Glue Spin from Lattice QCD

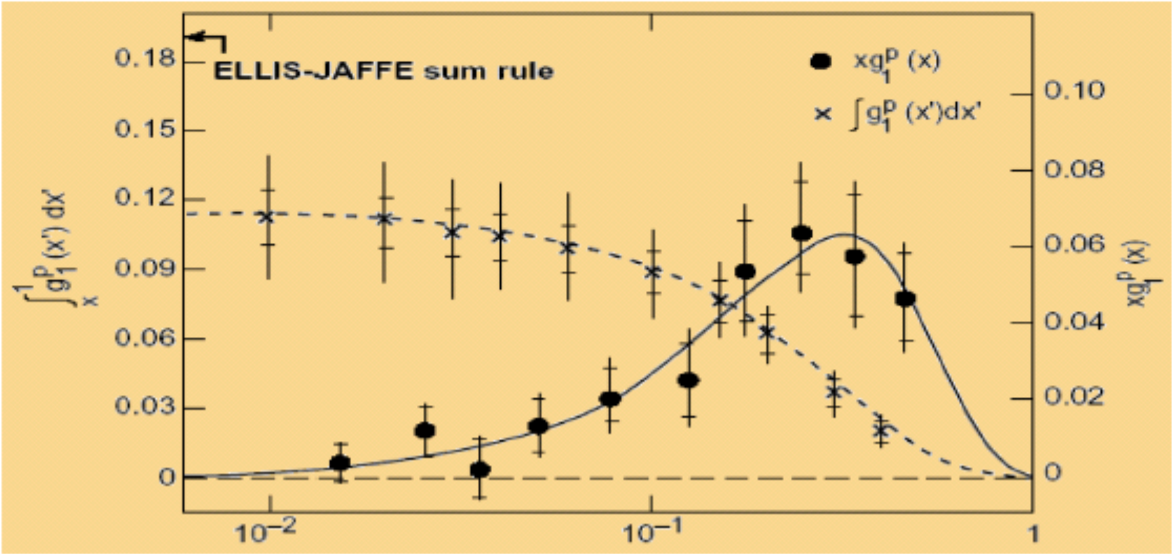
- Status of nucleon spin components
- Gauge field tensor operator
- Momentum and angular momentum sum rules
- Lattice results
- Quark spin from anomalous Ward identity
- Glue spin

GHP 2015, Baltimore, Apr. 8, 2015

Where does the spin of the proton come from?

Twenty 6 years since the "spin crisis"

- EMC experiment in 1988/1989 – "the plot":

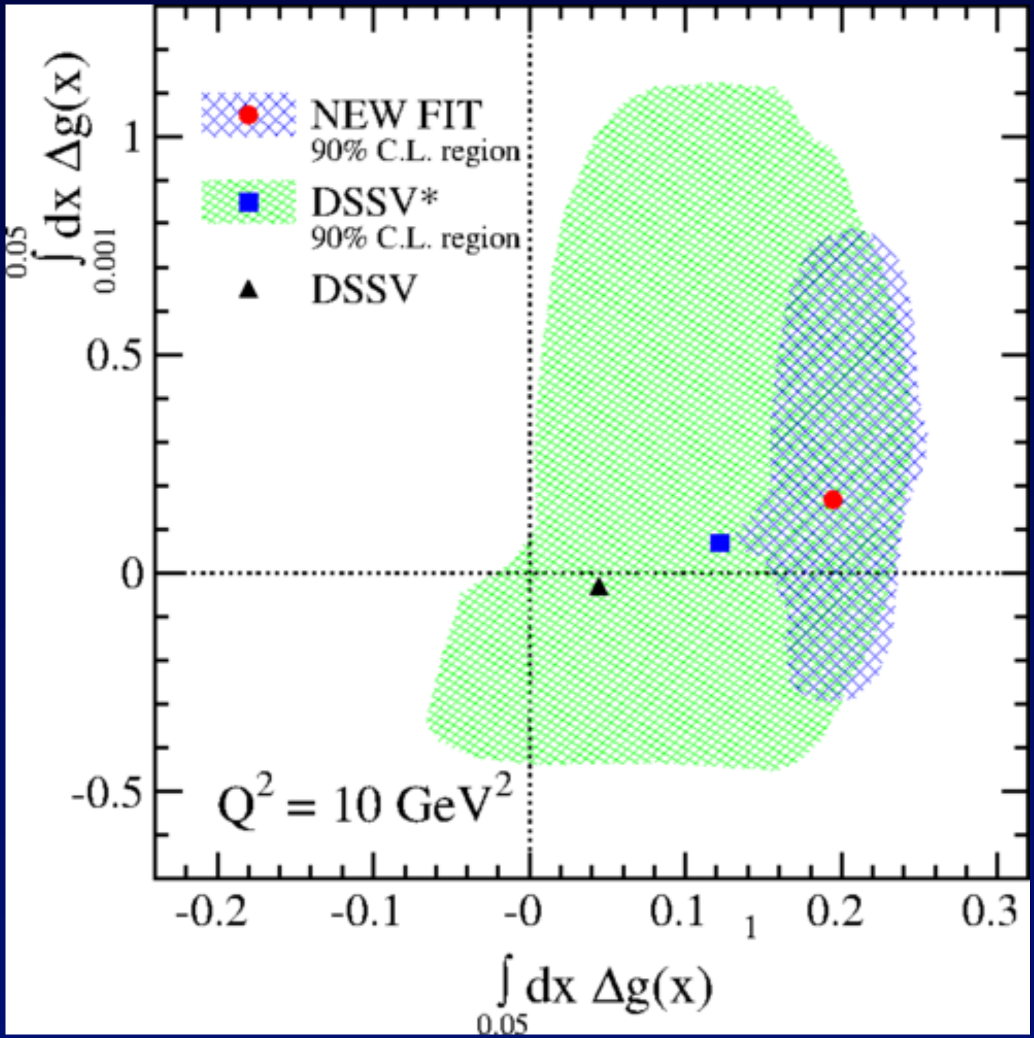


$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\|}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\|}\rangle$$

- "Spin crisis" or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

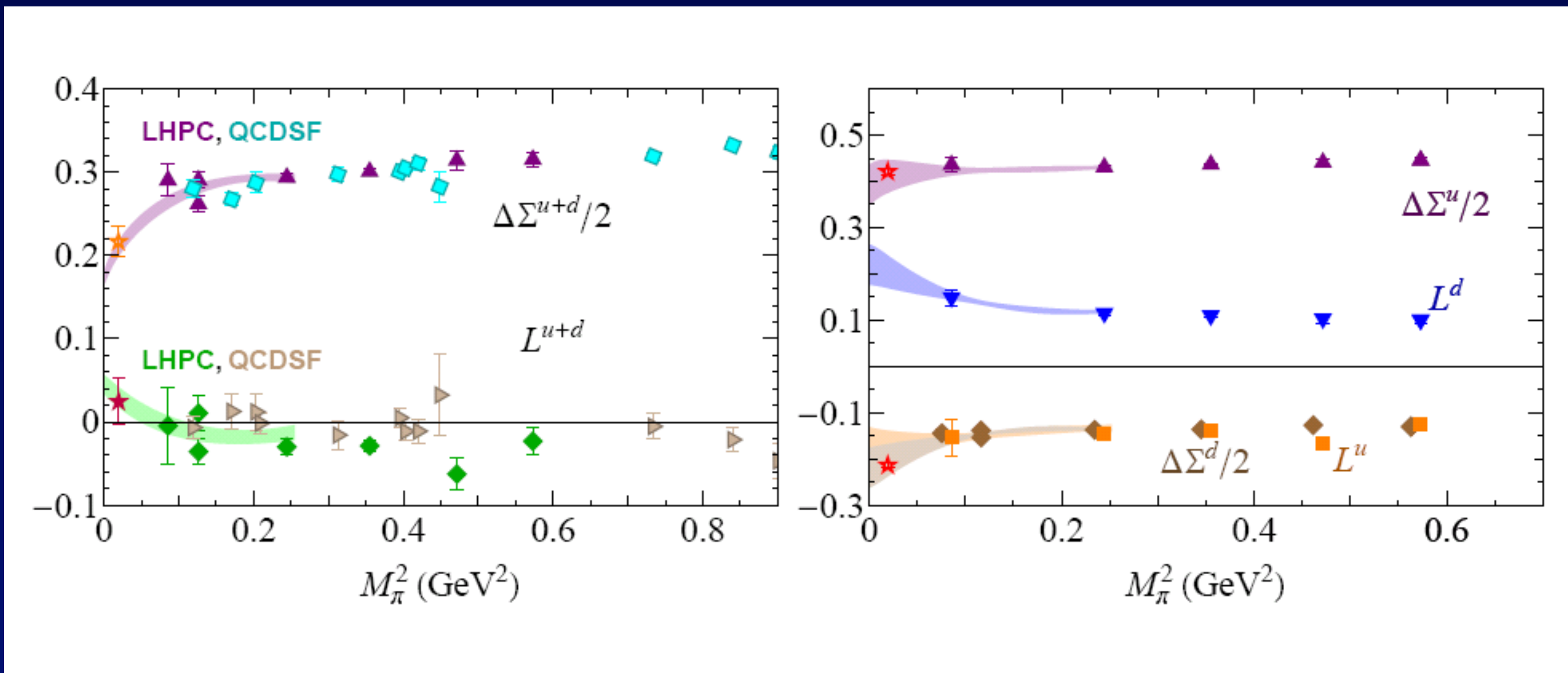
Glue Polarization ΔG



Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang,
PRL 113, 012001 (2014)

Quark Orbital Angular Momentum (connected insertion)



LHPC, S. Syritsyn et al., [1111.0718]
QCDSF, A. Sternbeck et al, [1203.6579]

Status of Proton Spin

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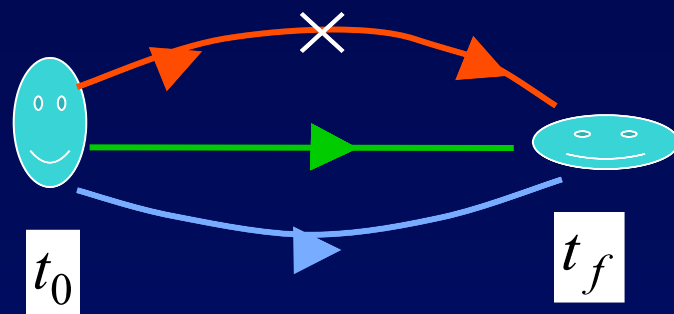
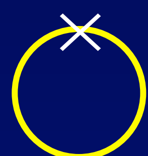
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF)→ ~ 0)
- Glue spin ~ 0.4 (STAR, COMPASS)
- From `proton spin crisis' to `missing spin search'.

Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon

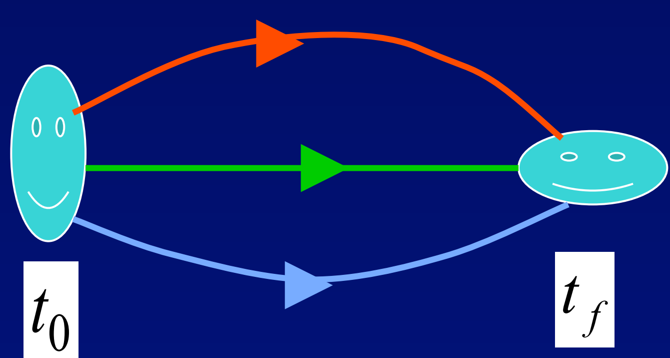
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$

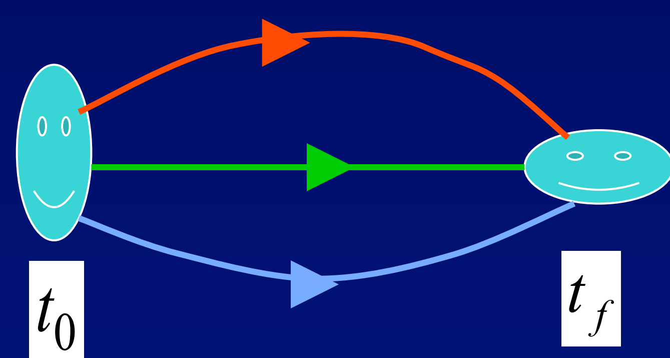




$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$

$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$





Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\left\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

$$\text{-i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ \left[\mathrm{OPE}\right] \rightarrow \left\langle x\right\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$\left.\begin{array}{l} Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1, \\[2ex] Z_q J_q^L + Z_g J_g^L = \dfrac{1}{2} \end{array}\right. \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
PRD arXiv:1403.7211

Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \;\; H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu}(x) + O(a^3),$$

$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_{\lambda}(c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2\right]}{(\sum_\mu s_\mu^2 + [\rho + \sum_\nu (c_\nu - 1)]^2)^{3/2}}$$

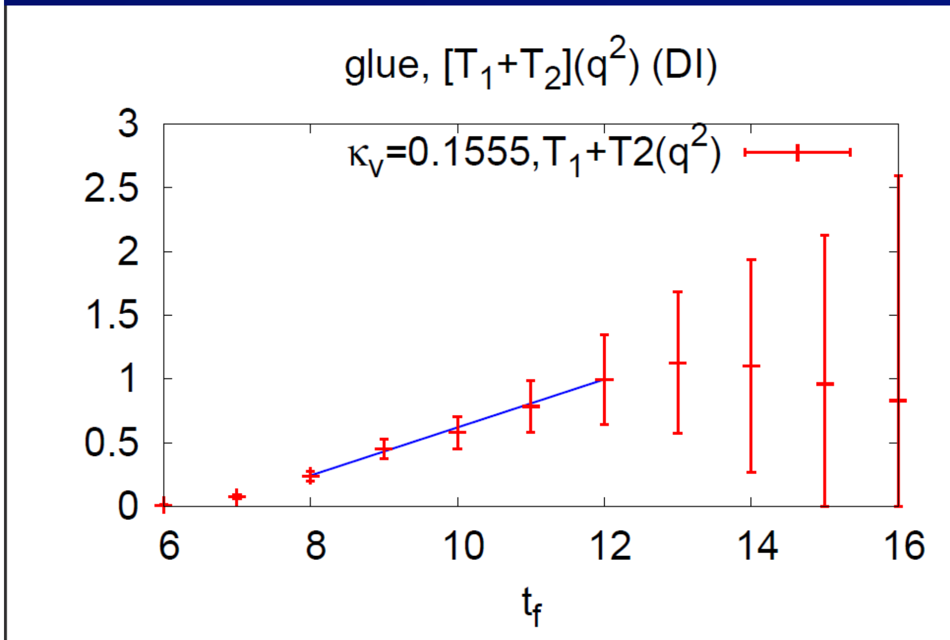
where, $r = 1,\ \ \rho = 1.368,\ \ \ c^T = 0.11157$

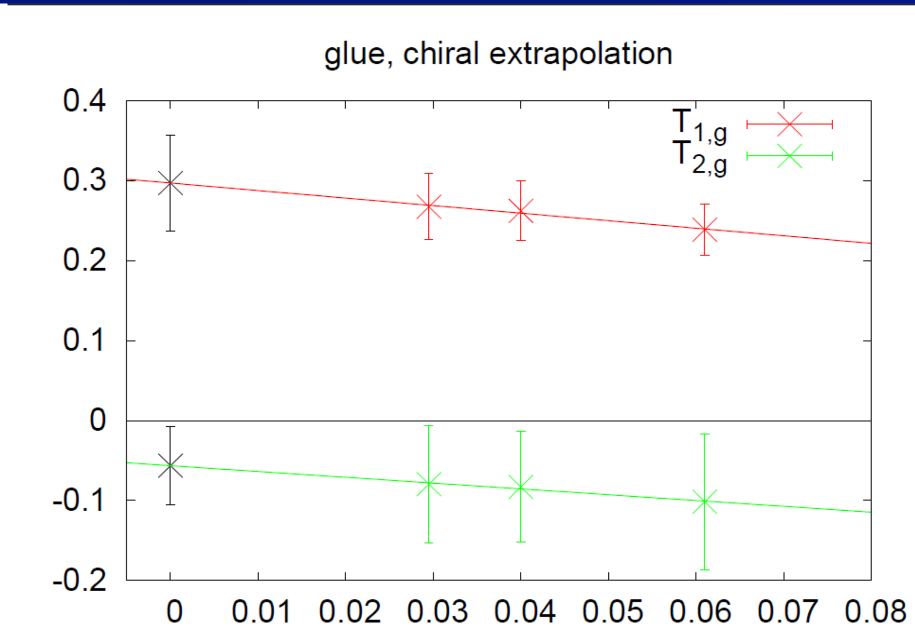
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

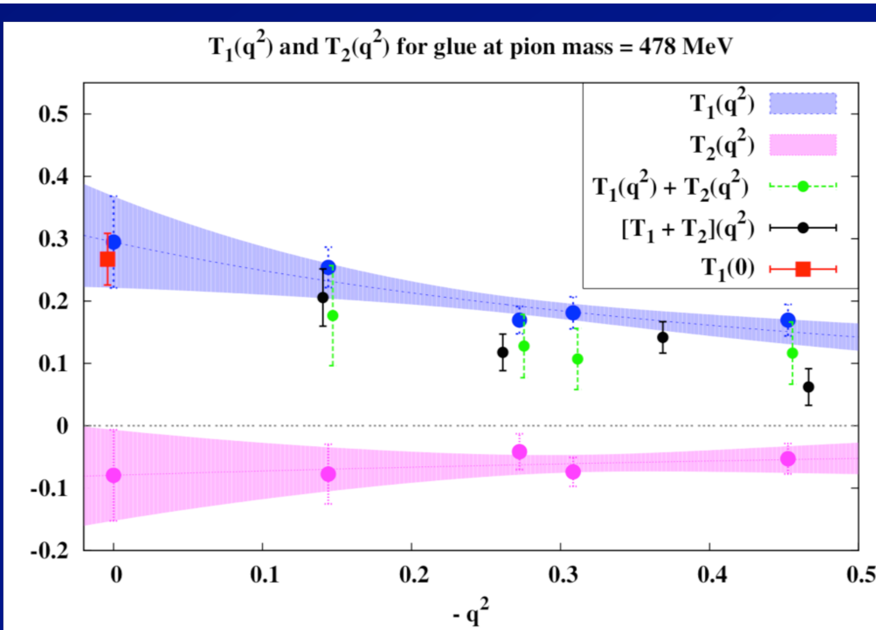
- Noise estimation $D_{ov}(x,x) \rightarrow \left\langle \eta_x^\dagger (D_{ov}\eta)_x \right\rangle$

  with Z_4 noise with color-spin dilution and some dilution in space-time as well.

Glue $T_1(q^2)$ and $T_2(q^2)$







M. Deka et al., PRD 91, 014501 (2015)
arXiv:1312.4816

(χ QCD Collaboration)

Quenched $16^3 \times 24$ lattice, β=6.0,
$m_\pi \geq$ 478 MeV, 500 configurations

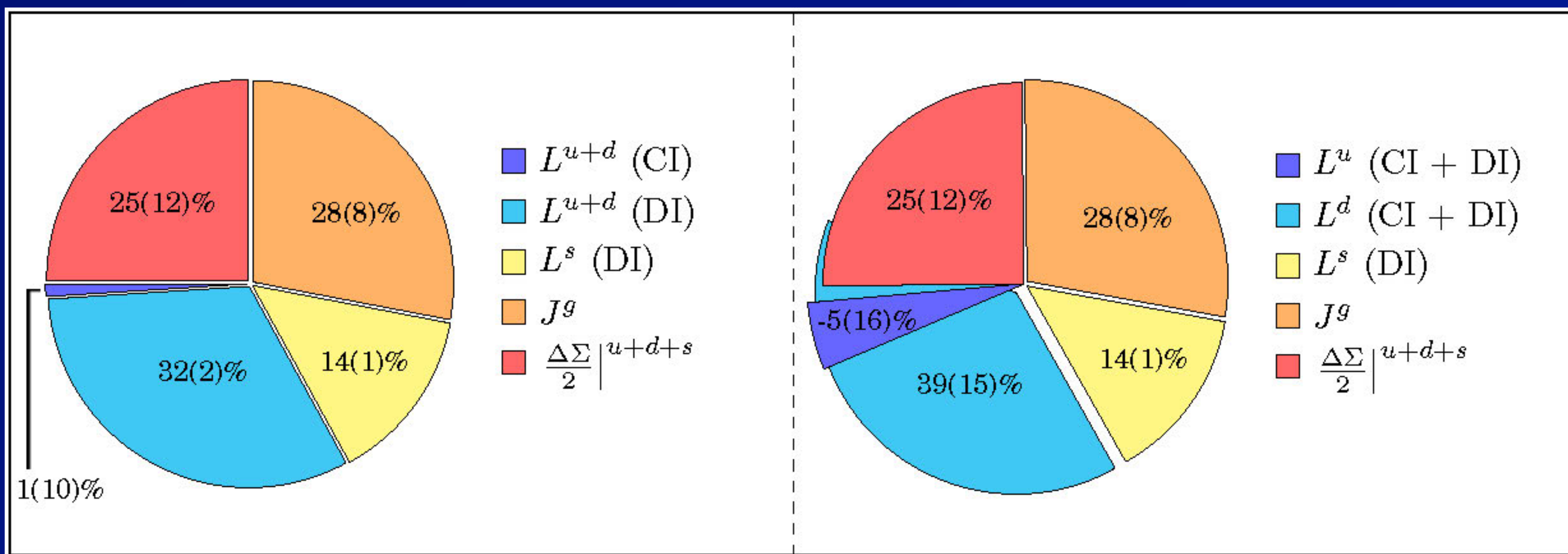
Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

$\overline{\mathrm{MS}}$ (2 GeV)

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-0.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, PRD 91, 014501 (2015))

pizza cinque stagioni



$\Delta q \approx 0.25;$

$2\,L_q \approx 0.47\ (0.01(\mathrm{CI})+0.46(\mathrm{DI}));$

$2\,\mathrm{J}_g \approx 0.28$

These are quenched results so far.

Quark Spin Calculation with Axial-vector current

- Recent calculation of strange quark spin with dynamical fermions
  - R. Babich et al. (1012.0562)

    $$\Delta s = -0.019(11)$$

  - QCDSF (G. Bali et al. 1112.3354) gives

    $$\Delta s = -0.020(10)(4)$$

  - M. Engelhardt (1210.0025)

    $$\Delta s = -0.031(17)$$

  - C. Alexandrou et al. (arXiv:1310.6339)

    $$\Delta s \sim -0.0227(34)$$

Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = i2mP + \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$

$$Z_A \langle p',s | A_\mu | p,s \rangle = \lim_{q\to 0} \frac{i|s|}{\vec{q}\cdot\vec{s}} \langle p',s | 2\sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q | p,s \rangle$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P=1$)
  - Overlap operator for $q(x) = -1/2\ \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ is renorm.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and captures the high modes from A_μ^0.
  - Direct check the origin of `proton spin crisis'.

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.137 fm

(O(a^2) extrapolation)

La ~ 2.8 fm
m_π ~ 330 MeV

24^3 x 64, a =0.115 fm

La ~ 5.5 fm
m_π ~ 140 MeV

48^3 x 96, a =0.115 fm

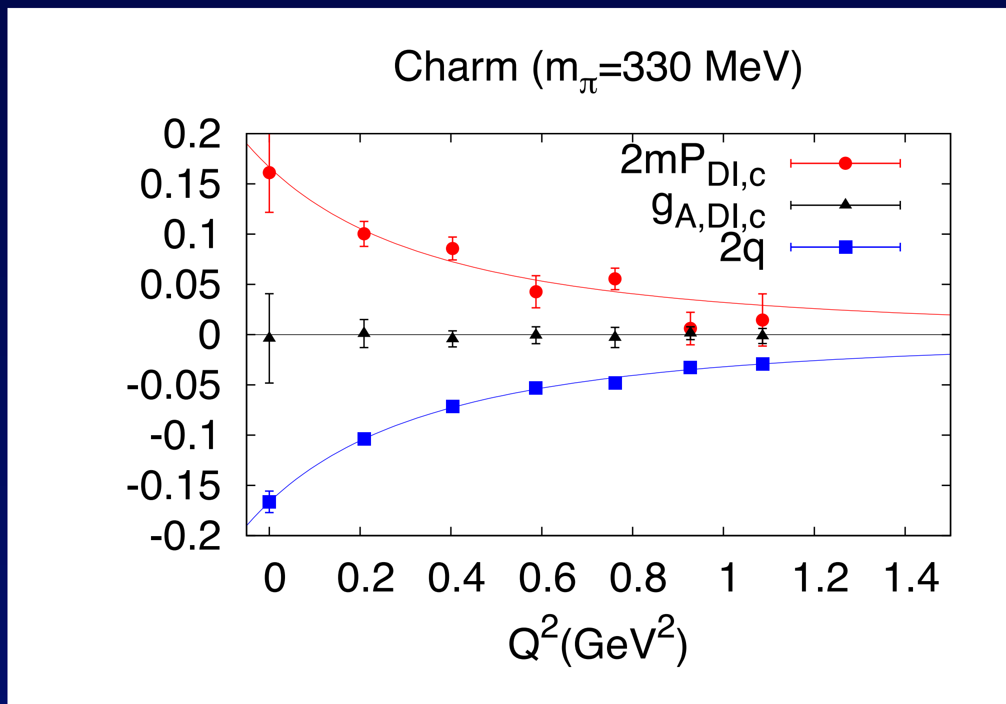
La ~ 2.7 fm
m_π ~ 295 MeV

32^3 x 64, a =0.085 fm

La ~ 5.5 fm
m_π ~ 140 MeV

64^3 x 128, a =0.085 fm

Disconnected Insertion for the Charm Quark



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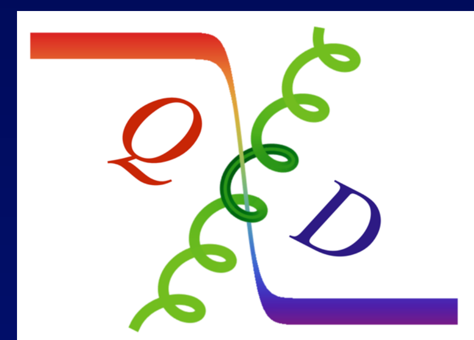
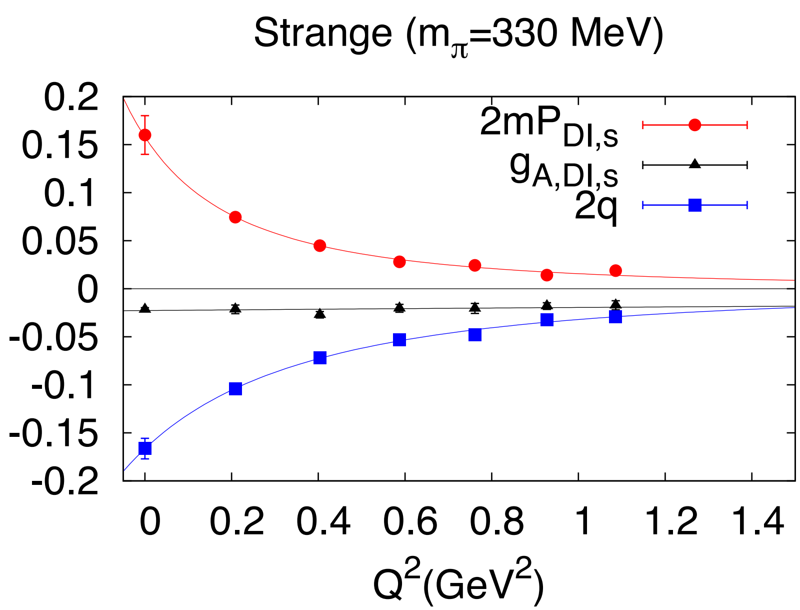
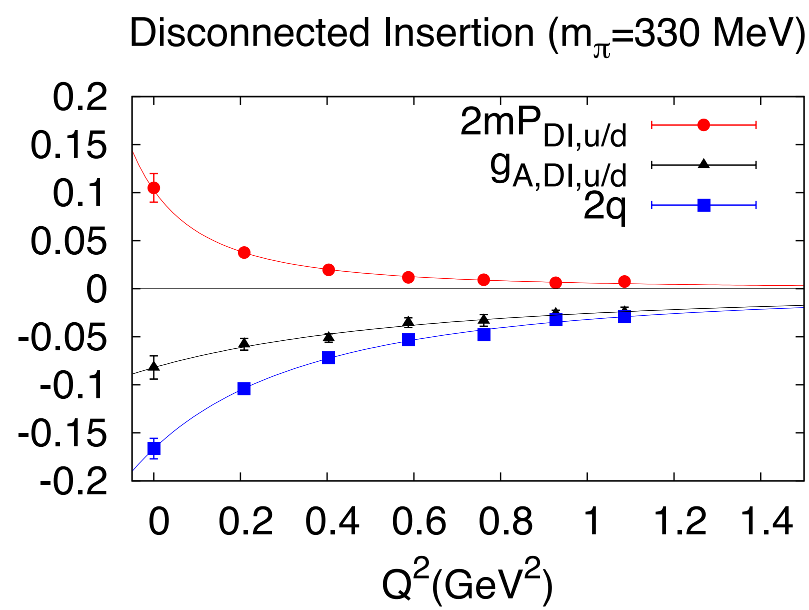


Y. Yang, M. Gong *et al*

- Topological term is large and negative
- Pseudoscalar term and the topological term cancel

Disconnected Insertion for the Strange and u/d Quarks





Strange

u/d (DI)

Quark Spin from AWI

Overlap fermion on 2+1 flavor $24^3 \times 64$ DWF lattice (L=2.8 fm)

g_A^0 compt	**m_π=330 MeV (m_V=m_{sea})**
$\Delta u+\Delta d$ (CI)	0.57(2)
Δc	~0
Δs	-0.05(1)
Δu(DI)=Δd(DI)	-0.11(2)
g_A^0	0.30(6)

The triangle anomaly (topological charge) is responsible for the smallness of quark spin in the proton (`proton spin crisis).

Controversy over Glue Spin S_g and Helicity ΔG -- Gauge Invariance and Frame Dependence

- Jaffe and Manohar

$S_g = \int d^3x \, \vec{E} \times \vec{A}$ in light-cone gauge $(A^+ = 0)$ and IMF frame.

- Collins, Soper; Manohar

$$\Delta G \, S^+ = \int dx \, \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) | PS \rangle$$

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

$$S_g = \int d^3x \, \vec{E} \times \vec{A}_{nC}, \quad A^\mu = A^\mu_{nC} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{nC} \to U^\dagger A^\mu_{nC} U, \quad A^\mu_{pure} \to U^\dagger A^\mu_{pure} U - \frac{i}{g} U^\dagger \partial^\mu U$$

$$D^i A^i_{nC} = \partial^i A^i_{nC} - ig[A^i, A^i_{nC}] = 0; \quad A_{nC} = A_{phys} = A_\perp, \quad A_{pure} = A_{||}$$

Gauge invariant decompostion

Gauge invariance issue

To make the canonical spin and orbital quantities gauge-invariant, typically the transverse part of the vector-potential is assumed (= Coulomb gauge):

$$\mathbf{A} \to \mathbf{A}_{\perp}, \quad \nabla \cdot \mathbf{A} = 0$$

But the transverse vector-potential is nonlocal:

$$\mathbf{A}(\mathbf{r}) \propto \int \frac{\nabla' \times \mathbf{B}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'$$

However, it becomes local and meaningful in the most important case of monochromatic optical fields:

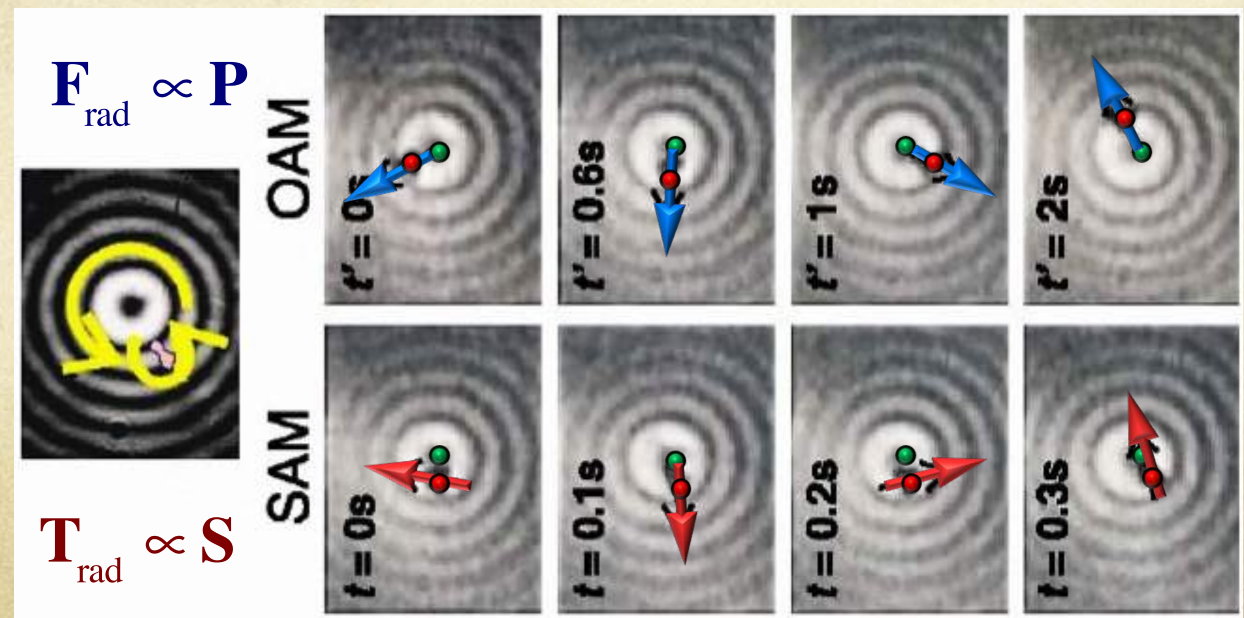
$$\boxed{\mathbf{A}(\mathbf{r}) \propto -i\omega \mathbf{E}(\mathbf{r})} \qquad \mathbf{O}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{O}(\mathbf{r}) e^{-i\omega t}\right]$$

Application to optical fields

Most importantly, the canonical momentum and spin densities **immediately appear in optical experiments**. They determine the radiation pressure and torque on a point electric dipole:

$$\boxed{\mathbf{F}^{\mathrm{rad}} \propto \mathrm{Im}(\alpha)\mathbf{P}\,, \quad \mathbf{T}^{\mathrm{rad}} \propto \mathrm{Im}(\alpha)\mathbf{S}}$$

Ashkin & Gordon (1983)
Canaguier-Durand (2013)
Bliokh *et al.* (2013, 2104)



Glue Helicity ΔG

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

$$\Delta G\ S^{+} = \int dx\ \frac{i}{2xP^{+}} \int \frac{d\xi^{-}}{2\pi}\ e^{-ixP^{+}\xi^{-}} \langle PS \,|\, F_{a}^{+\alpha}(\xi^{-}) L^{ab}(\xi^{-},0) \tilde{F}_{\alpha,b}^{+}(0) \,|\, PS \rangle$$

$$= \langle PS \,|\, \vec{E}^{a}(0) \times (\vec{A}^{a}(0) - \frac{1}{\nabla^{+}}(\vec{\nabla} A^{+,b}) L^{ba}(\xi^{-},0)\,|_{\xi^{-}=0} \,|\, PS \rangle\, |^{z};$$

It is shown that boosting $A_{\text{pure}}^{i,a}$ to the infinite momentum frame,

$$A_{\text{pure}}^{i,a}(\xi^{-}) = \frac{1}{\nabla^{+}} (\partial^{i} A^{+,b}) L^{ba}(\xi'^{-},\xi^{-})\,|_{\xi'^{-}=\xi^{-}}$$

- Therefore,

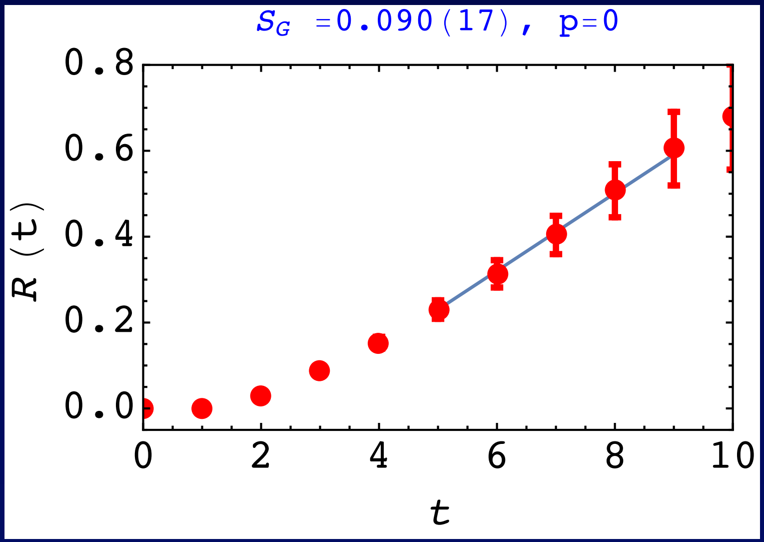
$$\Delta G\ S_{z} = \frac{\langle PS \,|\, \int d^{3}x\ (\vec{E} \times \vec{A}_{\text{phys}})_{z} \,|\, PS \rangle}{2E_{P}}$$

At infinite P

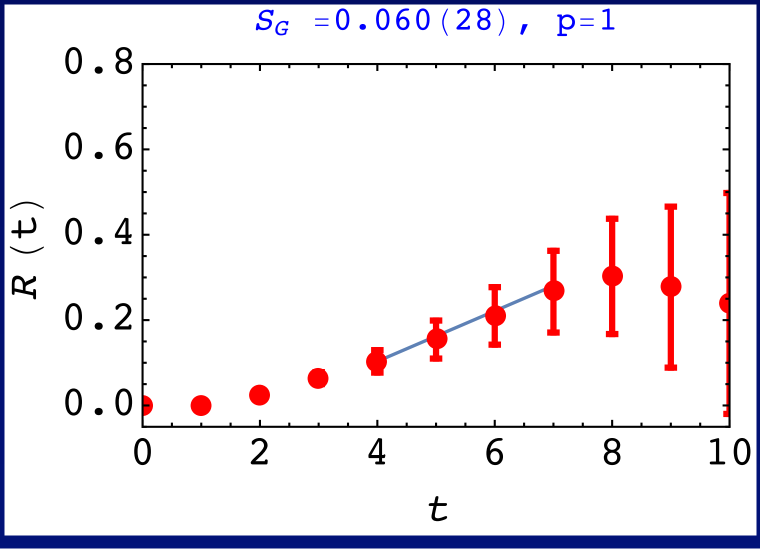
- S_g is gauge invariant but frame dependent

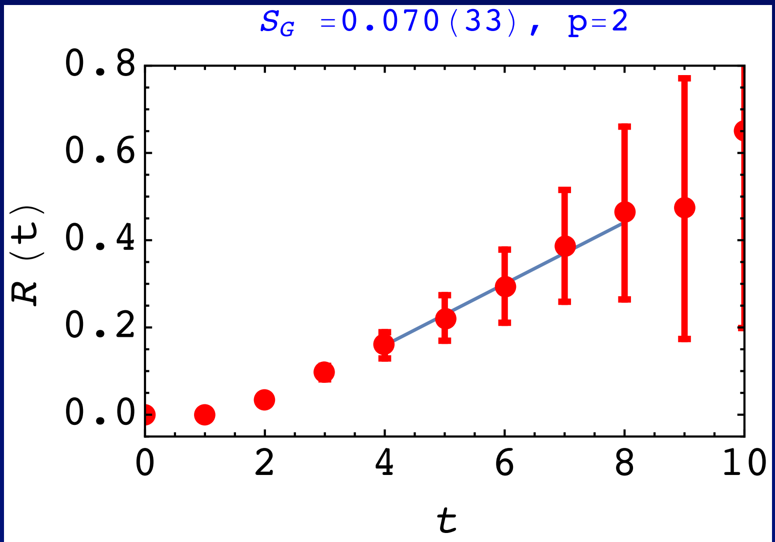
$$A_{\text{phys}} = g_{c}^{-1} A_{c} g_{c} \quad \rightarrow \quad \vec{S}_{g} = Tr(\vec{E} \times \vec{A}_{\text{phys}}) = Tr(g_{c} \vec{E} g_{c}^{-1} \times \vec{A}) = Tr(\vec{E}_{c} \times \vec{A}_{c})$$

Glue Spin at nucleon momenta at p =0, 460 and 920 MeV with overlap on 2+1 flavor 24^3 x 64 DWF lattice at sea m_π = 330 MeV

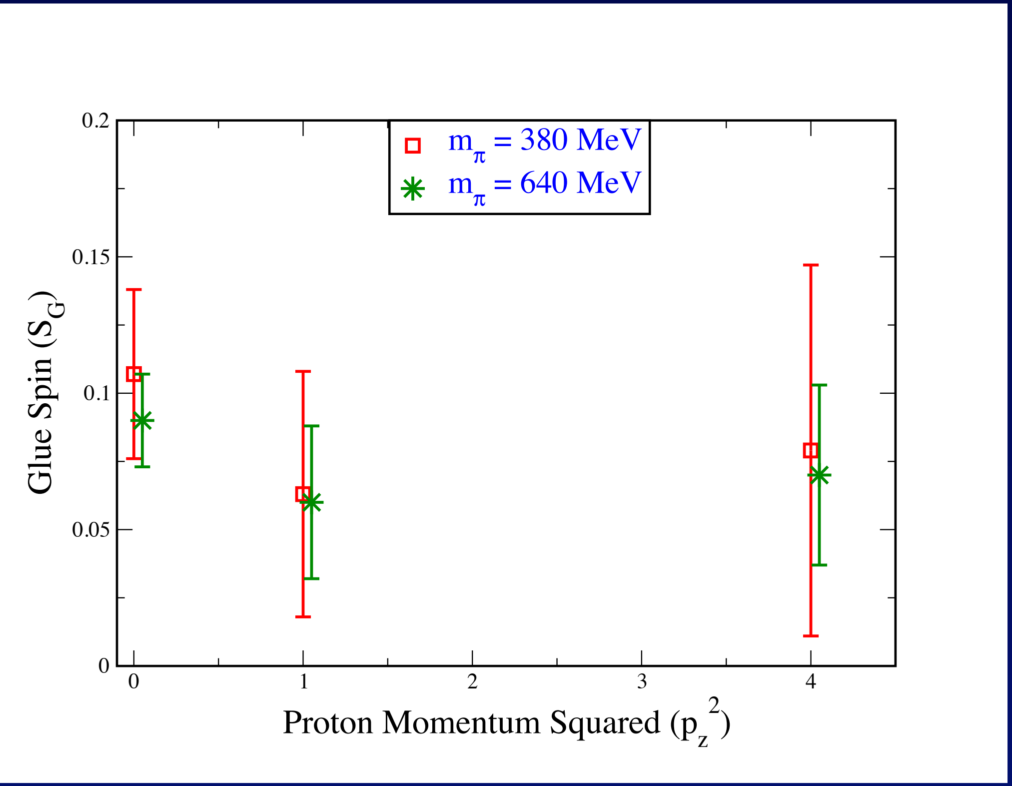


Matrix element from the slope in t





Glue spin in Coulomb gauge at p =0, 460 MeV, 920 MeV on the 24^3 x 64 lattice



Noisy, working on gauge smearing

Summary and Challenges

- Decomposition of proton spin into quark spin, quark orbital angular momentum, glue spin, and glue orbital angular momentum on the lattice is becoming feasible, pending on better understanding of the local glue spin operator.

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- `Proton Spin Crisis' is likely to be the second example of observable U(1) anomaly.

- Continuum limit at physical pion mass and with large lattice volume (5.5 fm) with chiral fermions. How large p needs to be for ΔG ?