Angular correlations in pA collisions from initial state physics

Vladimir Skokov

Western Michigan University

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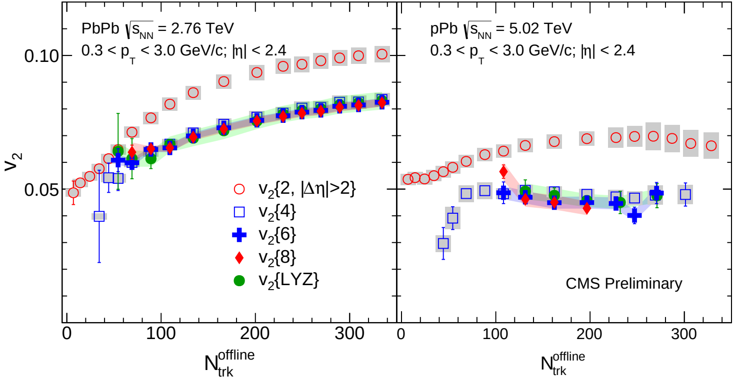
Outline

- **Motivation**: Experimental results for pA collisions at LHC
- **“Glasma” graph**: Azimuthal asymmetry from connected diagrams in dilute-dense limit and high order cumulants
- **Azimuthal anisotropy in McLerran-Venugopalan model**: numerical results
- **Quantum corrections:** JIMWLK evolution and fate of the anisotropy
- **Conclusions**

- Fourier components and cumulants analysis

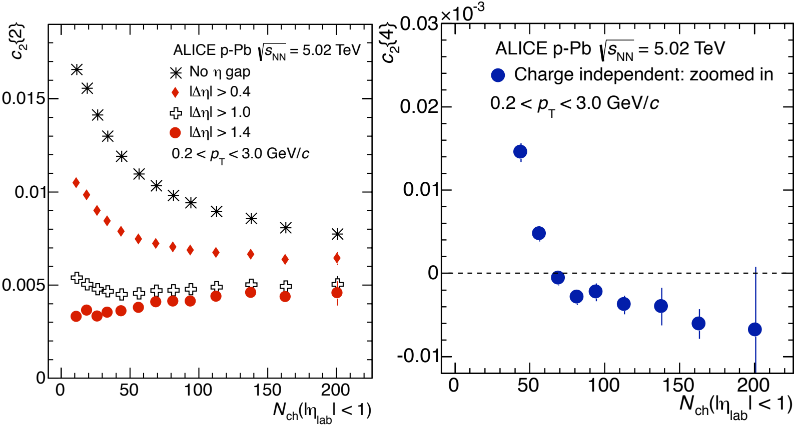


Hierarchy of harmonics $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ in high multiplicity events:
• expected in AA collisions ("hydrodynamic flow"); • surprising for pA?!

- Negative $c_2\{4\} \equiv -v_2^4\{4\}$ at high multiplicity

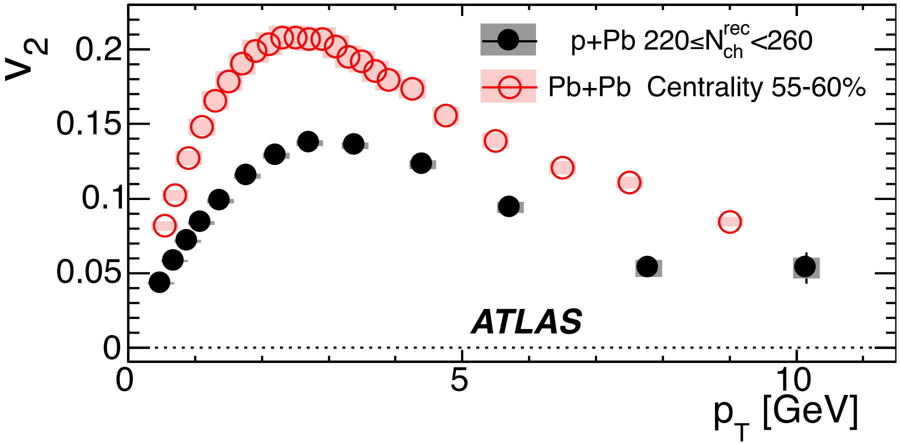
$$c_2\{4\} = \langle \exp[i2(\phi_1 - \phi_2 + \phi_3 - \phi_4)]\rangle - 2\langle \exp[i2(\phi_1 - \phi_2)]\rangle\langle \exp[i2(\phi_3 - \phi_4)]\rangle$$

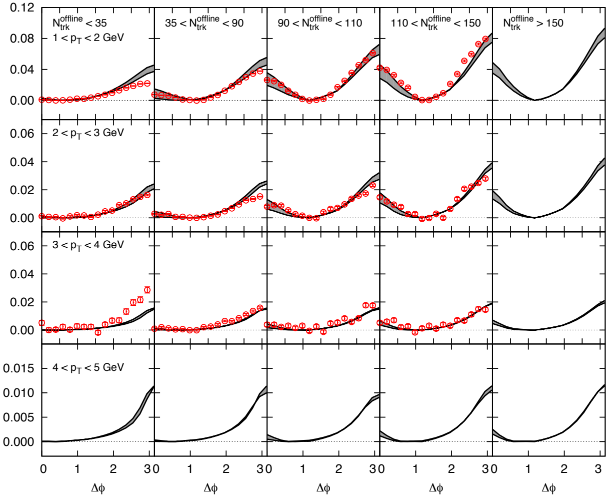
- Change of sign at $N \approx 60$.



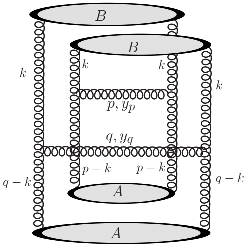
Two distinct regimes defined by sign of $c_2\{4\}$

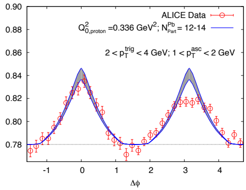
- Anisotropy persists to high momentum





Yield at $\Delta\phi = 0$:





K. Dusling, R. Venugopalan 1302.7018

Data is described well

- Particular case: high $k_\perp$ and dilute-dense limit
- It will be shown that the "glasma" graph approach are incapable to provide real $v_2\{4\}$

Two sources of anisotropy

single-particle anisotropy, $\mathcal{A}$ and "glasma" graph

$$v_2\{2\}^2 = c_2\{2\} \propto \mathcal{A}^2 + \frac{1}{4(N_c^2-1)}$$

$$v_2\{4\}^4 = -c_2\{4\} \propto \mathcal{A}^4 + \left(-\frac{1}{4(N_c^2-1)^3}\right)$$

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	positive		positive
$v_2\{4\}^4 = -c_2\{4\} \propto$	$\mathcal{A}^4$	+	$\left(-\frac{1}{4(N_c^2-1)^3}\right)$

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- It will be shown that the "glasma" graph approach are incapable to provide real $v_2\{4\}$

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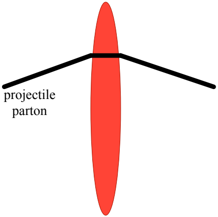
$v_2\{2\}^2 = c_2\{2\} \propto$	$\mathcal{A}^2$	$+$	$\frac{1}{4(N_c^2-1)}$
$v_2\{4\}^4 = -c_2\{4\} \propto$	$\mathcal{A}^4$ positive	$+$	$\left(-\frac{1}{4(N_c^2-1)^3}\right)$ negative

S-matrix for dilute-dense limit: no large N_c approximation



Goal: to find $v_2\{n\}$ in dilute-dense limit at high $k_\perp$

- In eikonal approximation, parton propagation is described by light-like Wilson line $V(\vec{x}) = \mathcal{P}\exp\left(ig\int dx^- A^+(x^-,\vec{x})\right)$

- S-matrix in momentum space: $\langle S_1(\vec{k}_\perp)\rangle = \frac{1}{d_\mathcal{R}}\left\langle \mathrm{tr}_\mathcal{R} V(\vec{k}_\perp)V^\dagger(\vec{k}_\perp)\right\rangle$ or can be obtained from

$$\langle S_1(\vec{k}_\perp)\rangle = \int d^2b\, d^2r\, e^{i\vec{r}_\perp\vec{k}_\perp}\, S_1(\vec{x}_\perp,\vec{y}_\perp),$$

where $\vec{x}_\perp = \vec{b}_\perp + \vec{r}_\perp/2$ and $\vec{y}_\perp = \vec{b}_\perp - \vec{r}_\perp/2$ and
$\langle S_1\rangle = \frac{1}{d_\mathcal{R}}\left\langle \mathrm{tr}_\mathcal{R} V(\vec{x}_\perp)V^\dagger(\vec{y}_\perp)\right\rangle$; $\quad k_\perp \propto 1/r_\perp$; $\quad \vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$
$d_\mathcal{R}$ is dimension of representation $\mathcal{R}$

$$S_A(\vec{r}) = \frac{N_c^2|S_F(\vec{r})|^2 - 1}{N_c^2 - 1}$$

$S_F(\vec{r})$ can be complex, while $S_A(\vec{r})$ is real.

Scattering cross section for dilute-dense limit II

- Scattering to high transverse momentum corresponds to small $|\vec{r}\,| \propto 1/k_\perp$. Gradient expansion of vector potential $A^+(x^-, \vec{x})$ gives (fundamental representation only)

$$\langle S_1(\vec{r}, \vec{b})\rangle - 1 = \left\langle \frac{(ig)^2}{2N_c} \operatorname{tr} \left(\vec{r} \cdot \vec{E}(\vec{b})\right)^2 + \frac{1}{2} \left[\frac{(ig)^2}{2N_c} \operatorname{tr} \left(\vec{r} \cdot \vec{E}(\vec{b})\right)^2 \right]^2 + O(r^6) \right\rangle$$

Light-cone electric field of target in covariant gauge
$E^i(\vec{b}) = \int dx^- F^{+i} = -\partial^i \int dx^- A^+(x^-, \vec{b})$.

- For m-quarks (only leading order is shown)

$$\langle S_m \rangle - 1 = \left(\frac{(ig)^2}{2N_c} \right)^m \left\langle \operatorname{tr}(\vec{r}_{1,\perp}\vec{E}_1)^2 \operatorname{tr}(\vec{r}_{2,\perp}\vec{E}_2)^2 \cdots \operatorname{tr}(\vec{r}_{m,\perp}\vec{E}_m)^2 \right\rangle; \quad \vec{E}_i = \vec{E}(\vec{b}_{i,\perp})$$

- By knowing $\langle \vec{E}(\vec{b}_1)\vec{E}(\vec{b}_2)\rangle$, one can compute S_m, cumulants, $c_n\{m\}$ and harmonics $v_n\{m\}$ of azimuthal anisotropy.

Event averaging corresponds to averaging over target ensemble $\langle \vec{E}(\vec{b}_1)\vec{E}(\vec{b}_2)\rangle$.

In McLerran-Venugopalan model

$$\frac{g^2}{N_c}\langle E_i^a(\vec{b}_1)E_j^b(\vec{b}_2)\rangle = \frac{1}{N_c^2-1}\delta^{ab}\delta_{ij}\,Q_s^2\,\Delta(\vec{b}_1-\vec{b}_2)$$

CUMUALNTS OF AZIMUTHAL ANISOTROPY I

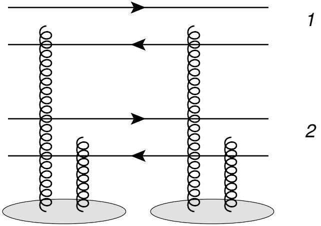
- Cumulants of azimuthal anisotropy can be readily computed. Cumulants are defined in such a way as to cancel disconnected pieces not associated with **single** particle azimuthal anisotropy. For example

$$c_n\{4\} = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle_{\rm conn} = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle - 2\langle e^{in(\phi_1-\phi_3)}\rangle^2$$

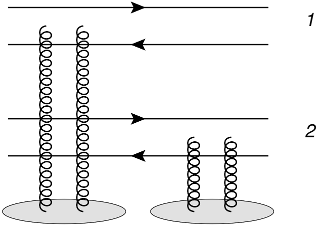
.

- In field theory, this corresponds to considering fully connected diagrams only

Connected graph



Disconnected graph



V. S. 1412.5191

Cumualnts of azimuthal anisotropy II

- There are $(2m-2)!!$ ways to contract $S_m(\vec{r}_1, \vec{b}_1, \dots, \vec{r}_m, \vec{b}_m)$ in fully connected way:

$$\langle S_m(\vec{r}_1, \vec{b}_1, \dots, \vec{r}_m, \vec{b}_m) - 1 \rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4} \right)^m \frac{1}{(N_c^2 - 1)^{m-1}}$$
$$\Delta(\vec{b}_1 - \vec{b}_2)\Delta(\vec{b}_2 - \vec{b}_1) \cdots \Delta(\vec{b}_{m-1} - \vec{b}_m)\Delta(\vec{b}_m - \vec{b}_1)$$
$$(\vec{r}_1\vec{r}_2)(\vec{r}_2\vec{r}_3) \cdots (\vec{r}_{m-1}\vec{r}_m)(\vec{r}_m\vec{r}_1) + \text{permutations.}$$

- Averaging with respect to impact parameters (for Gaussian $\Delta(\vec{b})$ with width R_c):

$$\langle S_m(\vec{r}_1, \dots, \vec{r}_m) - 1 \rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4} \right)^m \frac{1}{(N_c^2 - 1)^{m-1}} \frac{\xi^{m-1}}{m}$$
$$(\vec{r}_1\vec{r}_2)(\vec{r}_2\vec{r}_3) \cdots (\vec{r}_{m-1}\vec{r}_m)(\vec{r}_m\vec{r}_1) + \text{permutations.}$$

$\xi = S_c/S_p = 1/N_D$, $S_c = \pi R_c^2$, S_p is the proton radius.

- In large N_c, normalization of angular averages is defined by disconnected contribution

$$\langle S_m(\vec{r}_1, \dots, \vec{r}_m) - 1 \rangle^{\text{disc.}} \approx \left(-\frac{Q_s^2}{4} \right)^m \prod_{i=1}^{m} r_i^2 .$$

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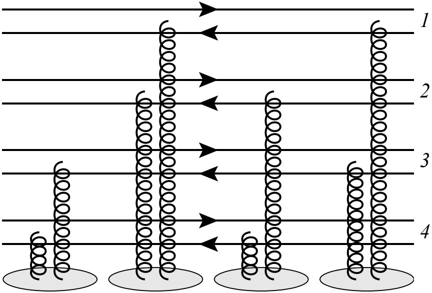
- Not all $(2m-2)!!$ terms contribute to cumulants. $m!!(m-2)!!$ nonzero terms are defined by all possible contractions of terms entering with opposite signs before ϕ's in $e^{2i(\phi_1+\phi_2+\cdots+\phi_n-\phi_{n+1}-\phi_{n+2}-\cdots-\phi_{2n})}$.
- For 4 particles, $e^{2i(\phi_1+\phi_2-\phi_3-\phi_4)}$

Graph that does not contribute



$\propto (\vec{r}_1\vec{r}_3)(\vec{r}_2\vec{r}_4)(\vec{r}_1\vec{r}_2)(\vec{r}_3\vec{r}_4)$

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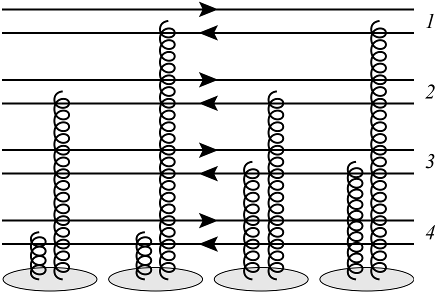
Graph that does contribute



$\propto (\vec{r}_1\vec{r}_3)(\vec{r}_1\vec{r}_4)(\vec{r}_2\vec{r}_3)(\vec{r}_2\vec{r}_4)$

$c_2\{m\}$ FROM CONNECTED DIAGRAMS

- Final result

$$c_2\{m\} = \frac{m!!(m-2)!!}{m\,2^m}\left(\frac{\xi}{N_c^2-1}\right)^{m-1}$$

- Suppressed by powers of $1/N_c^2$ and $\xi = S_c/S_p$.

- $c_2\{m\}$ are manifestly positive for any m.

- Same result remains true for adjoint representation (Casimir operators cancel in normalized observables).

$v_2\{m\}$ FROM CONNECTED DIAGRAMS I

- Harmonics are related to cumulants. Relation of flow coefficients to cumulants (see N. Borghini, P. M. Dinh and J. Y. Ollitrault 0105040)

$$v_2^{2k}\{2k\} = (-1)^{k+1} \times \text{(Numerical coefficient)} \times \mathrm{c}_2\{\mathrm{m} = 2\mathrm{k}\} = \kappa_{2\mathrm{k}} \mathrm{c}_2\{\mathrm{m} = 2\mathrm{k}\}$$

- First few κ_{2m} :

Order, $2m$	2	4	6	8	10	12	14
$1/\kappa_{2m}$	1	-1	–4	33	–456	9460	–274800

- Idea behind these numbers: if we have dominating **single** particle azimuthal anisotropy $v_2\{1\}$ then

$$v_2^m\{m\} = v_2^m\{1\} + \text{corrections}$$

Purpose for hydrodynamics: extract genuine $v_2\{1\}$ and suppress “non-flow”.

$v_2\{m\}$ FROM CONNECTED DIAGRAMS II

$$\kappa_{2m} = \frac{v_n^{2m}\{2m\}}{c_n\{2m\}}.$$

The notation and definitions from Borghini et al 0105040, expanding the generating equation up to order x^{2k}

$$\sum \frac{x^{2k}}{(k!)^2} \langle\langle |Q|^{2k} \rangle\rangle = \ln I_0(2x\langle Q \rangle).$$

and equating the coefficients of x^{2k} one obtains a relation between the cumulants and harmonics. Find the expansion of $\ln(I_0(2y))$ at $y = x\langle Q \rangle = 0$. The Bessel function can be represented as an infinite product $I_0(2y) = \prod_{k=1}^{\infty} \left(1 + \left(\frac{2y}{j_{0,k}}\right)^2\right)$ and thus

$$\ln(I_0(2y)) = \sum_{k=1}^{\infty} \ln\left(1 + \left(\frac{2y}{j_{0,k}}\right)^2\right) = \sum_{i=0}^{\infty} a_i y^{2i}, \quad a_i = \frac{(-1)^{i+1}}{i} \sum_{k=1}^{\infty} \left(\frac{2}{j_{0,k}}\right)^{2i}.$$

For details see Appendix of `V. S. 1412.5191`

$$\kappa_{2m} = \frac{v_n^{2m}\{2m\}}{c_n\{2m\}} = (-1)^{m+1} \left[m!(m-1)! \sum_{k=1}^{\infty} \left(\frac{2}{j_{0,k}}\right)^{2m} \right]^{-1}.$$

At large orders the sum can be approximated by the first term.

- Harmonics:

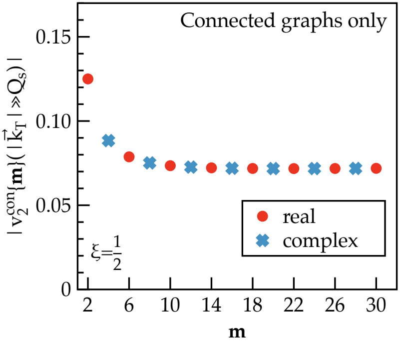
$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m}\left(\frac{\xi}{N_c^2-1}\right)^{m-1}; \qquad \beta_m = 2\sum_{k=1}^{\infty}\left(\frac{2}{j_{0,k}}\right)^m \approx 2\left(\frac{2}{j_{0,1}}\right)^m$$

- Explicitly for second and fourth order:

$$v_2^2\{2\} = \frac{1}{4}\frac{\xi}{N_c^2-1}; \qquad v_2^4\{4\} = -\frac{1}{4}\left(\frac{\xi}{N_c^2-1}\right)^3$$

- $v_2\{4\}$ is complex!
- $m \to \infty$:

$$\lim_{m\to\infty} |v_2\{m\}| = \frac{\xi}{N_c^2-1}\frac{j_{0,1}}{2}; \qquad j_{0,1} = 2.40483$$



- Hierarchy of $|v_2\{m\}|$.
- **Complex $v_2\{4k\}$, $k \in \mathcal{Z}$; including $v_2\{4\}$ and $v_2\{8\}$**
- Experiment: high multiplicity pA $c_2\{4\} < 0 \rightsquigarrow v_2\{4\} \in \mathcal{R}$
- **Theory: connected graph only** $c_2\{4\} > 0 \rightsquigarrow v_2\{4\} \in C$
- In order to describe high $k_\perp$ with IS effects, one needs **disconnected** graphs with single azimuthal anisotropy

V. S. 1412.5191

A. Dumitru, L. McLerran, V. S. 1410.4844

• For a hydro practitioner: “non-flow”. However, very different from conventional non-flow contributions (i.e. resonance decay): long-range in rapidity, approximate equality of high order harmonics $|v_2\{m\}|$. • We can learn something! Measure $|v_2\{m\}|$ at multiplicity below $N < 50$.

MV model for high energy

- Large-x valence partons are modeled by random, recoilless color charges $\rho^a(\vec{x}_\perp)$ creating semi-classical small-x gluon fields $A^a(\vec{x}_\perp)$.
- Gaussian distribution of sources

$$S_{\rm eff}[\rho^a] = \int dx^- d^2x_\perp \frac{\rho^a(x^-,\vec{x}_\perp)\,\rho^a(x^-,\vec{x}_\perp)}{2\mu^2}$$

- Weizsäcker-Williams fields:

$$A^{\mu a}(x^-,\vec{x}_\perp) = -\delta^{\mu +}\frac{g}{\nabla_\perp^2}\rho^a(x^-,\vec{x}_\perp)\,.$$

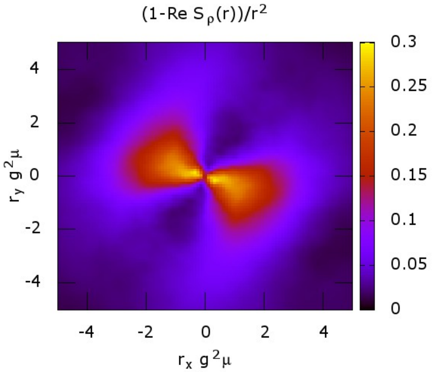
- Propagation of fundamental charge in this field

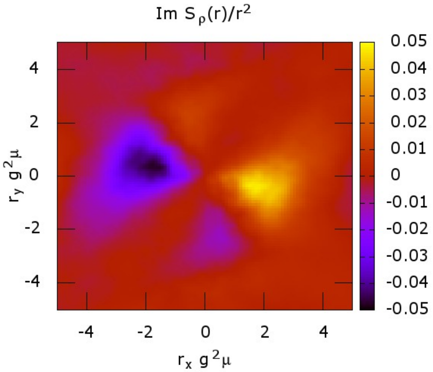
$$V(\vec{x}_\perp) = \mathbb{P}\exp\left\{-ig\int dx^- t^a A^{+a}(x^-,\vec{x}_\perp)\right\}$$

- S-matrix for scattering charge off given target field configuration

$$S_\rho(\vec{r}_\perp,\vec{b}_\perp) \equiv \frac{1}{N_c}{\rm tr}V^\dagger(\vec{x}_\perp)\,V(\vec{y}_\perp), \quad \vec{r}_\perp \equiv \vec{x}_\perp - \vec{y}_\perp\,, \;\; 2\vec{b}_\perp \equiv \vec{x}_\perp + \vec{y}_\perp$$

Details on numerical implementation:
A. Dumitru, V. S. 1411.6630
T. Lappi 0711.3039





- Normalization $1/r^2$ mimics LO of isotropic part of cross-section
- Real part is dominated by $\cos 2\phi_r$
- Imaginary part is dominated by $\cos \phi_r$

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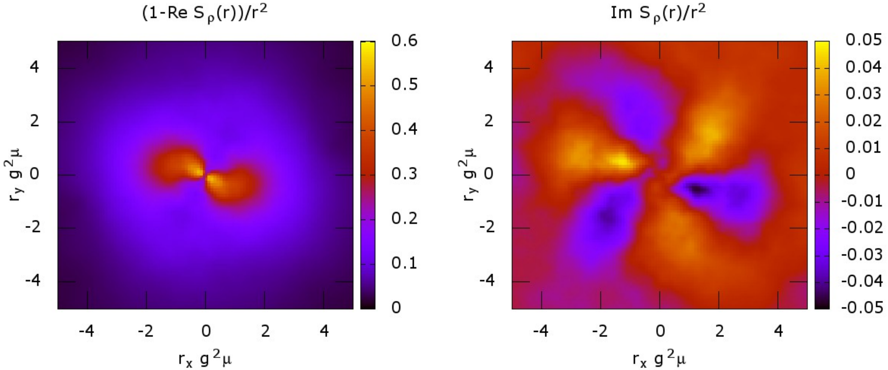
Single configuration II



- Higher orders are seen in real part
- Imaginary part is dominated by $\cos 3\phi_r$

This anisotropy is not correlated with spatial geometry of a collisions.

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Amplitudes: definition

- Goal is to extract amplitudes of azimuthal anisotropy
- In each event Fourier decomposition is performed:

$$1 - \mathrm{Re}\, S_\rho(\vec{r}_\perp) \equiv D_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp)\left(1 + \sum_{n=1}^{\infty} A'_{2n}(r_\perp)\cos(2n\phi_r)\right)$$

$$\mathrm{Im}\, S_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp)\sum_{n=0}^{\infty} A'_{2n+1}(r_\perp)\cos((2n+1)\phi_r)$$

- To cancel trivial random phase:

$$A_n(r_\perp) = \frac{\pi}{2}|A'_{2n}(r_\perp)|$$

- Results are presented in terms of Q_s defined by $\left\langle \mathcal{S}_\rho \right\rangle (r_s = \sqrt{2}/Q_s) \stackrel{!}{=} e^{-1/2}$

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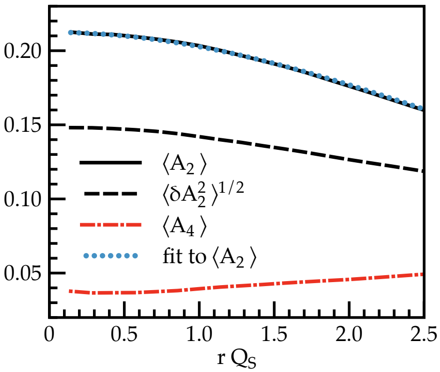

- Largest amplitude: $\langle A_2 \rangle$
- Finite $\langle A_2 \rangle$ at $r \to 0$; $\langle A_2 \rangle$ is approximately constant for $r < 1/Q_s$
- $\langle \delta A_2^2 \rangle^{1/2}$ is comparable to $\langle A_2 \rangle$: fluctuations are rather high
- $\langle A_4 \rangle$ is significantly smaller
- Fit of $\langle A_2 \rangle$ motivated by h_1^g of distribution of linearly polarized gluons (for an unpolarized target) introduced in TMD factorization

$$\delta^{ij} f_1^g(x, \vec{k}^2) + \left(\hat{k}^i \hat{k}^j - \frac{1}{2}\delta^{ij} \right) h_1^{\perp g}(x, \vec{k}^2) \, .$$

In MV (A. Metz and J. Zhou, 1105.1991):

$$h_1^{\perp g}(x, \vec{r}^2) \propto \frac{1}{r^2 Q_s^2} \left[1 - \exp\left(-\frac{r^2 Q_s^2}{4} \right) \right]$$

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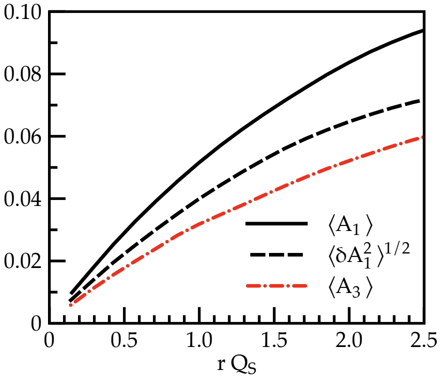
AMPLITUDES: ODD HARMONICS



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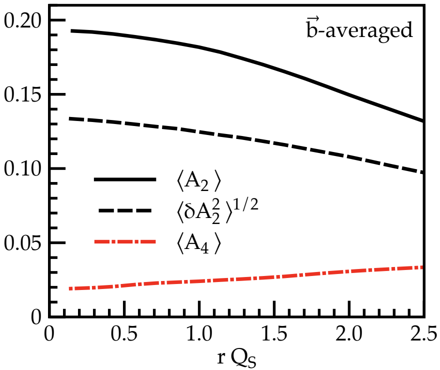
- Expectation value of Im S_ρ is 0
- However, odd harmonics are non-zero!
- At small r, $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero, as expected from analytic arguments

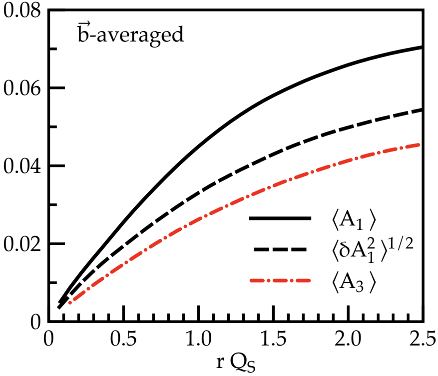
(A. Dumitru and A. Giannini, 1406.5781)

$$\text{Im } S_\rho \propto \alpha_s r^3 \cos \phi_r$$

Amplitudes: averaging over finite $\vec{b}_\perp$

$$\overline{D}_\rho(\vec{r}_\perp, \vec{b}_\perp) = \int \frac{d^2\vec{b}'_\perp}{\pi r_\perp^2}\, \Theta\left(r_\perp - |\vec{b}_\perp - \vec{b}'_\perp|\right) D_\rho(\vec{r}_\perp, \vec{b}_\perp)$$





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Beyond MV

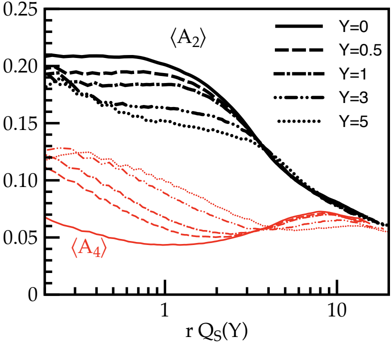
- Beyond MV: inclusion of quantum fluctuations.
- Previous **"mean-field" BK studies**
  showed that azimuthal anisotropy decreases exponentially in Y.
  `See A. Kovner & M. Lublinsky 1211.1928.`
  • Anisotropic initial conditions $1 - S \propto e^{-1/4 Q_S^2 r^2 (1 + \# \cos(2\phi))}$
  • BK equation assuming uniform distribution in impact parameter space

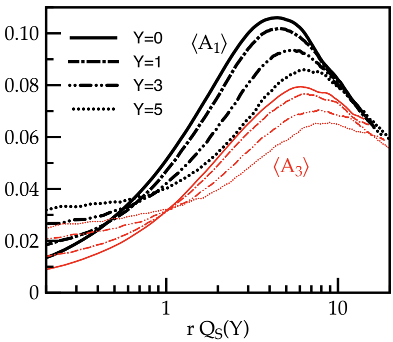
$$\partial_Y N(\vec{r}) = \frac{C_F \alpha(r^2)}{2\pi} \int d^2\vec{r}_1 K(\vec{r}, \vec{r}_1) \left(N(\vec{r}_1) + N(\vec{r}_2) - N(\vec{r}) - N(\vec{r}_1) N(\vec{r}_2) \right)$$

  • In this particular set up I was able to reproduce their conclusion: azimuthal anisotropy decays very fast with $Y = \ln(x_0/x)$.
  • Crucial assumption: no impact parameter space dependence
  • Even at level of initial condition (given by MV model), azimuthal anisotropy of $\mathcal{S}(\vec{r}, \vec{b})$ arises due to fluctuations of soft fields in transverse impact parameter plane.
- **JIMWLK** equation with both $\vec{r}$ and $\vec{b}$.

JIMWLK I

Evolution over a step ΔY in rapidity opens up phase space for radiation of gluons and modifies classical action. This is taken into account by functional renormalization group equation, JIMWLK.





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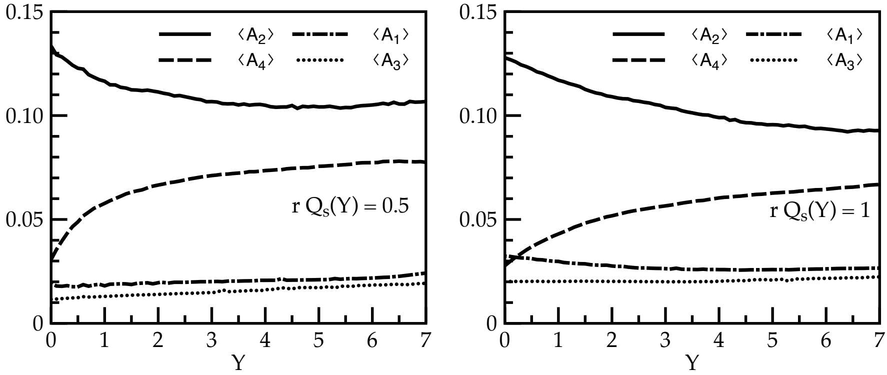
JIMWLK II



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SINGLE PARTICLE ANISOTROPY

- Kovner&Lublinsky: Re $S_\rho - 1 = \frac{(ig)^2}{2N_c} \mathrm{tr}(\vec{r}\vec{E})^2$; If projectile partons scatter off same electric field, they pick up same transverse momentum (independent on rapidity of partons).
- To take this into account, let modify $E - E$ correlator:

$$\frac{g^2}{2N_c} \left\langle \mathrm{tr}\, E_i(\vec{b}_{1\perp}) E_j(\vec{b}_{2\perp}) \right\rangle = \frac{1}{4} Q_s^2\, \Delta(\vec{b}_{1\perp} - \vec{b}_{2\perp}) \left(\delta^{ij} + 2\, \mathcal{A} \left(\hat{a}^i \hat{a}^j - \frac{1}{2} \delta^{ij} \right) \right)$$

- Angular distribution for scattering of single dipole, for fixed $\hat{a}$.

$$\left(\frac{1}{\pi} \frac{dN}{dk^2} \right)^{-1} \frac{dN}{d^2 k} = 1 - 2\mathcal{A} + 4\mathcal{A} (\hat{k} \cdot \hat{a})^2 \,.$$

Consequently, elliptic harmonic of single-particle distribution: $v_2 \equiv \left\langle e^{2i(\phi_k - \phi_a)} \right\rangle_{\hat{a}} = \mathcal{A}$.

- Repeating calculations, see details in †:

$$v_2^2\{2\} = c_2\{2\} = \xi \left(\mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right)$$

$$v_2^4\{4\} = -c_2\{4\} = \xi^3 \left(\mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right)$$

Factors of $\xi = 1/N_D$: partons scatter off domain with same $\vec{E}$ orientation.

A. Kovner, M. Lublinsky,1109.0347, 1211.1928;
† A. Dumitru, L. McLerran, V.S. 1410.4844;
A. Dumitru, A. Giannini 1406.5781
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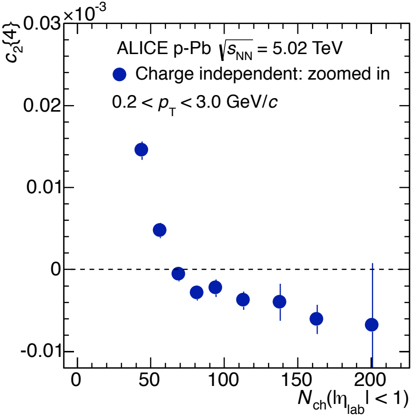
Single particle anisotropy



ALICE Coll. 1406.2474

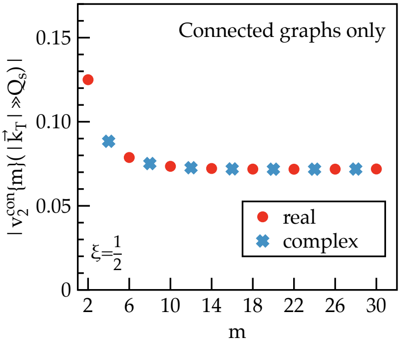
- $$c_2\{4\} = -\xi^3 \left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3} \right)$$
- Interpretation in terms of IS:
  - $N_{ch} < 60$: dominated by **connected** contribution to $c_2\{4\}$
  - $N_{ch} > 60$: dominated by single particle disconnected contribution to $c_2\{4\}$

Single particle anisotropy: illustration of hierarchy

- In general it is immensely difficult to derive general expression for $v_2\{m\}$.
- Limiting cases are easy:

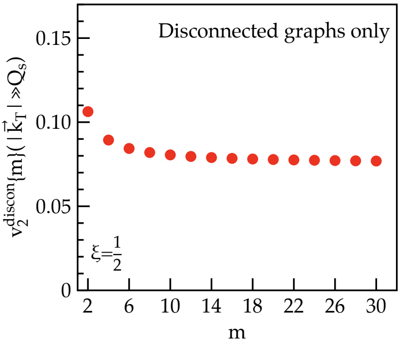
- Connected graphs dominate:

$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m}\left(\frac{\xi}{N_c^2-1}\right)^{m-1};$$



- $\mathcal{A}$(single particle anisotropy) dominates:

$v_2\{m\} = \xi^{1-1/m}\mathcal{A}$; at large m $v_2\{m\} = \xi\mathcal{A}$



ODD HARMONICS

- Single particle anisotropy:
  •[A] for fundamental representation S_F has odd harmonics, e.g. $\cos(3\phi)$;
  •[B] for adjoint representation S_A is manifestly real and thus can have only even harmonics (S_A)
- Two particle azimuthal anisotropy: [A] does not help much if there is an approximate quark—anti-quark symmetry of projectile wave function at small x. Indeed, two particle correlation function summed over qq, $q\bar{q}$, $\bar{q}q$ and $\bar{q}\bar{q}$ channels is C-even

$$S_2 \propto \left(\operatorname{tr} V^\dagger(\vec{x}_1)\, V(\vec{y}_1) + \operatorname{tr} V(\vec{x}_1)\, V^\dagger(\vec{y}_1)\right)\left(\operatorname{tr} V^\dagger(\vec{x}_2)\, V(\vec{y}_2) + \operatorname{tr} V(\vec{x}_2)\, V^\dagger(\vec{y}_2)\right)$$

  and so has even cumulants only.
- Obtaining non-zero $c_1\{2\}$ and $c_3\{2\}$ may require to account for (at least) one additional soft rescattering of (anti-) quarks besides their hard scattering from target shockwave.

Conclusions

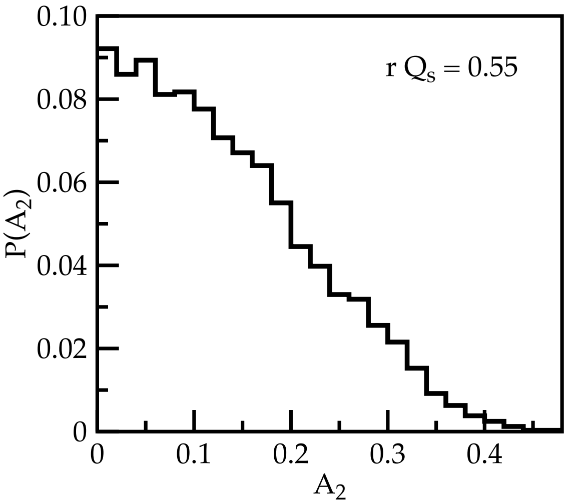
- Neglecting initial state one-particle azimuthal anisotropy: complex $v_2\{4\}$
- Initial state one-particle azimuthal anisotropy is present due to fluctuating valence quarks. This disconnected contribution to $c_2\{4\}$ results in real $v_2\{4\}$
- MV model:

  even amplitudes $\langle A_2 \rangle$ and $\langle A_4 \rangle$ of azimuthal anisotropy are approximately constant for $k_\perp > Q_s$ $(r_\perp < 1/Q_s)$;

  odd amplitudes $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero at $k_\perp \to \infty$ $(r_\perp \to 0)$

  $\langle A_2(k_\perp \sim Q_s) \rangle \approx 20\%$ $\quad \langle A_4(k_\perp \sim Q_s) \rangle \approx 4.7\%$
- Small-x evolution does not significantly modify anisotoropy
- Does $v_2\{m\}$ has a direct connection to TMD h_1^g?!
- No multiplicity bias were imposed in our studies.

Fluctuations of A_2



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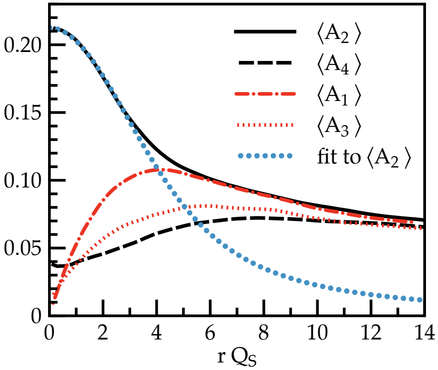
Amplitudes: large r



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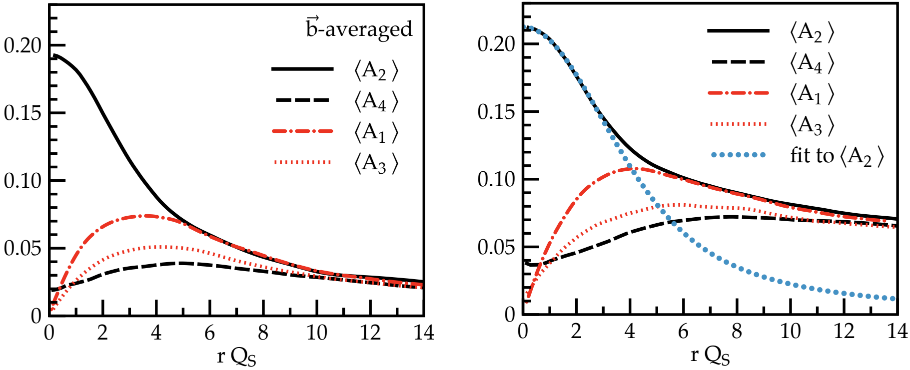
- Fit breaks down at large $r > 3Q_s$: analytical derivation involves adhoc IR cut-offs introduced arbitrary
- Amplitudes approach common non-zero function at large r: expected universal scale invariance of fluctuations of azimuthal dependence of S-matrix

AMPLITUDES: AVERAGING OVER FINITE $\vec{b}_\perp$



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