Hadron electric polarizability from lattice QCD

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Outline

- E&M polarizabilities and lattice QCD

- Background field method

- Neutral kaon and pion electric polarizability

- Neutron electric polarizability

- Future directions

Preliminaries

- The hadron mass changes when placed in a electromagnetic field

$$H_{\mathrm{em}} = -\,\boldsymbol{\mu}\cdot\boldsymbol{B} - \frac{1}{2}\alpha\boldsymbol{E}^2 - \frac{1}{2}\beta\boldsymbol{B}^2$$

$$-\frac{1}{2}\gamma_{\mathrm{E1}}\boldsymbol{\sigma}\cdot\boldsymbol{E}\times\dot{\boldsymbol{E}} - \frac{1}{2}\gamma_{\mathrm{M1}}\boldsymbol{\sigma}\cdot\boldsymbol{B}\times\dot{\boldsymbol{B}}$$

$$+\,\gamma_{\mathrm{E2}}\sigma_i E_{ij} B_j - \gamma_{\mathrm{M2}}\sigma_i B_{ij} E_j$$

$$-\frac{1}{12}\alpha_{\mathrm{E2}} E_{ij}^2 - \frac{1}{12}\beta_{\mathrm{M2}} B_{ij}^2 + \ldots$$

- μ - the magnetic dipole

- α & β - the electric & magnetic polarizability

- γ - spin polarizabilities

Electric polarizability

- The polarizability measures the dipole moment induced by the field

- The polarizabilities are measured in Compton scattering experiments for proton and from elastic and quasi-elastic Compton scattering on deuteron, lead, etc. for neutron

$$\vec{p} = \alpha \vec{E}$$
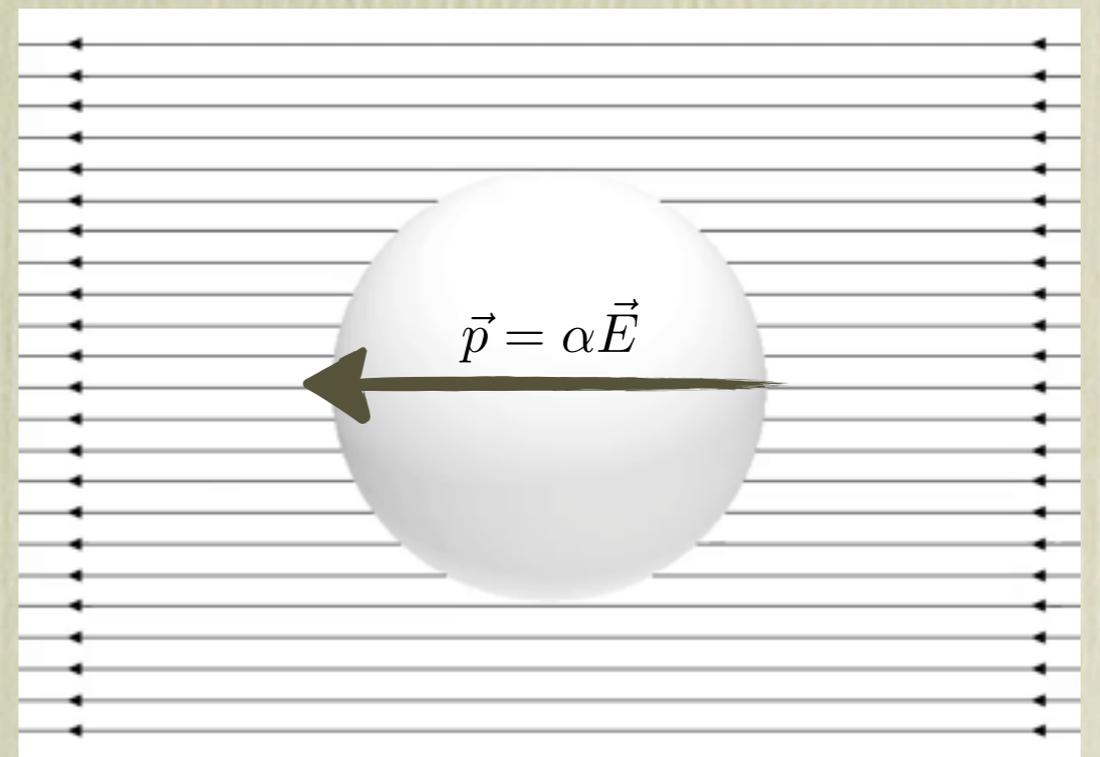
$$\Delta E = -\frac{1}{2}\alpha E^2$$

Lattice QCD primer

- The basic degrees of freedom are quarks and gluons interacting according to Quantum Chromodynamics.

- Quark and gluon fields are sampled on a discrete lattice: quarks at sites and glue on links.

- The action in **Euclidean** time is expressed in terms of the discrete fields.

- For numerical simulations the spatial volume and "temporal" extent is finite. Lattice spacing is tuned by changing interaction strength.

- The calculation is non-perturbative, at least in QCD sector.

$$\}a$$

$$\psi, \bar{\psi}$$

$$U_\mu = e^{-iagG_\mu}$$
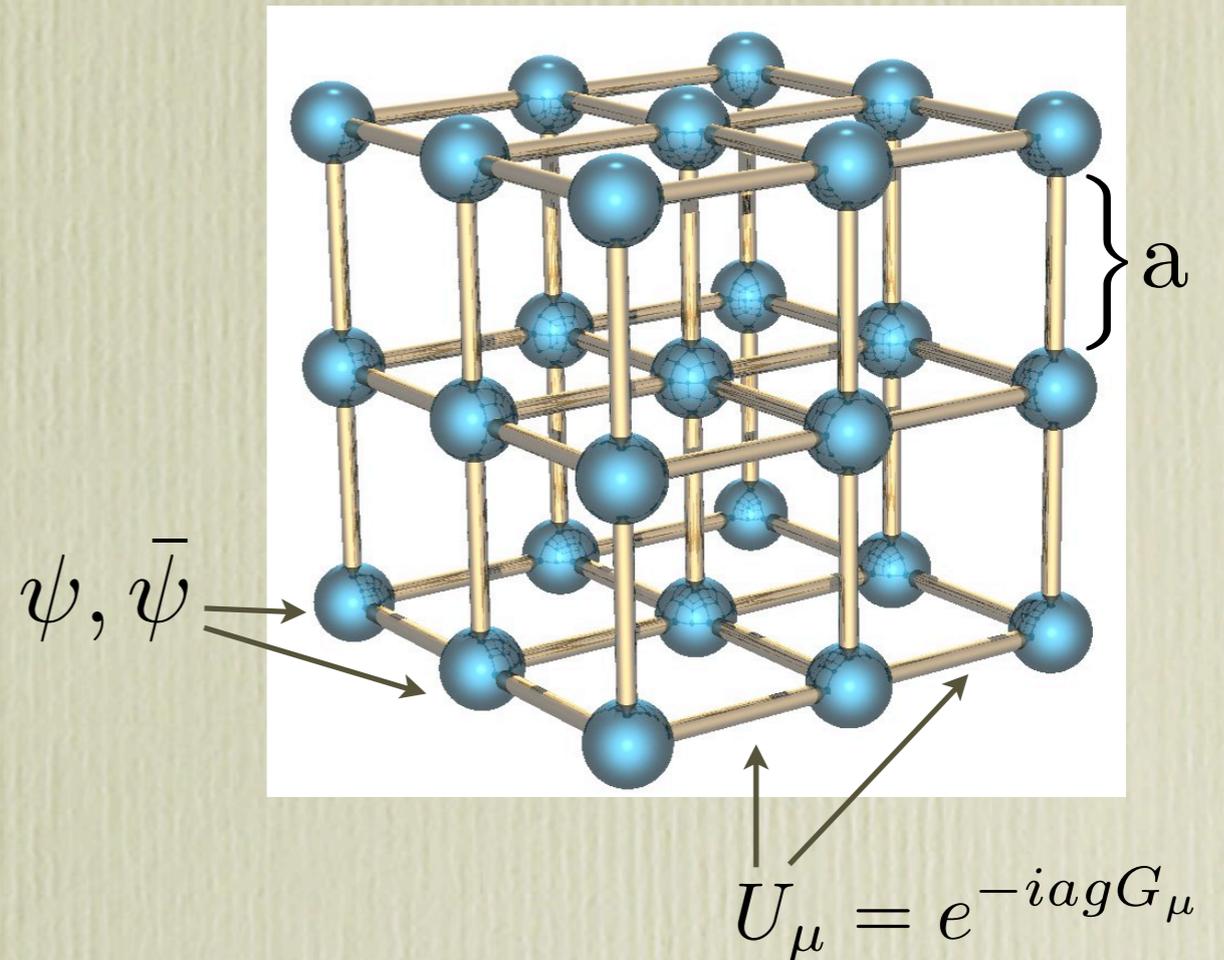
Sea and valence fermions

- Hadrons are created using quark composite functions.

- Hadron mass is extracted from correlation of the composite fields.

- Quark masses can be adjusted independently for **valence** and **sea** quarks.

- Quenched simulations: sea quarks infinitely heavy. Partially quenched: sea and valence quark masses are different.

- In lattice simulations quark masses are larger than physical ones for numerical reasons.

- Since quark masses are not observable, we use pion mass to determine how close we are to the physical point.
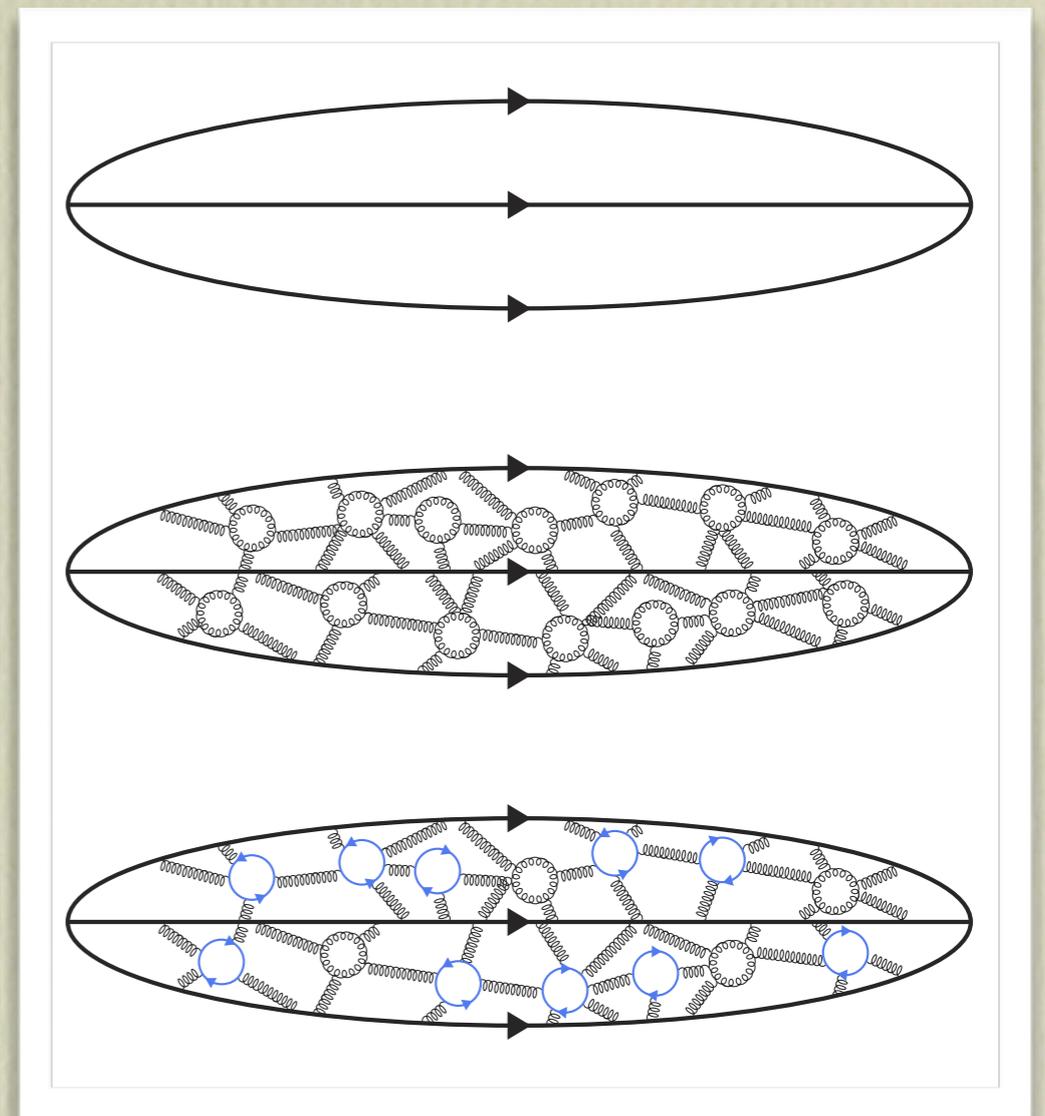
Electric field on the lattice

Background field method

- Introduce a background electric field

$$D_\mu = \partial_\mu - igG_\mu - \boxed{iqA_\mu}$$

- The U(1) field A_μ is static

- On the lattice this amounts to changing the links

$$U_\mu \to e^{-iqaA_\mu} U_\mu$$

- The polarizability is extracted from the mass shift

Extracting polarizability

- To introduce an electric field on the lattice we need to use a real phase factor in the exponential form

$$U_1 \to U_1 e^{-aqEt} \Rightarrow \Delta m = -\frac{1}{2}\alpha E^2$$

- The imaginary phase factor can also be used if we remember to flip the sign

$$U_1 \to U_1 e^{-iaqEt} \Rightarrow \Delta m = +\frac{1}{2}\alpha E^2$$

- Magnetic field is introduced using an imaginary phase factor

$$U_2 \to U_2 e^{-iaqBx_3} \Rightarrow \Delta m = -\frac{1}{2}\beta B^2$$

AA and F. X. Lee, PoS **LATTICE2008** (2008) 145.

Boundary conditions

- Lattice QCD is formulated in Euclidean time.

- This requires that the Hamiltonian of the system is bounded from below, i.e. there is a vacuum state of lowest energy.

- In the presence of a real electric field, the vacuum is no longer stable -- Schwinger instability against pair creation.

- In a finite volume box we can make the system stable by limiting the maximal distance between charges.

- We use Dirichlet boundary conditions in space to stabilize the system.

- Note that this instability exists even in a finite volume box if we use periodic boundary conditions.
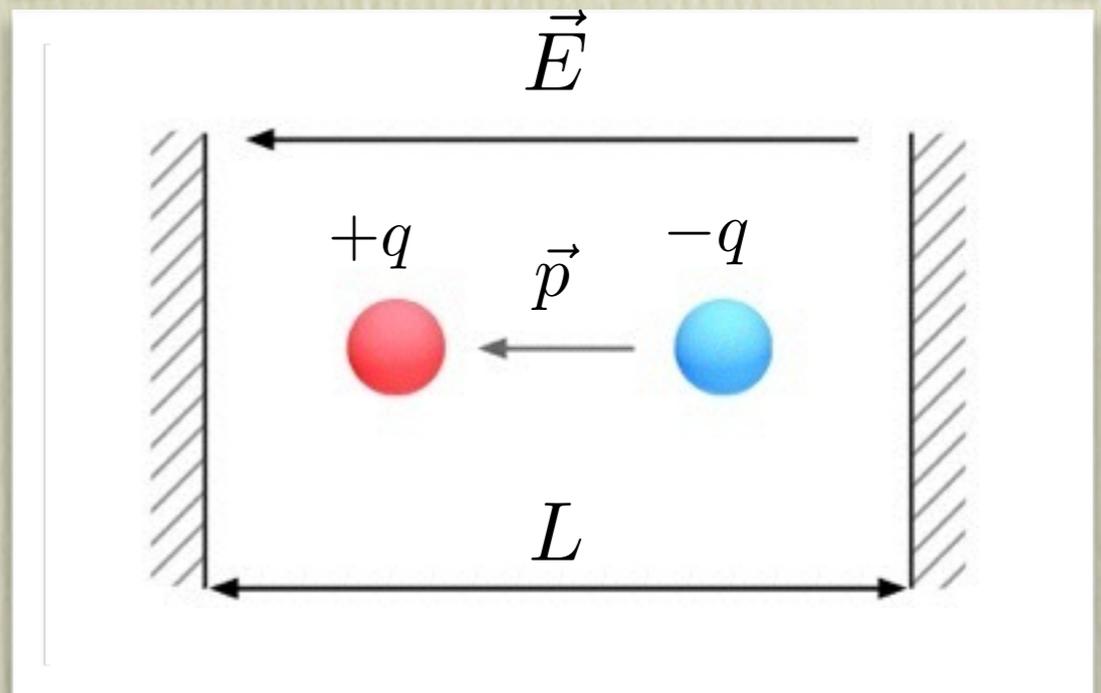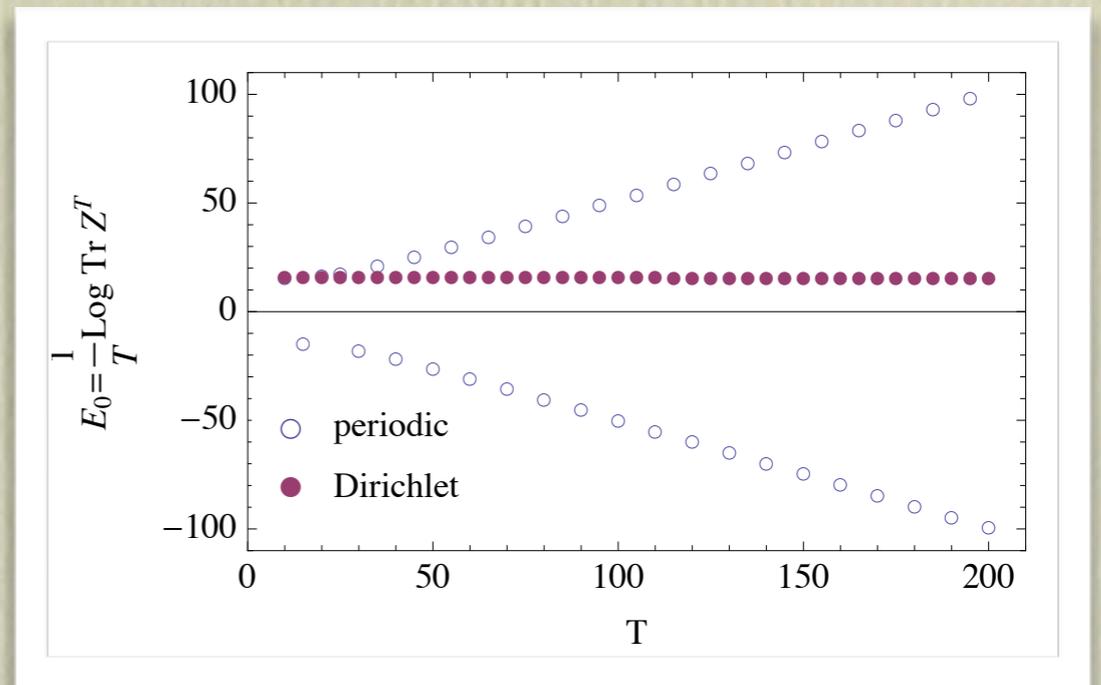
Dirichlet boundary conditions

- Dirichlet boundary conditions are equivalent to a hard wall in the direction of the electric field.

- The lowest energy for one-particle states corresponds to a non-zero momentum.

- The momentum is related to the size of the box and its magnitude decreases slowly with the size of the box.

- Finite volume effects are thus expected to be important and vanish slowly as we increase the size of the system. We can study the infinite volume limit by varying the dimension of the box only in the field direction.

- Energy shift due to polarizability is corrected due to hadron motion: δm = (E/m)δE.

$$p = \frac{\pi}{L}$$
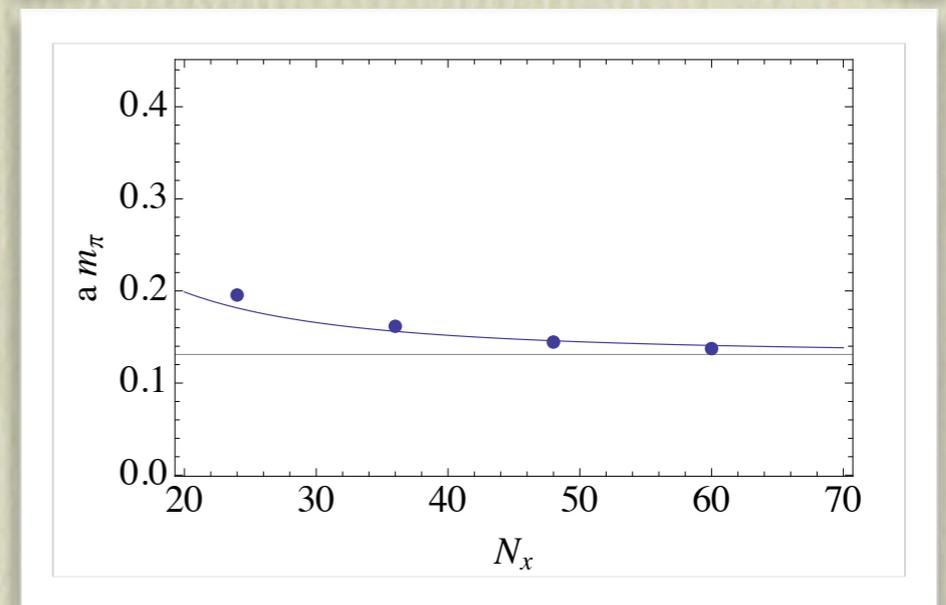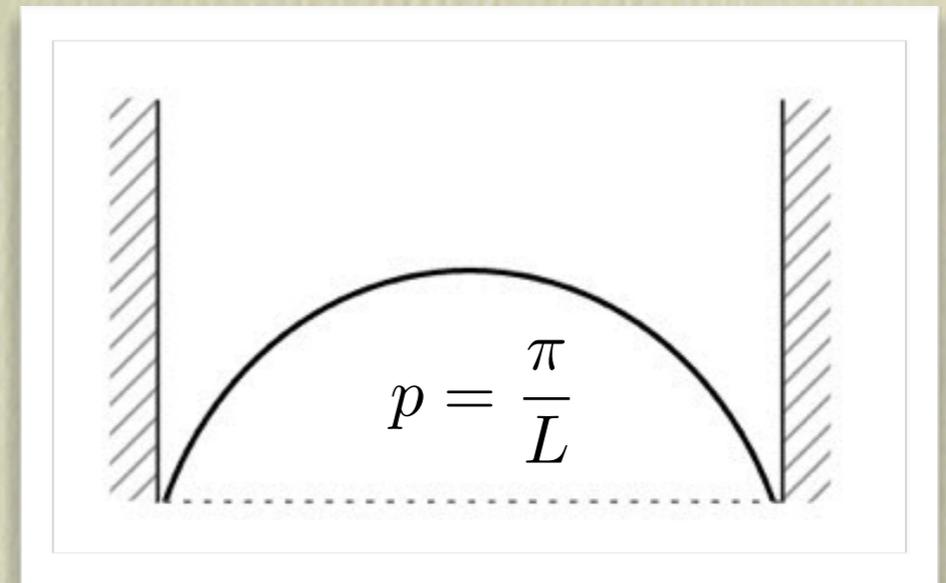
$$E_\pi(L) = \sqrt{m_\pi^2 + (\pi/L_x)^2}$$

Compton polarizability

$$\mathcal{L} = \bar{N}\left(i\gamma_\mu D^\mu - M\right)N + \left(-\frac{e\kappa}{4M}F_{\mu\nu}\right)\bar{N}\sigma^{\mu\nu}N + \left(\frac{e\kappa}{4M} - \frac{er_E^2}{6}\right)\partial^\nu F_{\mu\nu}\bar{N}\gamma_\mu N +$$

$$+ \left(\frac{\bar{\beta}}{4}F_{\mu\nu}F^{\mu\nu}\bar{N}N - \frac{\bar{\alpha}+\bar{\beta}}{4M}F_{\mu\alpha}F^{\nu\alpha}\bar{N}\gamma^\mu i\overleftrightarrow{D}_\nu N\right) + \dots$$

A. L'vov, *Int.J.Mod.Phys*. **A8** (1993) 5267

If we set the polarizabilities to zero in the Lagrangean above and use it to compute the energy of a non-relativistic neutron we have

$$E(\vec{p}) = \frac{\vec{p}^2}{2M} - \vec{\mu}\cdot\left(\vec{E}\times\frac{\vec{p}}{M}\right) + \frac{\mu^2}{2M}\vec{E}^2$$

When using Dirichlet boundary conditions the momentum is aligned with the electric field and the second term doesn't contribute. The last term contributes even when the momentum is zero and we have

W. Detmold et al, *Phys.Rev*. **D81** (2010) 054502

$$\boxed{\bar{\alpha} = \alpha + \frac{\mu^2}{M}}$$

Numerical results

Lattice parameters

- $24^3 \times 48$ nHYP clover ensemble -- pion mass 306MeV

  - 300 configurations -- 25 source points per config

  - Lattice spacing a=0.1245(16) fm

- $24^3 \times 64$ nHYP clover ensemble -- pion mass 227MeV

  - 450 configurations -- 18 source points per config

  - Lattice spacing a=0.1215(11) fm

- Electric field ~ 10^{21} V/m - $\eta = a^2qE = 0.0001$ -- imaginary

"Neutral Pion"

- The physical π^0 correlator has disconnected contributions in the presence of a background field

- The disconnected diagrams cancel only in the isospin limit -- the electric field breaks isospin symmetry

- Our calculation doesn't include disconnected contributions

- The particle we study is more like $\bar{d}u$ when u and d have the same charge

- In this version of QCD the pions are all uncharged and χPT predicts a flat behavior (to leading order).

- The "neutral pion" polarizability is expected to be small and positive.

W. Detmold, B. C. Tiburzi, and A. Walker-Loud, *Phys.Rev.* **D79** (2009) 094505

Neutral pion polarizability
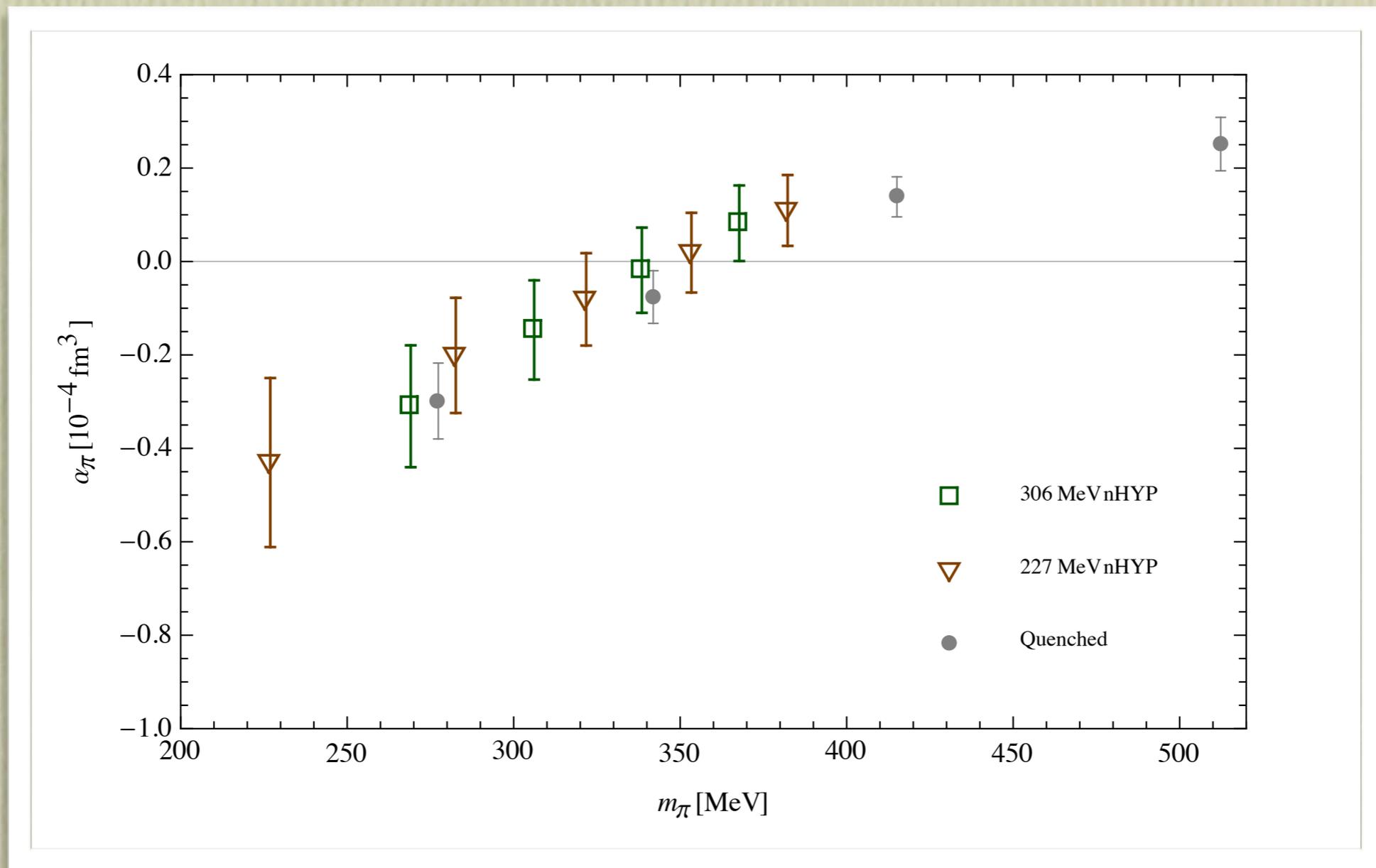
$$\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$

AA and F. X. Lee, PoS **LATTICE2009** 144

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506

Neutral pion polarizability

- We computed the neutral pion polarizability, neglecting the disconnected contributions, using dynamical quark backgrounds.

- We find that the presence of the dynamical quarks changes only slightly the results. The polarizability varies very little with the sea quark mass, but depends rather strongly on the valence mass.

- For m_π=227 MeV the value we find is about $^-$0.5 × 10$^-$4 fm^3.

- This seems to disagree with χPT expectations that the value is small, slightly positive and has little dependence on the quark mass.

- We identified two possible sources for this discrepancy: finite volume effects and neglecting the electric charge of the sea quarks. We found that the negativity does not go away in the infinite volume limit.

Neutral kaon polarizability

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506
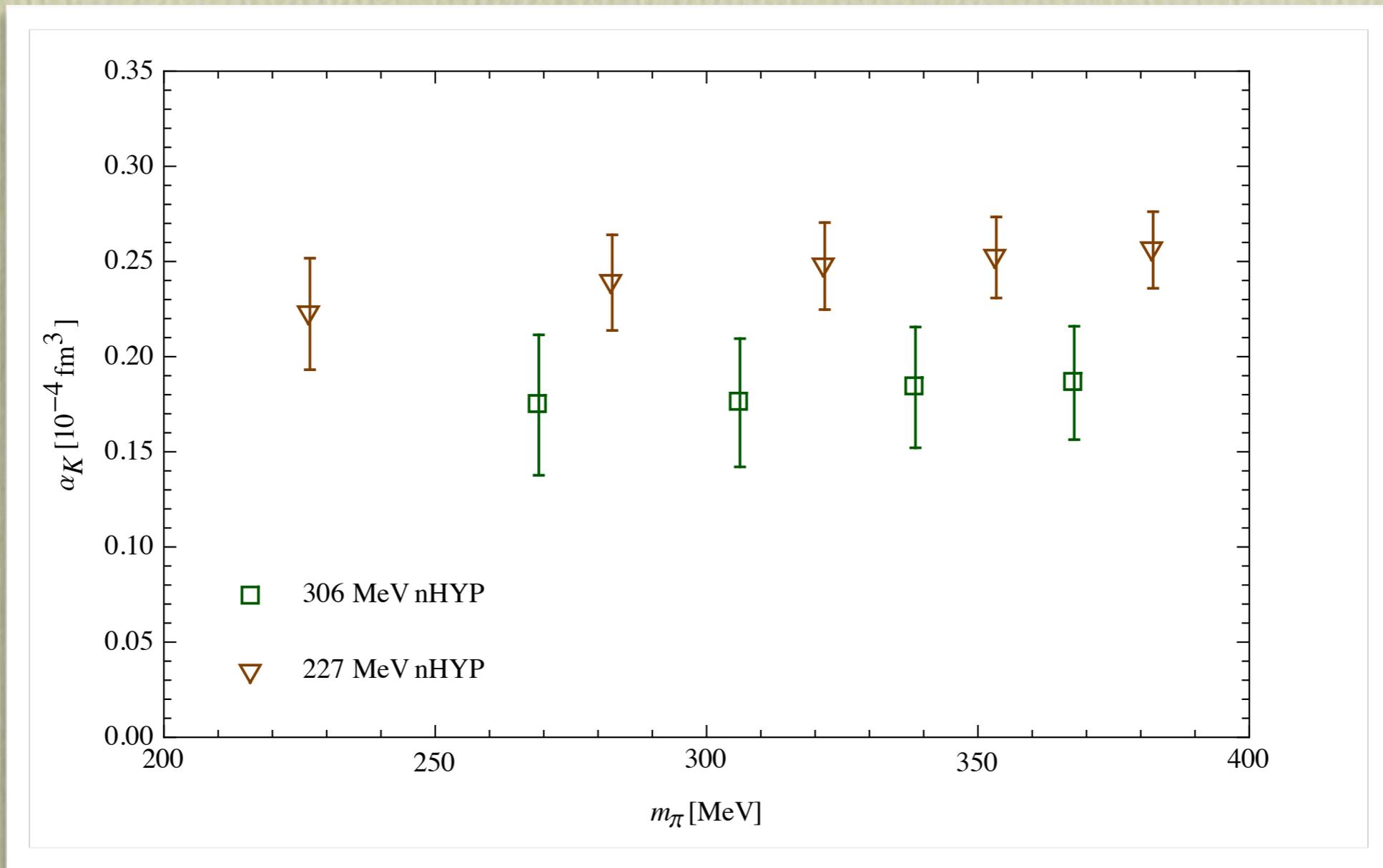
Neutral kaon polarizability
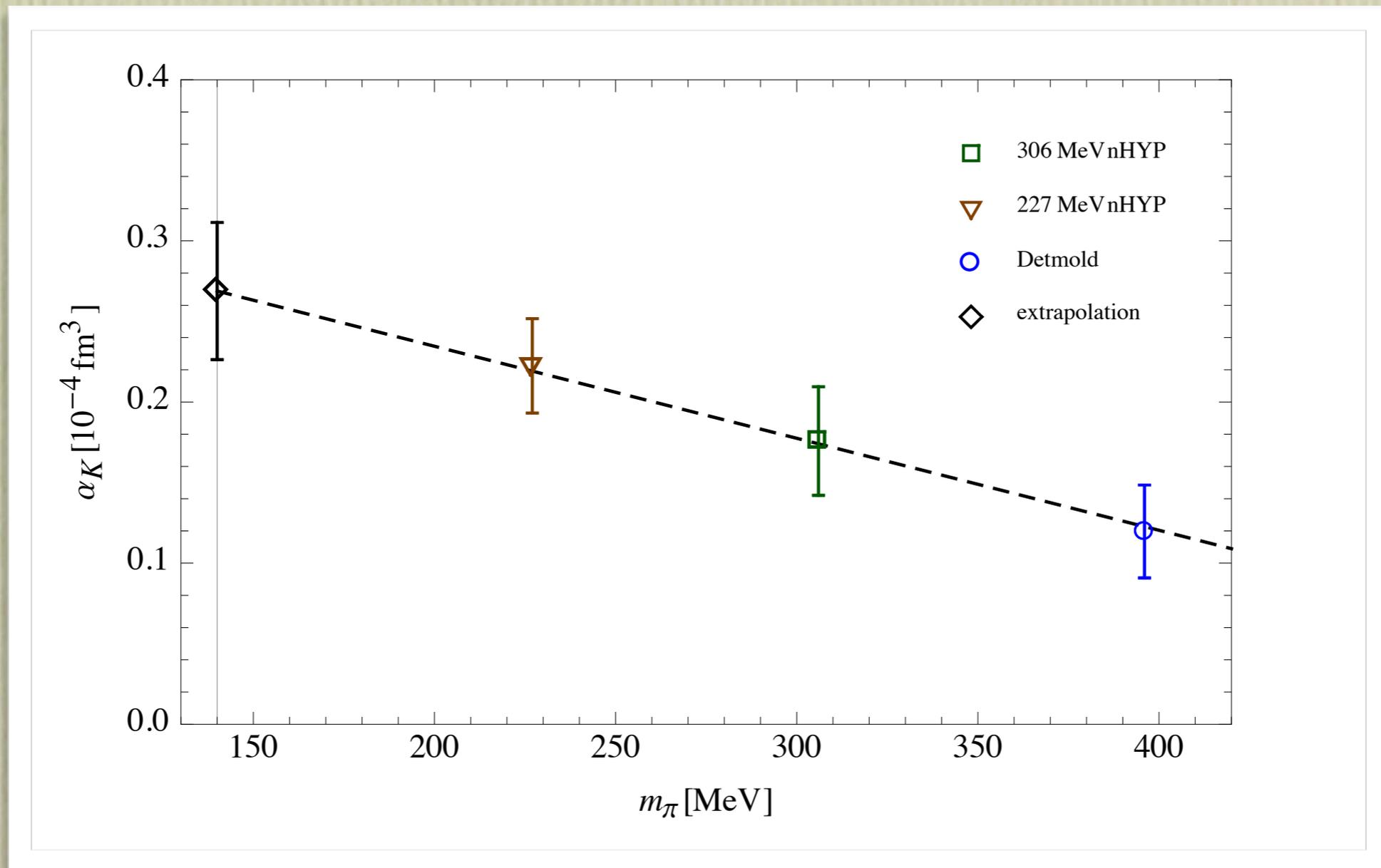
$$\alpha_K = 0.269(43) \times 10^{-4}\ \mathrm{fm}^3$$

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506
W. Detmold et al, Phys.Rev. **D79** (2009) 094505

Neutral kaon polarizability

- We computed the neutral kaon polarizability using dynamical quark backgrounds.

- The polarizability changes only slightly when we change the mass of the light quarks, both for valence and sea quarks.

- Since this parameter varies slowly with respect to the quark mass, we believe that the extrapolation to the physical point can be done with high confidence; we find a value of $0.27(4) \times 10^{-4}$ fm^3.

- The small value is in agreement with χPT expectations. This quantity receives no contribution to $O(p^4)$, the first non-zero contributions are at $O(p^6)$.

Neutron polarizability

AA and F. Lee, PoS **LAT2010** (2010) 131

M. Engelhardt, PoS **LAT2009** (2009) 128

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506

W. Detmold, B. Tiburzi, and A. Walker-Loud, Phys.Rev. **D81** (2010) 054502
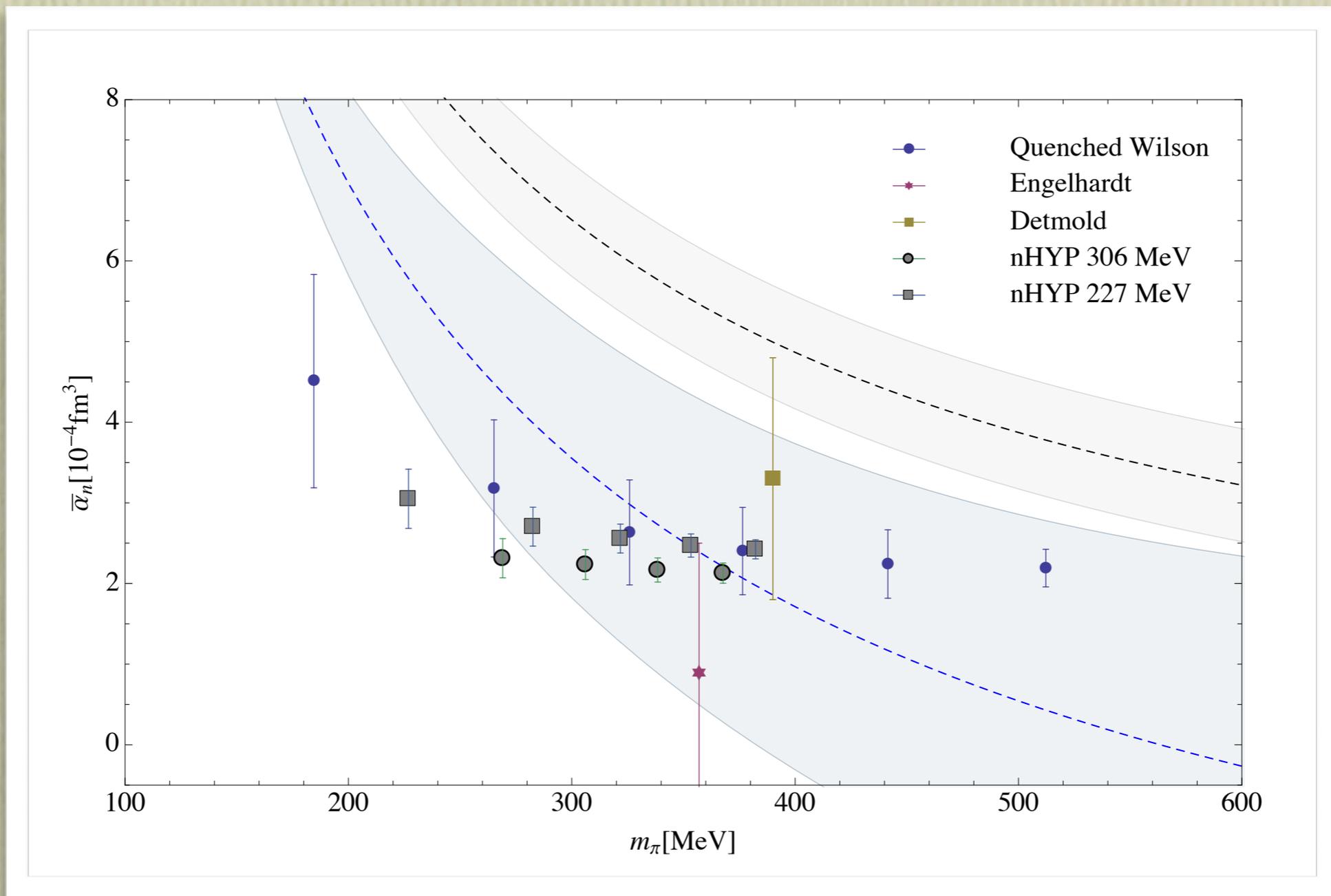
Neutron polarizability
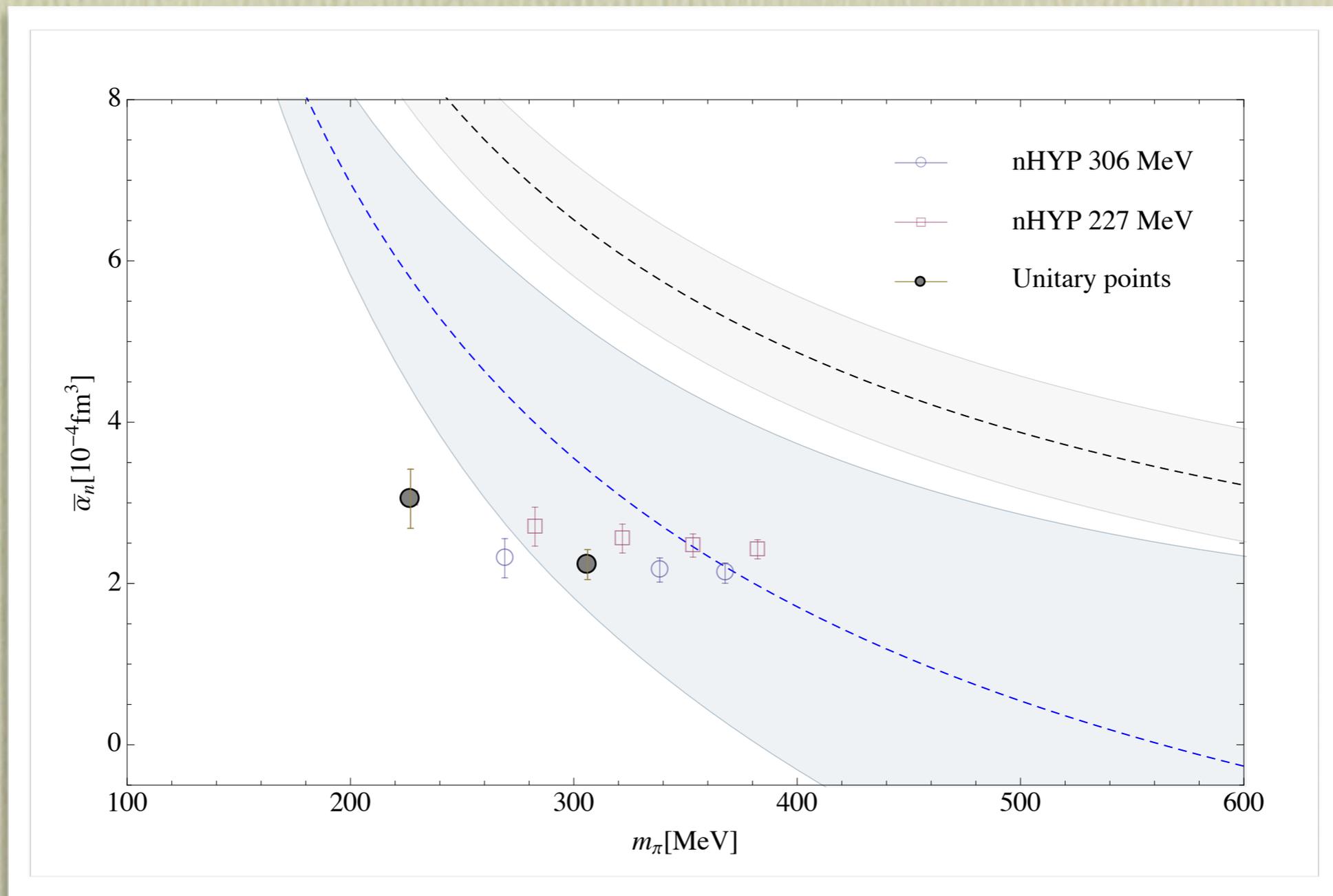
J. McGovern, D. Phillips, and H. Griesshammer, Eur. Phys.J. **A49** (2013) 12

M. Lujan, AA, W. Freeman, F. X. Lee, Phys.Rev. **D89** (2014) 074506

V. Lensky and V. Pascalutsa, Eur. Phys.J. **C65** (2010) 195

Neutron polarizability

- We computed the neutron polarizability using dynamical quark backgrounds.

- We find that the presence of the dynamical quarks increases slightly the polarizability, but the results are in rough agreement with the ones in the quenched study.

- It is clear that our results are not in agreement with χPT expectations: at 227 MeV pion mass the compute value is only 60% of the one predicted from χPT and more than 3 standard deviations away.

- We identified two possible sources for this discrepancy: finite volume effects and neglecting the electric charge of the sea quarks.

Finite volume effects
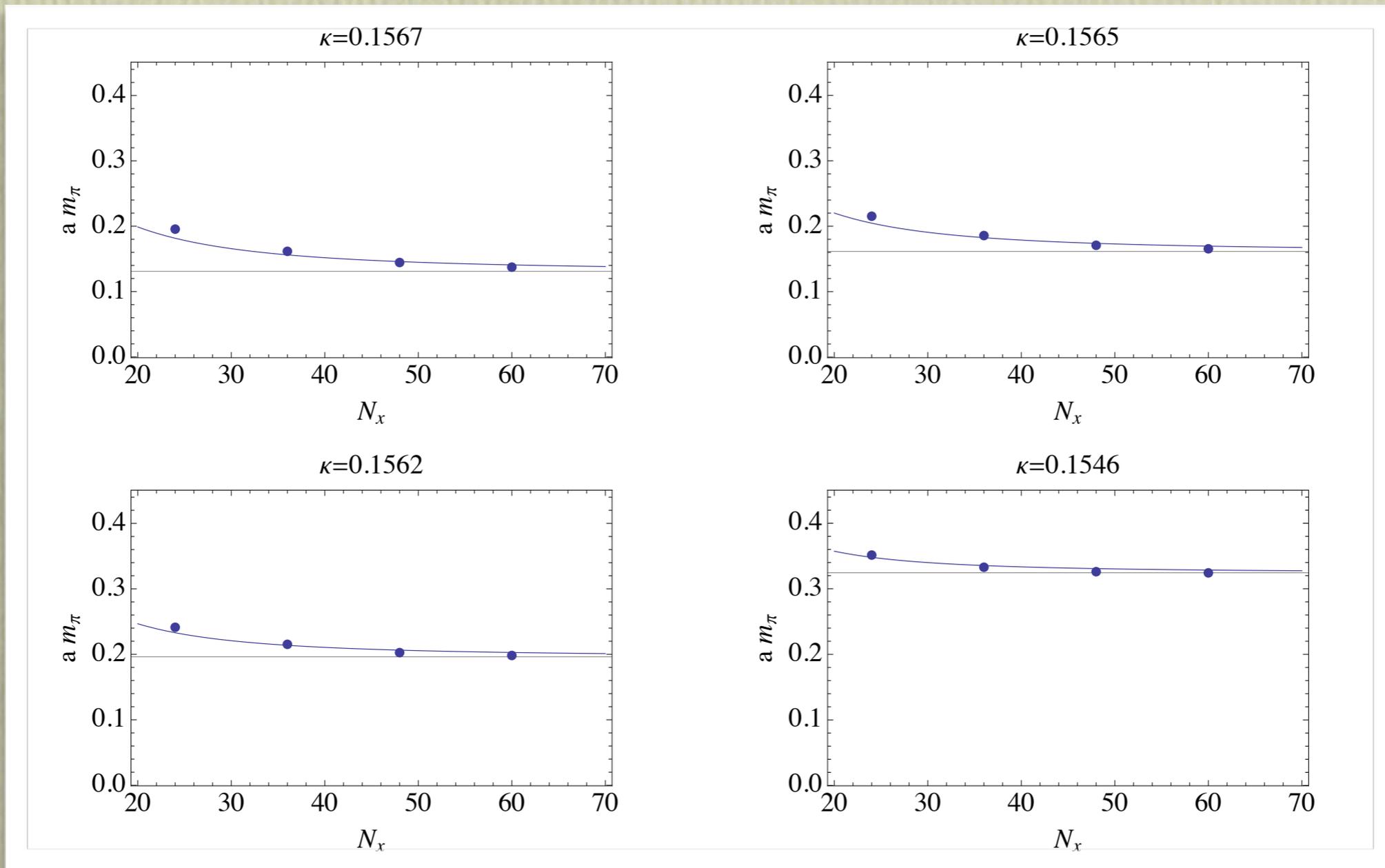
$$m_\pi(L) = \sqrt{m_\pi^2 + (\pi/L_x)^2}$$

Infinite volume extrapolation
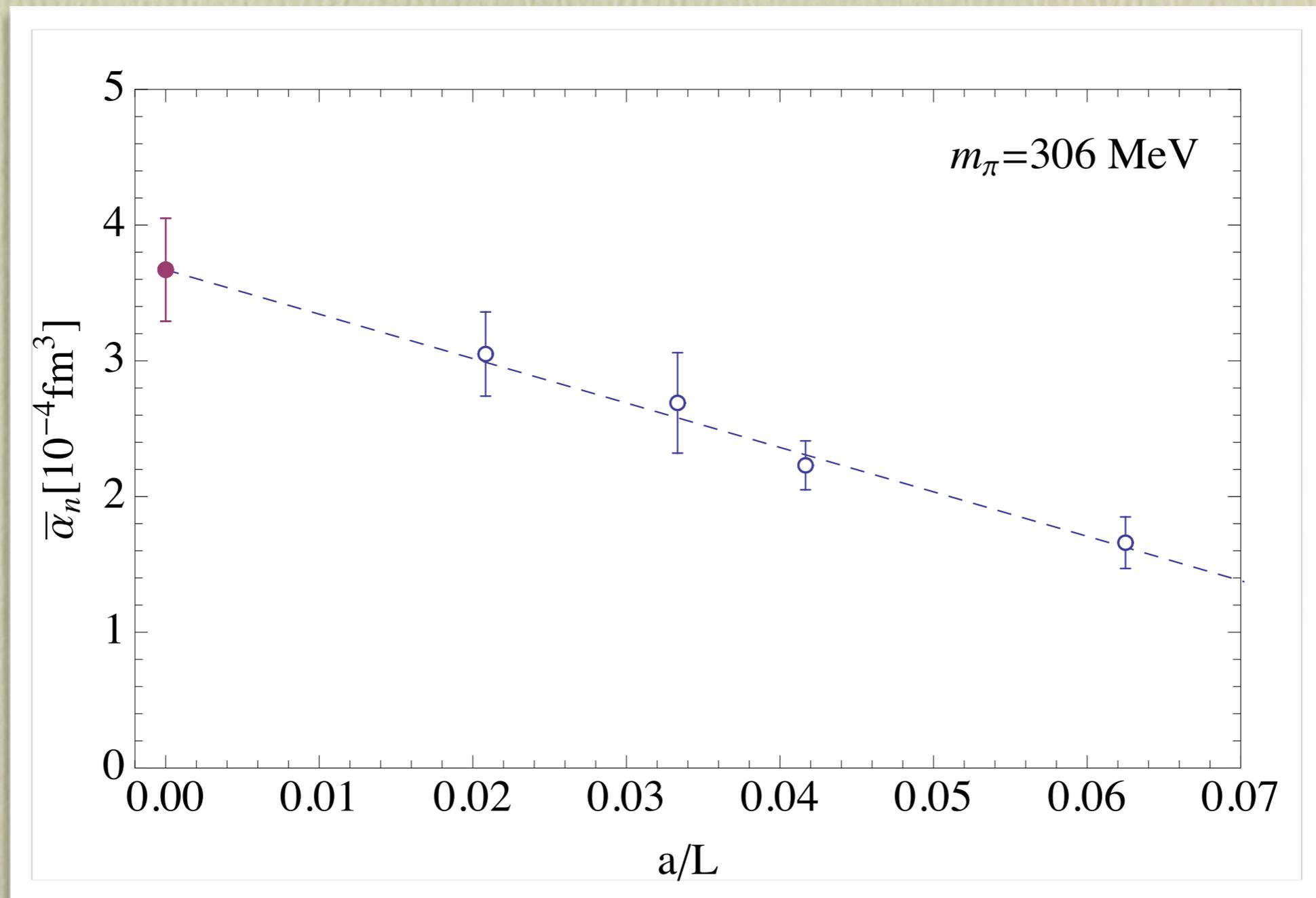
Four different lattice sizes: $16^3 \times 32$, $24^3 \times 48$, $30 \times 24^2 \times 48$, $48 \times 24^2 \times 48$

Infinite volume extrapolation
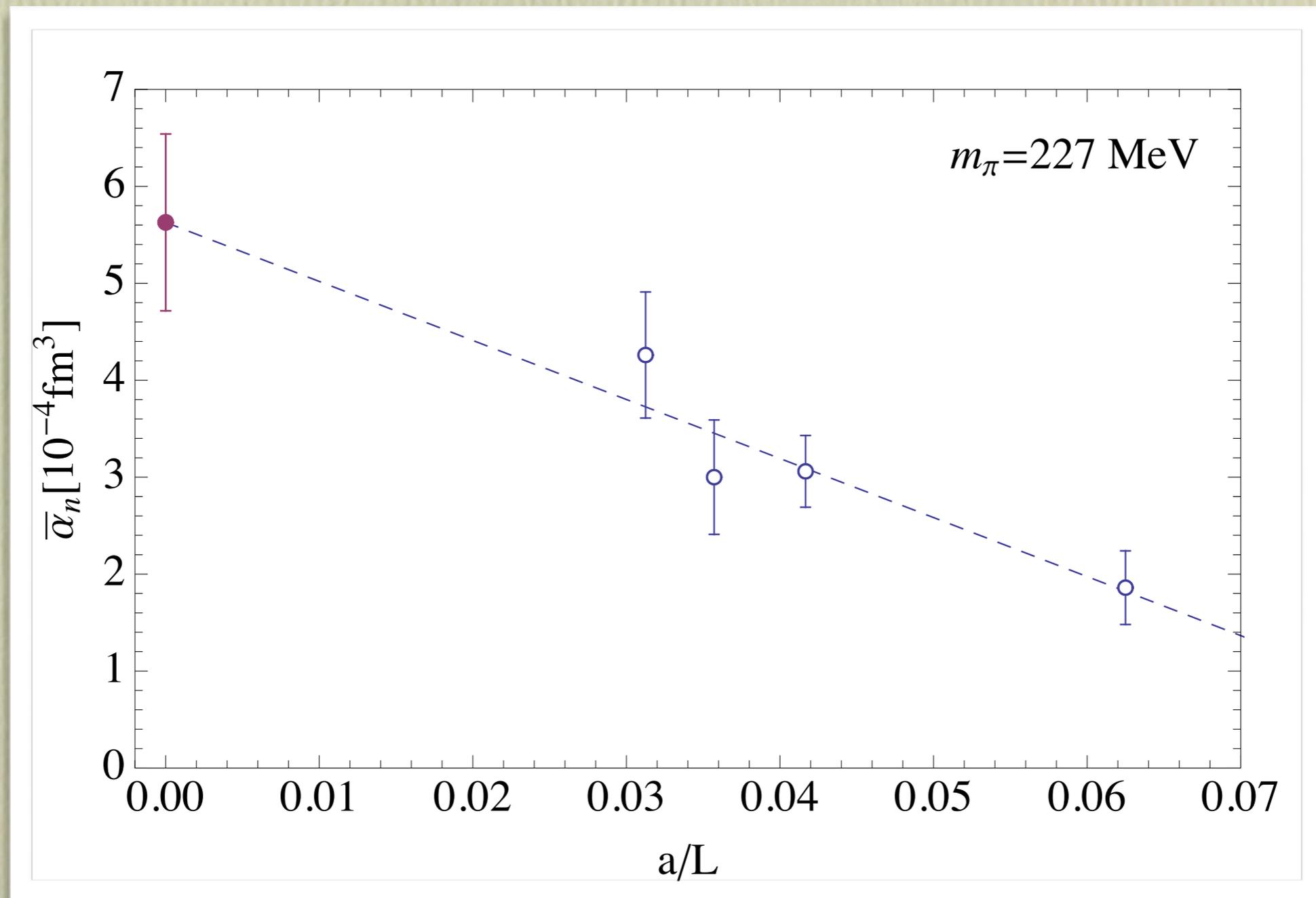
Four different lattice sizes: $16^3 \times 32$, $24^3 \times 64$, $28 \times 24^2 \times 64$, $32 \times 24^2 \times 64$

Infinite volume extrapolation
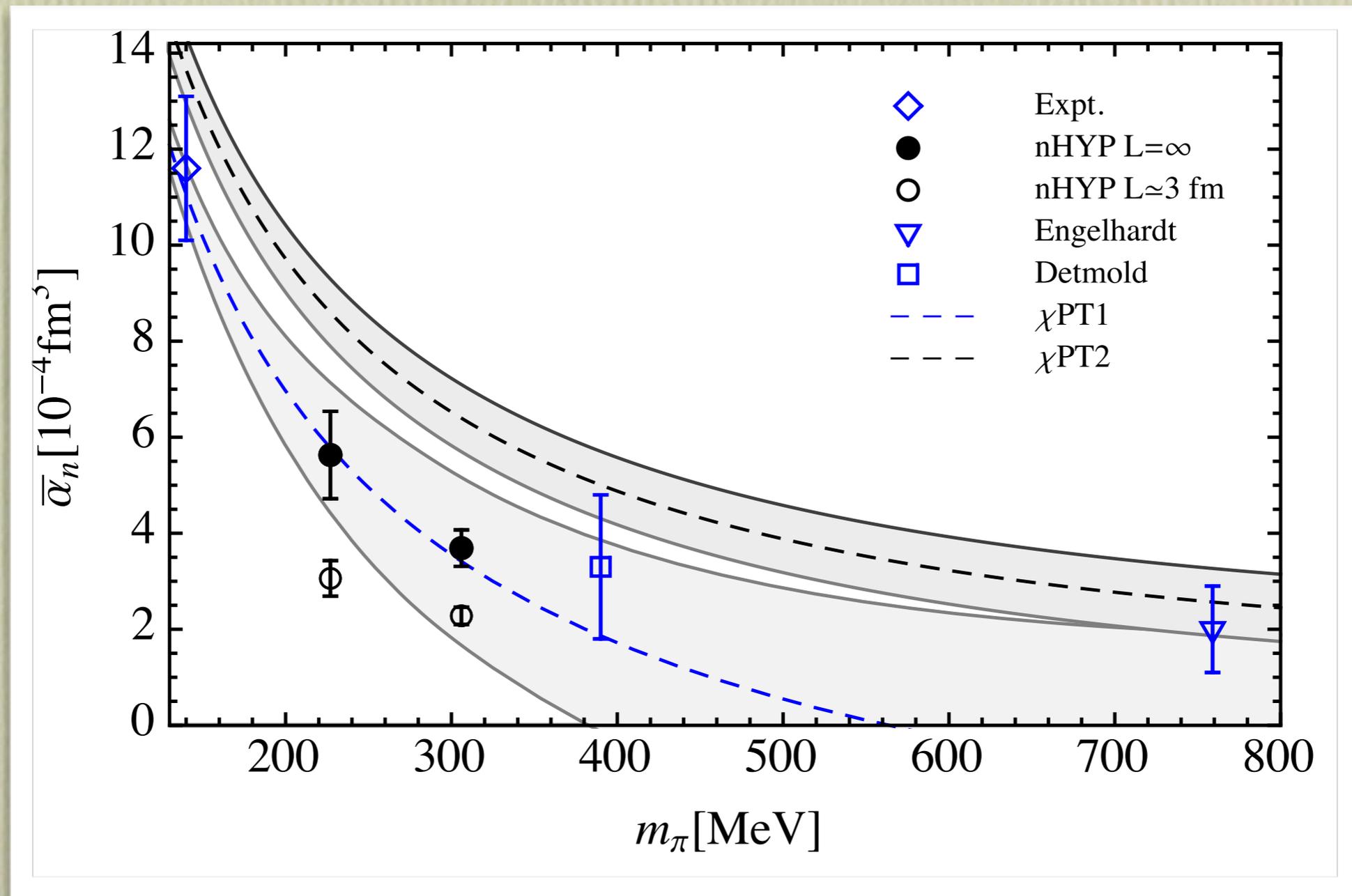
Future directions

Charging the sea quarks

- There are two types of dynamical effects:

  - quark-antiquark loop

  - background field effects on the sea quarks

- Include the dynamical effects of the electric field -- they become important at smaller quark masses

  - use the electric field in the generation of configurations -- not really feasible

  - use reweighting

  $$\langle G_E(t)\rangle_E = \frac{\left\langle G_E(t)\frac{\det M_E}{\det M_0}\right\rangle_0}{\left\langle\frac{\det M_E}{\det M_0}\right\rangle_0}$$

Reweighting

- We need to "charge" the sea quarks

- We use reweighting to exploit the correlation between the correlators with and without the field

- It turns out that standard estimators for the reweighting factor do not work

- We use a perturbative expansion

$$\langle G_E(t) \rangle_E = \frac{\left\langle G_E(t) \frac{\det M_E}{\det M_0} \right\rangle_0}{\left\langle \frac{\det M_E}{\det M_0} \right\rangle_0}$$

$$w(E) = \frac{\det M_E}{\det M_0} \approx e^{D_1 E + D_2 E^2}$$

$$D_1 = Tr M_0^{-1} \left. \frac{\partial M_E}{\partial E} \right|_{E=0}$$

$$D_2 = \frac{1}{2} Tr M_0^{-1} \left. \frac{\partial^2 M_E}{\partial E^2} \right|_{E=0} - \frac{1}{2} Tr \left(M_0^{-1} \left. \frac{\partial M_E}{\partial E} \right|_{E=0} \right)^2$$

W. Freeman, AA, F. X. Lee, M. Lujan , PoS **LAT2013** (2013) 288

W. Freeman, AA, M. Lujan, and F. X. Lee, Phys.Rev. **D90** (2014) 054507

Neutron polarizability

W. Freeman, AA, M. Lujan, and F. X. Lee, Phys.Rev. **D90** (2014) 054507

Charged hadrons

AA, M. Lujan, W. Freeman, and F. Lee, arXiv:1501.06516

W. Detmold, B. C. Tiburzi, and A. Walker-Loud, Phys.Rev. **D79**, 094505
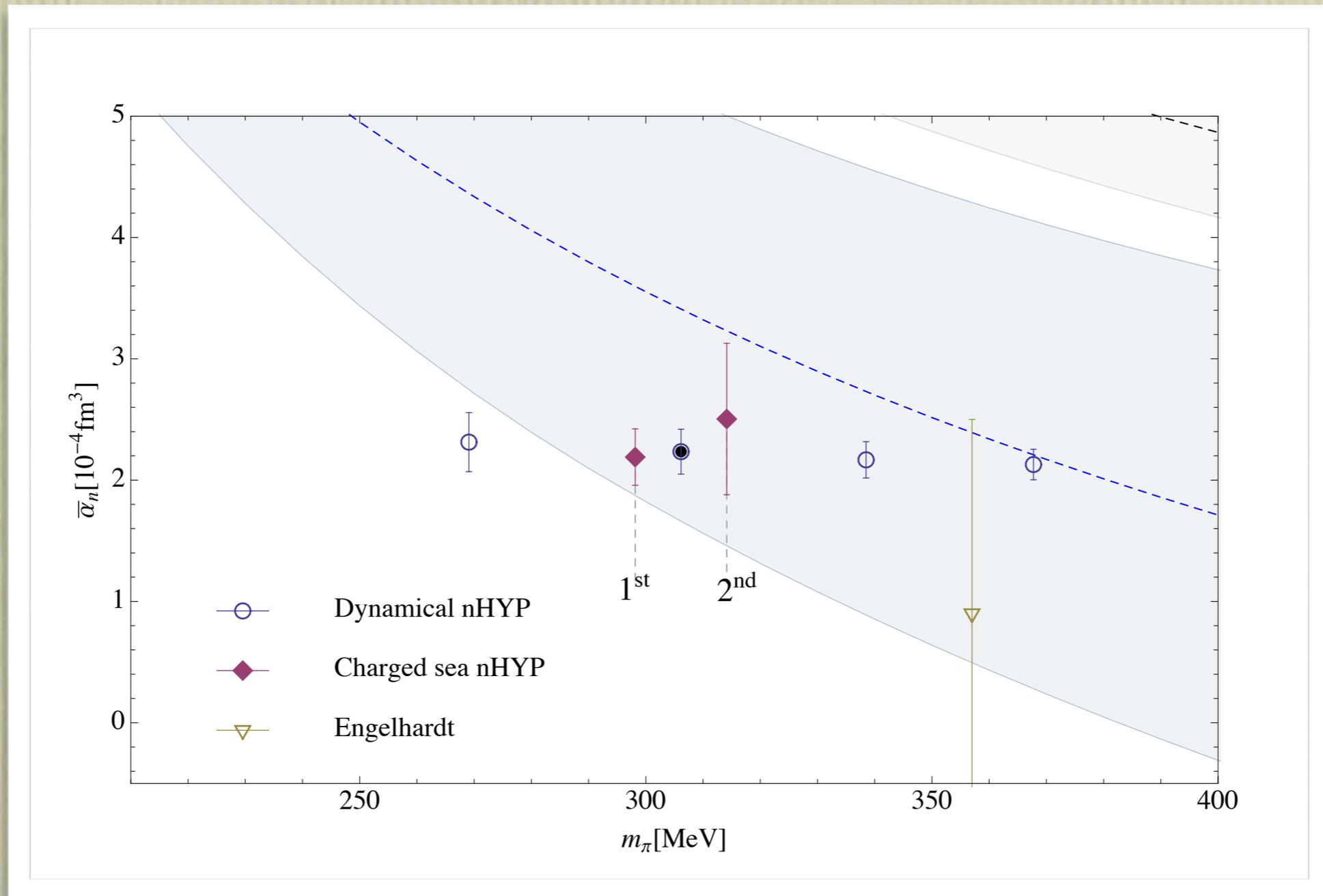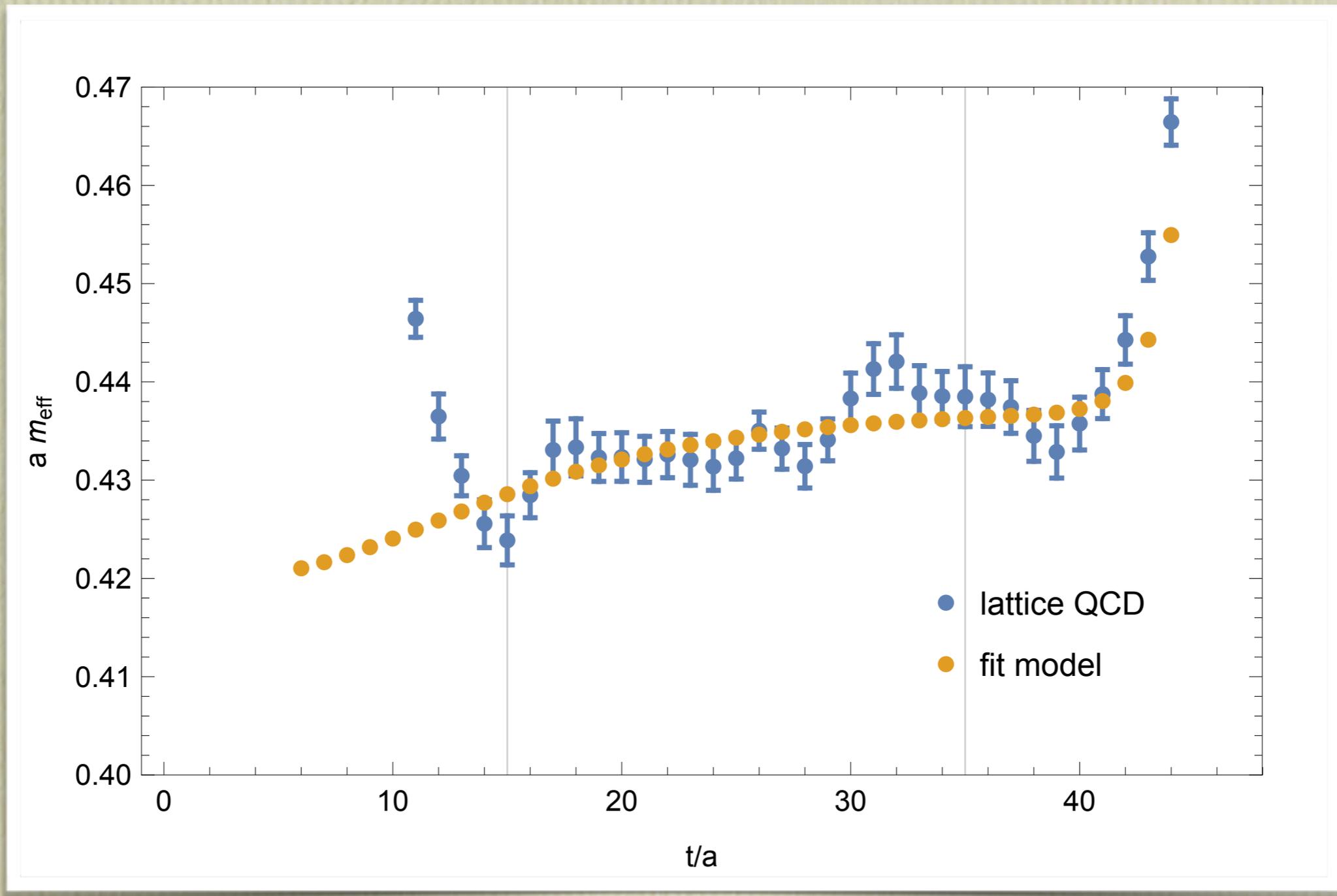
Conclusions and outlook

- We presented results for electric polarizability of neutral kaon, pion and neutron.

- We focused on electrical polarizability for neutron as a benchmark calculation.

- We presented lattice QCD results for pion mass as light as 227 MeV and box sizes of 3fm.

- After removing finite volume effects, we find that the polarizability rises as we approach the physical point at a rate similar to the one predicted by chiral perturbation theory.

- We need to include the effects of the electric field on the virtual quark-antiquark pairs using better estimators to reduce the errors.

- There are many challenges that need to be addressed as we move to charged hadrons, magnetic polarizability, spin polarizabilities, etc.