Quasi-Classical TMD's of an Unpolarized Heavy Nucleus

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with Yuri Kovchegov

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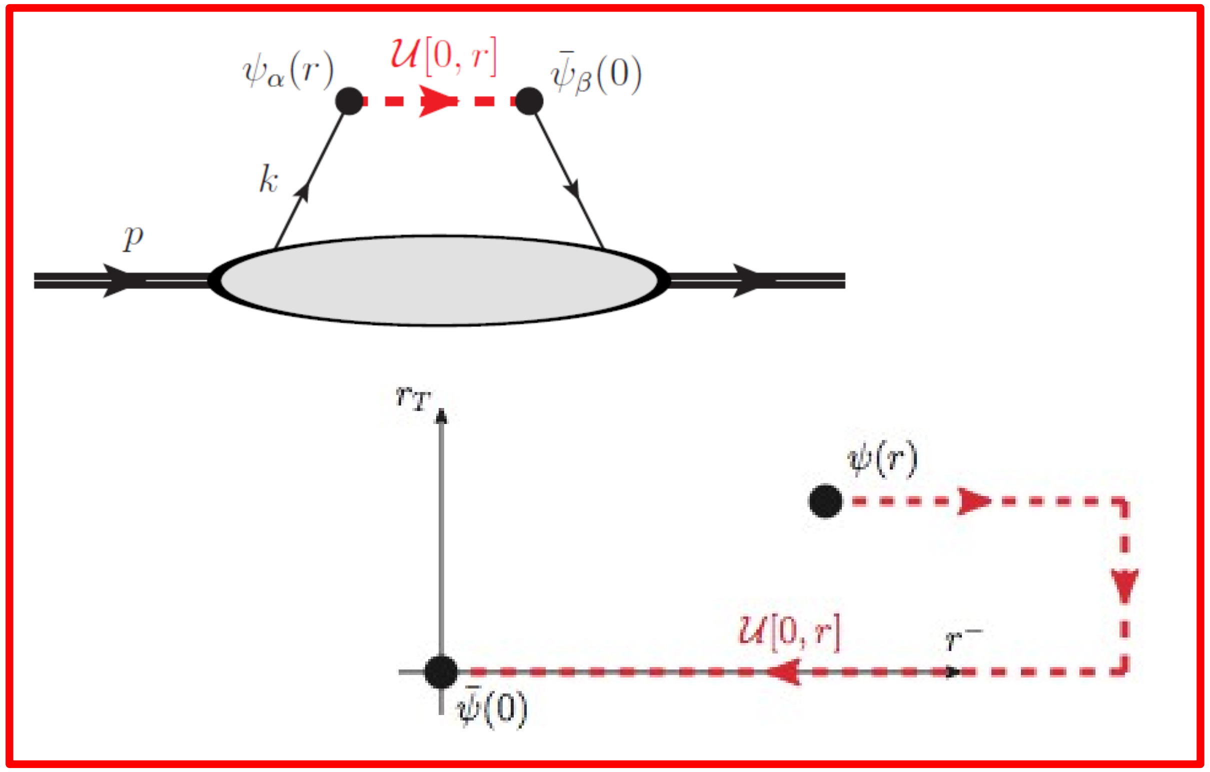
Yuri Kovchegov and M.S., Phys.Rev. D89 (2014) 5, 054035

and a paper in preparation

Quark TMD's



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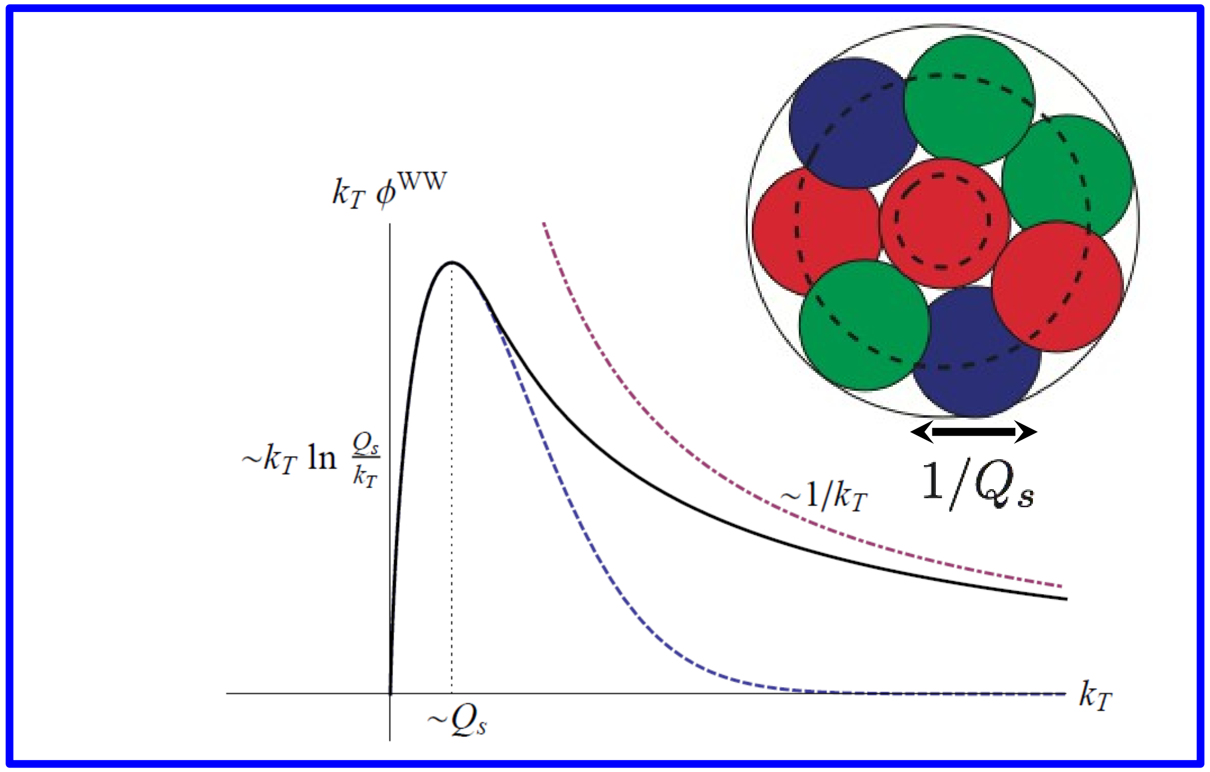
Classical Gluon Fields



- First-principles calculation simplifies the TMD's
- Quark structure of an unpolarized heavy nucleus
- Novel TMD mixing with predictive power

Leading TMD Quark Distributions

Quark structure of a spin-½ hadron: TMD quark correlation function:

$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3} \, e^{ik\cdot r} \, \langle h(p)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0, r] \, \psi_\alpha(r) \, |h(p)\rangle$$

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		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity; $h_{1T}^\perp$ = −
		$\frac{1}{2}\mathrm{Tr}[\phi\,\gamma^+]$	$\frac{1}{2}\mathrm{Tr}[\phi\,\gamma^+\gamma^5]$	$\frac{1}{2}\mathrm{Tr}[\phi\,\gamma^+\gamma_\perp^j\gamma^5]$

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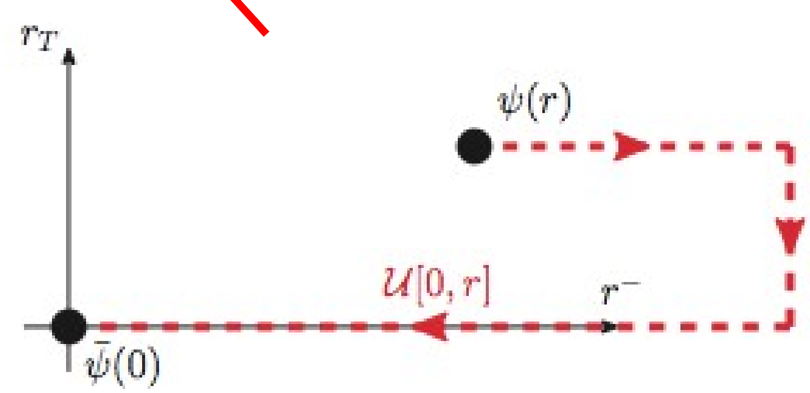


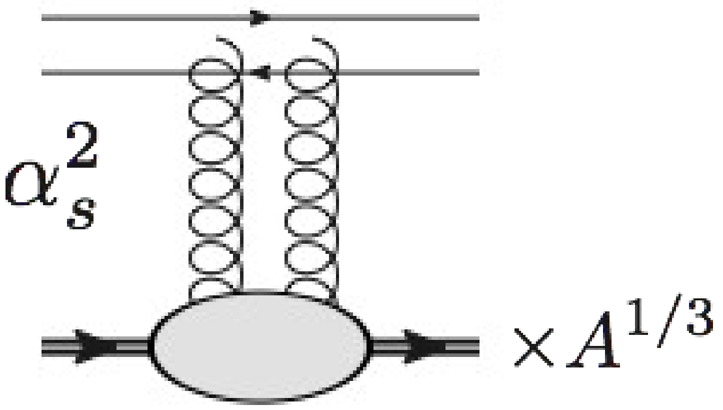
- Nontrivial staple-shaped gauge link
- Physical information about ISI / FSI

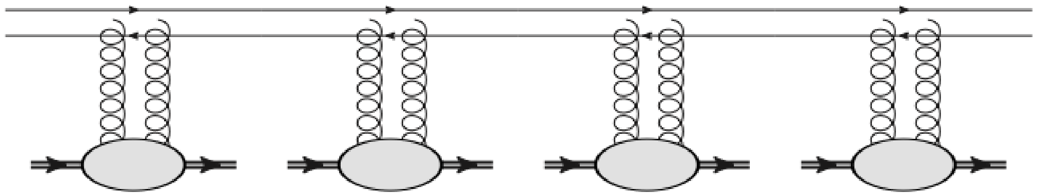
The Quasi-Classical Limit (MV Model)

High charge densities enhance the probability of multiple scattering.

- A sufficiently heavy nucleus generates a classical (Yang-Mills) gluon field:

$$A \gg 1 \qquad \alpha_s^2 A^{1/3} \sim \mathcal{O}(1)$$





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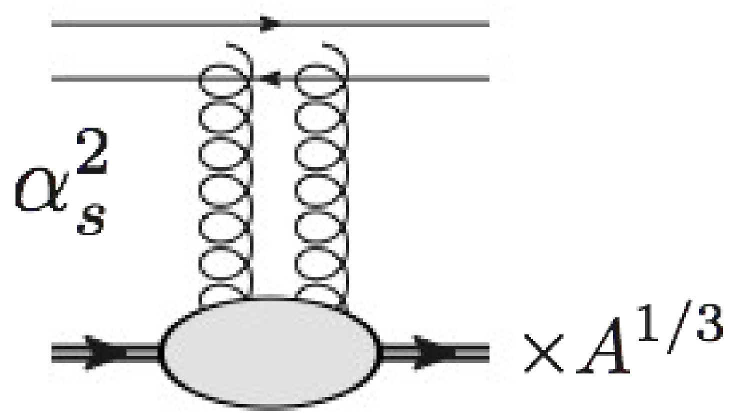

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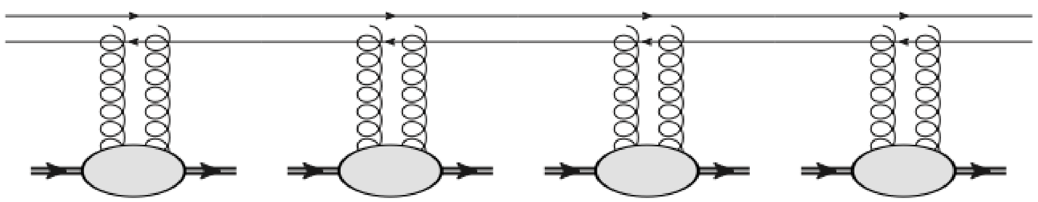
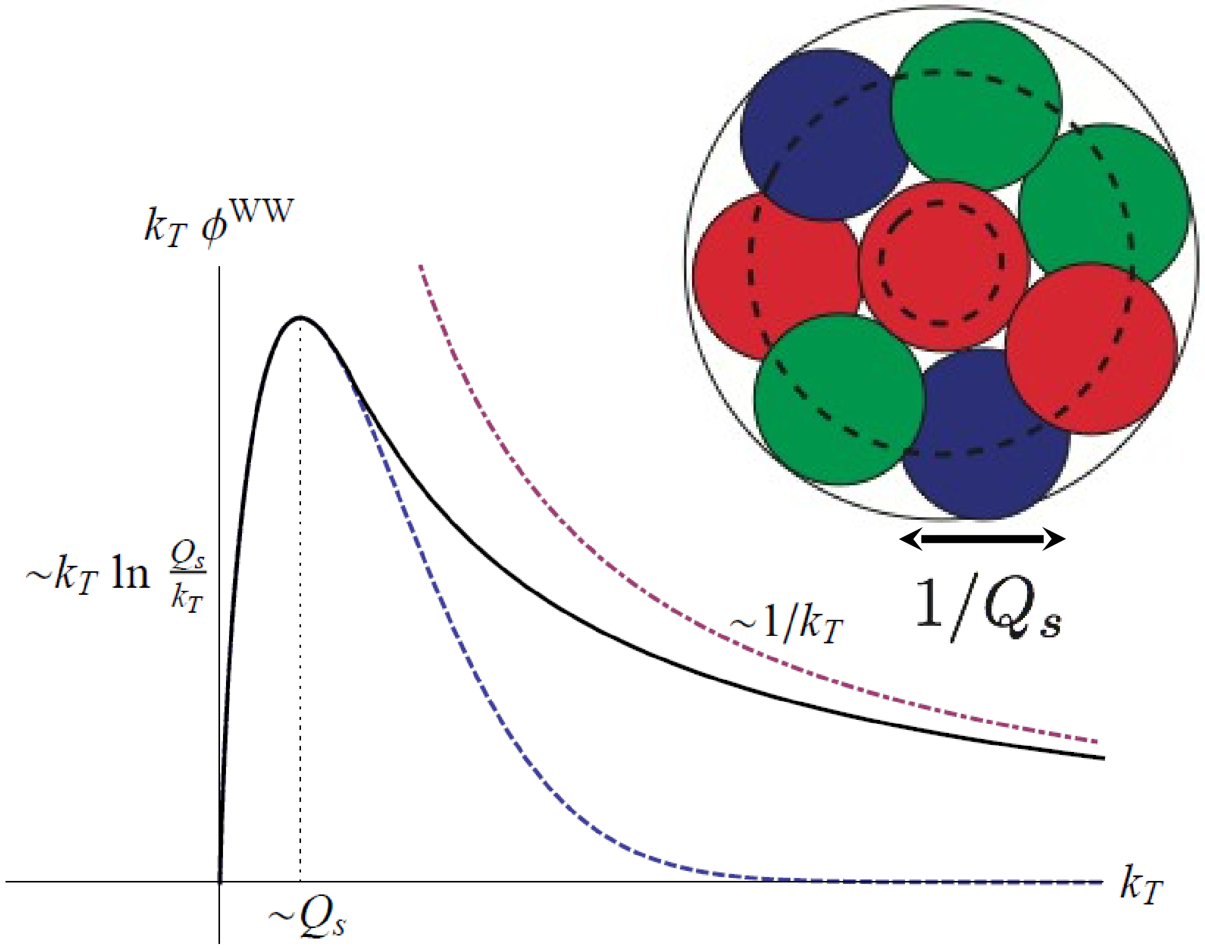


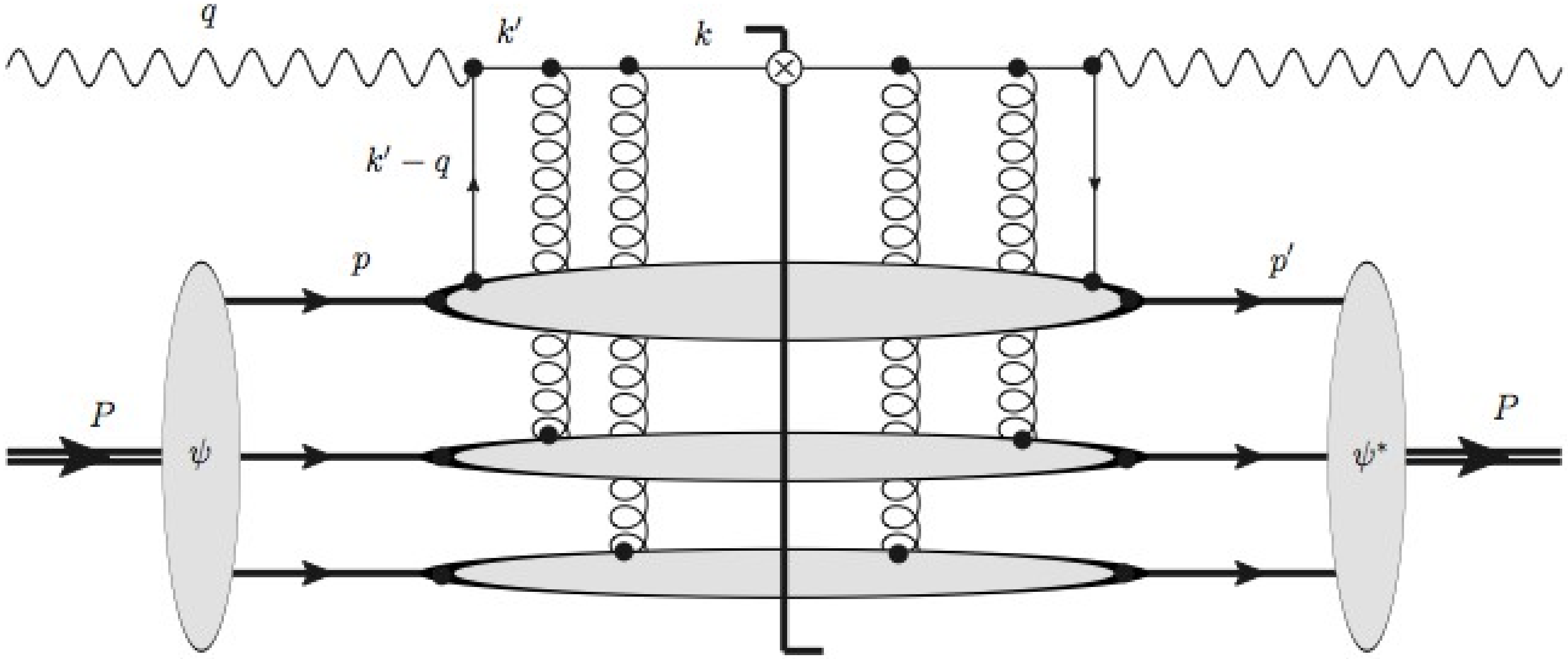
High density defines a characteristic momentum scale

$$Q_s^2(\vec{b}_\perp) \propto \alpha_s^2 T(\vec{b}_\perp) \sim \alpha_s^2 A^{1/3} \Lambda^2$$

- Defines a transverse correlation length for the gluon field
- Dynamical IR cutoff of nonperturbative physics



TMD's in the Quasi-Classical Limit



$$\langle A|\,\bar{\psi}_\beta(0)\,\mathcal{U}[0,r]\,\psi_\alpha(r)\,|A\rangle \approx$$

$$\approx \int d\Omega d\Omega' \; \Psi_N(\Omega)\Psi_N^*(\Omega')$$

$$\times \langle N(p')|\,\bar{\psi}_\beta(0)\,u[0,r]\,\psi_\alpha(r)\,|N(p)\rangle$$

$$\times \langle A-1|\,\mathcal{U}[0,r]\,|A-1\rangle$$

Light-front wave functions of the nucleons

Quark correlator of a nucleon up to $\mathcal{O}(\alpha_s)$

Gauge link calculated in the MV model

Quasi-Classical Factorization

$$\Phi_{\alpha\beta}(x, \vec{k}_\perp) = \frac{A}{(2\pi)^5} \sum_{\sigma\sigma'} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot\vec{r}_\perp}$$

Nuclear TMD's

$$\times W_{\sigma'\sigma}(p, b) [\phi^N_{\alpha\beta}(\hat{x}, \vec{k}'_\perp)]_{\sigma\sigma'} S^{[\infty^-, b^-]}_{(r_T, b_T)}$$

$$\hat{x} = \frac{P^+}{p^+} x$$

Nucleonic TMD's

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Wigner Distribution Of Nucleons

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Gauge Link (Classical)

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$$S^{[\infty^-,b^-]}_{(r_T,b_T)} = \exp\left[-\frac{1}{4}r_T^2 Q_s^2(b_T)\left(\frac{R^-(b_T)-b^-}{2R^-(b_T)}\right)\right]$$

Polarized Nucleons in an Unpolarized Nucleus

The spin of the nucleon enters through (2×2) spin density matrices.

$$W_{\sigma'\sigma} = W_{unp} \, [\mathbf{1}]_{\sigma'\sigma} + \vec{W}_{pol} \cdot [\vec{\sigma}]_{\sigma'\sigma}$$

$$W(p, b, S) = W_{unp}(p, b) + \vec{S} \cdot \vec{W}_{pol}(p, b)$$

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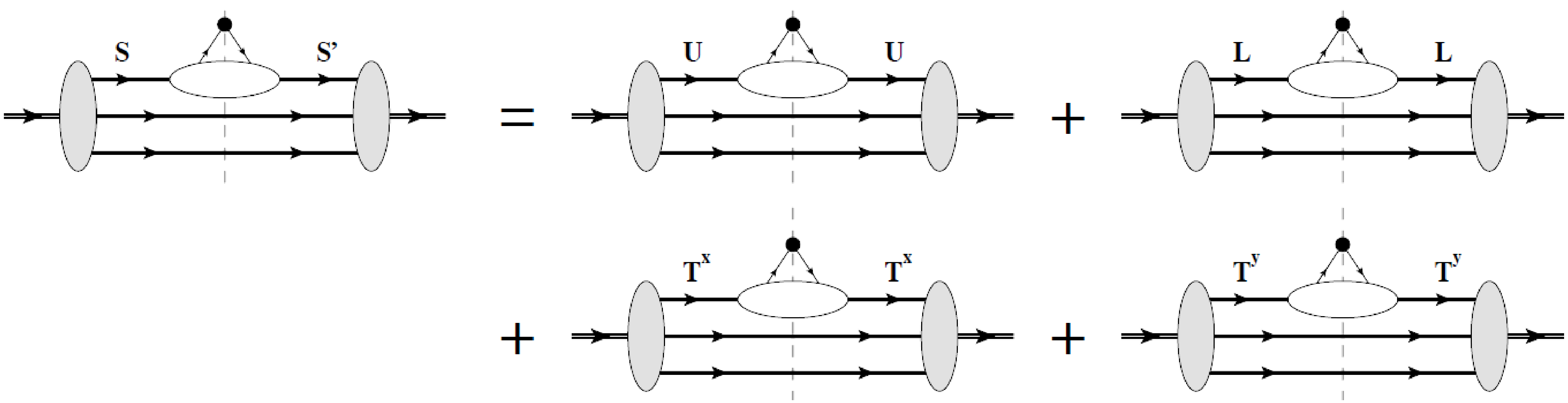
The intermediate nucleon spin can either be: unpolarized (U), longitudinally-polarized (L), or transversely-polarized (T).



Symmetries of the Nucleus

Since the Wigner distribution is built from only light-front wave functions, it has a high degree of symmetry:

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- Full 3D rotational symmetry in the rest frame (with non-relativistic motion)

$$W_{\sigma'\sigma}(\vec{p},\vec{b}) = \frac{1}{2(2\pi)^3 m} \int d^3(p-p') \, e^{+i(\vec{p}-\vec{p}')\cdot\vec{b}} \, \Psi_\sigma^N(\vec{p}^2) \, \Psi_{\sigma'}^{N*}(\vec{p}'^2)$$

$$\vec{p} = \left(\vec{p}_\perp \, , \, (Am)\left(\tfrac{p^+}{P^+} - \tfrac{1}{A}\right)\right) \qquad \vec{b} = \left(\vec{b}_\perp \, , \, -\tfrac{P^+ b^-}{Am}\right)$$

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- Gets integrated with other factors possessing 2D rotational symmetry about the beam axis

Parameterizing the Wigner Distribution

From 3D rotational invariance, parity, and time-reversal invariance:

$$W(\vec{p},\vec{b},\vec{S}) = W_{unp}[\vec{p}^{\,2},\vec{b}^{\,2},(\vec{p}\cdot\vec{b})^2] + \underline{\vec{S}\cdot(\vec{b}\times\vec{p})}\,W_{OAM}[\vec{p}^{\,2},\vec{b}^{\,2},(\vec{p}\cdot\vec{b})^2]$$

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The Wigner distribution is integrated over impact parameters with the gauge factor, which possesses 2D rotational invariance:

$$\int d^2b\, W(\vec{p}, \vec{b}, \vec{S})\, S(b_T)$$

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The maximum spin-orbit structure of an unpolarized nucleus is then:

$$W(\vec{p}, \vec{b}, \vec{S}) \Rightarrow W_{unp}[p_T^2, b_T^2; p_z^2, b_z^2] - b_z\,(\vec{p}_\perp \times \vec{S}_\perp)\, W_{OAM}[p_T^2, b_T^2; p_z^2, b_z^2]$$

Spin-Orbit Structure in the Quark Distribution

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In an unpolarized nucleus, the intermediate nucleons can only be unpolarized or transversely polarized

Spin-Orbit Structure in the Quark Distribution

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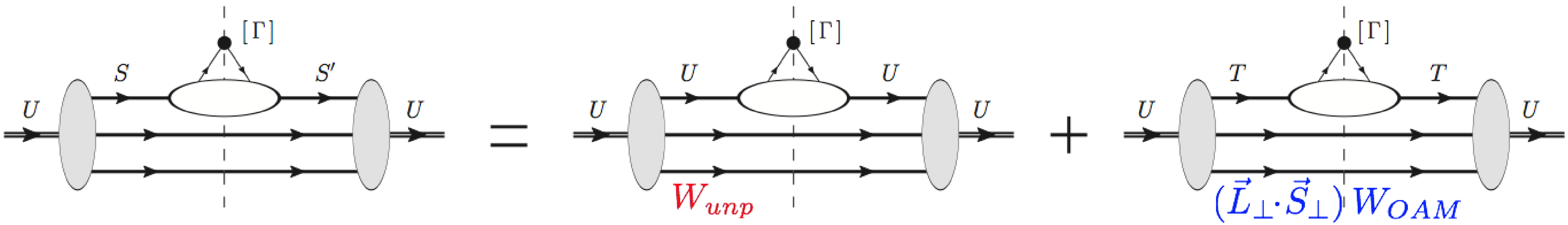

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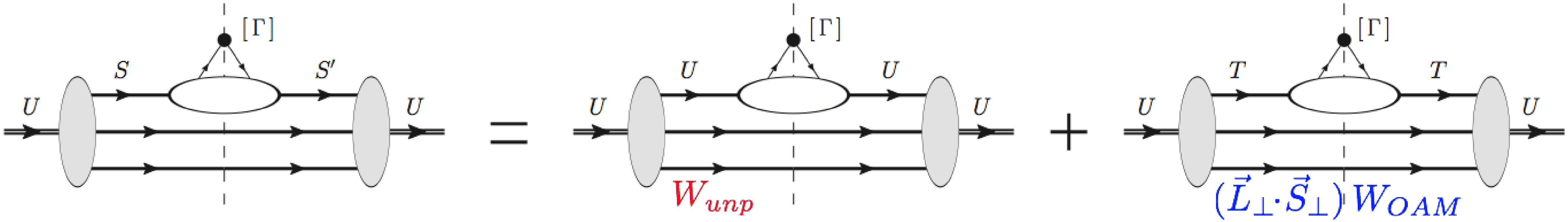
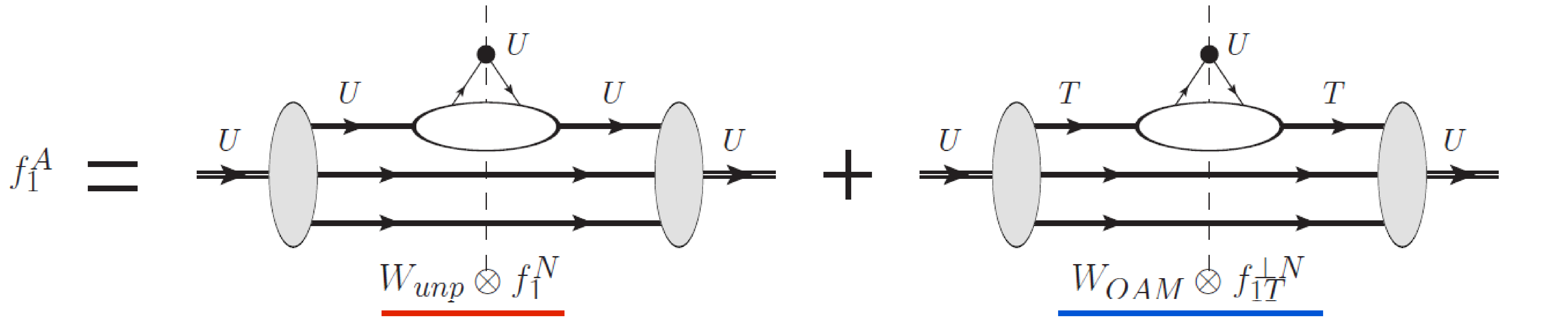


Two leading-twist quark TMD's for an unpolarized nucleus:

$$\tfrac{1}{2}\mathrm{Tr}[\Phi\,\gamma^+] = f_1^A$$ ← Unpolarized quark distribution

$$\tfrac{1}{2}\mathrm{Tr}[\Phi\,\gamma^+\gamma_\perp^j\gamma^5] = \epsilon_T^{ji}\,\frac{k_\perp^i}{Am} h_1^{\perp A}$$ ← Boer-Mulders function: (PT)-odd spin-orbit coupling

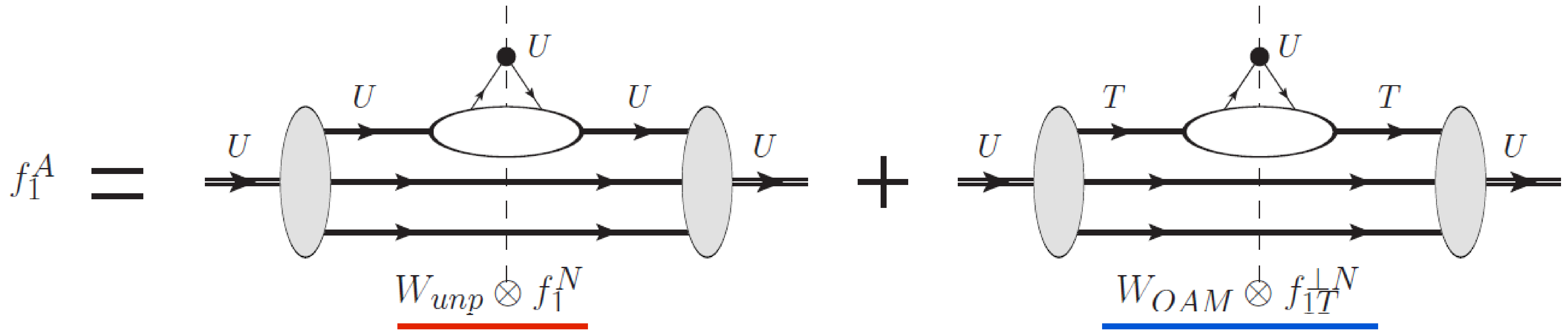
Unpolarized Quark Distribution



$$f_1^A(x, k_T) = \frac{2A}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2 r\, d^2 k'\, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot\vec{r}_\perp}\, S_{(r_T, b_T)}^{[\infty^-, b^-]}$$

$$\times \left(\underline{W_{unp}(p,b) f_1^N(\hat{x}, k'_T)} - \underline{\frac{P^+ b^-}{A m^2} (\vec{p}_\perp \cdot \vec{k}'_\perp) W_{OAM}(p,b) f_{1T}^{\perp N}(\hat{x}, k'_T)} \right)$$

Unpolarized Quark Distribution



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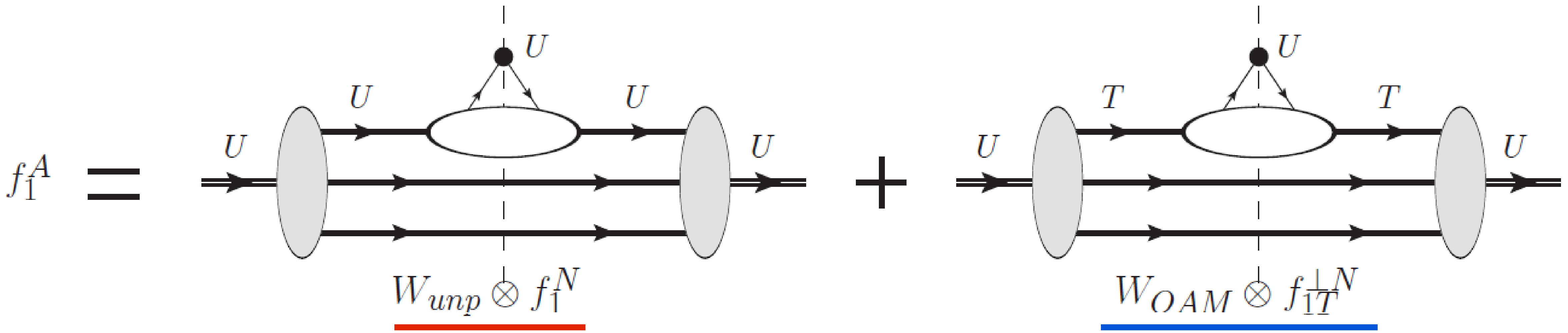
One channel builds up the unpolarized quark distribution of the nucleons:

$$f_1^N \to f_1^A \qquad (W_{unp})$$

Unpolarized Quark Distribution



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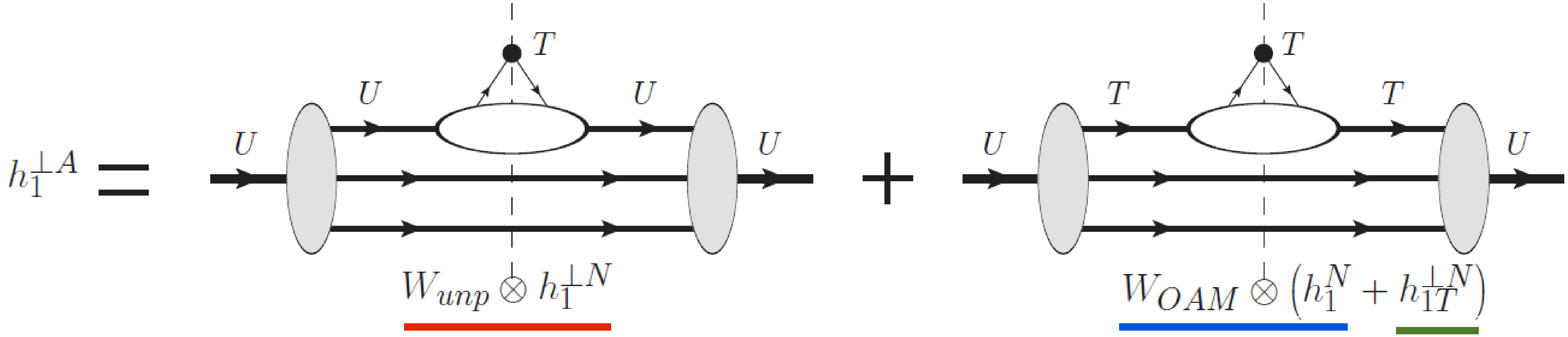
Another generates transversely polarized nucleons with OAM, and their Sivers function builds up the unpolarized quark distribution:

$$f_{1T}^{\perp N} \to f_1^A \qquad (W_{OAM})$$

Boer-Mulders Distribution

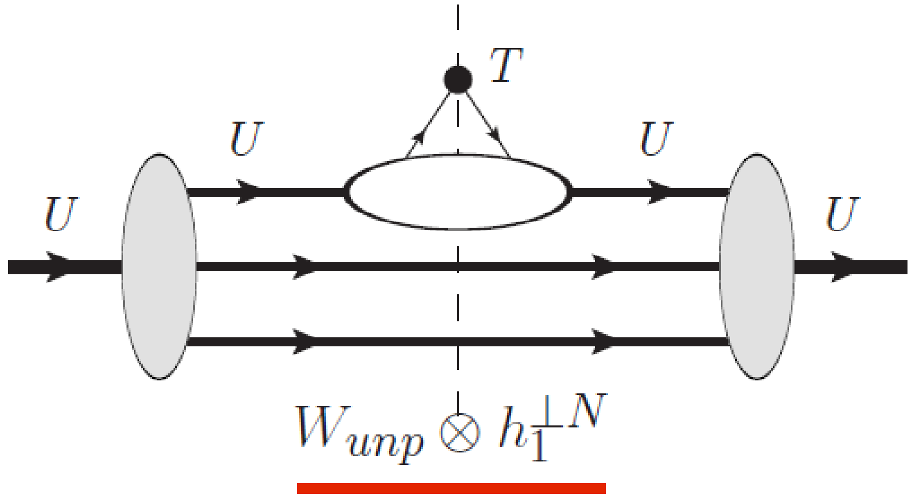
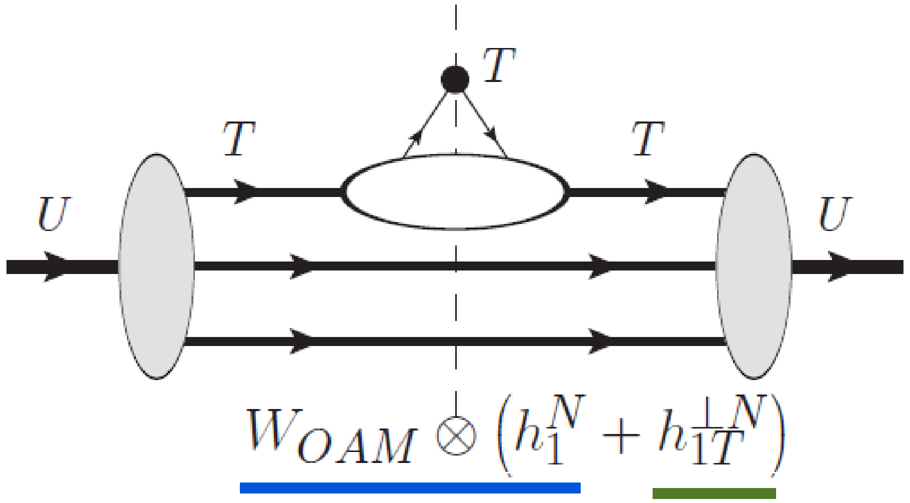


$$h_1^{\perp A}(x, k_T) = \frac{2A}{(2\pi)^5} \frac{Am}{k_T^2} \int d^{2+}p \, d^{2-}b \, d^2 r \, d^2 k' \, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot \vec{r}_\perp} \, S^{[\infty^-, b^-]}_{(r_T, b_T)}$$

$$\times \Bigg(\frac{\vec{k}_\perp \cdot \vec{k}'_\perp}{m} W_{unp}(p,b) h_1^{\perp N}(\hat{x}, k'_T) - \frac{P^+ b^-}{Am} (\vec{p}_\perp \cdot \vec{k}_\perp) W_{OAM}(p,b) h_1^N(\hat{x}, k'_T)$$

$$- \frac{P^+ b^-}{Am} \left(\frac{(\vec{p}_\perp \times \vec{k}'_\perp)(\vec{k}_\perp \times \vec{k}'_\perp)}{m^2} - \frac{k'^2_T \, (\vec{p}_\perp \cdot \vec{k}_\perp)}{2m^2} \right) W_{OAM}(p,b) h_{1T}^{\perp N}(\hat{x}, k'_T) \Bigg)$$

Boer-Mulders Distribution

$h_1^{\perp A} =$  $+$ 

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$$\times \left(\frac{\vec{k}_\perp \cdot \vec{k}'_\perp}{m} W_{unp}(p,b) h_1^{\perp N}(\hat{x}, k'_T) - \frac{P^+ b^-}{Am} (\vec{p}_\perp \cdot \vec{k}_\perp) W_{OAM}(p,b) h_1^N(\hat{x}, k'_T) \right.$$
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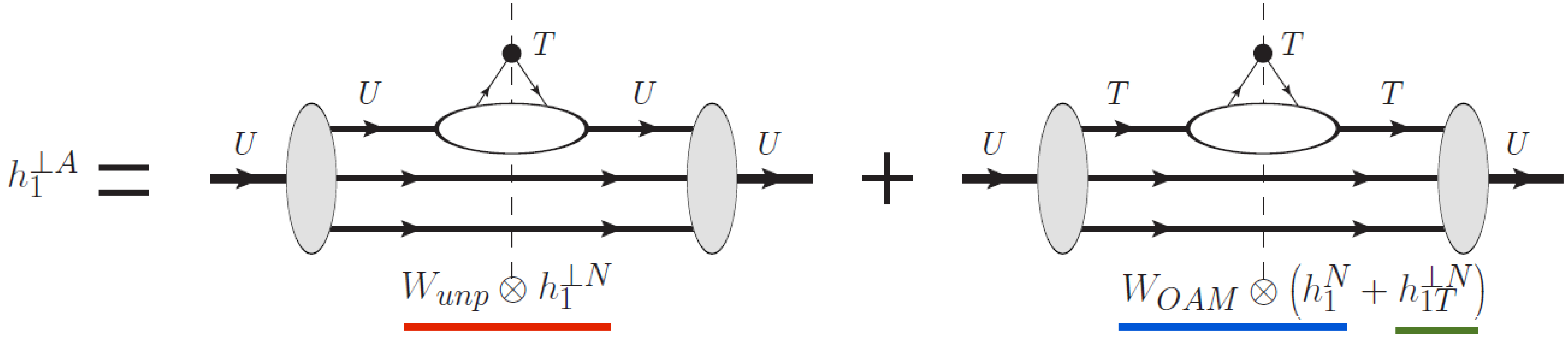
One channel builds up the Boer-Mulders function of the nucleons:

$$h_1^{\perp N} \to h_1^{\perp A} \quad (W_{unp})$$

Boer-Mulders Distribution



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Another channel generates transversely polarized nucleons with OAM, and their transversity or pretzelosity build up the Boer-Mulders function:

$$\left.\begin{array}{l} h_1^N \to h_1^{\perp A} \\ h_{1T}^{\perp N} \to h_1^{\perp A} \end{array}\right. (W_{OAM})$$

OAM and TMD Mixing

The presence of $(\vec{L} \cdot \vec{S})$ spin-orbit coupling induces nontrivial mixing between the nuclear and nucleonic TMD's.

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Liang, et. al, Phys. Rev. D77 (2008)

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PT - even (f_1^A); PT - odd ($h_1^{\perp A}$); PT - odd ($f_{1T}^{\perp N}$); PT - even (h_1^N, $h_{1T}^{\perp N}$)

PT - reversing gauge factor ($S_{(r_T, b_T)}^{[\infty^-, b^-]}$)

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PT - odd: $h_1^{\perp A} \sim (P^+ b^-) W_{OAM} \otimes S^{[\infty^-, b^-]}_{(r_T, b_T)} \otimes h_1^N$: PT - even

PT - odd: $h_1^{\perp A} \sim (P^+ b^-) W_{OAM} \otimes S^{[\infty^-, b^-]}_{(r_T, b_T)} \otimes h_{1T}^{\perp N}$: PT - even

PT - reversing gauge factor: $S^{[\infty^-, b^-]}_{(r_T, b_T)}$

Mixing depends directly on the multiple rescattering on the spectator nucleons:

- If the gauge factor is replaced by unity, the mixing vanishes:

$$\int_{-\infty^-}^{\infty^-} db^- \, b^- \, W_{OAM}\left((b^-)^2\right) = 0$$

Implications for an EIC

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- Once W_{OAM} is measured from the admixture of the Sivers function into the unpolarized quark distribution, this provides a prediction for the amount of admixture present in the nuclear Boer-Mulders function.
- In this way, measuring the mixing of TMD's provides direct access to the orbital angular momentum present in the nucleus.

Assumptions and Context

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In a similar manner, one can imagine constructing the TMD's of a dense proton from the calculated TMD's of its valence quarks. The proton should be highly relativistic and contain more structures than appeared here.

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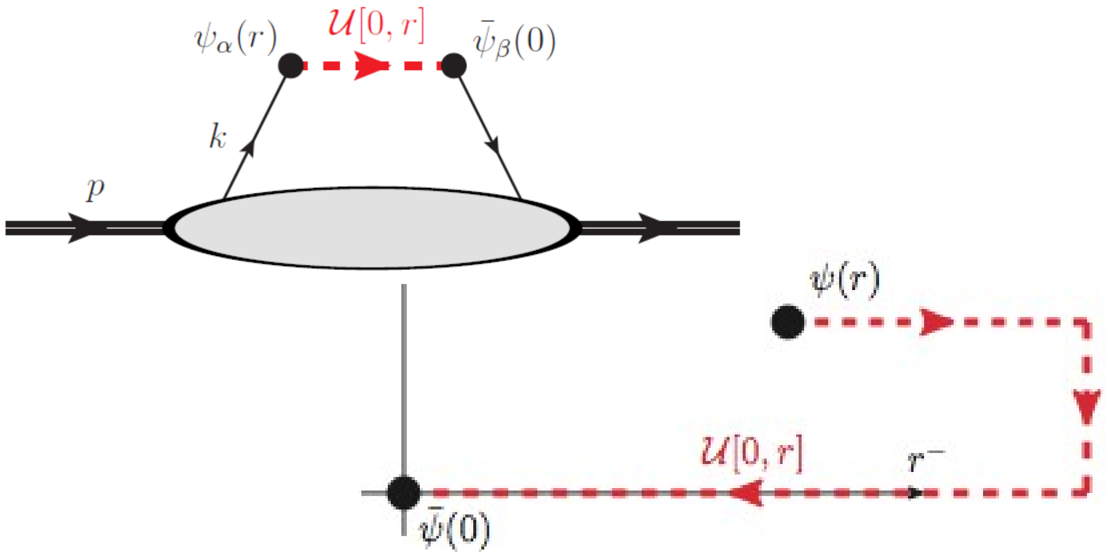
- Also apply to the "GTMD's" which generate both the TMD's and the GPD's

  ➡ The same spin-orbit couplings likely result in specific mixings in both the TMD and GPD sectors.

Summary

- The TMD quark distributions give additional insight into hadronic structure, but they are also sensitive to the gluon fields.



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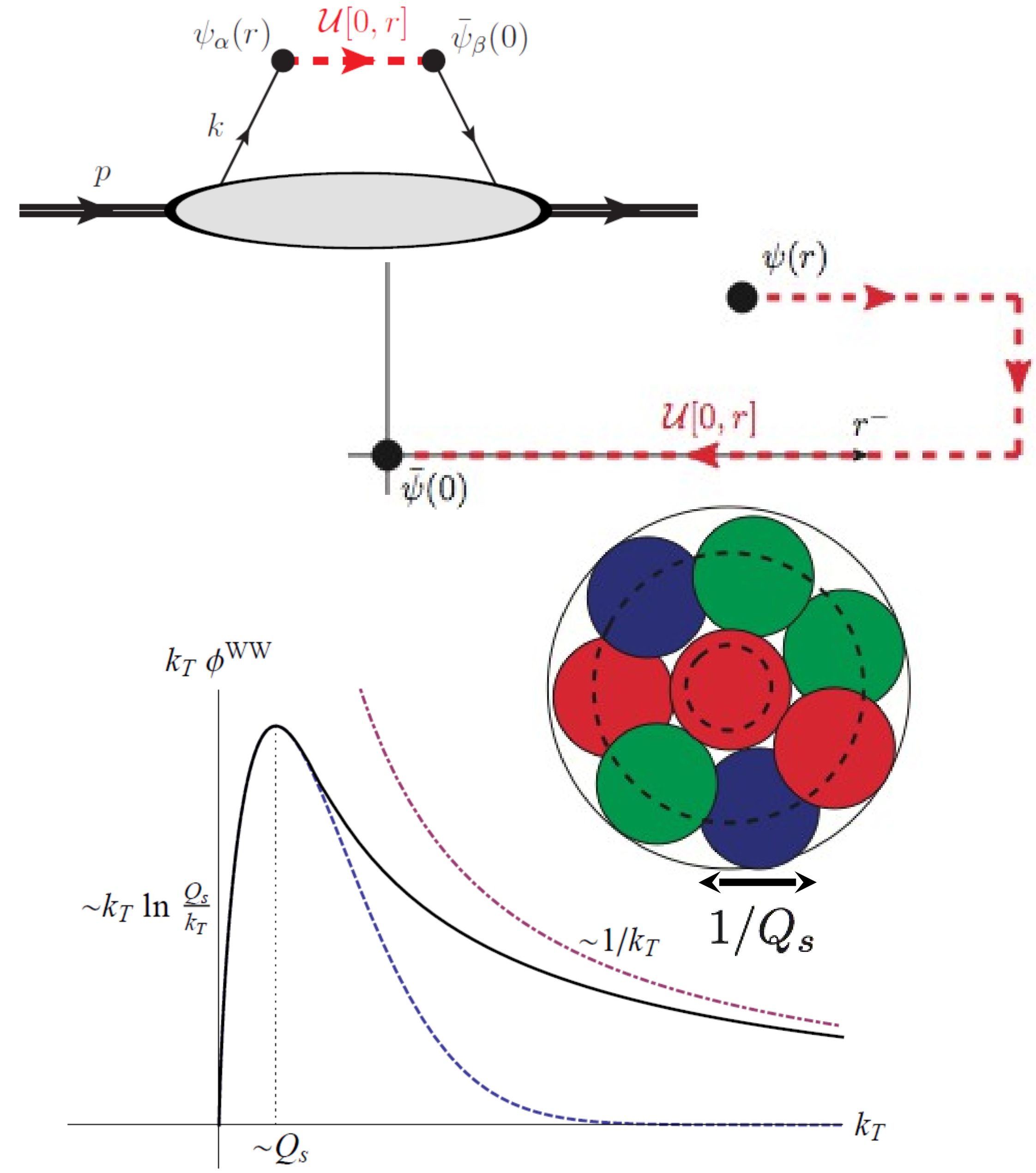

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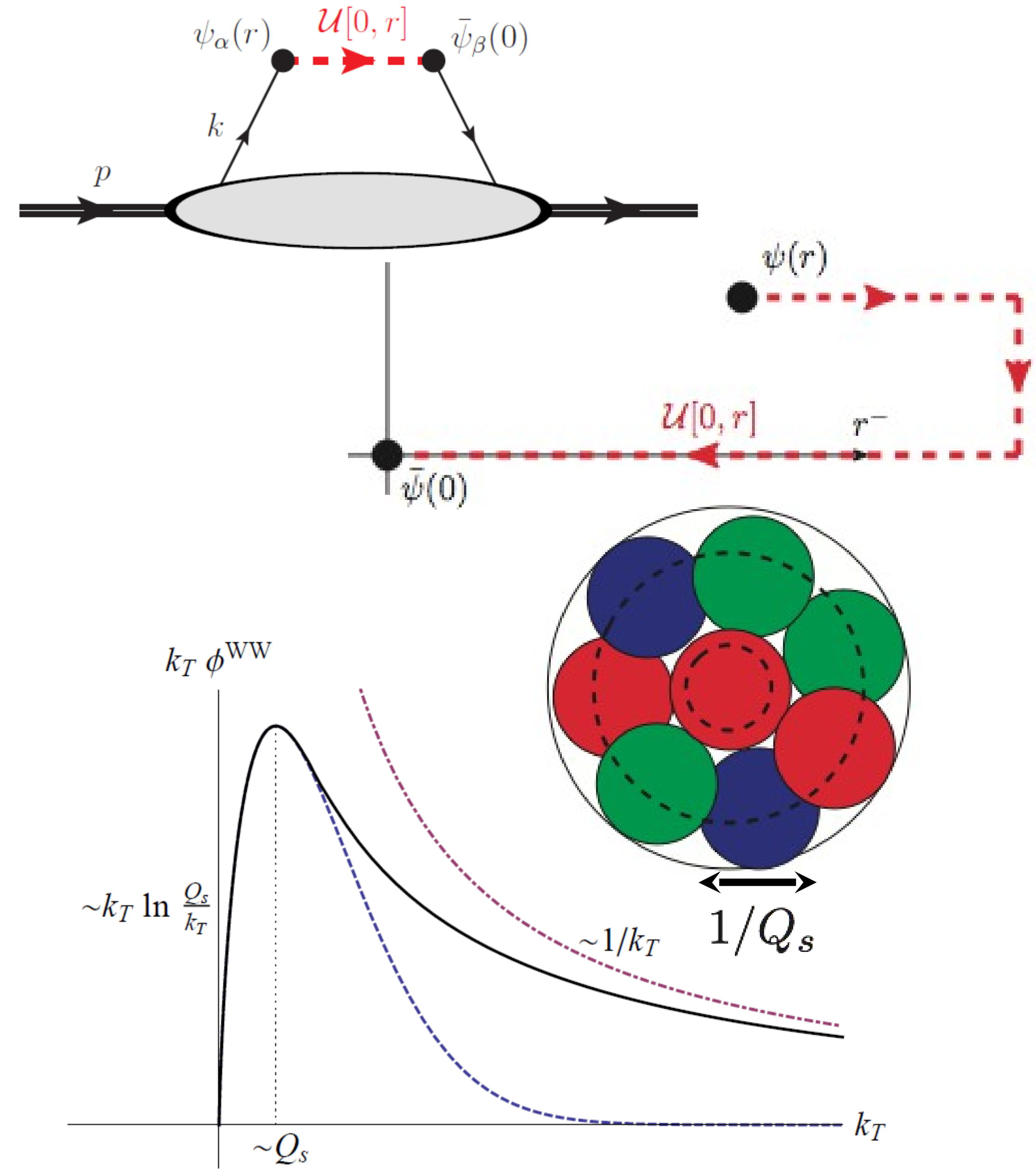

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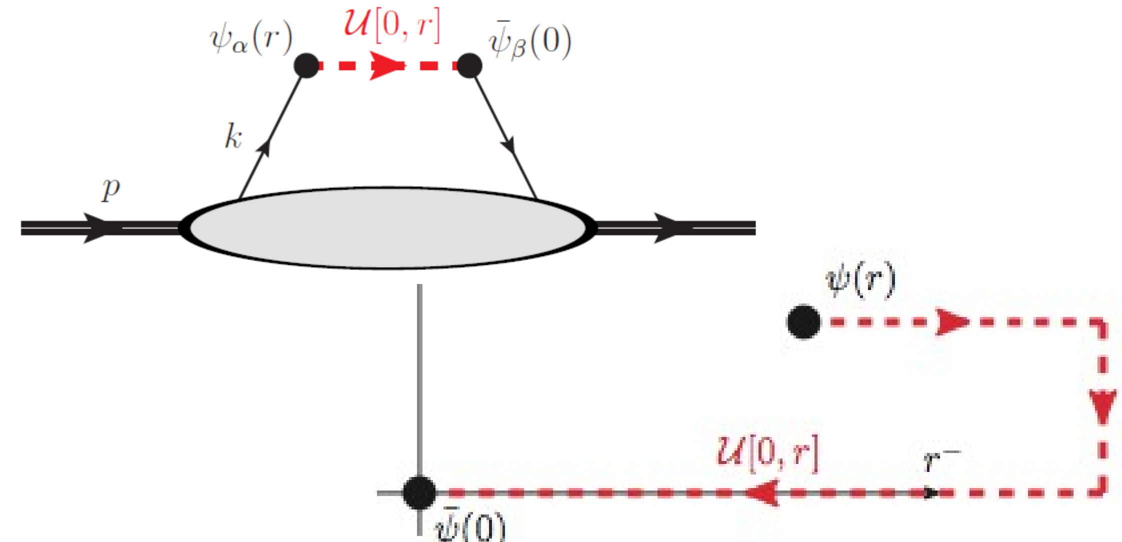

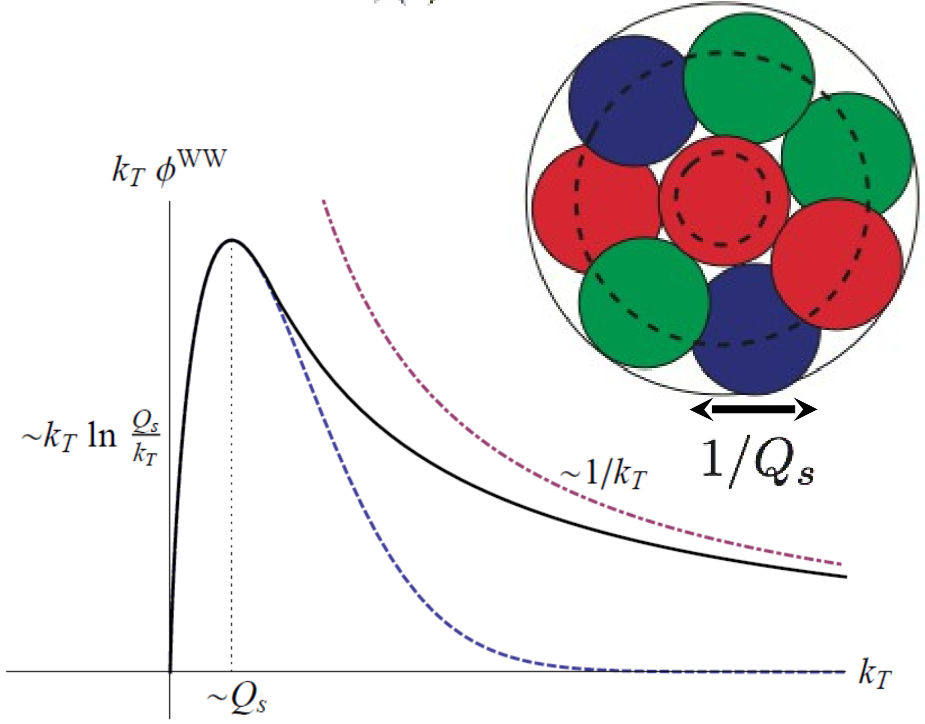

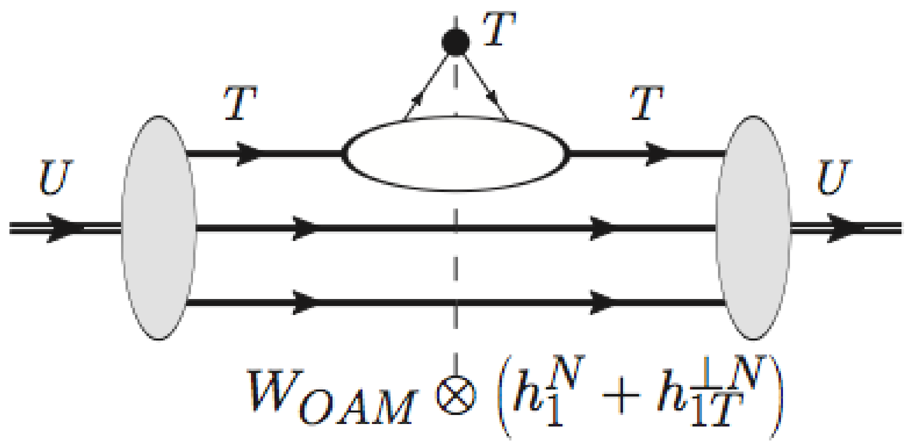

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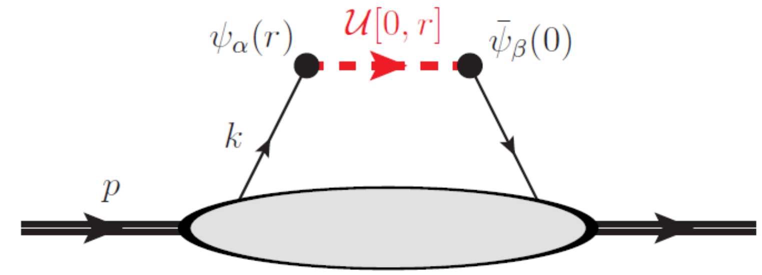
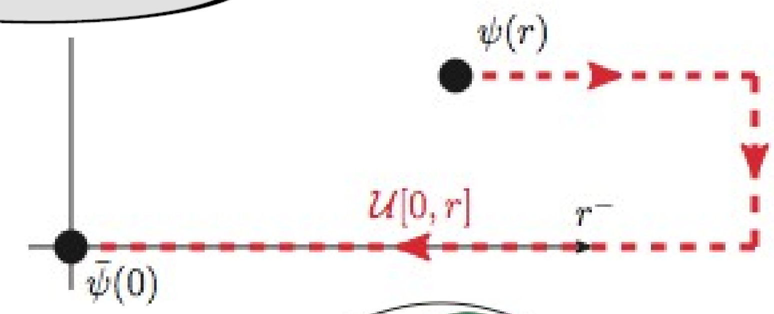
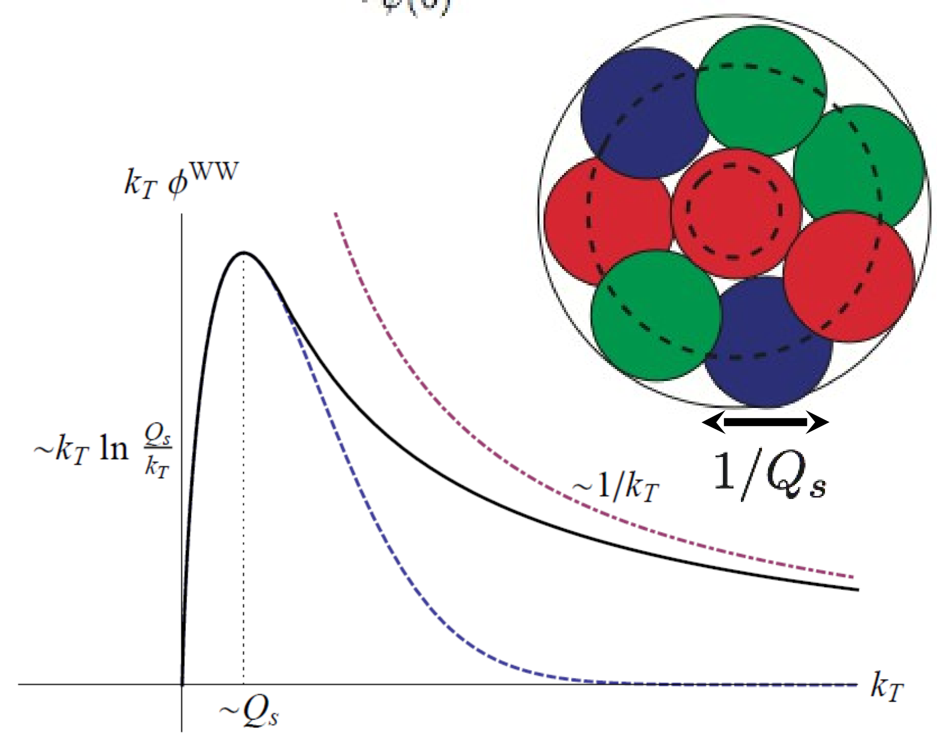
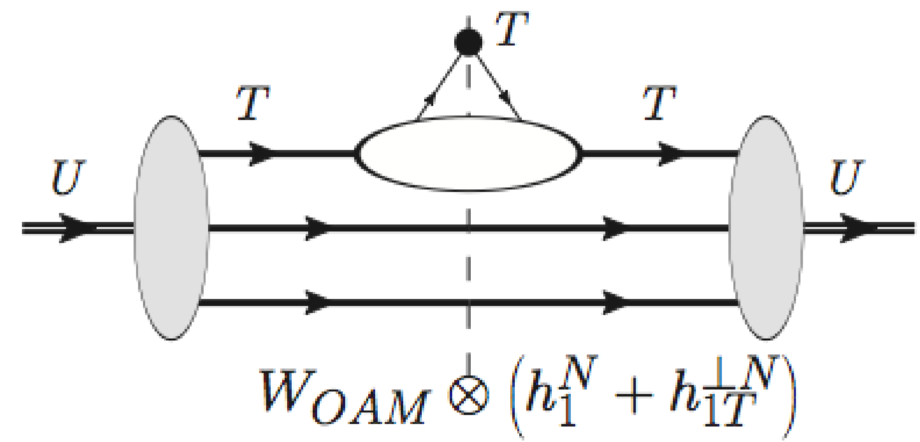
- This opens new doors to access spin-orbit structure in hadronic systems, both theoretically and experimentally.









Extra Slides

Non-Universality

The dependence on the direction of the gauge link violates universality.

- PT symmetry, for example, is an exact symmetry of the collinear PDF's.
- But for the TMD distributions, PT symmetry alters the trajectory of the gauge link from future-pointing to past-pointing.
- The TMD distributions measured in Semi-Inclusive Deep Inelastic Scattering can differ by a sign from the ones measured in the Drell-Yan process.

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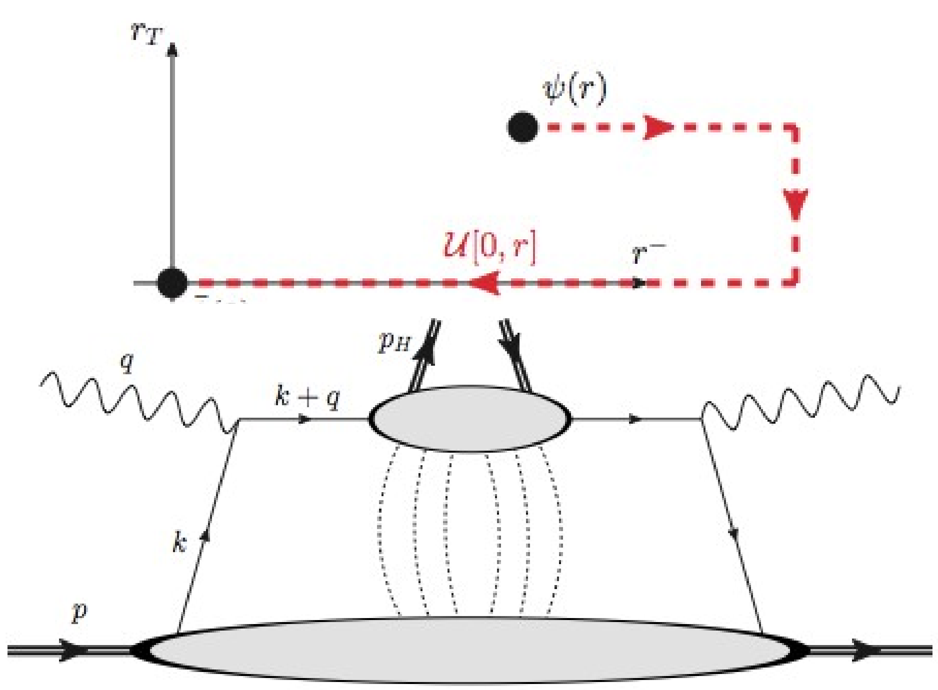
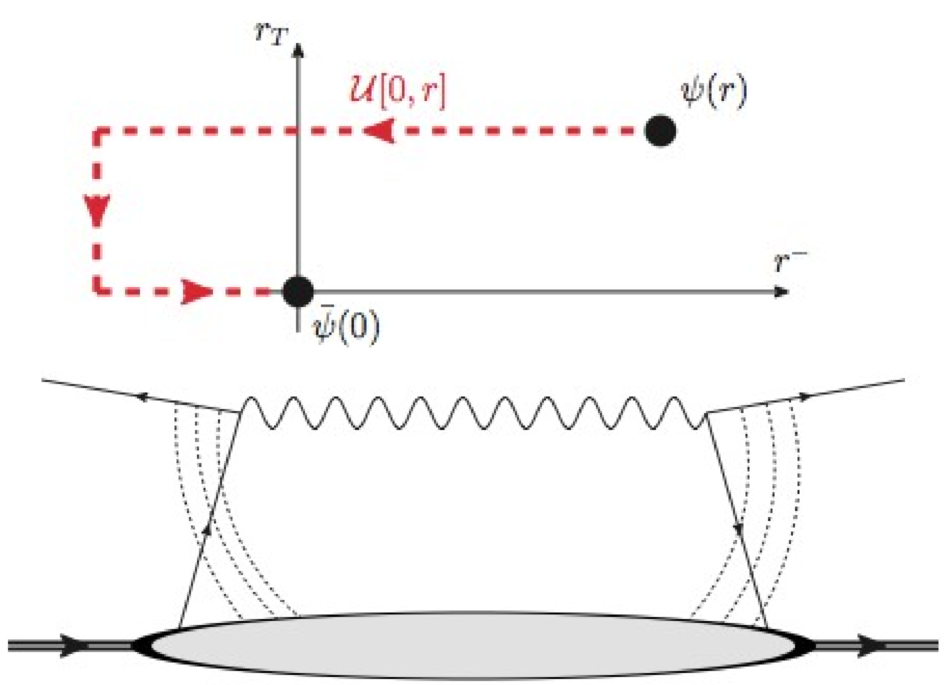
Future-Pointing

Final-State Interactions





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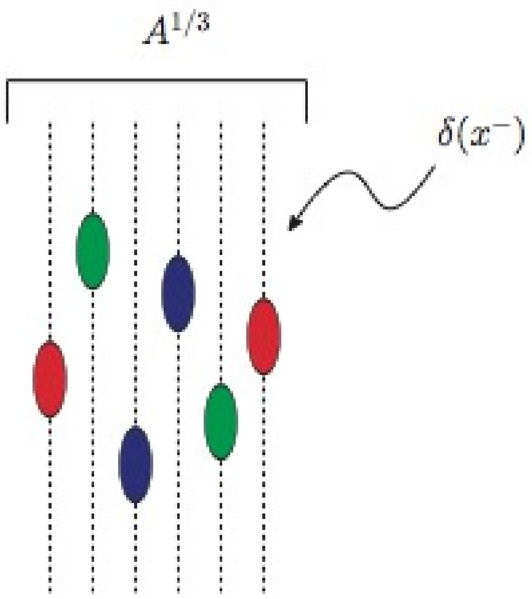
Past-Pointing

Initial-State Interactions

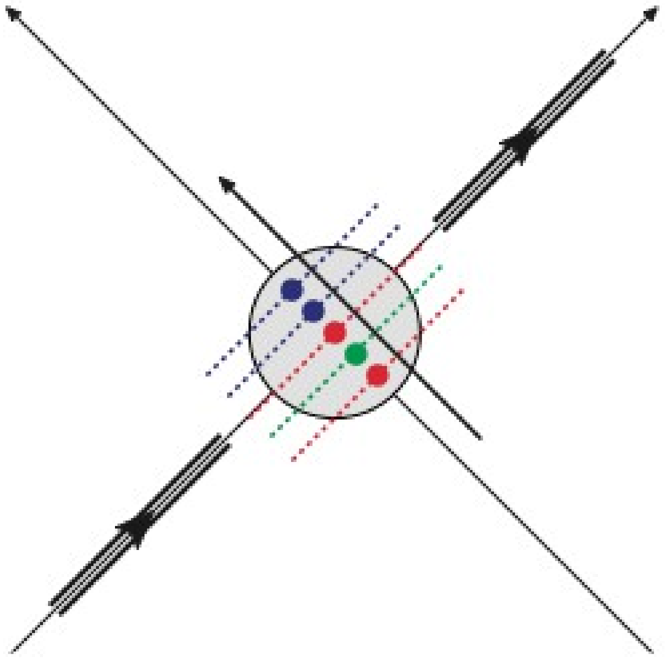
The McLerran-Venugopalan Model

- In Feynman gauge, the classical field of each nucleon is localized along the x^- axis:



$$A^{+a}(x^+, x^-, \vec{x}_\perp) = \frac{g}{2\pi} T^a \delta(x^-) \ln(x_T \Lambda)$$

- The nucleons have a small but finite separation in x^-, so each nucleon's color field is generated independently.



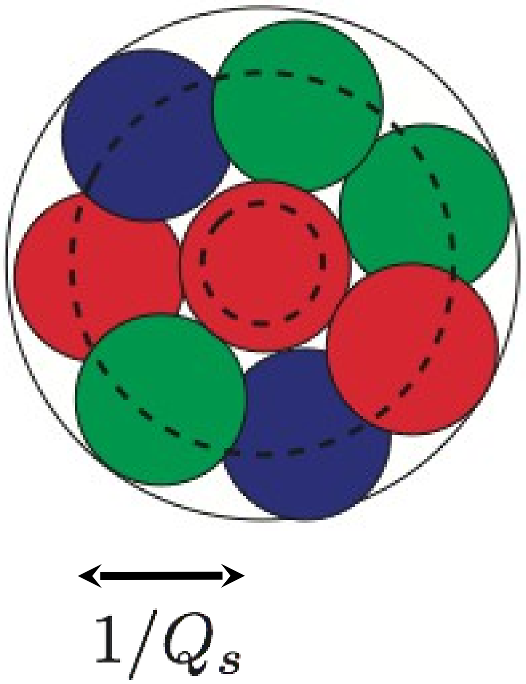
- When the projectile crosses the nucleus, it undergoes a random walk in the color space of the nucleons and in the transverse momentum the nucleon field delivers.

- The typical transverse momentum a projectile acquires from crossing the nucleus defines the saturation scale Q_s

$$Q_s^2(\vec{b}_\perp) \propto \alpha_s^2 T(\vec{b}_\perp) \sim \alpha_s^2 A^{1/3} \Lambda^2$$

Gluon Saturation

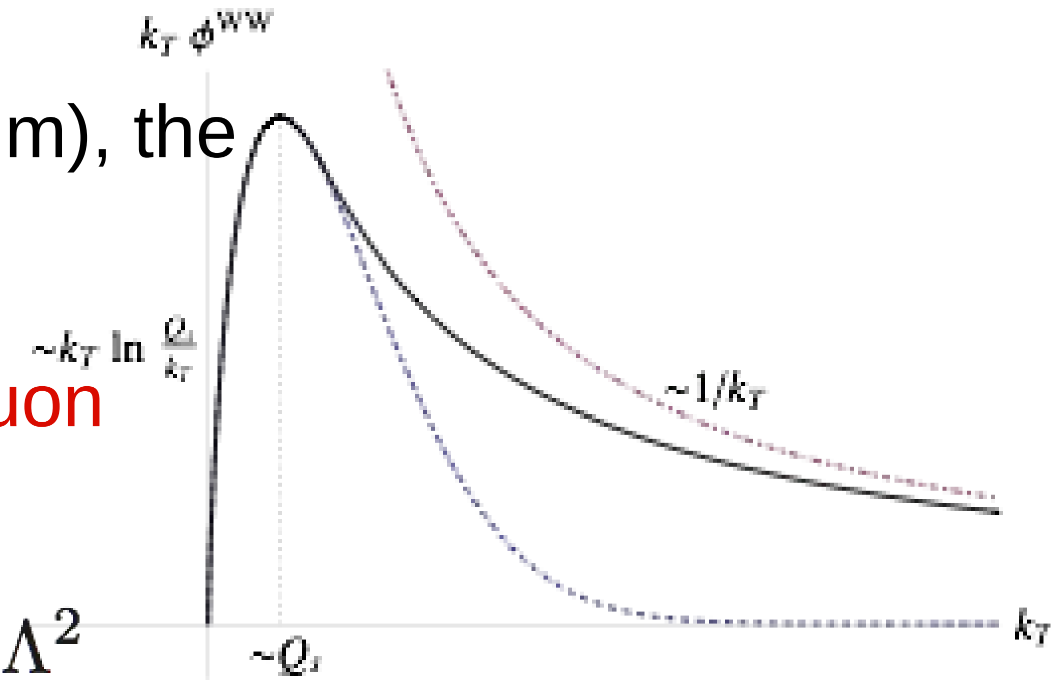
- The inverse saturation scale defines a correlation length in the transverse plane over which the color fields are correlated.



- The color fields over short distances is qualitatively different from the fields over longer distances:

➡ At short distances (large transverse momentum), the gluon field is correlated and matches the field of a single color source.

➡ Over long distances (low transverse momentum), the gluon field is uncorrelated and screened.



- The saturation scale dynamically cuts off the gluon distribution in the IR.

- If the charge density is high enough that $Q_s^2 \gg \Lambda^2$ then the process can be calculated perturbatively.

Covariant Light-Front Perturbation Theory

Light-front wave functions are quantized at fixed "light-front time" $x^+ = ct + z$

- Even though they don't depend on the collision axis, they do have a built in preferred axis of their own (z)
- These wave functions are optimized for describing high-energy states with a preferred collision axis: boost-invariant, 2D rotationally invariant, etc.
- 3D rotations are "dynamical": they couple to the interaction Hamiltonian, changing the particle content of the state and requiring an exact solution.

A proper description of rotations in the light-front formalism requires "covariant light-front perturbation theory"

Carbonell, et. al, Phys. Rept. 300 (1998)

- Keeps the quantization axis arbitrary instead of using the z axis .
- To preserve Lorentz covariance, must rotate the quantization axis as well!
- In general, relativistic LFWF depend on the direction of the quantization axis.

➡ They do not possess 3D rotational invariance in the kinematic variables....

Nucleons with Non-Relativistic Motion

But in the non-relativistic limit $c \to \infty$, the light-front quantization condition reduces down to the equal-time quantization condition: *Carbonell, et. al, Phys. Rept. 300 (1998)*

$$(ct + \vec{x}\cdot\hat{n} = const) \;\to\; (ct = const)$$

➡ Nonrelativistic LFWF are equivalent to equal-time WF, which have no dependence on the special direction $\hat{n}$.

If the nucleons move non-relativistically in the nucleus, then their WF do possess 3D rotational invariance in the nuclear rest frame!

In the non-relativistic limit:

$$W_{\sigma'\sigma}(\vec{p},\vec{b}) = \frac{1}{2(2\pi)^3 m}\int d^3(p-p')\, e^{+i(\vec{p}-\vec{p}')\cdot\vec{b}}\, \Psi^N_\sigma(\vec{p}^{\,2})\, \Psi^{N*}_{\sigma'}(\vec{p}^{\,\prime 2})$$

where the vector quantities are

$$\vec{p} = \left(\vec{p}_\perp \,,\, (Am)(\tfrac{p^+}{P^+} - \tfrac{1}{A})\right) \qquad\qquad \vec{b} = \left(\vec{b}_\perp \,,\, -\tfrac{P^+ b^-}{Am}\right)$$