Nucleon structure from Lattice QCD at nearly physical quark masses

Gunnar Bali for RQCD

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Outline

- Importance of proton structure beyond QCD
- Lattice QCD set-up
- Mass: σ-terms
- Spin: The Δq's and g_A
- Other couplings
- Momentum fraction: $\langle x \rangle_{u-d}$
- Summary

Protons in use e.g. at the LHC

What is known about parton distribution functions?

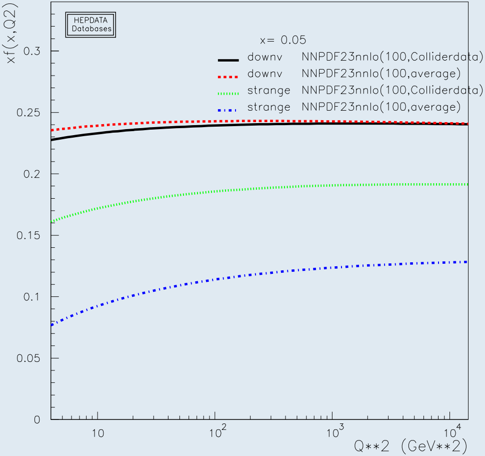
The *u* and *d* PDFs are well-known from experiment, e.g., at DESY.
Strangeness and gluonic PDFs have much larger uncertainties.

Generated using

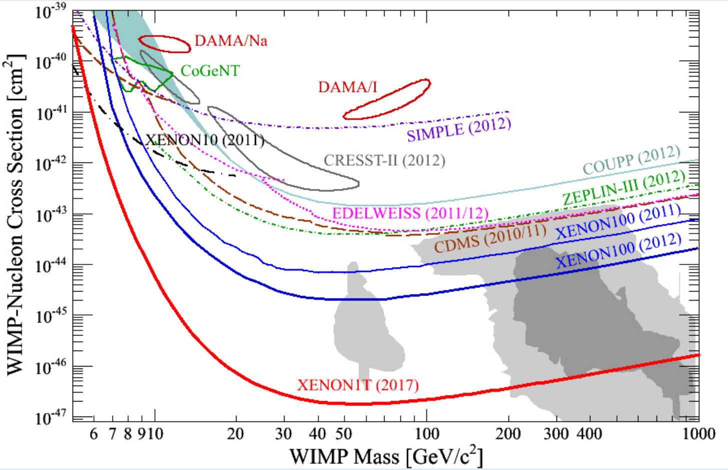
`http://hepdata.cedar.ac.uk/pdfs`

from the NNPDF2.3 data set.

NNPDF: R D Ball et al,
NPB 867 (13) 244



Nucleons as dark matter probes: XENON1T at Gran Sasso



y-scale of shaded areas depends on scalar couplings $m_q\langle N|\bar{q}q|N\rangle$.

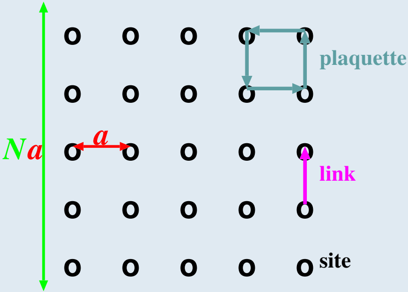
Proton structure calculations are...

- ... essential to constrain beyond-the-Standard-Model (BSM) dark matter candidates, relating predictions to experimental limits.
- ... important to predict cross-sections for processes on the quark-gluon level. Experiment e.g. unable to directly measure strangeness and gluon PDFs.
- ... needed to relate QCD to low energy effective theories that are also relevant for precision experiments.

Here I concentrate on

- How is the mass distributed among the partons? (scalar couplings)
- How is the spin distributed? (axial couplings)
- Proton-neutron transition couplings. (g_S, g_T, $\tilde{g}_T$, g_P, g_P^*)
- How is the momentum distributed? (moments of PDFs)

Lattice QCD



typical values:

$a^{-1} = 2$–5 GeV, $Na = 2$–7 fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a representative ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: discretized $\mathscr{L}_{QCD} = \frac{1}{16\pi\alpha_L(a)} FF + \bar{q}_f(\not{D} + m_f(a))q_f$

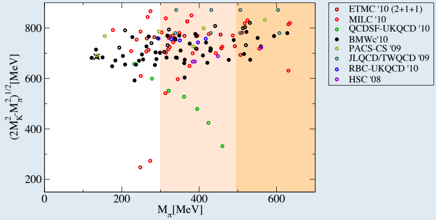
$$
\begin{aligned}
m_N^{\text{latt}} = m_N^{\text{phys}} &\longrightarrow a \\
m_\pi^{\text{latt}}/m_N^{\text{latt}} = m_\pi^{\text{phys}}/m_N^{\text{phys}} &\longrightarrow m_u(a) \approx m_d(a) \\
\cdots
\end{aligned}
$$

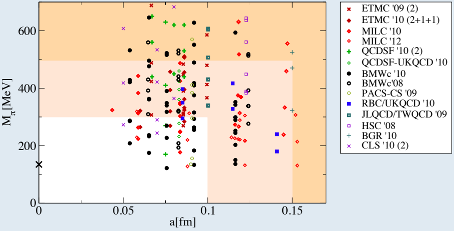
Output: hadron masses, matrix elements, decay constants, etc...

Required:

1. $L = Na \to \infty$: FSE suppressed with $\exp(-Lm_\pi) \Rightarrow Lm_\pi \gtrsim 4$.
2. $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) helps for m_{ud} but m_{ud}^{latt} must be sufficiently small to start with ($m_\pi \lesssim 200\,\text{MeV}$?).
3. $a \to 0$: functional form known: $\mathcal{O}(a^2), \mathcal{O}(\alpha_s a) \Rightarrow \approx 4$ lattice spacings.

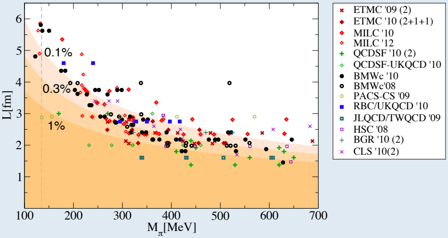
Landscape of recent lattice simulations





Figures taken from

C Hoelbling, arXiv:1410.3403



Computational challenges

Cost of simulation is proportional to

- number of points: $(L/a)^4$
- condition number of linear system: $1/m_\pi^2$
- $L^{1/2}/m_\pi$ in (Omelyan) time integration within hybrid Monte Carlo
- $1/a^{\geq 2}$ critical slowing down (autocorrelations)

Adjusting $L \propto 1/m_\pi$ this means:

$$\text{cost} \propto \frac{1}{a^{\geq 6}\, m_\pi^{7.5}}$$

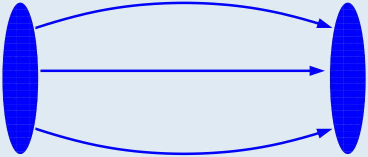
In addition: for baryonic observables at small m_π serious signal/noise problem.

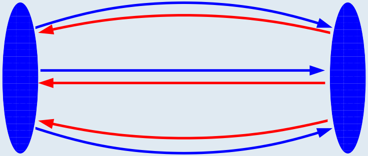
State of the art: $64^3 \times 128$ sites, corresponding to $\approx (4 \times 10^9)^2$ (sparse) complex matrices.

Tremendous progress in Hybrid Monte Carlo, solver, noise reduction.

HW Hamber, E Marinari, G Parisi, C Rebbi, NPB225 (83) 475 (Appendix B)

GP Lepage 89, http://inspirehep.net/record/287173

$C_N(t) \sim$  $\sim \exp(-m_N t)$

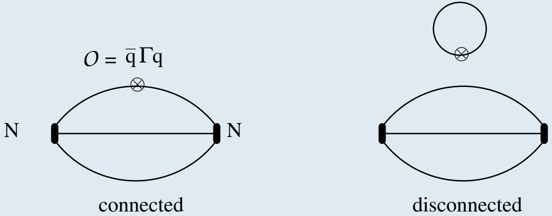
$[\Delta C_N(t)]^2 \sim$  $\sim \exp(-3 m_\pi t)$

$$\frac{\Delta C_N(t)}{C_N(t)} \sim \exp\left[\left(m_N - \frac{3}{2} m_\pi\right) t\right]$$

"Self-averaging" over many source points increases statistics.
Becomes increasingly important towards small m_π.

Three point functions

Evaluate $\langle N|\bar{q}\Gamma q|N\rangle$ (Lines: quark "propagators" M_{xy}^{-1}, $M = \not{D} + m_q$)



$q \in \{u, d\}$: both quark-line connected and disconnected terms.

$q = s$: only the disconnected term.

"Connected" requires only 12 rows (spin $\times$ colour) of M^{-1}.

"Disconnected" $12N^3$ rows (timeslice): stochastic "all-to-all" methods.

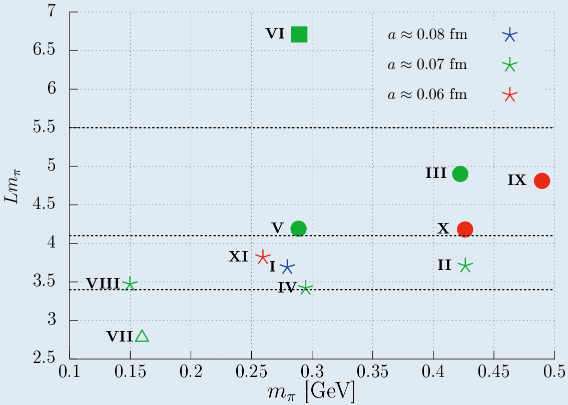
"Disconnected" cancels ($m_u = m_d$, $\not{Q}\not{E}\not{D}$) from isovector combinations: "proton minus neutron", i.e. $\langle p|(\bar{u}\Gamma u - \bar{d}\Gamma d)|p\rangle = \langle p|\bar{u}\Gamma d|n\rangle$.

Action and ensembles

- $N_f = 2$ NP improved Sheikholeslami-Wilson fermions, Wilson glue.
- Lm_π up to 6.7, a down to 0.06 fm, m_π down to 150 MeV.
- Two lattice spacings around $m_\pi \approx 290$ MeV, three around 425 MeV.
- 300–600 Wuppertal=Gauss smearing iterations on top of APE smearing.

#	β	a/fm	κ	V	m_π/MeV	Lm_π	$n_{\rm conf}$	$t_{\rm sink}/a$
I	5.20	0.081	0.13596	$32^3 \times 64$	280	3.69	1986(4)	13
II	5.29	0.071	0.13620	$24^3 \times 48$	426	3.71	1999(2)	15
III			0.13620	$32^3 \times 64$	423	4.90	1998(2)	15,17
IV			0.13632	$32^3 \times 64$	295	3.42	2023(2)	7,9,11,13,15,17
V				$40^3 \times 64$	289	4.19	2025(2)	15
VI				$64^3 \times 64$	289	6.71	1232(2)	15
VII			0.13640	$48^3 \times 64$	160	2.78	3442(2)	15
VIII				$64^3 \times 64$	150	3.47	1593(3)	9,12,15
IX	5.40	0.060	0.13640	$32^3 \times 64$	490	4.81	1123(2)	17
X			0.13647	$32^3 \times 64$	426	4.18	1999(2)	17
XI			0.13660	$48^3 \times 64$	259	3.82	2177(2)	17

Ensembles II



$Lm_\pi \approx 6.7$: ■

$Lm_\pi > 4.1$: ● ●

$Lm_\pi > 3.4$: * * *

$Lm_\pi \approx 2.8$: △

Decomposition of the proton (and pion) mass I

$$m_N = \underbrace{\sum_{q\in\{u,d,s,\ldots\}} m_q \langle N|\bar{q}\,\mathbb{1}\,q|N\rangle}_{\text{quarks}} + \underbrace{\left\langle N \left| \frac{1}{8\pi\alpha_L}(\mathbf{E}^2 - \mathbf{B}^2) + \sum_q \bar{q}\mathbf{D}\cdot\gamma q \right| N \right\rangle}_{\text{gluon interactions (Eucl. spacetime)}}$$

$$+ \underbrace{\frac{1}{4}\left(m_N - \sum_q m_q \langle N|\bar{q}\,\mathbb{1}\,q|N\rangle\right)}_{\text{trace anomaly}}$$

VEV $\langle 0|\bar{q}q|0\rangle$ is understood to be subtracted from $\langle N|\bar{q}q|N\rangle$.

Pion-nucleon σ-term: $\sigma_{\pi N} = m_u\langle N|\bar{u}u|N\rangle + m_d\langle N|\bar{d}d|N\rangle = \sigma_u + \sigma_d$.

Scalar particles (Higgs, neutralino etc.) couple $\propto$ quark matrix elements.

Decomposition of the proton (and pion) mass II

$$\sigma_\pi = m_{ud}\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle = m_{ud}\frac{\partial m_\pi}{\partial m_{ud}} \underbrace{=\frac{m_\pi}{2}}_{\text{GMOR}} + \mathcal{O}(m_\pi^3)\,.$$

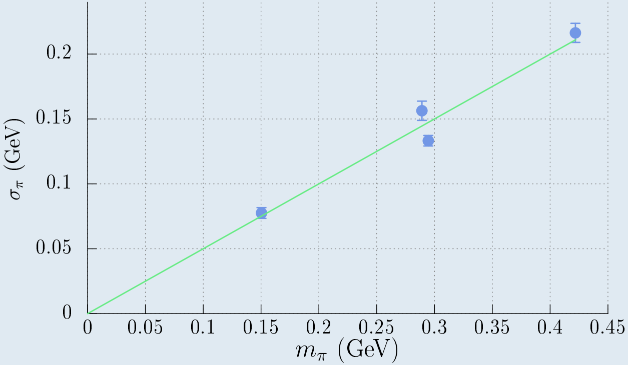
Therefore:

$$m_\pi \approx \underbrace{\frac{1}{2}m_\pi}_{\sigma_\pi} + \underbrace{\frac{3}{8}m_\pi}_{\text{gluon interactions}} + \underbrace{\frac{1}{8}m_\pi}_{\text{trace anomaly}}$$

σ_π can be further decomposed into valence and sea quark contributions.
Wilson fermions: singlet and non-singlet mass renormalization constants differ by $r_m > 1 \Rightarrow$ "valence" > "connected":

$$r := \frac{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle^{\text{sea}}}{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle} = r_m\left(\frac{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle^{\text{dis}}_{\text{lat}}}{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle_{\text{lat}}} - 1\right) + 1$$

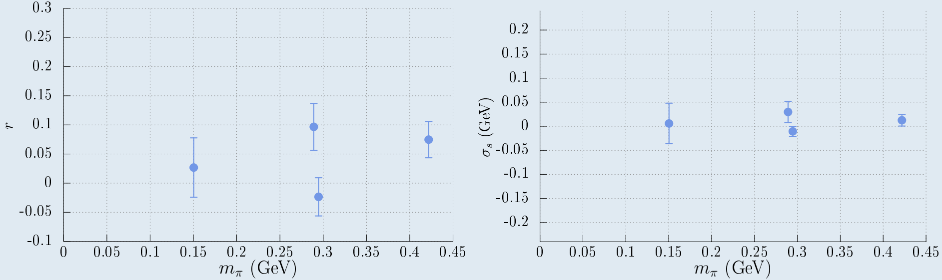
Pion mass: σ_π compared to $m_\pi/2$



[S Collins, D Richtmann]

The theoretical expectation $\sigma_\pi \approx m_\pi/2$ is confirmed.

Pion mass: light sea quark and strange quark contribs.

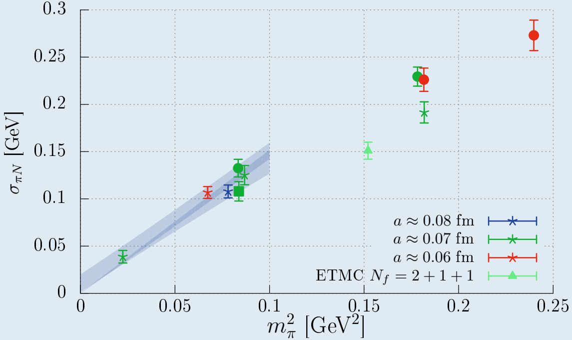


Less than $\sim 10\%$ of σ_π (or $\sim 5\%$ of the mass) is due to sea quarks.

Strange quarks are negligible too.

Nevertheless, $r_m = Z_m^{\text{singlet}}/Z_m^{\text{nonsinglet}} > 1$ means at $a \approx 0.071$ fm about 30% of the signal originates from the disconnected contribution. So this needs to be computed even for the valence quark contribution.

$\sigma_{\pi N}$ for the nucleon



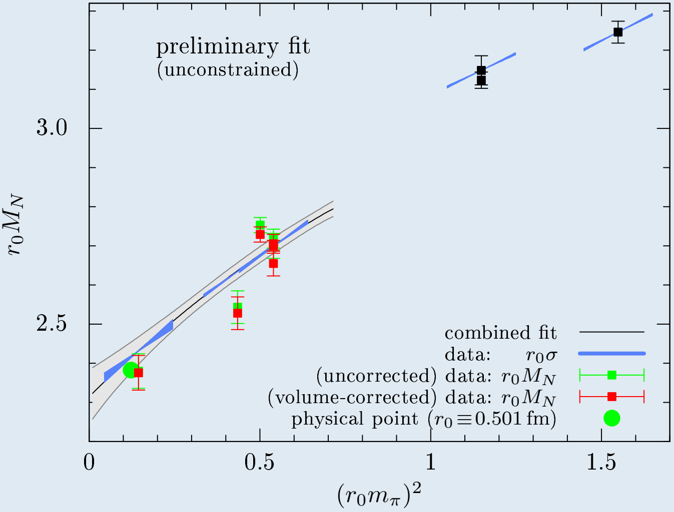
$Lm_\pi \approx 6.7$: ■
$Lm_\pi > 4.1$: ● ●
$Lm_\pi > 3.4$: * * *

[S Collins]

The non-vanishing light quark masses are directly responsible for only ≈ 35 MeV of the nucleon mass but for 68 MeV of the pion mass!

This may not be too surprising since $m_N \not\to 0$ as $m_{ud} \to 0$.

Chiral extrapolation of the nucleon mass



The scalar matrix elements $m_q\langle N|\bar{q}q|N\rangle$ determine the coupling of the nucleon to scalar particles at zero recoil:

$$\frac{f_N}{m_N} \approx \sum_{q\in\{u,d,s\}} f_{T_q}\frac{\alpha_q}{m_q} + \frac{2}{33-6} f_{T_G} \sum_{q\in\{c,b,t,\ldots\}} \frac{\alpha_q}{m_q}\,.$$

Cross section $\propto |f_N|^2$. Higgs example: $\alpha_q \propto m_q/m_W$.

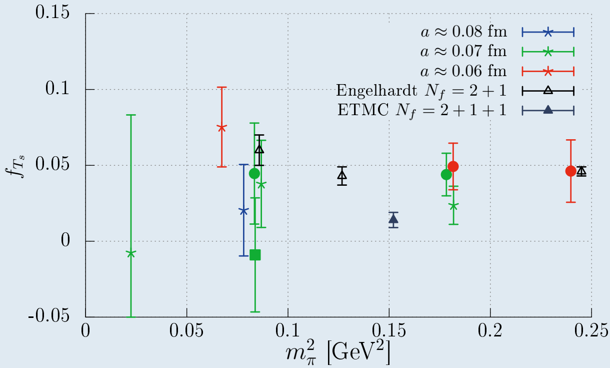
$$f_{T_q} \equiv \frac{m_q\langle N|\bar{q}q|N\rangle}{m_N}$$

are the contributions of the light quark masses to the proton mass and

$$f_{T_G} \approx 1 - \sum_{q\in\{u,d,s\}} f_{T_q}\,.$$

Little about f_{T_q} is known experimentally.

Scalar strangeness content



[QCDSF: GB et al, arXiv:1111.1600,
RQCD: S Collins et al, in preparation]: NPI Wilson
[M Engelhardt, arXiv:1210.0025]: domain wall on staggered
[ETMC, C Alexandrou et al, arXiv:1309.7768]: twisted mass

Spin of the nucleon

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_{q,\bar{q}} L_q + J_g \ :$$

Ji decomposition into the contributions of the (longitudinal) quark spins

$$\Delta\Sigma = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} + \cdots \ ,$$

the (longitudinal) quark and antiquark orbital angular momenta $L_q = J_q - \frac{1}{2}\Delta q$ and the (longitudinal) gluon total angular momentum J_g.

Naïve non-relativistic SU(6) quark model: $\Delta\Sigma = 1$, $L_q = J_g = \Delta s = 0$.
Relativistic quark models: $\Delta\Sigma \sim 0.6$, $L_{\text{quarks}} \sim 0.2$.

I will say nothing about the Jaffe and Manohar decomposition:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_{\text{quarks}} + \Delta G + \mathcal{L}_g \quad \left(J_g \neq \Delta G + \mathcal{L}_g, J_q \neq \frac{1}{2}\Delta q + \mathcal{L}_q\right) .$$

The total quark angular momenta $J_q = \frac{1}{2}\Delta q + L_q$ can be extracted from generalized form factors at $q^2 = 0$:

$$J_q + J_{\bar{q}} = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right] ,$$

where $A_{20}^q(q^2)$ and $B_{20}^q(q^2)$ are obtained from matrix elements of local quark bilinears of the form

$$\left\langle N, s', p+q \left| \bar{q}\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2\}} q \right| N, s, p \right\rangle .$$

Then

$$L_q = J_q - \frac{1}{2}\Delta q , \qquad J_g = \frac{1}{2} - \sum_{q,\bar{q}} J_q .$$

Individual quark spin contributions ($q \in \{u, d, s\}$)

$$(\Delta q + \Delta \bar{q})\ s_\mu = \frac{1}{m_N} \langle N, s \,|\bar{q}\gamma_\mu\gamma_5 q|\, N, s\rangle = F_A^q(0) = \tilde{A}_{10}^q(0)$$

Axial charges:

$$a_3 = -s_\mu \frac{1}{m_N} \langle N, s|\bar{\psi}\gamma_\mu\gamma_5\lambda_3\psi|N, s\rangle = \Delta u - \Delta d = g_A$$

$$a_8 = -s_\mu \frac{\sqrt{3}}{m_N} \langle N, s|\bar{\psi}\gamma_\mu\gamma_5\lambda_8\psi|N, s\rangle$$

$$= \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\Delta s - 2\Delta\bar{s}$$

$$a_0(Q^2) = -s_\mu \frac{1}{m_N} \langle N, s|\bar{\psi}\gamma_\mu\gamma_5 \mathbb{1}\psi|N, s\rangle$$

$$= \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} = \Delta\Sigma(Q^2)\,.$$

$\psi = (u, d, s)^t$, λ_j are Gell-Mann flavour matrices.

$a_3 = g_A$ known from neutron β decay, assuming isospin symmetry.

a_8 usually estimated from hyperon β decay, assuming $\mathrm{SU}(3)_F$ symmetry.

Extraction of the Δq's from experiment

DIS gives spin structure functions of proton and neutron $g_1^{p,n}(x, Q^2)$.

First moment related to a_i's via OPE (leading twist):

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 \mathrm{d}x\, g_1^{p,n}(x, Q^2) \quad = \quad \frac{1}{36}\left[(a_8 \pm 3a_3)C_{NS} + 4a_0 C_S\right]$$

Use models to extrapolate g_1 from experimental $x_{\min}$ to $x = 0$!
$C_{S/NS} = C_{S/NS}(\alpha_s(Q^2))$.

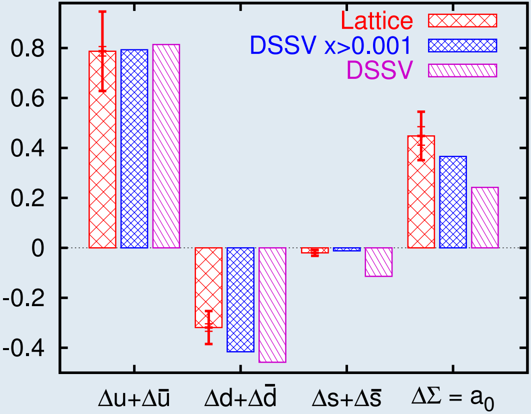
Combinations of a_i give Δq's, e.g., $(\Delta s + \Delta\bar{s})(Q^2) = \frac{1}{3}[a_0(Q^2) - a_8]$

SIDIS allows for direct measurements of the $\Delta q(x)$ but requires fragmentation functions.
[COMPASS, arXiv 1001.4654]

	Naive Extrap.	combined with DSSV
$(\Delta s + \Delta\bar{s})(5\,\mathrm{GeV}^2)$	$-0.02 \pm 0.02 \pm 0.02$	$-0.10 \pm 0.02 \pm 0.02$

DSSV: [de Florian et al, arXiv:0904.3821]

No continuum limit, $m_\pi \approx 290$ MeV $\Rightarrow$ add 20 % systematic error.



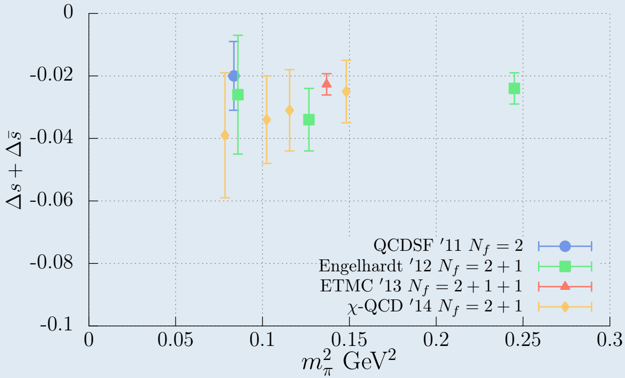
[QCDSF: GB et al, 1112.3354]

Result in the $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 7.4$ GeV^2:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.45(4)(9)$$

$$\Delta s = -0.020(10)(4)$$

Comparison of recent lattice calculations



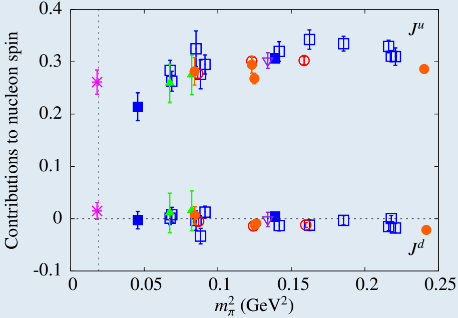
Consistency between different determinations: small $\Delta s + \Delta\bar{s}$.

ETMC result shows statistical accuracy that is possible. Systematics!

[QCDSF: GB et al, 1112.3354; M Engelhardt, 1210.0025; ETMC: A Abdel-Rehim et al, 1310.6339; χQCD: Y Yang et al, unpublished.]

$J_q + J_{\bar{q}} = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$

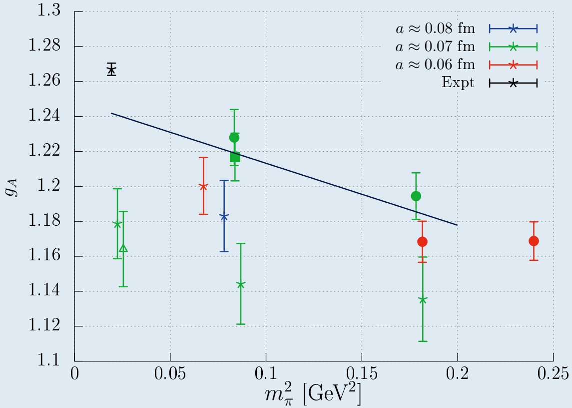
From Lattice 14 review [M Constaninou, 1411.0078]



[LHPC: S Syritsyn et al, 1111.0718 ($N_f = 2+1$); QCDSF/UKQCD: A Sternbeck et al, 1203.6579 ($N_f = 2$); ETMC: C Alexandrou et al, 1104.1600, unpublished ($N_f = 2$); ETMC: C Alexandrou et al, 1303.5979 ($N_f = 2+1+1$).]

$\triangledown$: disconnected contribution included.

$g_A = \Delta u - \Delta d$



$Lm_\pi \approx 6.7$: ■

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$Lm_\pi \approx 2.8$: △

[S Collins, R Rödl]

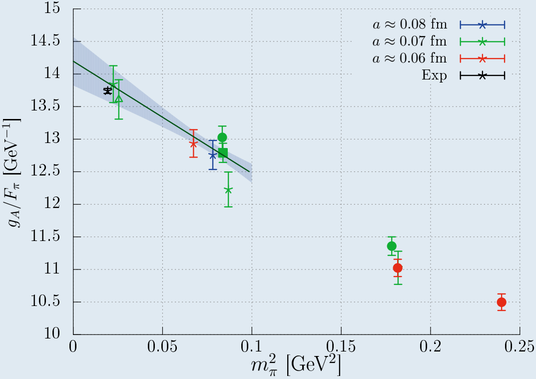
Comparing similar volumes: no significant discretization effects.

$m_\pi \approx 425$ MeV: g_A increases by $\approx 5\%$ with $Lm_\pi \approx 3.7 \rightarrow 4.9$

$m_\pi \approx 290$ MeV: g_A up by $\approx 6\%$ with $Lm_\pi \approx 3.4 \rightarrow 4.2$, then constant.

$m_\pi \approx 150$ MeV: No difference between $Lm_\pi \approx 2.8$ and $Lm_\pi \approx 3.5$.

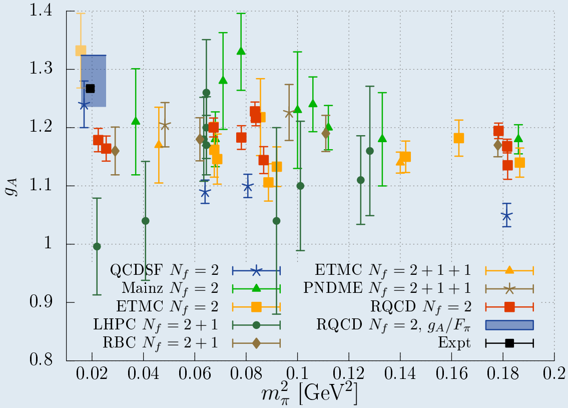
Finite volume effects predicted by χPT similar for g_A and F_π
$\Longrightarrow$ follow `QCDSF: R Horsley et al, arXiv:1302.2233` and plot ratio



Extrapolation to physical point: $g_A/F_\pi = 13.88(29)\ \text{GeV}^{-1}$
Expt: $g_A/F_\pi = 13.797(34)\ \text{GeV}^{-1}$.

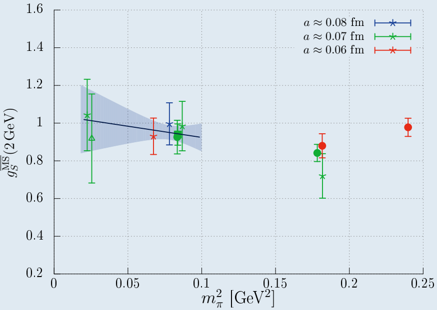
Using $F_\pi(\text{expt}) = 92.21$ MeV we obtain $g_A = 1.280(27)(35)$
Expt: $g_A = 1.2670(35)$.

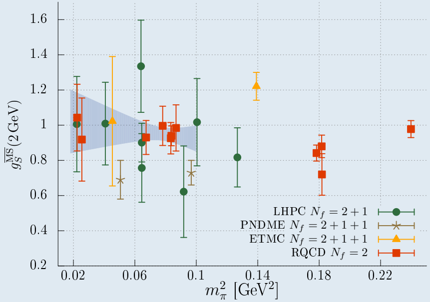
g_A: summary of recent lattice results



QCDSF: 1302.2233
Mainz: 1311.5804
ETMC 2: 1312.2874
LHPC: 1209.1687
RBC/UKQCD: 1309.7942
ETMC 2+1+1: 1303.5979
PNDME: 1306.5435
RQCD: 1412.7336

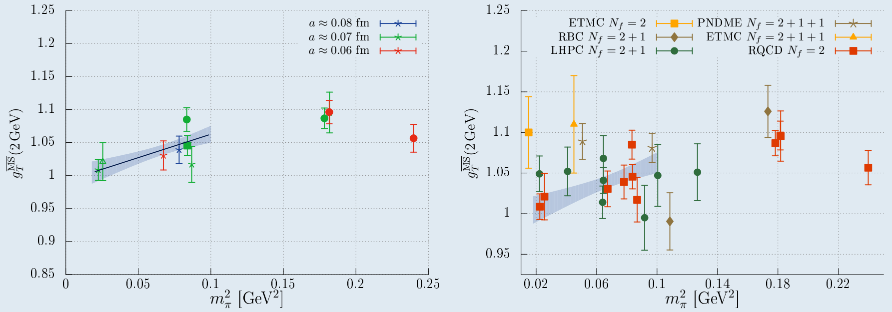
Isovector scalar charge





LHPC: 1206.4527, PNDME: 1306.5435, ETMC: 1411.3494, RQCD: 1412.7336

Isovector tensor charge



ETMC 2: 1311.4670, RBC: 1003.3387, LHPC: 1206.4527,
PNDME: 1306.5435, ETMC 2+1+1: 1311.4670, RQCD: 1412.7336

General remark: we vary a^2 only by a factor 1.8 $\Rightarrow$ we cannot exclude lattice spacing effects of up to $0.071^2/(0.081^2 - 0.060^2) \cdot \Delta g \approx 1.7 \cdot \Delta g$.

Isovector electromagnetic formfactors

$$\langle p|\bar{u}\gamma_\mu d|n\rangle = \bar{u}_p(\mathbf{p}_\mathrm{f})\left[g_V(q^2)\gamma_\mu + \frac{\tilde{g}_T(q^2)}{2m_N} i\sigma_{\mu\nu}q^\nu\right]u_n(\mathbf{p}_\mathrm{i})$$

Dirac FF: $g_V(q^2) = F_1^p(q^2) - F_1^n(q^2) \xrightarrow{q^2\to 0} 1$

Pauli FF: $\tilde{g}_T(q^2) = F_2^p(q^2) - F_2^n(q^2) \xrightarrow{q^2\to 0} \kappa_p - \kappa_n \approx 3.7058901(5)$

$$g_V(Q^2) = 1 - \frac{r_1^2}{6}Q^2 + \mathcal{O}(Q^4), \quad \tilde{g}_T(Q^2) = \tilde{g}_T(0)\left[1 - \frac{r_2^2}{6}Q^2 + \mathcal{O}(Q^4)\right]$$

Proton radius:

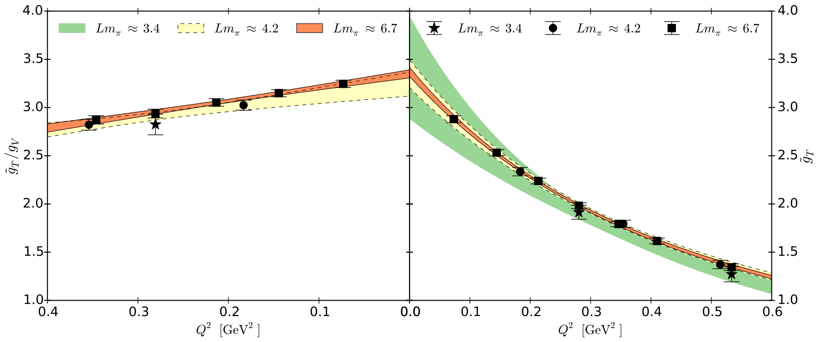
$$r_p^2 \approx r_1^2 + \frac{3\tilde{g}_T(0)}{2m_N^2}.$$

Dipole fit to determine the induced tensor charge $\tilde{g}_T = \tilde{g}_T(0)$:

$$\tilde{g}_T(Q^2) = \frac{\tilde{g}_T(0)}{(1 + r_2^2 Q^2/12)^2}.$$

Extrapolation of the Pauli formfactor at $m_\pi = 290\,\mathrm{MeV}$

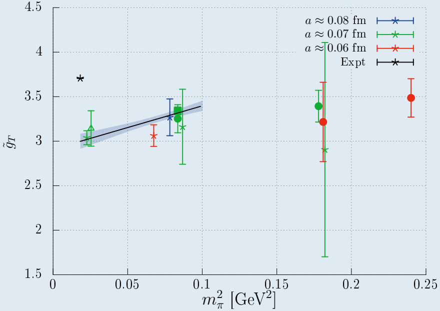
Difference between magnetic moment anomalies $\tilde{g}_T(0) = \kappa_p - \kappa_n$.

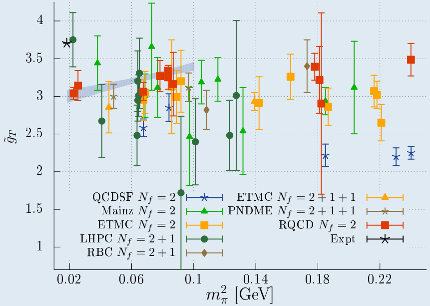


Extrapolation error decreases with smaller $Q^2_{\mathrm{min}} = \pi^2/L^2$. Therefore, invisible FSE for $Lm_\pi > 3.4$ at $m_\pi = 290$ MeV ($L > 2.3$ fm) do not necessarily imply they are irrelevant within the smaller statistical errors at $m_\pi = 150$ MeV ($L > 4.5$ fm).

Induced isovector tensor charge

Extrapolating in the usual way... however, FSE are unquantifiable at the lightest mass point and $\mathcal{O}(a)$ improvement is not yet included.

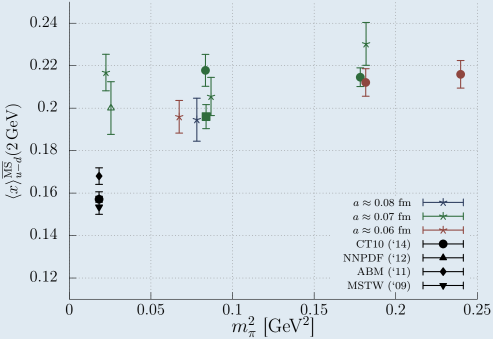




QCDSF: 1106.3580, Mainz: 1311.5804 + 1411.4804,
ETMC 2: 1102.2208, LHPC: 1404.4029, RBC: 0904.2039,
ETMC 2+1+1: 1303.5979, PNDME: 1306.5435, RQCD: 1412.7336

Isovector quark momentum fraction: $\langle x \rangle^{\overline{\rm MS}}_{u-d}(2\,{\rm GeV})$

[RQCD: GB et al, arXiv:1408.6850] $N_f = 2$



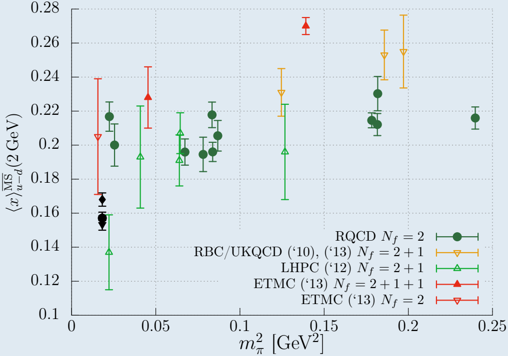
$Lm_\pi \approx 6.7$: ■
$Lm_\pi > 4.1$: ● ●
$Lm_\pi > 3.4$: * * *
$Lm_\pi \approx 2.8$: △

Mild dependence on V, m_π.
Renormalised non-perturbatively.
$\mathcal{O}(a)$ leading errors, a varied from 0.08 to 0.06 fm.

Improvement on earlier calculations which suffered from excited state contamination $\langle x \rangle^{\overline{\rm MS}}_{u-d}(2\,{\rm GeV}) \sim 0.25$.

Near physical point but more work needs to be done — lattice spacing dependence?

$\langle x \rangle^{\overline{\rm MS}}_{u-d}(2\,{\rm GeV})$: summary of recent lattice results



RQCD: GB et al, 1408.6850; RBC/UKQCD: Y Aoki et al, arXiv:1003.3387; LHPC: J Green et al, arXiv:1209.1687; ETMC 2+1+1: C Alexandrou et al, arXiv:1312.2874; ETMC 2: C Alexandrou et al, arXiv:1303.5979.

PDFs from
S Alekhin et al, 1310.3059; CT10: J Gao et al, 1302.6246; NNPDF: R Ball et al 1207.1303; A Martin et al 0905.3531.

[ETMC, arXiv:1410.8761]: disconnected contributions small ⇒ predictions for $\langle x \rangle^{\overline{\rm MS}}_q(2\,{\rm GeV})$ soon. Mixing between quarks and glue!

Summary

- Lattice can contribute to many quantities that are hard to constrain by experiment, e.g., $\sigma_{\pi N}$, f_{T_s}, g_S, g_T.
- Lattice calculations are important to determine the spin content of the nucleon: Δq, $\Delta\Sigma$, J_q, $\langle x\rangle_{\Delta q}$, ...
- In the past disconnected quark line diagrams were often omited and differences quoted: g_A, $\langle x\rangle_{u-d}$, ..., but no Δs, $\Delta\Sigma$, J_q, $\langle x\rangle_q$ etc.
- Improved methods now allow for the calculation of these contributions.
- g_A seems to approach the physical value, once $Lm_\pi > 4$.
- $\langle x\rangle_{u-d}$ comes out 20% bigger than expected.
  lattice spacing effects? Renormalization?
- Precision physics requires an extrapolation $a \to 0$. For quite a few quantities however errors of 20% are acceptable.
- High Mellin moments almost impossible to compute $\Rightarrow$ recent interest also in "quasi" parton distribution functions.