QCD at nonzero temperature: T_c and the equation of state

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Collaborations

Part of this research was done by the HotQCD Collaboration:

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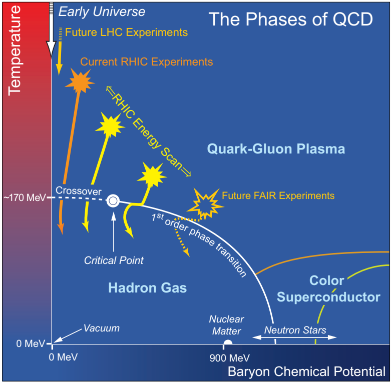
A. Bazavov *et al.* (HotQCD), Phys. Rev. D85 (2012) 054503 [arXiv:1111.1710]
A. Bazavov *et al.* (HotQCD), Phys. Rev. D90 (2014) 094503 [arXiv:1407.6387]

Part was done by the Wuppertal-Budapest Collaboration:
Y. Aoki, S. Borsányi, S. Dürr, Z. Fodor, C. Hoelbling, S.D. Katz, S. Krieg, C. Ratti, K.K. Szabó.

Y. Aoki *et al.*, Phys. Lett. B643 (2006) 46 [arXiv:hep-lat/0609068]
Y. Aoki *et al.*, JHEP 0906 (2009) 088 [arXiv:0903.4155]
S. Borsányi *et al.*, JHEP 1009 (2010) 073 [arXiv:1005.3508]
S. Borsányi *et al.*, Phys. Lett. B730 (2014) 99 [arXiv:1309.5258]

Introduction: Phases of QCD

Here is an illustration or our current understanding of the phase diagram of QCD, with the behavior at low temperature and large baryon chemical potential μ_b most speculative.



The challenge:

- Explain this from first principles
- Put numbers to the transitions & crossover
- Determine properties of the different phases

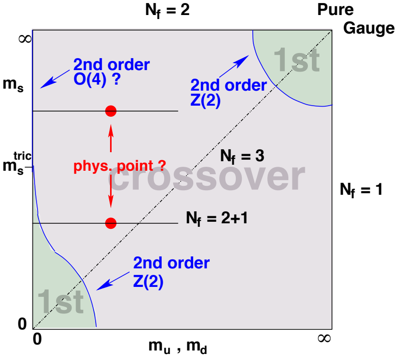
The method: Lattice QCD

In this talk, I will restrict myself to the left axis of this phase diagram, the part with $\mu_B = 0$.

Phases of QCD

There is a transition/crossover from a confined phase with (spontaneously) broken chiral symmetry at low temperature to a quark-gluon plasma phase with restored chiral symmetry at high temperature.

With lattice QCD we can vary the quark masses.



Find/conjecture:

- a crossover at physical quark masses
  Y. Aoki *et al.*, Nature 443 (2006) 675 [arXiv:hep-lat/0611014]
- a transition for $m_u = m_d = 0$
  - possibly second order in O(4) unversality class
  - possibly first order in a small sliver of small m_l

  Location of m_s^{tric} is not yet known.

Lattice QCD setup

Lattice QCD starts from the path integral:

$$\begin{aligned}\langle \mathcal{O}(U,\psi,\overline{\psi})\rangle &= \frac{1}{Z}\int dU_\mu d\overline{\psi} d\psi \mathcal{O}(U,\psi,\overline{\psi}) \mathrm{e}^{-S_G(U)+\overline{\psi}M(U)\psi} \\ &= \frac{1}{Z}\int dU_\mu \mathcal{O}(U,M^{-1}(U)) \mathrm{e}^{-S_G(U)} \det M(U)\ .\end{aligned}$$

Gauge fields, $U_{x,\mu} = \mathrm{e}^{igaA_{x,\mu}}$, are on links, quarks, ψ and $\overline{\psi}$, on sites.

The Gaussian integration over the quark fields has been carried out, giving $\det M(U)$.

The integration over the gauge links is done by Monte Carlo methods.

Generically,

$$S_G(U) \longrightarrow \int d^4x\, \frac{1}{2} \mathrm{Tr}\left(F_{\mu\nu}^2\right) + \mathcal{O}(a^2)\,.$$

By judicious choices the $\mathcal{O}(a^2)$ can be improved to $\mathcal{O}(a^4)$, classically – $\mathcal{O}(\alpha_s^n a^2)$ with quantum fluctuations taken into account.

Fermion action

Similarly, for the (staggered) fermion discretization,

$$\overline{\psi}M(U)\psi \longrightarrow \int d^4x\,\overline{\psi}\,(\not{D}+m)\,\psi + \mathcal{O}(a^2)\,.$$

Fermions are difficult to put on a lattice while preserving the chiral symmetry. So called doublers, named "tastes" appear, naively 16.

Staggered fermions reduce this to 4, while preserving a partial chiral symmetry.

Staggered fermions are the least computationally demanding.

However, at finite lattice spacing, due to interaction with ultraviolet gluons taste symmetry is broken: only one pion is a Goldstone boson; the others get additional mass contributions of $\mathcal{O}(\alpha_s a^2)$.

Improvements try to reduce both generic lattice artifacts and those from taste symmetry breaking.

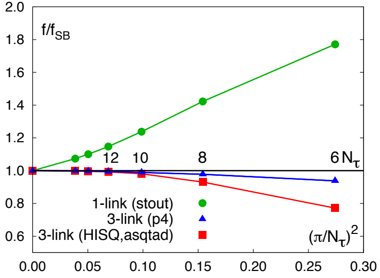
Fermion action improvements

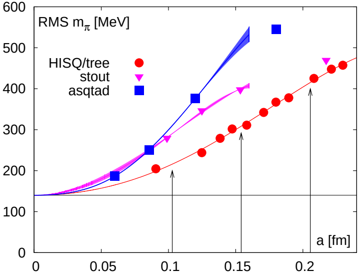
Four different improved staggered fermions are in use:

- stout (WB) — taste symmetry breaking, only
- p4 (Bielefeld, HotQCD)
- asqtad (MILC, HotQCD)
- HISQ, highly improved staggered quarks (HPQCD, MILC, HotQCD)

Generic lattice artifacts affect the high temperature behavior, illustrated in the free energy [left figure].

The right figure shows the effect of taste symmetry breaking:
$m_\pi^{\mathrm{RMS}} = m^{\mathrm{Goldstone}} + \mathcal{O}(\alpha_s^k a^2)$ — Goldstone pion fixed at physcial mass.





Finite temperature transition

In the chiral limit, $m_l = 0$, — the strange quark mass is kept at its physcial value — the condensate is an order parameter,

$$\langle\bar{\psi}\psi\rangle_l = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}\,; \qquad \chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2}\,.$$

The latter is the chiral susceptibility. Both need (UV) subtractions ($\sim m/a^2$ for the condensate, $\sim 1/a^2$ for the susceptibility) and multiplicative renormalization (which is avoided in the combination $m\langle\bar{\psi}\psi\rangle$ or $m^2\chi_m$)

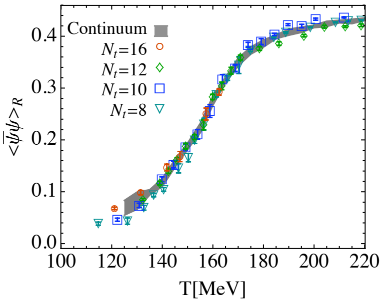
$$\langle\bar{\psi}\psi\rangle_R = \frac{m_l}{T^4}\left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]\,, \qquad \Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}\,.$$

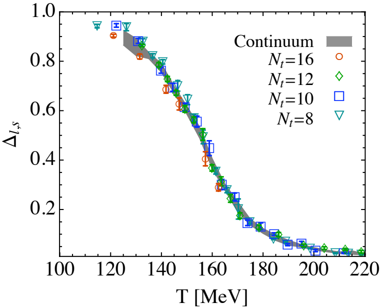
$$\frac{\chi_R(T)}{T^4} = \frac{m_s^2}{T^4}\left[\chi_{m,l}(T) - \chi_{m,l}(T=0)\right]\,.$$

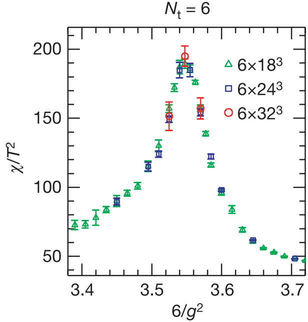
The temperature is $T = 1/(aN_t)$ and the volume $V = (aN_s)^3$.

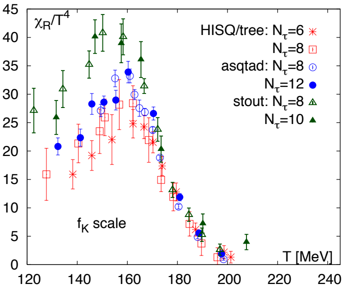
The transition/crossover can be obtained from the inflection point of the condensate or the peak of the susceptibility.

Condensate and susceptibility









Scaling analysis

More systematic than locating peak or inflection points is a proper scaling analysis based on

$$\frac{f}{T^4} = -\frac{1}{VT^3} \ln Z \equiv f_{sing}(t, h) + f_{reg}(T, m_l, m_s)\,,$$

where $f_{sing}(t, h) = h^{1+1/\delta} f_s(z)$, with scaling variables,

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c}\,, \qquad h = \frac{1}{h_0} H = \frac{1}{h_0}\frac{m_l}{m_s}\,, \qquad z = \frac{t}{h^{1/\beta\delta}}\,.$$

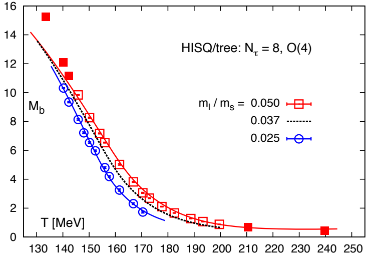
β and δ are critical exponents, known for 3d O(4) and O(2).
For the order parameter and susceptibility one has

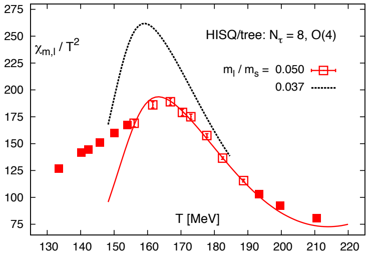
$$M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle}{T^4} = h^{1/\delta} f_G(z) + \text{reg}\,, \qquad \frac{m_s^2 \chi_{m,l}}{T^4} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) + \text{reg}\,,$$

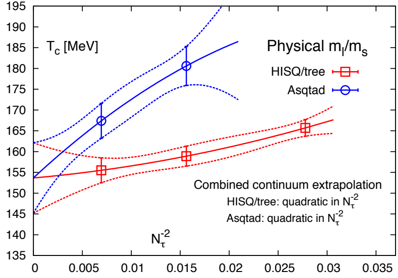
with $f_G(z)$ and $f_\chi(z)$ known functions.

In a “scaling fit,” T_c, t_0 and h_0 serve as fit parameters.

Scaling analysis







HotQCD collaboration, from fit:

$$T_c = 154 \pm 8 \pm 1 \,\text{MeV}\,.$$

Wuppertal-Budapest collaboration:

$$T_c = 155 \pm 3 \pm 3 \,\text{MeV}\,,$$

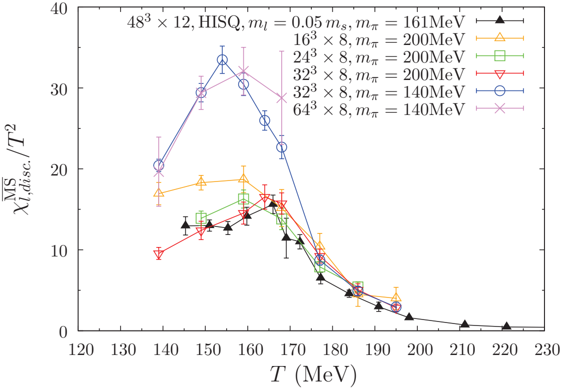
$$T_c = 157 \pm 3 \pm 3 \,\text{MeV}\,,$$

$$T_c = 147 \pm 2 \pm 3 \,\text{MeV}\,,$$

from $\langle \bar\psi\psi \rangle_R$, $\Delta_{l,s}$, and χ'_R/T^4.

T_c from domain-wall fermions

A theoretically cleaner, but computationally (much) more expensive fermion discretization than staggered fermions: domain-wall fermions.



The result from the (disconnected) chiral susceptibility, at one lattice spacing ($N_t = 8$), $T_c = 155(1)(8)\,\mathrm{MeV}$, confirms the staggered continuum result.
T. Bhattacharya *et al.* (HotQCD), Phys. Rev. Lett. 113 (2014) 082001 [arXiv:1402.5175]

QCD equation of state

Lattice QCD computations of the EoS usually start with the trace anomaly, or interaction measure,

$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\epsilon - 3p}{T^4} = \frac{I}{T^4} = T\frac{\partial}{\partial T}(p/T^4) = -\frac{1}{VT^3}\frac{\partial \ln Z}{\partial \ln a} .$$

which can be computed from local expectation values and "β-functions."
The computation requires subtraction of UV divergencies, using zero temperature measurements,

$$\Delta(X) = \langle X \rangle_T - \langle X \rangle_0 ,$$

which, at the same time, normalize the pressure to zero at $T = 0$. Thus

$$\frac{\epsilon - 3p}{T^4} = N_t^4 R_\beta(\beta) \left\{ -\Delta(s_G) + R_m(\beta) \left[2am_l \Delta(\bar{\psi}_l \psi_l) + am_s \Delta(\bar{\psi}_s \psi_s) \right] \right\} ,$$

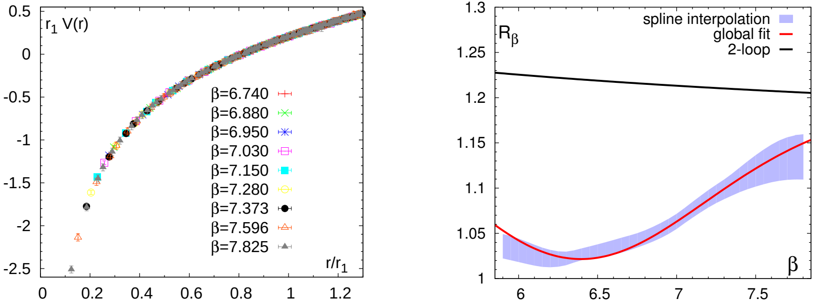
where s_G denotes the gauge action density and R_X are β-functions.

Scale setting

The lattice scale is determined from the heavy quark potential via

$$\left(r^2 \frac{\mathrm{d}V_{\bar{q}q}(r)}{\mathrm{d}r}\right)_{r=r_1} = 1.0\,, \qquad R_\beta(\beta) = -a\frac{\mathrm{d}\beta}{\mathrm{d}a} = (r_1/a)(\beta)\,\left(\frac{\mathrm{d}(r_1/a)(\beta)}{\mathrm{d}\beta}\right)^{-1}\,,$$

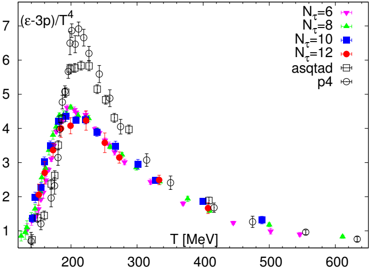
$$R_m(\beta) = \frac{1}{m(\beta)}\frac{\mathrm{d}m(\beta)}{\mathrm{d}\beta}\,.$$

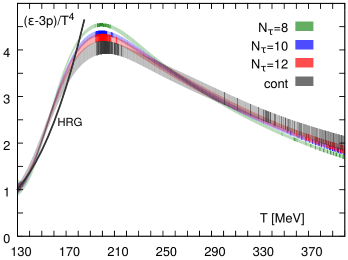


Alternatively, the scale can be set from f_K with a similar β-functions.

EoS results

HotQCD data with the HISQ/tree action compared to earlier results (with $N_t = 8$) on the left. The differences are due to the reduced taste symmetry breaking lattice artifacts (disappearing at high temperatures).





Right: continuum extrapolation of the HISQ/tree data with a spline interpolation

$$\frac{\epsilon - 3p}{T^4} = A + \frac{B}{N_t^2} + \sum_{i=1}^{5} \left[C_i + \frac{D_i}{N_t^2} \right] S_i(T)\,,$$

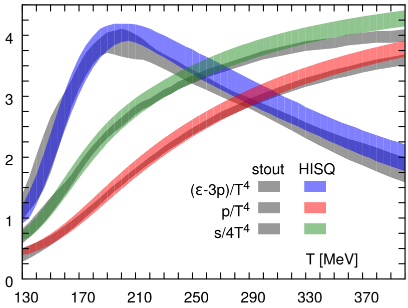
where the $S_i(T)$ are B-splines, and the fit uses two internal knots.

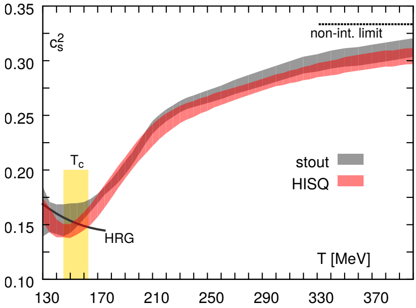
EoS results

The pressure is determined by integration from the trace anomaly

$$\frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} dT' \frac{\epsilon - 3p}{T'^5} .$$

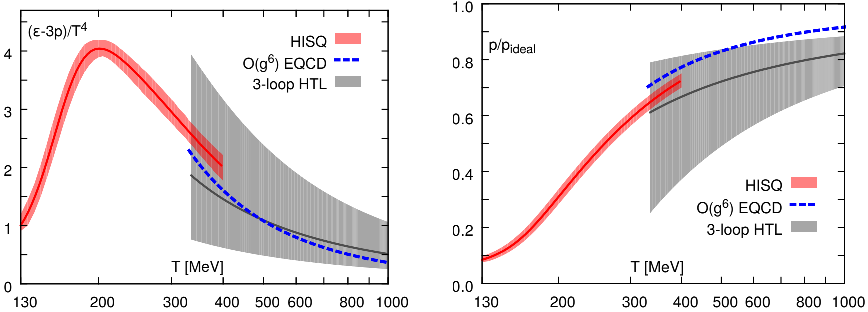
The energy density, entropy $s = (\epsilon + p)/T$, and speed of sound $c_s^2 = \partial p/\partial\epsilon$ can be obtained as well. The results of the HotQCD and WB collaborations agree within about 1 sigma over the temperature range considered.





Approach to the perturbative limit

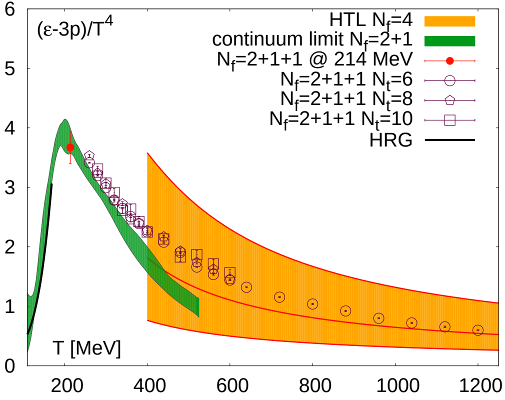
At high temperature, a weakly interacting quark-gluon gas: should be treatable in perturbation theory. Away from $T = \infty$ ($g = 0$), resummations and/or dimensional reduction is needed.



- Electrostatic QCD (EQCD) – dimensional reduction, M. Laine and Y. Schröder, Phys. Rev. D73 (2006) 085009 [arXiv:hep-ph/0603048]
- Three-loop hard thermal loop expansion, N. Haque *et al.*, JHEP 1405 (2014) 027 [arXiv:1402.6907]

EoS with dynamical charm

The Wuppertal-Budapest and the MILC Collaboration have started to investigate the influence of dynamical charm quarks on the EoS.



Preliminary results indicate the influence of dynamicl charm becoming visible around $T \sim 300\,\mathrm{MeV}$.

S. Borsányi *et al.* (HotQCD), arXiv:1410.7917

Conclusions

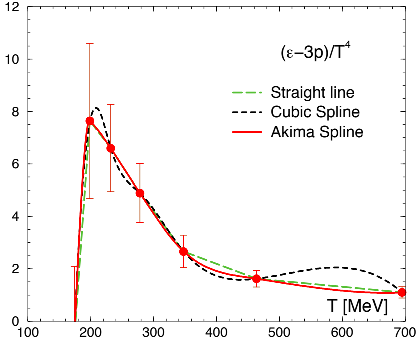
- Computations with staggered fermions by the HotQCD and Wuppertal-Budapest Collaborations established the QCD finite temperature transition/crossover at $T_c = 154 \pm 9\,\mathrm{MeV}$.
- The two collaborations also agree on the QCD equation of state for $130\,\mathrm{MeV} \leq T \lesssim 400\,\mathrm{MeV}$ within about 1 sigma.
- For $T \lesssim 145\,\mathrm{MeV}$ the lattice QCD EoS agrees well with the hadron resonance gas model result.
- The energy density at the crossover temperature is $\epsilon_c \simeq 300\,\mathrm{MeV}/\mathrm{fm}^3 \approx 2\epsilon_{nuclear} \approx \frac{2}{3}\epsilon_{proton}$.
- Other thermodynamic quantities, like entropy density and the velocity of sound are easily obtained as well. The velocity of sound has a minimum at $T \sim 146\,\mathrm{MeV}$.
- A dynamical charm quark appears to lead to visible effects on the EoS for $T \gtrsim 300\,\mathrm{MeV}$.

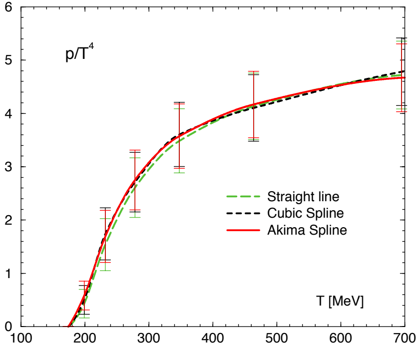
Extra

EoS in the fixed scale approach

Temperature is changed by varying N_t with other parameters fixed.
Advantage: only one "zero temperature" (large N_t) simulation needed.

Improved Wilson quarks, 2+1 flavors, at heavy m_{ud}, corresponding to $m_\pi/m_\rho \simeq 0.63$, and one lattice spacing, $a \simeq 0.07$ fm.



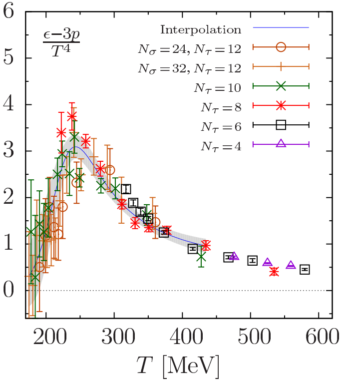


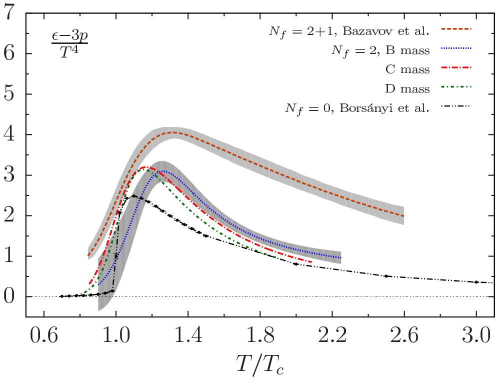
T. Umeda *et al.*, Phys. Rev. D85 (2012) 094508 [arXiv:1202.4719]

EoS with twisted mass Wilson fermions

2 flavors, with N_t up to 12, three quark masses, corresponding to $m_\pi \simeq 360$, 430, and 640 MeV.

Left: tree-level corrected interaction measure for $m_\pi \simeq 360$ MeV.
Right: Comparison with quenched and (2+1)-flavor continuum results.





F. Burger *et al.*, Phys. Rev. D91 (2015) xxxxxx [arXiv:1412.6748]

S. Borsányi *et al.*, JHEP 1207 (2012) 056 [arXiv:1204.6184]

HISQ/tree – numerical setup

- Line of constant physics $m_l = m_s/20$ (physical $m_l = m_s/27$), $m_\pi = 160$ MeV
- Statistics (in molecular dynamics time units):

$T > 0$		$T = 0$	
$24^3 \times 6$	30-40K	$24^3 \times 32$	5-20K
$32^3 \times 8$	30-100K	32^4, $32^3 \times 64$	10-30K
$40^3 \times 10$	100-200K	48^4	5-14K
$48^3 \times 12$	50-100K	$48^3 \times 64$	8-12K
		64^4	8K

HISQ/tree – scale setting

Fit a/r_1 data with the Ansatz:

$$\frac{a}{r_1} = \frac{c_0 f(\beta) + c_2 (10/\beta) f^3(\beta)}{1 + d_2 (10/\beta) f^2(\beta)},$$

$$f(\beta) = (b_0 (10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0))$$

$$c_0 = 43.1281 \pm 0.2868$$
$$c_2 = 343236 \pm 41191$$
$$d_2 = 5513.84 \pm 754.821$$