Two-Gluon Correlations in Heavy-Light Ion Collisions

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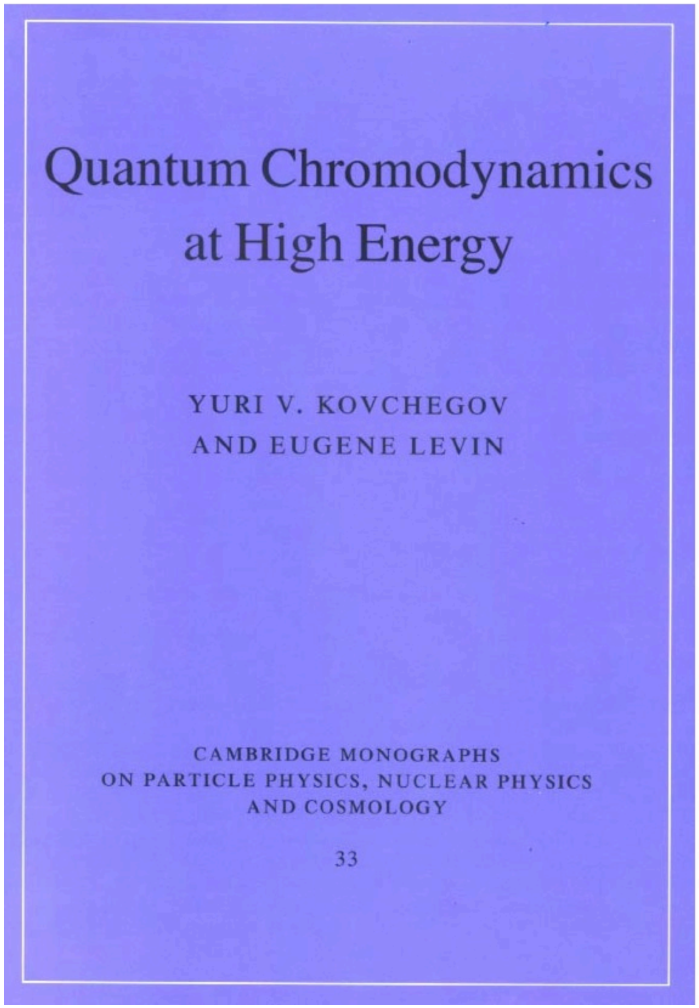
Outline

- Long-range rapidity correlations in hadronic and nuclear collisions: general discussion
- Two-gluon production in heavy-light ion collisions:
  - Geometric correlations
  - Long-range rapidity correlations
    - Even harmonics
    - Energy independence
    - Initial vs final state correlations
  - Perturbative HBT correlations
- k_T-factorization
- Conclusions

Introduction to saturation physics



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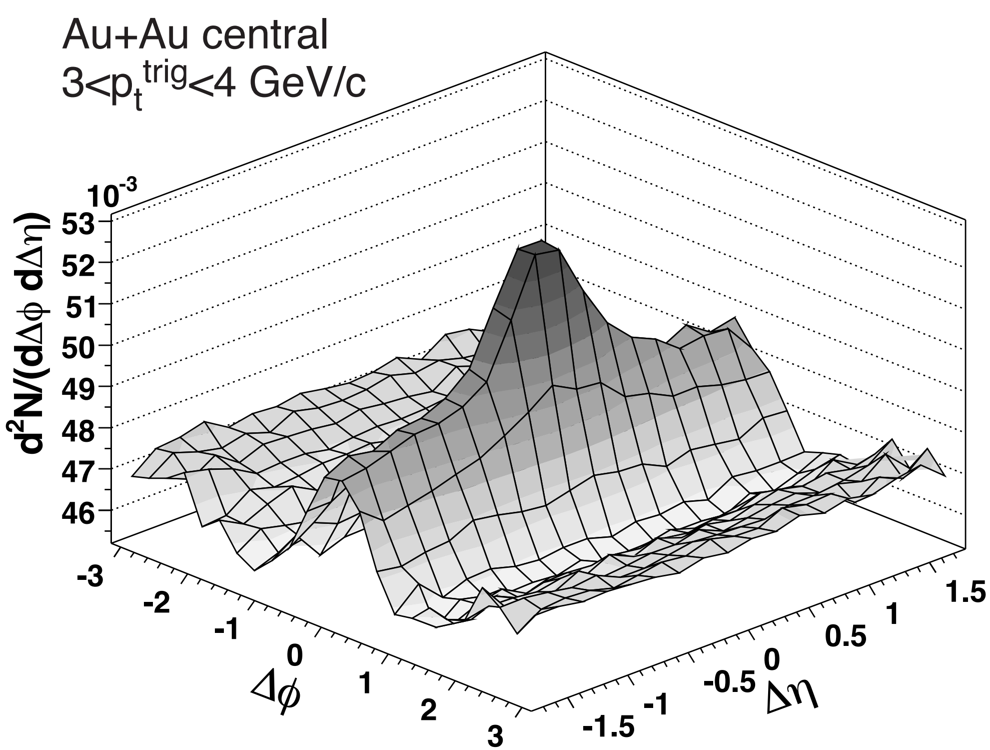
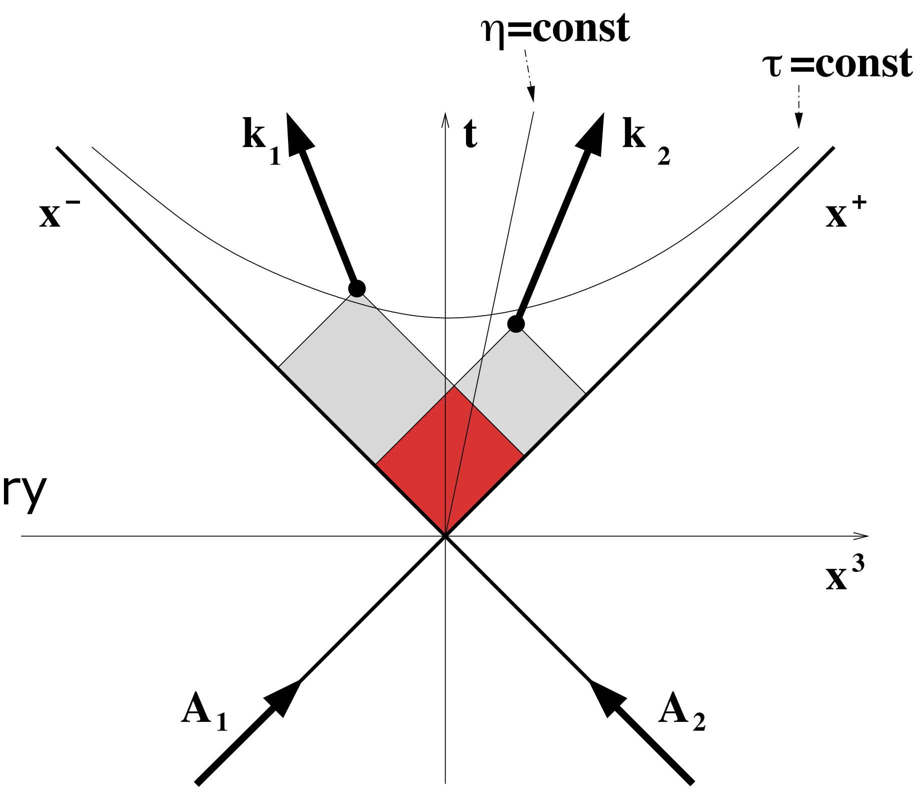
Long-range rapidity correlations

Ridge in heavy ion collisions

- Heavy ion collisions, along with high-multiplicity p+p and p+A collisions, are known to have long-range rapidity correlations, known as 'the ridge':



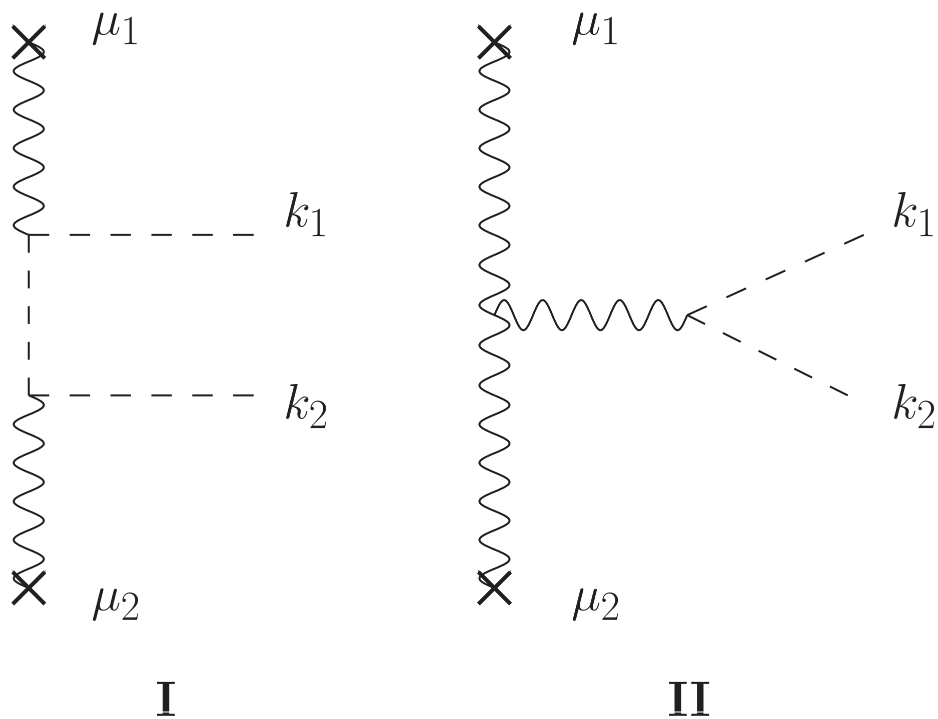
Origin of rapidity correlations



Causality demands that long-range rapidity correlations originate at very early times (cf. explanation of the CMB homogeneity in the Universe)

Gavin, McLerran, Moschelli ‘08;
Dumitru, Gelis, McLerran, Venugopalan ‘08.

Correlations in AdS shock wave collisions



$$C(k_1, k_2) \sim \cosh(4\,\Delta y)$$

H. Grigoryan, Yu.K. ‘10

Correlations in AdS shock wave collisions

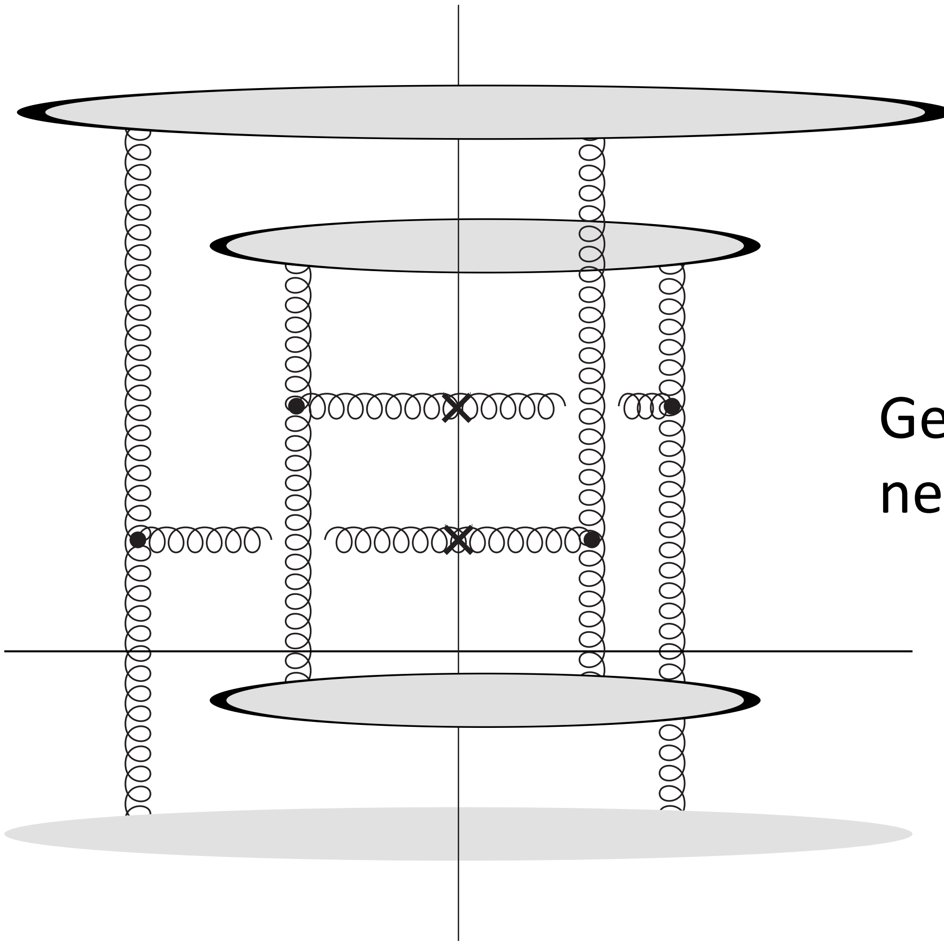
$$C(k_1, k_2) \sim \cosh(4\,\Delta y)$$

- Correlations grow with rapidity interval???
- It is possible that higher-order corrections in shock wave strengths will modify this result, making it closer to real life.
- However, such corrections are important at later times, and are less likely to affect the long-range rapidity correlation…
- This could be another argument in favor of weakly-coupled dynamics in the early stages of heavy ion collisions.

Ridge in CGC

- There are two explanations of the ridge in CGC:
  - Long-range rapidity-independent fields are created at early times, with correlations generated soon after and with azimuthal collimation produced by radial hydro flow. (Gavin, McLerran, Moschelli '08)
  - Both long-range rapidity correlations and the azimuthal correlations are created in the collision due to a particular class of diagrams referred to as the "Glasma graphs".

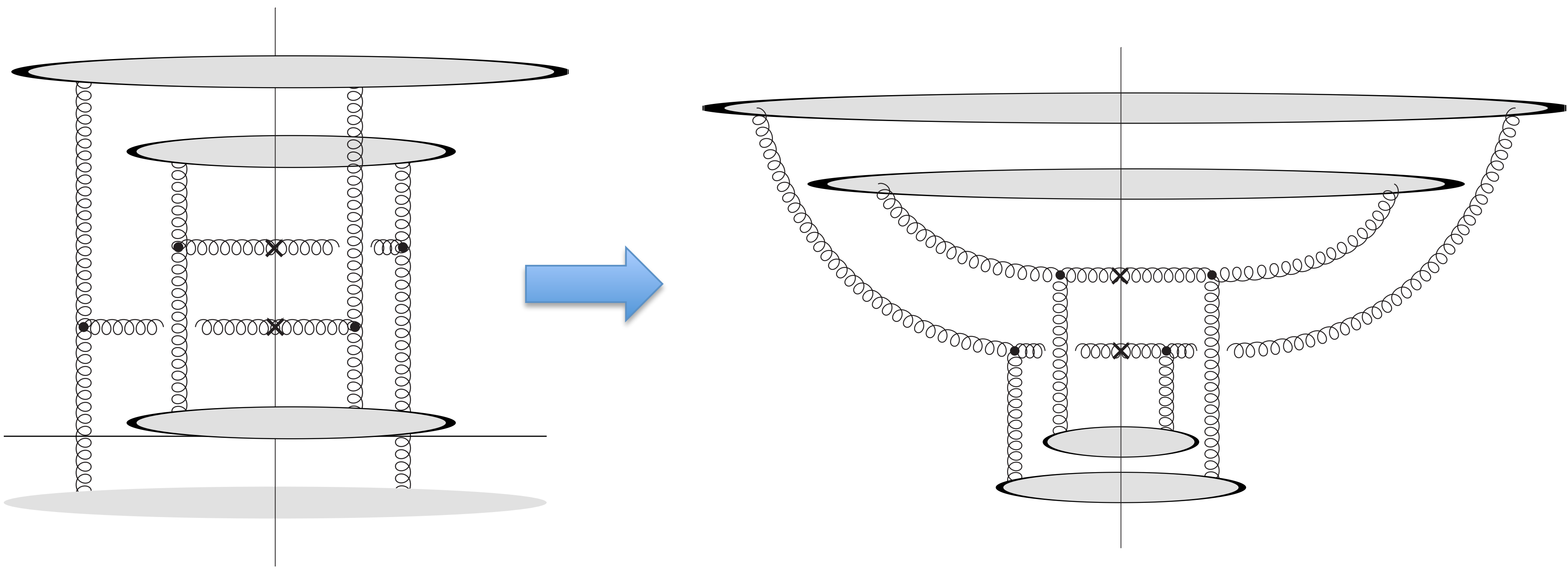
Glasma graphs



Generate back-to-back and near-side azimuthal correlations.

Dumitru, Gelis, McLerran, Venugopalan '08.

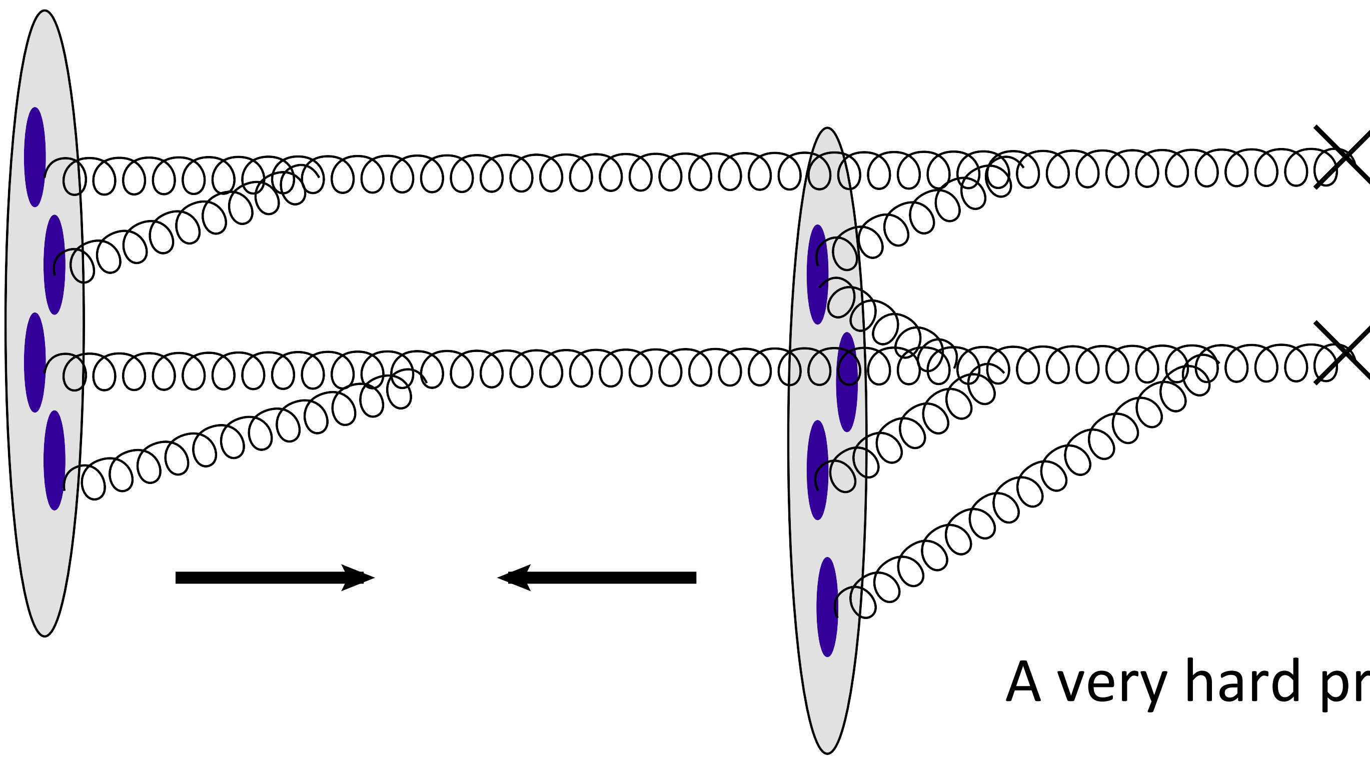
Glasma graphs in LC gauge



Glasma graphs are one of the many rescattering diagrams when two nucleons with a gluon each scatter on a nuclear target.

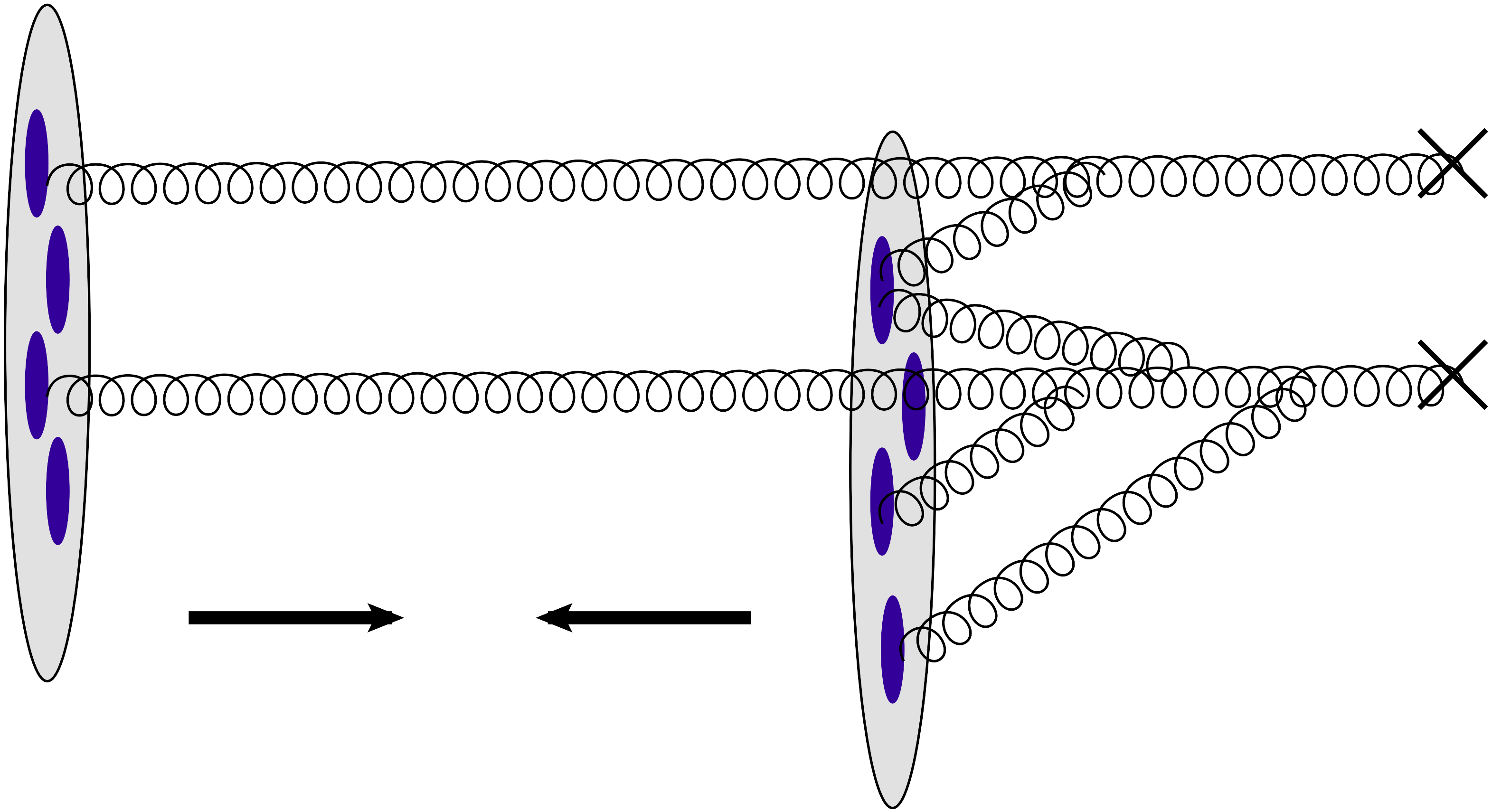
What to calculate?

- To systematically include Glasma graphs in the CGC formalism it would be great to solve the two-gluon inclusive production problem in the MV model, that is, including multiple rescatterings in both nuclei to all orders (the two produced gluons only talk to each other through sources):



A very hard problem!

Heavy-Light Ion Collisions



Little steps for the little feet: consider multiple rescatterings only in one of the two nuclei.

Double gluon production in heavy-light ion collisions

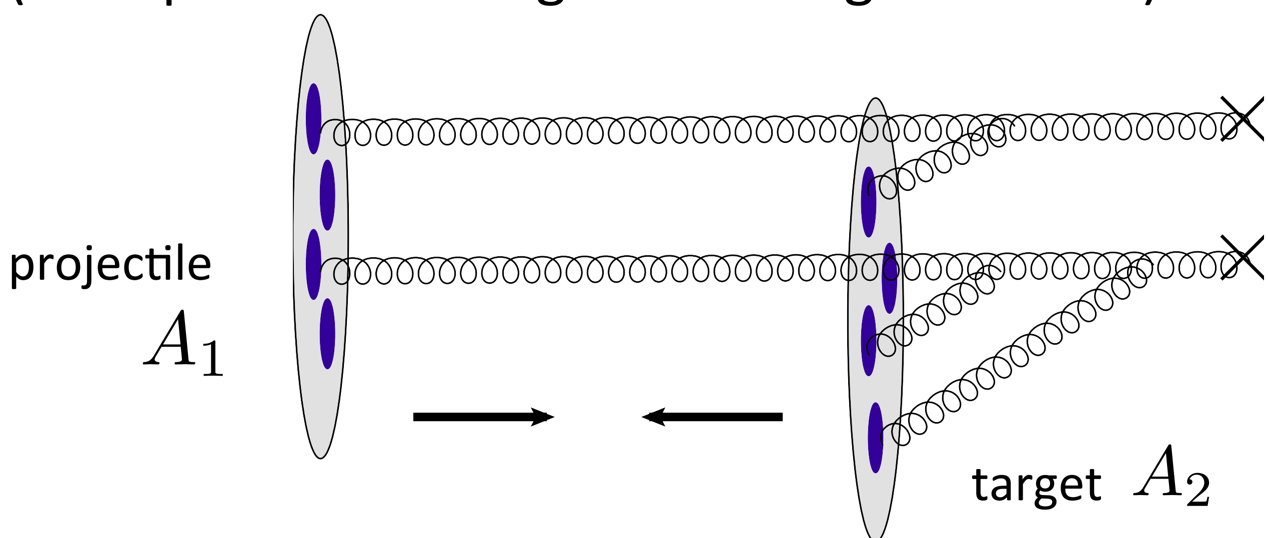
A. Setting up the problem: geometric correlations

Two-gluon production

- We want to calculate two gluon production in A_1+A_2 collisions with $1 << A_1 << A_2$ resumming all powers of

$$\alpha_s^2 \, A_2^{1/3} \sim 1 \qquad \text{while} \qquad \alpha_s^2 \, A_1^{1/3} \ll 1$$

(multiple rescatterings in the target nucleus)



- The gluons come from different nucleons in the projectile nucleus as A_1>>1 and this is enhanced compared to emission from the same nucleon.

Applicability region

- The saturation scales of the two nuclei are very different:

$$\Lambda_{QCD} \ll Q_{s1} \ll Q_{s2}$$

- We are working above the saturation scale of the smaller nucleus:

$$k_T \gg Q_{s1}$$

- We thus sum all multiple rescatterings in the larger nucleus, $Q_{s2}/k_T \sim 1$, staying at the lowest non-trivial order in $Q_{s1}/k_T \ll 1$.

- Multiple interactions with the same nucleon in either nucleus are suppressed by $\Lambda_{QCD}/k_T \lll 1$.

Production cross-section

- The single- and double-inclusive cross sections can be written as

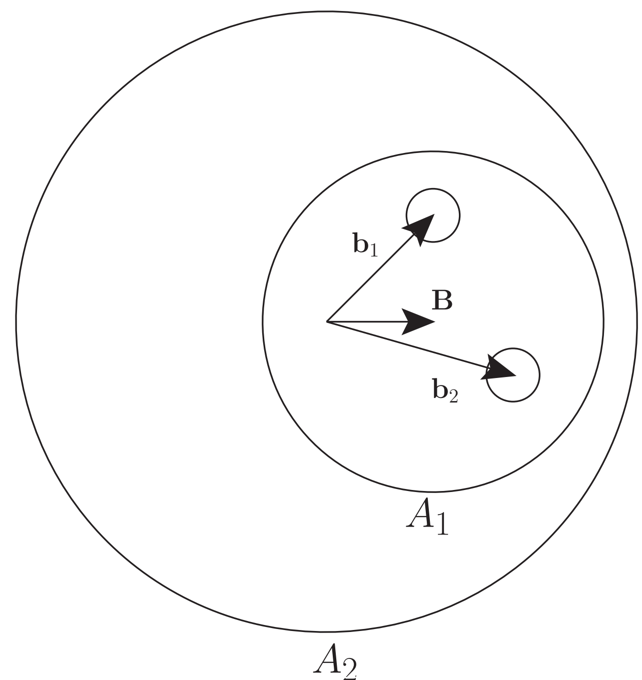
$$\frac{d\sigma}{d^2k\,dy} = \int d^2B\,d^2b\,|\Psi_I(\mathbf{B}-\mathbf{b})|^2\,\left\langle \frac{d\sigma^{pA_2}}{d^2k\,dy\,d^2b} \right\rangle$$

$$\frac{d\sigma}{d^2k_1dy_1\,d^2k_2dy_2} = \int d^2B\,d^2b_1\,d^2b_2\,|\Psi_{II}(\mathbf{B}-\mathbf{b}_1,\mathbf{B}-\mathbf{b}_2)|^2\,\left\langle \frac{d\sigma^{pA_2}}{d^2k_1dy_1d^2b_1}\,\frac{d\sigma^{pA_2}}{d^2k_2dy_2d^2b_2} \right\rangle$$

- Assume a large nucleus with uncorrelated nucleons (MV/Glauber model). Then the single- and double-nucleon wave functions are (with T_1 the nuclear profile function)

$$|\Psi_I(\mathbf{b})|^2 = T_1(\mathbf{b})$$

$$|\Psi_{II}(\mathbf{b}_1,\mathbf{b}_2)|^2 = T_1(\mathbf{b}_1)\,T_1(\mathbf{b}_2)$$



Geometric Correlations

- Assume uncorrelated interaction with the target:

$$\left\langle \frac{d\sigma^{pA_2}}{d^2k_1 dy_1 d^2b_1} \frac{d\sigma^{pA_2}}{d^2k_2 dy_2 d^2b_2} \right\rangle \approx \left\langle \frac{d\sigma^{pA_2}}{d^2k_1 dy_1 d^2b_1} \right\rangle \left\langle \frac{d\sigma^{pA_2}}{d^2k_2 dy_2 d^2b_2} \right\rangle$$

- For cross sections we have

$$\frac{d\sigma}{d^2k\, dy} = \int d^2B\, d^2b\, T_1(\mathbf{B}-\mathbf{b}) \left\langle \frac{d\sigma^{pA_2}}{d^2k\, dy\, d^2b} \right\rangle$$

$$\frac{d\sigma}{d^2k_1 dy_1\, d^2k_2 dy_2} = \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b_1})\, T_1(\mathbf{B}-\mathbf{b_2}) \left\langle \frac{d\sigma^{pA_2}}{d^2k_1 dy_1 d^2b_1} \right\rangle \left\langle \frac{d\sigma^{pA_2}}{d^2k_2 dy_2 d^2b_2} \right\rangle$$

- Clearly

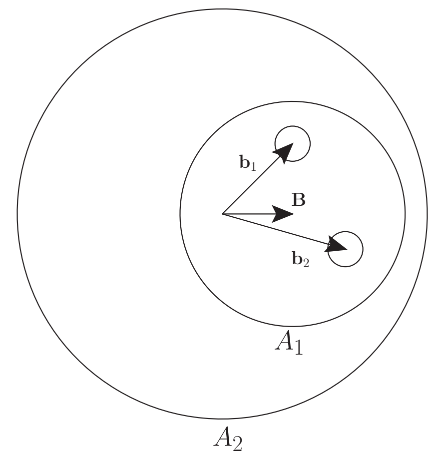
$$\frac{d\sigma}{d^2k_1\, dy_1\, d^2k_2\, dy_2} \not\propto \frac{d\sigma}{d^2k_1\, dy_1} \frac{d\sigma}{d^2k_2\, dy_2}$$

- Correlations due to the integration over the impact parameter B
  -> Geometric correlations!

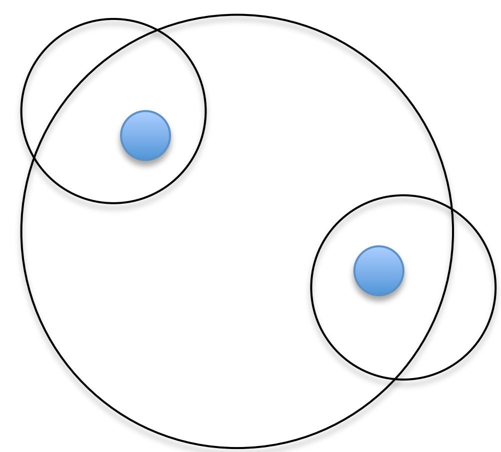
Geometric correlations: physical meaning

- In the same even the two nucleons are always within the smaller nucleus diameter from each other (in transverse

  nucleus.



$$C \propto \frac{\dfrac{d\sigma}{d^2k_1\, dy_1\, d^2k_2\, dy_2}}{\dfrac{d\sigma}{d^2k_1\, dy_1}\ \dfrac{d\sigma}{d^2k_2\, dy_2}}$$



Geometric correlations at fixed B

- are zero as

$$\frac{d\sigma}{d^2k_1\,dy_1\,d^2k_2\,dy_2\,d^2B} = \frac{d\sigma}{d^2k_1\,dy_1\,d^2B}\,\frac{d\sigma}{d^2k_2\,dy_2\,d^2B}$$

  which is clear from

$$\frac{d\sigma}{d^2k\,dy} = \int d^2B\,d^2b\,T_1(\mathbf{B}-\mathbf{b})\,\left\langle \frac{d\sigma^{pA_2}}{d^2k\,dy\,d^2b} \right\rangle$$

$$\frac{d\sigma}{d^2k_1dy_1\,d^2k_2dy_2} = \int d^2B\,d^2b_1\,d^2b_2\,T_1(\mathbf{B}-\mathbf{b_1})\,T_1(\mathbf{B}-\mathbf{b_2})\,\left\langle \frac{d\sigma^{pA_2}}{d^2k_1dy_1d^2b_1} \right\rangle \left\langle \frac{d\sigma^{pA_2}}{d^2k_2dy_2d^2b_2} \right\rangle$$

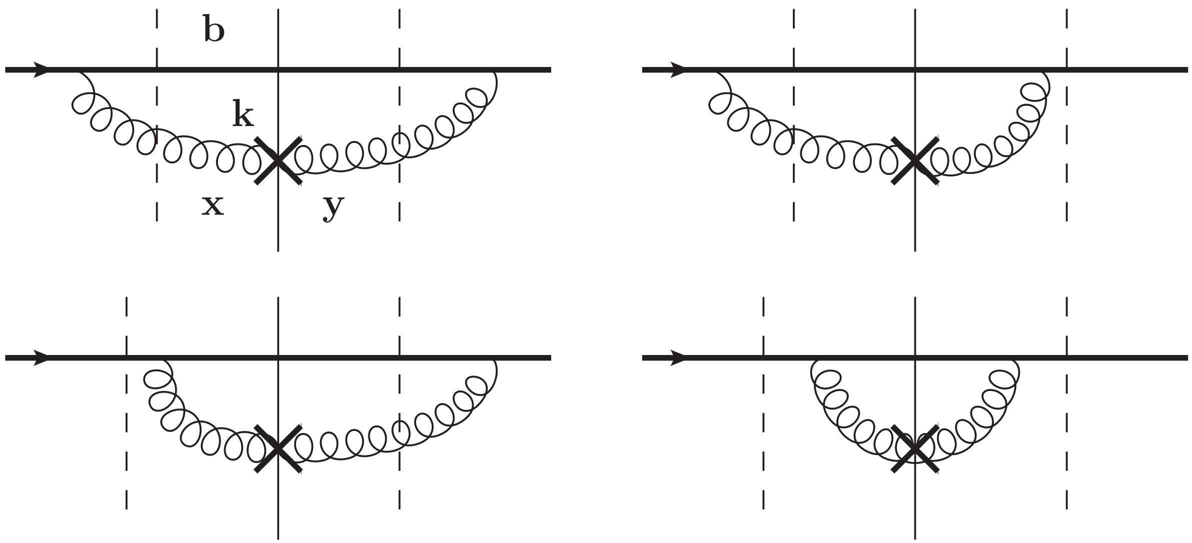
- Note that direction of B has to be fixed (to remove the geometric correlations), in other words the *vector* B should be fixed with respect to *vectors* k_1 and k_2. Maybe hard to do, but perhaps possible.

B. Two-gluon production

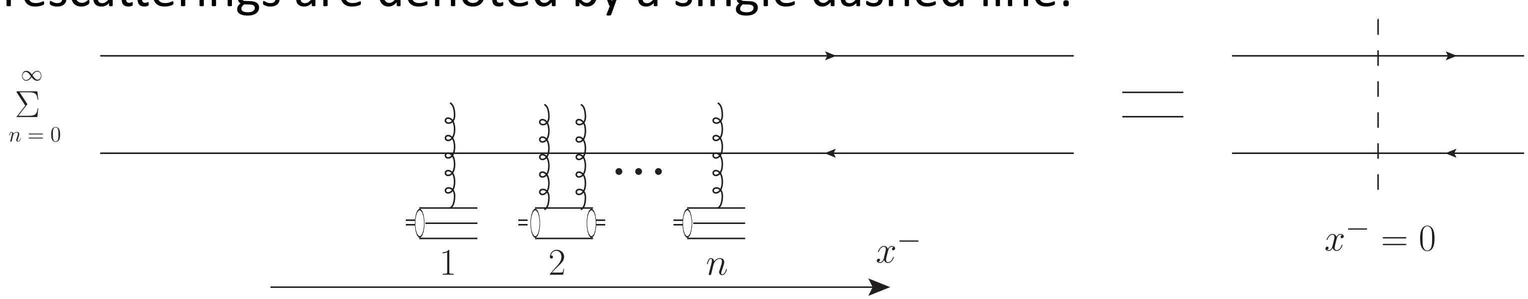
(i) Single gluon production in pA

Single gluon production in pA

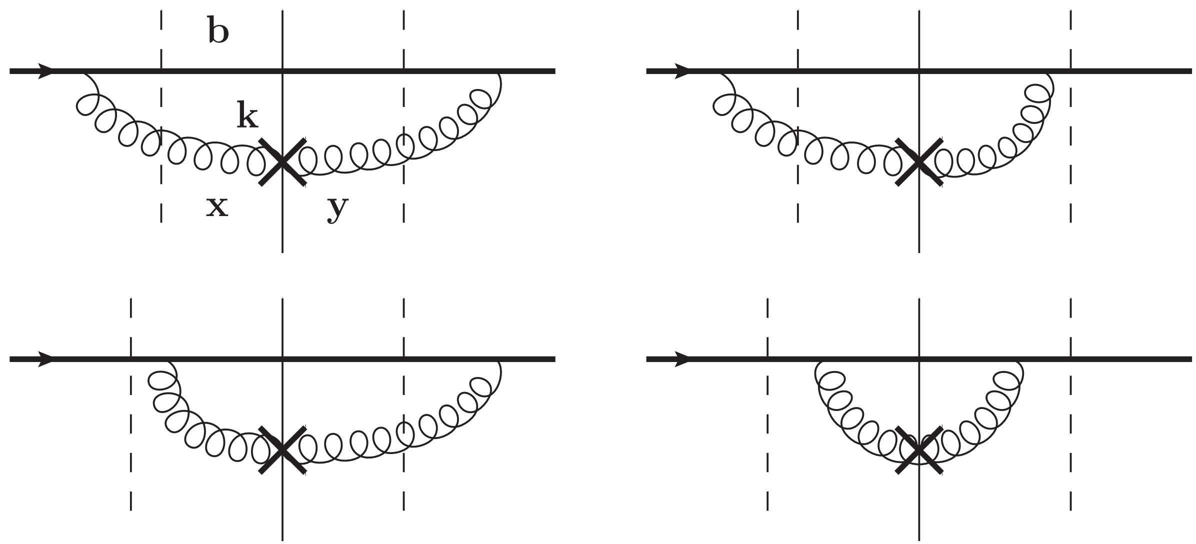
Model the proton by a single quark (can be easily improved upon).
The diagrams are shown below (Yu.K., A. Mueller '97):



Multiple rescatterings are denoted by a single dashed line:



Single gluon production in pA



The gluon production cross section can be readily written as (U = Wilson line in **adjoint** representation, represents gluon interactions with the target)

$$\left\langle \frac{d\sigma^{pA_2}}{d^2k\,dy\,d^2b} \right\rangle = \frac{\alpha_s\,C_F}{4\,\pi^4} \int d^2x\,d^2y\,e^{-i\,\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\frac{\mathbf{x}-\mathbf{b}}{|\mathbf{x}-\mathbf{b}|^2} \cdot \frac{\mathbf{y}-\mathbf{b}}{|\mathbf{y}-\mathbf{b}|^2}$$

$$\times\,\left\langle \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{y}}^\dagger]\;-\;\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{b}}^\dagger]\;-\;\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}}U_{\mathbf{y}}^\dagger]\;+\;1 \right\rangle$$

Forward dipole amplitude

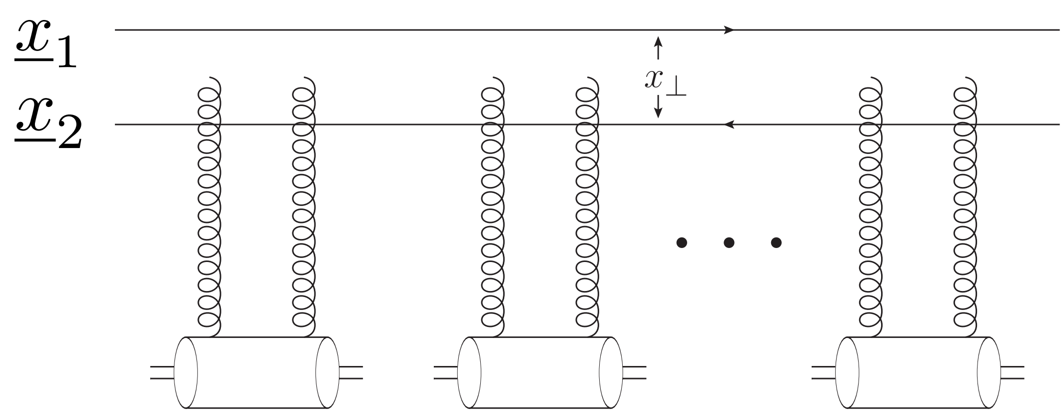
- The eikonal quark propagator is given by the Wilson line

$$V(\underline{x}) = \mathrm{P}\exp\left[i\,g\int_{-\infty}^{\infty} dx^{+}\,A^{-}(x^{+},x^{-}=0,\underline{x})\right]$$

- The eikonal quark propagator is given by the Wilson line

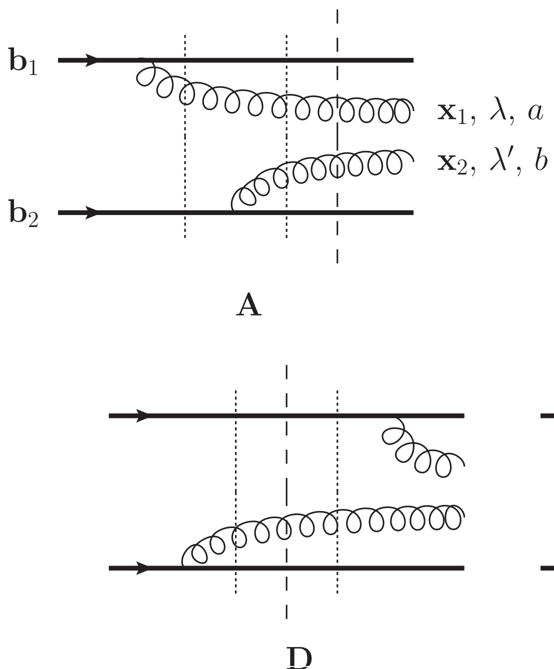
$$x^{\pm} = \frac{t \pm z}{\sqrt{2}}$$

$$N(\underline{x}\ ,\underline{x}\ ) = 1 \quad \frac{1}{N}\ \mathrm{tr}\ V(\underline{x}\ )\,V\ (\underline{x}\ )$$



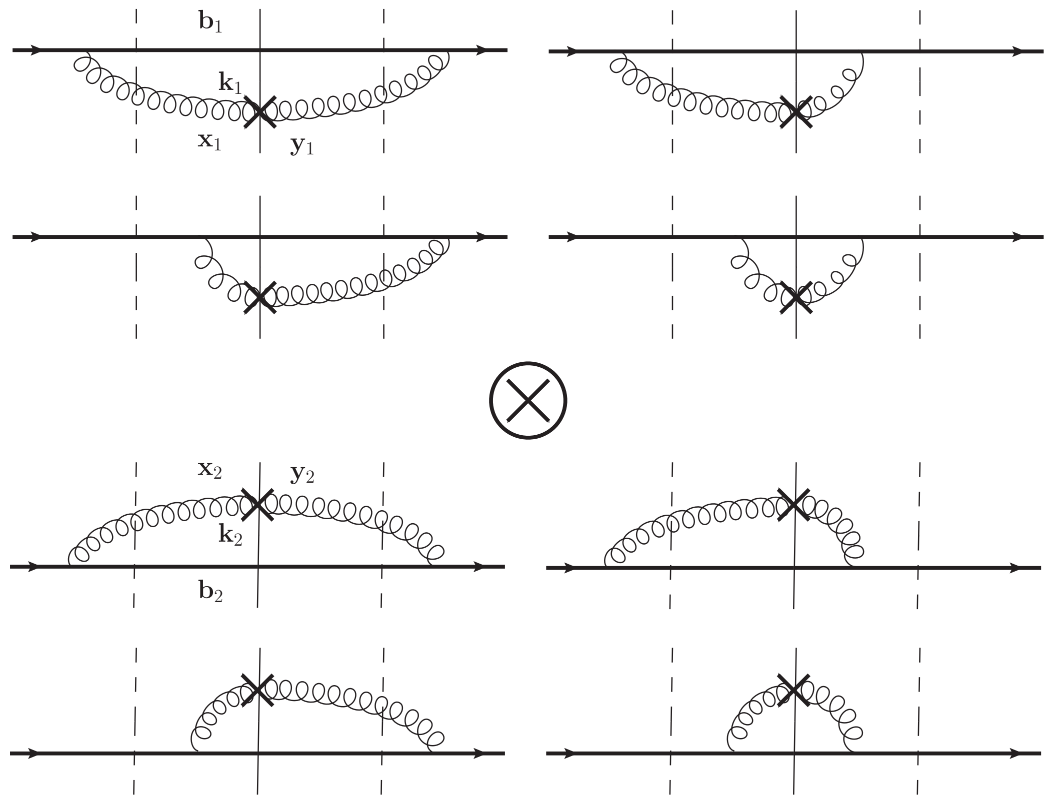
(ii) Two-gluon production in heavy-light ion collisions

The process



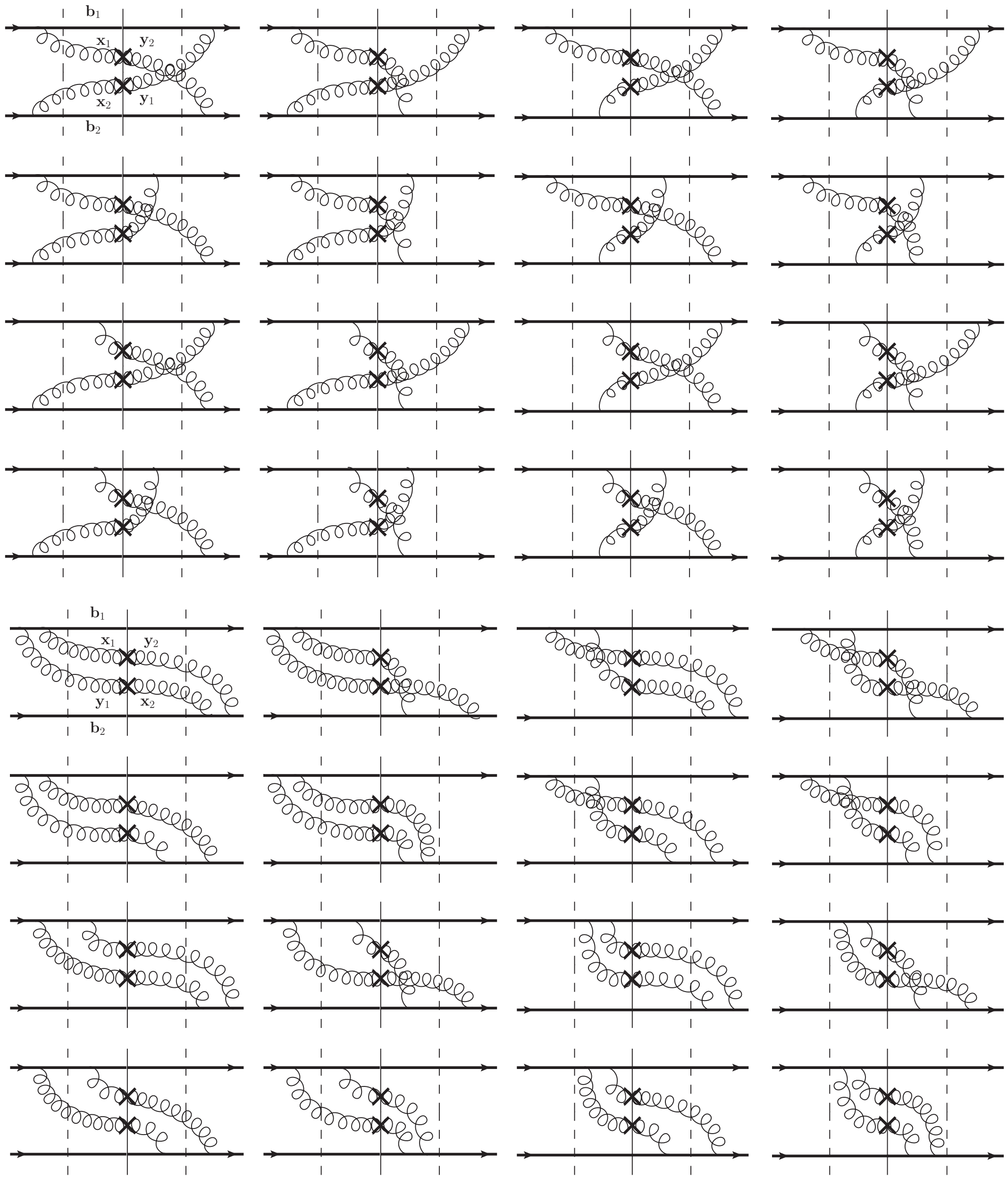
Solid horizontal lines = quarks in the incoming nucleons.
Dashed vertical line = interaction with the target.
Dotted vertical lines = energy denominators (ignore).

Amplitude squared



This contribution to two-gluon production looks like one-gluon production squared, with the target averaging applied to both.

Amplitude squared



These contributions to two-gluon production contain cross-talk between the emissions from different nucleons.

Two-gluon production cross section

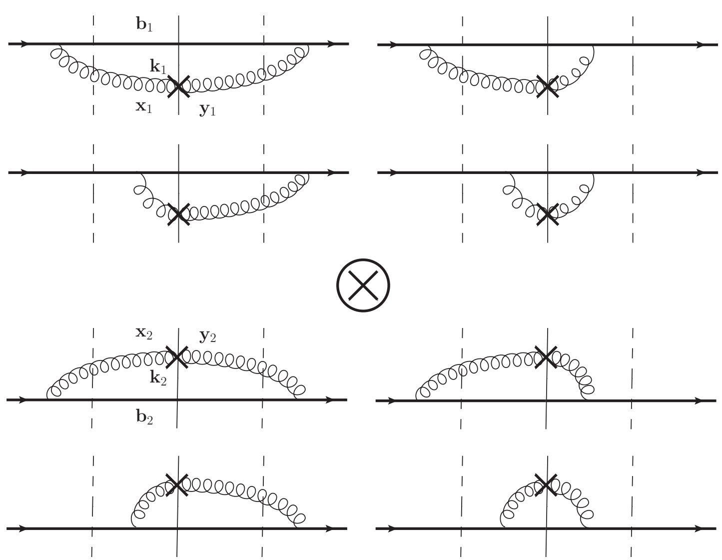
- “Squaring” the single gluon production cross section yields

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$

$$\times\ \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\ \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$

$$\times\ \left\langle\left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U_{\mathbf{y}_1}^\dagger]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U_{\mathbf{b}_1}^\dagger]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U_{\mathbf{y}_1}^\dagger]\ +\ 1\right)\right.$$

$$\left.\times\ \left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U_{\mathbf{y}_2}^\dagger]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U_{\mathbf{b}_2}^\dagger]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U_{\mathbf{y}_2}^\dagger]\ +\ 1\right)\right\rangle$$



(cf. Kovner & Lublinsky, ‘12)

Two-gluon production cross section

- The "crossed" diagrams give

$$
\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2
$$

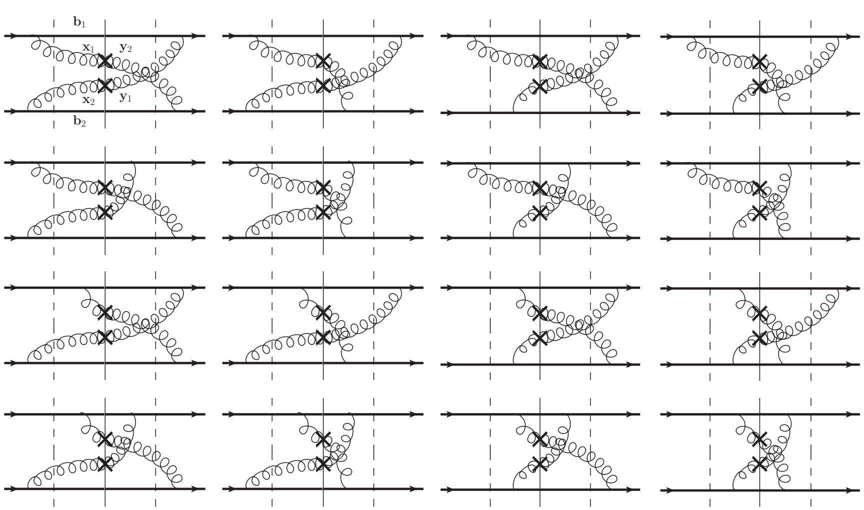
$$
\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]
$$

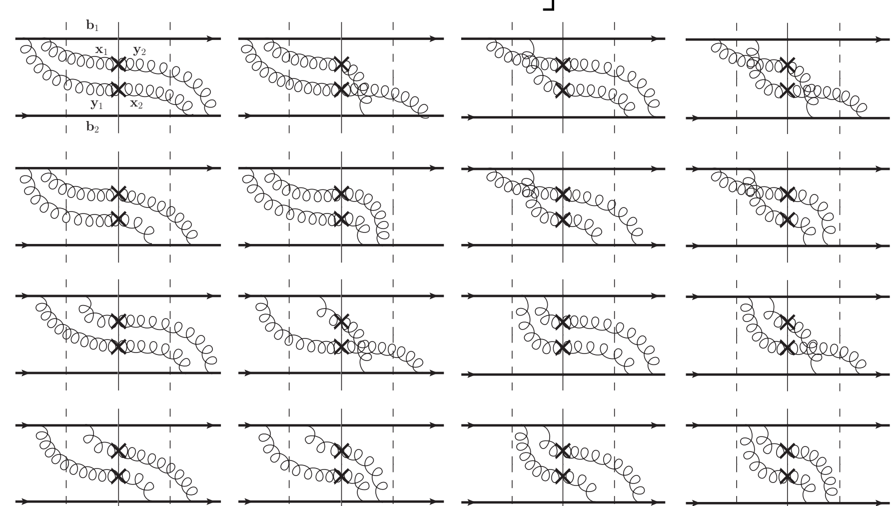
$$
\times\ \frac{16\,\alpha_s^2}{\pi^2}\,\frac{C_F}{2N_c}\ \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}\ \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}
$$

$$
\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)
$$

$$
+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)
$$

$$
+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]
$$





Two-gluon production cross section

- The "crossed" diagrams give

$$\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2$$

$$\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]$$

$$\times \frac{16\,\alpha_s^2}{\pi^2}\,\frac{C_F}{2N_c}\,\frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}\,\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}$$

$$\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)$$

$$+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)$$

$$+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]$$

- We introduced the adjoint color-dipole and color quadrupole amplitudes:

$$S_G(\mathbf{x}_1,\mathbf{x}_2,y) \equiv \frac{1}{N_c^2-1}\,\left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2}] \right\rangle$$

$$Q(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) \equiv \frac{1}{N_c^2-1}\,\left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2} U_{\mathbf{x}_3} U^\dagger_{\mathbf{x}_4}] \right\rangle$$

Two-gluon production: properties

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times\ \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\ \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times\left\langle\left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{b}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U^\dagger_{\mathbf{y}_1}]\ +\ 1\right)\right.$$
$$\left.\times\left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{b}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U^\dagger_{\mathbf{y}_2}]\ +\ 1\right)\right\rangle$$

- Note that if the interaction with the target factorizes,

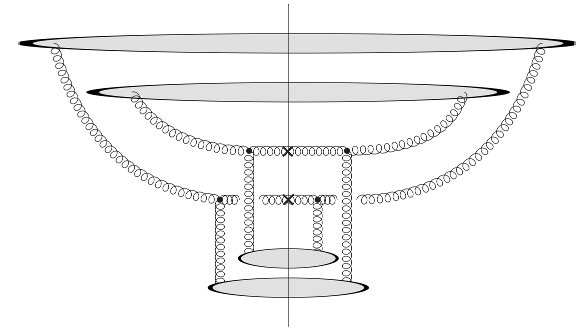
$$\left\langle Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}]\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}]\right\rangle\Big|_{\text{large}-N_c,\,\text{large}-A_2} \approx \left\langle Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}]\right\rangle\ \left\langle Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}]\right\rangle$$

  we still have **the geometric correlations**.

- The geometric correlations lead to non-zero cumulants! (Like everything that depends on geometry.)

Two-gluon production: properties

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\;\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times \Big\langle \left(\frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_1}U_{\mathbf{y}_1}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_1}U_{\mathbf{b}_1}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{b}_1}U_{\mathbf{y}_1}^\dagger] + 1\right)$$
$$\times \left(\frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_2}U_{\mathbf{y}_2}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_2}U_{\mathbf{b}_2}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{b}_2}U_{\mathbf{y}_2}^\dagger] + 1\right)\Big\rangle$$



- If we expand the interaction with the target to the lowest non-trivial order, one reproduced the contribution of the ‘glasma’ graphs:

$$\left.\frac{d\sigma_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2}\right|_{LO} = \frac{\alpha_s^2}{4\,\pi^4}\int d^2B\, d^2b\, [T_1(\mathbf{B}-\mathbf{b})]^2\, \frac{Q_{s0}^4(\mathbf{b})}{\mathbf{k}_1^2\,\mathbf{k}_2^2}\int_\Lambda \frac{d^2l}{(\mathbf{l}^2)^2}\left[\frac{1}{(\mathbf{k}_1-\mathbf{l})^2\,(\mathbf{k}_2+\mathbf{l})^2} + \frac{1}{(\mathbf{k}_1-\mathbf{l})^2\,(\mathbf{k}_2-\mathbf{l})^2}\right]$$

away-side correlations

$$\sim \frac{1}{(\mathbf{k}_1+\mathbf{k}_2)^2}$$

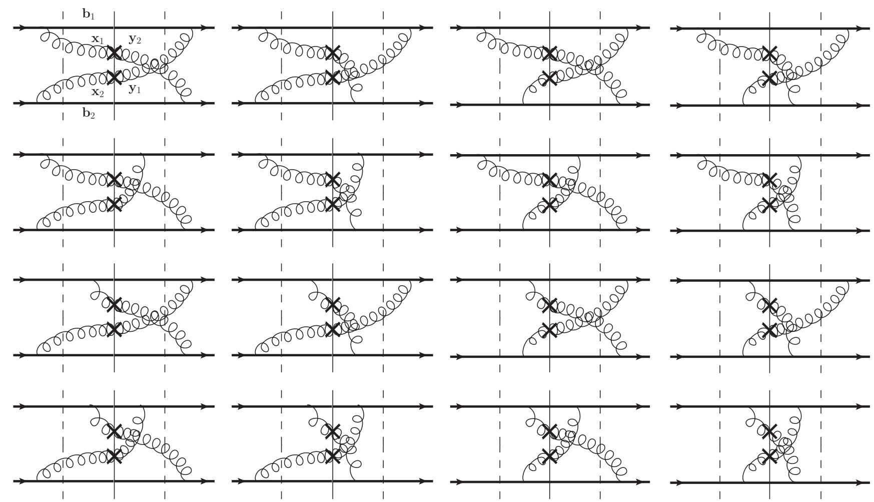
near-side correlations

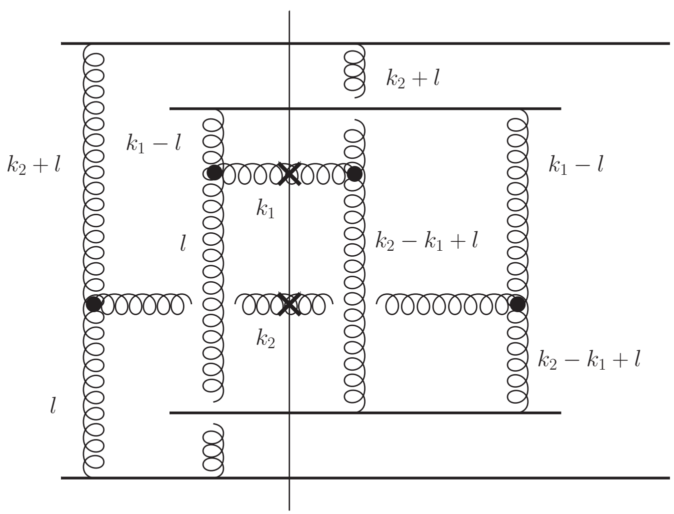
$$\sim \frac{1}{(\mathbf{k}_1-\mathbf{k}_2)^2}$$

(cf. Dumitru, Gelis, McLerran, Venugopalan ’08)

Two-gluon production: properties

- Crossed diagrams at lowest nontrivial order





$$\sim \cos 2\Delta\phi \left[\frac{1}{(\mathbf{k}_1 - \mathbf{k}_2)^2} + \frac{1}{(\mathbf{k}_1 + \mathbf{k}_2)^2} \right]$$

Stronger correlations!

Two-gluon production: properties

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times\ \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\ \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times\ \Bigg\langle\Bigg(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{b}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U^\dagger_{\mathbf{y}_1}]\ +\ 1\Bigg)$$
$$\times\ \Bigg(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{b}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U^\dagger_{\mathbf{y}_2}]\ +\ 1\Bigg)\Bigg\rangle$$

- The cross section is symmetric under (ditto for the “crossed” term)

$\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$ (just coordinate relabeling)

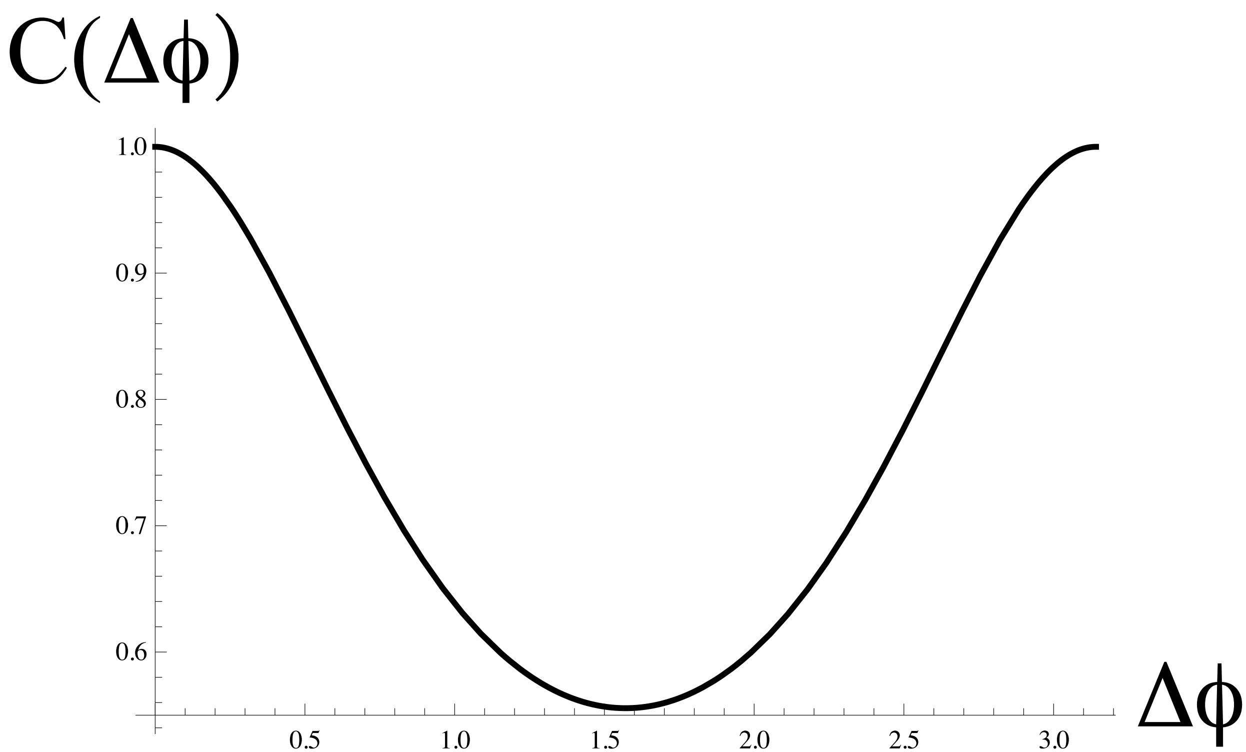
$\mathbf{k}_2 \rightarrow -\mathbf{k}_2$ as $Tr\left[U_{\mathbf{x}}\,U^\dagger_{\mathbf{y}}\right] = Tr\left[U_{\mathbf{y}}\,U^\dagger_{\mathbf{x}}\right]$

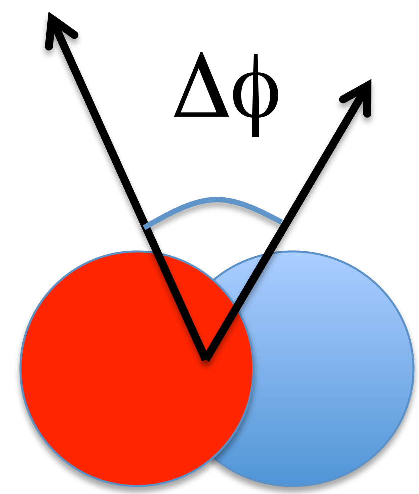
- Hence the correlations generate only even azimuthal harmonics

$$\sim \cos 2\,n\,(\phi_1 - \phi_2)$$

Correlation function

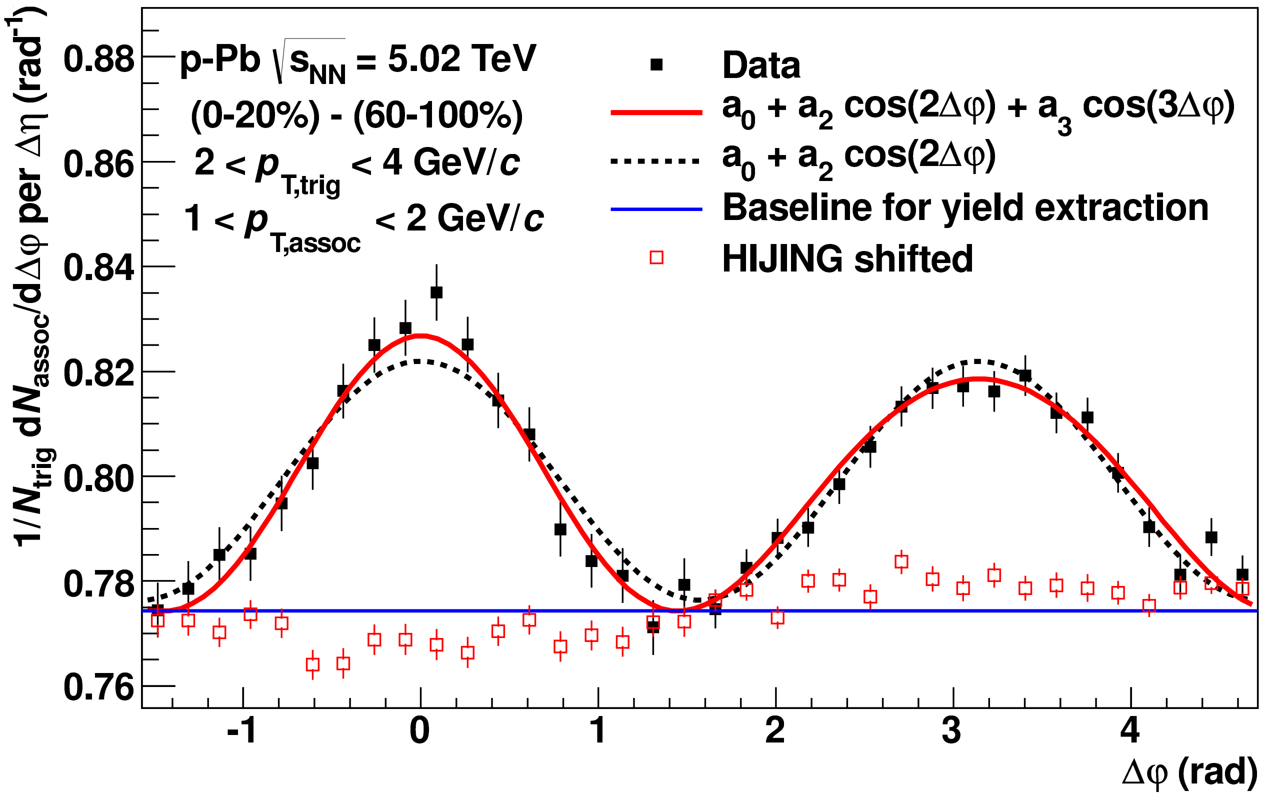
May look like this (a toy model; two particles far separated in rapidity, jets subtracted, pA and AA):





Dumitru, Gelis, McLerran, Venugopalan ’08; Kovner, Lublinsky ‘10; Yu.K., D. Wertepny ‘12; Lappi, Srednyak, and Venugopalan ‘09

LHC p+Pb data from ALICE



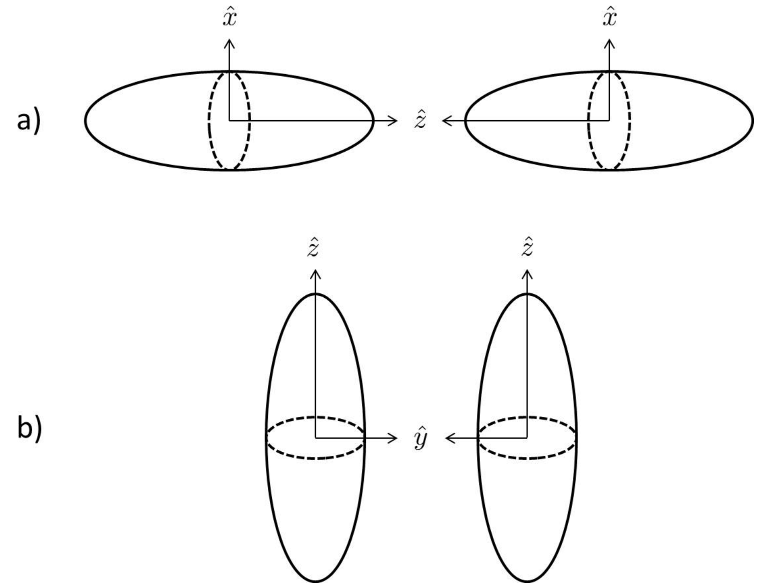
- These are high-multiplicity collisions: it is possible that quark-gluon plasma is created in those collisions, with the hydrodynamics contributing to these correlations.

- Saturation approach is lacking the odd harmonics, like cos (3 $\Delta\phi$) , etc. Can they be generated by corrections to the leading-order CGC calculation?

Initial vs final-state correlations: U+U

- Hydro conventional wisdom: eccentricity is higher in side-by-side collisions, and the flow harmonics are larger.

- Saturation approach: local saturation scale is larger in tip-on-tip collisions, making correlations stronger as well.



$$\frac{C_{tip-on-tip}(\mathbf{k}_1, y_1, \mathbf{k}_2, y_2)\big|_{LO}}{C_{side-on-side}(\mathbf{k}_1, y_1, \mathbf{k}_2, y_2)\big|_{LO}} = \frac{1}{\lambda} \approx 1.26 \quad (\text{for } U+U)$$

Qualitatively different behavior in the two approaches!

$$\rho(\vec{\mathbf{r}}) = \rho_0 \, e^{-\frac{x^2}{R^2} - \frac{y^2}{R^2} - \frac{\lambda^2}{R^2} z^2}$$

Energy (In)dependence

- "Glasma graphs" correlation function has the same number of Q_s factors in the numerator and the denominator:

$$C(\mathbf{k}_1, y_1, \mathbf{k}_2, y_2)\Big|_{LO} \propto \frac{\int d^2B\, d^2b\, [T_1(\mathbf{B}-\mathbf{b})]^2\, Q_{s2}^4(\mathbf{b})}{\int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, Q_{s2}^2(\mathbf{b}_1)\, Q_{s2}^2(\mathbf{b}_2)}$$

- Loosely-speaking, since each saturation scale is proportional to a power of energy,

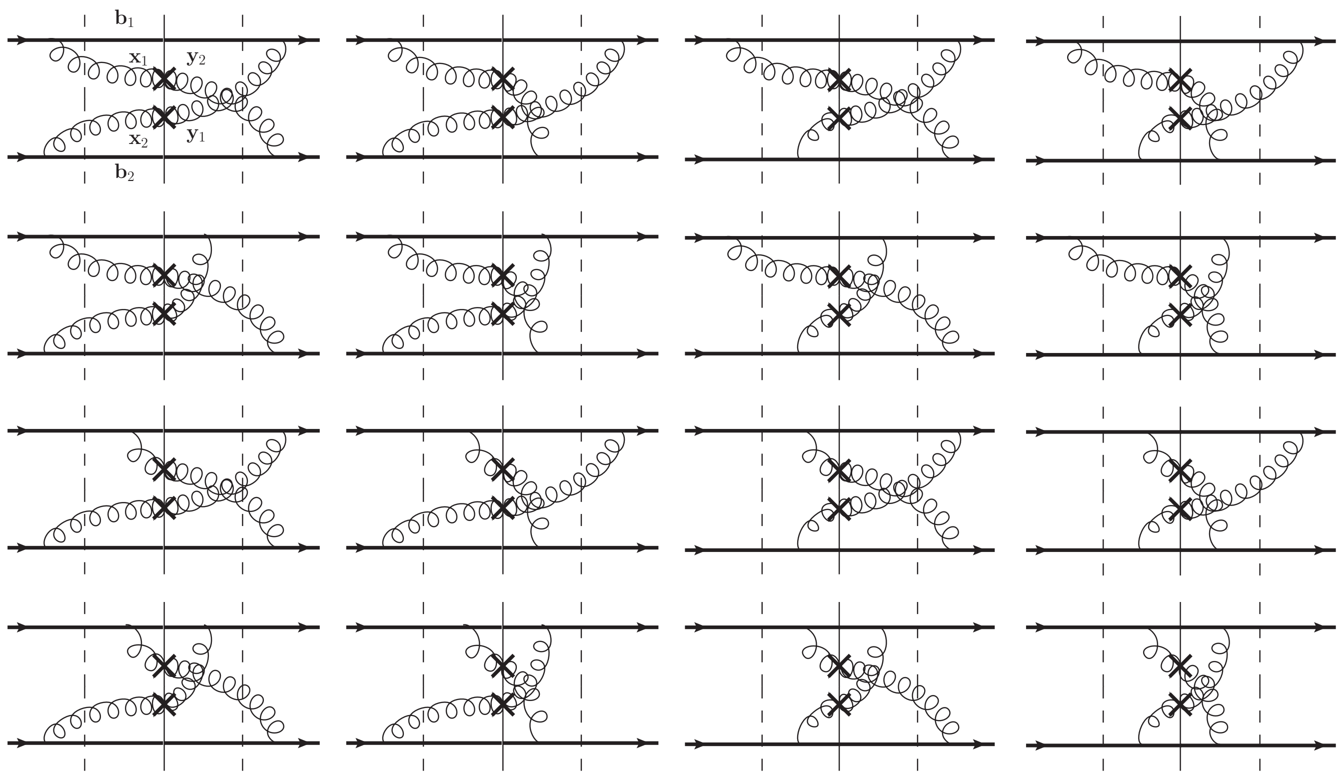
$$Q_s^2 \sim s^\lambda$$

the energy dependence cancels, and the correlation function is (almost) **energy-independent!**

C. HBT correlations

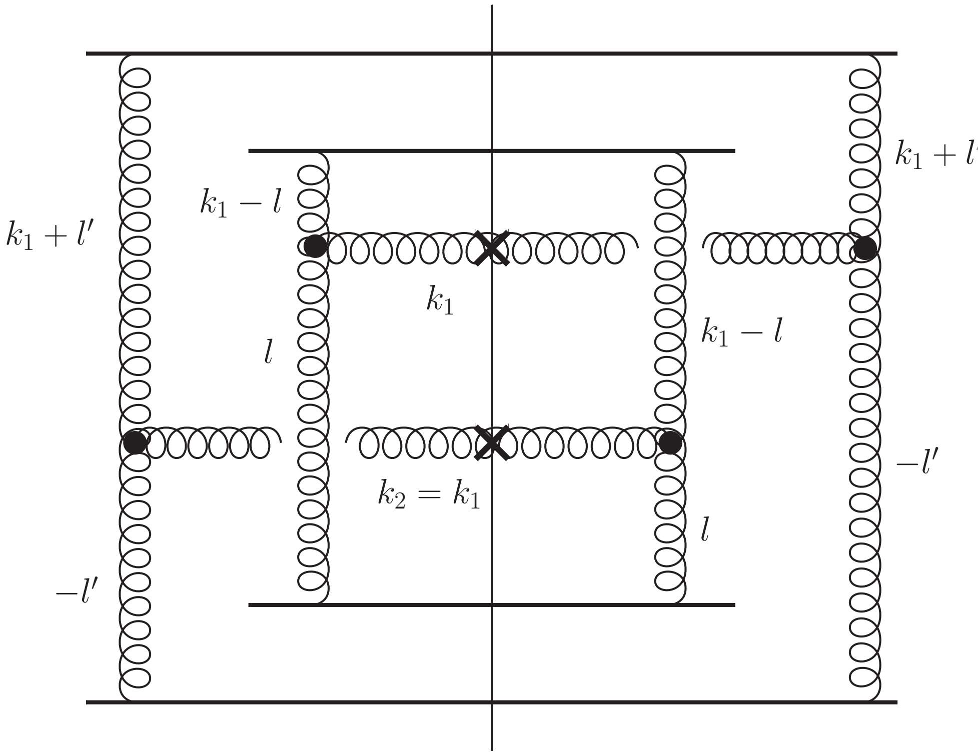
HBT diagrams

- There is another contribution coming from the "crossed" diagrams



HBT diagrams

- They give HBT correlations (with R_{long} =0 due to Lorentz contraction)



$$\sim \delta^2(\mathbf{k}_1 - \mathbf{k}_2)$$

- Just like the standard HBT correlations

$$|\Psi_1(\mathbf{k}_1)\,\Psi_2(\mathbf{k}_2) + \Psi_1(\mathbf{k}_2)\,\Psi_2(\mathbf{k}_1)|^2 \to \Psi_1(\mathbf{k}_1)\,\Psi_2(\mathbf{k}_2)\,\Psi_1^*(\mathbf{k}_2)\,\Psi_2^*(\mathbf{k}_1) + c.c. + \ldots$$

- Possibly fragmentation would break phase coherence making these perturbative HBT correlations not observable.

Back-to-back HBT?

- Note that all our formulas are symmetric under

$$\mathbf{k}_2 \to -\mathbf{k}_2$$

- Therefore, the HBT correlation is accompanied by the identical back-to-back HBT correlation

$$\sim \delta^2(\mathbf{k}_1 + \mathbf{k}_2)$$

- Note again that this correlation may be destroyed in hadronization.

k_T-Factorization?

In search of factorization

- Since a lot of CGC 'ridge' phenomenology was done using a k_T-factorized expression, it is natural to try to see whether our two-gluon production cross section can be written in a factorized form.

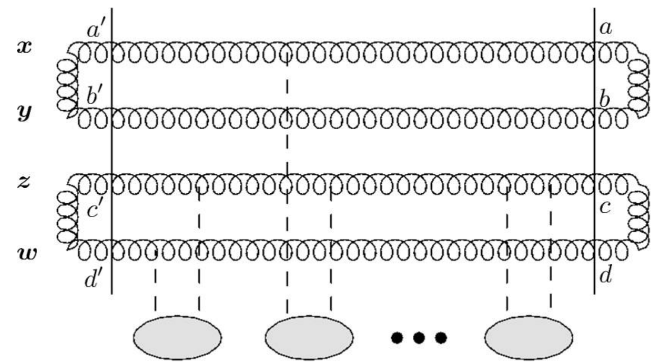
- The answer is 'yes', but factorization does not come naturally.

Two-gluon Distribution Functions

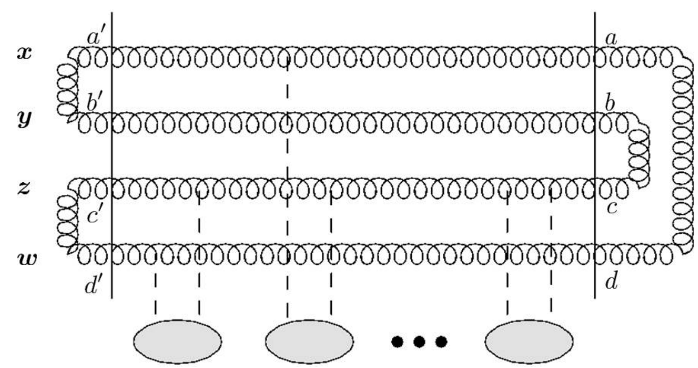
- Unintegrated single-gluon distribution (gluon TMD) can be defined through a (gluon) dipole operator N_G.

$$\left\langle \frac{d\phi_{A_1}(\mathbf{q},y)}{d^2b} \right\rangle_{A_1} = \frac{C_F}{\alpha_s (2\pi)^3} \int d^2r\, e^{-i\mathbf{q}\cdot\mathbf{r}}\ \nabla^2_{\mathbf{r}}\ N_G(\mathbf{b}+\mathbf{r},\mathbf{b},y)$$

- The two-gluon ditributions can be defined either through a double-trace or the quadrupole operators:



Final State



Final State

Two-gluon Distribution Functions

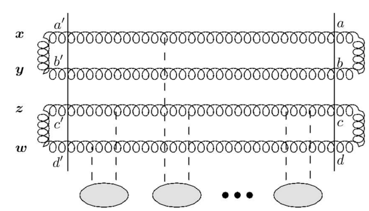
- The two-gluon distributions are:

$$\left\langle \frac{d\phi^{D,Q}_{A_2}(\mathbf{q}_1, \mathbf{q}_2, y)}{d^2 b_1 \; d^2 b_2} \right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s (2\pi)^3} \right)^2 \int d^2 r_1 \; d^2 r_2 \, e^{-i\mathbf{q}_1 \cdot \mathbf{r}_1 - i\mathbf{q}_2 \cdot \mathbf{r}_2} \; \nabla^2_{\mathbf{r}_1} \; \nabla^2_{\mathbf{r}_2} \; N_{D,Q}(\mathbf{b}_1 + \mathbf{r}_1, \mathbf{b}_1, \mathbf{b}_2 + \mathbf{r}_2, \mathbf{b}_2, y)$$

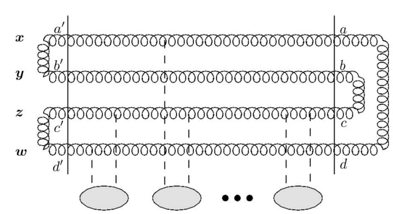
where the double-trace and quadrupole operators are

$$N_D(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}, Y) = \frac{1}{(N_c^2 - 1)^2} \left\langle \mathrm{Tr}\left[1 - U_{\mathbf{x}} U^\dagger_{\mathbf{y}}\right] \mathrm{Tr}\left[1 - U_{\mathbf{z}} U^\dagger_{\mathbf{w}}\right] \right\rangle_{A_2} (Y)$$

$$N_Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}, Y) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{Tr}\left[\left(1 - U_{\mathbf{x}} U^\dagger_{\mathbf{y}}\right) \left(1 - U_{\mathbf{z}} U^\dagger_{\mathbf{w}}\right)\right] \right\rangle_{A_2} (Y)$$



Final State



Final State

k_T-Factorization

- After some algebra the two-gluon inclusive production cross section can be written in a factorized form:

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2\, \left\langle \frac{d\phi_{A_1}(\mathbf{q}_1, y=0)}{d^2(\mathbf{B}-\mathbf{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\mathbf{q}_2, y=0)}{d^2(\mathbf{B}-\mathbf{b}_2)} \right\rangle_{A_1}$$
$$\times \left\{ \left\langle \frac{d\phi^D_{A_2}(\mathbf{q}_1-\mathbf{k}_1, \mathbf{q}_2-\mathbf{k}_2, y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} + \left[\frac{\mathcal{K}(\mathbf{b}_1, \mathbf{b}_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q}_1, \mathbf{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\mathbf{q}_1-\mathbf{k}_1, \mathbf{q}_2-\mathbf{k}_2, y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} + (\mathbf{k}_2 \to -\mathbf{k}_2) \right] \right.$$

with a somewhat involved (but known) coefficient function K.

- The expression is rather convoluted, and the function K depends on the nucleons' positions… The expression is different from what was used in phenomenology.

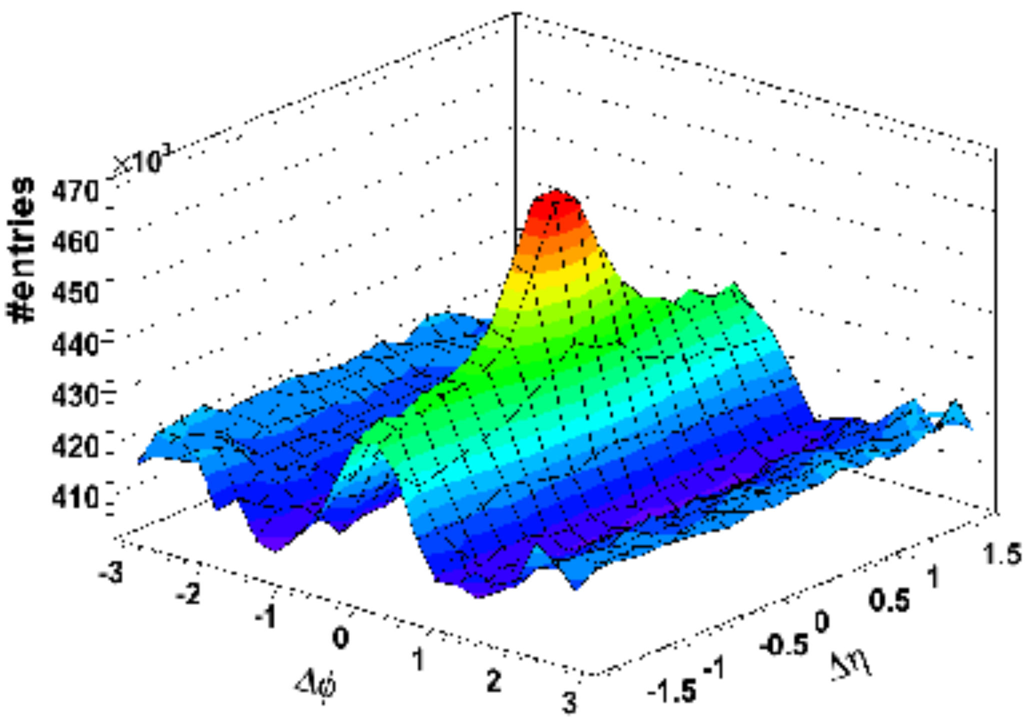
Conclusions

- We completed a calculation of the two-gluon inclusive production cross section in nuclear collisions including saturation effects in one nucleus to all orders (heavy-light ion collisions).
- Correlations we see:
  - Geometric correlations ✓
  - HBT correlations (along with b2b HBT ones)
  - Away-side correlations ✓
  - Near-side long-range rapidity correlations ✓
    (✓ = long-range in rapidity)
- Near- and away-side correlations are identical to all orders in saturation effects: this is mainly consistent with the p+Pb data from LHC. Odd harmonics are missing.
- Long-range rapidity correlations are (almost) energy-independent.
- k_T-factorization can be obtained with 2 types of double-gluon distributions.
- It seems like geometric correlations are hard to remove by cumulants, since they depend on non-local geometry.

Backup Slides

Ridge in heavy ion collisions

- Heavy ion collisions, along with high-multiplicity p+p and p+A collisions, are known to have long-range rapidity correlations, known as 'the ridge':



This conclusion is consistent with the data

Long-Range Rapidity Correlations in Heavy-Light Ion Collisions

Yuri V. Kovchegov, Douglas E. Wertepny

(Submitted on 5 Dec 2012)

We study two-particle long-range rapidity correlations arising in the early stages of heavy ion collisions in the saturation/Color Glass Condensate framework, assuming for simplicity that one colliding nucleus is much larger than the other. We calculate the two-gluon production cross section while including all-order saturation effects in the heavy nucleus with the lowest-order rescattering in the lighter nucleus. We find four types of correlations in the two-gluon production cross section: (i) geometric correlations, (ii) HBT correlations, (iii) back-to-back correlations, and (iv) near-side azimuthal correlations which are long-range in rapidity. The geometric correlations (i) are due to the fact that nucleons are correlated by simply being confined within the same nucleus and may lead to long-range rapidity correlations for the produced particles without strong azimuthal angle dependence. Somewhat surprisingly, long-range rapidity correlations (iii) and (iv) have exactly the same amplitudes along with azimuthal and rapidity shapes: one centered around \Delta \phi = \pi\ with the other one centered around \Delta \phi = 0 (here \Delta \phi\ is the azimuthal angle between the two produced gluons). We thus observe that the early-time CGC dynamics in nucleus-nucleus collisions generates azimuthal non-flow correlations which are qualitatively different from jet correlations by being long-range in rapidity. If strong enough, they have the potential of mimicking the elliptic (and higher-order even-harmonic) flow in the di-hadron correlators: one may need to take them into account in the experimental determination of the flow observables.

This conclusion is consistent with the data

Long-range angular correlations on the near and away side in p-Pb collisions at sqrt(sNN) = 5.02 TeV

ALICE Collaboration

(Submitted on 10 Dec 2012)

Angular correlations between charged trigger and associated particles are measured by the ALICE detector in p-Pb collisions at a nucleon-nucleon centre-of-mass energy of 5.02 TeV for transverse momentum ranges within 0.5 < pT,assoc < pT,trig < 4 GeV/c. The correlations are measured over two units of pseudorapidity and full azimuthal angle in different intervals of event multiplicity, and expressed as associated yield per trigger particle. Two long-range ridge-like structures, one on the near side and one on the away side, are observed when the per-trigger yield obtained in low-multiplicity events is subtracted from the one in high-multiplicity events. The excess on the near-side is qualitatively similar to that recently reported by the CMS collaboration, while the excess on the away-side is reported for the first time. The two-ridge structure projected onto azimuthal angle is quantified with the second and third Fourier coefficients as well as by near-side and away-side yields and widths. The yields on the near side and on the away side are equal within the uncertainties for all studied event multiplicity and pT bins, and the widths show no significant evolution with event multiplicity or pT. These findings suggest that the near-side ridge is accompanied by an essentially identical away-side ridge.