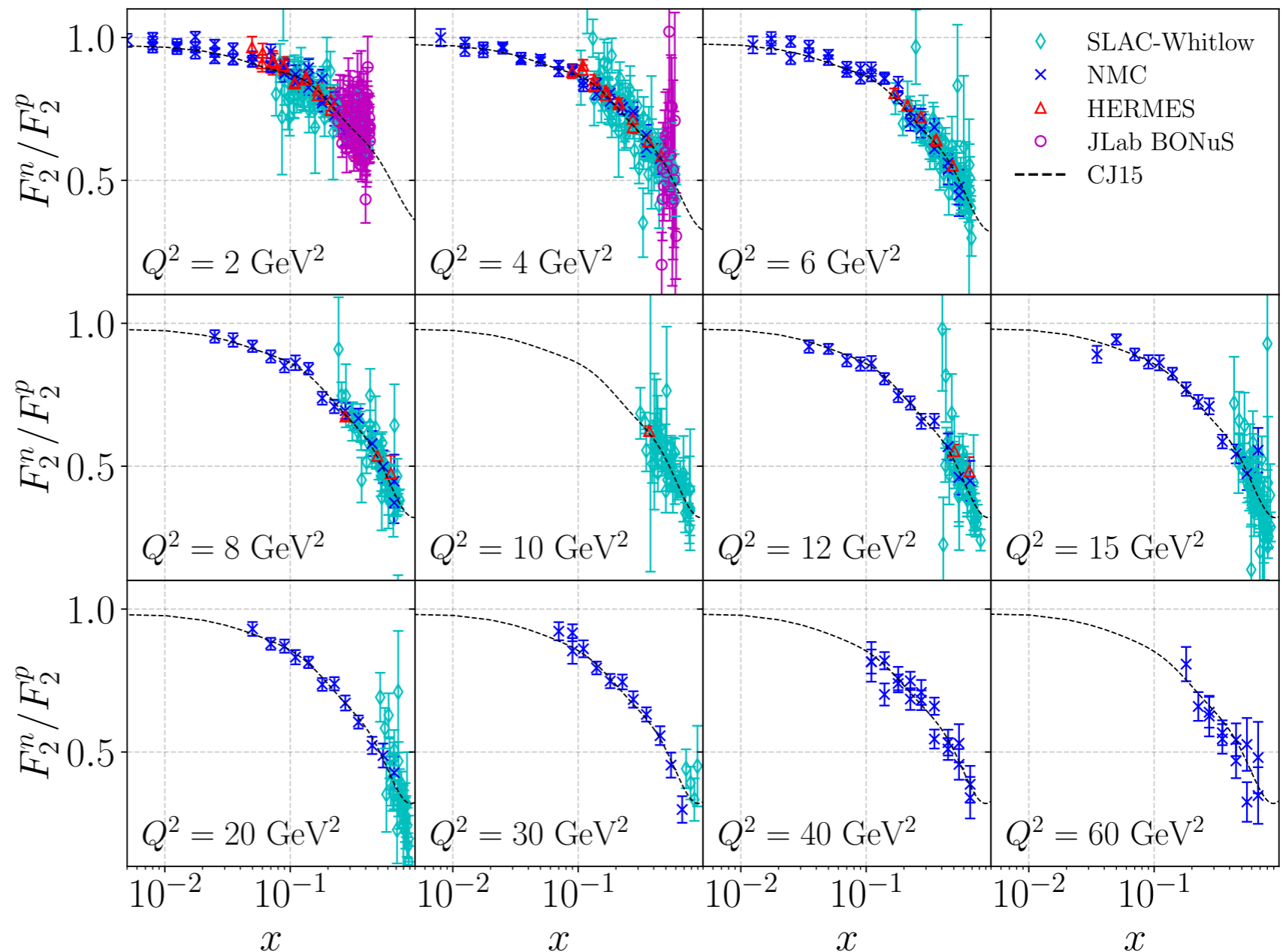
$$F_{2,n}/F_{2p}$$

Using  $d/p$  data

$$\hat{R}_{n/p}^{(0)} = \frac{2\hat{R}_{d/p}^{exp,(0)}}{R_{d/N}^{CJ}} - 1$$

Using  $n/d$  data (BoNUS)

$$\hat{R}_{n/p}^{(0)} = \frac{\hat{R}_{n/d}^{exp,(0)} R_{d/N}^{CJ}}{2 - \hat{R}_{n/d}^{exp,(0)} R_{d/N}^{CJ}}$$



# Extraction of neutron $F_2$ structure function

$$F_{2,n}/F_{2p}$$

*What about MARATHON data set??*

Jefferson Lab Hall A Collab., PRL 128 (2022) 13

# Extraction of neutron $F_2$ structure function

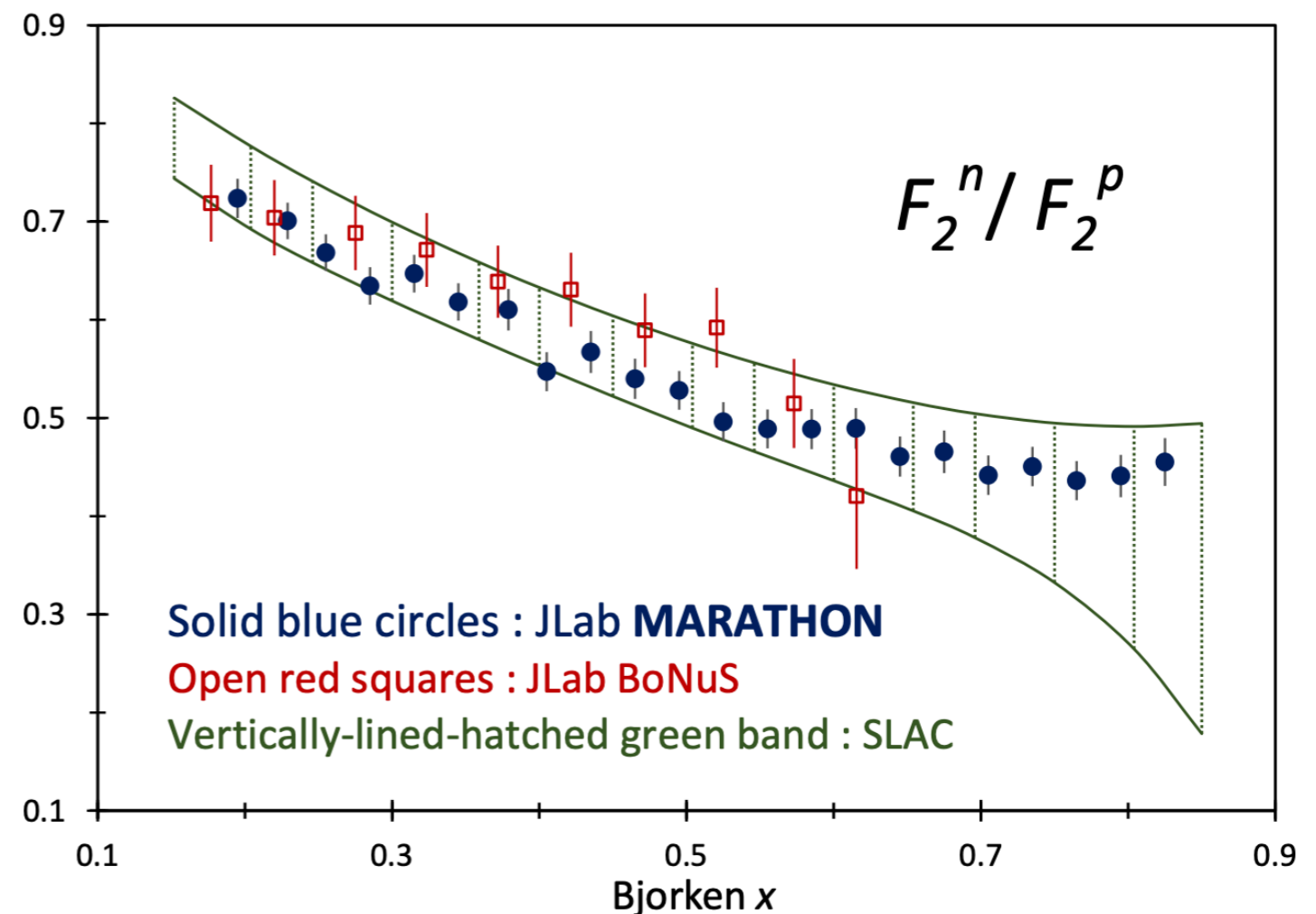
$$F_{2,n}/F_{2p}$$

What about MARATHON data set??

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$$\frac{F_2^n}{F_2^p} = \frac{2\mathcal{R}_{ht} - F_2^h/F_2^t}{2F_2^h/F_2^t - \mathcal{R}_{ht}}$$

- A=3 not available in CJ framework
- Experimental analysis based on AKP framework



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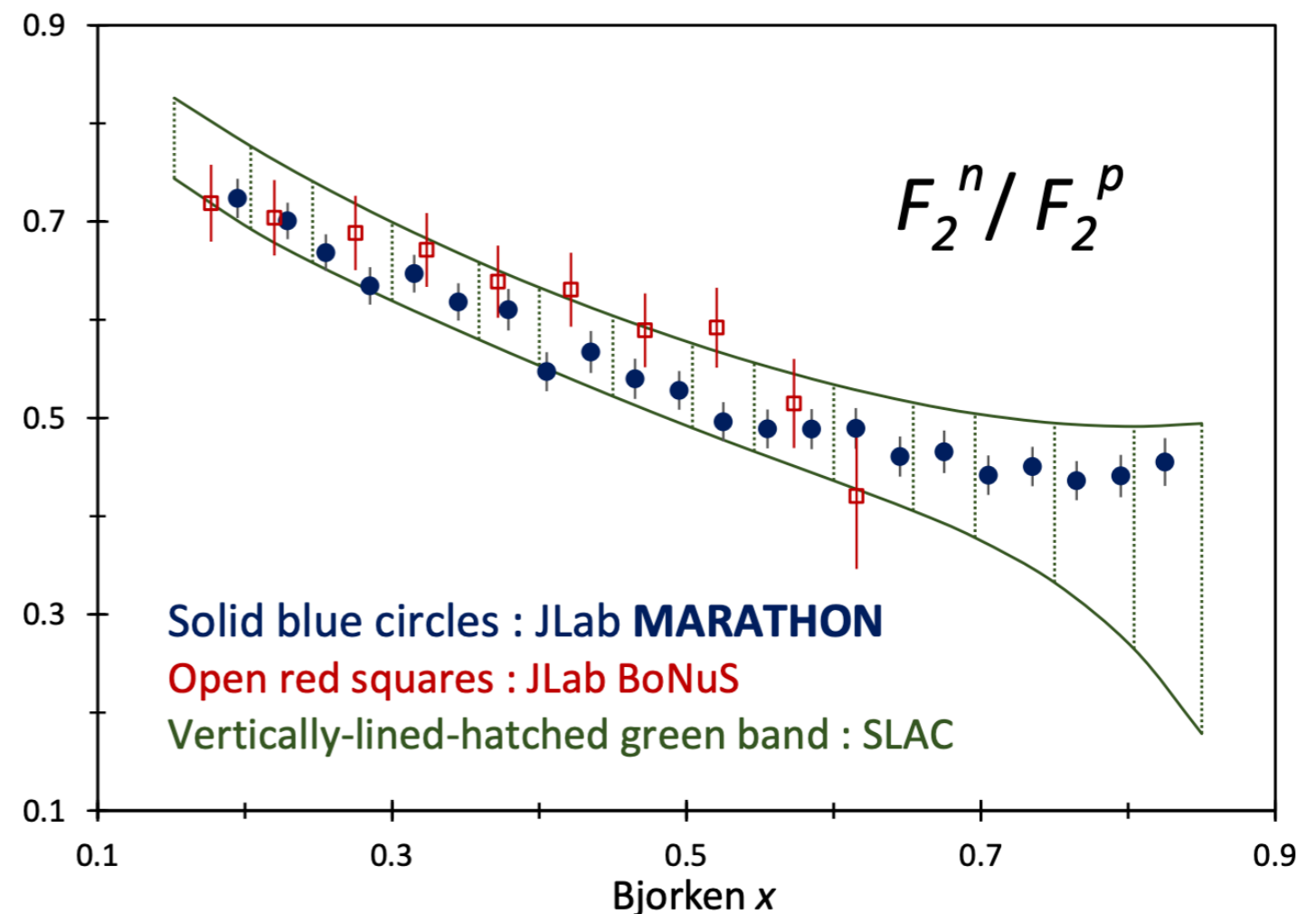
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**Be careful!**

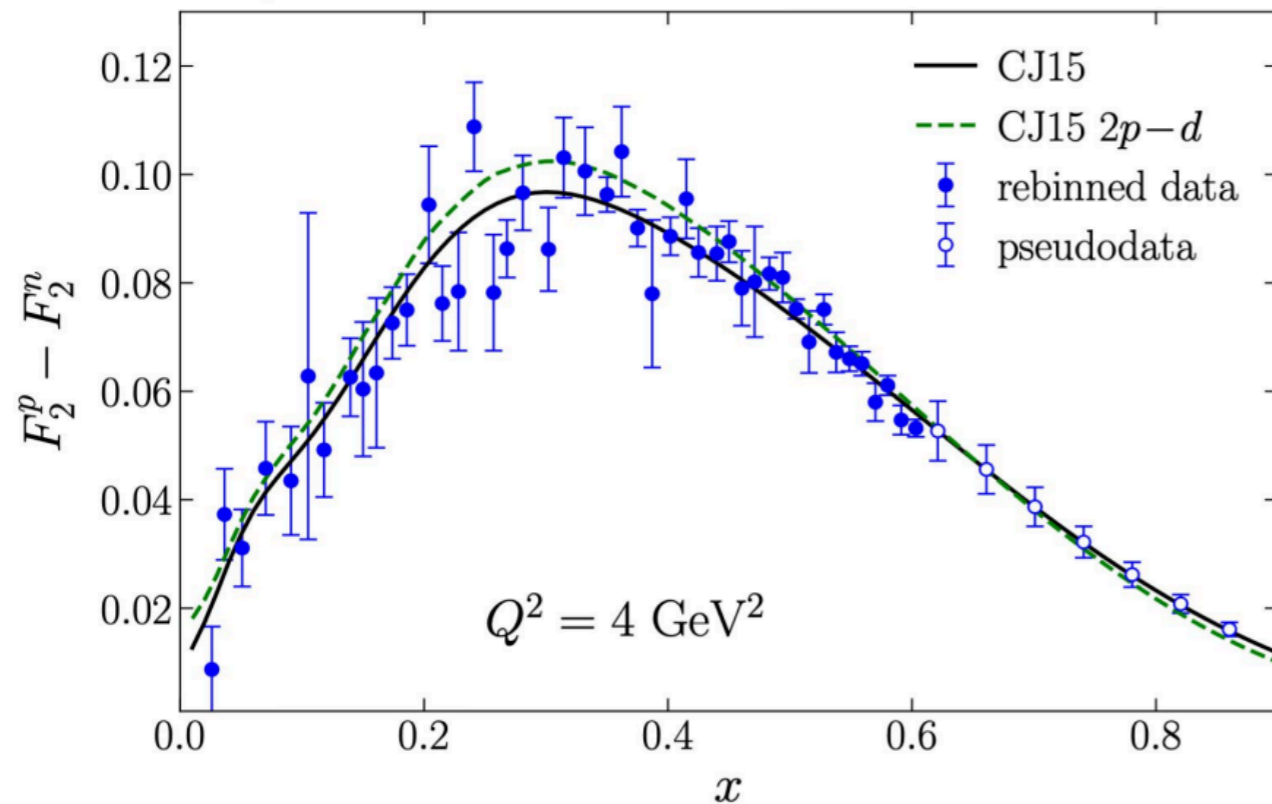
Systematic uncertainties  
in treatment of HT and  
off-shell corrections



# Application: GSR

## Gottfried sum rule

$$S_G(Q^2) = \int_0^1 \frac{dx}{x} [F_2^p(x, Q^2) - F_2^n(x, Q^2)]$$

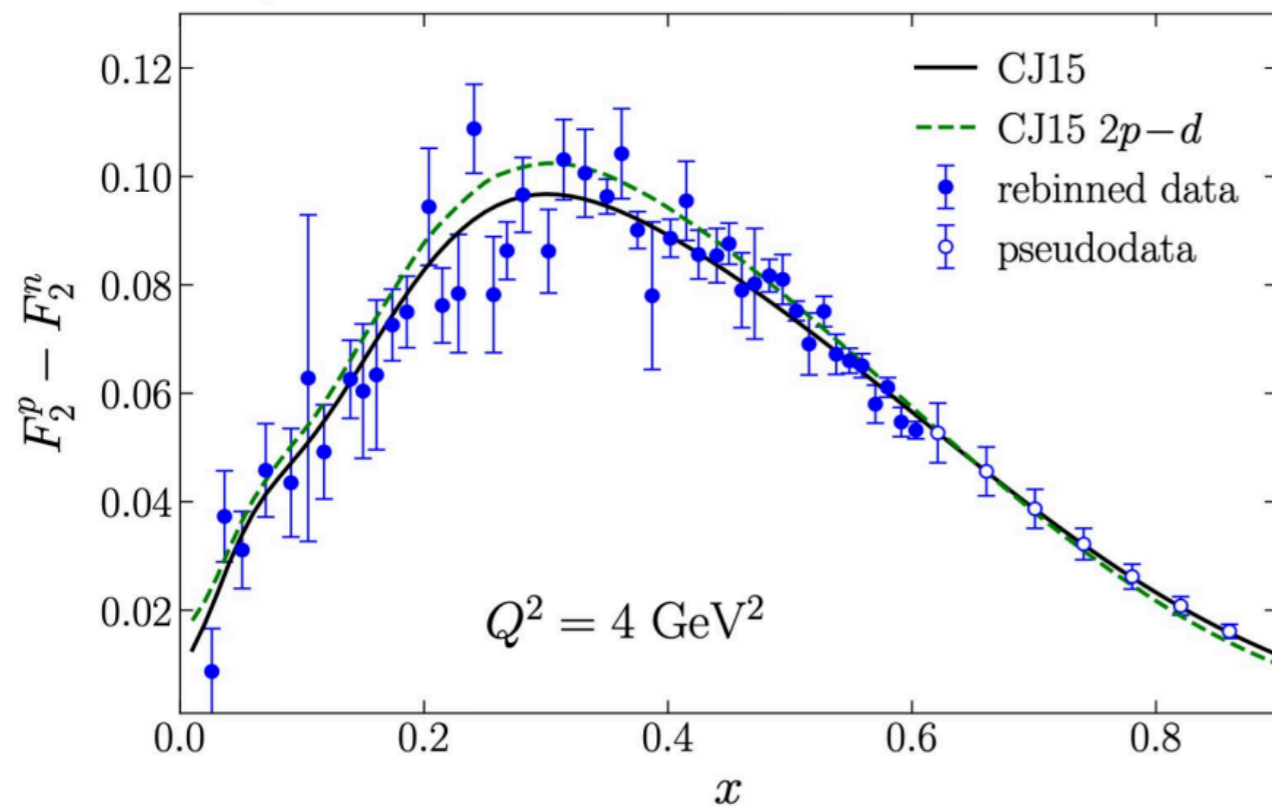


- $x < 0.01$ : Regge theory
- $0.01 < x < 0.6$ : Exp. data
- $x > 0.6$ : CJ15 model

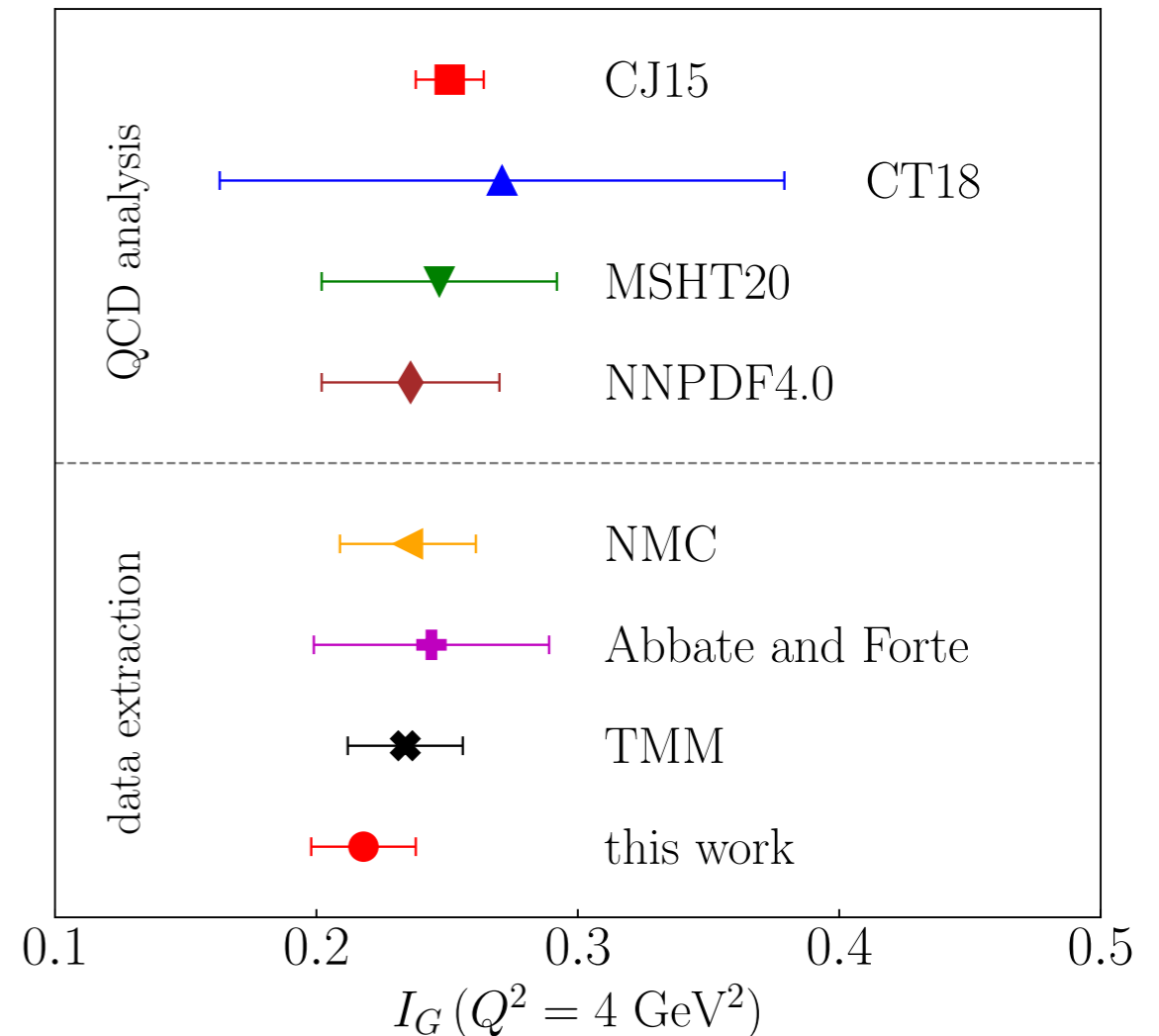
# Application: GSR

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**GOOD AGREEMENT!**

# Application: non-singlet moments

## Nachtmann moment

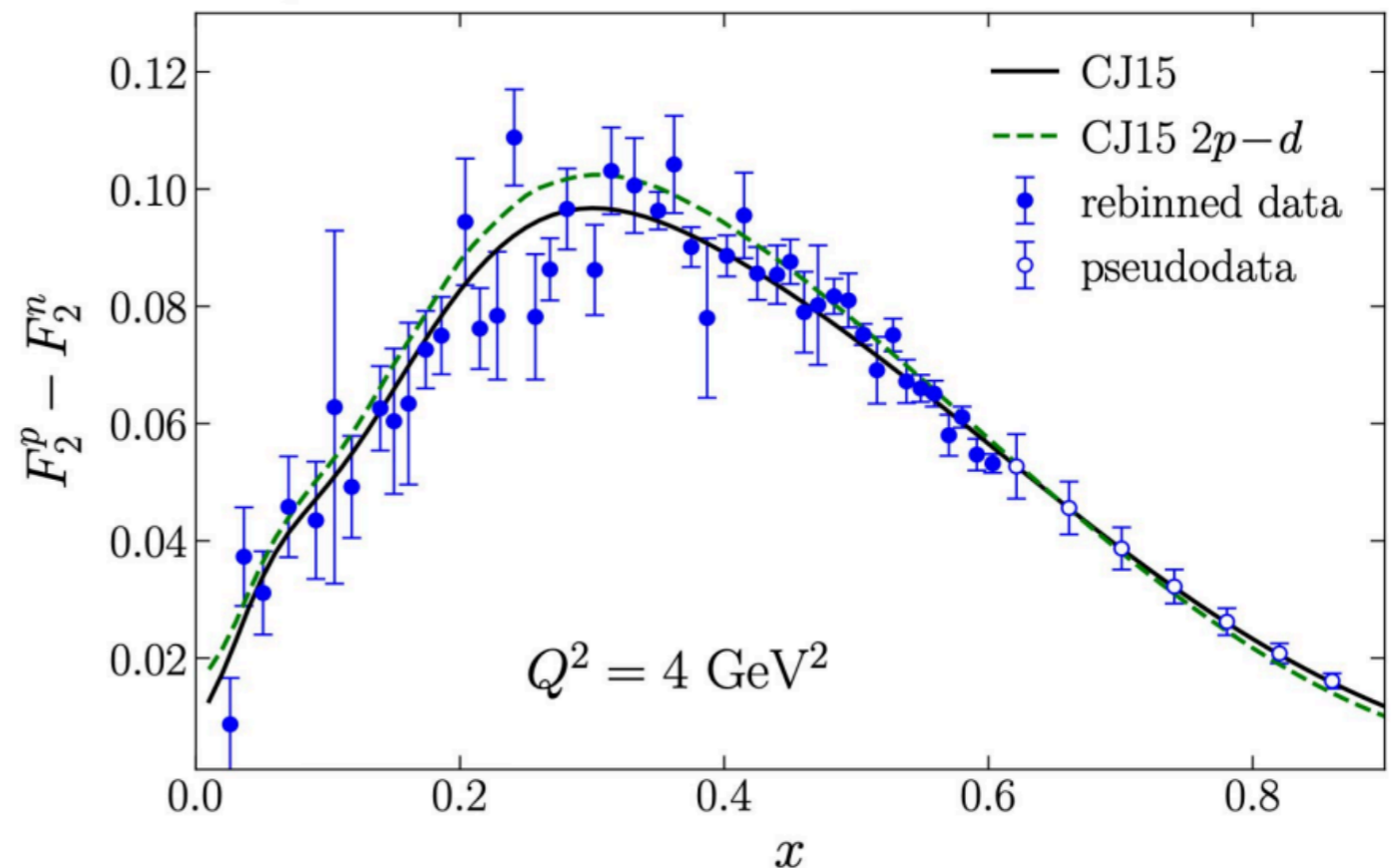
$$M_2^{p-n}(Q^2) = \int_0^1 dx \frac{\xi^3}{x^3} \left[ \frac{3 + 9r + 8r^2}{20} \right] F_2^{p-n}(x, Q^2)$$

$\xi$  = Nachtmann variable

$$r = \sqrt{1 + 4x^2 \frac{M_N^2}{Q^2}}$$

- Accounts for TMC
- Isoscalar  $F_2$  from exp. data
- Relation to the non-singlet moment

$$\frac{3}{C_2} M_2^{p-n} = \langle x \rangle_{u^+ - d^+} + \text{HT}$$

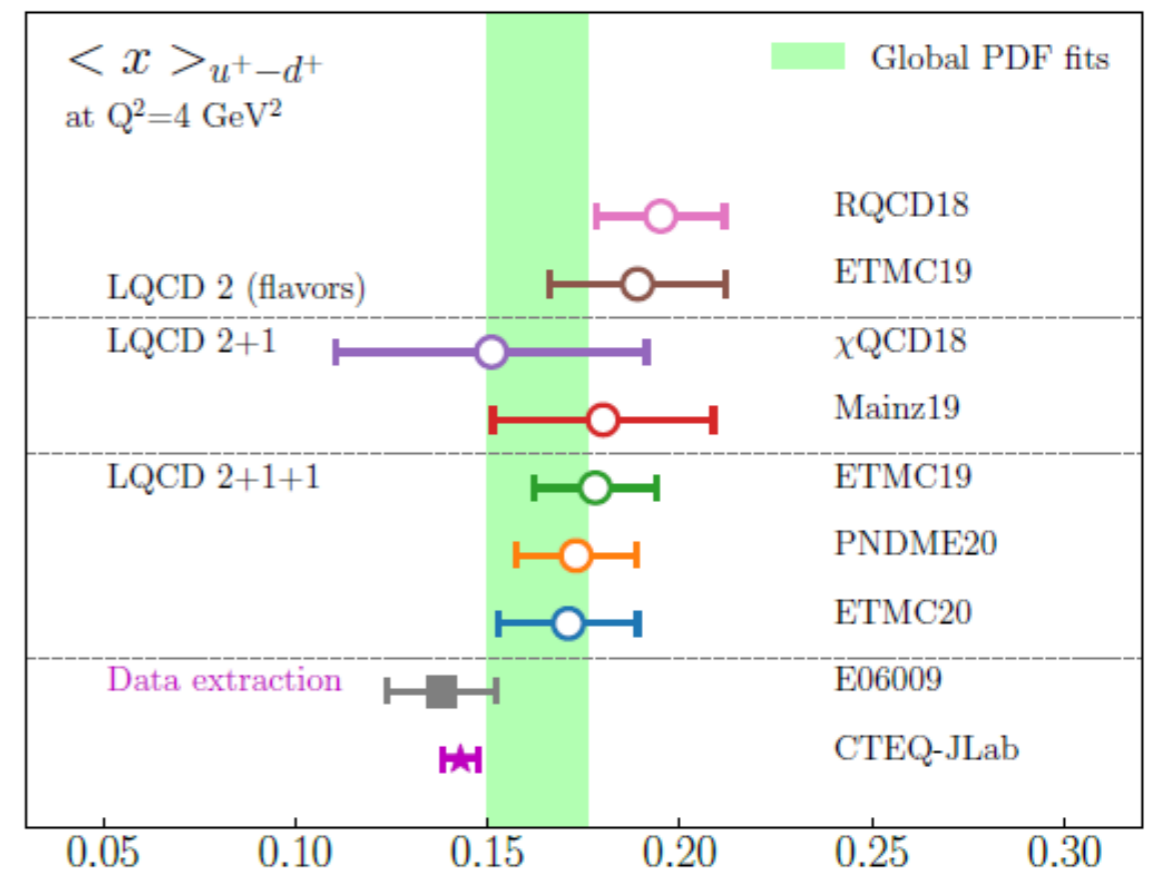
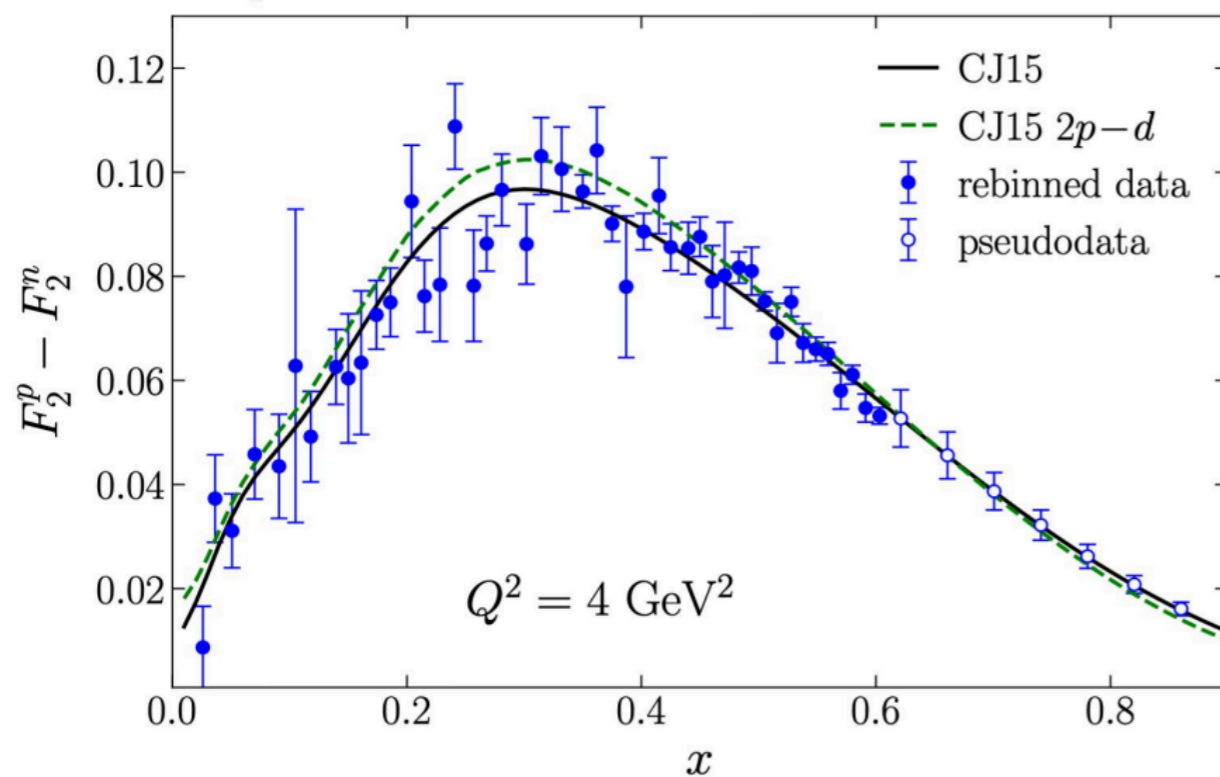




# Application: non-singlet moments

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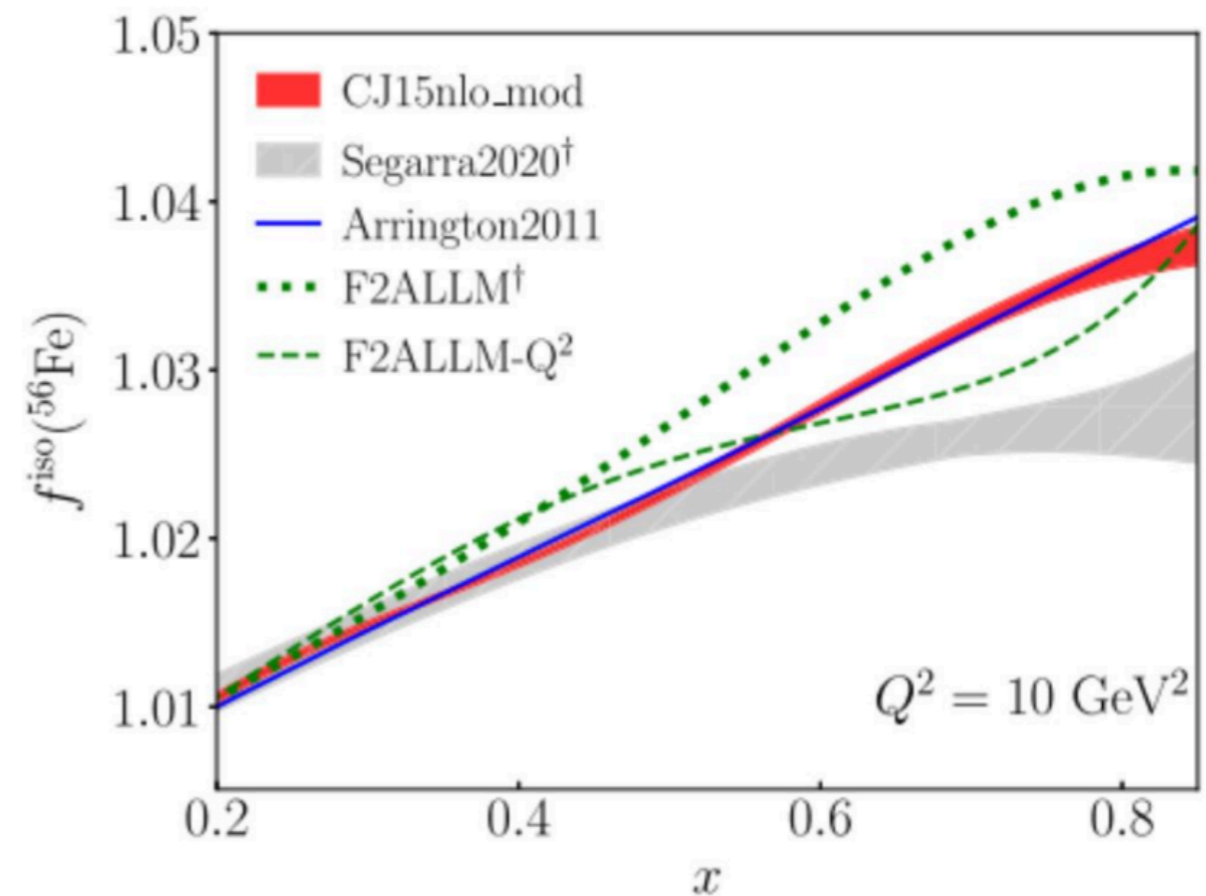
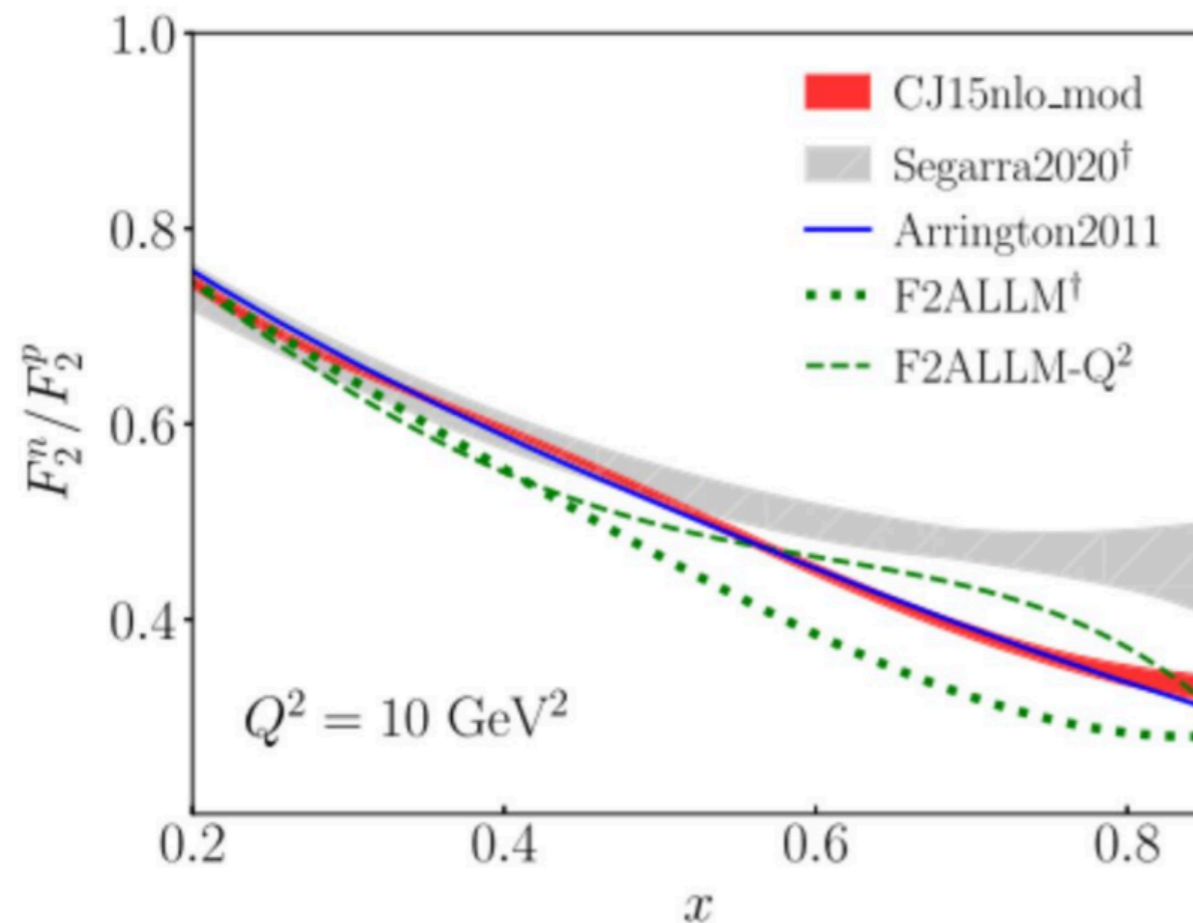
$$\langle x \rangle_{u+-d+} = \int_0^1 dx x [u(x) + \bar{u}(x) - d(x) - \bar{d}(x)]$$

# Application: isoscalar corrections

$$f_A^{iso}(x, Q^2) \simeq \left(\frac{A}{2}\right) \frac{1 + F_2^n / F_2^p}{Z + N F_2^n / F_2^p}$$

**EMC effect**

CJ15nlo\_mod



# Open database on GitHub

<https://github.com/JeffersonLab/CJ-database/>

## CJ Unpolarized DIS Database Homepage [↗](#)

Reference: [arXiv:2309.16851](https://arxiv.org/abs/2309.16851).

See also

- CTEQ-JLab collaboration [website](#).
- [note](#) for reduced cross section and F2 calculation.

## World DIS data tables [↗](#)

World **proton** and **deuteron** data of unpolarized DIS cross sections, F2 structure functions, and the longitudinal to transverse cross section ratio R are collected or extracted from various experiments. Data were collected for the CJ global fit and related analysis. Now open for general use. See details under the [data](#) directory.

## Neutron F2 extraction [↗](#)

Based on the collected F2 data, we performed a data-driven extraction of **neutron F2** and **neutron-to-proton F2n/F2p ratio** within the CJ15 framework (see eq. 7-9 in reference for details). Data from all experiments are cross-normalized and combined into a single Excel file, both in the original kinematics, as well as rebinned in Q<sup>2</sup>. Check the [f2n](#) directory.

## Structure function grids [↗](#)

Within CJ framework, we calculated various structure functions (F2, F3, FL, etc) at given x, Q<sup>2</sup> grids. Results are provided under folder [SFN\\_grids](#) in the [LHAPDF](#) format. An example plotting script is available at `src/plot_sfn.py`

## LHAPDF grids

$F_2, F_L, F_3$

$\gamma, \gamma^Z, Z$   
w/, w/o HT

Experiment	$\sigma_r$	F2	R
SLAC-Whitlow	p: <a href="#">10014</a>	p: <a href="#">10010</a>	p: <a href="#">10064</a>
	d: <a href="#">10015</a>	d: <a href="#">10011</a>	d: <a href="#">10065</a>
	d/p: <a href="#">10034</a>	d/p (*): <a href="#">10034</a>	
SLAC-Whitlow(rebinned)		rebinned p: <a href="#">10012</a>	
		rebinned d: <a href="#">10013</a>	
SLAC-E140			d: <a href="#">10066</a>
SLAC-E140x	p: <a href="#">10037</a>	p: <a href="#">10035</a>	p: <a href="#">10067</a>
	d: <a href="#">10038</a>	d: <a href="#">10036</a>	d: <a href="#">10068</a>
NMC	p: <a href="#">10022</a>	p: <a href="#">10020</a>	
	d: <a href="#">10040</a>	d: <a href="#">10039</a>	
	d/p: <a href="#">10021</a>	d/p (*): <a href="#">10021</a>	
BCDMS	p: <a href="#">10018</a>	p: <a href="#">10016</a>	p: <a href="#">10069</a>
	d: <a href="#">10019</a>	d: <a href="#">10017</a>	d: <a href="#">10070</a>
JLab E06-009 (includes E04-001, E02-109)	d: <a href="#">10042</a>	d: <a href="#">10041</a>	d: <a href="#">10071</a>
JLab E94-110	p: <a href="#">10044</a>	p: <a href="#">10043</a>	p: <a href="#">10074</a>
<a href="#">JLab E03-103</a>	p: 10047	p: 10045	
	d: 10048	d: 10046	
JLab E99-118	p: <a href="#">10052</a>	p: <a href="#">10049</a>	p: (A)
	d: <a href="#">10053</a>	d: <a href="#">10050</a>	p-d: (A)
	d/p: <a href="#">10054</a>	d/p: <a href="#">10051</a>	
JLab JLCCE96	p: <a href="#">10055</a>	p: <a href="#">10072</a>	
	d: <a href="#">10056</a>	d: <a href="#">10073</a>	
<a href="#">JLab E00-116</a>	p: 10003	p: 10001	
	d: 10004	p: 10002	
CLAS6	p: <a href="#">10059</a>	p: <a href="#">10057</a>	
	d: <a href="#">10060</a>	d: <a href="#">10058</a>	
BONUS		n: <a href="#">10061</a>	
		n/d: <a href="#">10033</a>	
HERA I+II <a href="#">HERMES</a>	p: <a href="#">10026 - 10032</a>		
	p: 10007	p: 10005	
	d: 10008	d: 10006	
E665	d/p: 10009		
		p: <a href="#">10062</a>	
		d: <a href="#">10063</a>	

# Conclusions and Outlook

- We provided a **data-driven extraction of F2n** from  $d$  and  $p$  data, with our best knowledge of HT and nuclear effects

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- **F2n grids** are available also in **LHAPDF format**
- The F2n data set has been used for different applications



Backup

# Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{F_{2,n}}{F_{2,p}} = \frac{n}{p} \xrightarrow{x \rightarrow 1} \frac{4d + u}{4u + d} \simeq \frac{1}{4}$$

**Mult HT**

$$C_p(x) = C_n(x) = C(x)$$

$$\frac{(4d + u)(1 + C/Q^2)}{(4u + d)(1 + C/Q^2)} \simeq \frac{1}{4}$$

**Add HT**

$$H_p(x) = H_n(x) = H(x)$$

$$\frac{4d + u + H/Q^2}{4u + d + H/Q^2} \simeq \frac{u + H/Q^2}{4u + H/Q^2}$$

$$\text{expansion in } \frac{H}{uQ^2} \simeq \frac{1}{4} + \boxed{3 \frac{H}{16uQ^2}} + p.s$$

**Bias in n/p function**

# Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{n}{p} \xrightarrow{x \rightarrow 1} \frac{1}{4} \quad \text{LT} \quad \text{Mult HT} \quad C_p(x) = C_n(x) = C(x)$$

**Add HT**

$$\boxed{H_p(x) \neq H_n(x)}$$

$$\frac{u + H_n/Q^2}{4u + H_p/Q^2}$$

$$\simeq \frac{1}{4} + \frac{4H_n - H_p}{16uQ^2} + p.s$$

$$\begin{array}{l} \nearrow H_p(x) = H_n(x) \\ \longrightarrow H_p(x) = 2H_n(x) \end{array}$$

$$\frac{1}{4} + 3 \frac{H}{16uQ^2}$$

$$\frac{1}{4} + \frac{H}{16uQ^2}$$

structure function  
is smaller

**Mult HT**

$$\boxed{C_p(x) \neq C_n(x)}$$

$$\frac{u + \tilde{H}_n/Q^2}{4u + \tilde{H}_p/Q^2}$$

same as Add

**Bias not present!**

# Results in the CJ fitting framework

## Case 1: isospin symmetry

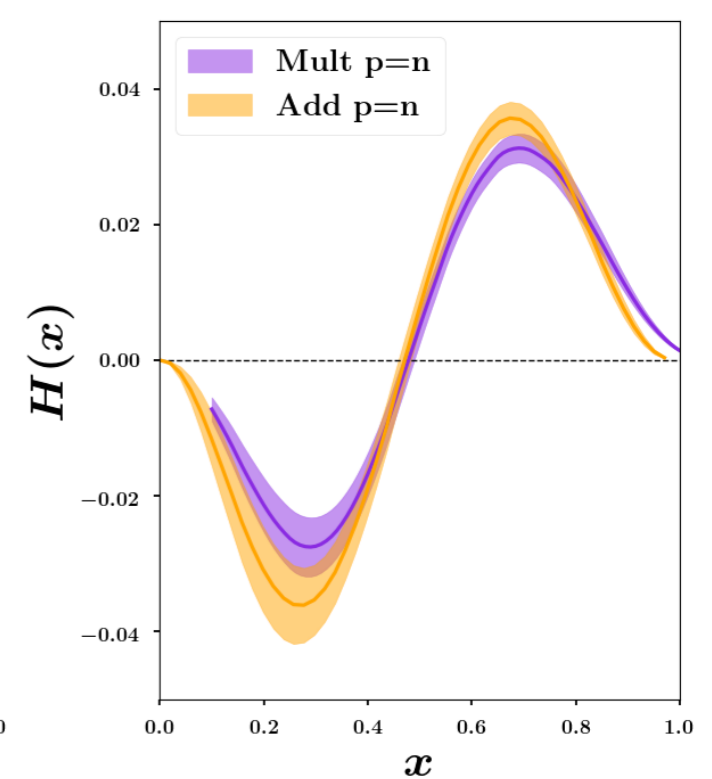
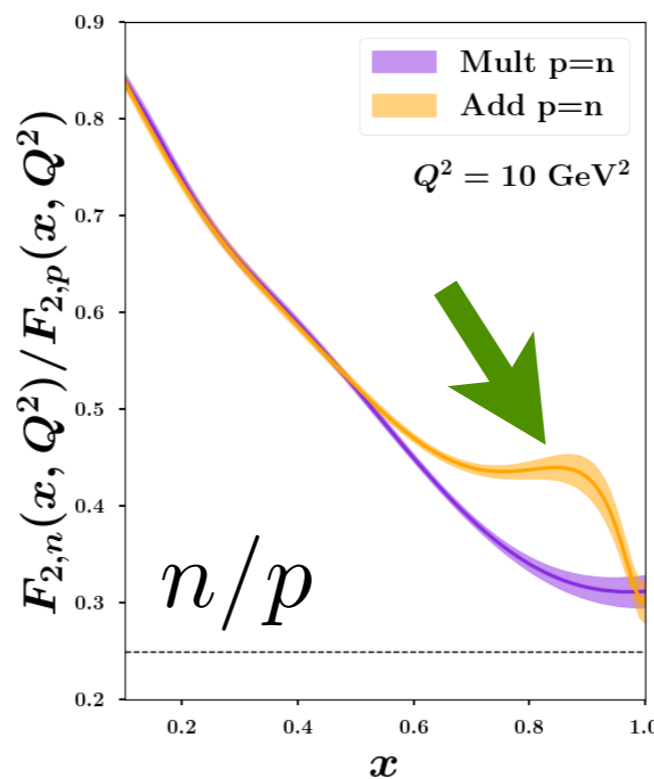
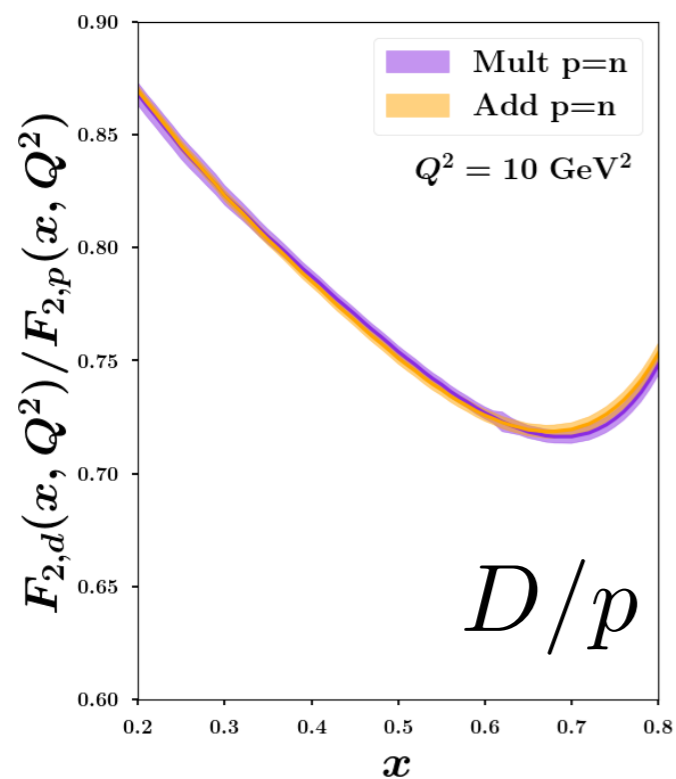
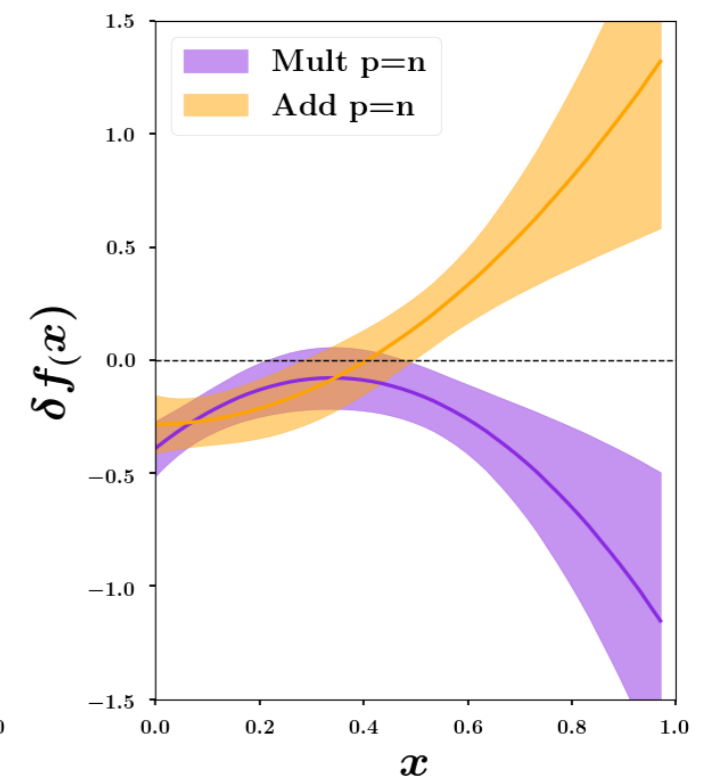
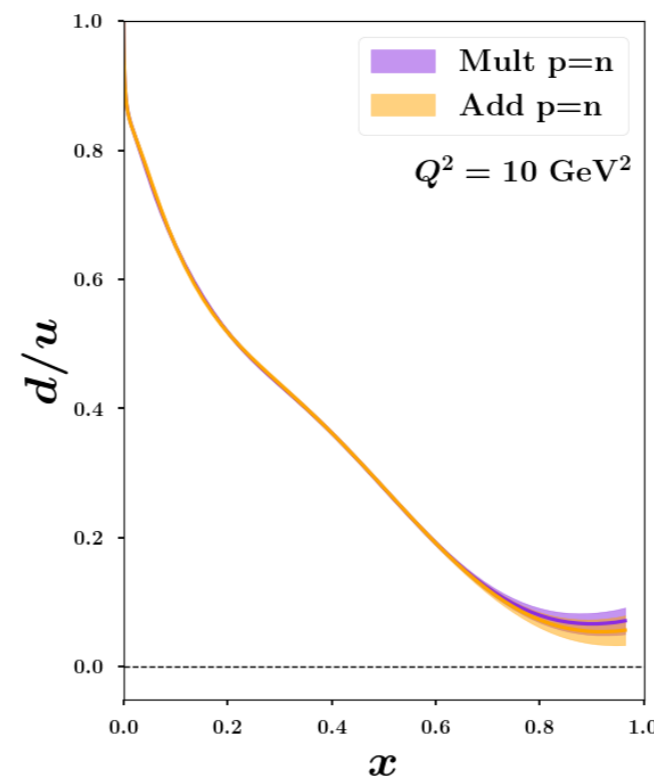
Add HT

Unnaturally large  $n/p$

BUT smaller  $d/u$  than Mult

**Bias identified**

Off-shell compensates  $n/p$  bias



# Results in the CJ fitting framework

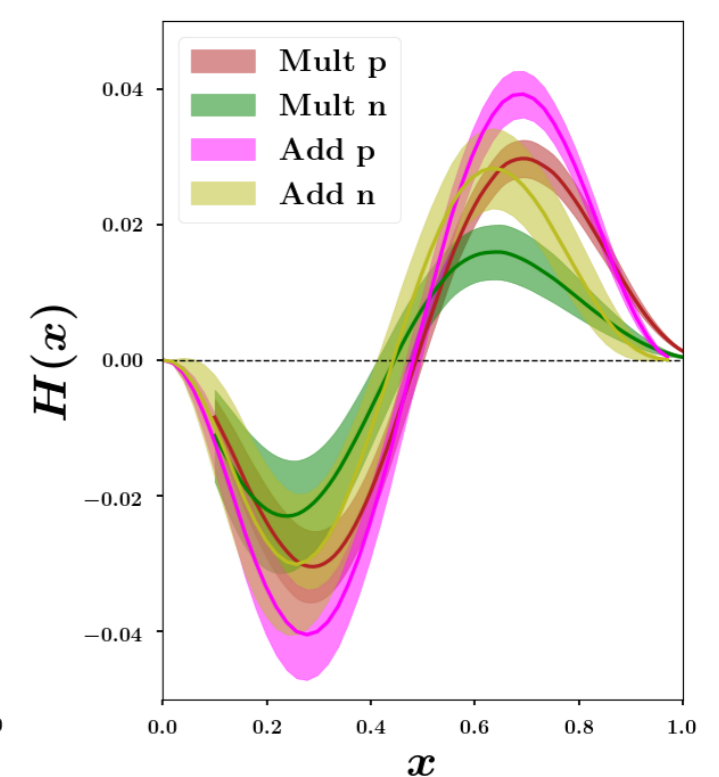
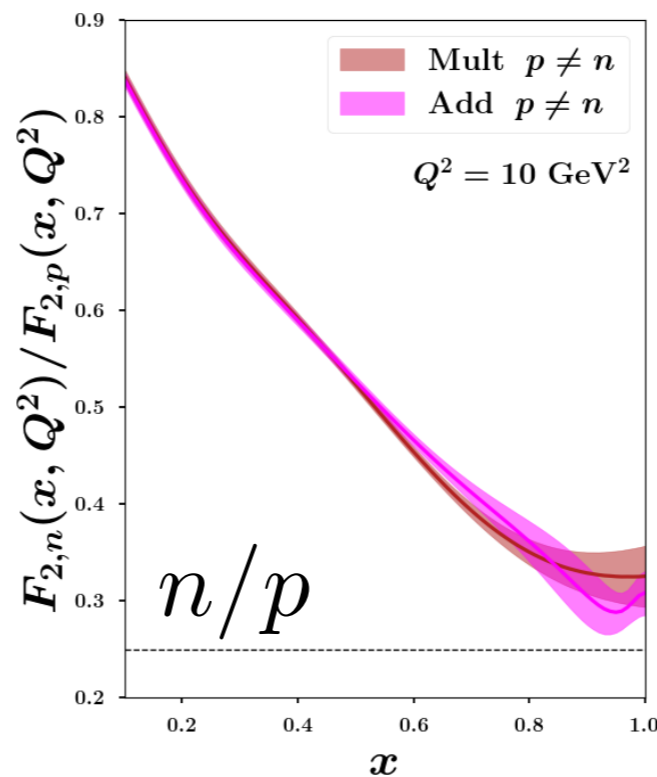
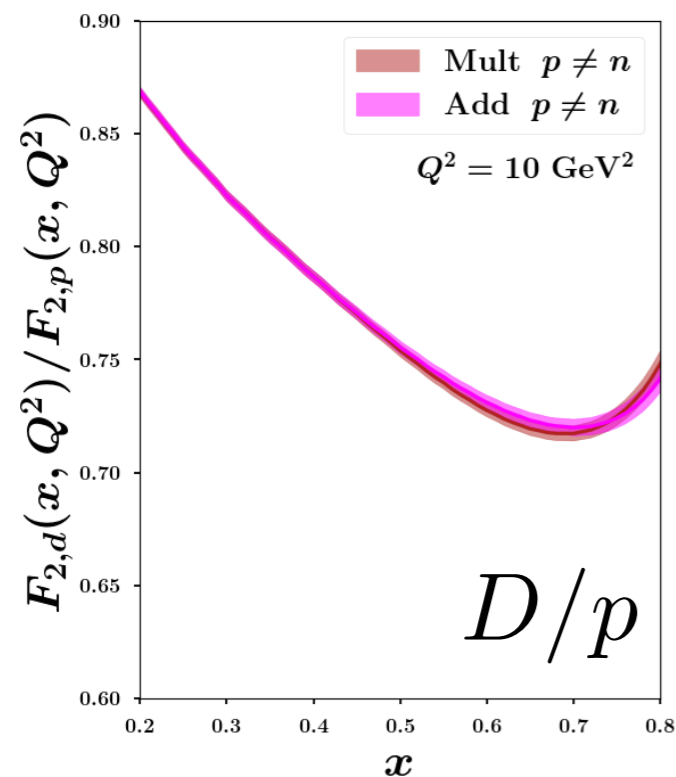
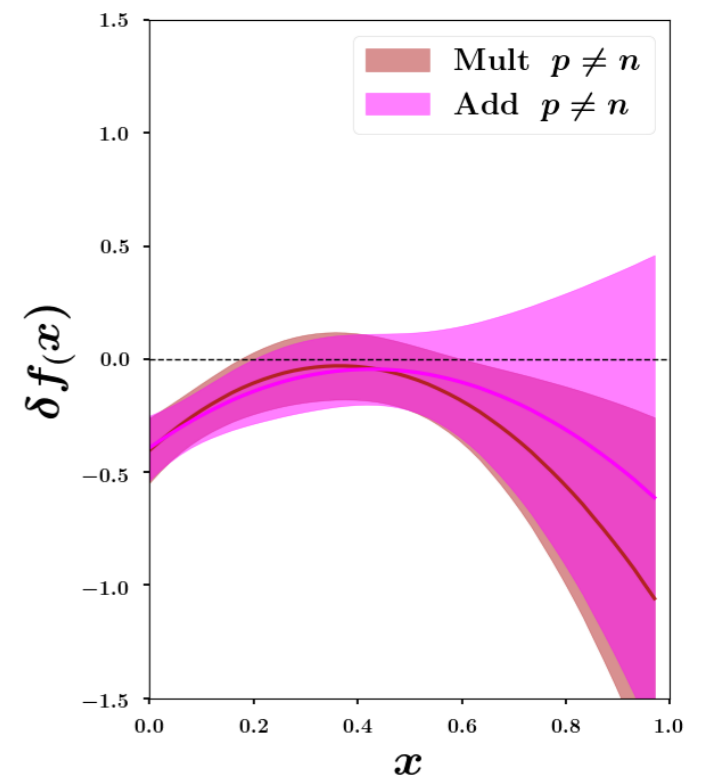
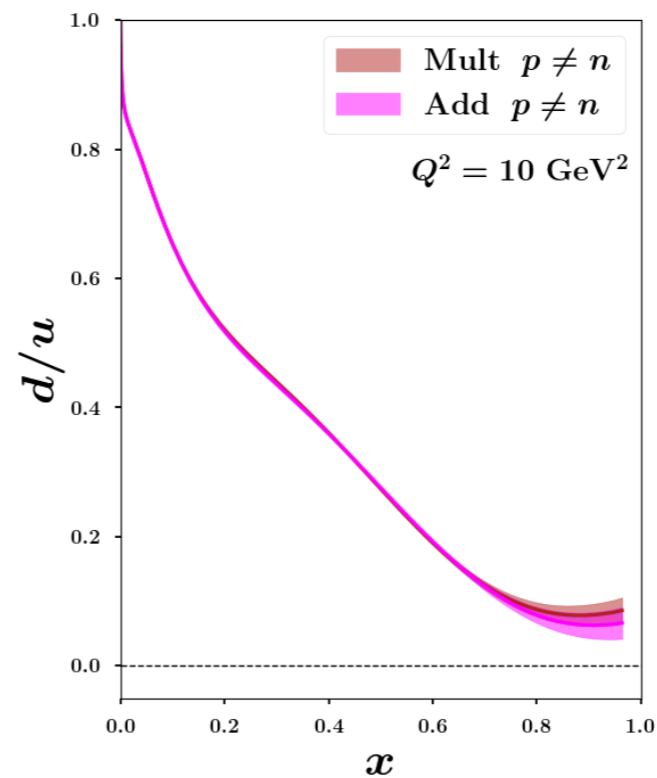
## Case 2: isospin breaking

Compatible  $n/p$

$$H_n(x) \simeq \frac{1}{2}H_p(x)$$

## Bias removed

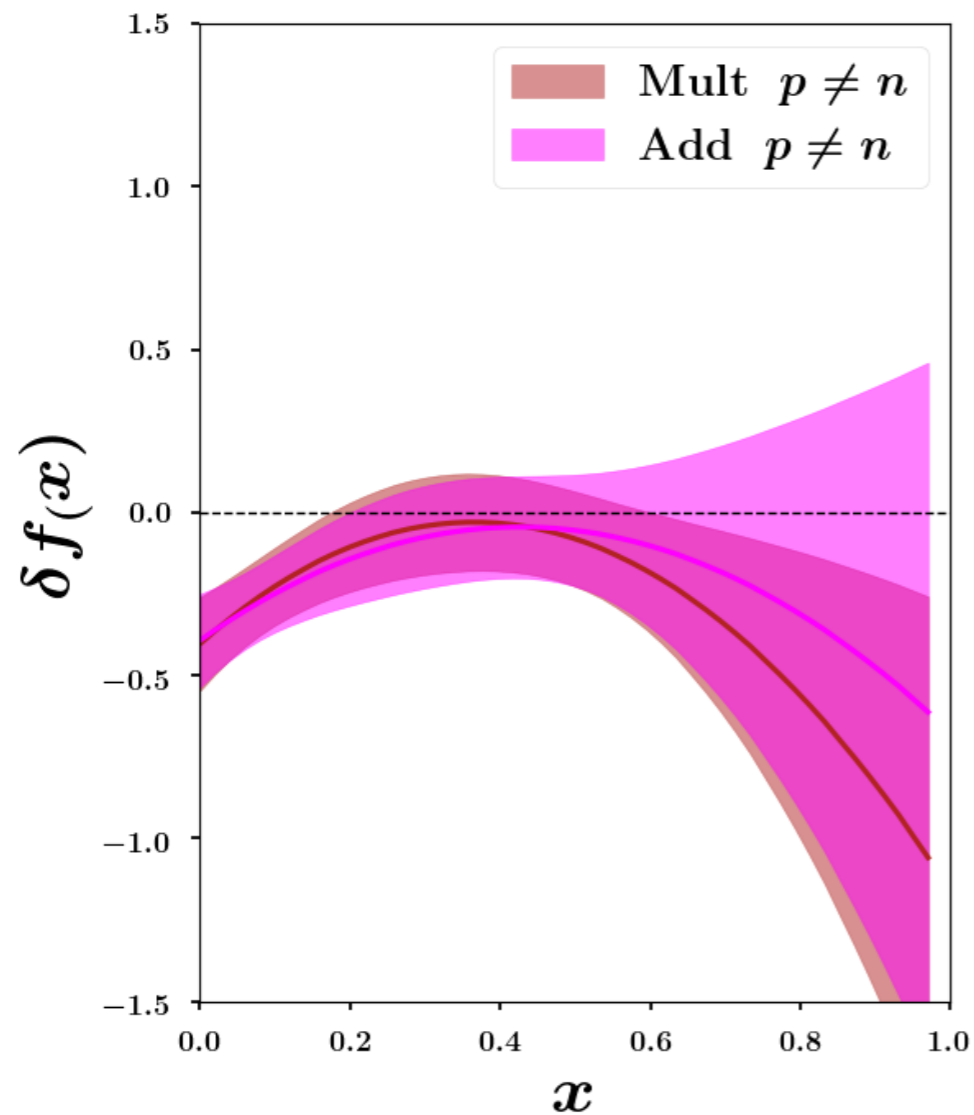
No need of compensation by off-shell  
Theory calculation confirmed!



# Results in the CJ fitting framework

**After removing the bias**

$$\delta f(x) \simeq 0$$



Is the nucleon inside the deuterium  
almost on-shell?

Need  $A=3$  data to assess flavour  
dependence of off-shell function

MARATHON data  
Adams, et al., PRL 128 (2022)

# Other extractions of the off-shell correction

**AKP**

Alekhin, Kulagin, Petti, PRD 107 (2023)

No significant differences seen between HT Add and Mult

DISCLAIMER: off-shell function parametrized at the structure function level ( $\delta F$ )  
and many other differences in the implementation

Fit to  $A=3$  data:  $\delta F_p(x) \simeq \delta F_n(x)$  (baseline)

**JAM**

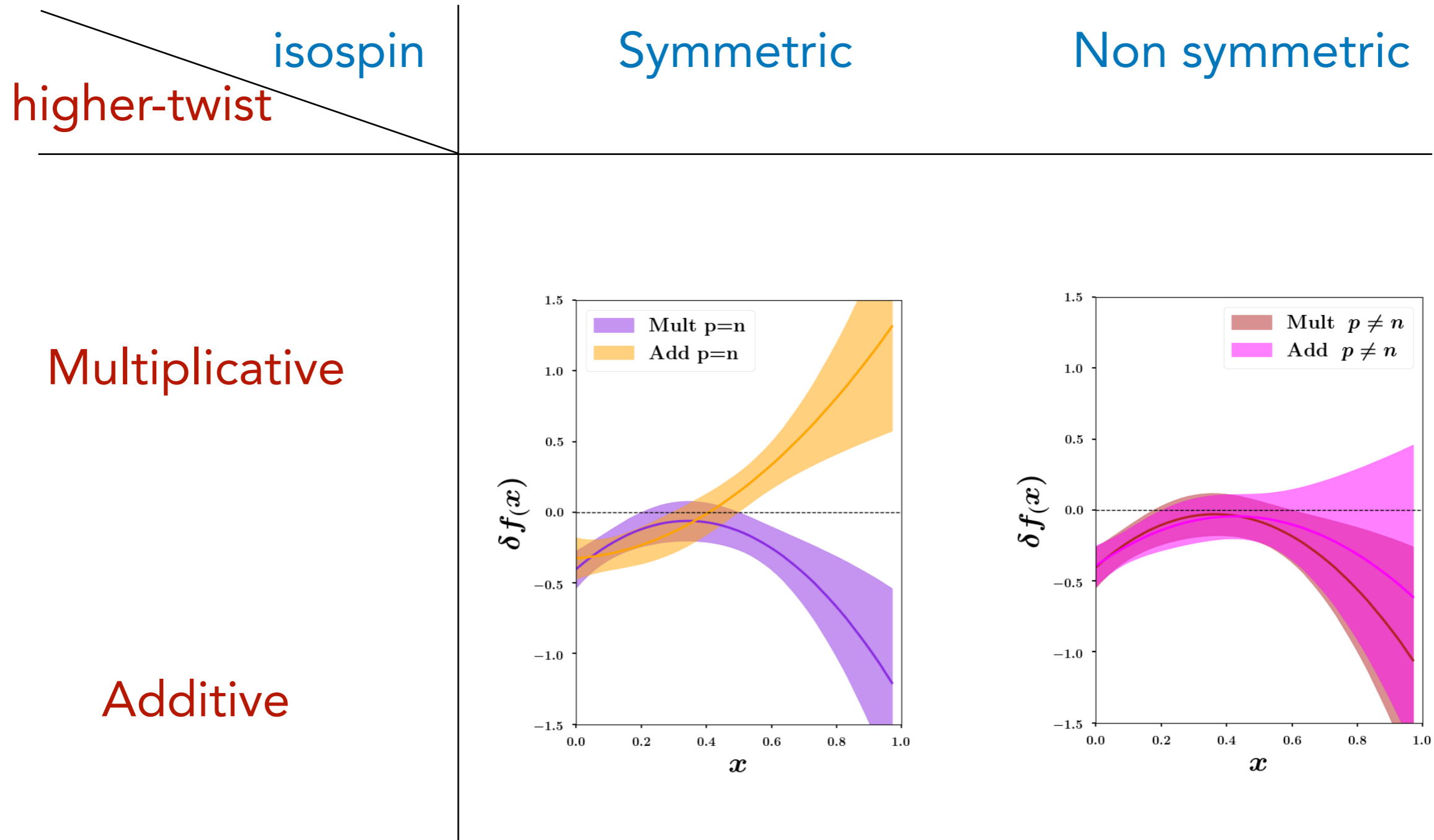
JAM Collaboration, PRL 127 (2021)

Only multiplicative HT considered

DISCLAIMER: off-shell function parametrized at the pdf level ( $\delta f$ )  
but many differences in the implementation

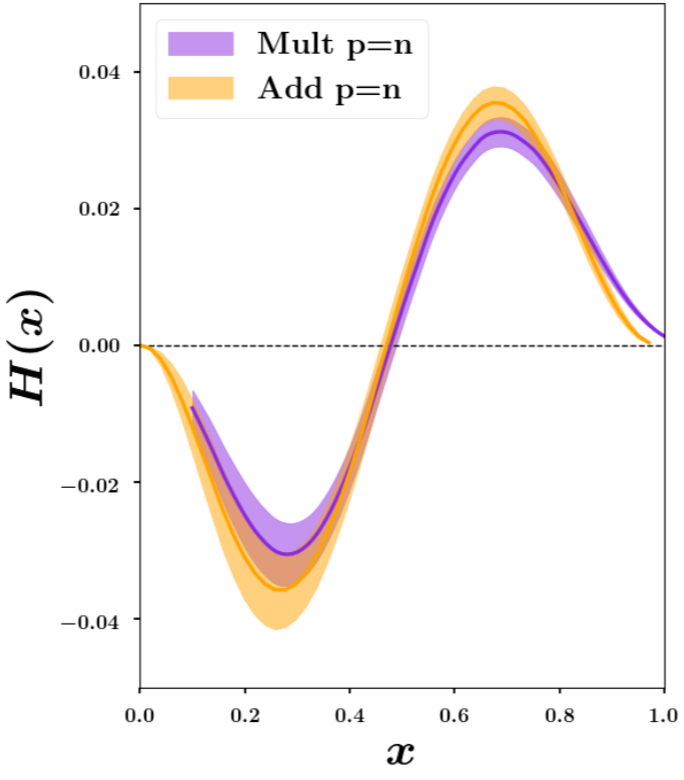
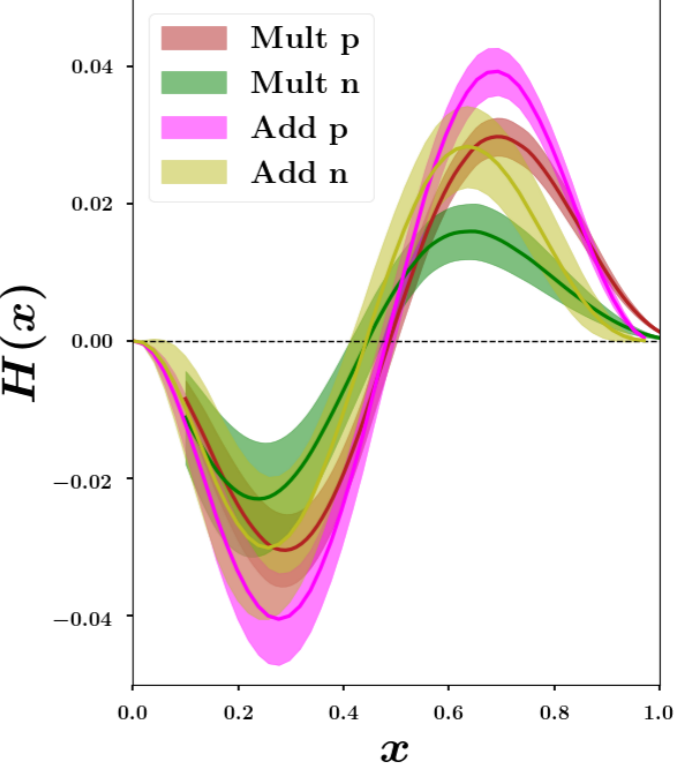
Fit to  $A=3$  data:  $\delta f_u(x) \neq \delta f_d(x)$

# Off-shell table



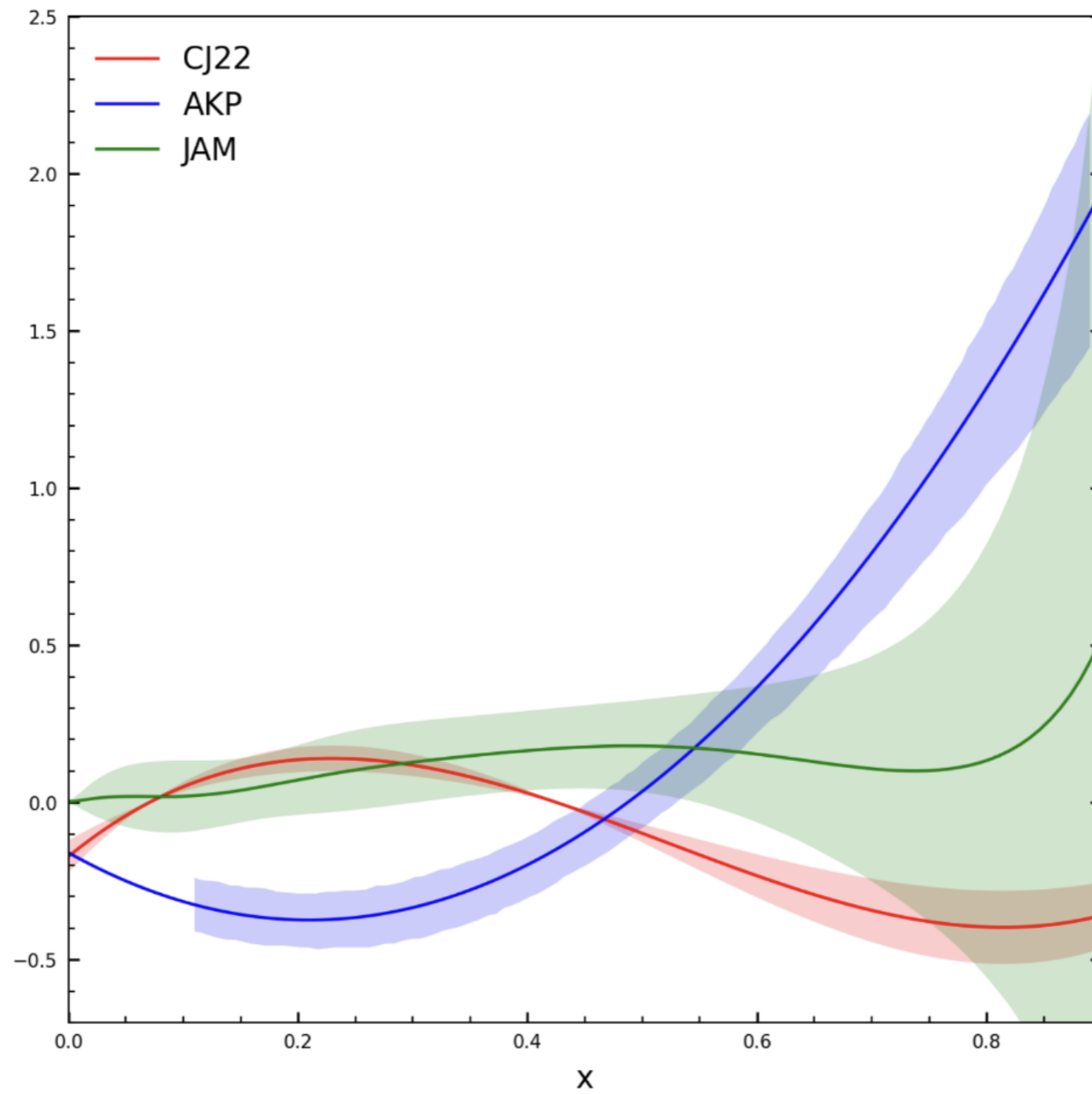


# Higher-Twist table

	isospin	Symmetric	Non symmetric
higher-twist			
Multiplicative		$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$ 	$\delta\tilde{H} = F_{2,N}(x, Q^2) \delta H_{\text{Mult}}(x)$ 
Additive			

# AKP vs CJ

$\delta f$



# AKP results

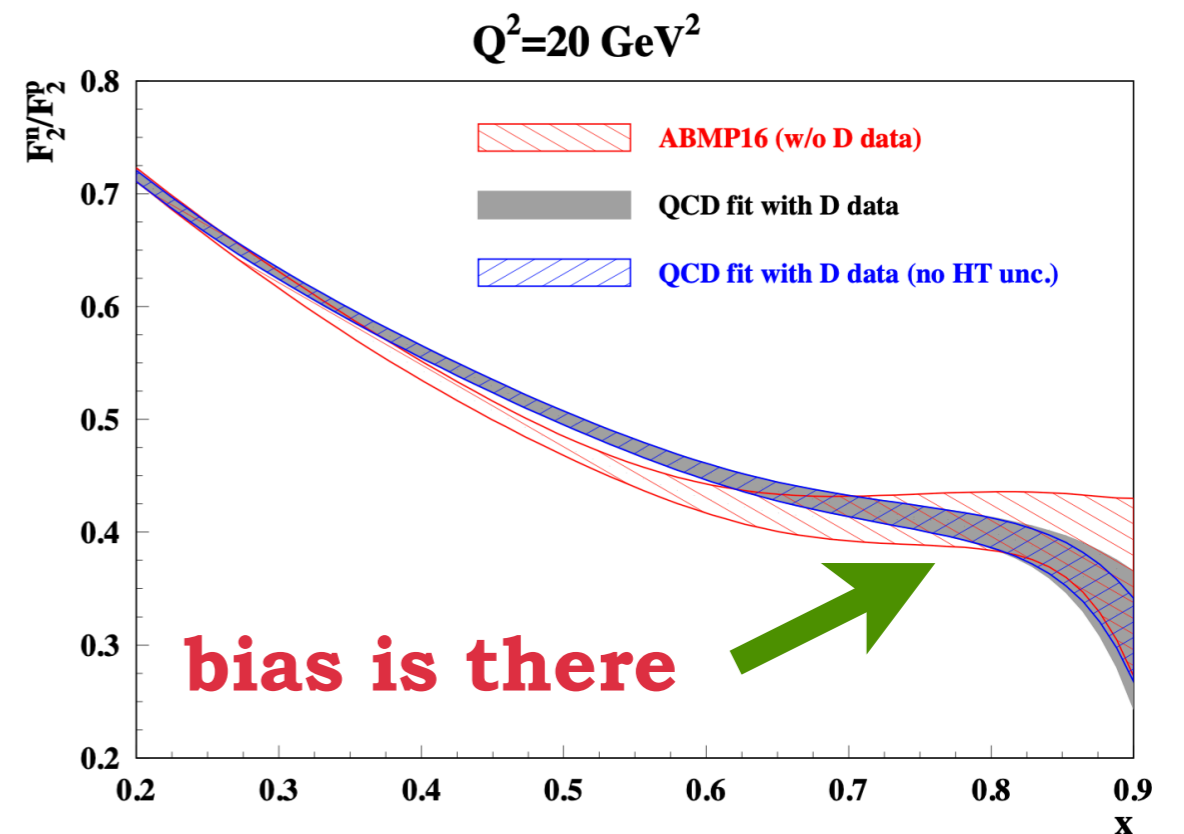
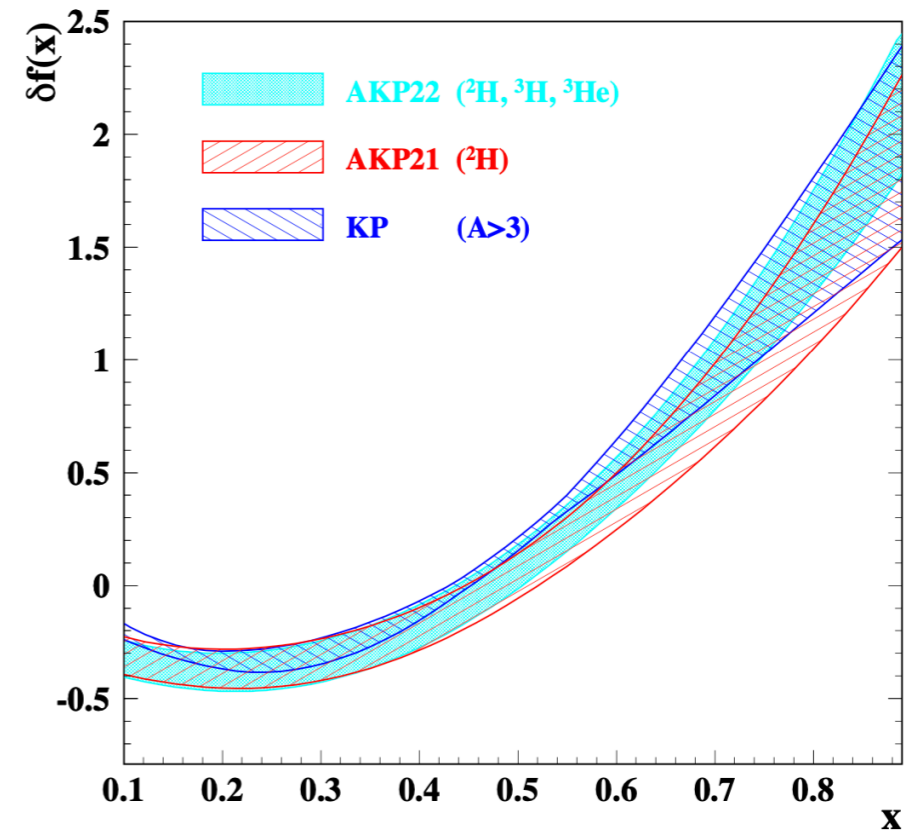
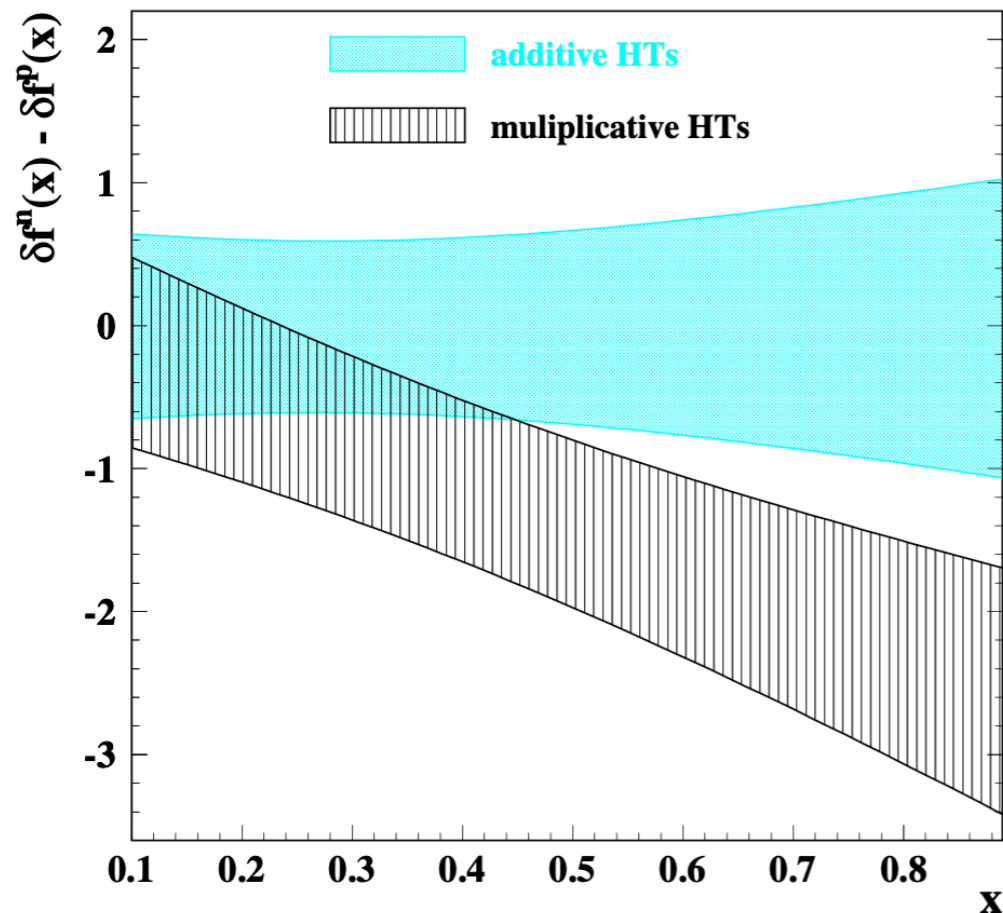
**AKP**

Alekhin, Kulagin, Petti, PRD 107 (2023)

Add HT ( $p=n$ ) as baseline choice

$H_2, H_T$  parametrized

Fit to  $A=3$  data  $\delta F_p \delta F_n$

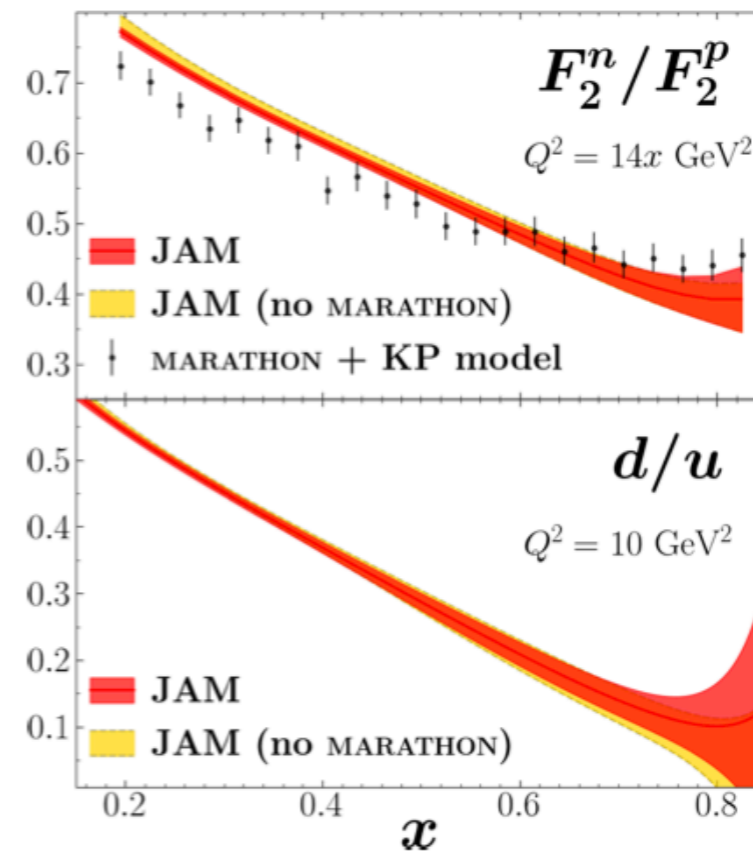
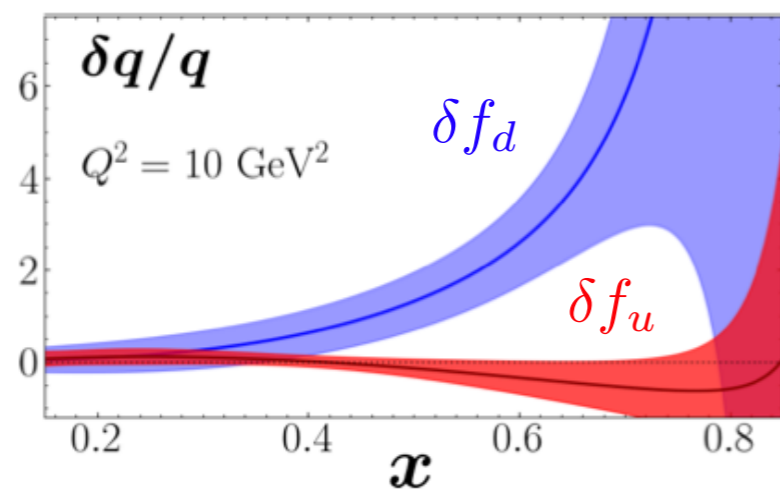


# JAM results

**JAM** Fit including  $A=3$  data  $\delta f_u$   $\delta f_d$

JAM Collaboration, PRL 127 (2021)

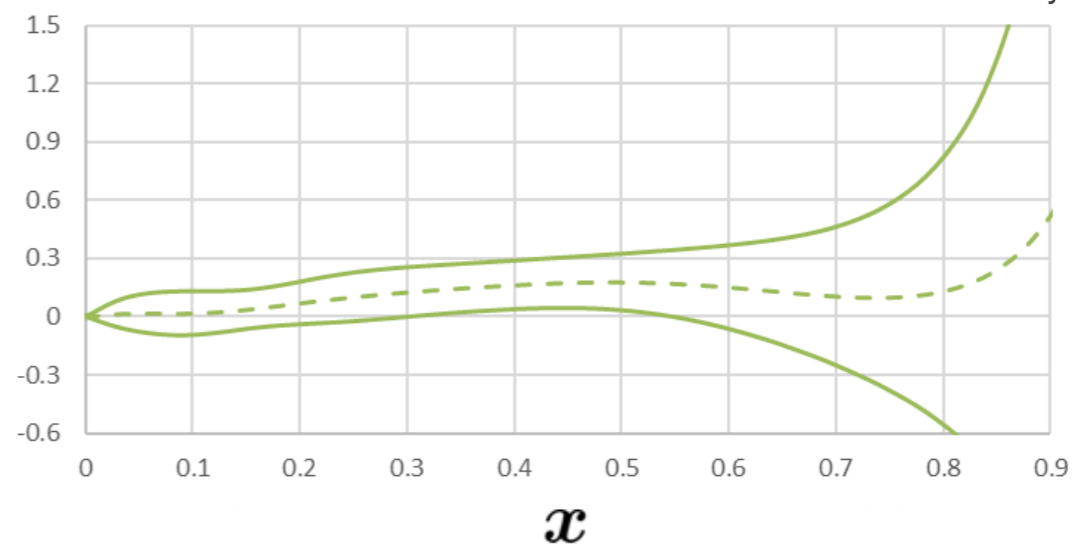
Mult HT (p=n) as default choice



Isoscalar offshell function (JAM)

Courtesy of C. Cocuzza

$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u + d}$$



# Some implementation differences

Theoretical choices  $\longrightarrow$

Corrections (increasing-x)  $\downarrow$

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST $x < 0.1$	(same)
smearing	Paris	AV18	AV18 $x > 0.1$	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----