Viscous corrections from nonlinear transport

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Outline

I. The problem of dissipative (δf) corrections

II. A couple δf models

III. Test them against full kinetic theory

IV. Conclusions and future steps

Hydrodynamics

conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad , \qquad \partial_\mu N_B^\mu(x) = 0$$

additional EoM for dissipative fluids:

$$T^{\mu\nu}(x) = T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x)$$

$$N^\mu(x) = N^\mu_{ideal}(x) + \delta N^\mu(x)$$

$$\delta \dot{T}^{\mu\nu} = A^{\mu\nu}(\{T^{\alpha\beta}, N^\gamma\}) \quad , \qquad \delta \dot{N}^\mu = B^\mu(\{T^{\alpha\beta}, N^\gamma\})$$

(e.g. Israel-Stewart '79)

utilizes equation of state ($p(e, n_B)$, $T(e, n_B)$), transport coeffs (η, ζ, κ_B), relaxation times (τ_η, τ_ζ, τ_κ)

some issues:
- **initial conditions needed $\leftrightarrow$ thermalization**
- **how to "turn" hydro off at late times**
- **very small systems - quantum effects?** DM, Wang, Greene, arXiv:1404.4119
- **is it really hydro?** He et al, arXiv:1502.05572

Hydro $\rightarrow$ particles

hydro gives N^μ & $T^{\mu\nu}$, but experiments measure particles

$$N^\mu(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu f_i(p,x)$$

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(p,x)$$

- **in local equilibrium (ideal hydro) - "one to one"**

$$T^{\mu\nu}_{LR}(x) = diag(e,p,p,p) \qquad \Leftrightarrow \qquad f_{eq,i}(x,p) = \frac{g_i}{(2\pi)^3} \frac{1}{e^{(p^\mu u_\mu - \mu_i)/T} + a}$$

- **near local equilibrium (viscous hydro) - "few to many"**

$$\begin{array}{r} T^{\mu\nu}(x) = T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x) \\ N^\mu(x) = N^\mu_{ideal(x)} + \delta N^\mu(x) \end{array} \qquad \Leftarrow \quad f_i(x,p) = f_{eq,i}(x,p) + \delta f_i(x,p)$$

$\Rightarrow$ **question of δf (even for single-species systems!)**

δf models, for shear

$$\delta T^{\mu\nu} = \pi^{\mu\nu} \qquad\qquad (\pi^{\mu\nu} u_\nu = 0,\ u_\nu \pi^{\mu\nu} = 0,\ \pi^\mu{}_\mu = 0)$$

purely spatial (in LR), symmetric, traceless

Navier-Stokes: $\pi^{\mu\nu} = \eta \underbrace{\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial \cdot u)\right]}_{2\sigma^{\mu\nu}}$

matter for: - hadron distributions (fluid$\rightarrow$ hadron gas conversion)
- E&M rates from QGP
- bound states in plasma ...

Grad ansatz

From "14-moment" expansion in kinetic theory

$$\delta f^{Grad}(x,\vec{p}) = \frac{1}{2T(x)^2[e(x)+p(x)]} p^\mu p^\nu \pi_{\mu\nu}(x)\, f_{eq}(x,\vec{p})$$

Popular for hydro because for multicomponent system the same form

$$\delta f_i = \frac{1}{2T^2(e+p)} p^\mu p^\nu \pi_{\mu\nu}\, f_{eq,i}$$

only needs knowledge of the full $T^{\mu\nu}$ and automatically yields $\sum_i T_i^{\mu\nu} = T^{\mu\nu}$.
Also used by Israel & Stewart (IS) to derive causal viscous hydrodynamics.

Can generalize to any power - "IS with p^α":

$$\delta f = \frac{C(\alpha)}{2T^2(e+p)} \left(\frac{p\cdot u}{T}\right)^{\alpha-2} p^\mu p^\nu \pi_{\mu\nu}\, f_{eq}$$

Weakness: negative f at sufficiently high momenta ($\pi^{\mu\nu}$ not pos definite)

Covariant transport

(on-shell) phase-space density $f(x, \vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p}$

transport equation:

$$p^\mu \partial_\mu f_i(x,p) = C^i_{2\to 2}[\{f_j\}](x,p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x,p) + \cdots$$

with, e.g.,

$$C^i_{2\to 2} = \frac{1}{2} \sum_{jkl} \int_{234} (f^k_3 f^l_4 - f^i_1 f^j_2) W^{ij\to kl}_{12\to 34} \qquad \left(\int_j \equiv \int \frac{d^3p_j}{2E_j}\ , \qquad f^k_a \equiv f^k(x, p_a) \right)$$

thermalizes (in box), fully causal and stable

near hydro limit transport coeffs & relaxation times:

$\eta \approx 1.2T/\sigma$, $\qquad \tau_\pi \approx 1.2\lambda_{tr}$

Linear response (for transport)

late-time, near-equilibrium behavior universal, and can be studied systematically de Groot, et al ('70s)... Arnold, Moore, Yaffe, JHEP 0011... Denicol et al PRD85 ('12)...

operationally: linearize in $\delta f = f - f_{eq}$, and put $\partial_t \to 0$

$$p \cdot \nabla f_{eq} = C[f_{eq}, \delta f]$$

integral eqn. relates δf to gradients in the system (one way to derive transport coeffs)

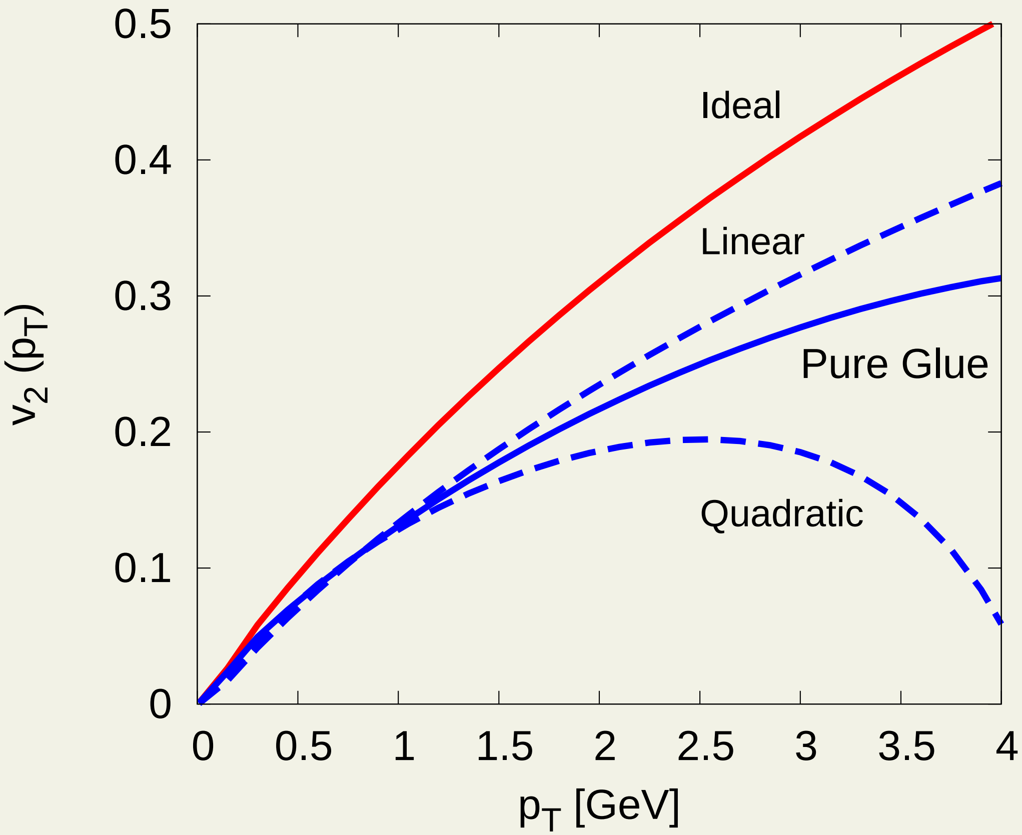
For shear, solution is of the form

$$\delta f(x, \vec{p}) = \chi(\frac{p \cdot u}{T})\, \hat{p}_\mu \hat{p}_\nu \frac{\sigma^{\mu\nu}}{T} f_{eq}$$

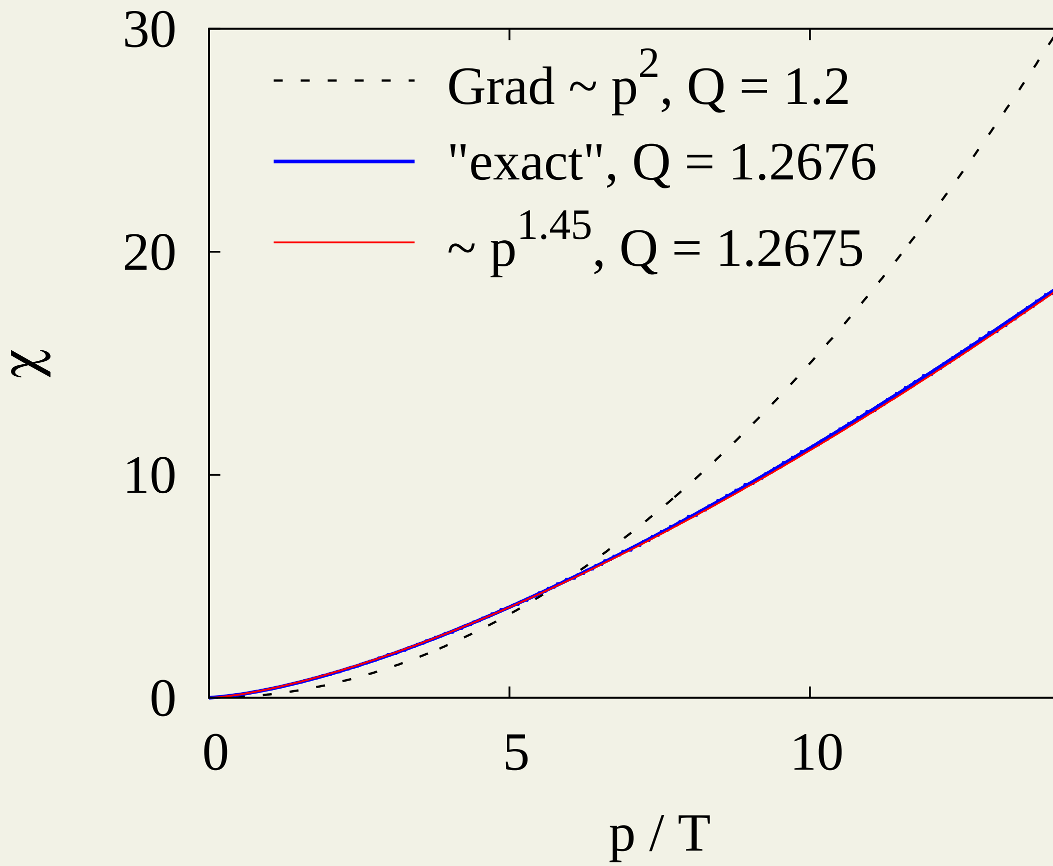
$\Rightarrow$ **nontrivial momentum dependence, but negativity problem remains**

theory preference for $\chi \sim p^{1.5}$

massless quark-gluon gas, forward-peaked cross sections Dusling, Moore, Teaney, PRC ('09)

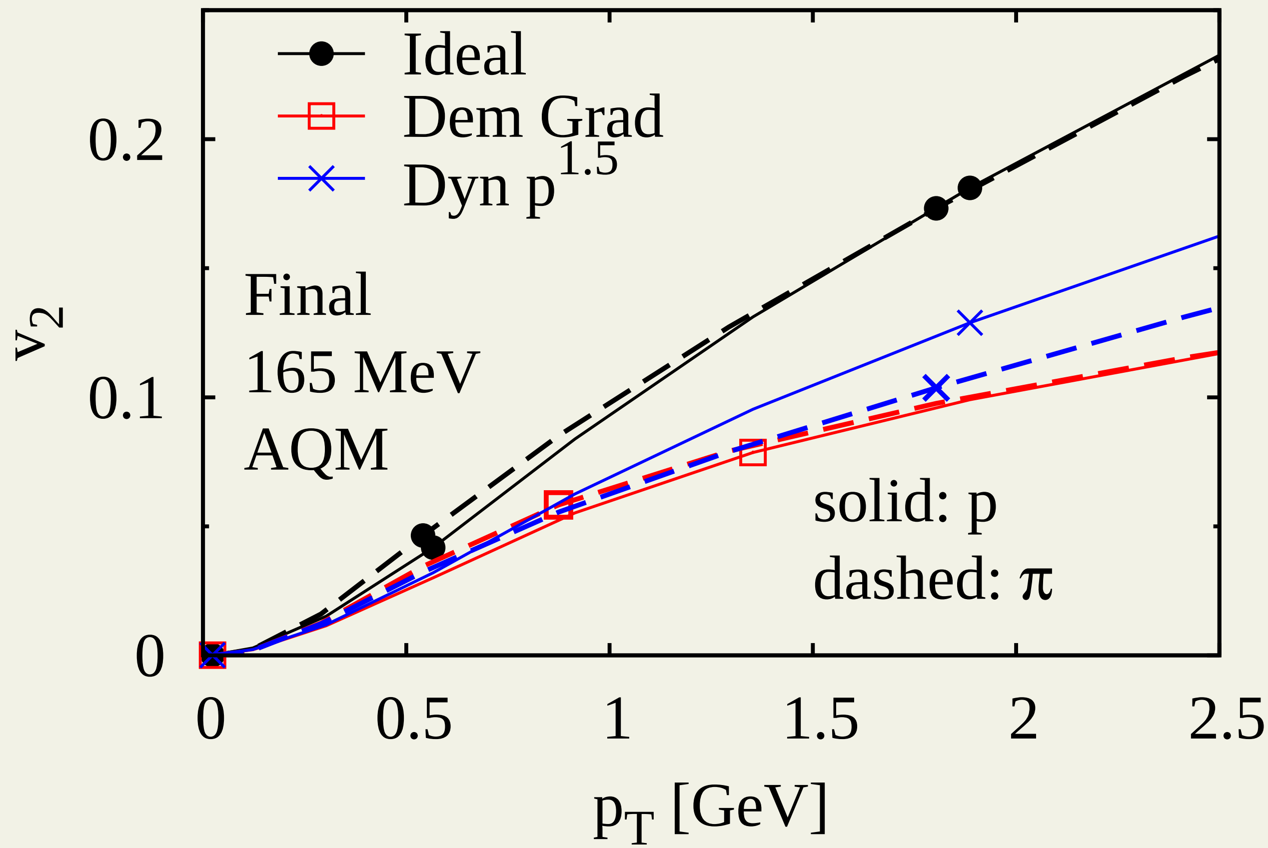


massless gas, isotropic $\sigma = const$ DM, JPG38 ('12)



Au+Au @ RHIC, b=7 fm - conversion to hadron gas with ≈ 50 species

DM & Wolff, arXiv:1407.6413



pion-proton flow splitting due to different δf_i (additive quark model rates)

SR ansatz

Strickland & Romatschke, PRD 71 ('05):

$$f(p_T, p_z) = N e^{-\sqrt{E^2+p_z^2}/\Lambda} = N \exp\left[-\frac{p_T}{\Lambda}\sqrt{\mathrm{ch}^2\xi + a\,\mathrm{sh}^2\xi}\right] \qquad (\xi \equiv \eta - y)$$

- N, Λ, a **fixed by** N^0**,** T^{00}**,** T^{zz}

- **matches free streaming solution in 0+1D** $[a = (\tau/\tau_0)^2]$

more generally Tinti & Florkowski, PRC89 ('14)

$$f = N \exp\left[-\frac{1}{\Lambda}\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}\right]$$

- **always positive, also: linear for small momenta**

Relaxation time approx (RTA)

Instead of Boltzmann, try

$$p \cdot \partial f(x, \vec{p}) = (p \cdot u) \frac{f_{eq}(x, \vec{p}) - f(x, \vec{p})}{\tau_{REL}}$$

Conserved particle number, energy & momentum, if $f_{eq}(f)$ chosen well. τ_{REL} must be matched to some dynamical aspect, e.g., the viscosity.

In late-time near-equilibrium regime ($\partial_t \to 0$): $\quad p \cdot \nabla f_{eq} = -\tau_{REL}(p \cdot u)\delta f$

so with shear: $\delta f = \dfrac{\tau_{REL}}{2(p \cdot u)T} p_\mu p_\nu \sigma^{\mu\nu} f_{eq} = \dfrac{\tau_{REL}}{2} \dfrac{T}{(p \cdot u)} \dfrac{p_\mu p_\nu}{T^2} \dfrac{\pi^{\mu\nu}}{\eta} f_{eq}$

and from $\displaystyle\int \frac{d^3p}{E} p^\mu p^\nu \delta f = \pi^{\mu\nu} \quad \Rightarrow \quad \tau_{REL} = \frac{5\eta}{4nT}$

Notice that $\delta f / f_{eq} \sim p^1$ (linear) in this case.

GOAL: test these models against full, nonlinear kinetic theory.

i) obtain transport solutions with isotropic $2 \to 2$ interactions (MPC/Grid transport code)

ii) from f, determine $T^{\mu\nu}$

iii) study how well δf models reconstruct f from the $T^{\mu\nu}$ alone

MPC/Grid

similar to BAMPS - not a cascade

- $2 \to 2$ **and** $3 \leftrightarrow 2$ **with test particles on a spatial grid**

  pair/triplet collisions with probability

$$P_{2\to X} = \frac{\sigma_{2\to X} v_{rel} \Delta t}{V_{cell}} \ , \qquad P_{3\to Y} = \frac{K_{3\to Y} \Delta t}{V_{cell}^2}$$

  5 numerical knobs: **cell sizes** $(d_x,\ d_y,\ d_z/d_\eta)$**, timestep** Δt**, subdivision** ℓ

  still action at distance $\to$ **violates locality, causality, covariance**

  $\Rightarrow$ but more flexible than earlier MPC/Cascade

- **we also have a version for parallel computers (MPI)**

massless system, 0+1D Bjorken expansion (transversely uniform)

$\Rightarrow f(p_T, \xi \equiv \eta - y, \tau)$

$$u^\mu_{BJ} = (\mathrm{ch}\eta, 0, 0, \mathrm{sh}\eta), \qquad (p \cdot u) = p_T \,\mathrm{ch}\xi$$

$$T^{\mu\nu}_{ideal,LR} = p(\tau)\,\mathrm{diag}(3,1,1,1), \qquad \pi^{\mu\nu}_{LR} = \pi_L(\tau)\,\mathrm{diag}(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 1)$$

dynamics governed by: $K(\tau) \equiv \dfrac{\tau_{exp}}{\tau_{sc}} = \dfrac{(\partial\cdot u)}{\lambda_{tr,MFP}} = \frac{2}{3}\tau n \sigma_{TOT} = \frac{2}{3}\tau_0 n_0 \sigma_{TOT}$

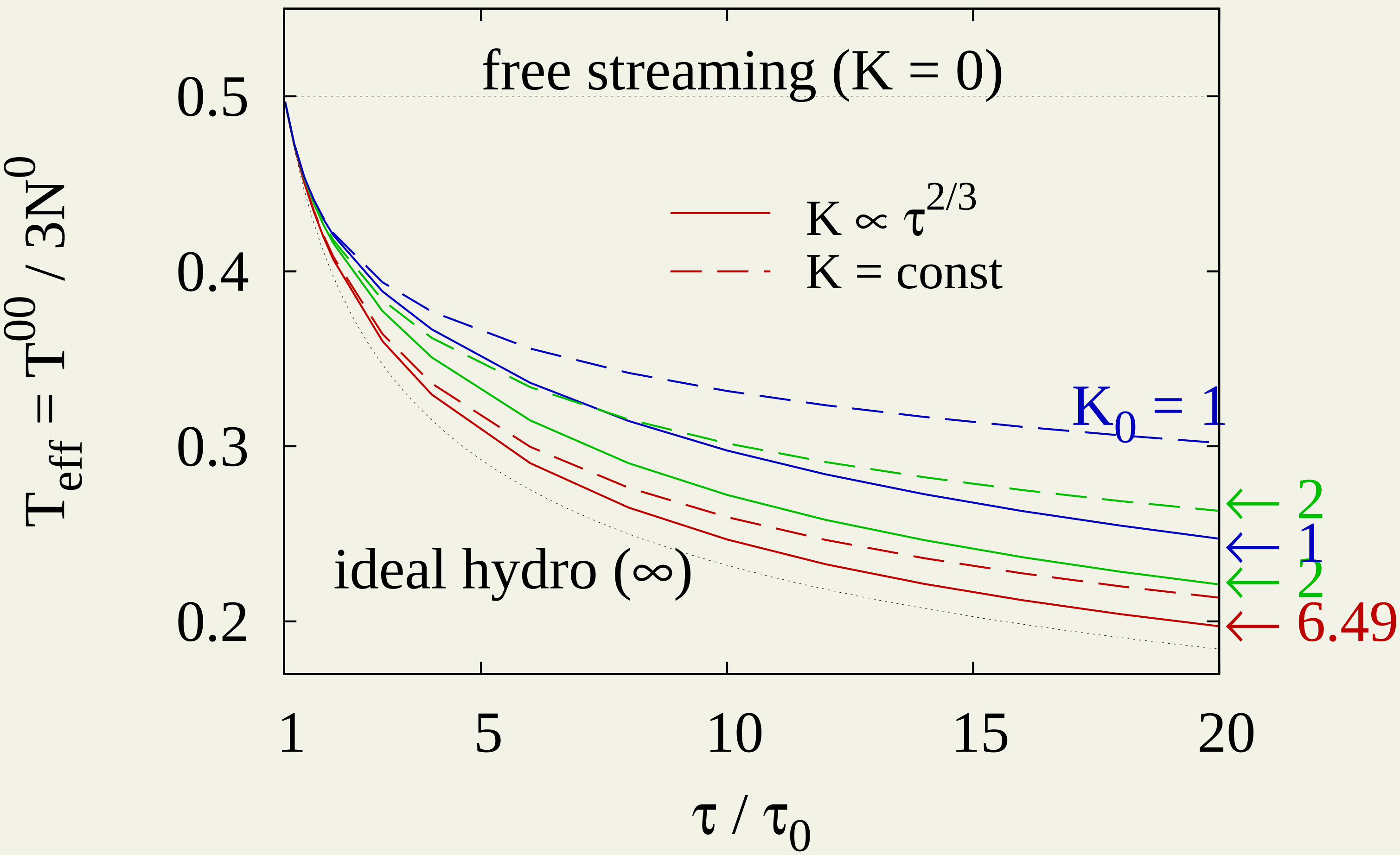
for $\boldsymbol{\eta/s \approx const}$**,** $\sigma_{TOT} \sim \tau^{2/3} \Rightarrow K(\tau) \propto \tau^{2/3}$ DM & Huovinen, PRC79 ('09)

for $\boldsymbol{\sigma = const}$**,** $K(\tau) = const$

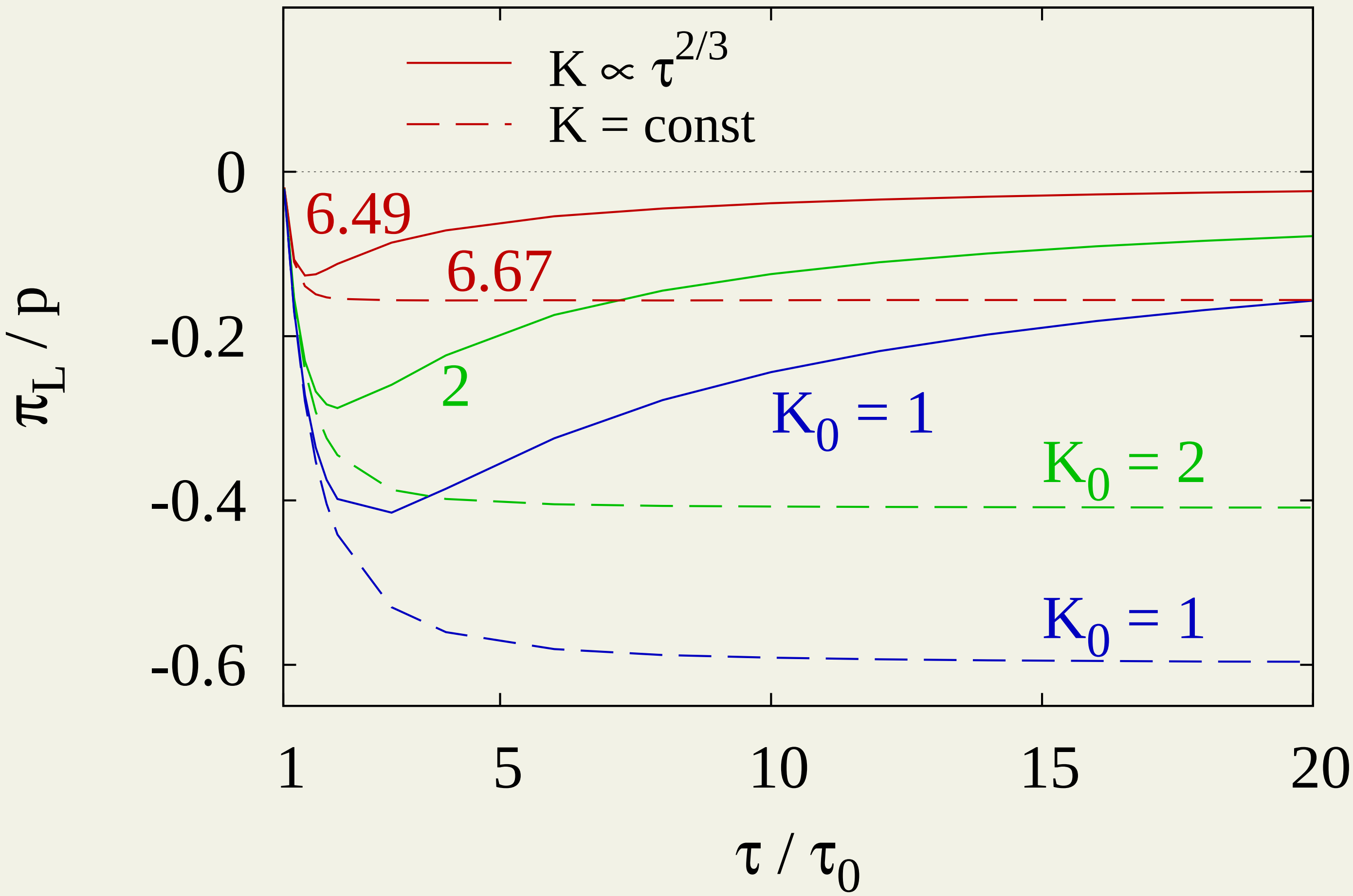
with $\eta = 1.26757..\dfrac{T}{\sigma_{TOT}}$**, we have** $K_0 \approx 0.2 T_0 \tau_0 \dfrac{s}{\eta}(\tau_0)$ **(used** $s = 4n$**)**

and typically $\tau_0 T_0 \sim 1$**, so** $\eta/s = 1/(4\pi)$ **corresponds to** $K_0 \sim 2$

T_{eff} vs τ - cooling due to $p\,dV$ work Gyulassy, Pang, Zhang, NPA626 ('97)



viscous pressure correction vs τ - just like DM & Huovinen, PRC79 ('09)



First look at $f(p_T, \xi)$ with SR and Grad(IS) $\to$ plot $\dfrac{f_{rec}}{f_{trans}}$

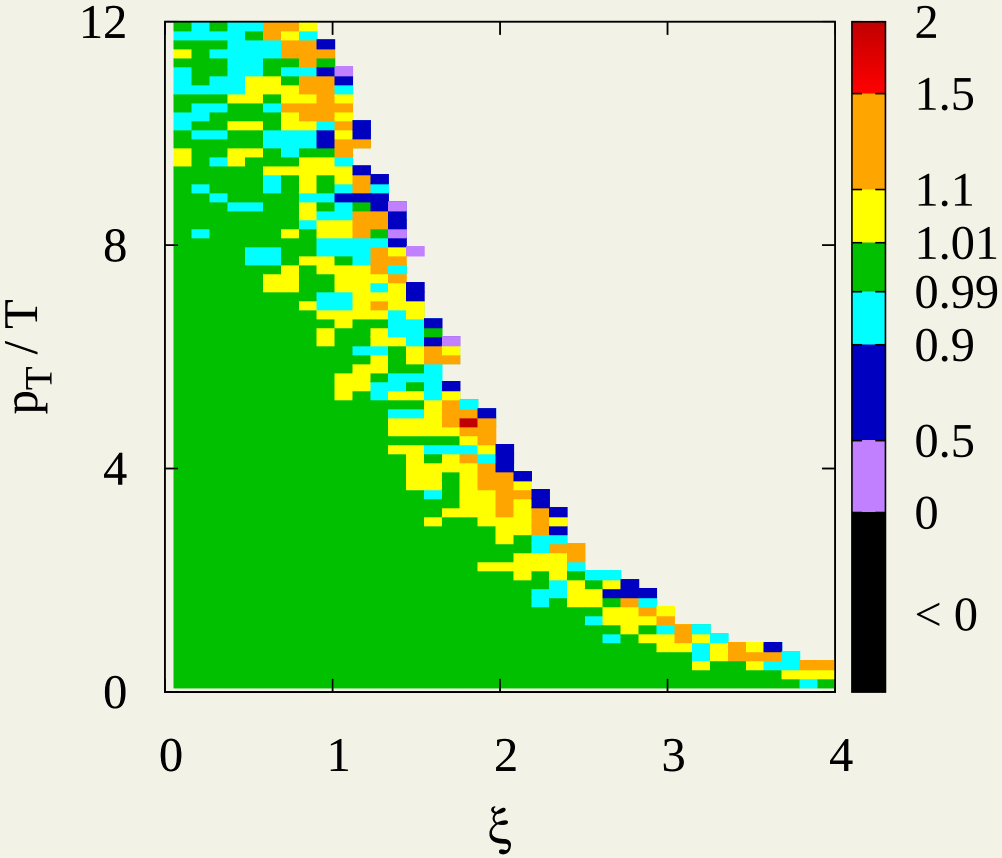
Generic features illustrated for $\eta/s \sim 1/(4\pi)$
[$K_0 = 2$, $K(\tau) \propto \tau^{2/3}$]

Color coding:

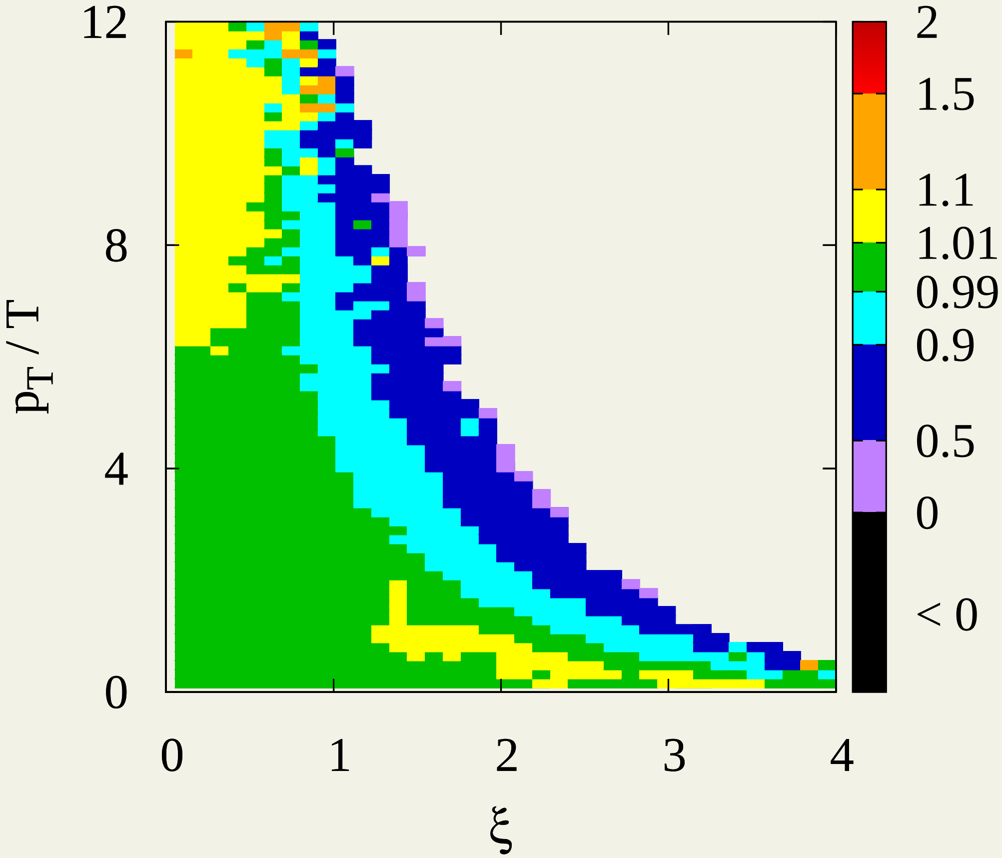
$+100$%
$+50$%
$+10$%
± 1%
-10%
-50%
-100%
< 0

$$\frac{\tau}{\tau_0} = 1.02, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.52, \quad \frac{\pi_L}{p} = -0.02\right)$$

SR ansatz

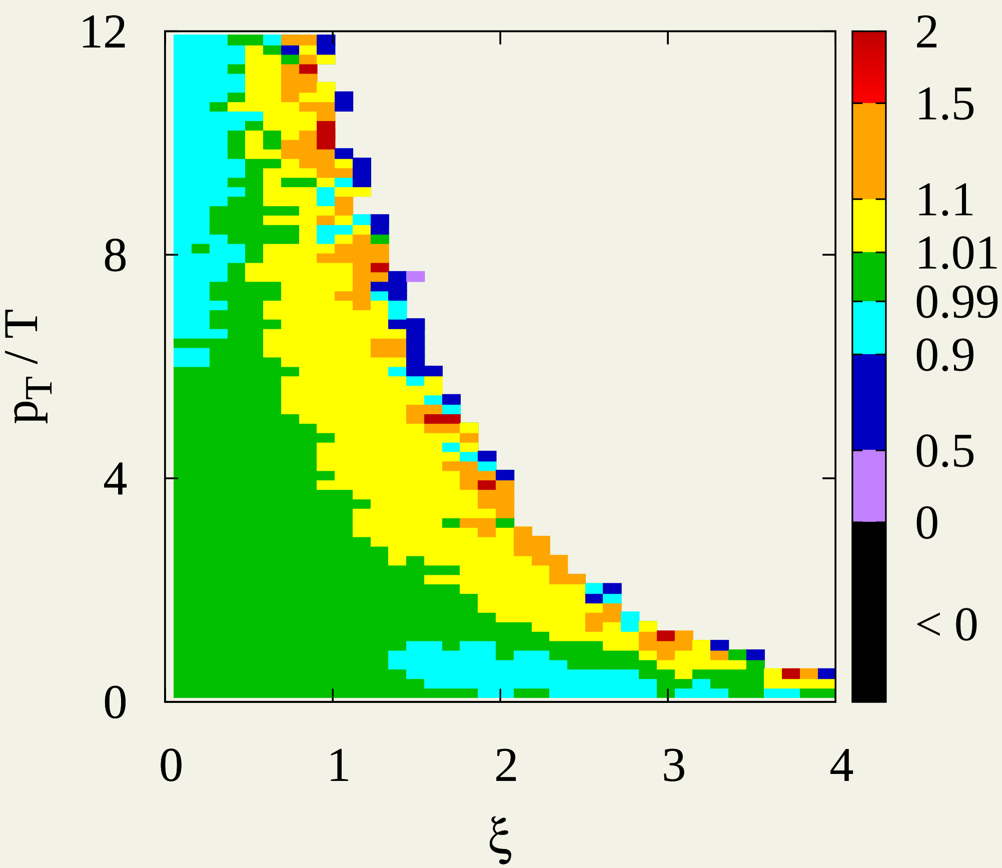


Grad (IS)

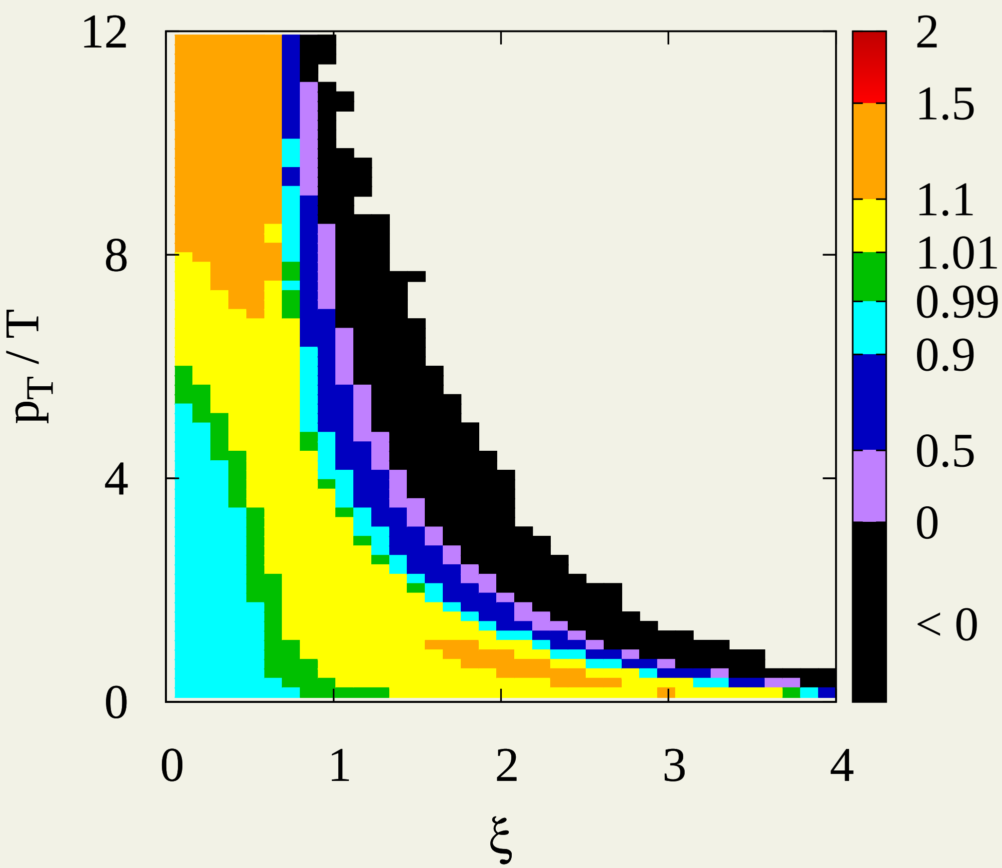


$$\frac{\tau}{\tau_0} = 1.2, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.47, \quad \frac{\pi_L}{p} = -0.15\right)$$

SR ansatz

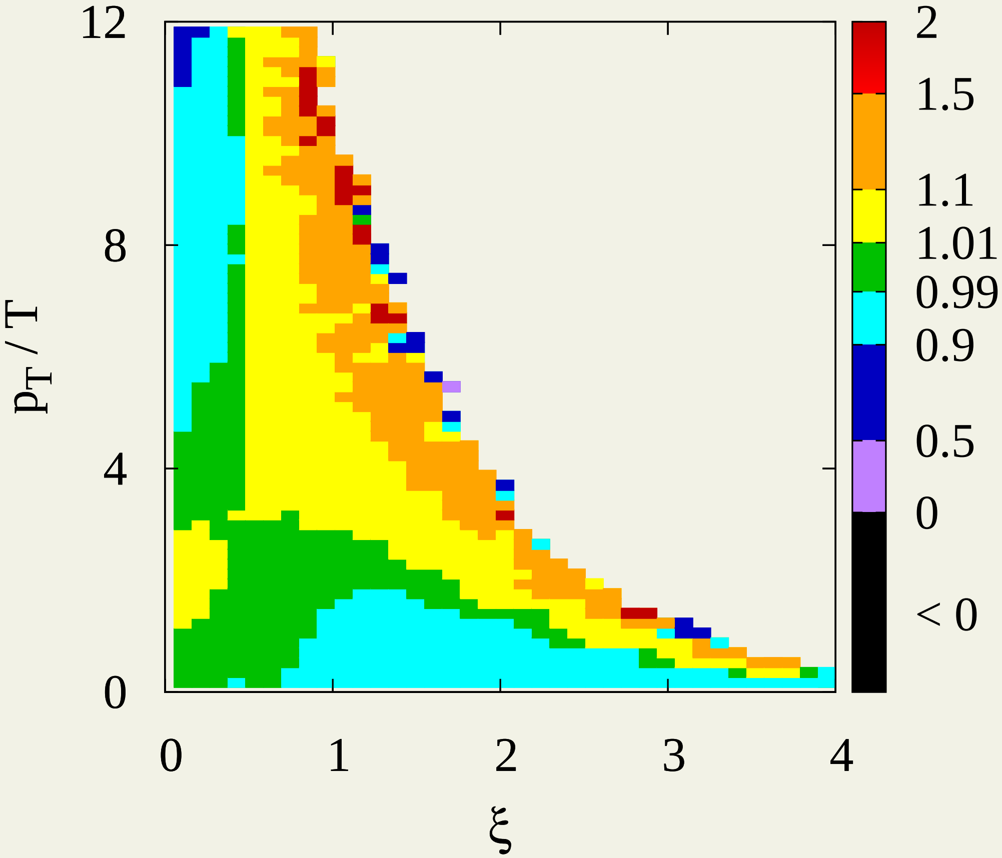


Grad (IS)

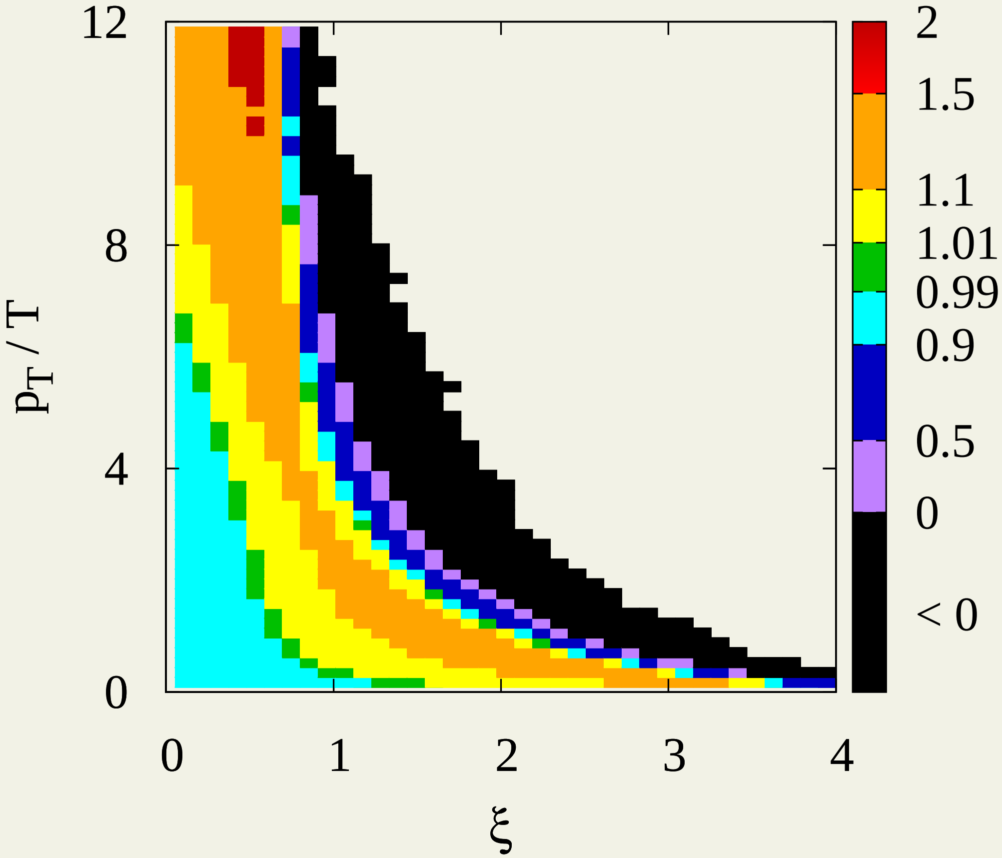


$$\frac{\tau}{\tau_0} = 1.4, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.42, \quad \frac{\pi_L}{p} = -0.23\right)$$

SR ansatz



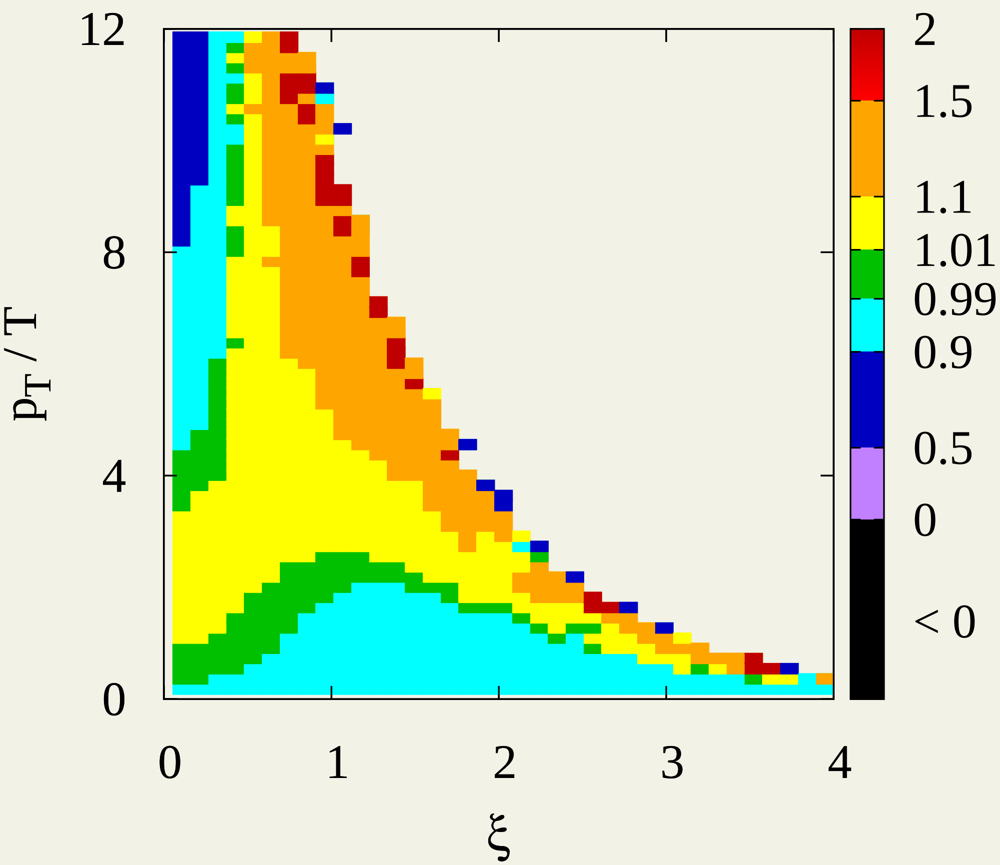
Grad (IS)

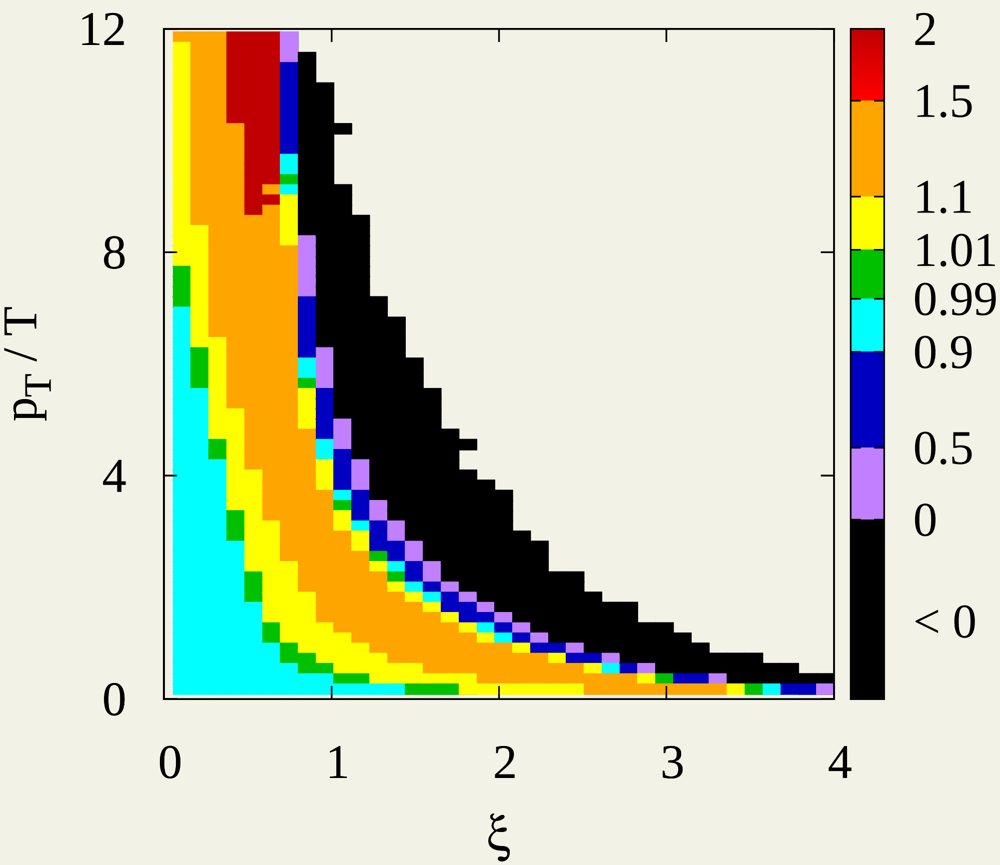


$$\frac{\tau}{\tau_0} = 1.6, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.38, \quad \frac{\pi_L}{p} = -0.27\right)$$

SR ansatz

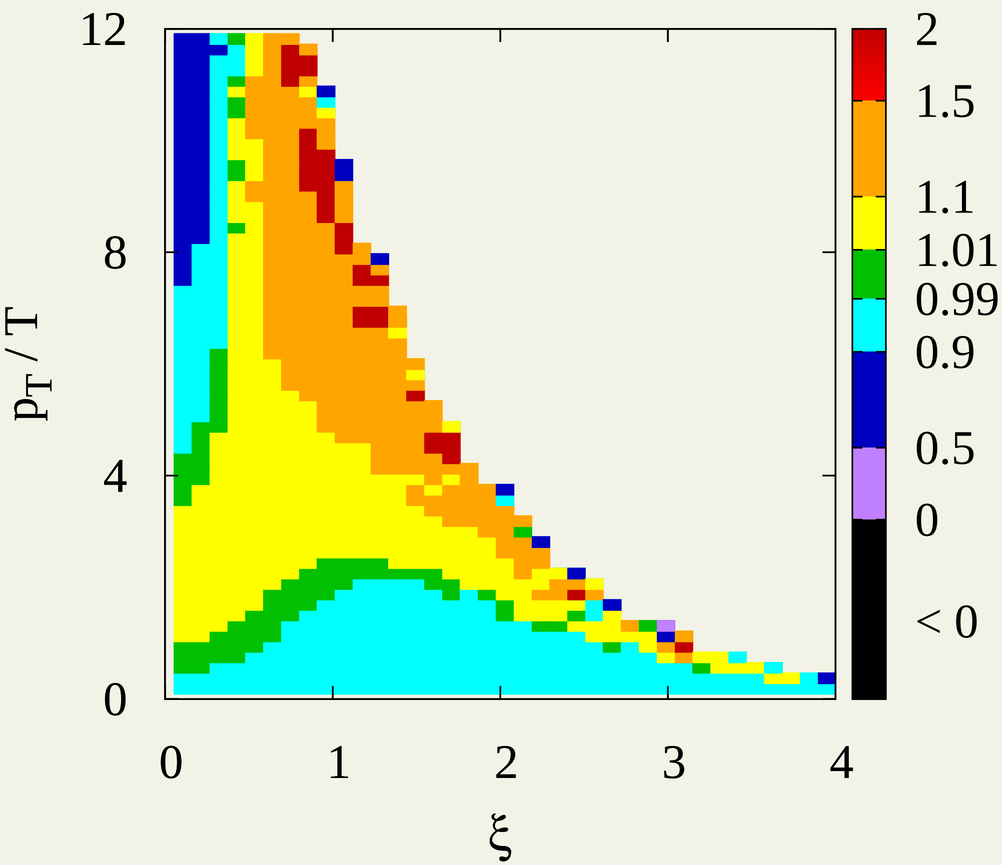
Grad (IS)



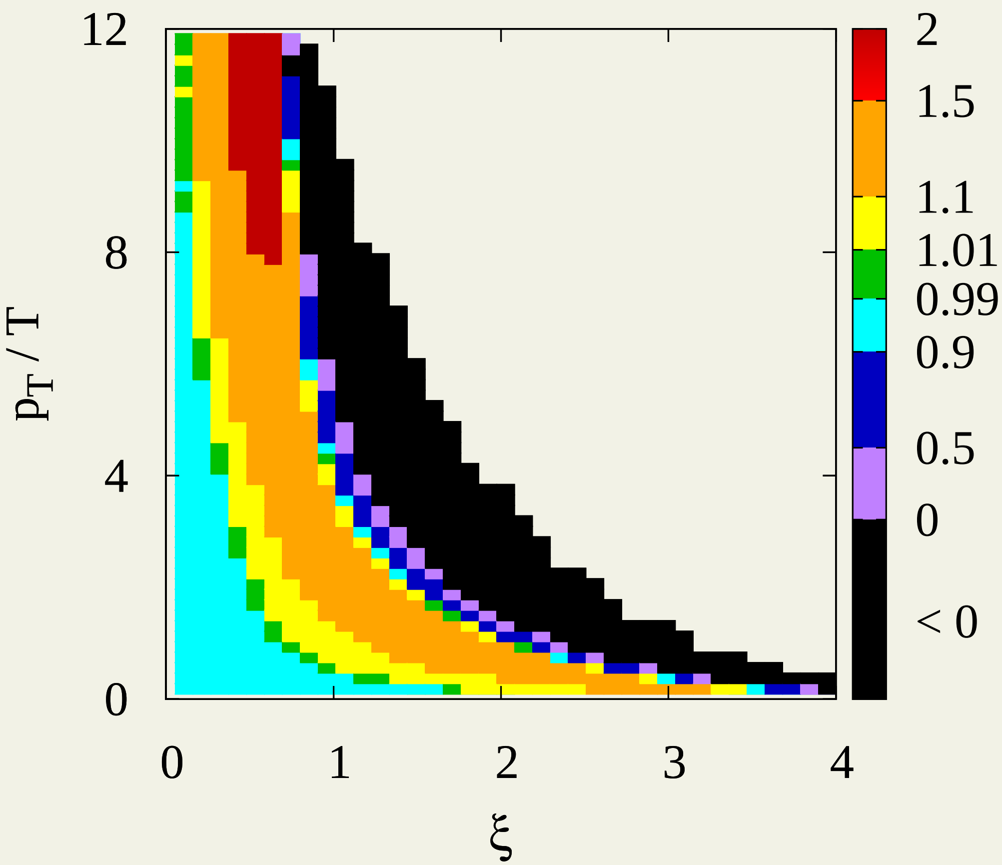


$$\frac{\tau}{\tau_0} = 1.8, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.36, \quad \frac{\pi_L}{p} = -0.28\right)$$

SR ansatz

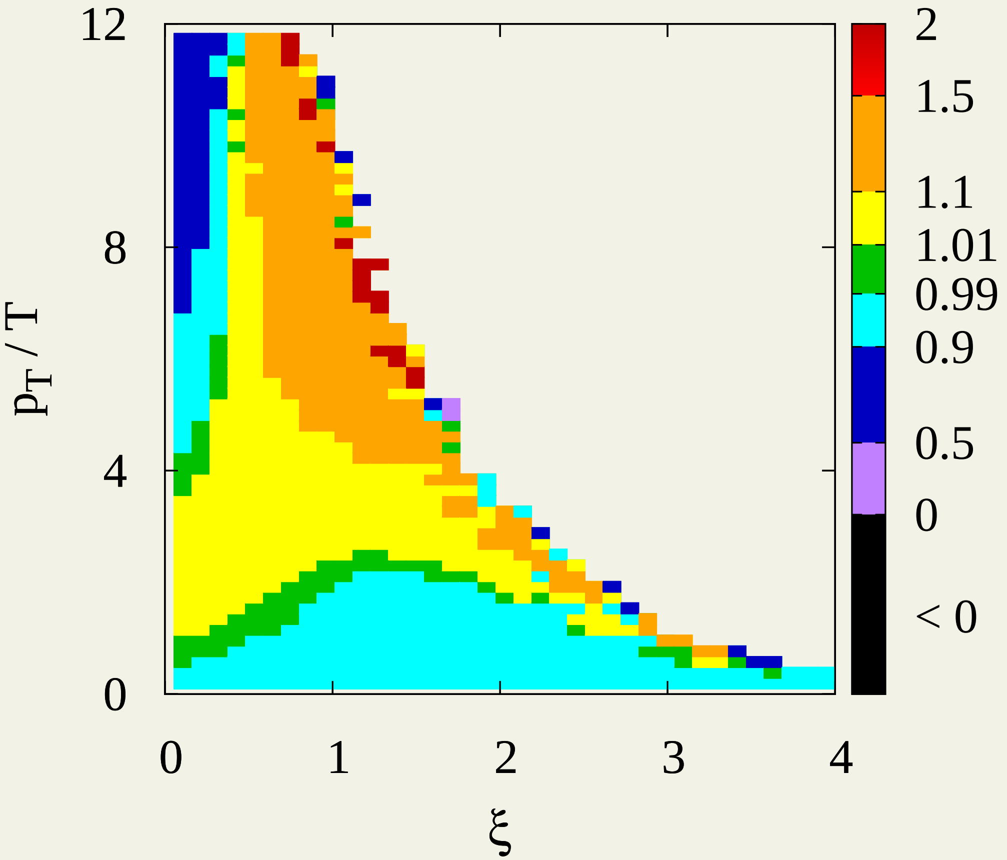


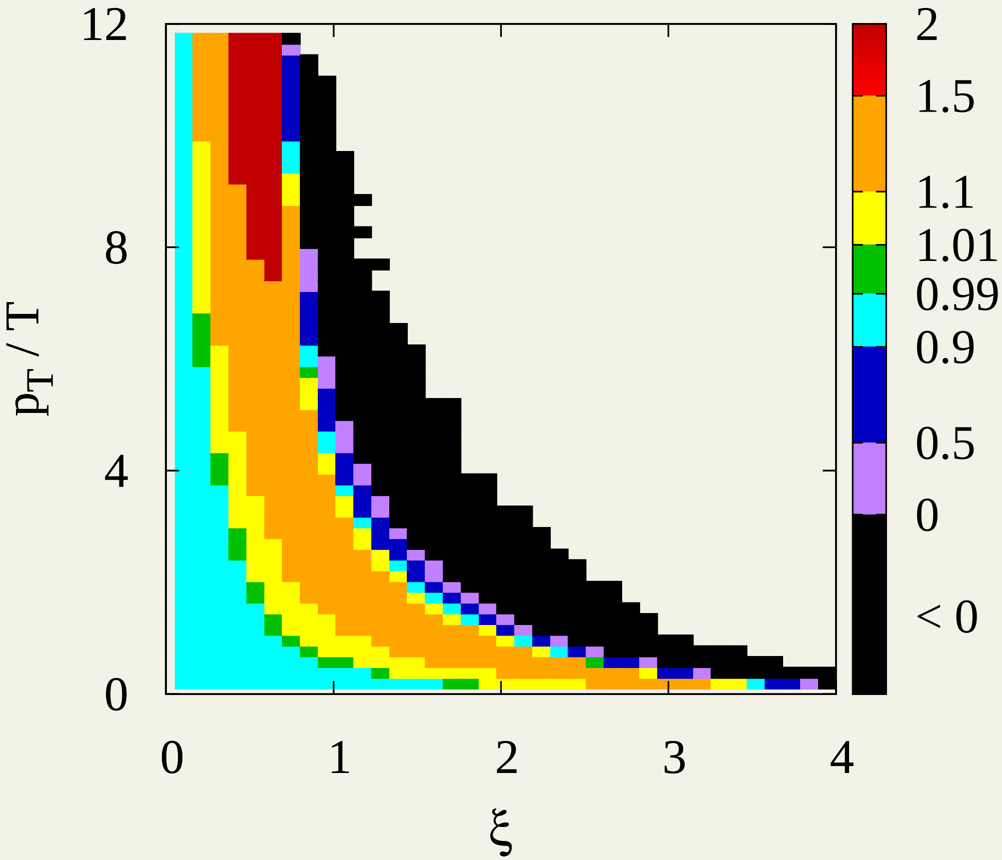
Grad (IS)



$$\frac{\tau}{\tau_0} = 2, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.33, \;\; \frac{\pi_L}{p} = -0.29\right)$$

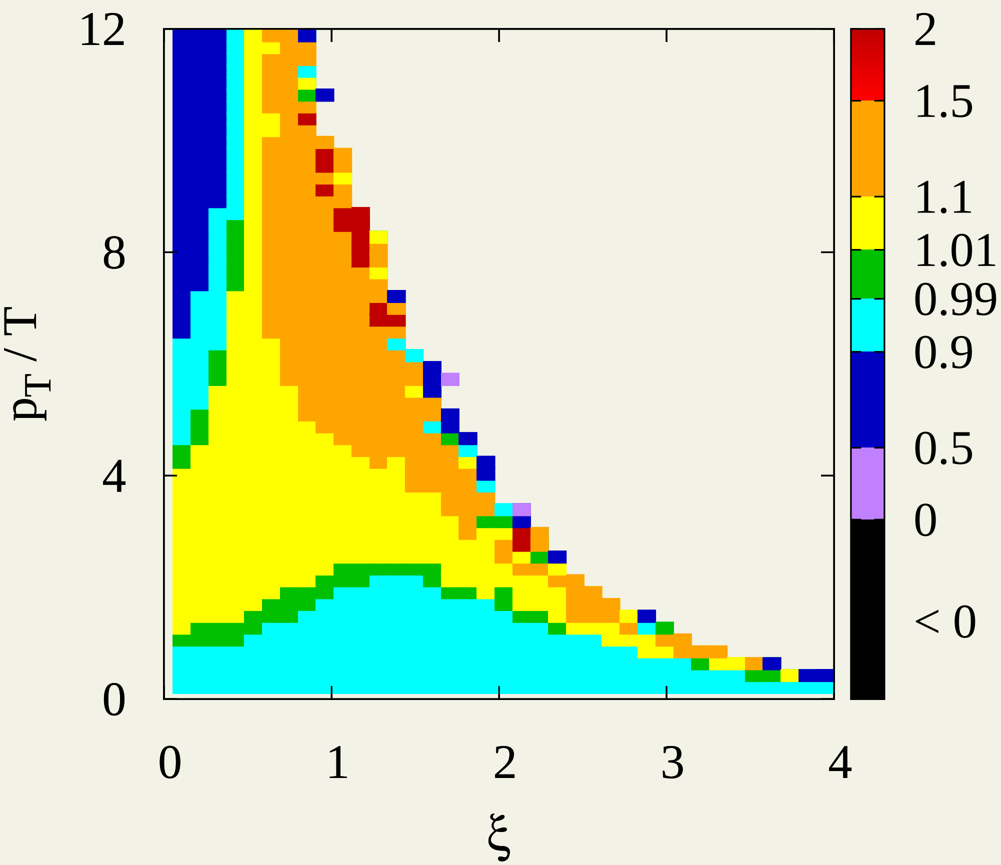
SR ansatz **Grad (IS)**

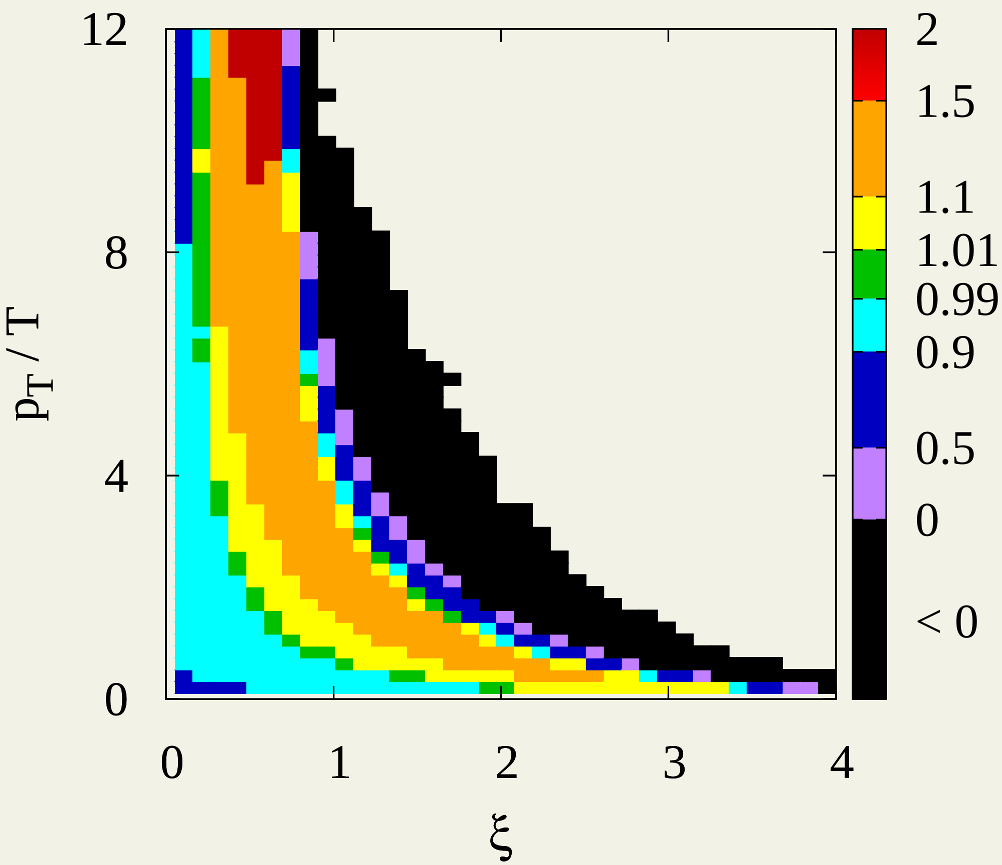




$$\frac{\tau}{\tau_0} = 3, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.25, \quad \frac{\pi_L}{p} = -0.26\right)$$

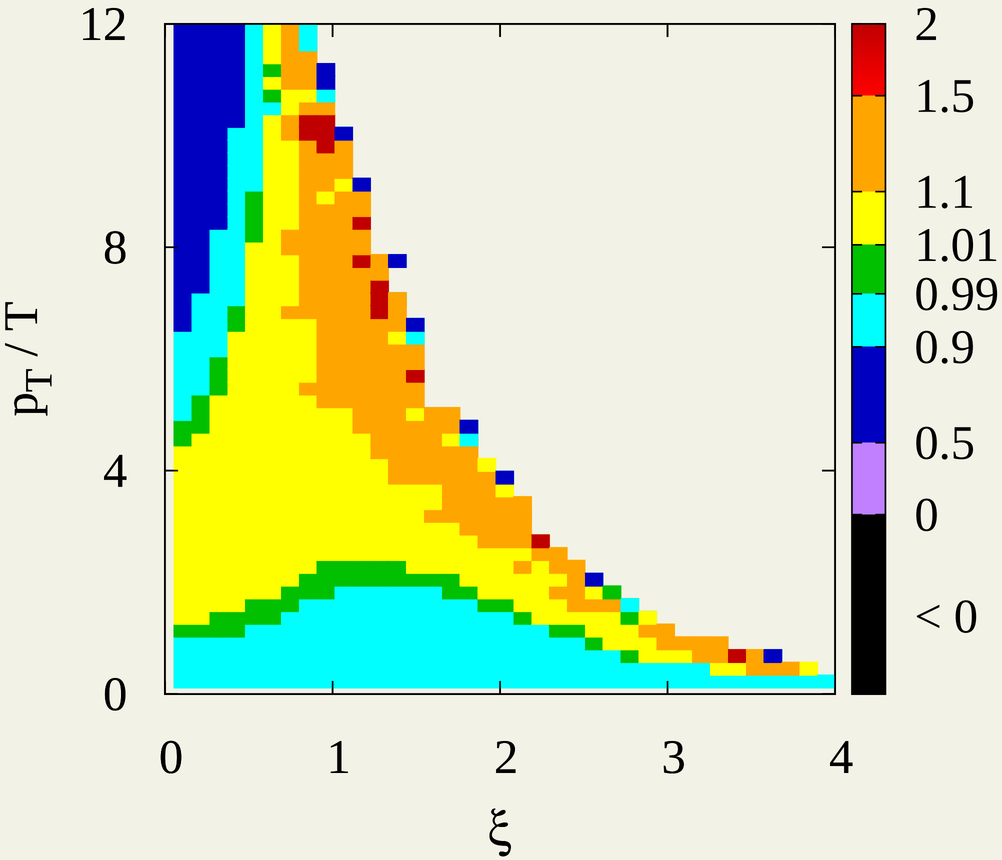
SR ansatz **Grad (IS)**

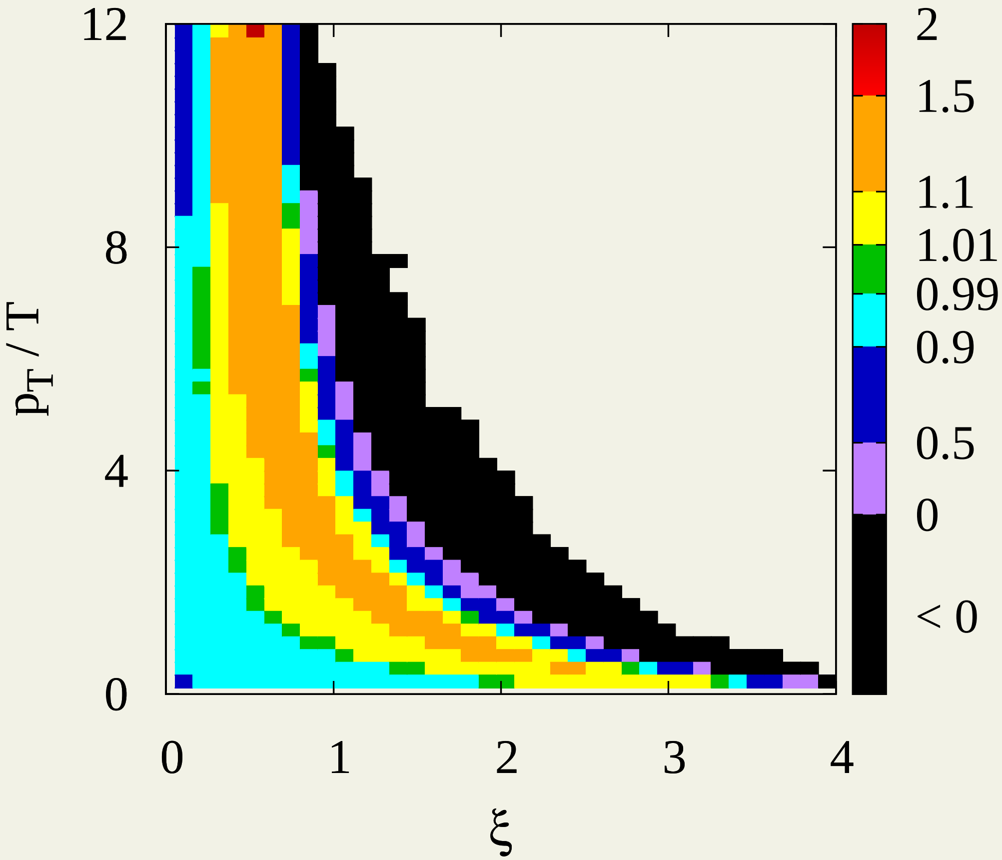




$$\frac{\tau}{\tau_0} = 4, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.21, \;\; \frac{\pi_L}{p} = -0.22\right)$$

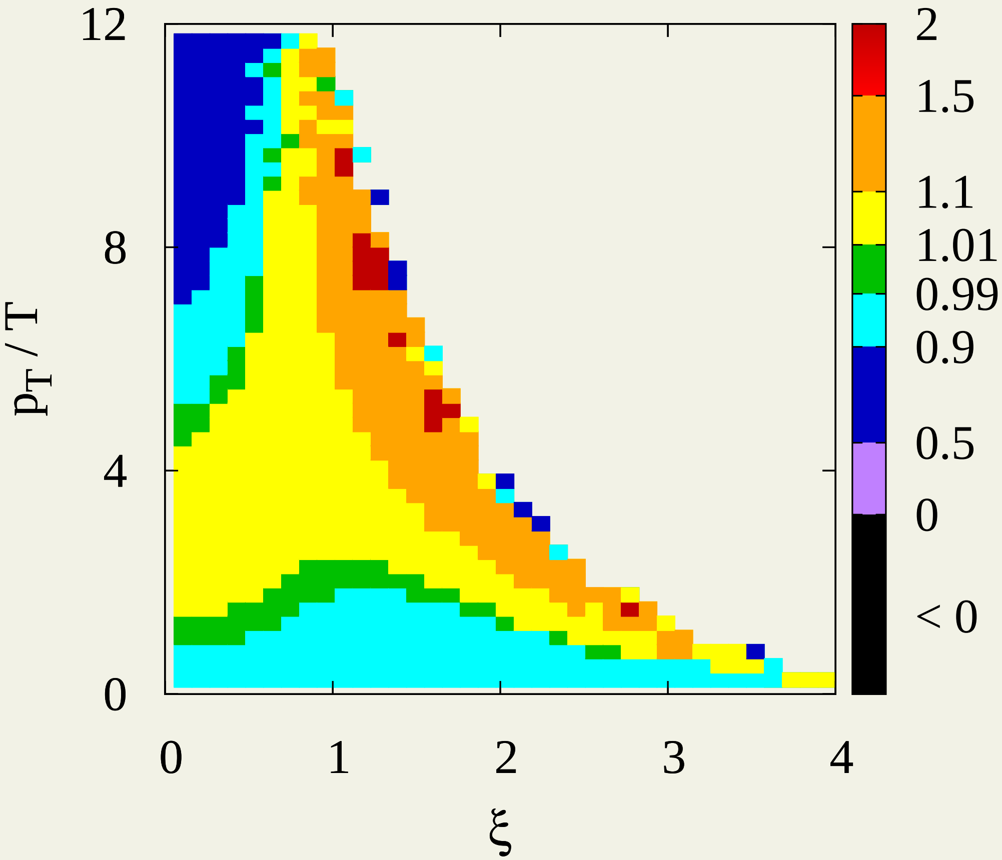
SR ansatz **Grad (IS)**



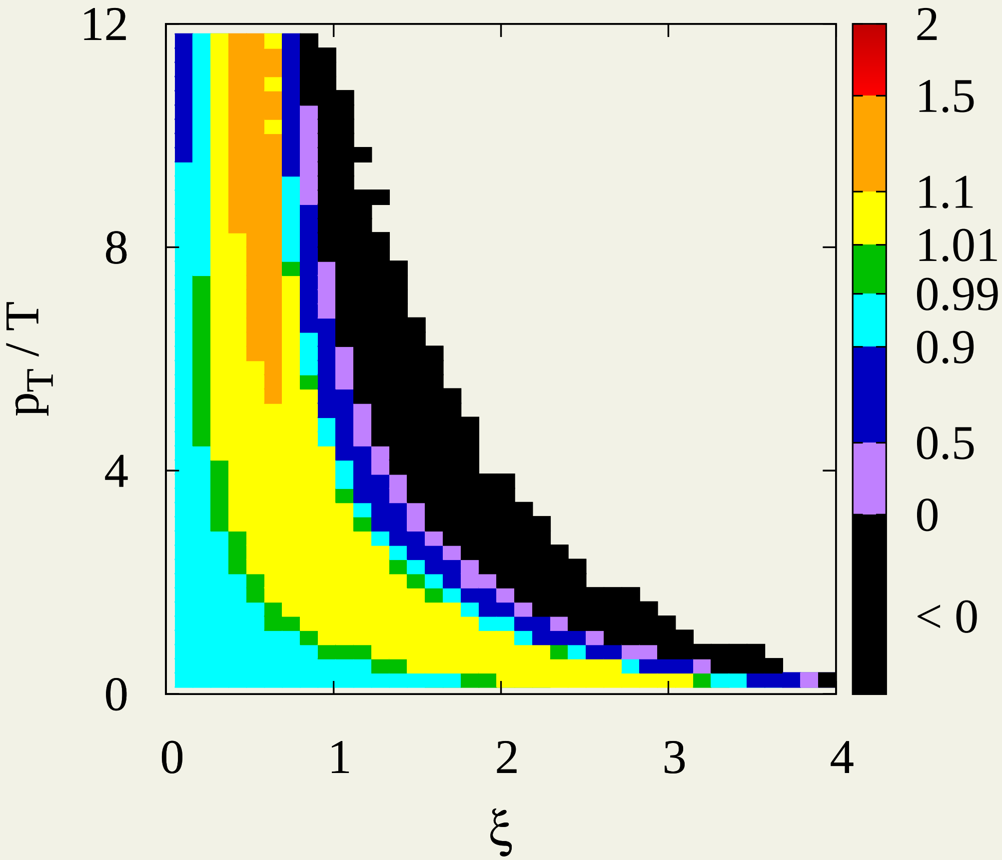


$$\frac{\tau}{\tau_0} = 6, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.16, \;\; \frac{\pi_L}{p} = -0.17\right)$$

SR ansatz

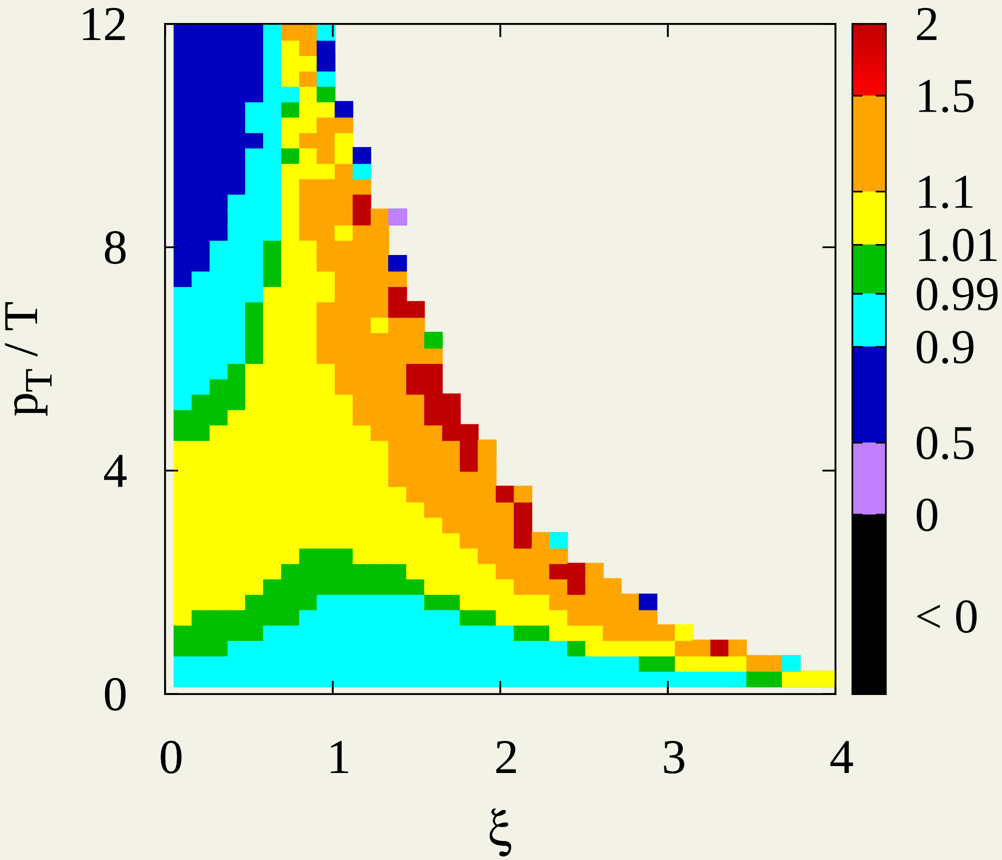


Grad (IS)



$$\frac{\tau}{\tau_0} = 8, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.13, \ \ \frac{\pi_L}{p} = -0.14\right)$$

SR ansatz



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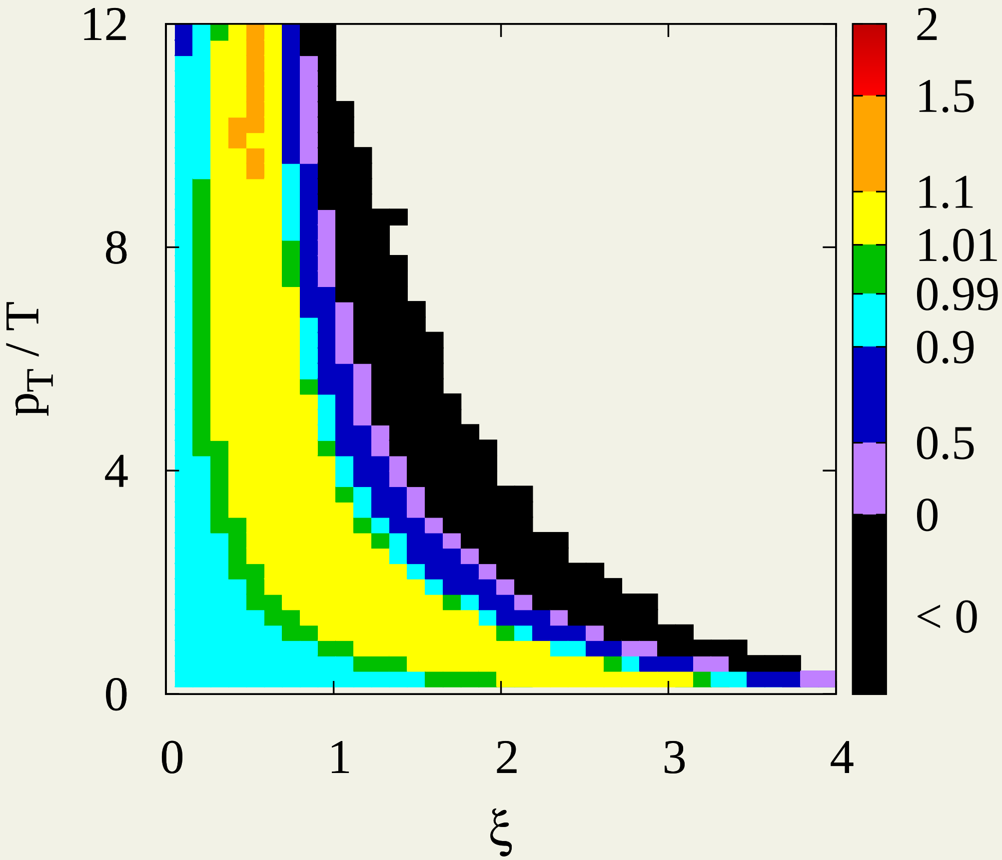
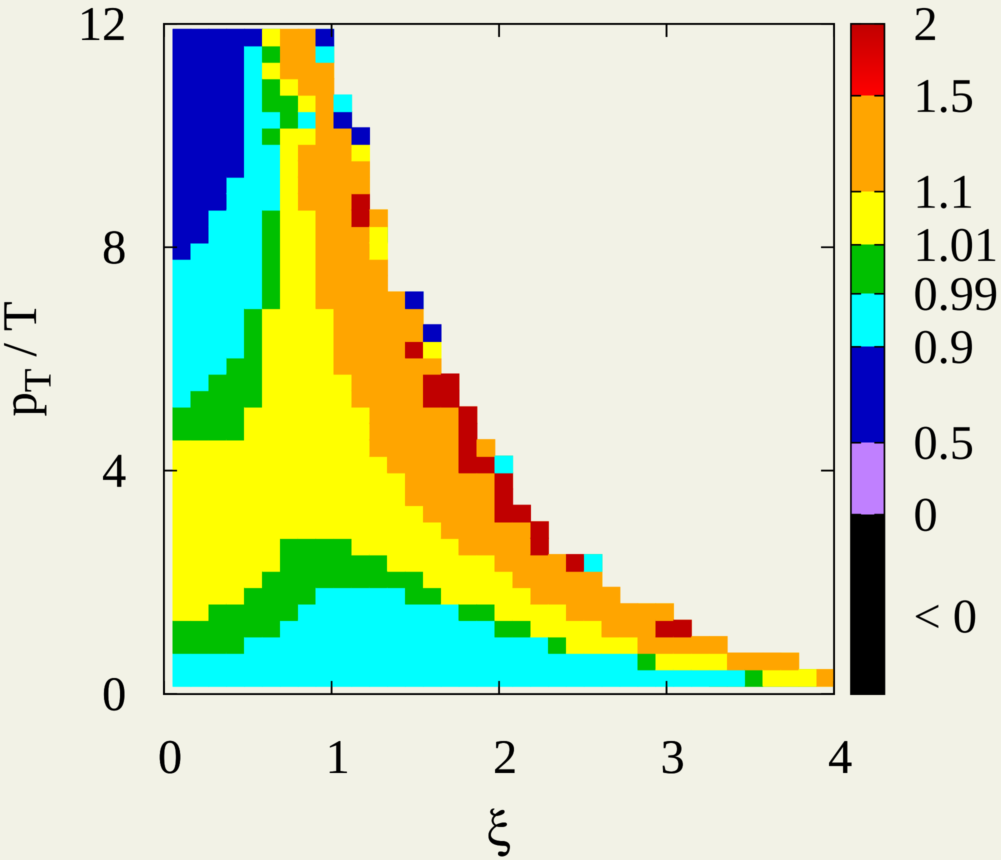


$$\frac{\tau}{\tau_0} = 10, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.11, \;\; \frac{\pi_L}{p} = -0.12\right)$$

SR ansatz



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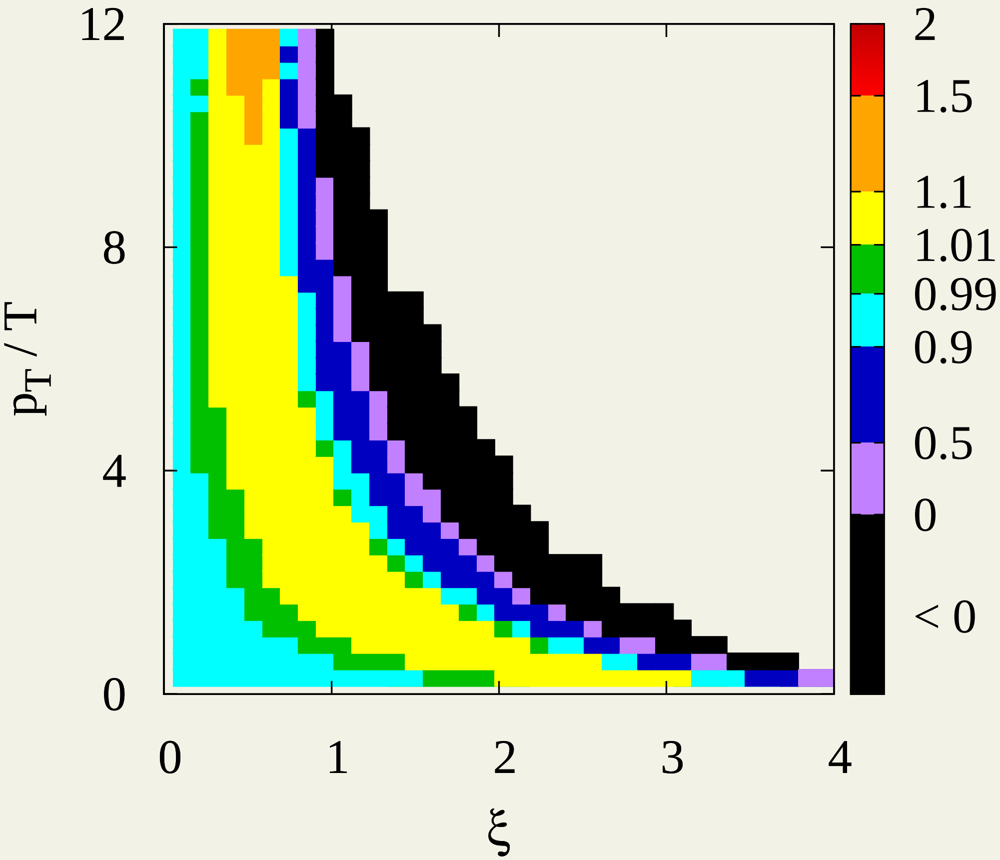
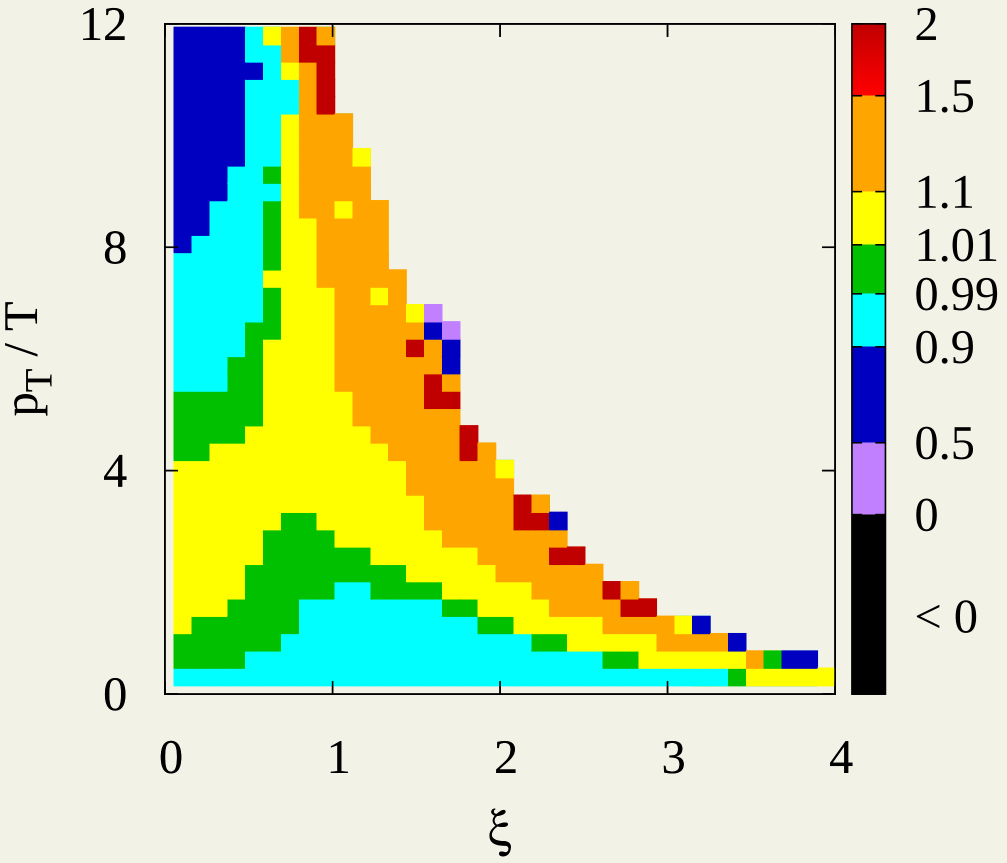
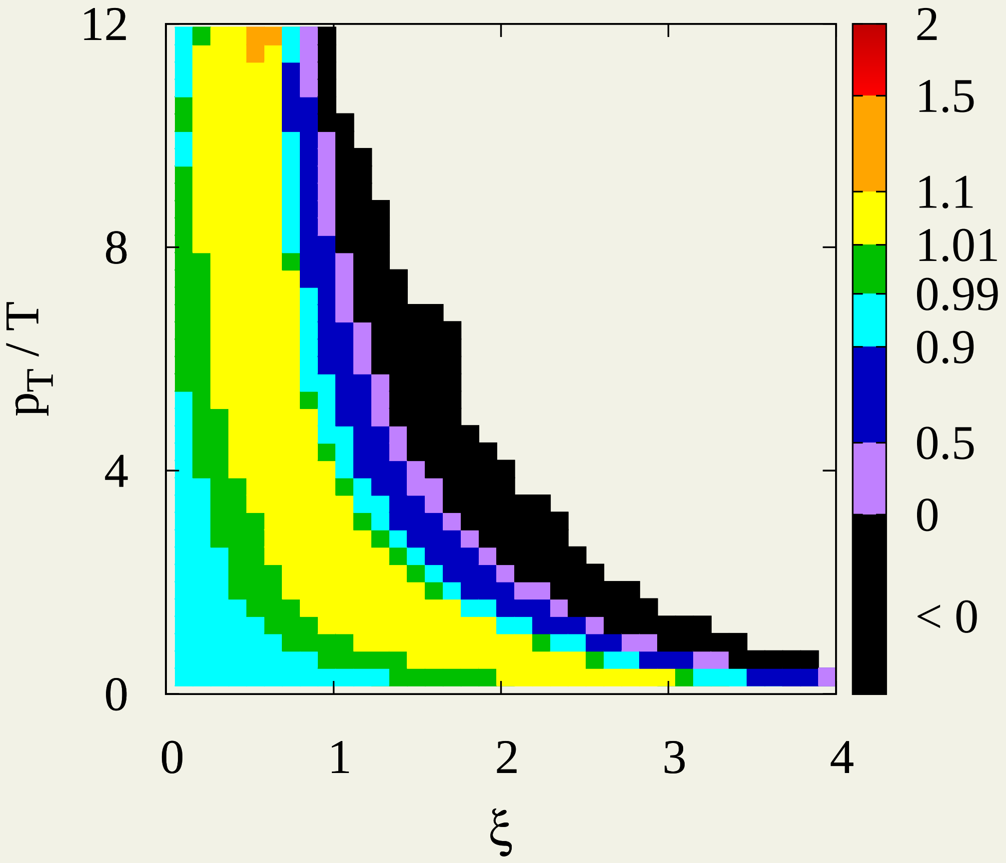


$$\frac{\tau}{\tau_0} = 12, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.10, \ \frac{\pi_L}{p} = -0.11\right)$$

SR ansatz

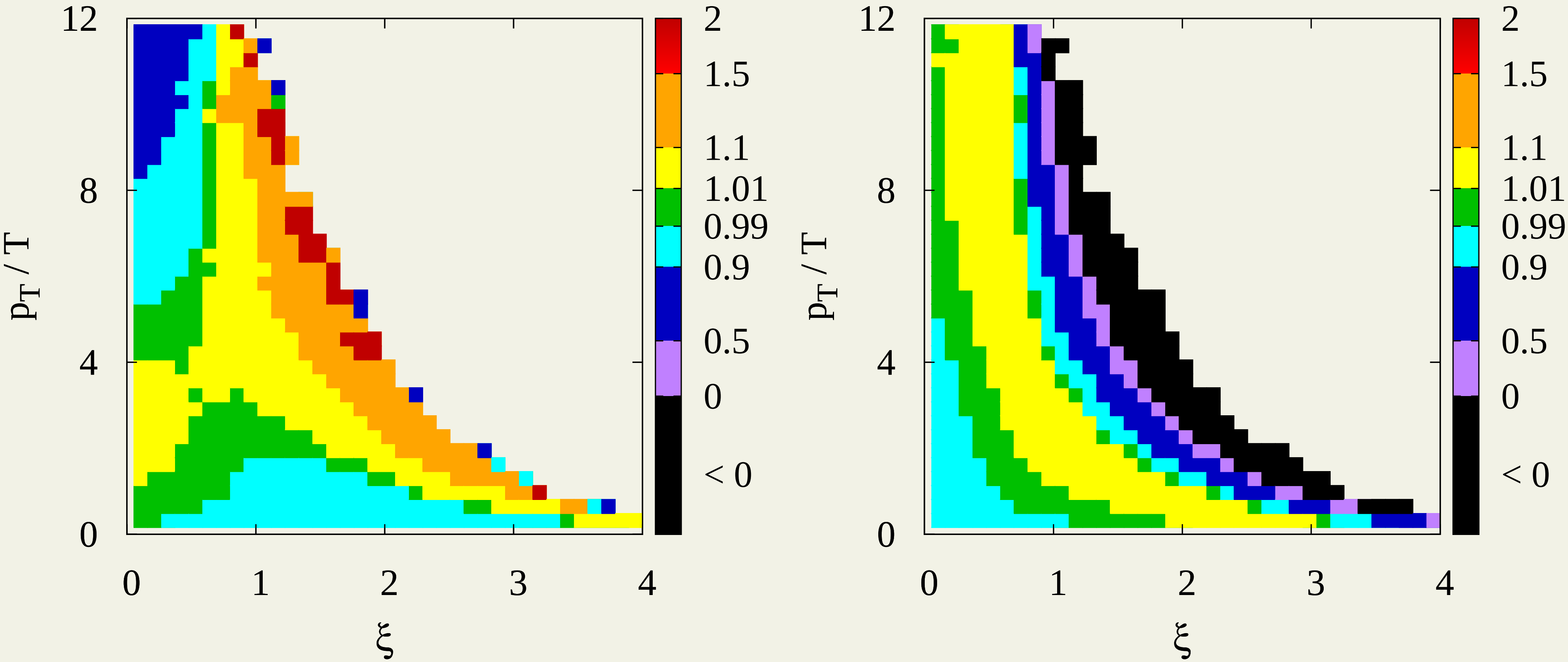


Grad (IS)



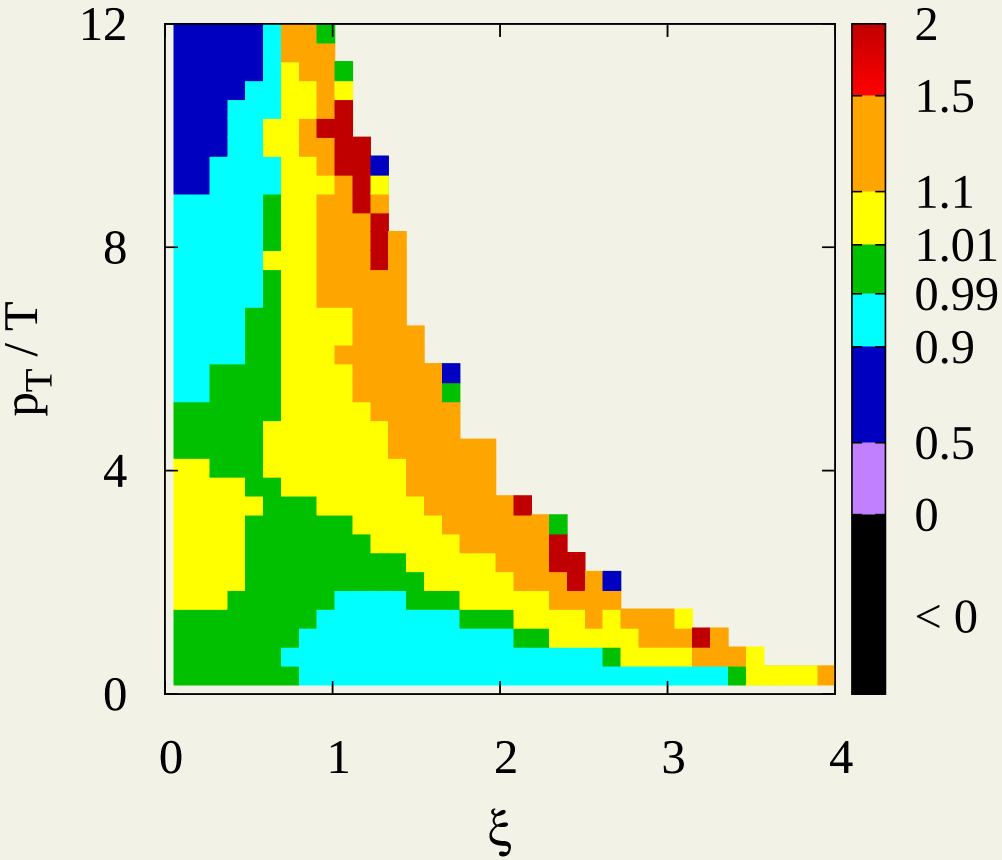
$$\frac{\tau}{\tau_0} = 14, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.09, \quad \frac{\pi_L}{p} = -0.10\right)$$

SR ansatz **Grad (IS)**

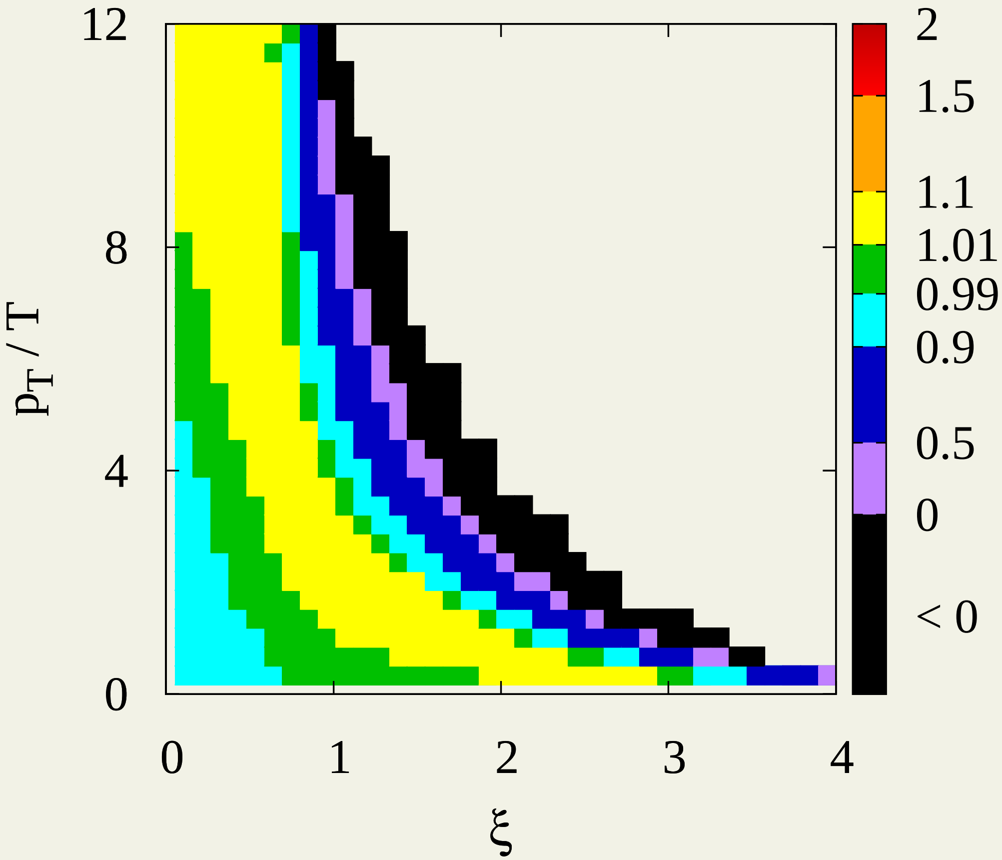


$$\frac{\tau}{\tau_0} = 16, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.08, \quad \frac{\pi_L}{p} = -0.09\right)$$

SR ansatz

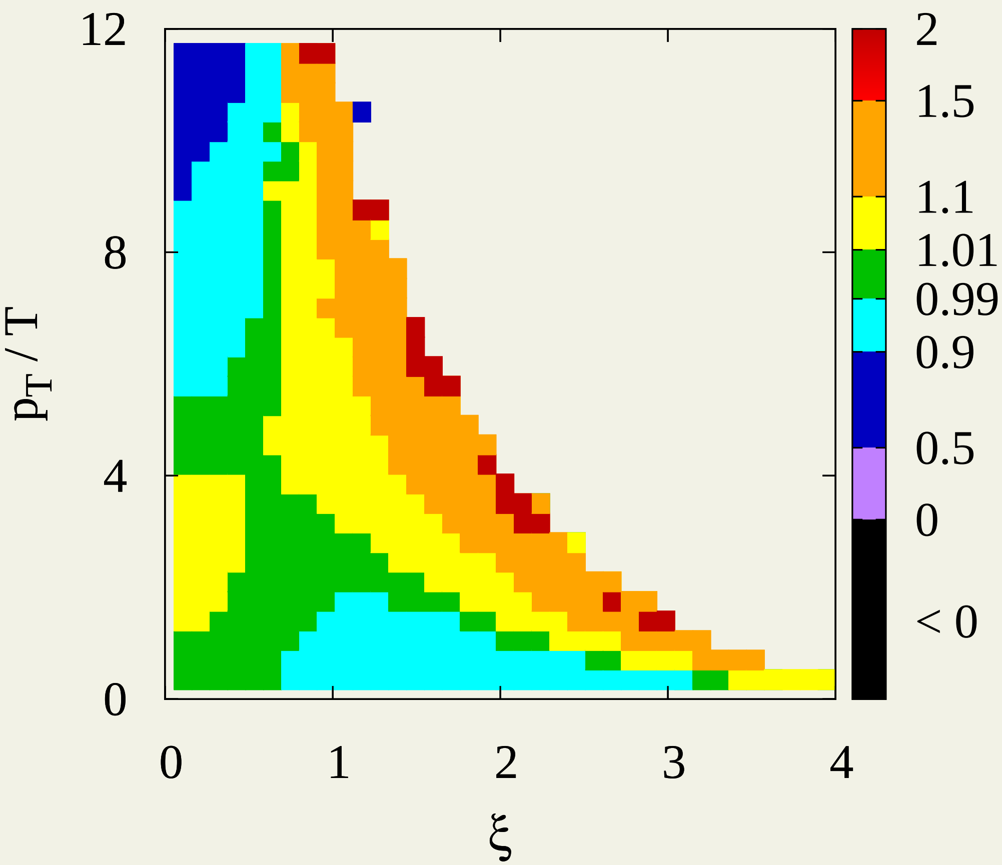


Grad (IS)

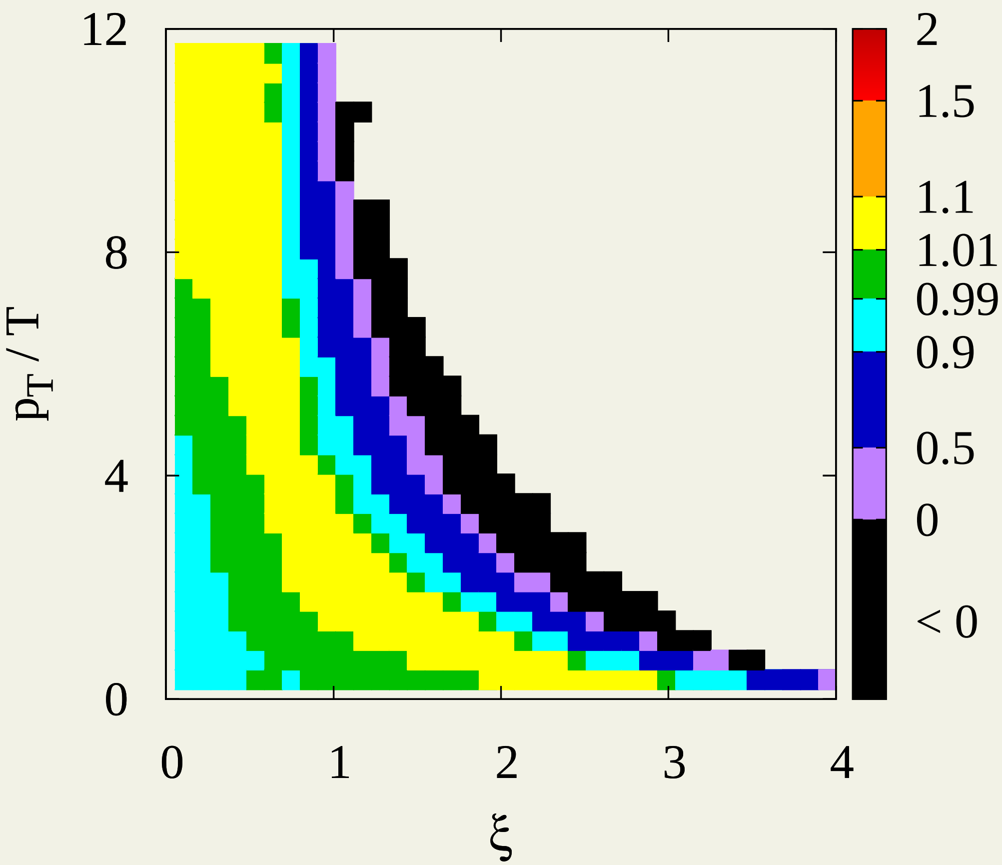


$$\frac{\tau}{\tau_0} = 18, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.08, \quad \frac{\pi_L}{p} = -0.08\right)$$

SR ansatz

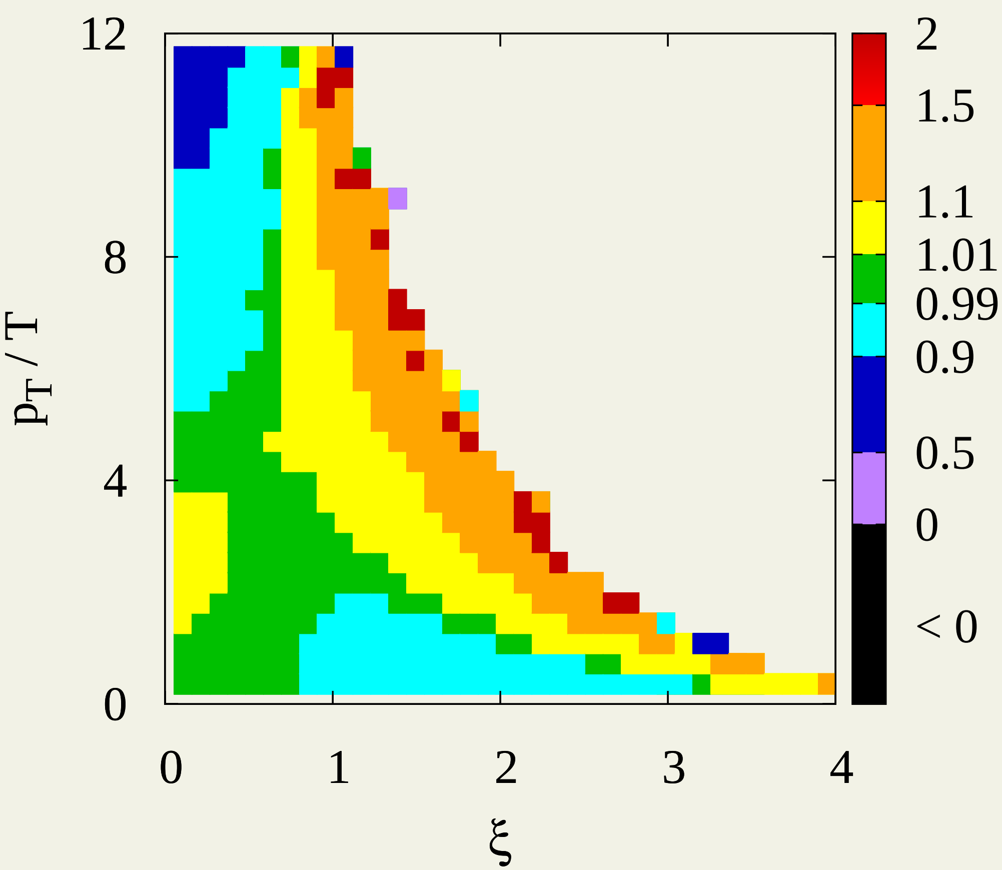


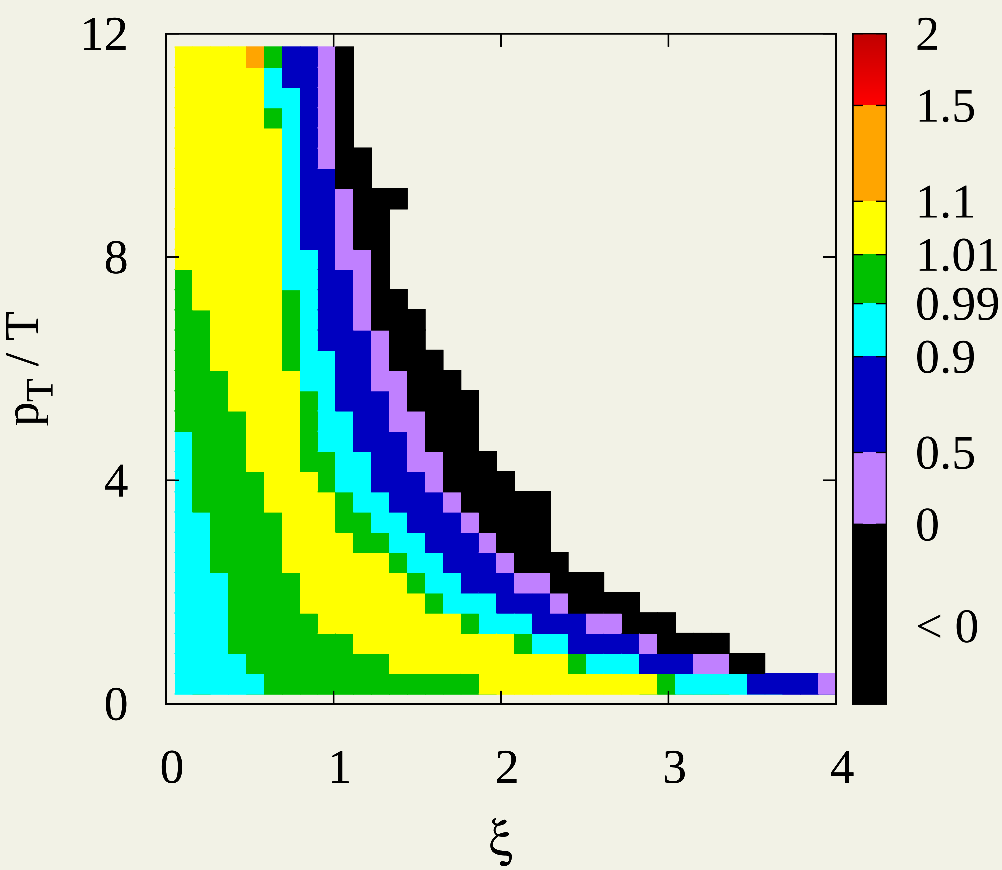
Grad (IS)



$$\frac{\tau}{\tau_0} = 20, \quad K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3} \qquad \left(\frac{\tau_{REL}}{\tau_{exp}} = 0.07, \ \frac{\pi_L}{p} = -0.08\right)$$

SR ansatz **Grad (IS)**





$\Rightarrow$

At early times or high ξ, SR looks better

At high pT and small ξ , Grad looks better.

Now compare four models:

a) SR, b) IS/Grad, c) IS with $p^{1.5}$, d) IS with p^1 (RTA)

study viscosity dependence $\frac{\eta}{s} \sim 0.03 - 0.2$ $[K_0 = 1, 2, 6.49]$

$$\frac{\boldsymbol{\eta}}{\boldsymbol{s}} \sim \mathbf{0.2} \qquad [K_0 = 1]$$

$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 1\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.93,\quad \frac{\pi_L}{p} = -0.17\right)$$

$+100\%$

$+50\%$

$+10\%$

$\pm 1\%$

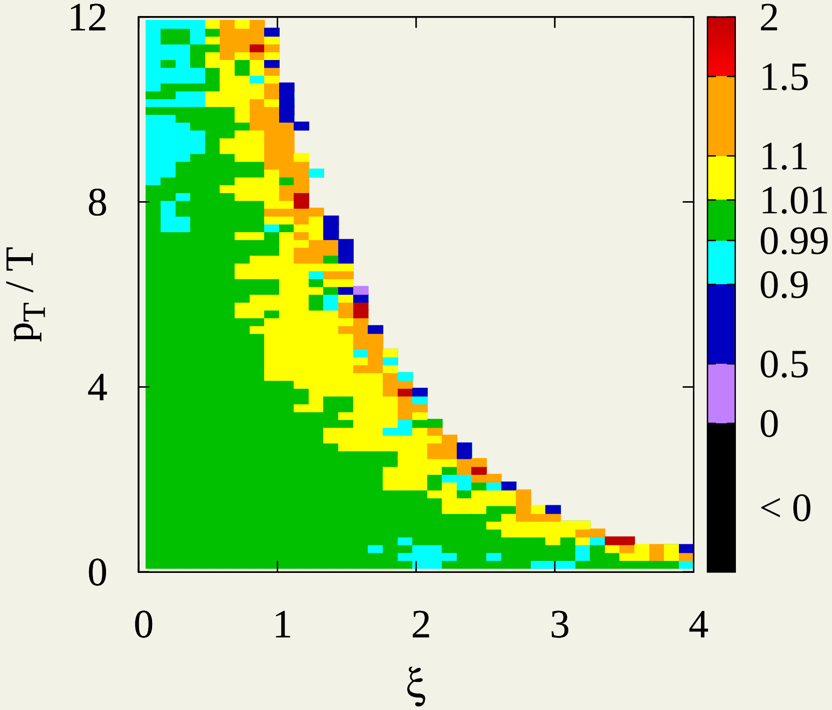
-10%

-50%

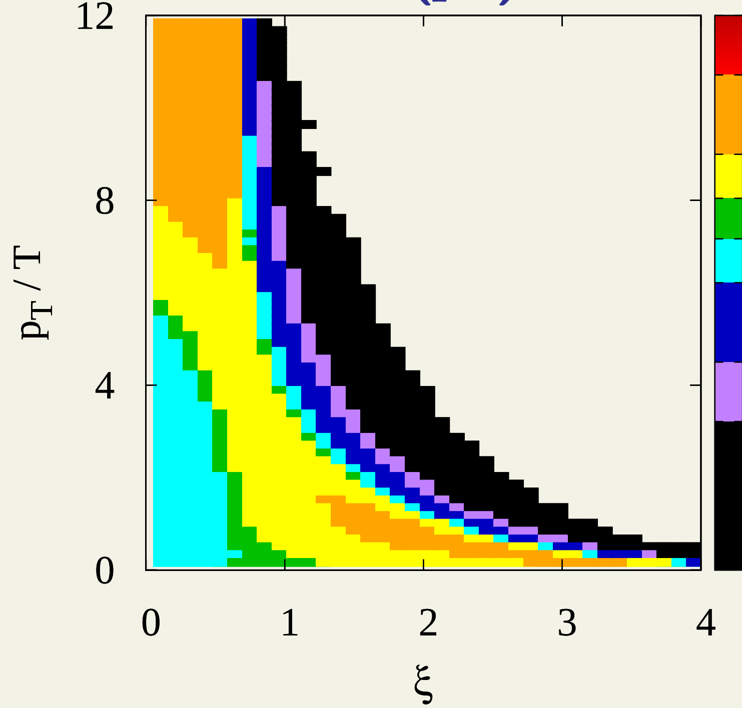
-100%

< 0

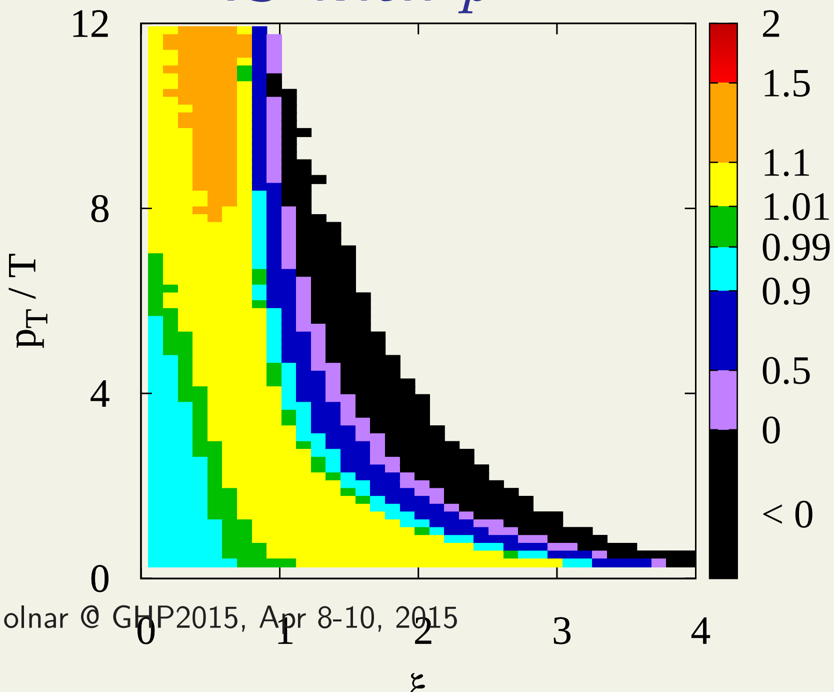
SR ansatz



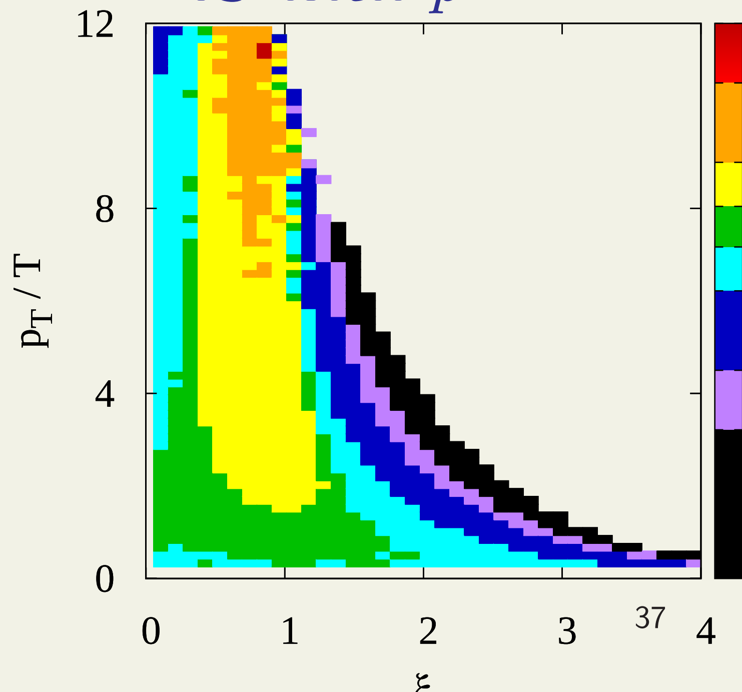
Grad IS (p^2)



IS with $p^{1.5}$



IS with p^1



$$\frac{\tau}{\tau_0} = 3$$

$$K = 1\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.51,\right.$$

$$\left.\frac{\pi_L}{p} = -0.41\right)$$

+100%

+50%

+10%

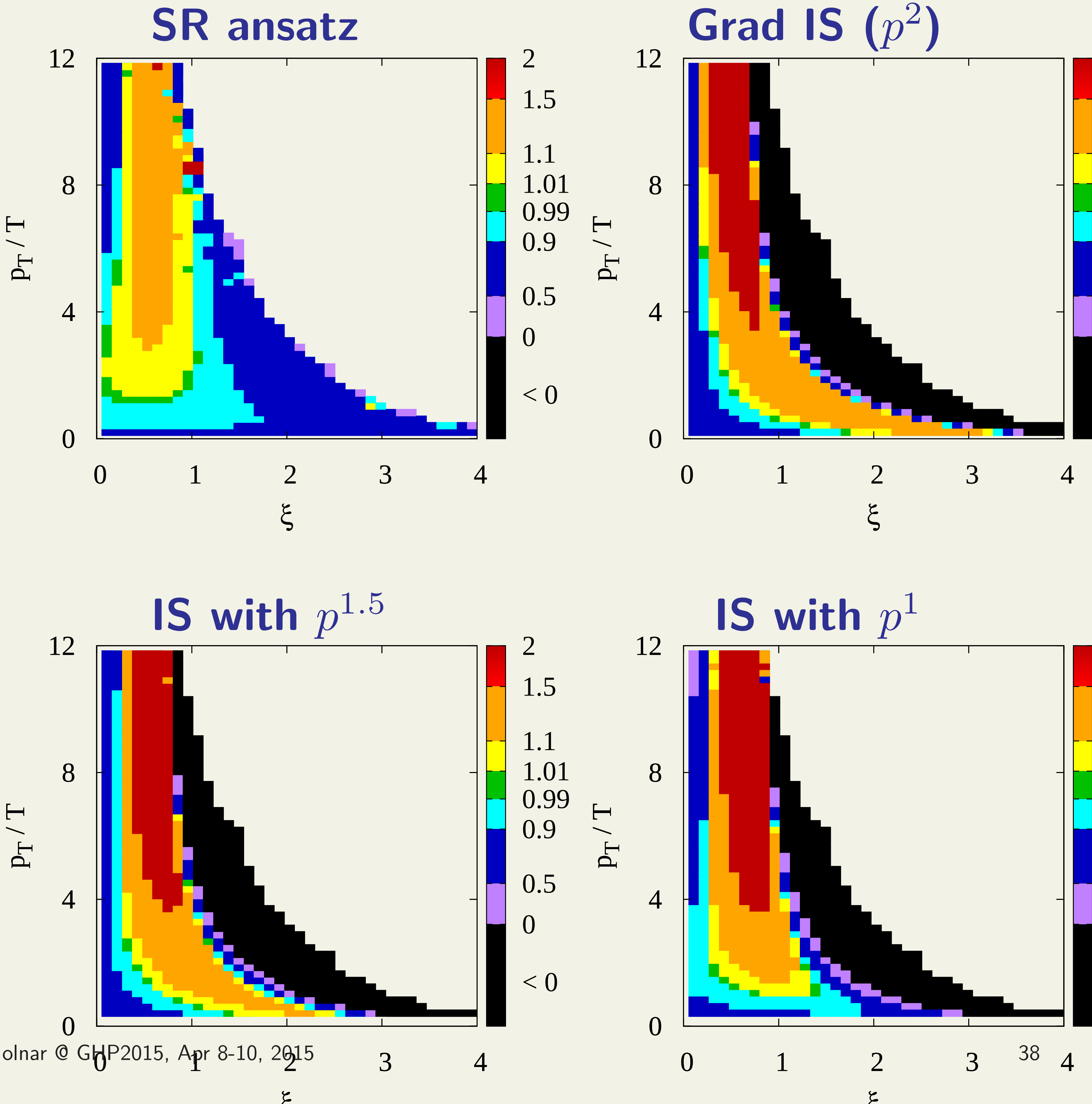
±1%

−10%

−50%

−100%

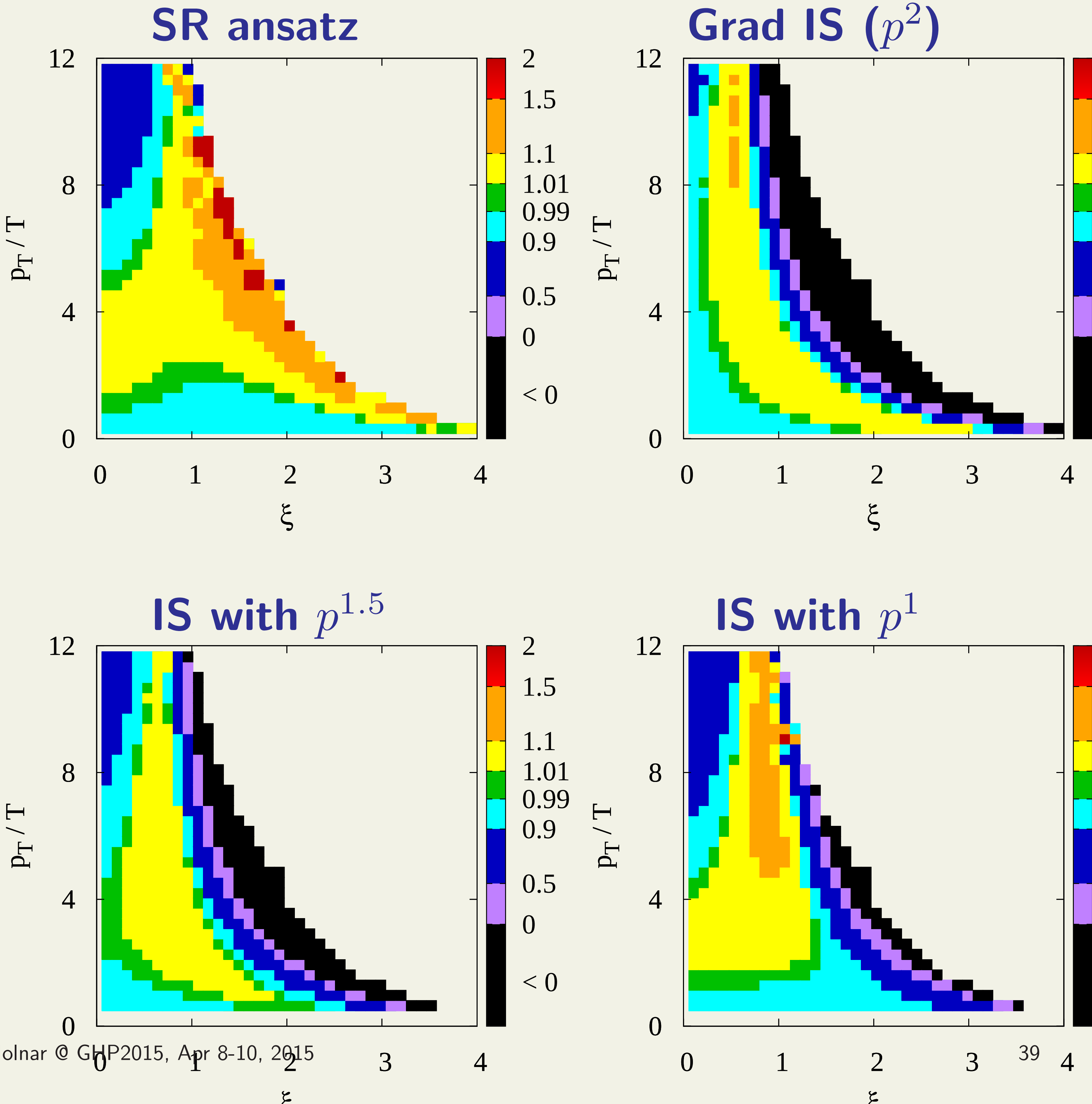
< 0



$$\frac{\tau}{\tau_0} = 20$$

$$K = 1\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 14, \quad \frac{\pi_L}{p} = -0.16\right)$$



+100%

+50%

+10%

±1%

−10%

−50%

−100%

< 0

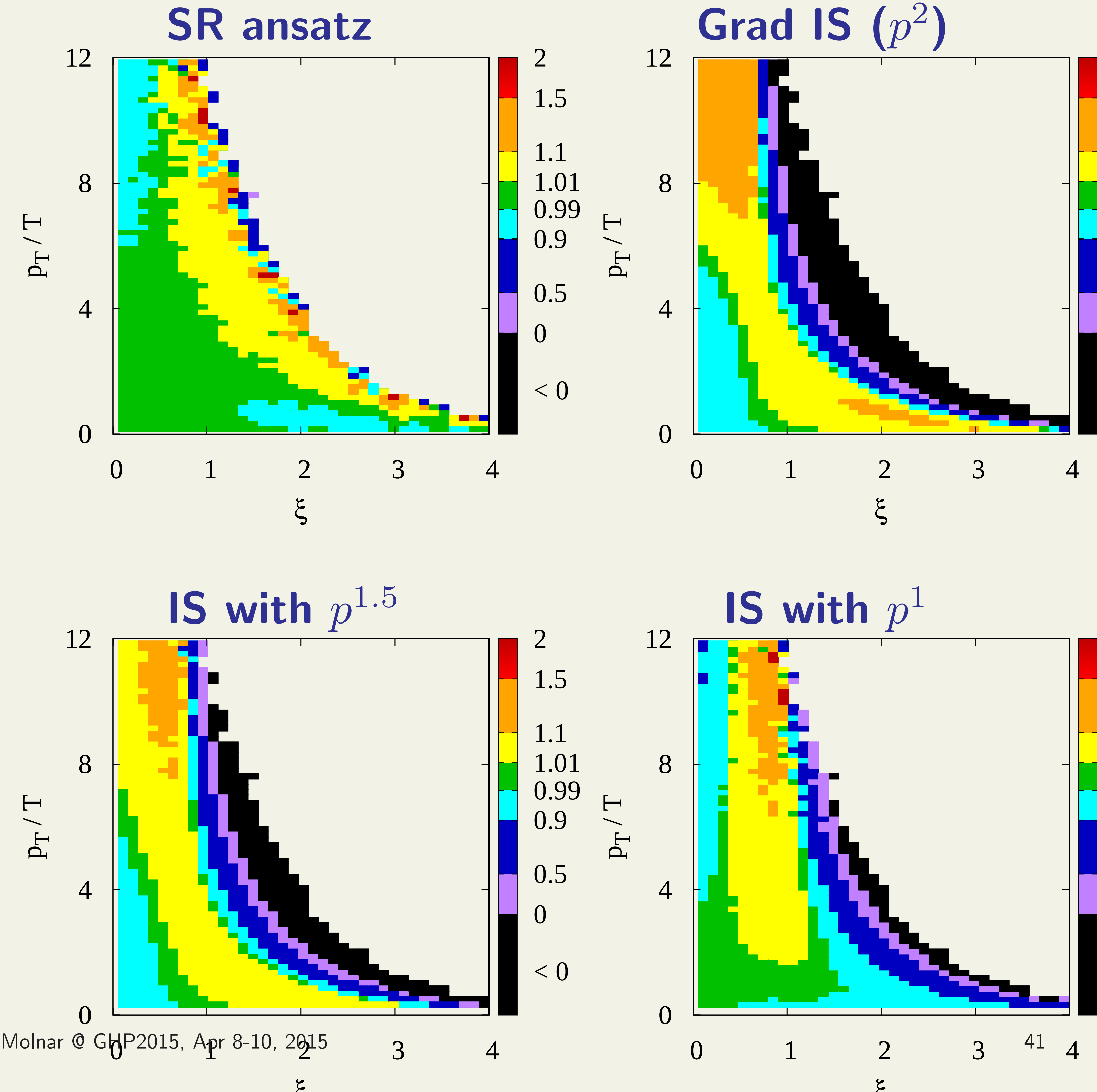
$$\frac{\boldsymbol{\eta}}{\boldsymbol{s}} \sim \mathbf{0.1} \qquad [K_0 = 2]$$

$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.47,\right.$$

$$\left.\frac{\pi_L}{p} = -0.15\right)$$



$+100\%$
$+50\%$
$+10\%$
$\pm 1\%$
-10%
-50%
-100%
< 0

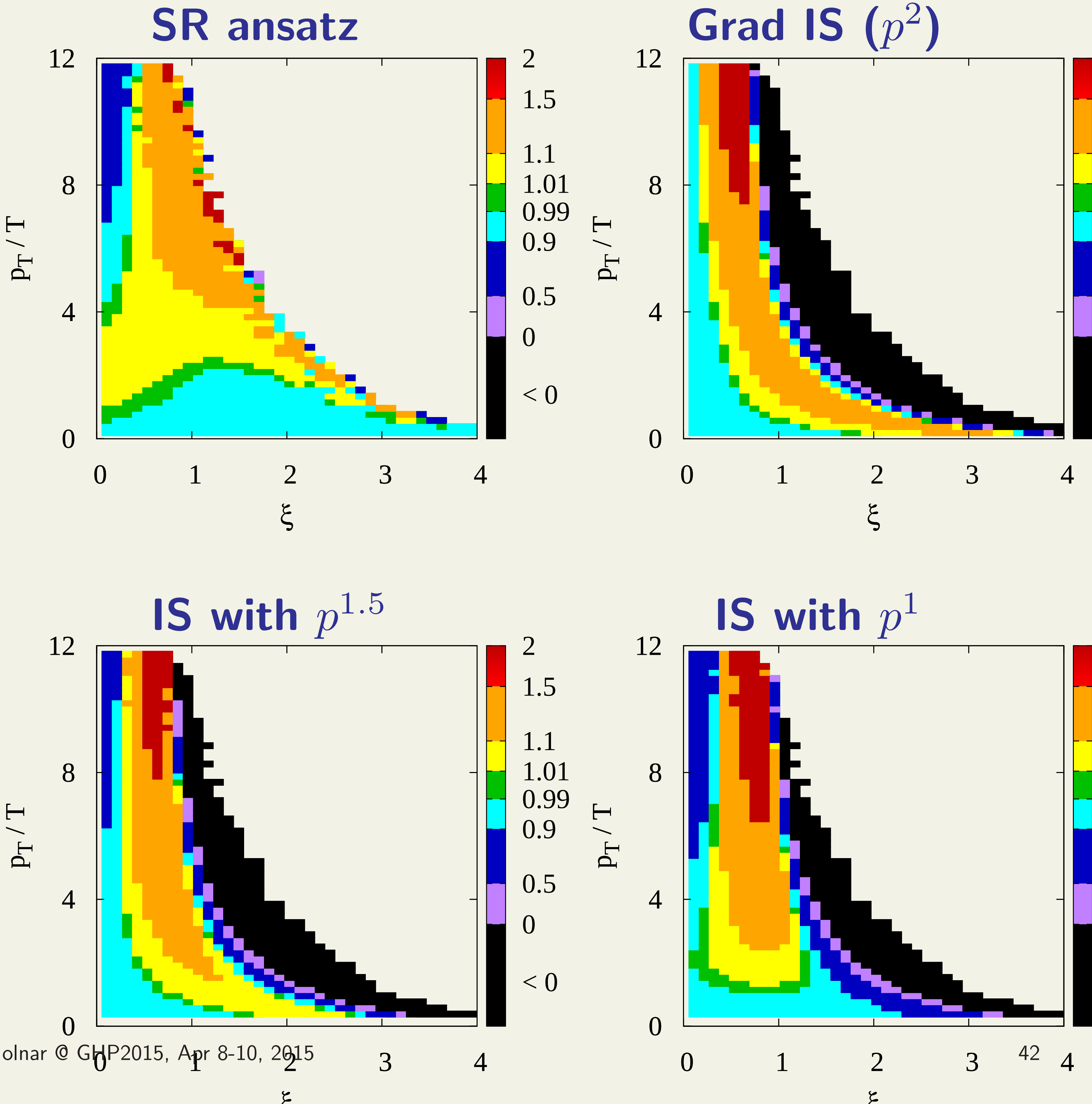
$$\frac{\tau}{\tau_0} = 2$$

$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.33,\right.$$

$$\left.\frac{\pi_L}{p} = -0.29\right)$$

+100%
+50%
+10%
±1%
−10%
−50%
−100%
< 0

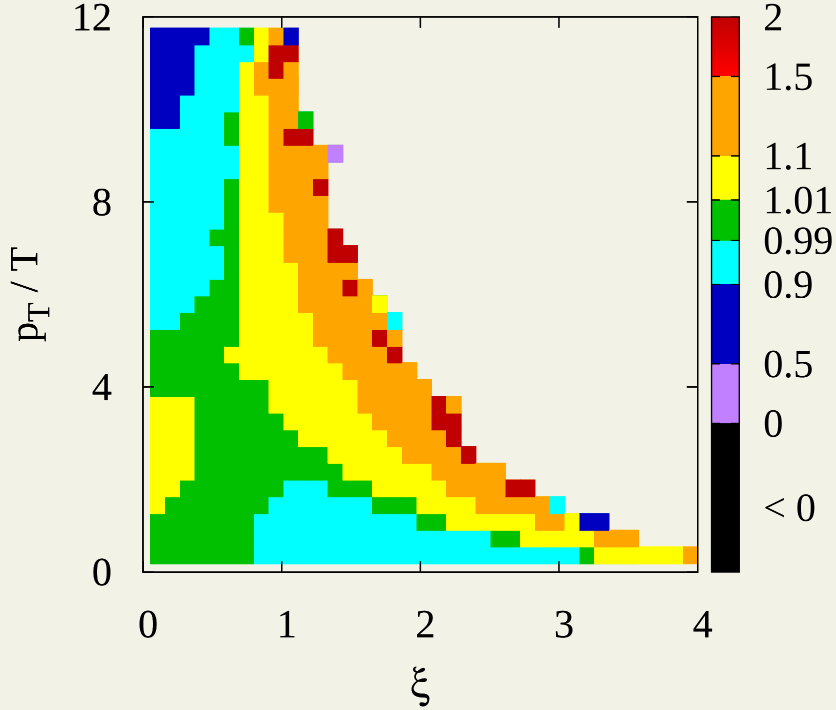


$$\frac{\tau}{\tau_0} = 20$$

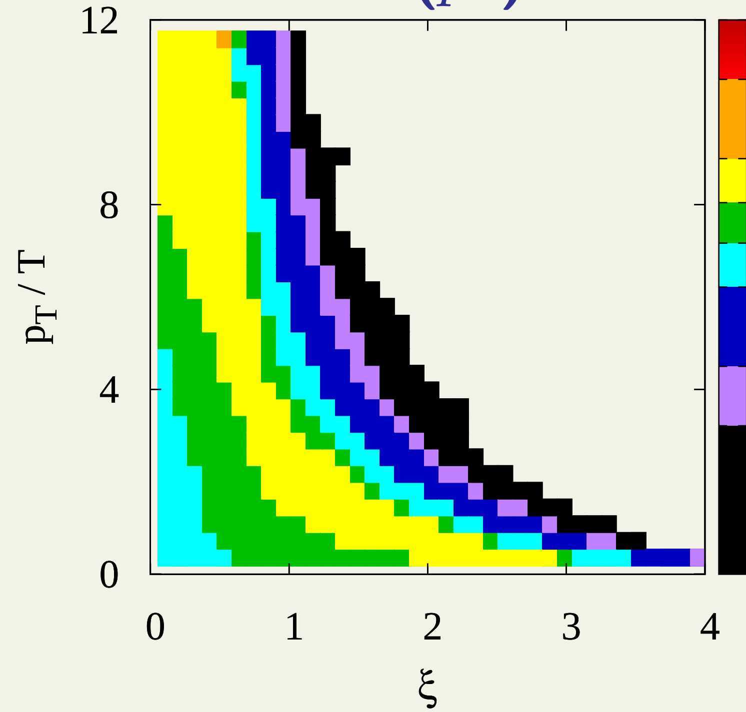
$$K = 2\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.07, \quad \frac{\pi_L}{p} = -0.08\right)$$

SR ansatz

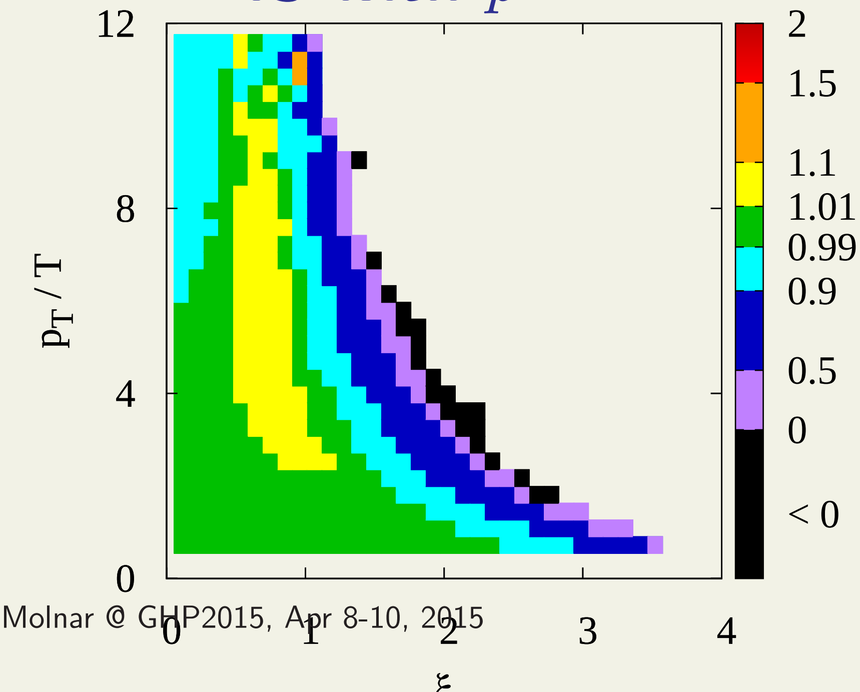


Grad IS (p^2)

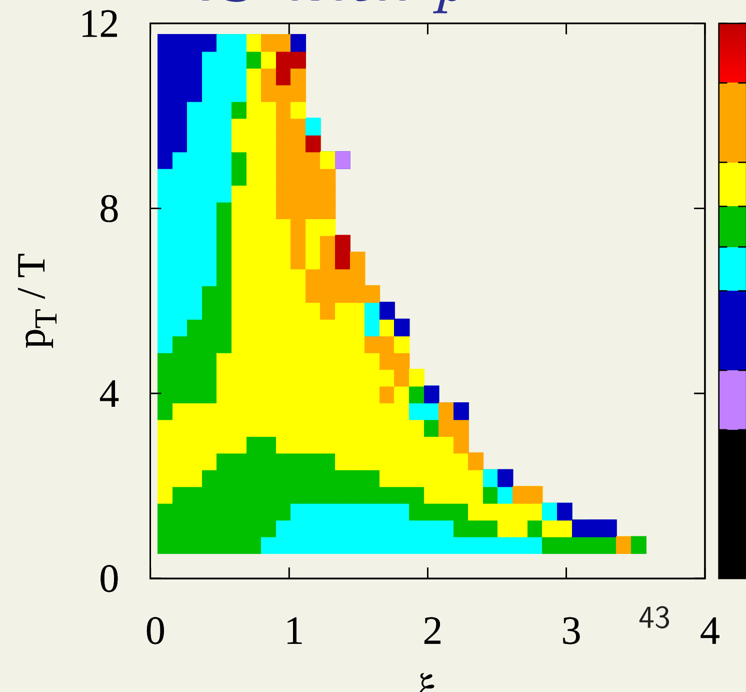


+100%
+50%
+10%
±1%
−10%
−50%
−100%
< 0

IS with $p^{1.5}$



IS with p^1



$$\boldsymbol{\frac{\eta}{s} \sim 0.03} \qquad [K_0 = 6.49]$$

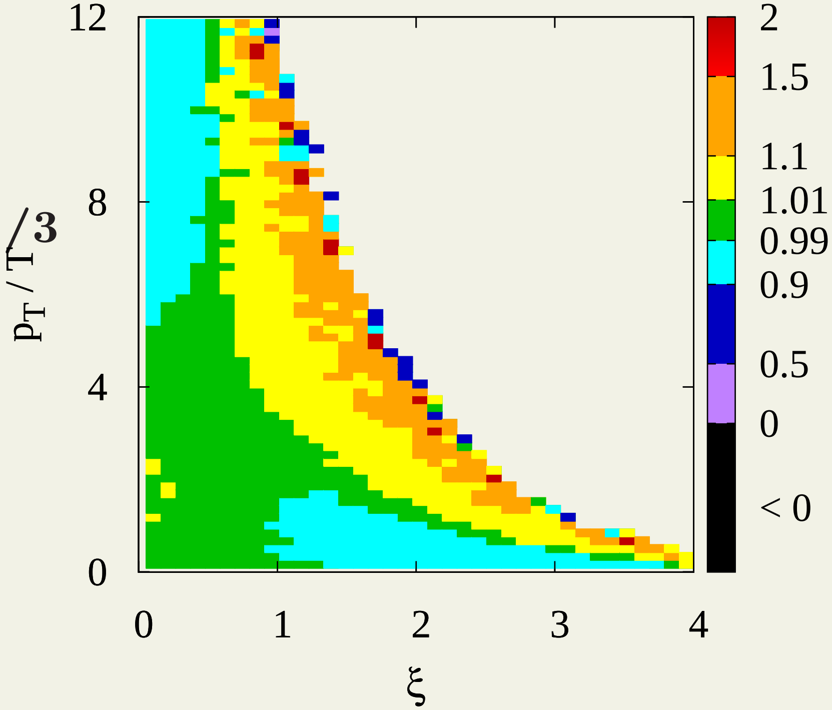
$$\frac{\tau}{\tau_0} = 1.2$$

$$K = 6.49 \left(\frac{\tau}{\tau_0}\right)^{2/3}$$

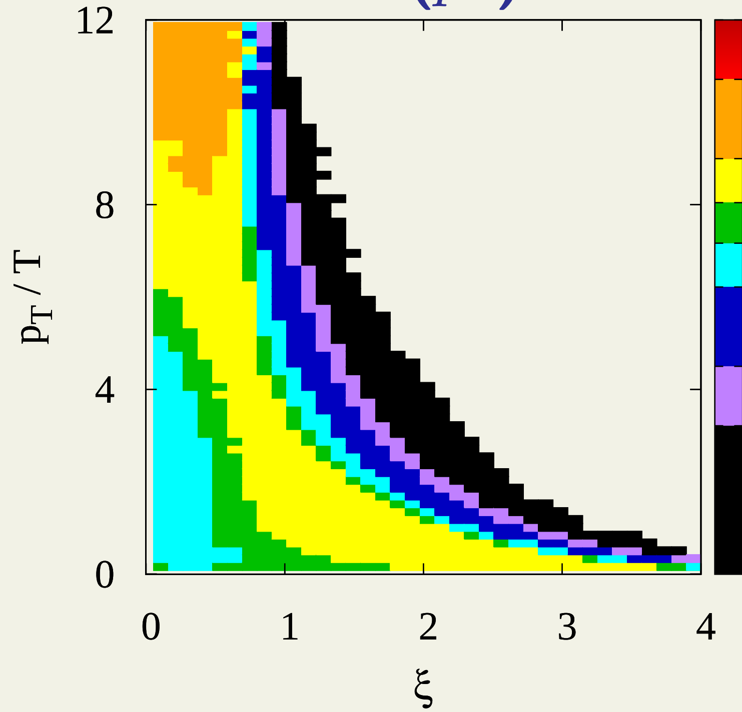
$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.14, \quad \frac{\pi_L}{p} = -0.11\right)$$

+100%
+50%
+10%
±1%
−10%
−50%
−100%
< 0

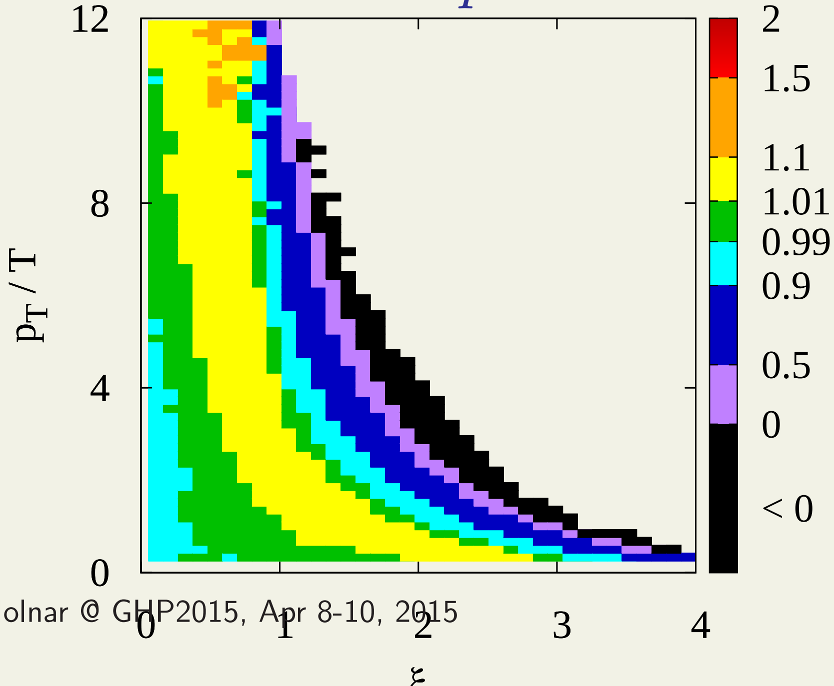
SR ansatz



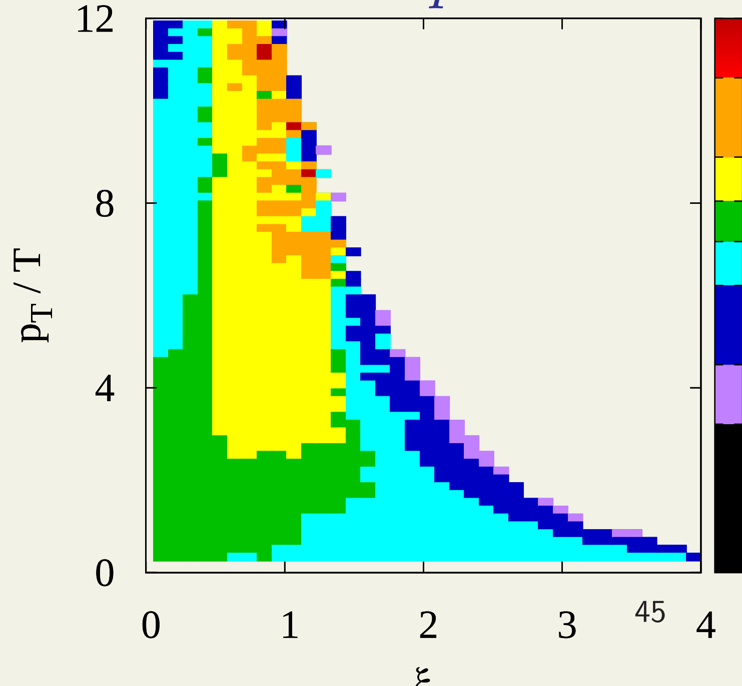
Grad IS (p^2)



IS with $p^{1.5}$



IS with p^1



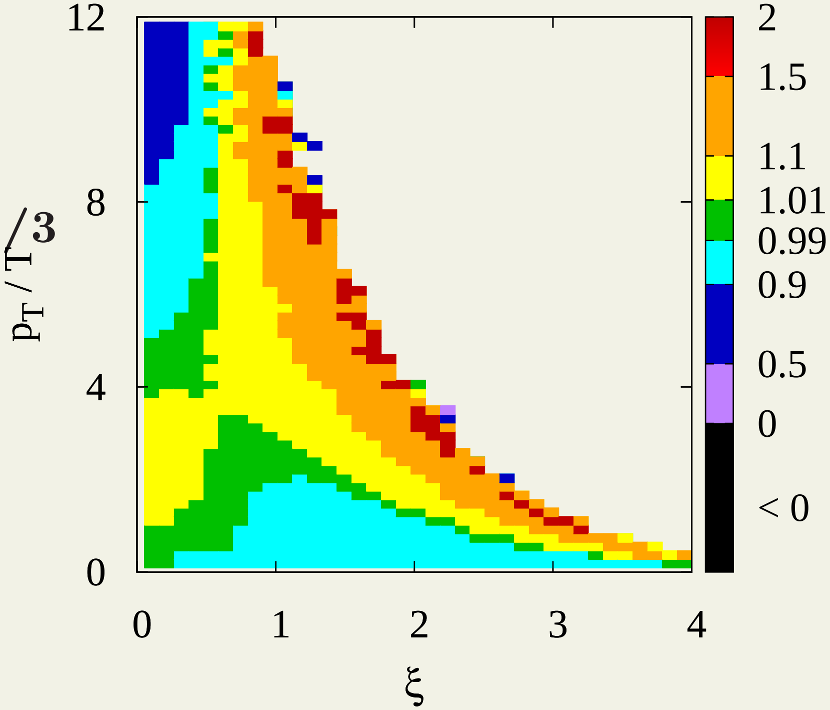
$$\frac{\tau}{\tau_0} = 1.6$$

$$K = 6.49\left(\frac{\tau}{\tau_0}\right)^{2/3}$$

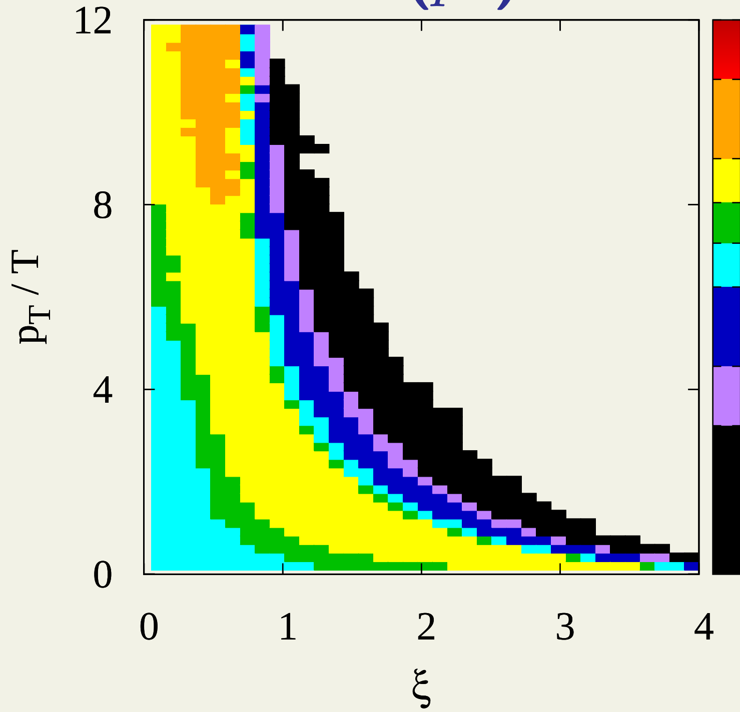
$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.12,\right.$$

$$\left.\frac{\pi_L}{p} = -0.12\right)$$

SR ansatz



Grad IS (p^2)



+100%

+50%

+10%

±1%

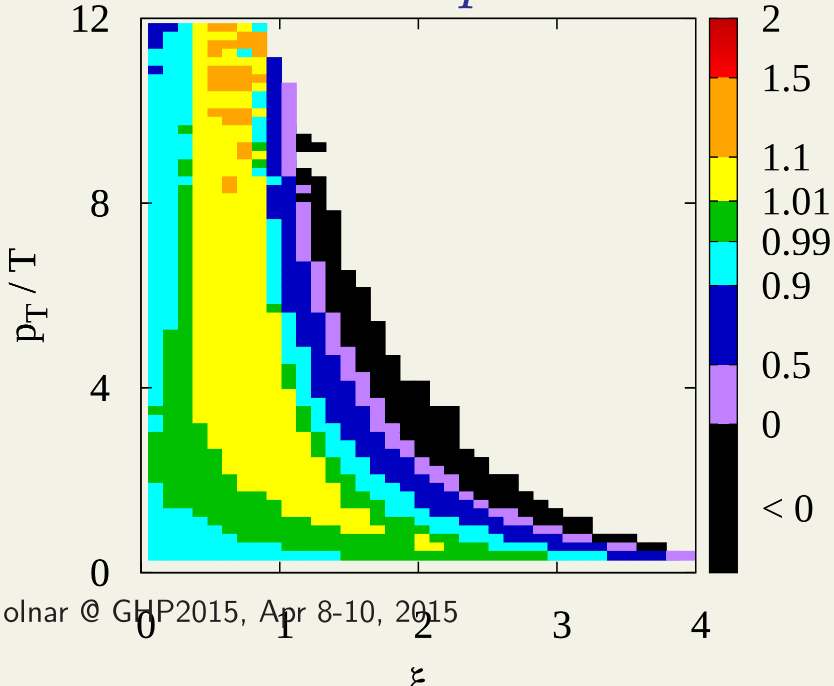
−10%

−50%

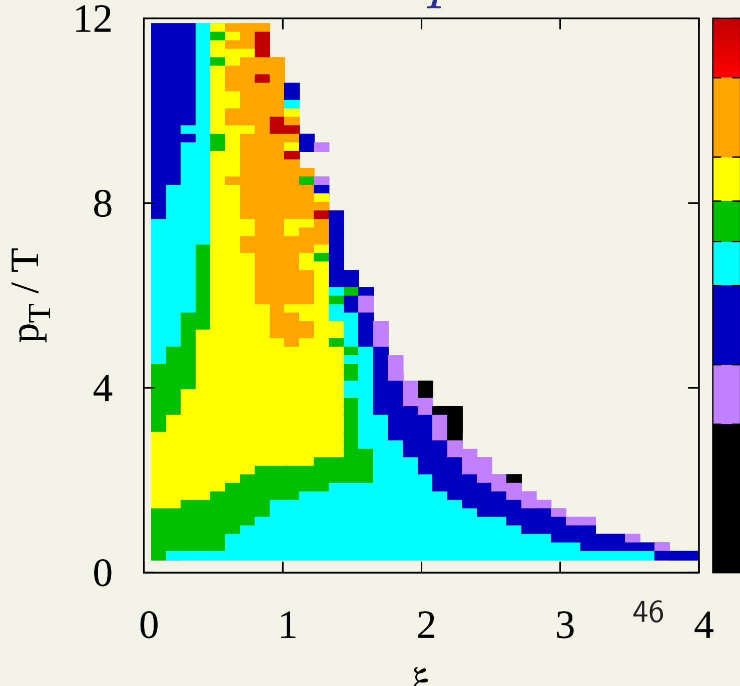
−100%

< 0

IS with $p^{1.5}$



IS with p^1



$$\frac{\tau}{\tau_0} = 20$$

$$K = 6.49 \left(\frac{\tau}{\tau_0}\right)^{2/3}$$

$$\left(\frac{\tau_{REL}}{\tau_{exp}} = 0.02, \quad \frac{\pi_L}{p} = -0.02\right)$$

$+100\%$

$+50\%$

$+10\%$

$\pm 1\%$

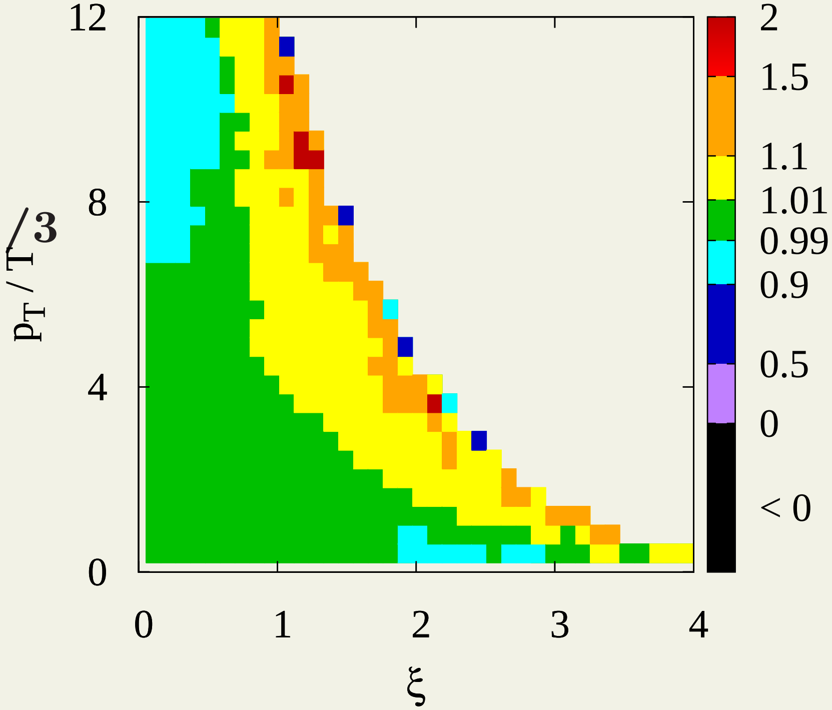
-10%

-50%

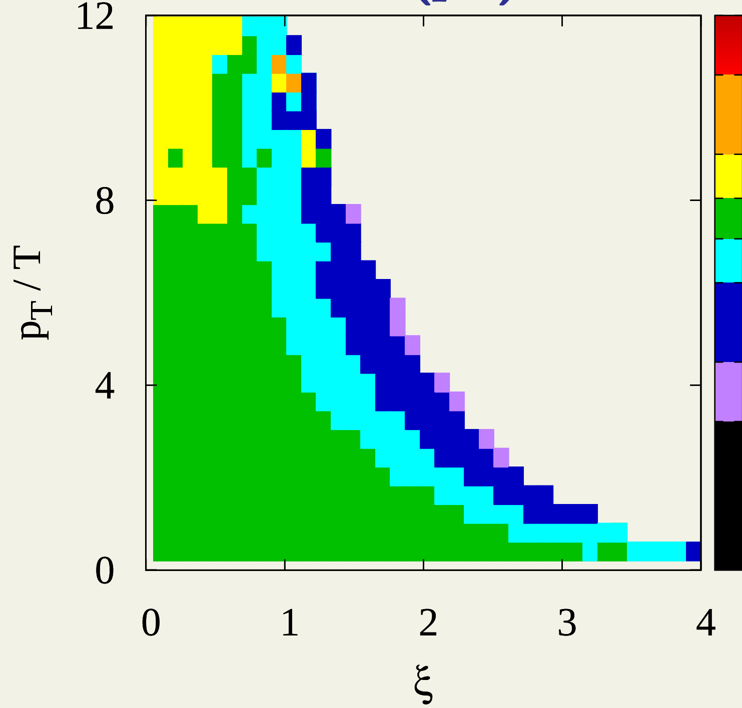
-100%

< 0

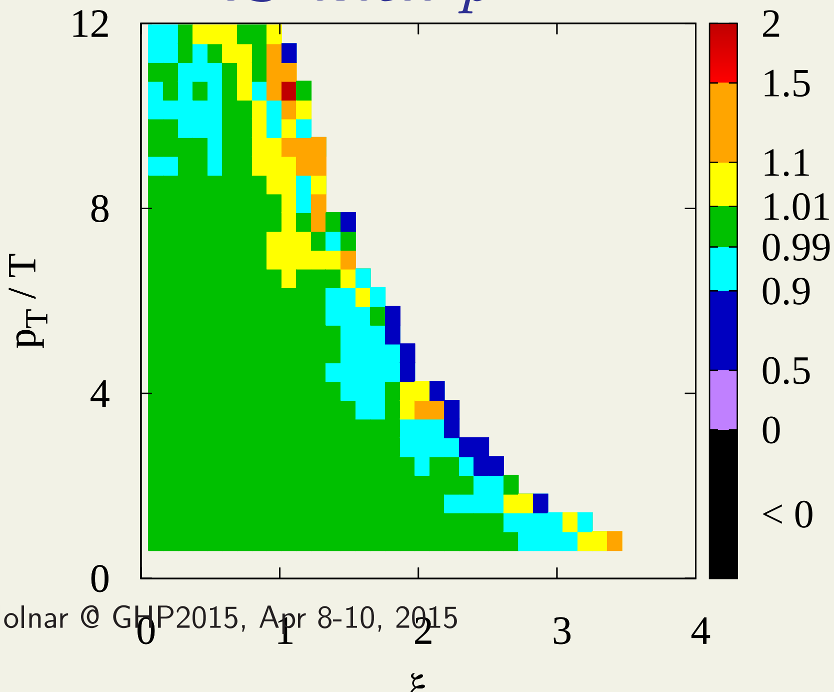
SR ansatz



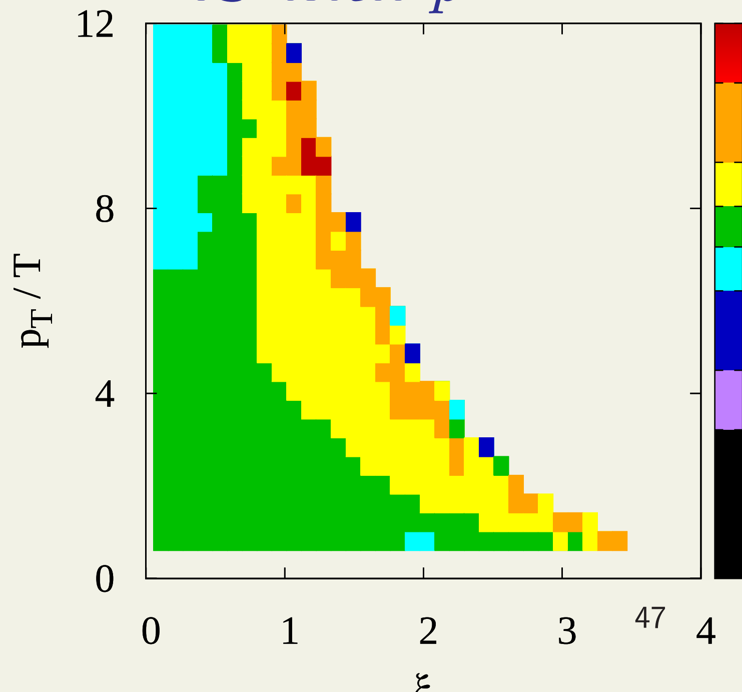
Grad IS (p^2)



IS with $p^{1.5}$



IS with p^1



$\Rightarrow$

For high shear viscosity, SR looks more accurate

For small viscosity and late times, IS with $p^{1.5}$ works best

(linear response indeed works when $\tau_{REL} \ll \tau_{exp}$)

SR is generally very similar to IS with linear p^1

Now make 1D projections:

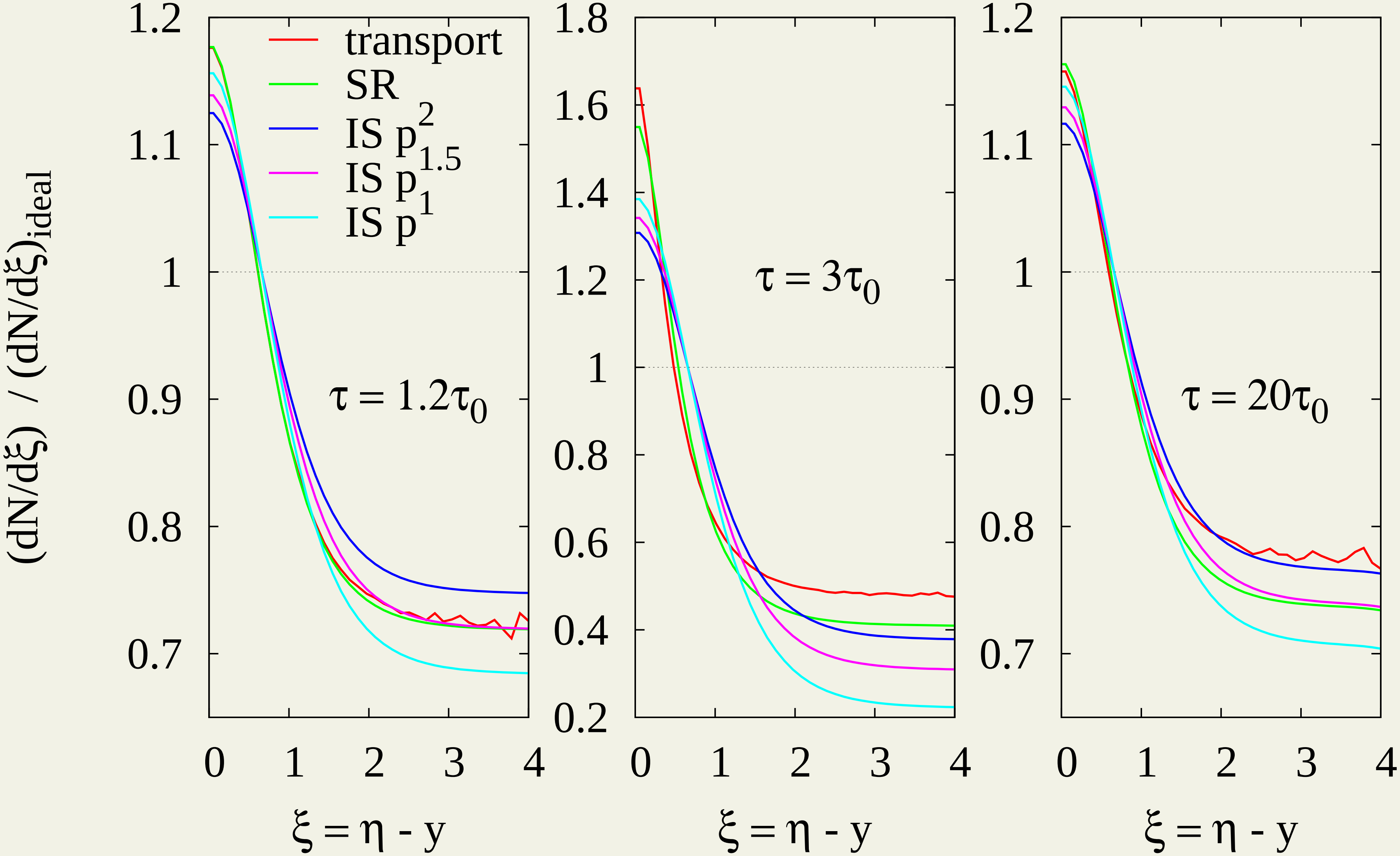
rapidity distribution: $$\frac{dN}{d\xi}(\tau) = A_T\, \tau\, \mathrm{ch}\xi \int d^2p_T\, p_T\, f$$

spectra: $$\frac{dN}{2\pi p_T dp_T}(\tau) = A_T\, \tau\, p_T \int d\xi\, \mathrm{ch}\xi\, f$$

Rapidity distributions

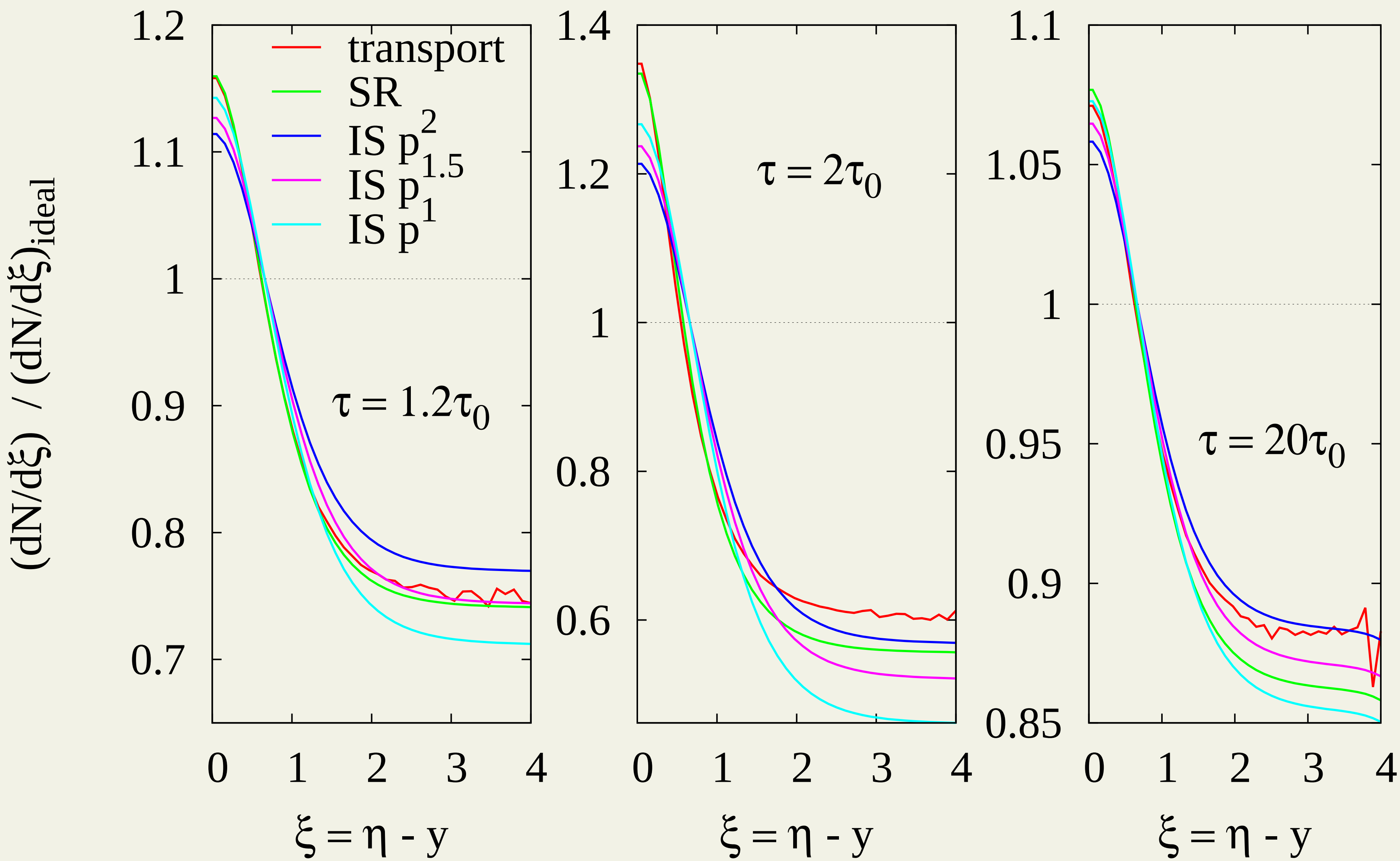
normalize to $\left.\dfrac{dN}{d\xi}\right._{thermal} = \dfrac{1}{2\mathrm{ch}^2\xi}$

$K_0 = 1$



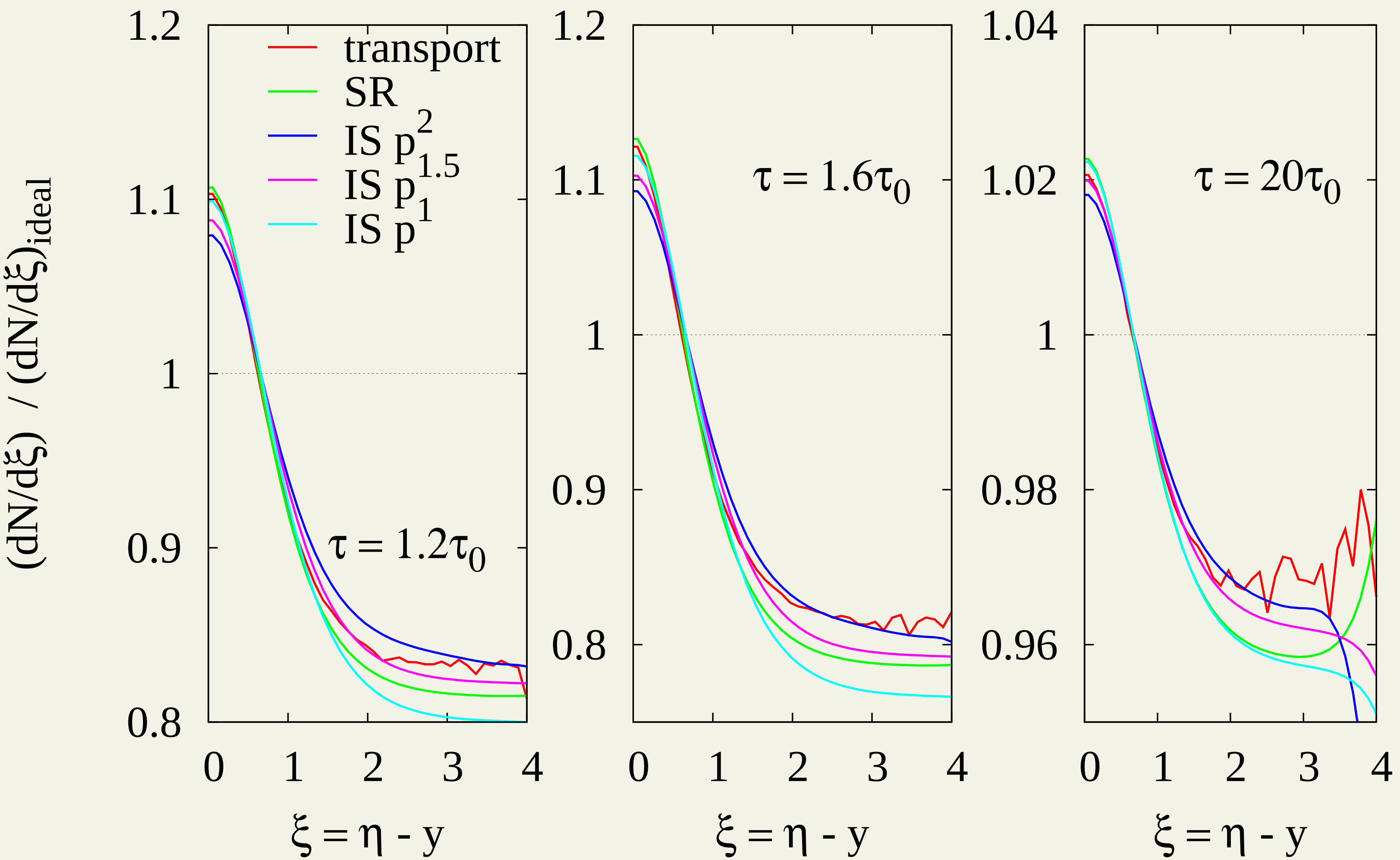
(dN/dξ) / (dN/dξ)ideal
transport
SR
IS p2
IS p1.5
IS p1
τ = 1.2τ0
τ = 3τ0
τ = 20τ0
ξ = η - y
ξ = η - y
ξ = η - y

$K_0 = 2$



(dN/dξ) / (dN/dξ)ideal
transport
SR
IS p2
IS p1.5
IS p1
τ = 1.2τ0
τ = 2τ0
τ = 20τ0
ξ = η - y
ξ = η - y
ξ = η - y
1.2
1.1
1
0.9
0.8
0.7
1.4
1.2
1
0.8
0.6
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4

$K_0 = 6.49$



(dN/dξ) / (dN/dξ)_ideal
transport
SR
IS p^2
IS p^1.5
IS p^1
τ = 1.2τ_0
τ = 1.6τ_0
τ = 20τ_0
ξ = η - y
ξ = η - y
ξ = η - y
1.2
1.1
1
0.9
0.8
1.2
1.1
1
0.9
0.8
1.04
1.02
1
0.98
0.96
0
1
2
3
4

$\Rightarrow$

For low ξ, SR works best.

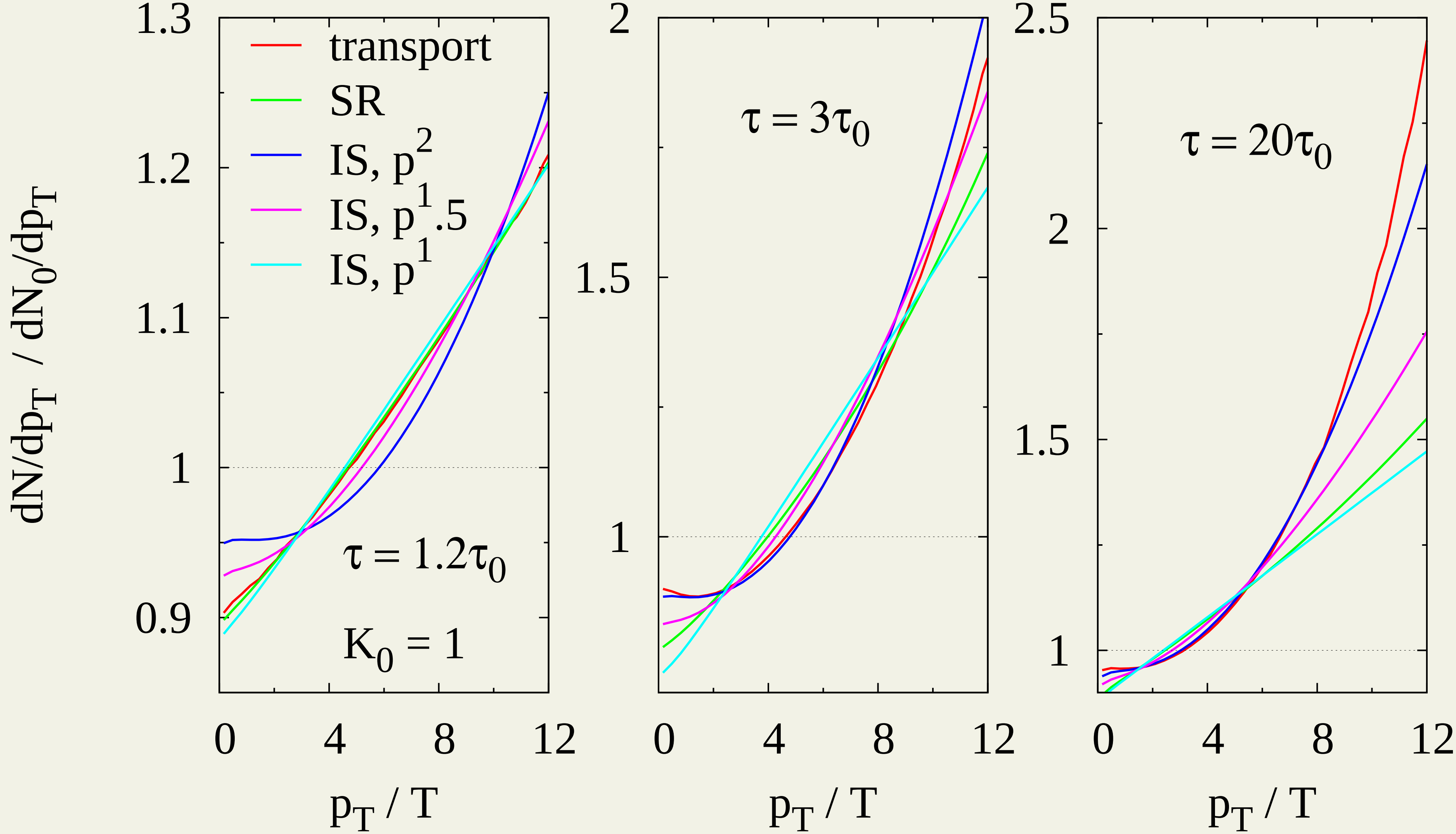
For high ξ:

at early times with higher viscosity, SR is best

at late times, or with low viscosity, Grad (IS) is best

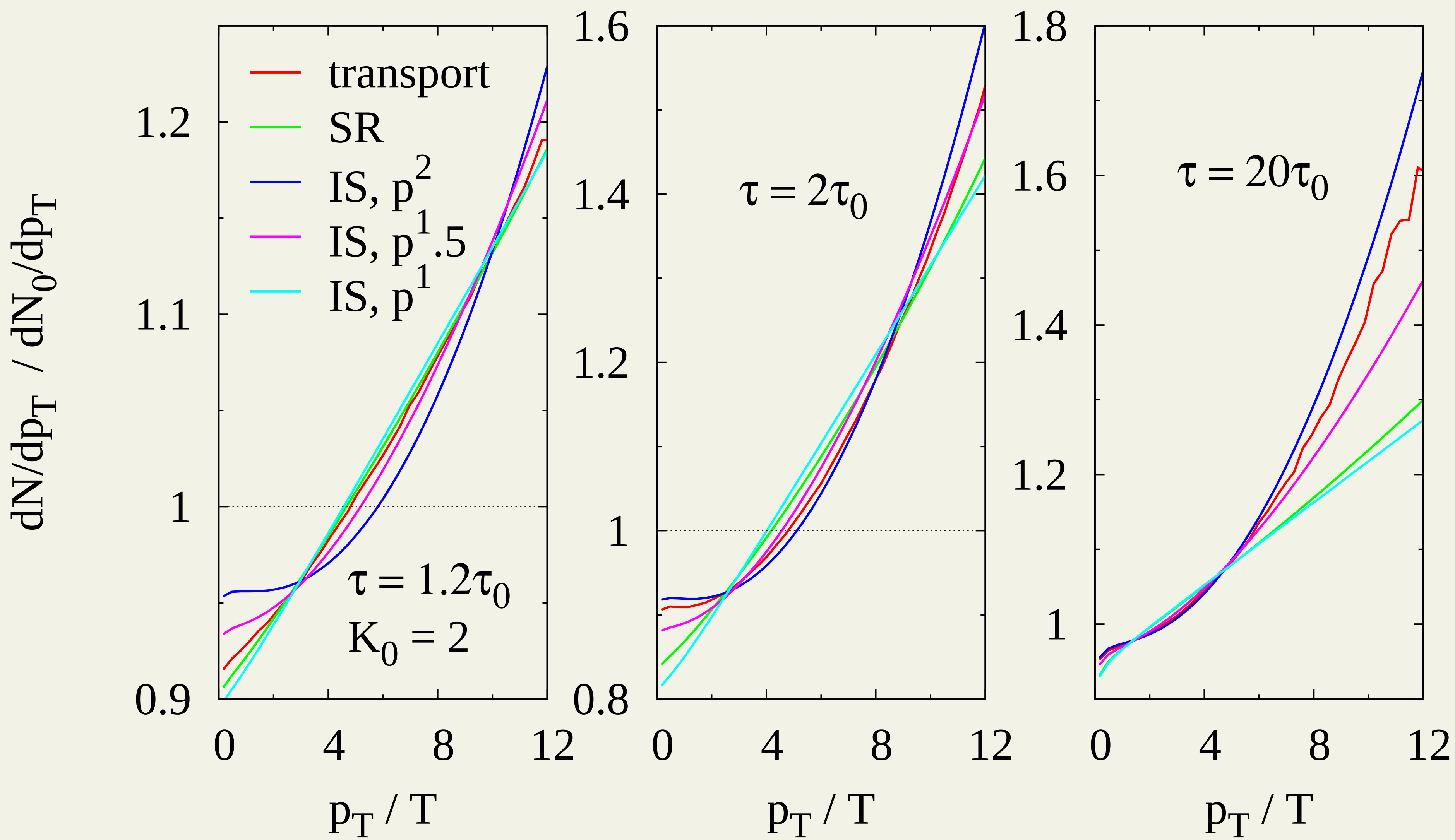
Now check spectra, normalize to ideal thermal (f_0).

$K_0 = 1$



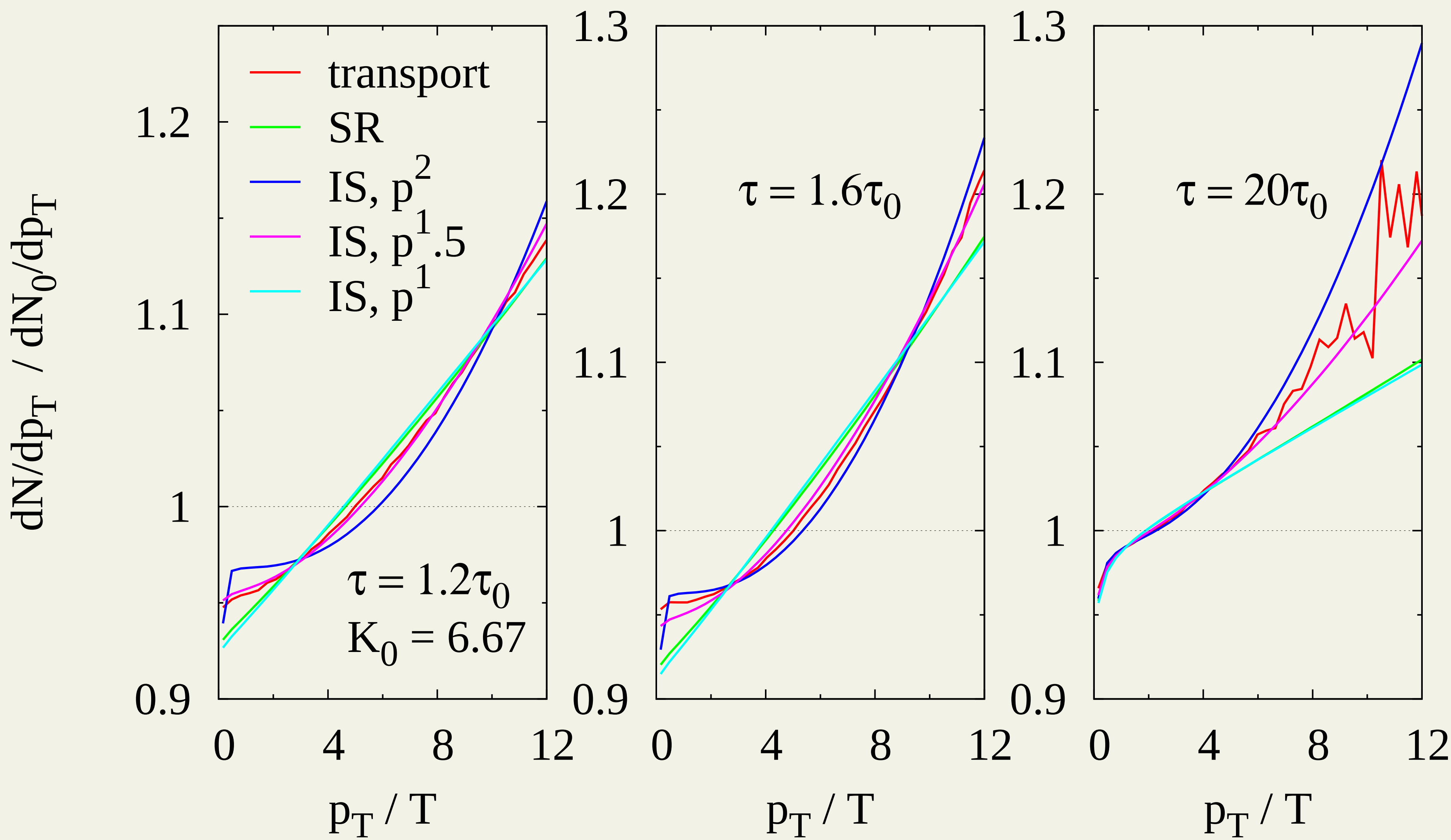
transport
SR
IS, p^2
IS, p^1.5
IS, p^1
τ = 1.2τ_0
K_0 = 1
τ = 3τ_0
τ = 20τ_0
dN/dp_T / dN_0/dp_T
p_T / T
1.3
1.2
1.1
1
0.9
2
1.5
2.5
0
4
8
12

$K_0 = 2$



dN/dp_T / dN_0/dp_T
transport
SR
IS, p^2
IS, p^1.5
IS, p^1
τ = 1.2τ_0
K_0 = 2
τ = 2τ_0
τ = 20τ_0
0.9
1
1.1
1.2
0.8
1.2
1.4
1.6
1.8
0
4
8
12
p_T / T
p_T / T
p_T / T

$K_0 = 6.49$



transport
SR
IS, p^2
IS, p^1.5
IS, p^1
τ = 1.2τ_0
K_0 = 6.67
τ = 1.6τ_0
τ = 20τ_0
dN/dp_T / dN_0/dp_T
p_T / T
0.9
1
1.1
1.2
1.3
0
4
8
12

$\Rightarrow$

At early times and for higher viscosity, SR does best.

At late times, for not too low viscosity quadratic(!) does best.

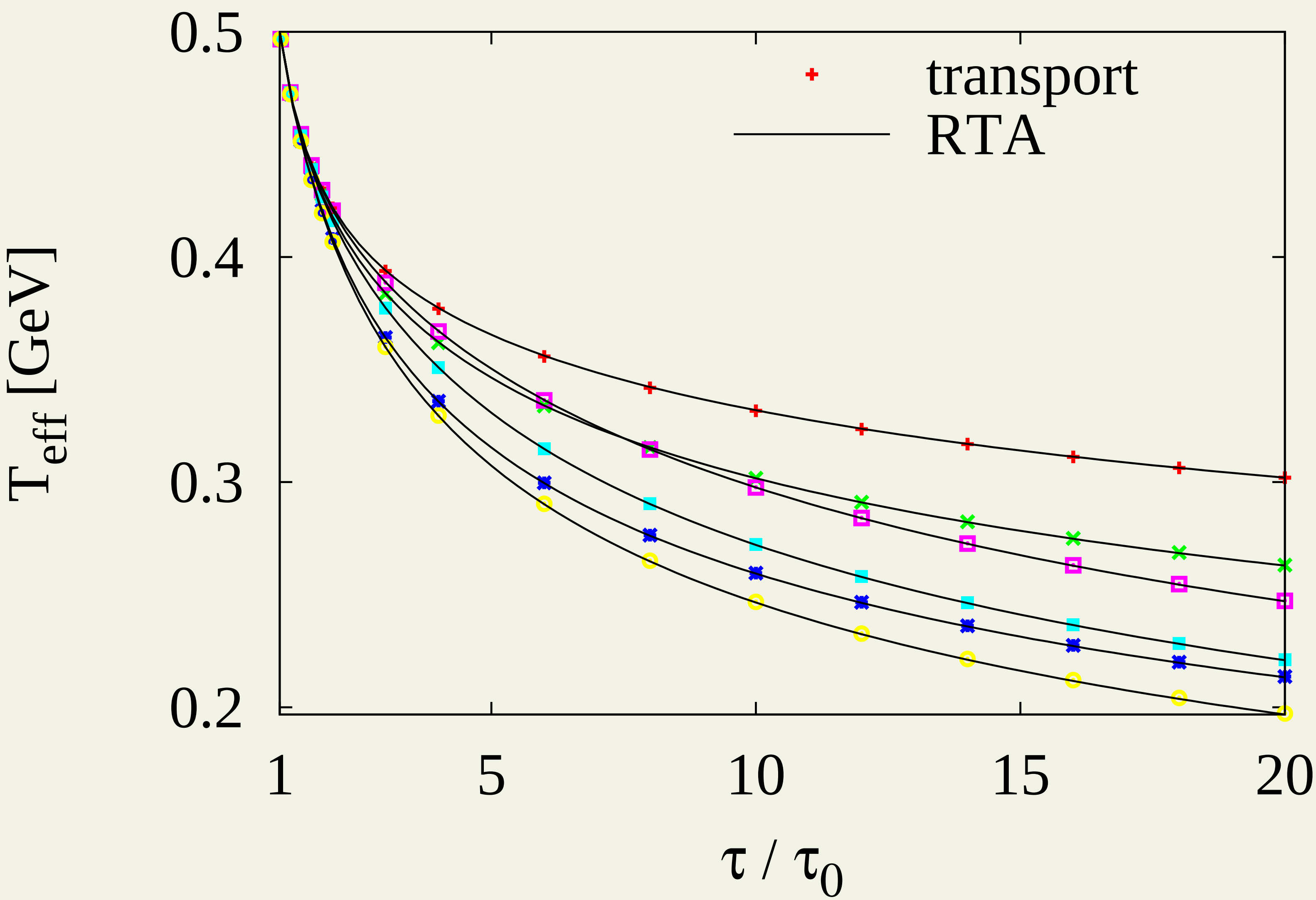
At low viscosity $p^{1.5}$ works best (linear response valid).

Finally, let us look at the RTA more carefully

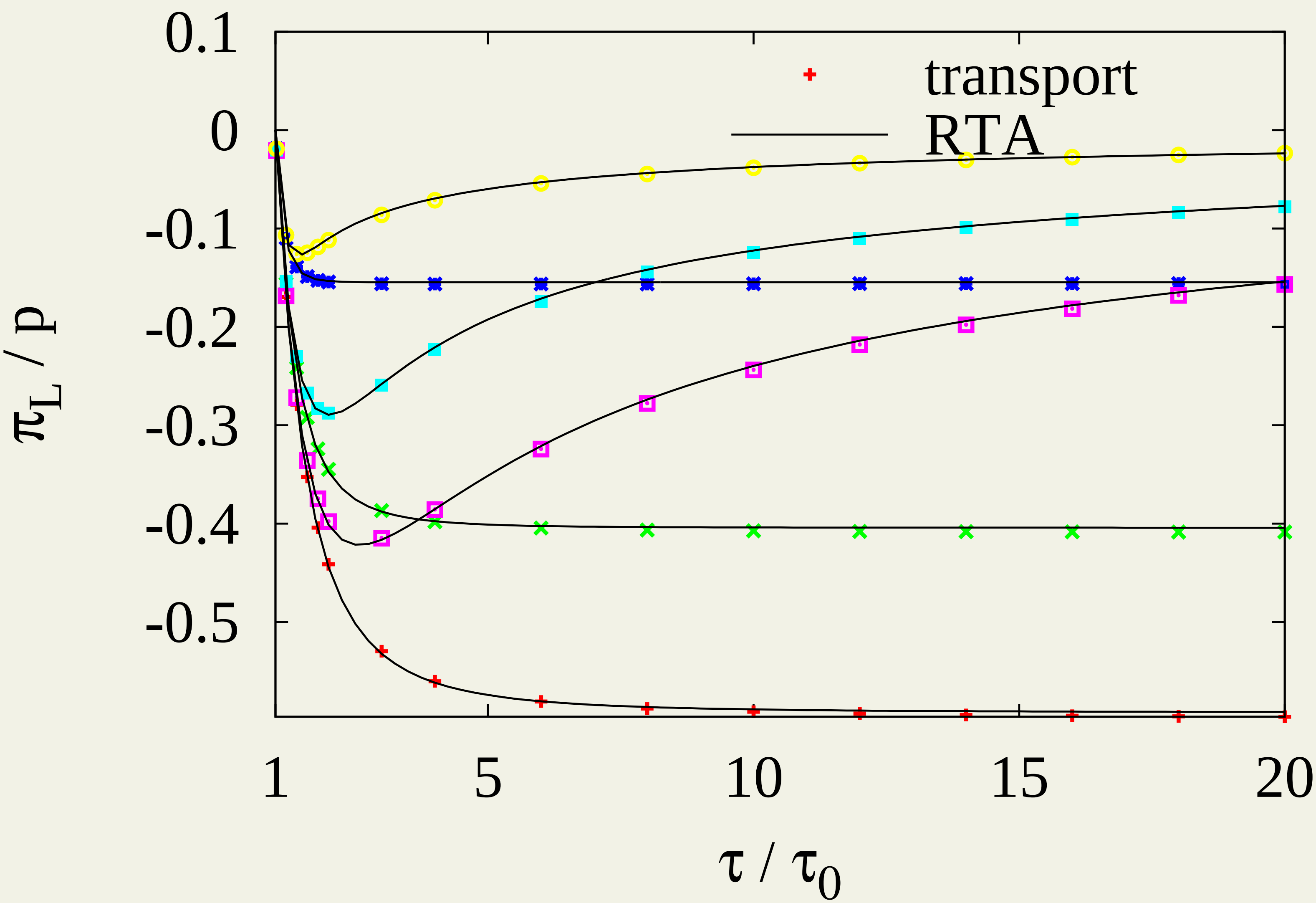
Match to viscosity:

$$\tau_{REL} = \frac{5\eta}{4nT} = 1.05631.. \frac{\tau}{K(\tau)} \qquad \sim \qquad \tau_{sc}$$

RTA well reproduces cooling - T_{eff} vs τ



and also well reproduces shear stress vs τ



$+100\%$

$+50\%$

$+10\%$

$\pm1\%$

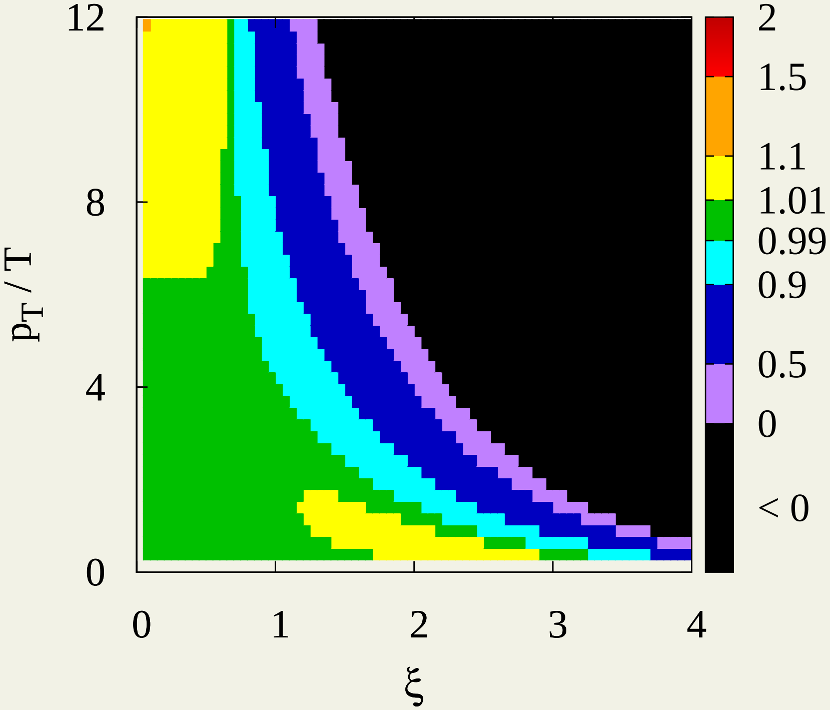
-10%

-50%

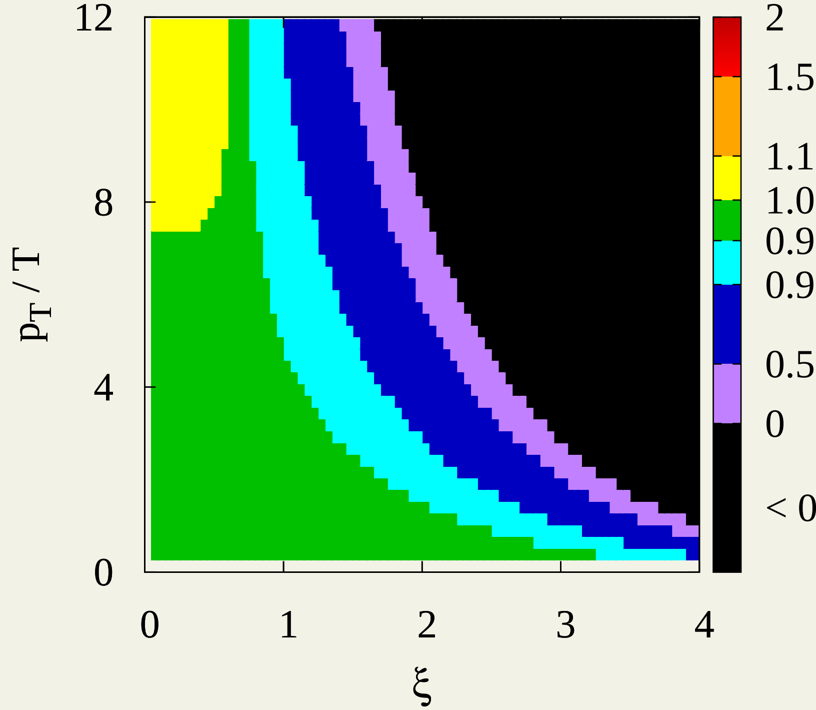
-100%

< 0

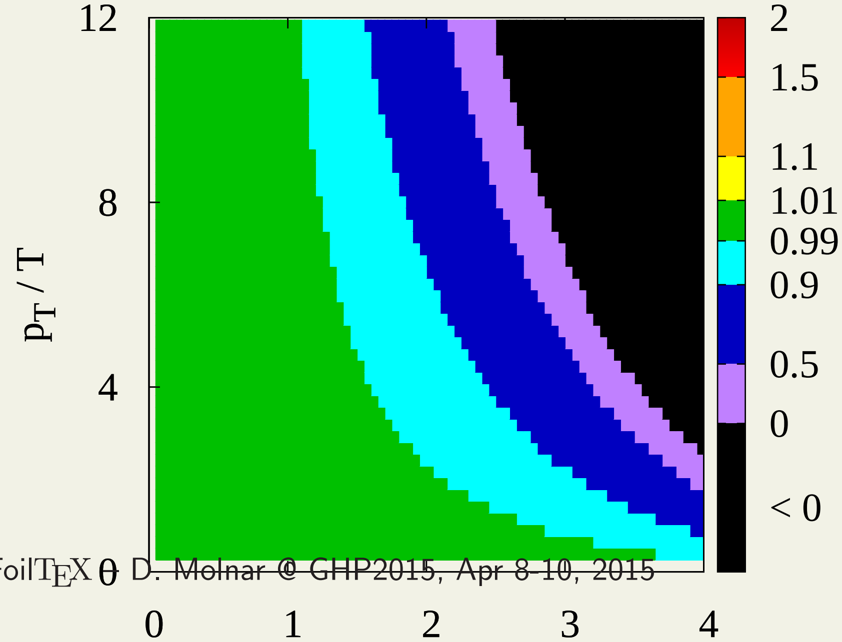
f_{Grad} / f_{RTA}



IS with $p^{1.5}$ / f_{RTA}



IS with p^1 / f_{RTA}



despite its linear δf in linear response regime ($\tau = 20\tau_0$, $\eta/s \sim 0.03$)

Summary

- If you want particles from dissipative hydro, you must use some model of δf. Several such models have been proposed, and they should be tested, e.g., against full kinetic theory.

- NO FREE LUNCH: we compared Grad ($\sim p^2$), linearized Boltzmann, the Strickland-Romatschke ansatz, and relaxation time equation in the linear response regime in a simple 0+1D Bjorken scenario, and found none of these universally accurate.

  We do confirm good applicability of the linearized response approach (which does provide universal answers), but only at late times $\tau/\tau_0 \gtrsim 10$ and for fairly small $\eta/s \lesssim 0.05$.

- At present the choice of δf is a theoretical uncertainty. Perhaps we should look for observables that are less sensitive to it..

Many more comparisons can be done (e.g., 2+1D for elliptic flow, more complex interactions, multicomponent case, ...). Send us your δf's.

Now... if you only wanted positivity

$$1 + \phi(p) \;\to\; e^{\phi(p)} \;\to\; e^{\tanh\phi(p)} \;\to$$

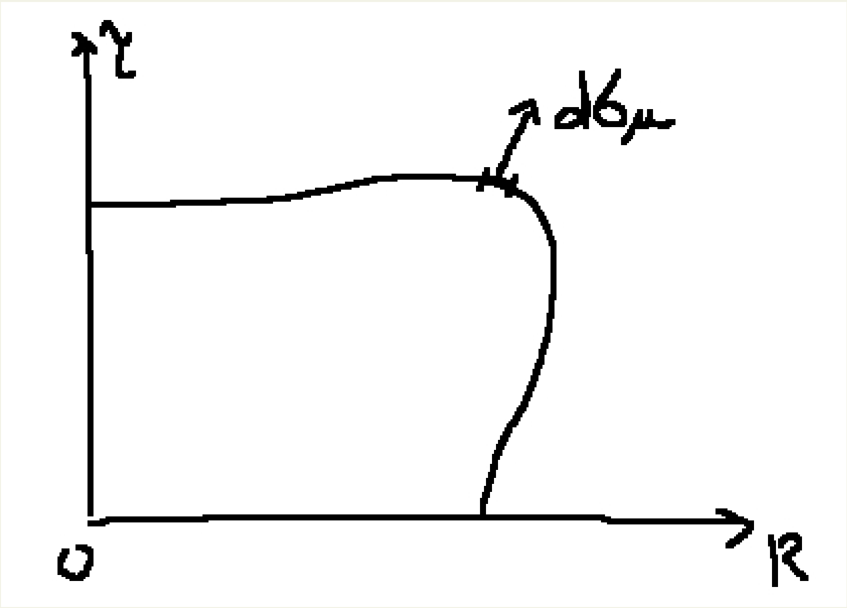
$$\to\; e^{\alpha\tanh\frac{\phi(p)}{\alpha}} \;\to\; e^{\alpha(p)\tanh\frac{\phi(p)}{\alpha(p)}} \;\to \ldots$$

lots of options

Cooper-Frye freezeout

Cooper & Frye, PRD10 ('74)

<u>Assumed</u> sudden transition to a gas on a 3D hypersurface (e.g., $T = const$)



$$E\, dN = p^\mu d\sigma_\mu(x)\, d^3p\, f_{gas}(x, \vec{p})$$

(covariant analog of

$$\frac{dN}{d^3x d^3p} = f(\vec{x}, \vec{p}, t_{fo})$$

for a $t = const$ surface)

conserves energy-momentum and charges locally

BUT: - **negative contributions possible $p \cdot d\sigma < 0$**
- **arbitrariness in choice of HS & self-consistency problem**