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Quarkonium at T>0 from lattice QCD spectral functions

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References:

A. R., T. Hatsuda, S. Sasaki: Phys.Rev.Lett. 108 (2012) 162001

Y. Burnier, A.R.: Phys.Rev.Lett. 111 (2013) 182003

S.Kim, P. Petreczky, A.R.: Phys.Rev. D 91 (2015) 054511

Y. Burnier, O. Kaczmarek, A. R.: Phys. Rev. Lett. 114 (2015) 082001

Motivation: Heavy-Ion Collisions

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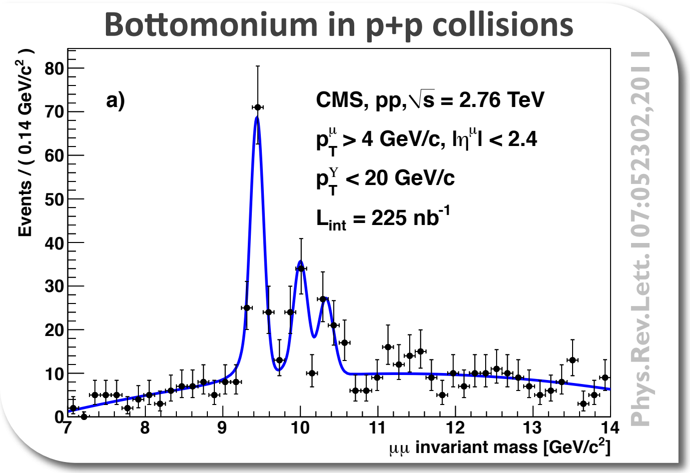
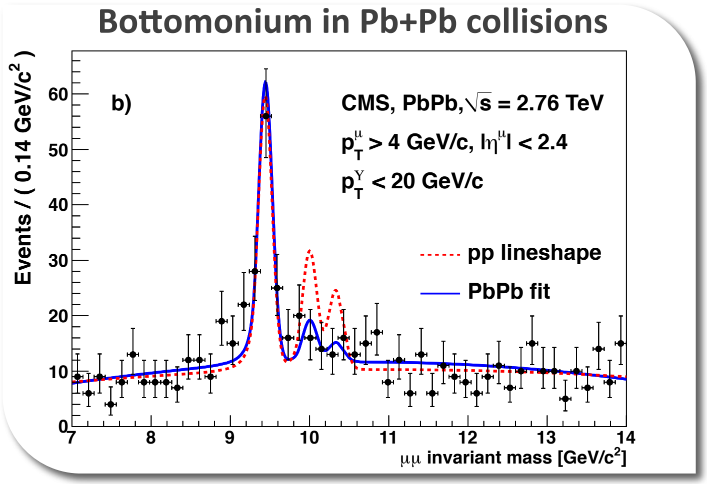
- At RHIC & LHC: Precision era of relativistic heavy-ion collision experiments
- Our interest: probes susceptible to medium but distinguishable $Q_{probe} >> T_{med}$

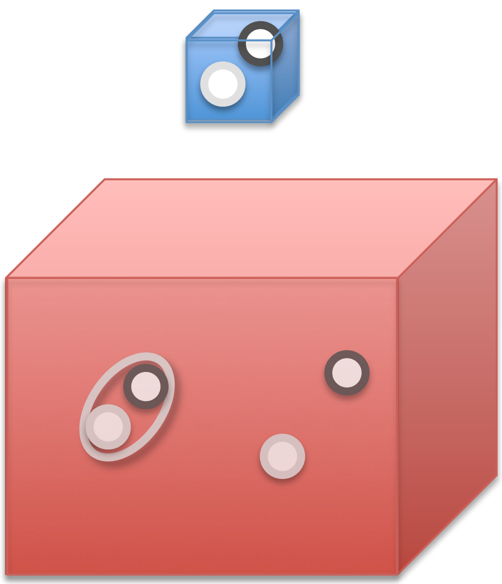
Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q >> T_{med}$



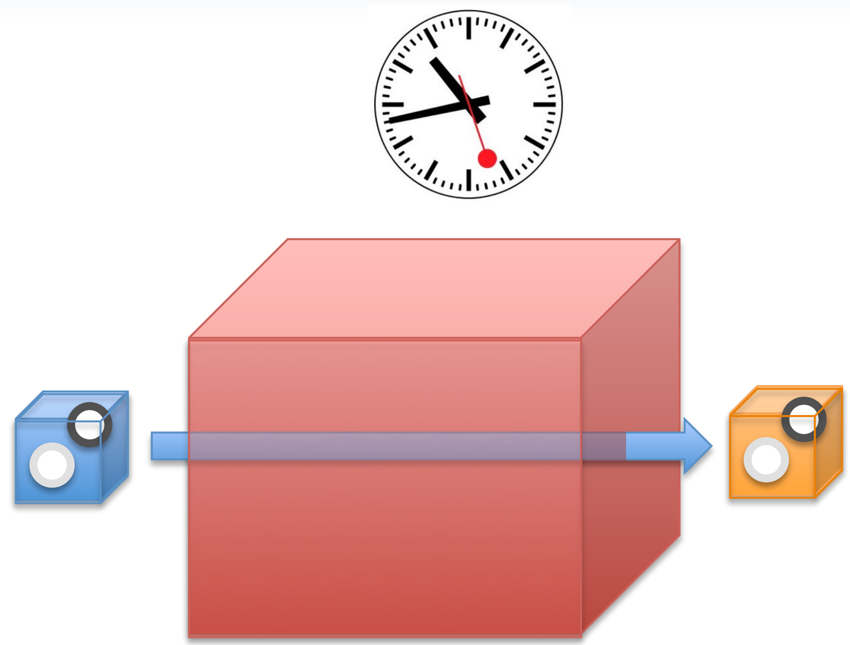


- Extract properties of the QGP from observed vector channel Bottomonium yields
- Need 1st principles understanding of $b\bar{b}$ modification in a strongly coupled QGP

Two limiting scenarios



T. Matsui and H. Satz: Phys.Lett. B178 (1986) 416



QQbar as Open-Quantum System

Kinetically equilibrated heavy quarks

presence of in-medium bound eigenstates?

modern approach: lattice QCD meson spectra

S.Kim, P. Petreczky, A.R.: Phys.Rev. D91 (2015) 054511

Dynamical: approach to equilibrium

redistribution of states over time?

consistent potential based description

Y. Burnier, O. Kaczmarek, A. R.: Phys. Rev. Lett. 114, 082001

Heavy Quarks on the Lattice

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- Effective field theory from scale separation: $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

QCD	NRQCD
Dirac fields	**Pauli fields**
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$ $\xi^\dagger(x), \xi(x)$

$$\bar{q}(x), q(x), A^\mu(x)$$

$$L_{NRQCD} = \chi^\dagger\left(iD_t + \frac{D_i^2}{2M_Q} + \ldots\right)\chi + \xi^\dagger(\ldots)\xi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{q}(\ldots)q$$

- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau)=\langle\chi(\tau)\chi^\dagger(0)\rangle$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau)\left(1 - \frac{\mathbf{p}_{\mathrm{lat}}^2}{4M_Q a} + \ldots\right)G(\mathbf{x}, \tau)$$

well behaved if $M_Q a > 1.5$

Davies, Thacker Phys.Rev. D45 (1992)

- $^3S_1$ (Υ) and $^3P_1$ (χ_{b1}) channel correlators $D(\tau)$ from products of heavy quark propagators $G(\tau)$

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) G_{x\tau} O^\dagger(\mathbf{x}_0, \tau_0) G^\dagger_{x\tau}\rangle_{med} \qquad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

Thacker, Lepage Phys.Rev. D43 (1991)

N_f=2+1 light HISQ Flavor Medium

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- Light d.o.f. (gluons, u d s quarks) represented by HotQCD configurations
  A. Bazavov et. al., Phys. Rev. D 85 (2012) 054503
  - 48^3x**12** with relatively light pions $M_\pi\sim$ 161MeV and a T_C=159±3MeV

HotQCD HISQ/tree action $48^3 \times N_\tau$ $m_{u,d}/m_s = 0.05$

β	6.664	6.700	6.740	6.770	6.800	6.840	6.880
a[fm]	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau = 12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
a[fm]	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
$M_b a$	2.187	2.107	2.030	1.956	1.835	1.753	1.559
$T/T_C(N_\tau = 12)$	1.151	1.194	1.240	1.286	1.371	1.436	1.614

- Important property for the use with lattice NRQCD: $2.759 > M_b a > 1.559 > 1.5$
- Temperature changed by variation of the lattice spacing 140MeV < T < 249MeV
  For a study based on the fixed scale approach see: FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097, JHEP 1111 (2011) 103
- Low temperature configurations available at b=6.664, 6.8, 6.95, 7.28

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A Novel Bayesian Approach

- Inversion of Laplace transform required to obtain spectra from correlators

$$D(\tau) = \int_{2M_Q}^{\infty} d\omega \, \exp[-\omega\tau] \, \rho(\omega)$$

$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i] \, \rho_l \, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

- Give meaning to problem by incorporating prior knowledge: Bayesian approach

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

  - Bayes theorem: Regularize the naïve χ^2 functional P[D|ρ] through a prior P[ρ|I]

$$P[\rho|D, I] \propto P[D|\rho] \; P[\rho|I]$$

  - New prior enforces: ρ positive definite, smoothness of ρ, result independent of units

$$P[\rho|I] \propto e^S \qquad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

Y.Burnier, A.R.
PRL 111 (2013) 18, 182003

  - **Different from Maximum Entropy Method**: S not entropy, no more flat directions

$$\frac{\delta}{\delta\rho} P[\rho|D, I]\Big|_{\rho=\rho^{BR}} = 0$$

  - No apriori restriction on the search space
  - Convergence to unique global extremum

Bottomonium T>0 Correlators

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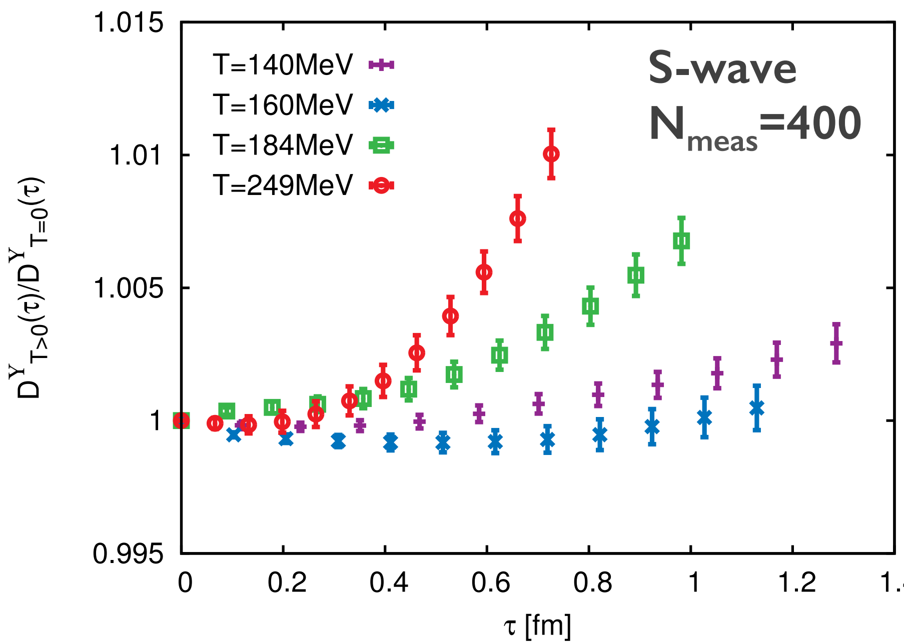
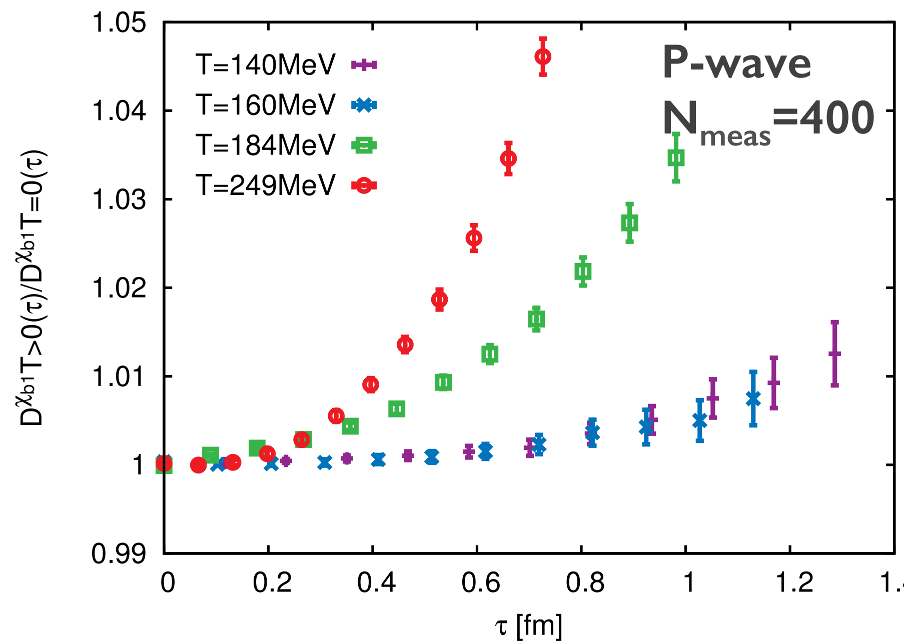


S-wave at most 1% change



P-wave at most 5% change

S.Kim, P.Petreczky, A.R. Phys.Rev. D 91 (2015) 054511

- Statistically significant in-medium modification above T=160MeV
- Side remark: similar qualitative and quantitative behavior for η_b and h_b (scalar)

S-wave Spectral Functions At T>0

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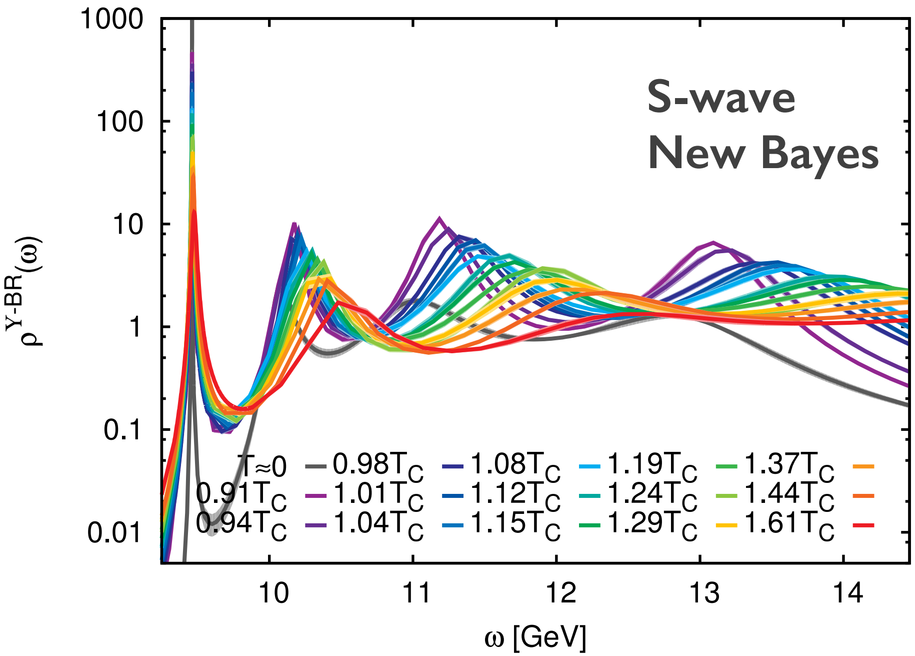

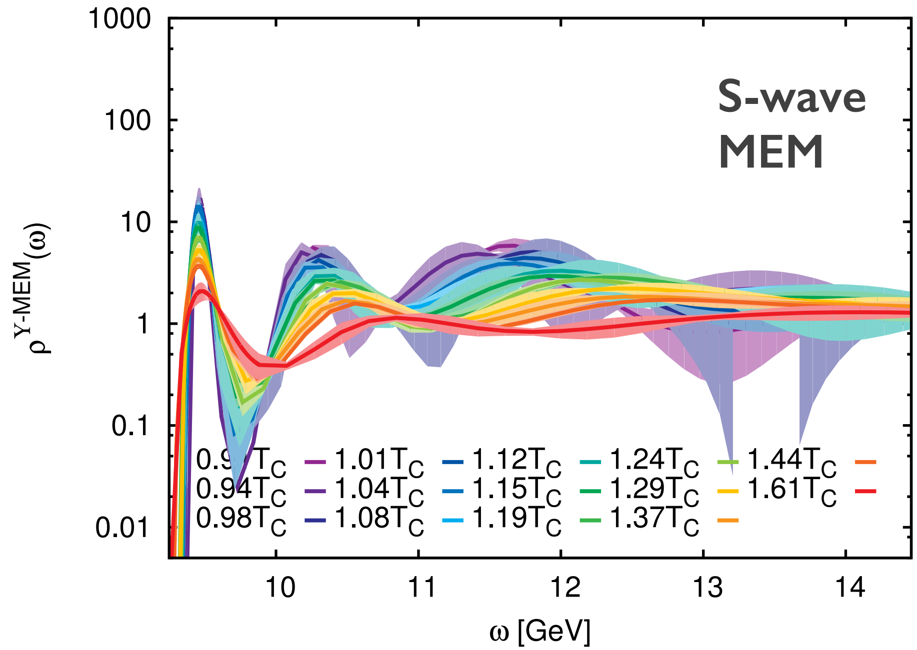

S.Kim, P.Petreczky, A.R. Phys.Rev. D 91 (2015) 054511

- Bayesian reconstruction:

N_ω=1200 I_ω=[-1,25] β^{num}=20 N_{jack}=10

m_l=const 512 bit precision, Δtol=10^{-60}

- New Bayesian method resolves peaks much better than MEM
- Well defined **ground state peak present up to 1.61T$_C$**

P-wave Spectral Functions At T>0

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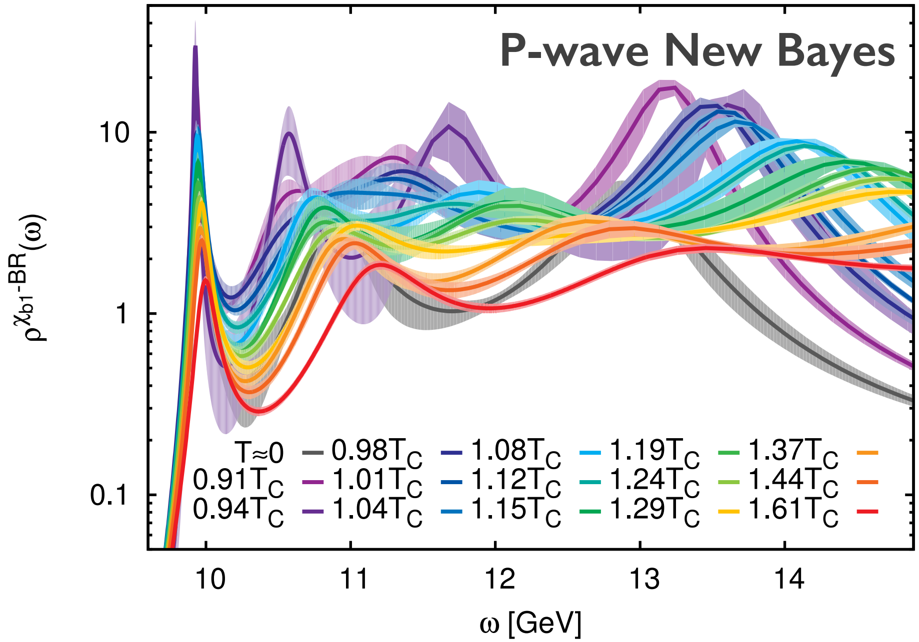
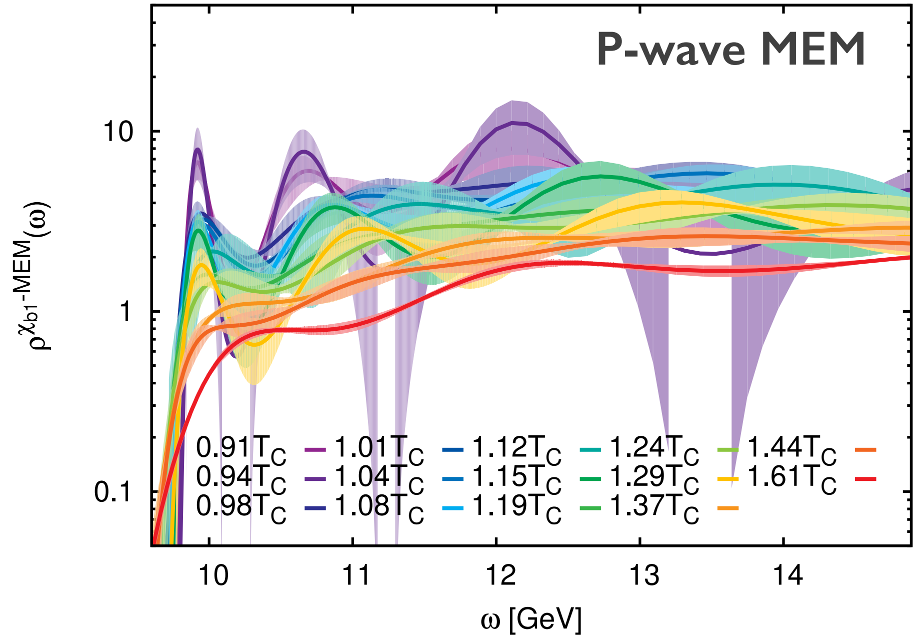


Ground state peak well defined up to $T=1.61T_C$



Ground state peak disappears for $T>1.29T_C$

S.Kim, P.Petreczky, A.R. Phys.Rev. D 91 (2015) 054511

- Worse signal to noise ratio leads to larger Jackknife errors than for S-wave
- New approach finds well defined peak up to highest T investigated 249 MeV

MEM result similar to FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097

S- & P-wave Survival At T=249MeV

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- Inspection by eye insufficient: systematic comparison to **non-interacting spectra**

Analytically known, no peaked features

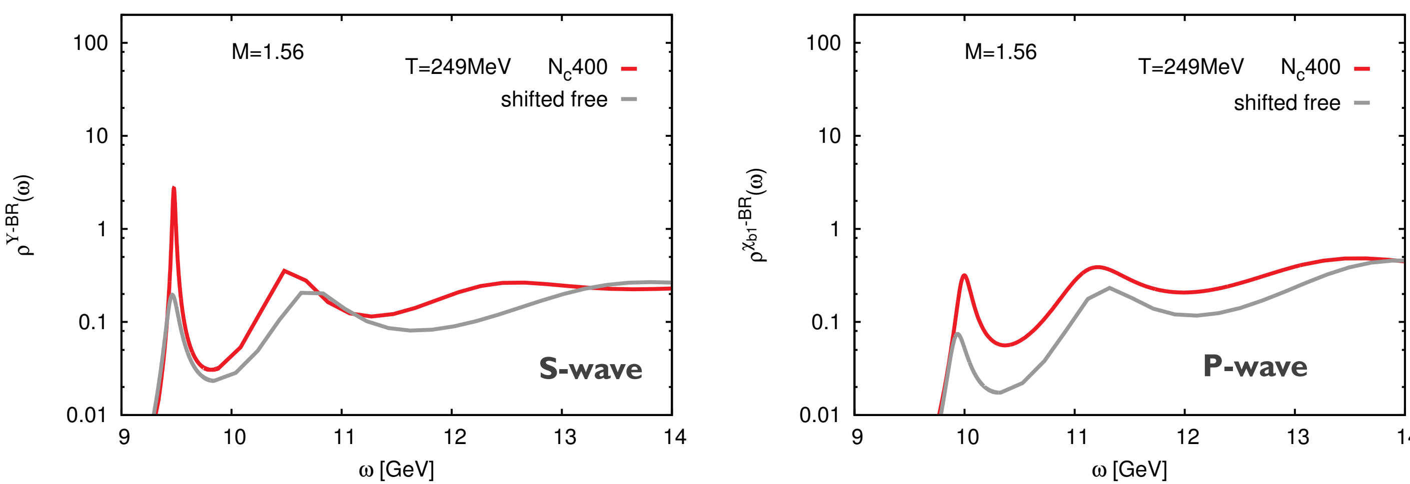
Numerically: Reconstruct from free NRQCD (U_μ=1)

$$a_\tau E_{\mathbf{p}} = -\log\left(1 - \frac{\mathbf{p}_{\mathrm{lat}}^2}{8M_b a_s}\right)$$

$$\rho_S(\omega) = \frac{4\pi N_c}{N_s^2} \sum_{\mathbf{p}} \delta(\omega - 2E_{\mathbf{p}})$$

G.Aarts et. al., JHEP 1111 (2011) 103

- Expectation: Presence of wiggly features due to numerical **Gibbs ringing**



S.Kim, P.Petreczky, A.R.
Phys.Rev. D 91 (2015) 054511

- At T=249 MeV: Ground state peak still significantly stronger than numerical ringing

Defining the heavy quark potential

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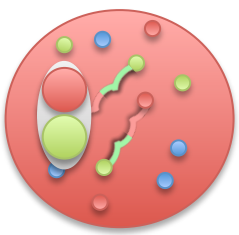
- Effective field theory

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

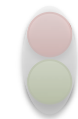
Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory



QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$	$\psi_S(R,t), \psi_O(R,t)$
$\bar{q}(x), q(x), A^\mu(x)$		$i\partial_t \psi_S = \left(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\right)\psi_S$

Quantum mechanics



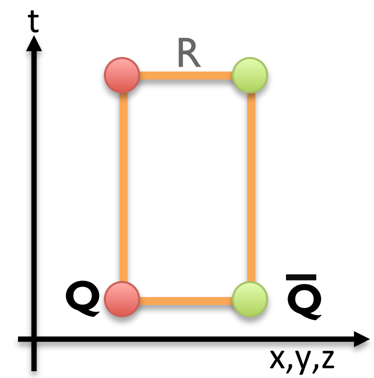
  - Only in static limit: R is well defined, can introduce small corrections systematically

- Matching between QCD and pNRQCD in the static limit

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{\rm pNRQCD} \equiv W_\square(R,t) = \mathrm{Tr}\left(\exp\left[-i\int_\square dx^\mu A_\mu(x)\right]\right)$$

$$i\partial_t W_\square(R,t) \stackrel{t>>t_{\rm med}}{=} V^{QCD}(R) W_\square(R,t)$$

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)}$$



Extracting V^{QCD} from lattice QCD

- On the lattice real-time observables not directly accessible!
- How to connect to the Euclidean domain: **spectral functions**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\, \boxed{\rho_\square(R,\omega)}$$

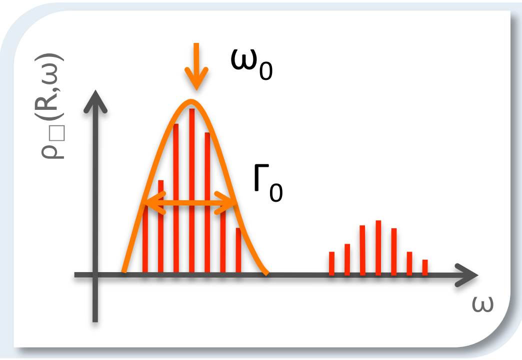
A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}$$

Bayesian spectral reconstruction

Y.Burnier, A.R. PRL 111 (2013) 182003

- Relation between spectrum and potential from the symmetries of $W_\square(R,t)$



$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \ldots$$

$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

Summary: V^{QCD} from the lattice

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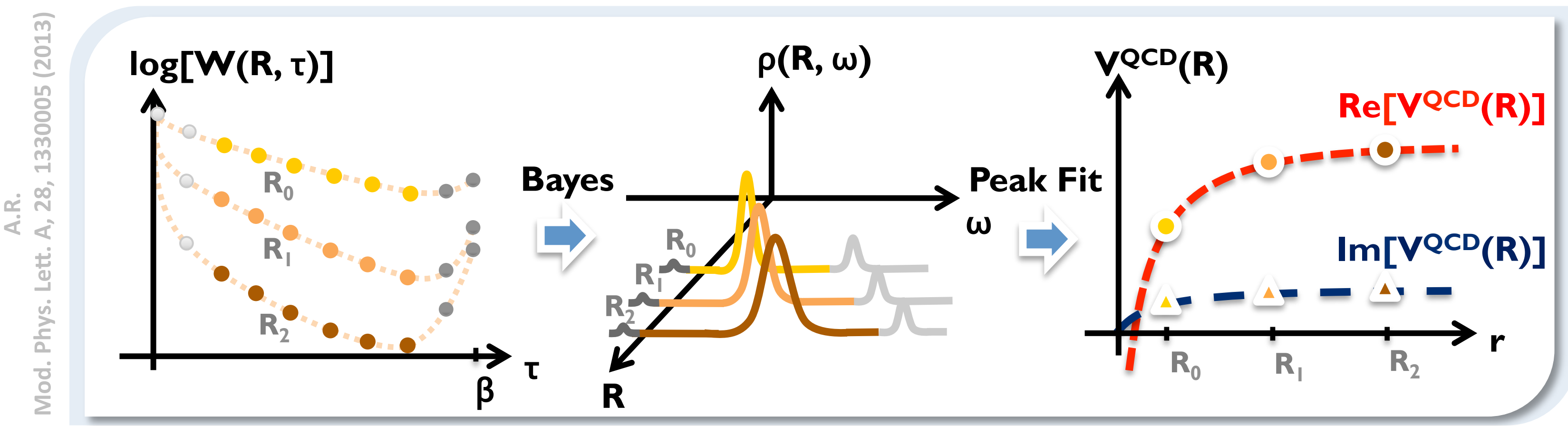
- From lattice QCD correlators to the complex heavy quark potential



- Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
  Practical reason: Absence of cusp divergences, hence less suppression along τ

V^{QCD} in quenched lattice QCD

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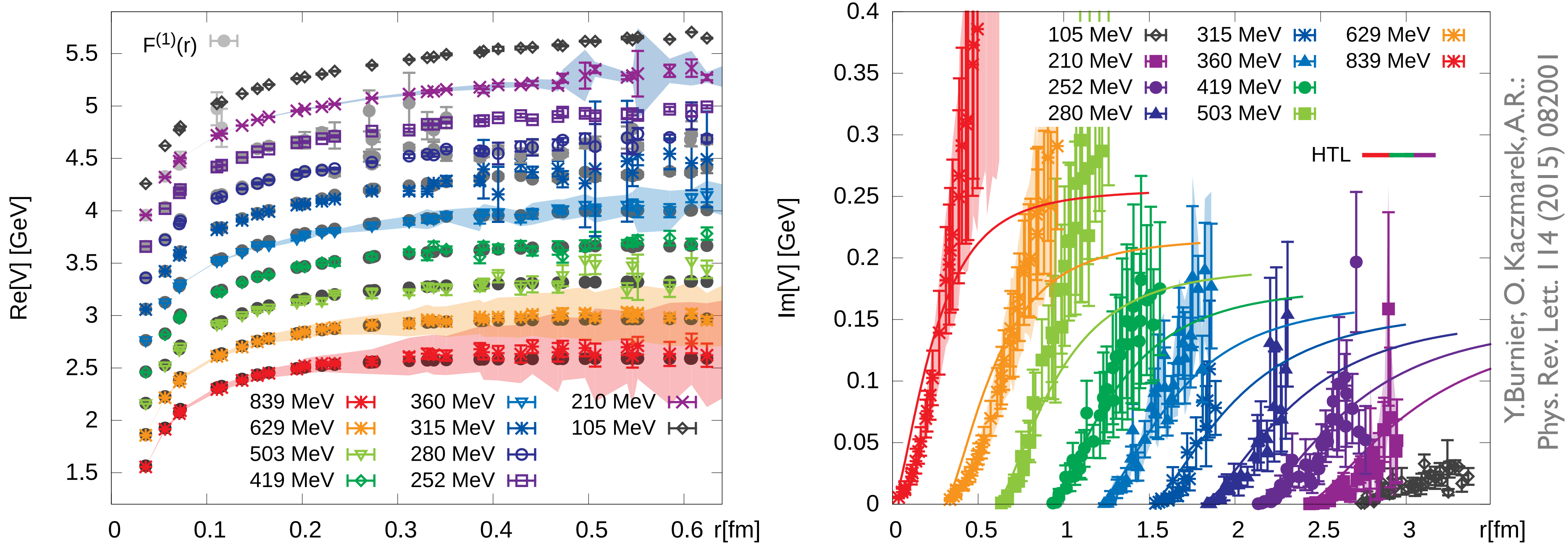
Fixed scale approach: $\beta=7.0$ $\xi=a_s/a_\tau=4$ $a_s=0.039$fm $N_\tau=192$-24



- $Re[V^{QCD}]$: smooth transition from confining to Debye screened behavior
- First principles support: Color singlet free energies lie close to $Re[V^{QCD}]$

$$F^{(1)}(R) = -\frac{1}{\beta}\log\big[W_{||}(R,\tau=\beta)\big]_{CG}$$

- $Im[V^{QCD}]$ for small R: good agreement with HTL prediction down to $1.17T_C$

Re[V^{QCD}] in full lattice QCD

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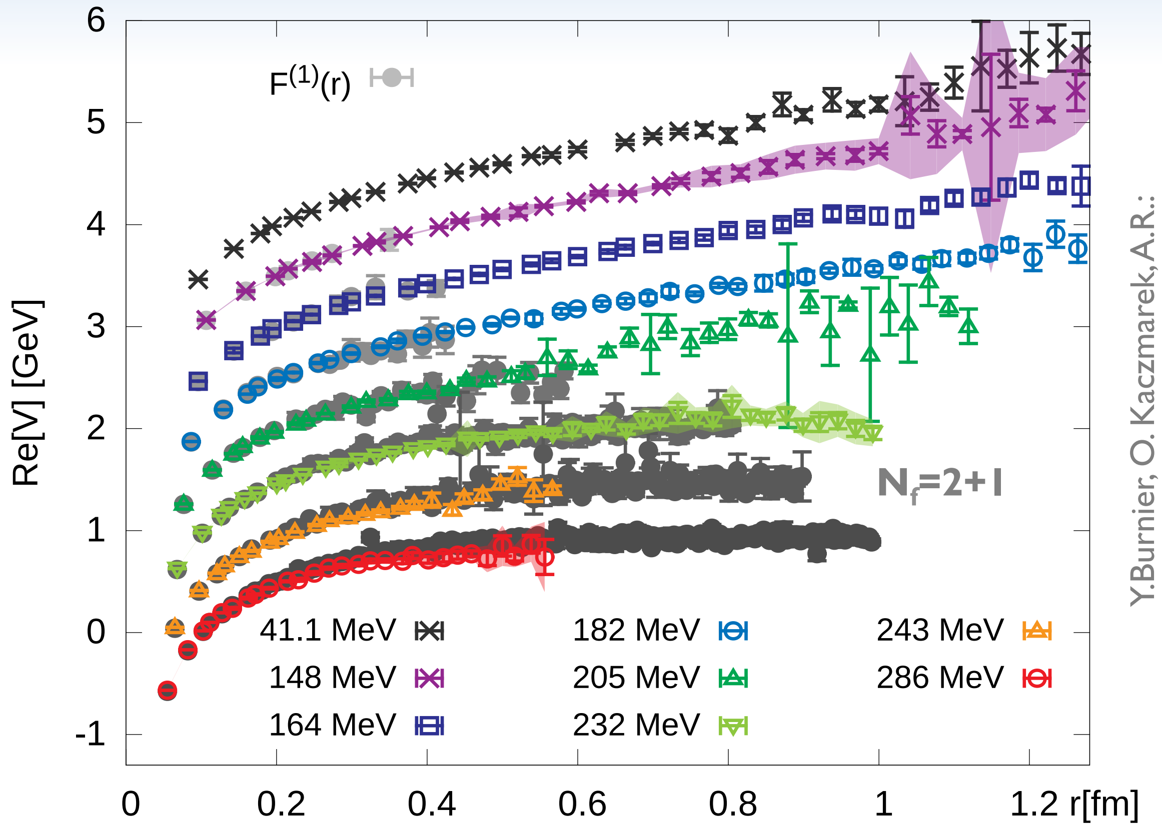
HotQCD Nf=2+1 lattices asqtad action
β =6.664 – 7.480 ξ =a_s/a_τ=1 N_τ=12
A. Bazavov et. al. PRD 85 (2012) 054503



Y.Burnier, O. Kaczmarek, A.R.:
Phys. Rev. Lett. 114 (2015) 082001

- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)
- Qualitatively similar to quenched case (confinement to Debye screening)
- Also here close agreement between color singlet free energies $F^{I}(r)$ and Re[V^{QCD}]

Conclusions

- Bottomonium is a precision probe of the QGP produced in heavy-ion collisions
- New Bayesian spectral reconstruction improves lattice QCD based investigations
- NRQCD bottomonium in a realistic thermal medium (HISQ - HotQCD)
  - In-medium modification of correlators above T=160MeV [up to 1% (Υ) and 5% (χ_{b1})]
  - A systematic comparison between free and interacting spectra show:

    S-wave and P-wave ground state survive up to at least T=249MeV
- Effective field theory derived potential for static quarks from T>0 QCD
  - Re[V] and Im[V] can be determined from the spectral structure of Wilson loops
  - New Bayesian method makes quantitative evaluation on the lattice possible

    first principles model selection: $F^1(R)$ close to Re[V], Im[V] of same order than HTL

Thank you for your attention