Resonances in coupled-channel scattering from lattice QCD

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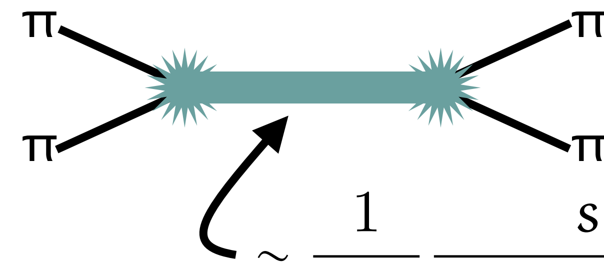
Based on work in collaboration with J.J. Dudek, R.G. Edwards and C.E. Thomas.

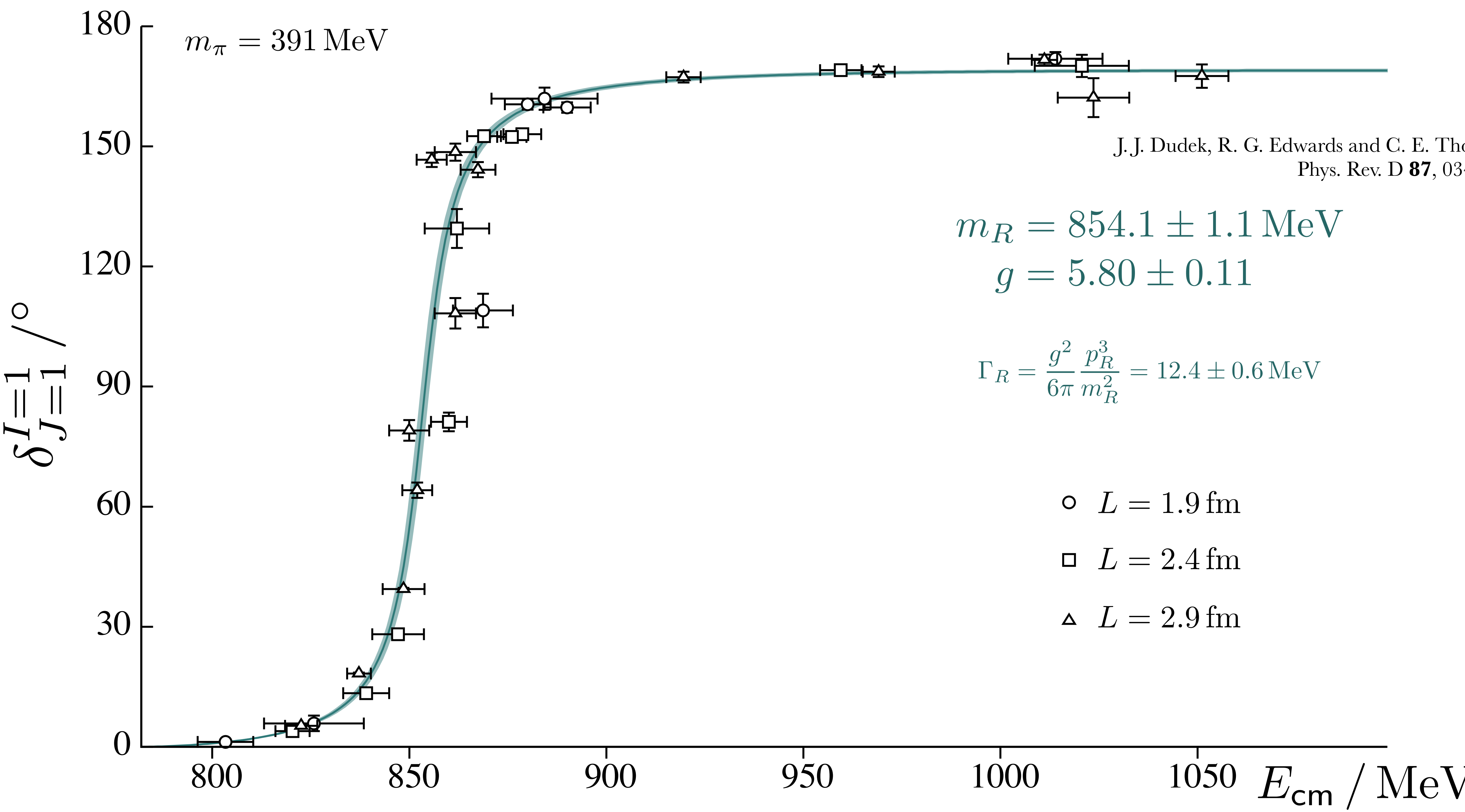
APS GHP meeting
Baltimore, MD.
10th April 2015.

Resonances from QCD



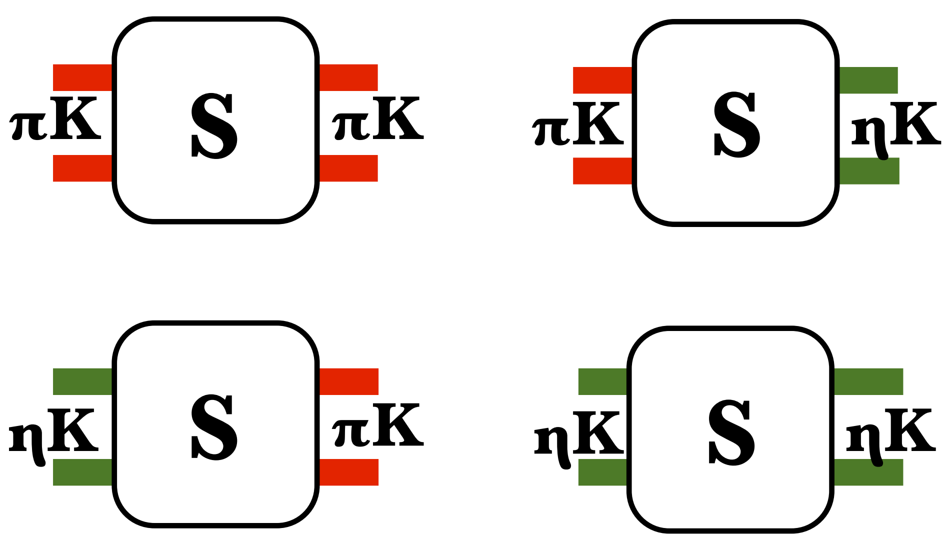
$$\sim \frac{1}{\rho(s)} \frac{s^{\frac{1}{2}}\Gamma(s)}{m_R^2 - s - i s^{\frac{1}{2}}\Gamma(s)}$$

$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k_{cm}^3}{E_{cm}^2}$$



Coupled-channel scattering

- Most physical resonances couple to multiple channels.
- To understand the physical spectrum, applying coupled-channel methods will be essential.
- We consider here **πK**, where **ηK** can also contribute in I=1/2.
- The physical amplitudes have resonances in several partial waves



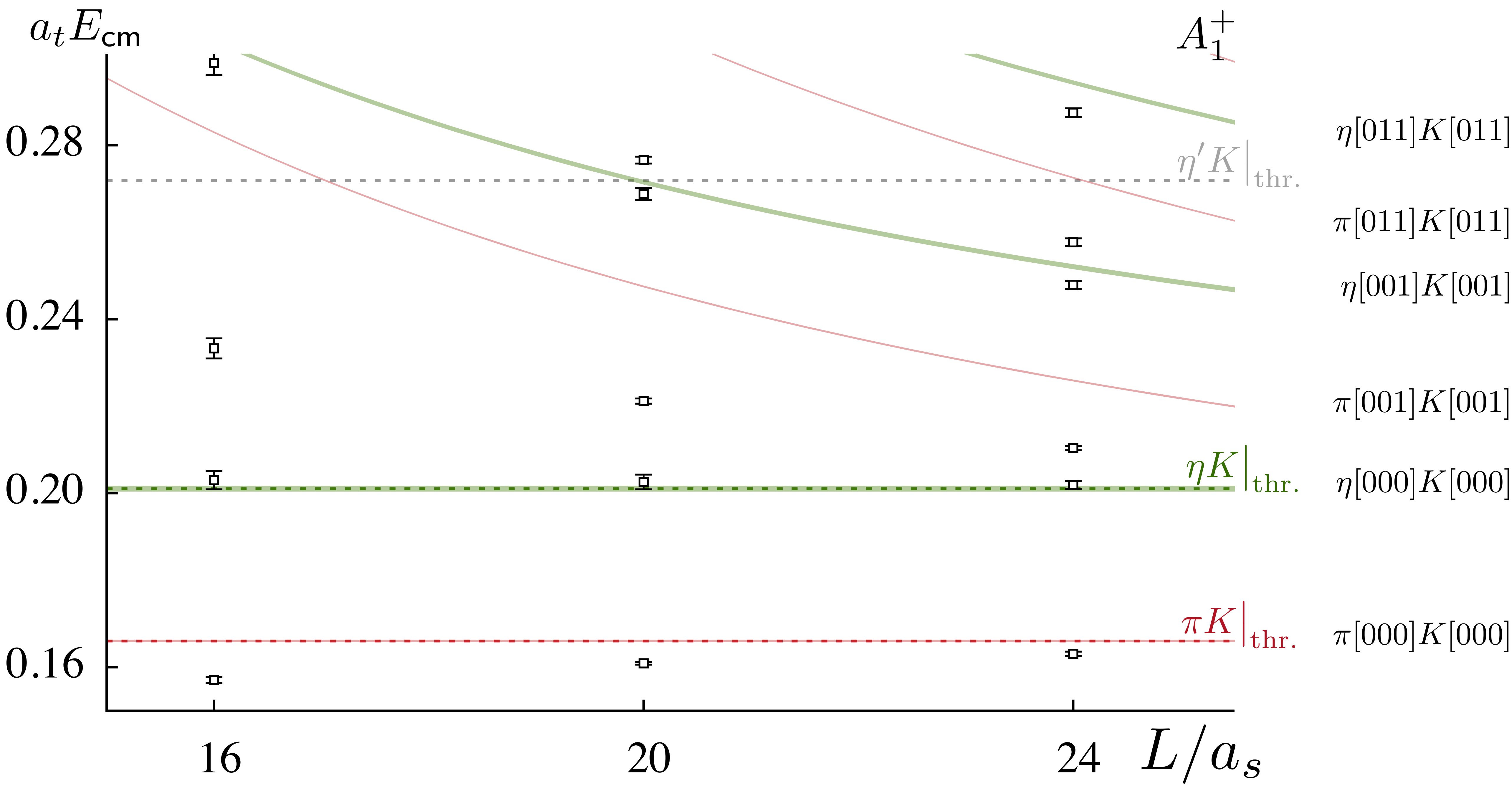
$J^P=0^+$	$\kappa(700)$, $K_0^*(1430)$, ...
$J^P=1^-$	$K^*(892)$, ...
$J^P=2^+$	$K_2^*(1430)$, ...

Aim: Obtain the scattering S-matrix from Lattice QCD

Coupled-channel scattering

Main ingredients to obtain the finite volume spectra:

- Anisotropic lattices - finer temporal spacing.
- Distillation (Peardon *et al* 2009).
- A large basis of $\bar{q}q$-like constructions.
- Pairs of optimised meson operators.
- Variational method to obtain the spectrum (Michael 1985, Lüescher & Wolff 1990).



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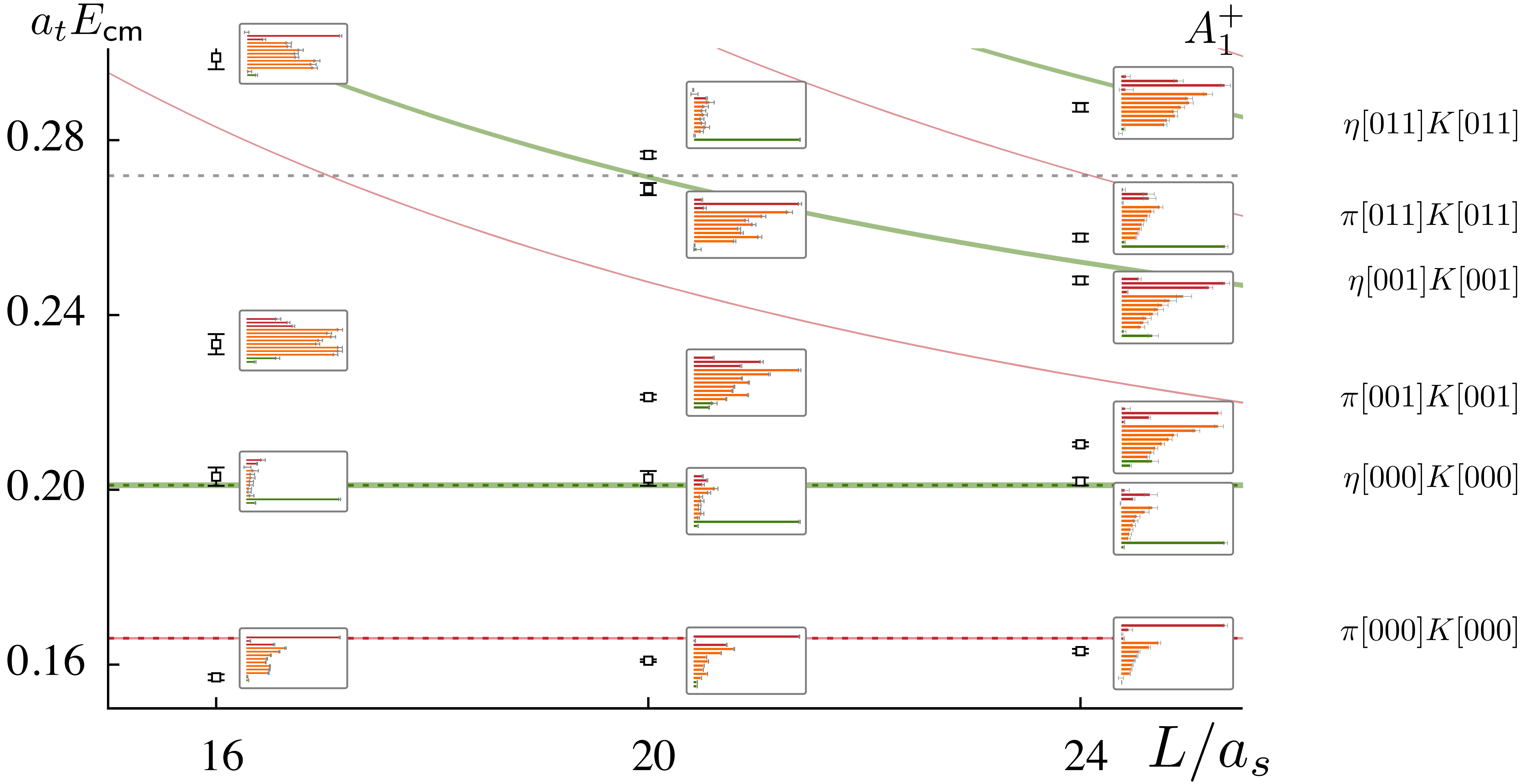

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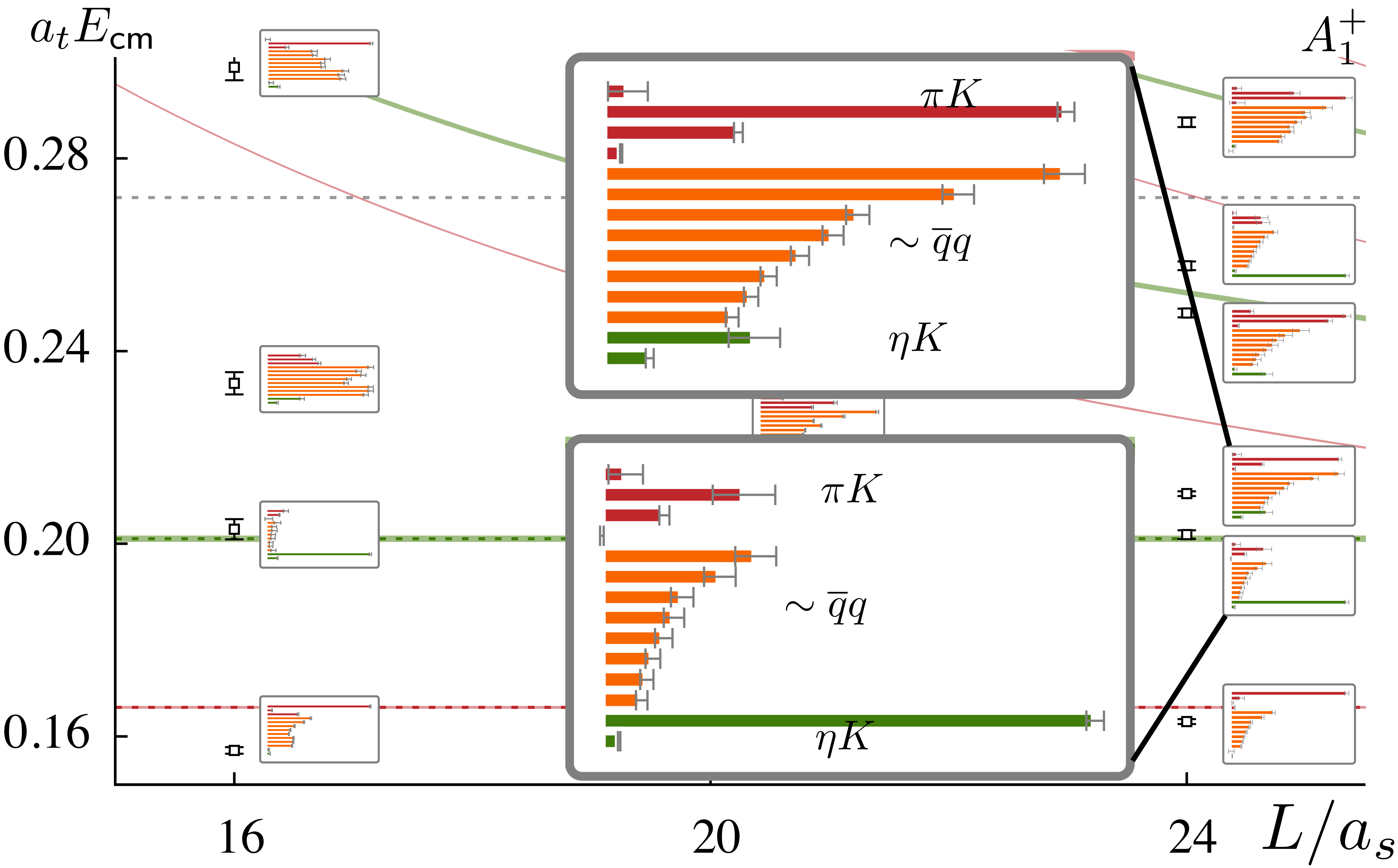

Coupled-channel extensions of Lüscher's method

$$\det\left[t_{ij}^{-1}(E) + \mathcal{M}_{ij}(E)\right] = 0$$

Many contributors:
Lüscher
Gottlieb & Rummukainen
Christ, Kim, & Yamazaki
Kim, Sachrajda & Sharpe
He, Feng & Liu
Bernard, Lage, Meissner, and Rusetsky
Leskovec & Prelovsek
Briceño & Davoudi
Hansen & Sharpe
Gockeler *et al*
Guo, Dudek, Edwards & Szczepaniak
Briceño, Davoudi, Luu
+ ...

Coupled-channel extensions of Lüscher's method

$$\det\left[t_{ij}^{-1}(E) + \mathcal{M}_{ij}(E)\right] = 0$$

infinite volume scattering
t-matrix

known finite-volume
functions

$$S = 1 + 2i\rho\, t$$

diagonal in partial waves,
mixes channels

diagonal in channels,
mixes partial waves

Coupled-channel extensions of Lüscher's method

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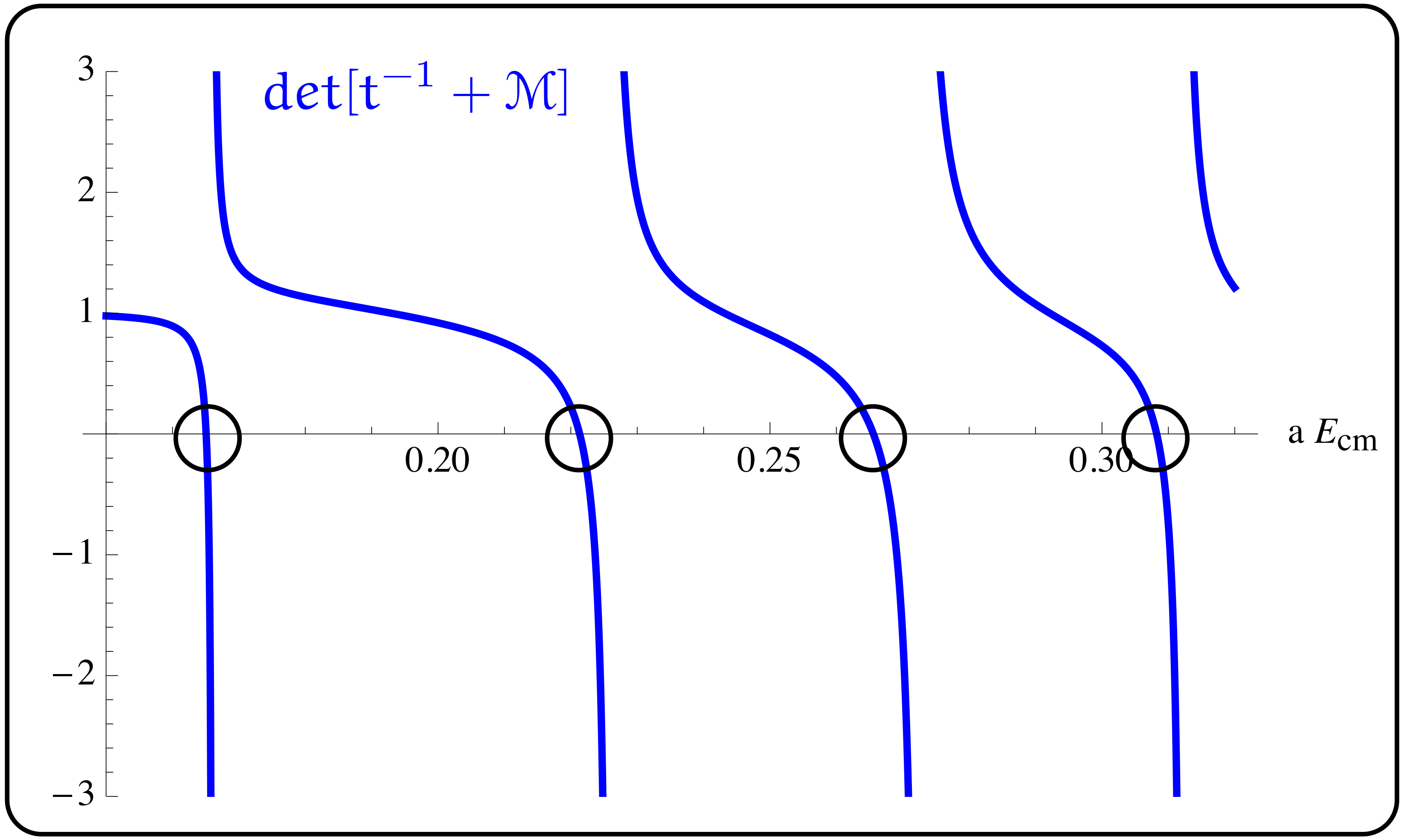


Coupled-channel scattering

Problem: Three or more unknowns for each energy level, eg:

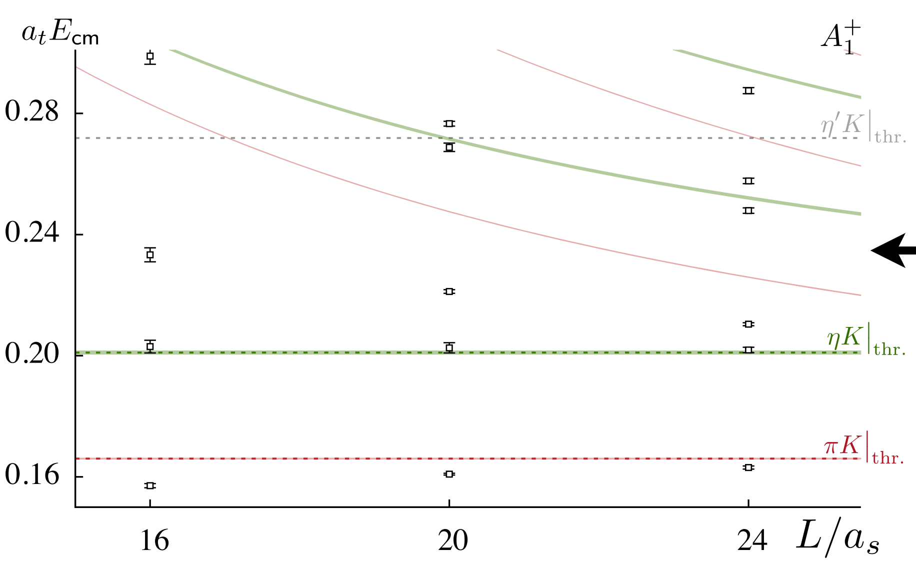
(...and even more with higher partial waves)

$$S_{11} = \eta\, e^{2i\,\delta^{\pi K}}$$

$$S_{22} = \eta\, e^{2i\,\delta^{\eta K}}$$

2x2 complex matrix (or more) but only one equation.
No one-to-one relation from energy levels to amplitudes

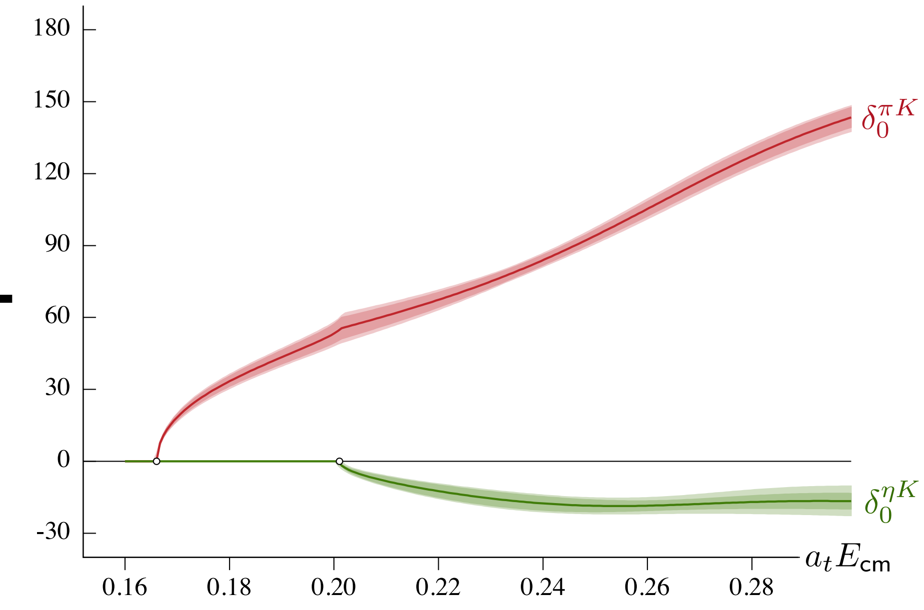
Finite volume energy levels



?

Extensions of Lüscher's method

Scattering Amplitudes



Coupled-channel scattering

Solution: Parameterise t-matrix, constrain parameters using many energy levels
E.g.: K-matrix (it's essential that we preserve unitarity)

$$S_{ij} = \delta_{ij} + 2i\left(\rho_i \rho_j\right)^{\frac{1}{2}} t_{ij}$$

$$[S^\dagger S]_{ij} = \delta_{ij}$$

$$\rightarrow \mathrm{Im}[t^{-1}]_{ij} = -\rho_i \delta_{ij}$$

- K-matrix contains everything that isn't constrained by unitarity

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) - i\delta_{ij}\rho_i(s)$$

- K must be real for real s. One option for two channel scattering:

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K}\end{bmatrix} + \begin{bmatrix}\gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K}\end{bmatrix}$$

- m, g, γ are real free parameters. Simple to add more - more poles, or a polynomial in s.
- Simple to generalise to scattering with non-zero angular momentum.
- Can improve model by adding extra physically motivated properties - eg: Chew-Mandelstam phase space.

Coupled-channel scattering

- Describe t-matrix using K-matrix in S-wave only → obtain a spectrum.
- Minimise a χ^2 to obtain the best agreement between the K-matrix and lattice energies.

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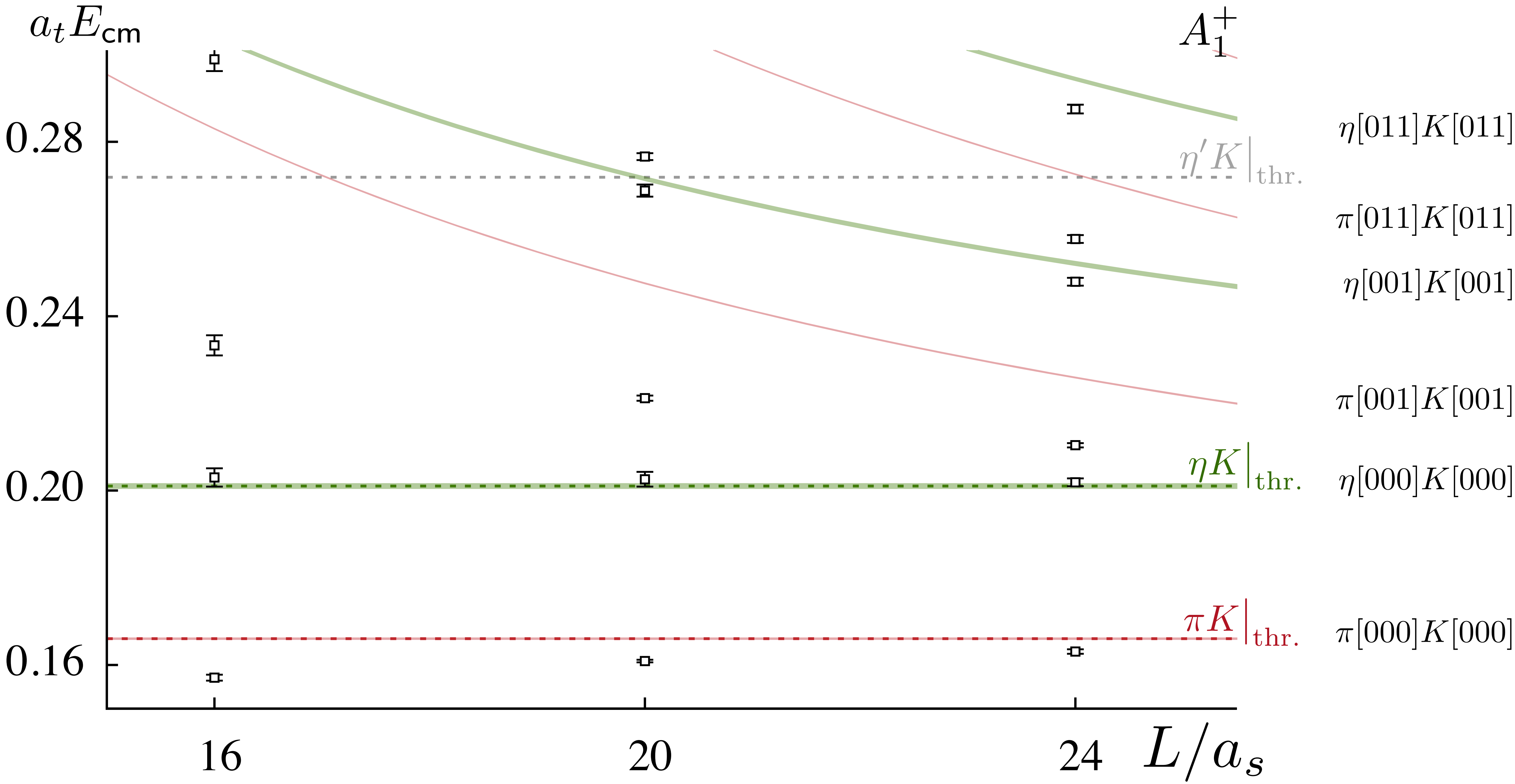

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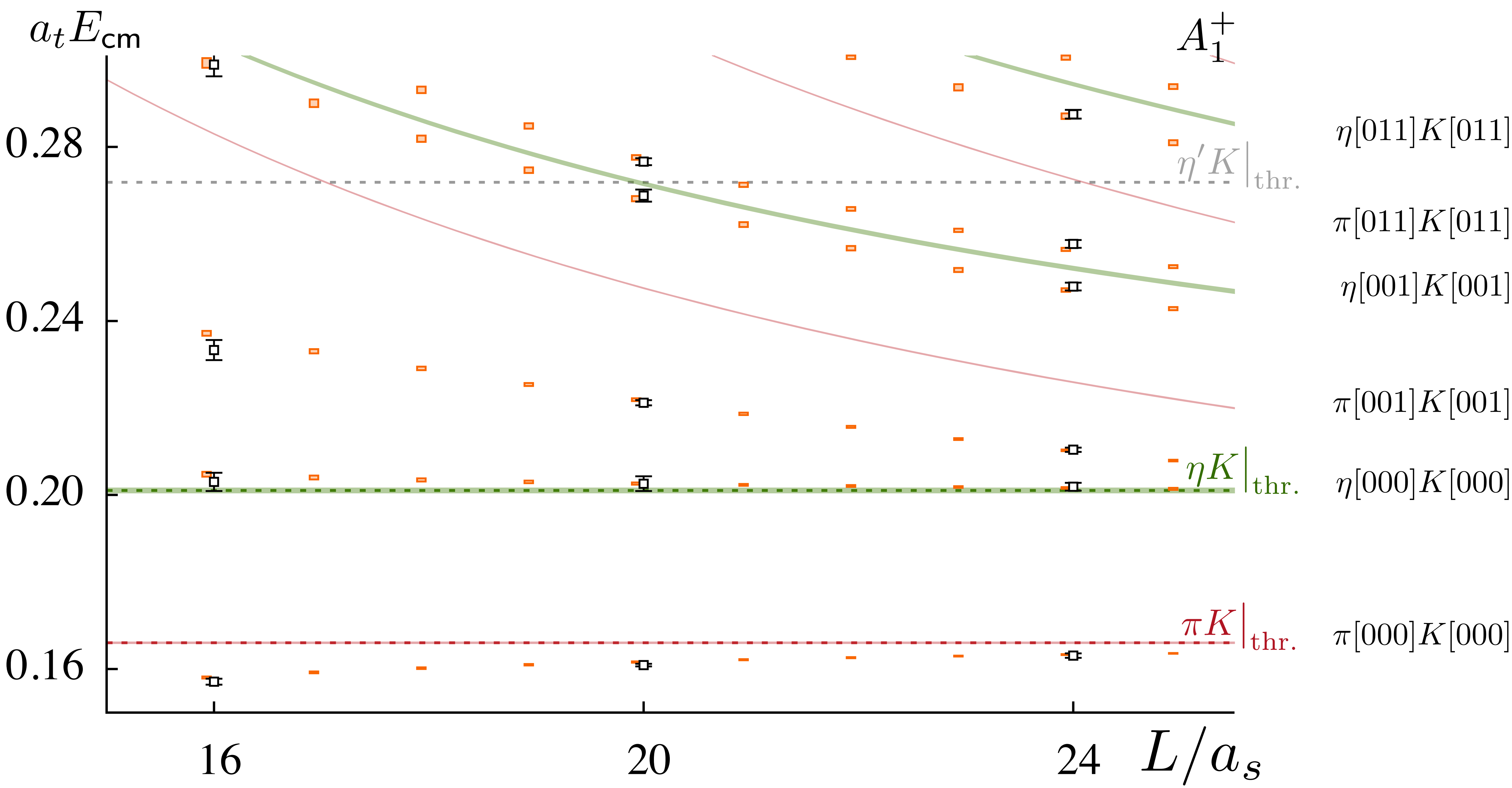
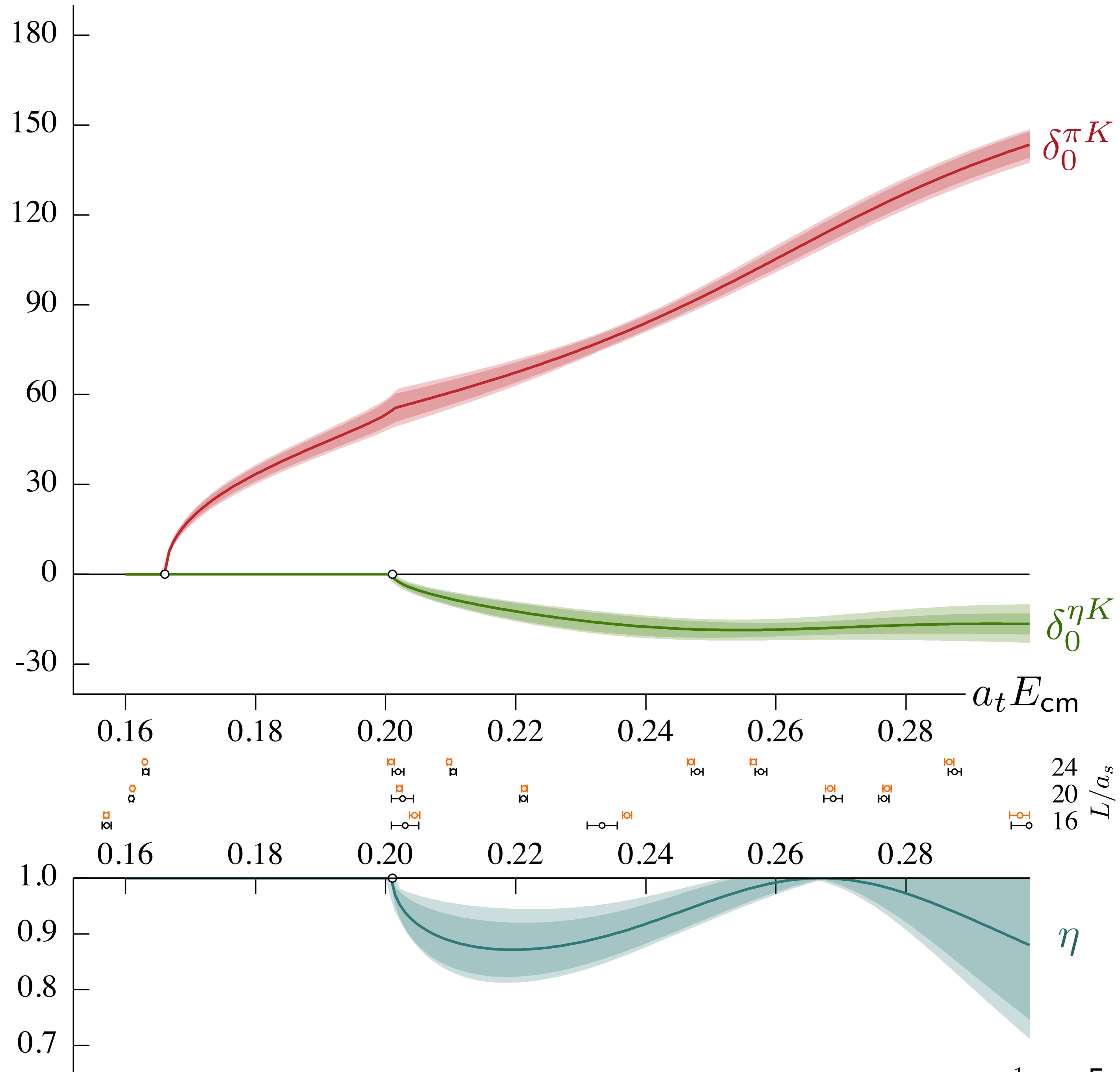
$$\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71$$



S-wave amplitudes



$$S_{11} = \eta\, e^{2i\,\delta^{\pi K}}$$

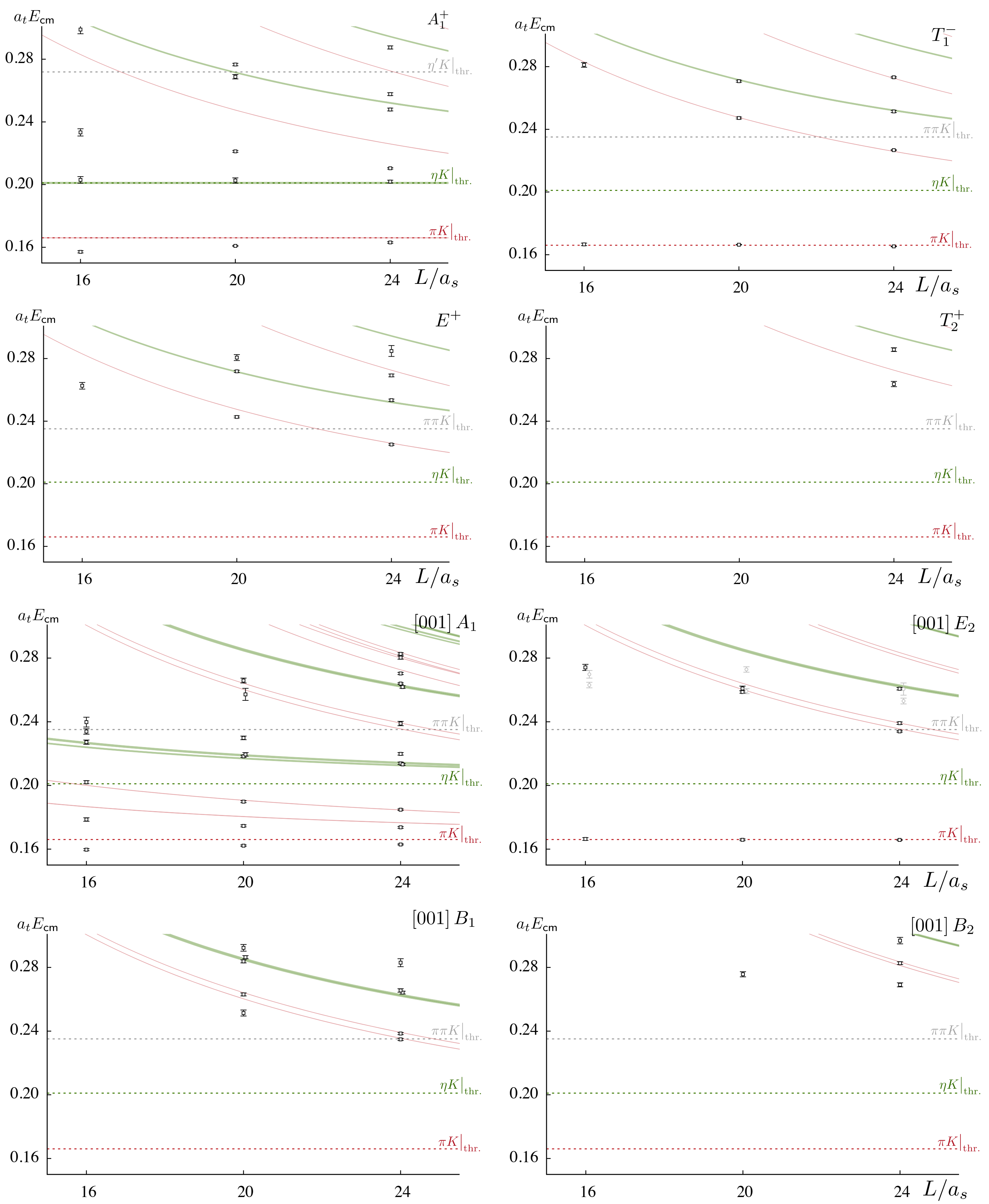
$$S_{22} = \eta\, e^{2i\,\delta^{\eta K}}$$

- Broad resonance in S-wave πK.
- ηK coupling is small.
- 3 subthreshold points, naturally included in an energy-level fit.

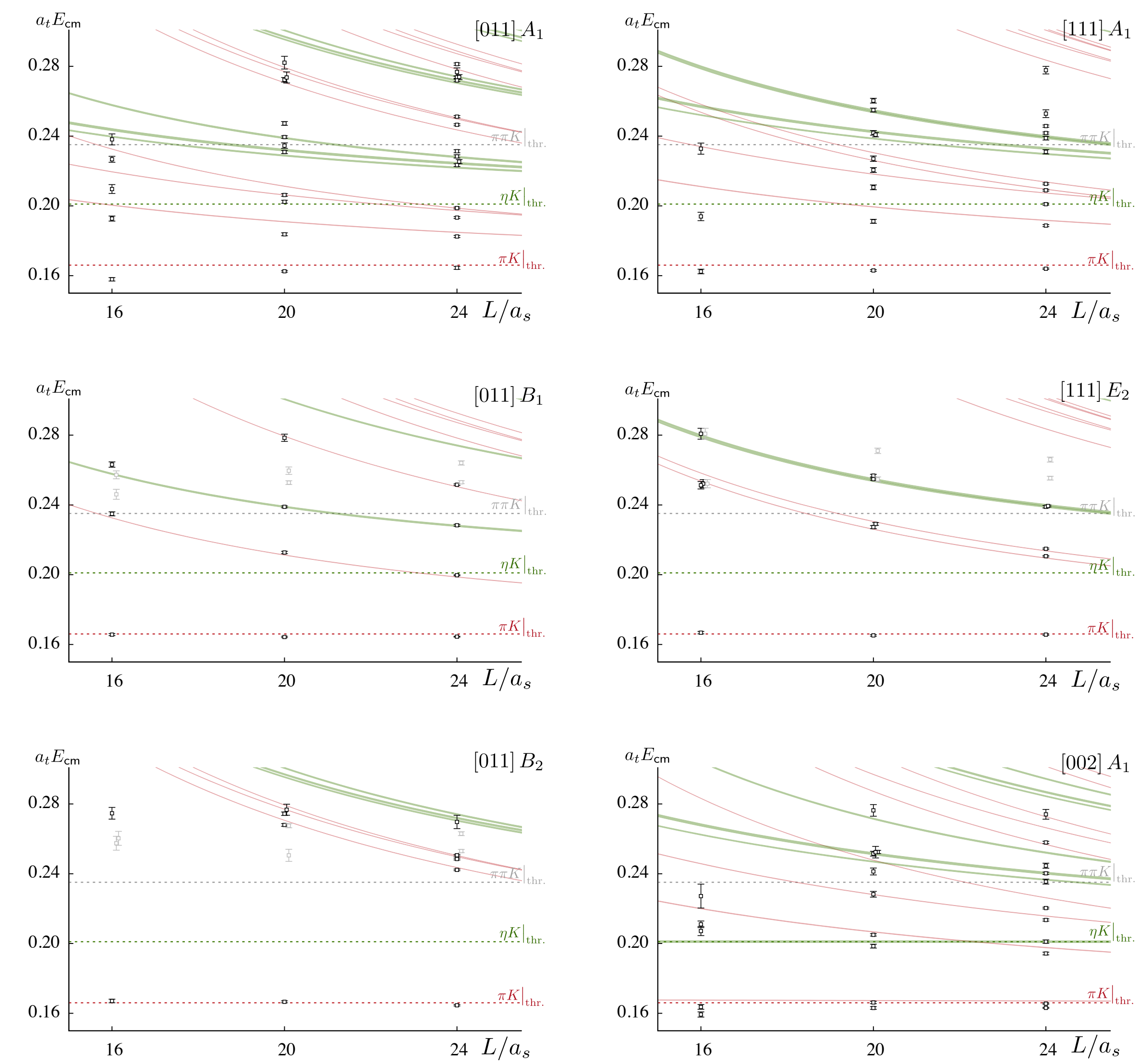
$$\begin{aligned}
m &= (0.2466 \pm 0.0020 \pm 0.0009) \cdot a_t^{-1} \\
g_{\pi K} &= (0.165 \pm 0.006 \pm 0.002) \cdot a_t^{-1} \\
g_{\eta K} &= (0.033 \pm 0.010 \pm 0.003) \cdot a_t^{-1} \\
\gamma_{\pi K,\pi K} &= 0.184 \pm 0.054 \pm 0.030 \\
\gamma_{\pi K,\eta K} &= -0.52 \pm 0.20 \pm 0.06 \\
\gamma_{\eta K,\eta K} &= -0.37 \pm 0.07 \pm 0.05
\end{aligned}
\quad
\begin{bmatrix}
1 & 0.35 & -0.38 & 0.17 & 0.27 & -0.19 \\
 & 1 & -0.05 & -0.16 & 0.85 & 0.08 \\
 & & 1 & 0.26 & -0.11 & 0.64 \\
 & & & 1 & 0.10 & 0.25 \\
 & & & & 1 & 0.05 \\
 & & & & & 1
\end{bmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71\,.$$

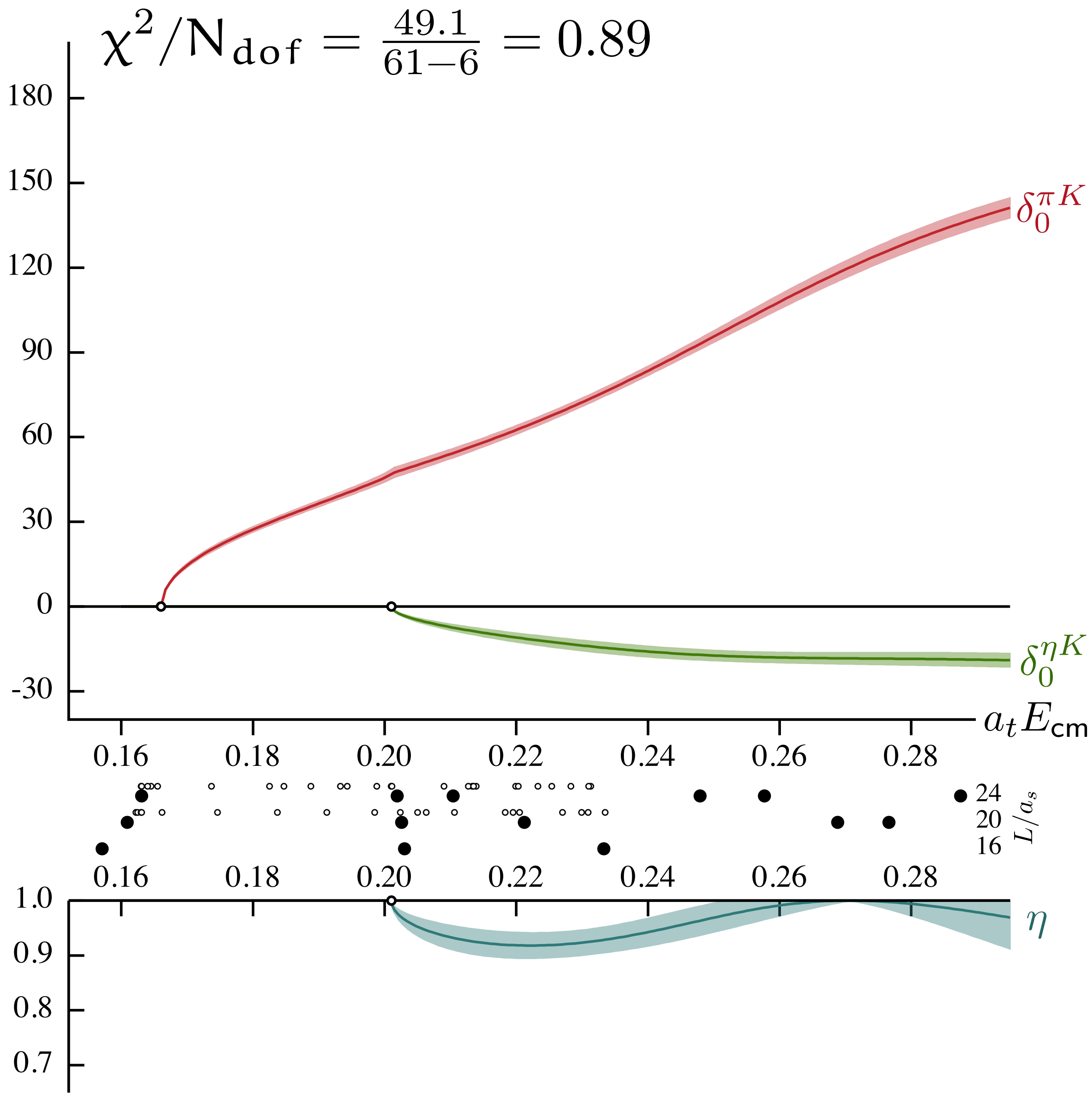
More energy levels

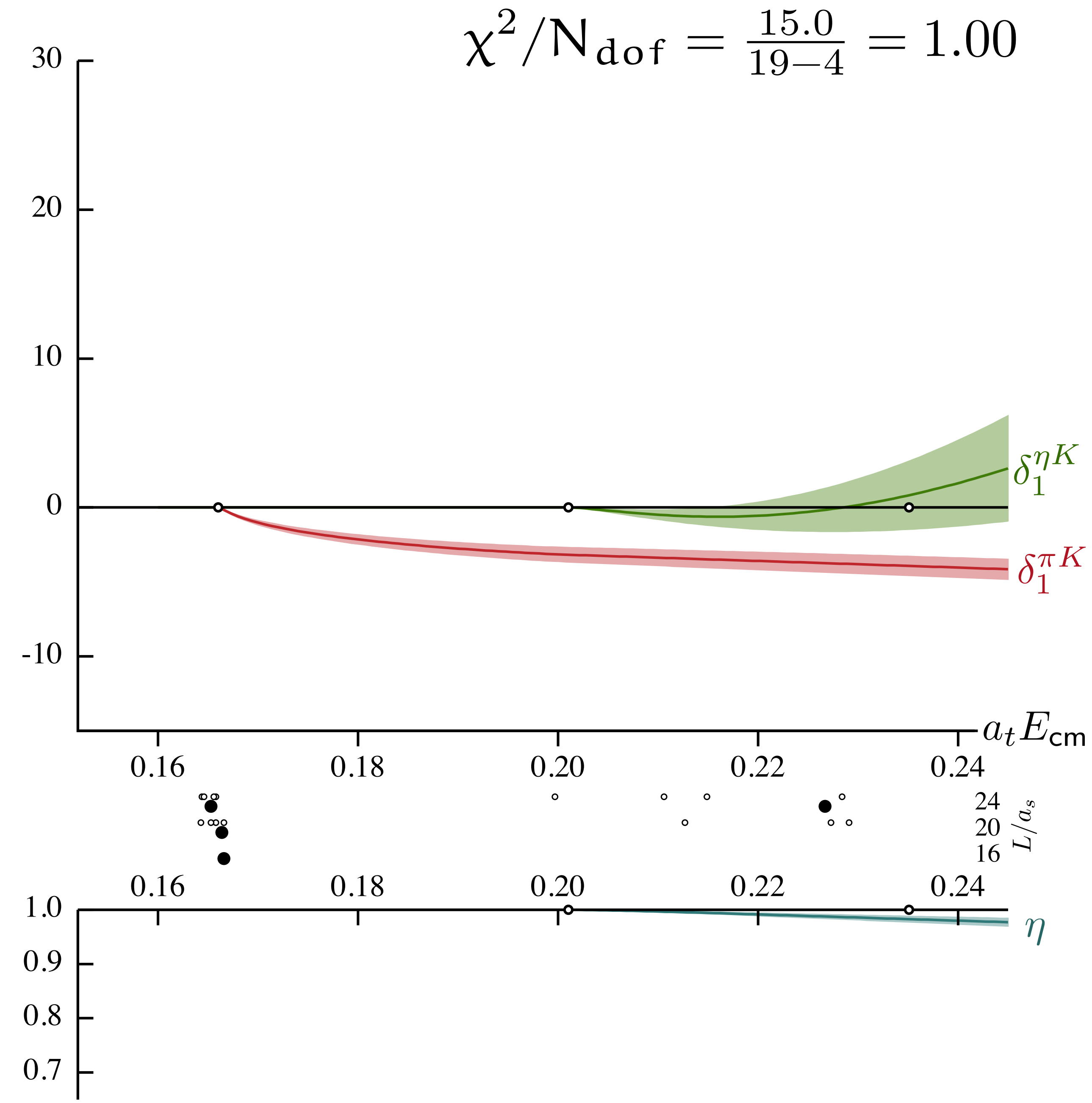


- Many more energy levels from irreps where the mesons are **moving with respect to the lattice**.
- More than 100 usable levels.
- Less symmetry. More **mixing of partial waves**, requiring simultaneous solutions.



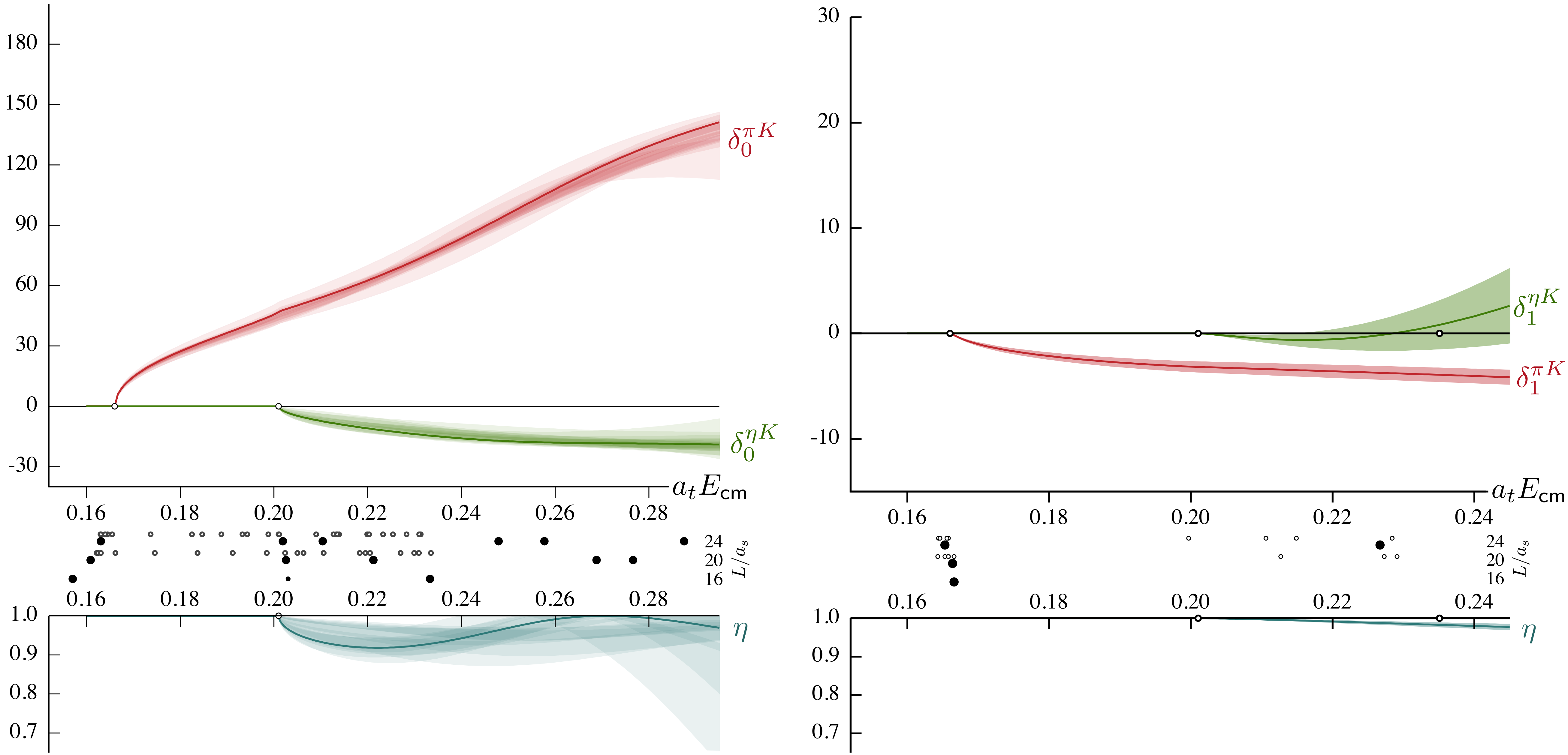
S+P-waves from 80 energy levels





- K-matrix parameterisations in S and P wave.
- Separate fits and global fits yield consistent results.
- D-wave is negligible in this region.

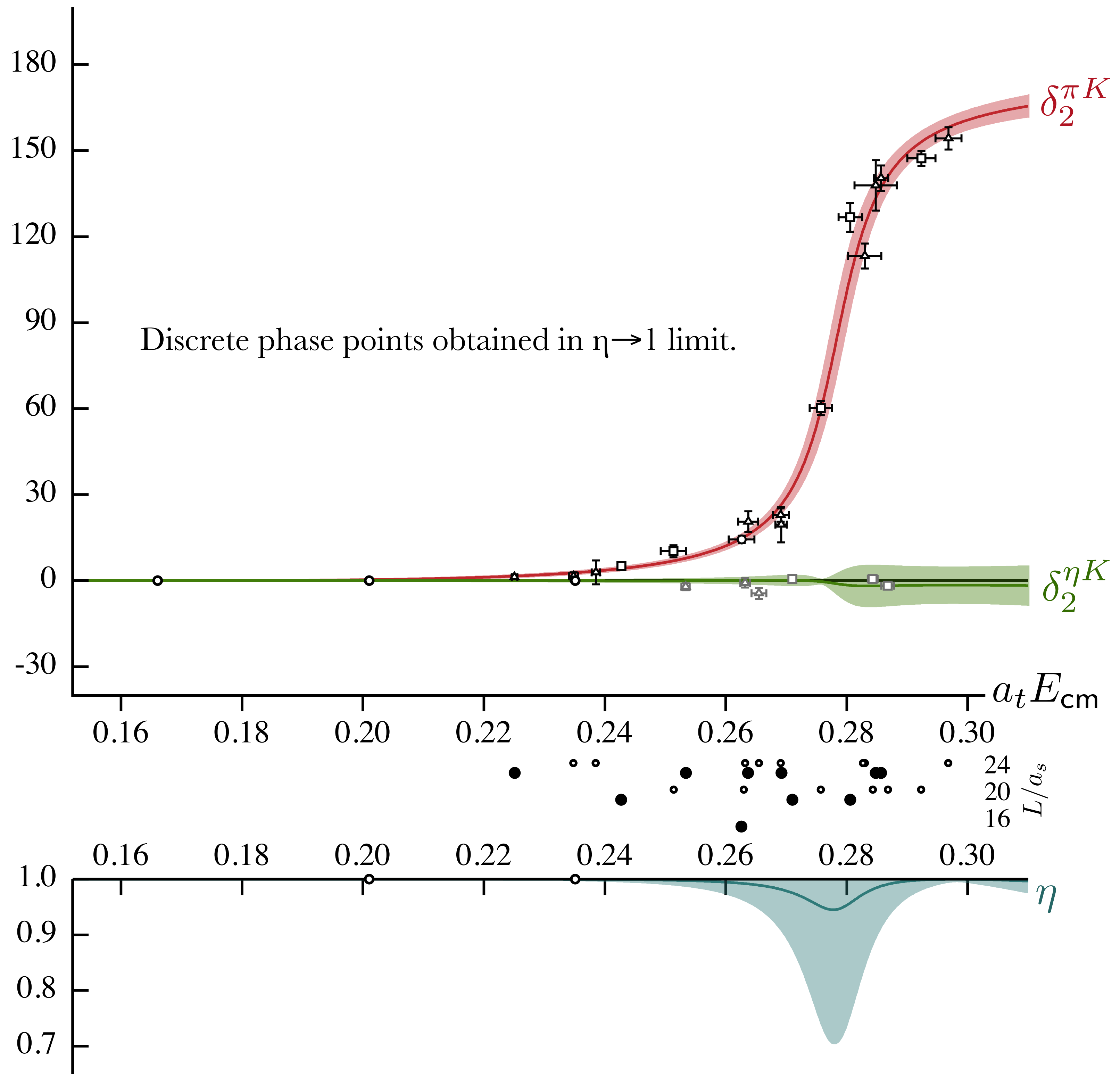
Parameterisation variation



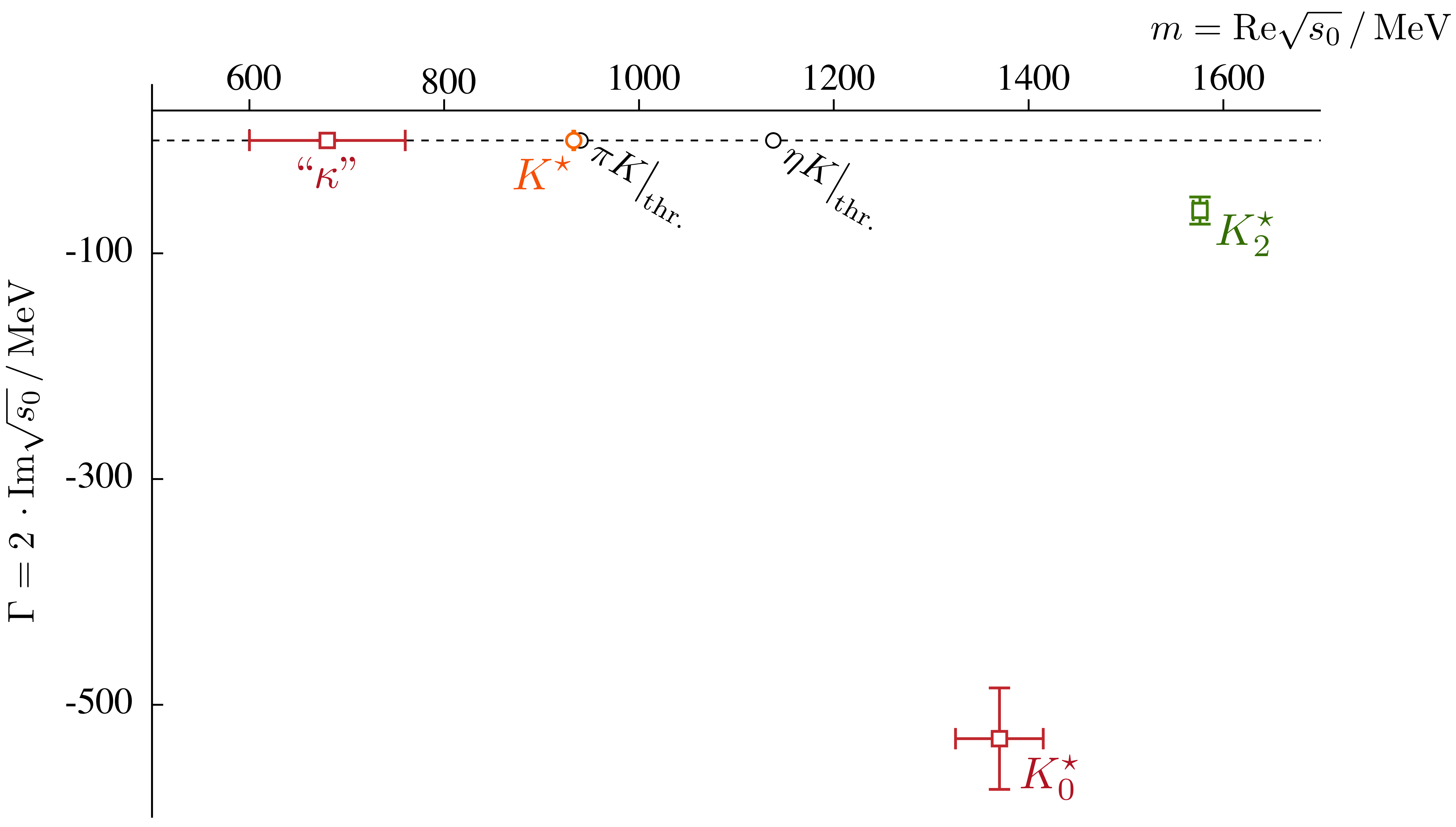
- K-matrix parameterisations in *S* and *P* wave.
- Separate fits and global fits yield consistent results.
- *D*-wave is negligible in this region.

Narrow *D*-wave resonance

- Many other energy levels containing scattering amplitude information.
- Using only irreps with $\mathcal{J}$=2 and higher (E^+, $T_2{}^+$, [100]$B_{1,2}$) we find a narrow resonance:
- Fit to energies.
- In $\mathcal{J}\geq 1$ scattering the lowest threshold is $\pi\pi K$ at $a_t E_{cm}$=0.235.
- Ideally requires 3-body formalism. Although not strictly rigorous, we can apply the 2→2 formalism anyway.



S-matrix poles



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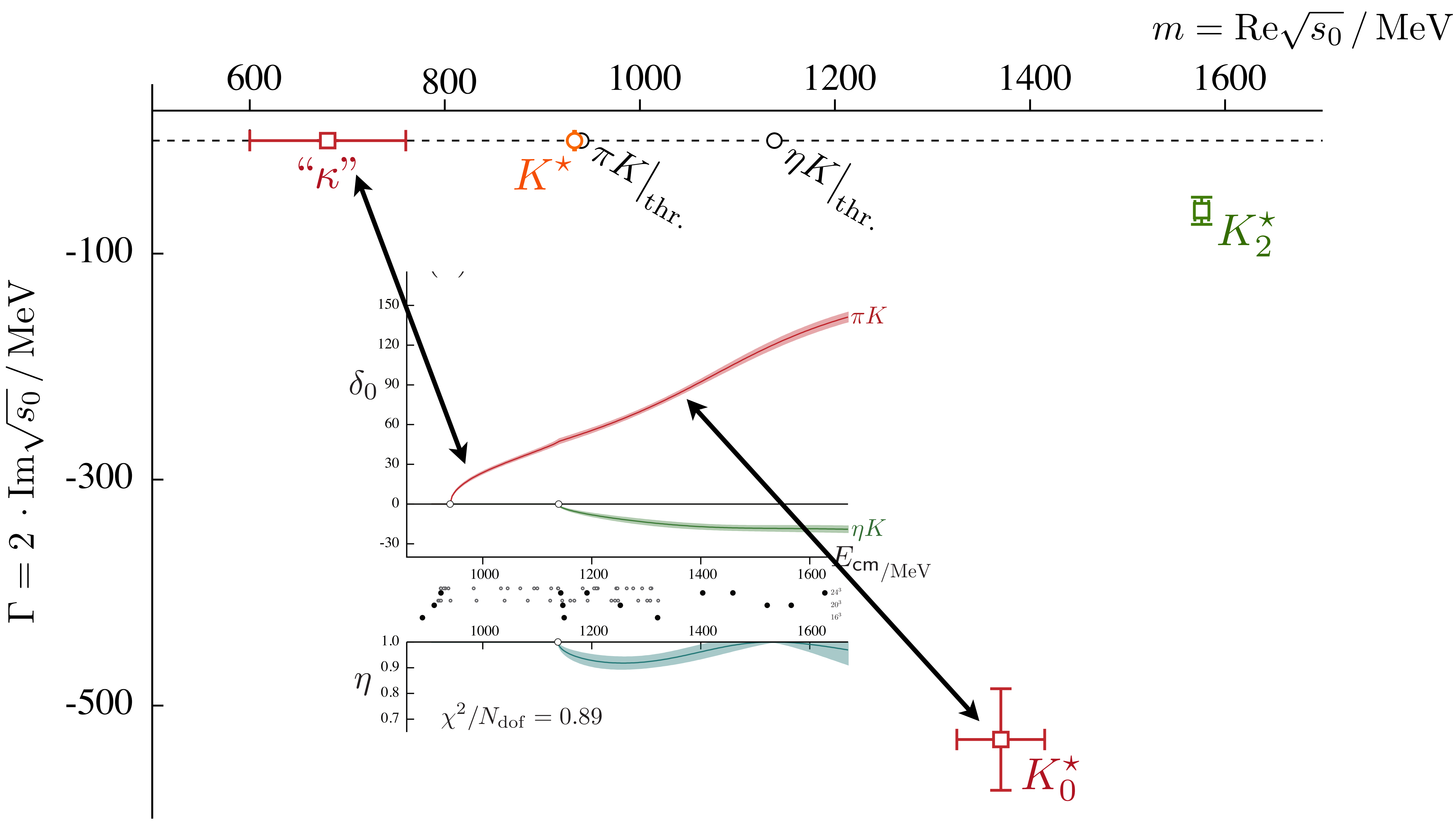

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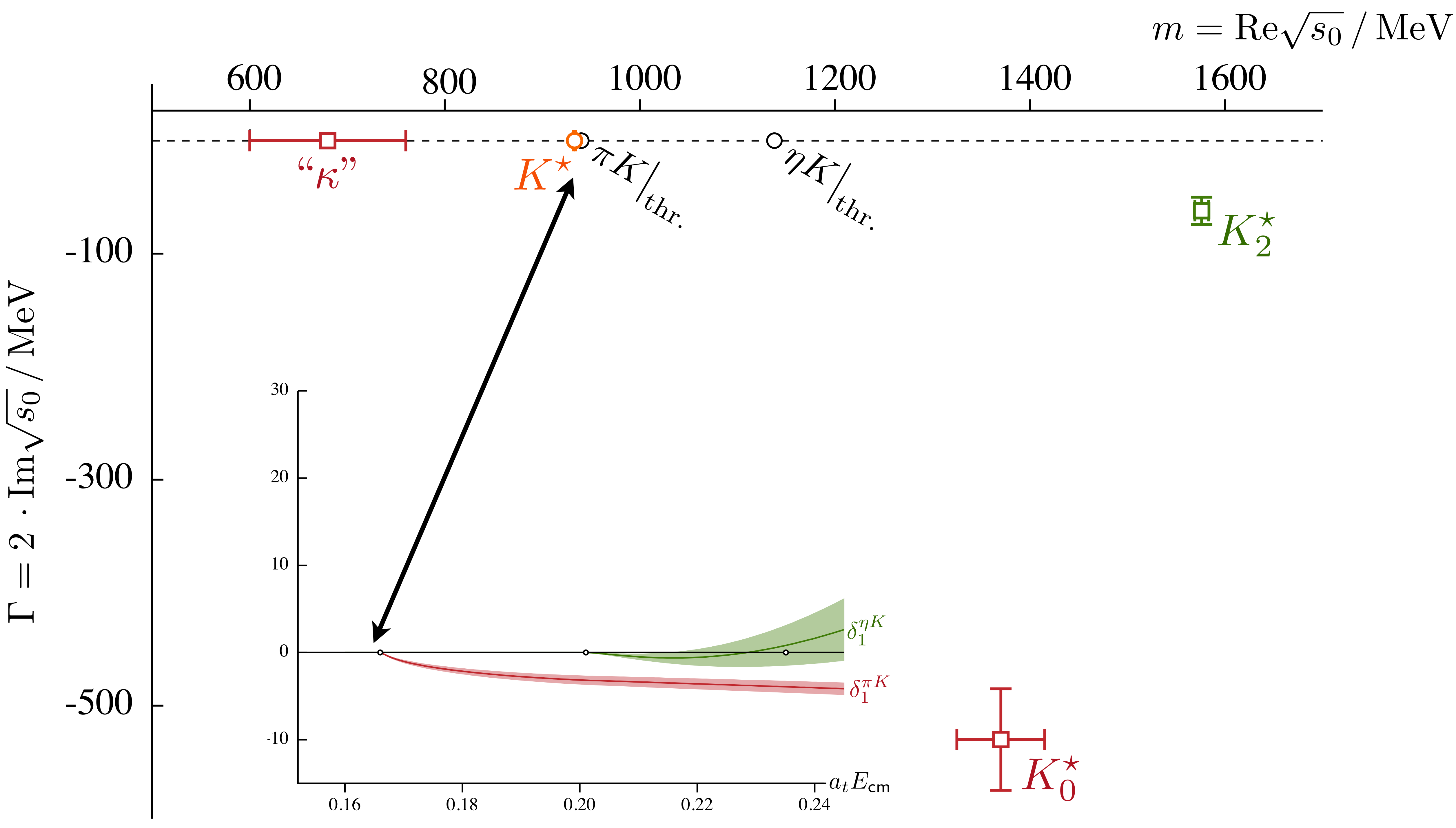

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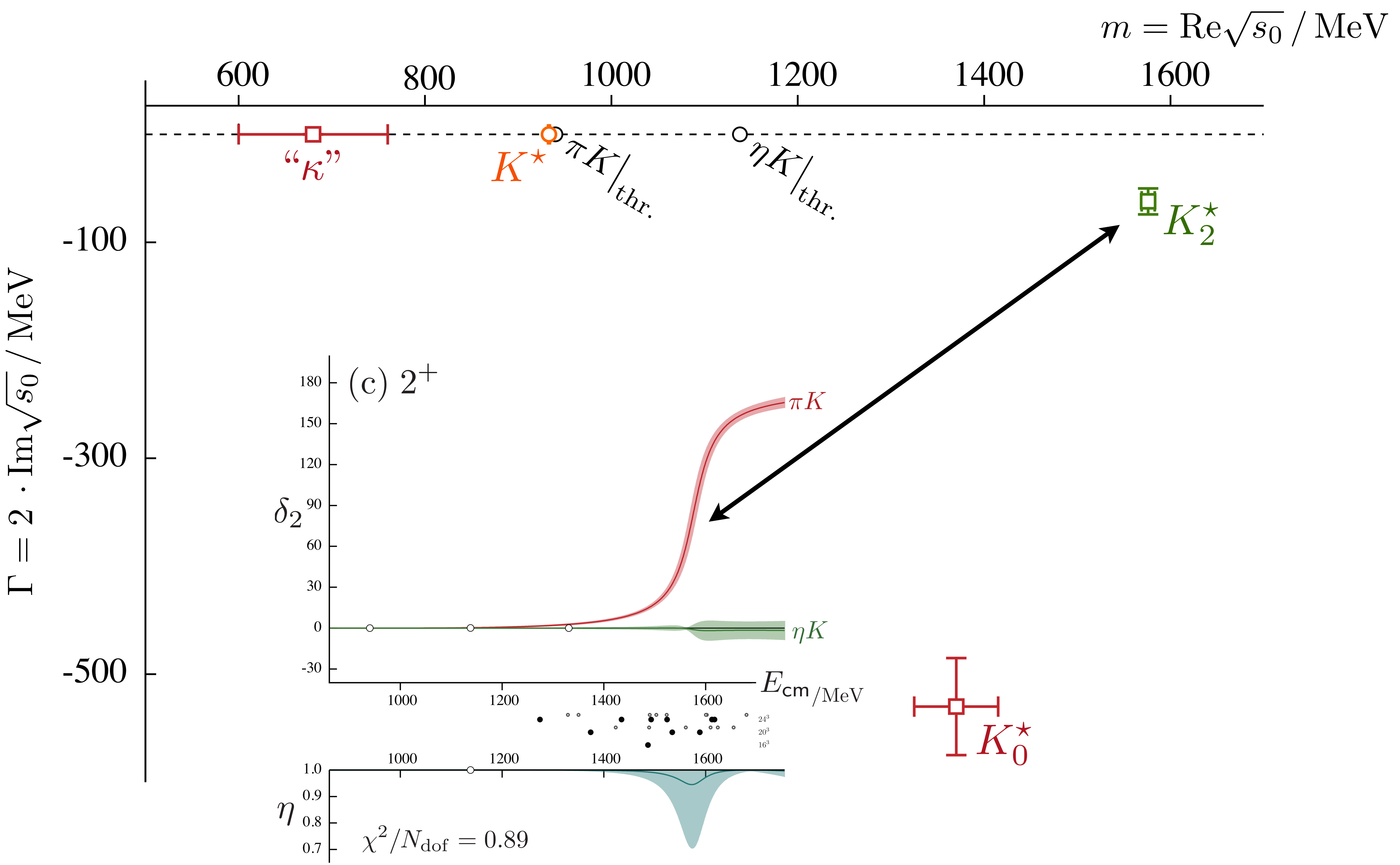


Summary

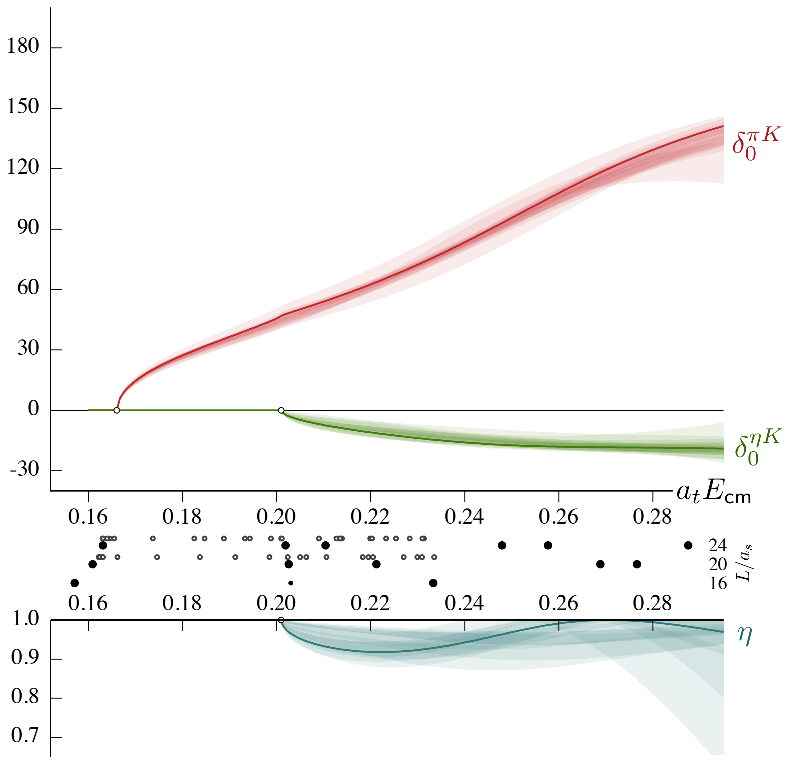
- Coupled-channel scattering amplitudes can be obtained from QCD using lattice methods.
- Using extensions of Lüscher's method, we were able to connect finite volume energy levels to infinite volume scattering amplitudes.
- There are many exciting possibilities for future calculations using similar methods:

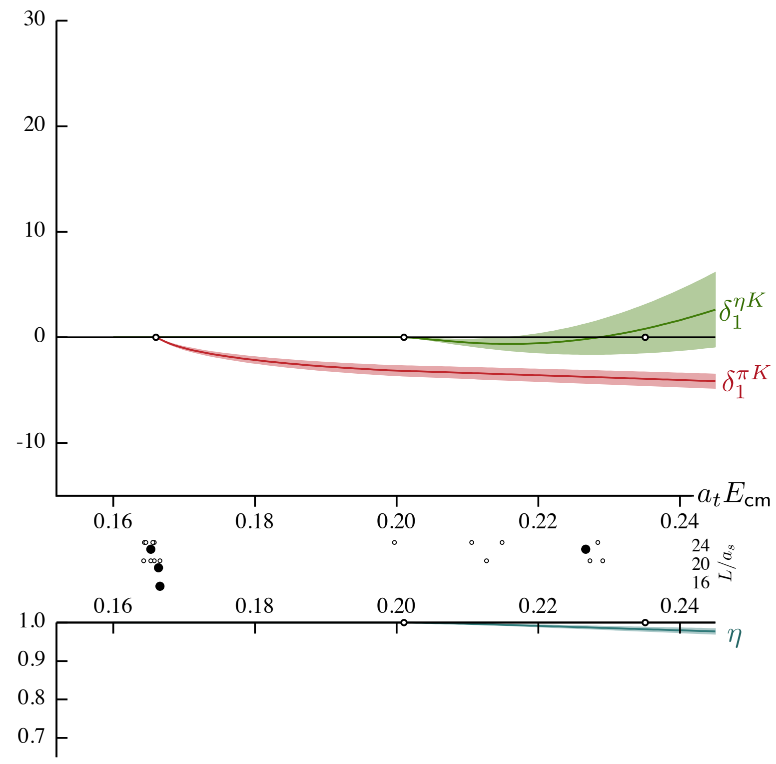
  Strongly coupled systems like the $a_0(980)$ and $f_0(980)$ are under investigation.

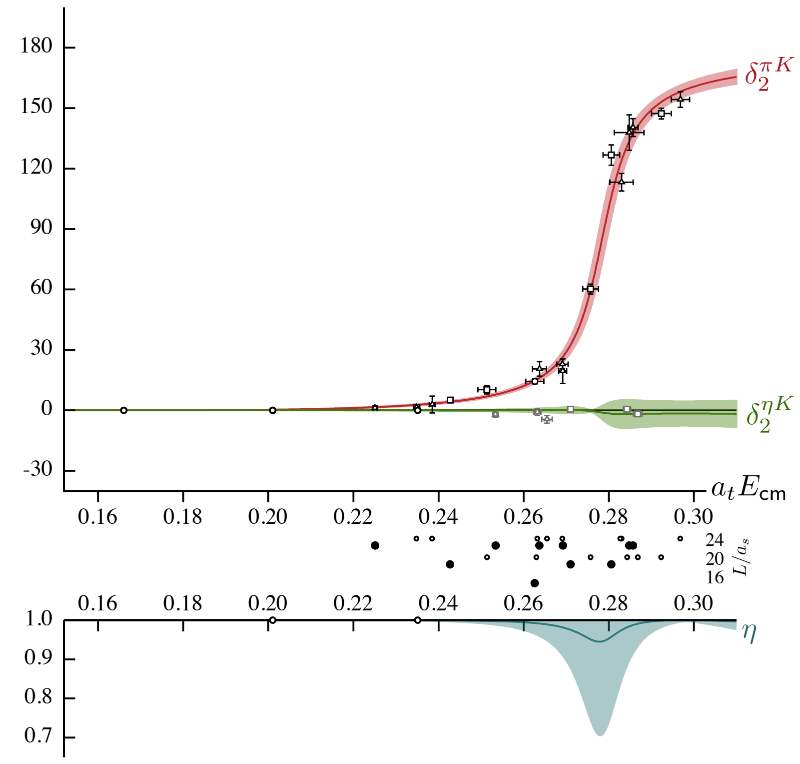
  Investigations into πγ ➞ ππ and similar processes are underway.

  Channels involving charm quarks are also under investigation by European collaborators.

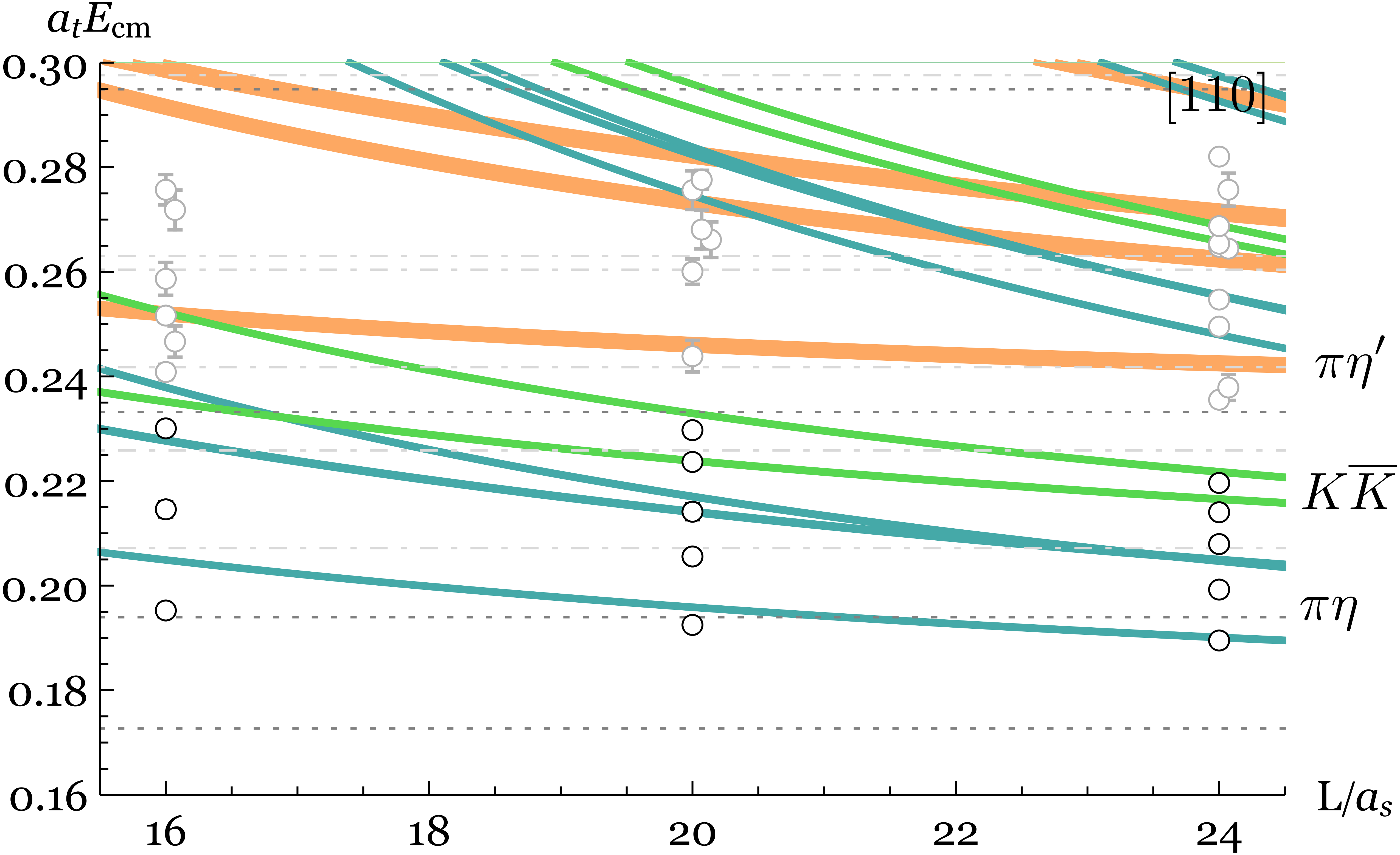
- Further in the future: πN ➞ πN, γN➞ πN. Multiparticle scattering, exotics.





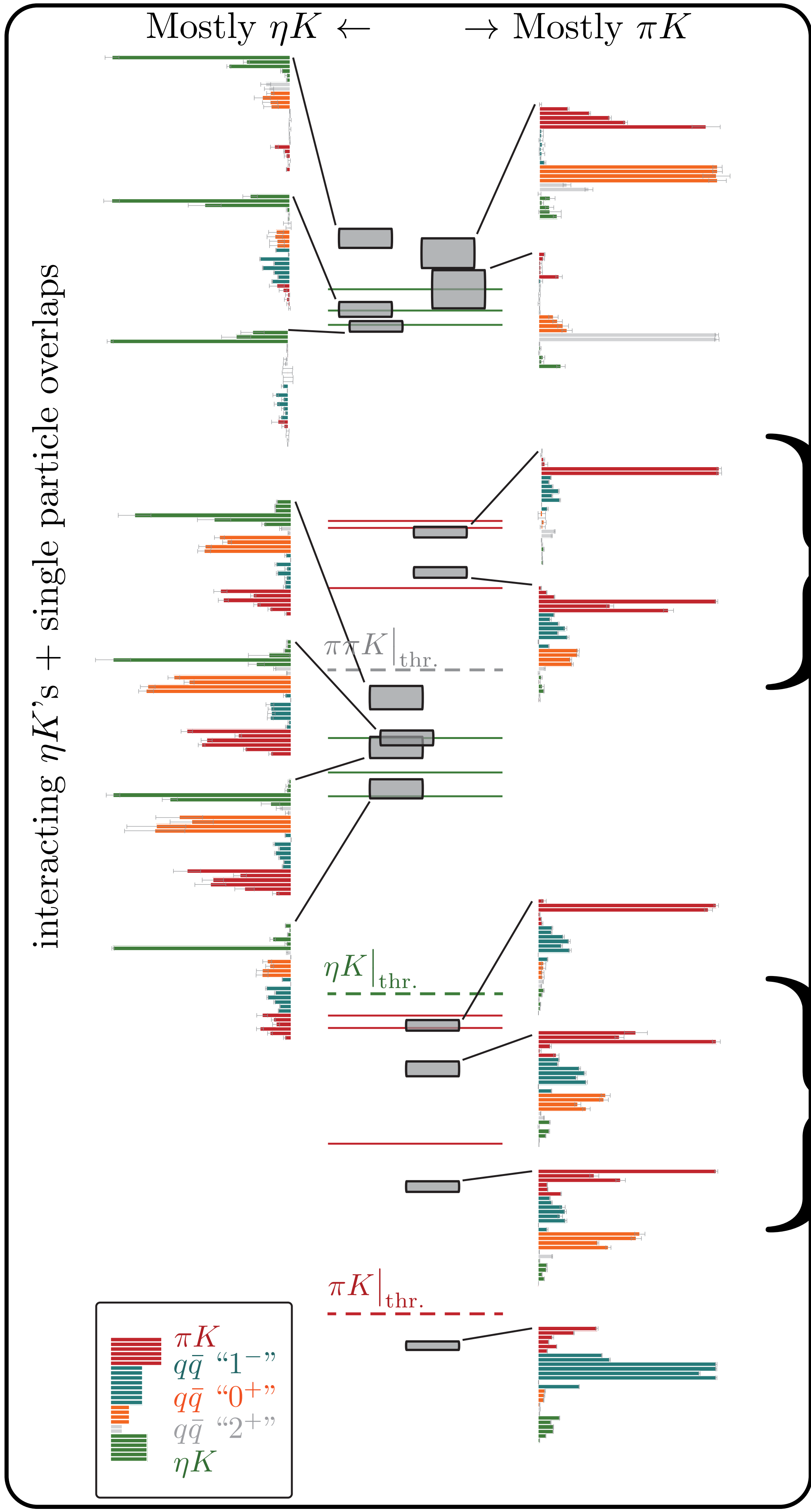


Coming soon: πη-KK̄-πη'



Backup slides

$[011]A_1$



$\sim J^P = 2^+$

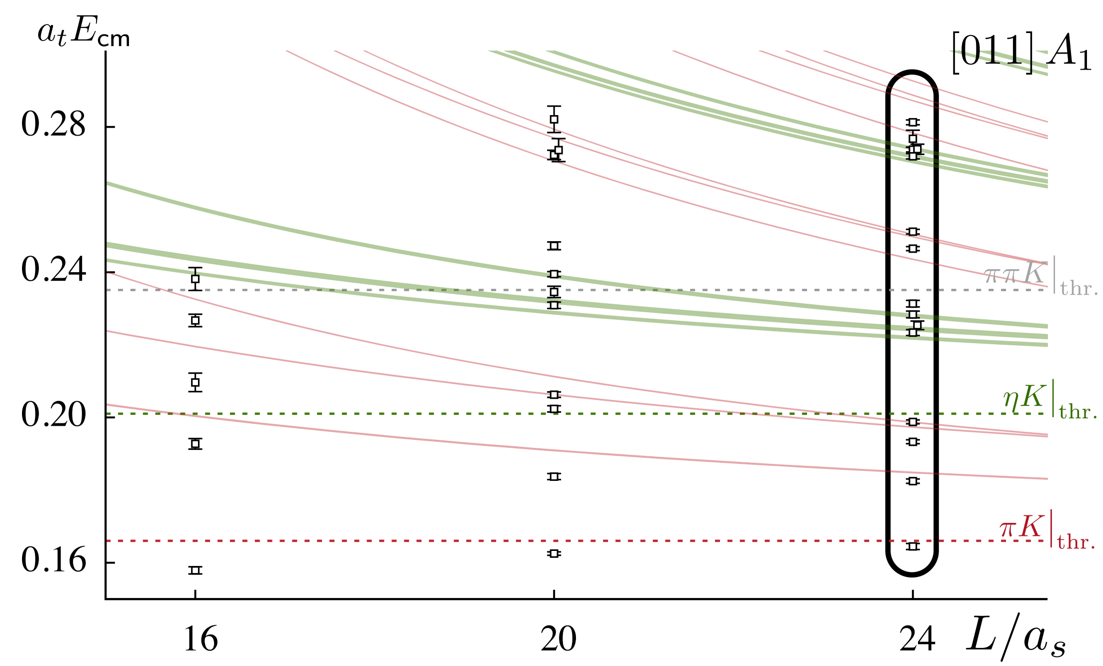
interacting πK's + single particle overlaps

$\sim J^P = 0^+$
+ interactions

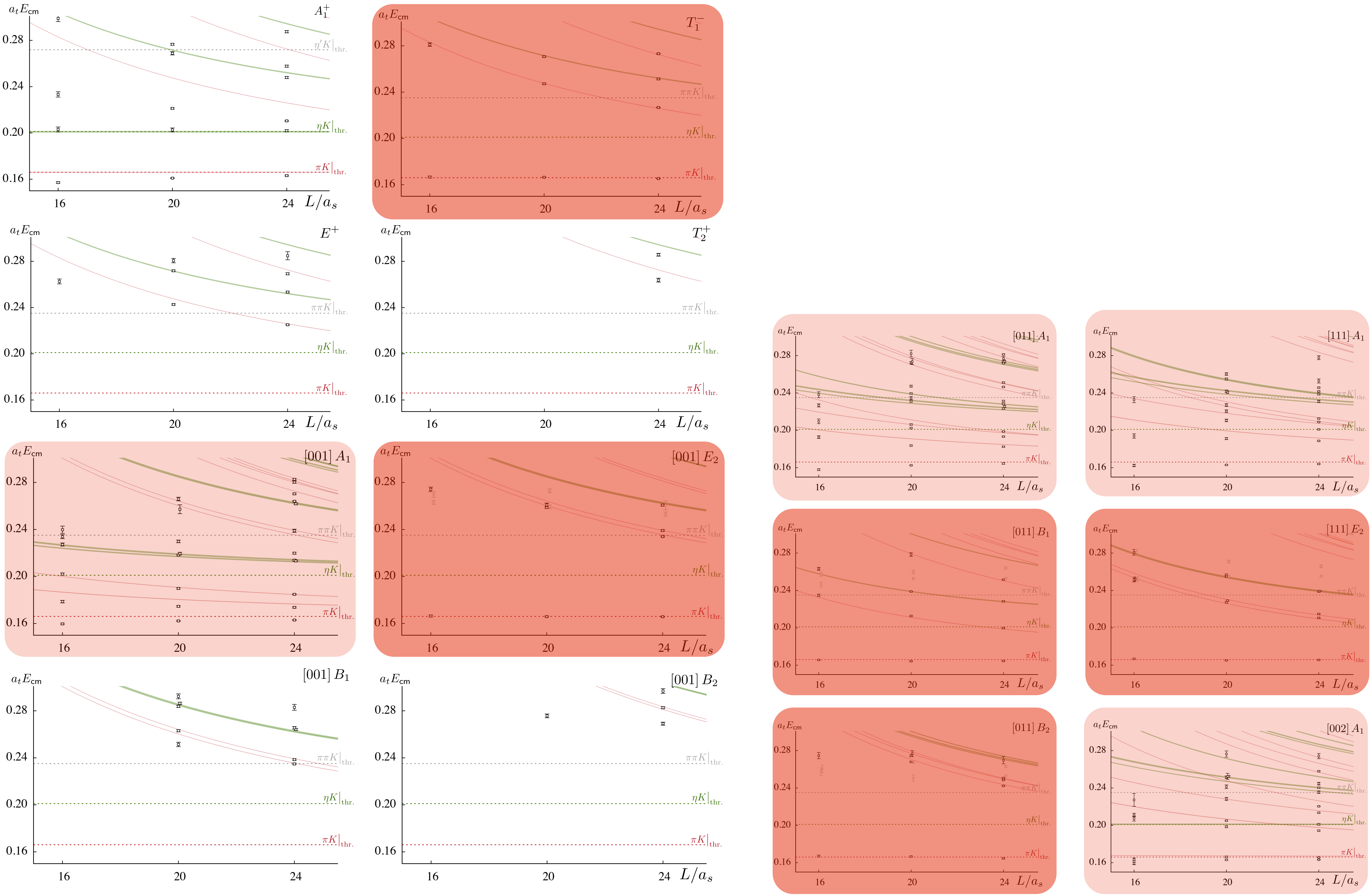
interacting πK's + single particle overlaps

$\sim J^P = 1^-$

- Overlaps ~ guide to resonant content $Z_i^{\mathfrak{n}} = \langle \mathfrak{n} | \mathcal{O}_i^\dagger | 0 \rangle$
- Shifted πK-like and ηK-like states
- $J^P=1^-$ state near to πK threshold, $J^P=2^+$ state, extra $J^P=0^+$.
- Considerable partial-wave mixing. $[011]\ A_1$ contains $J^P=0^+$, 1, 2, ...



P-wave contributions



P-wave near-threshold state

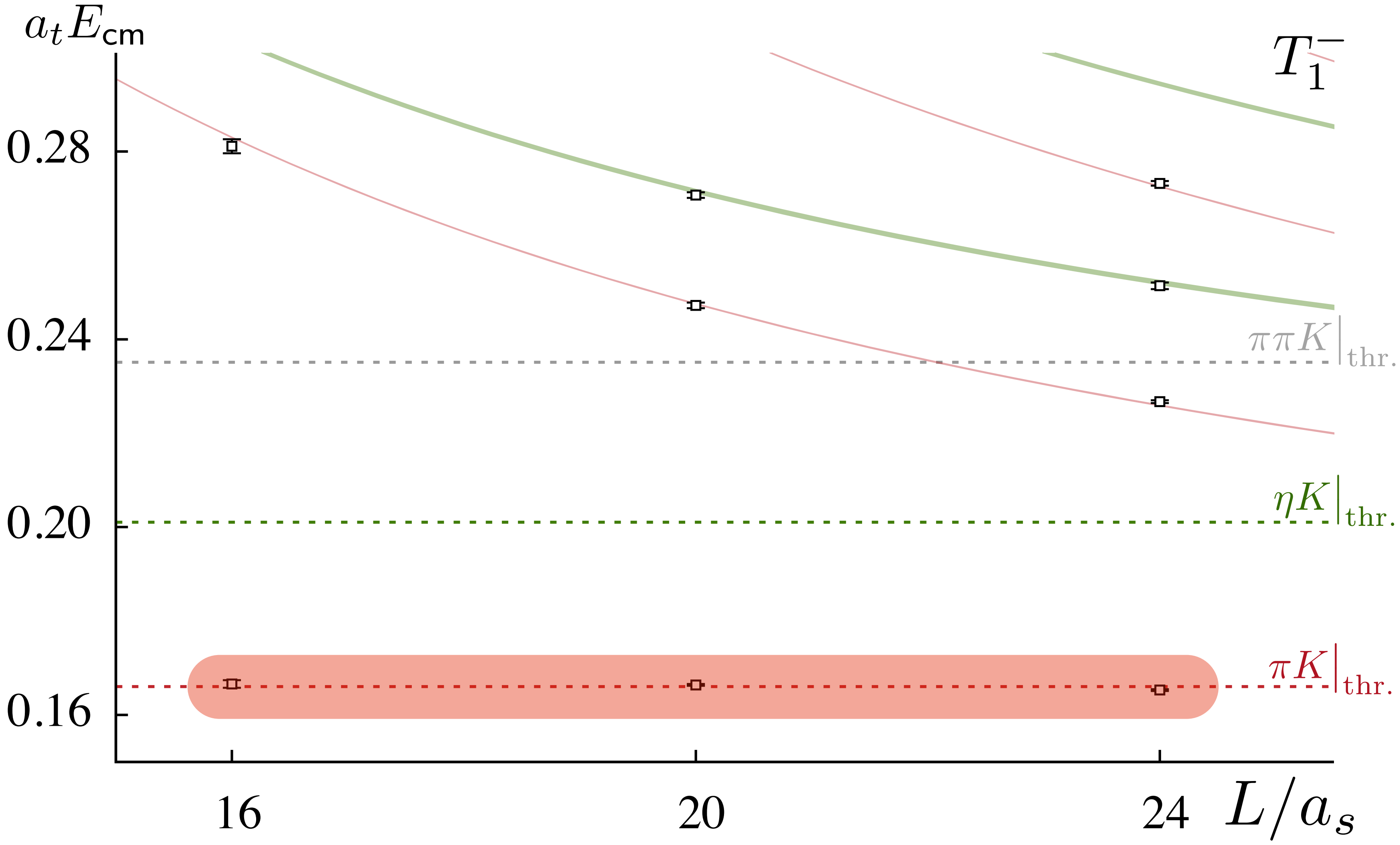
Elastic scattering just above πK threshold, no ηK to consider.

The irreps with P-wave overlap:

T_1^-, [001] A_1, [001] E_2, [011] A_1, [011] $B_{1,2}$, [111] A_1, [111] E_2, [002] A_1

all have an "extra" level near πK threshold.

Fitting the energy levels using an elastic Breit-Wigner in πK:



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$$\Gamma(s) = \frac{g_R^2}{6\pi}\frac{k_{cm}^3}{E_{cm}^2}$$

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all have an "extra" level near πK threshold.

$$k^3 \cot\delta_1 = (m_R^2 - s)\frac{6\pi s^{\frac{1}{2}}}{g_R^2}$$

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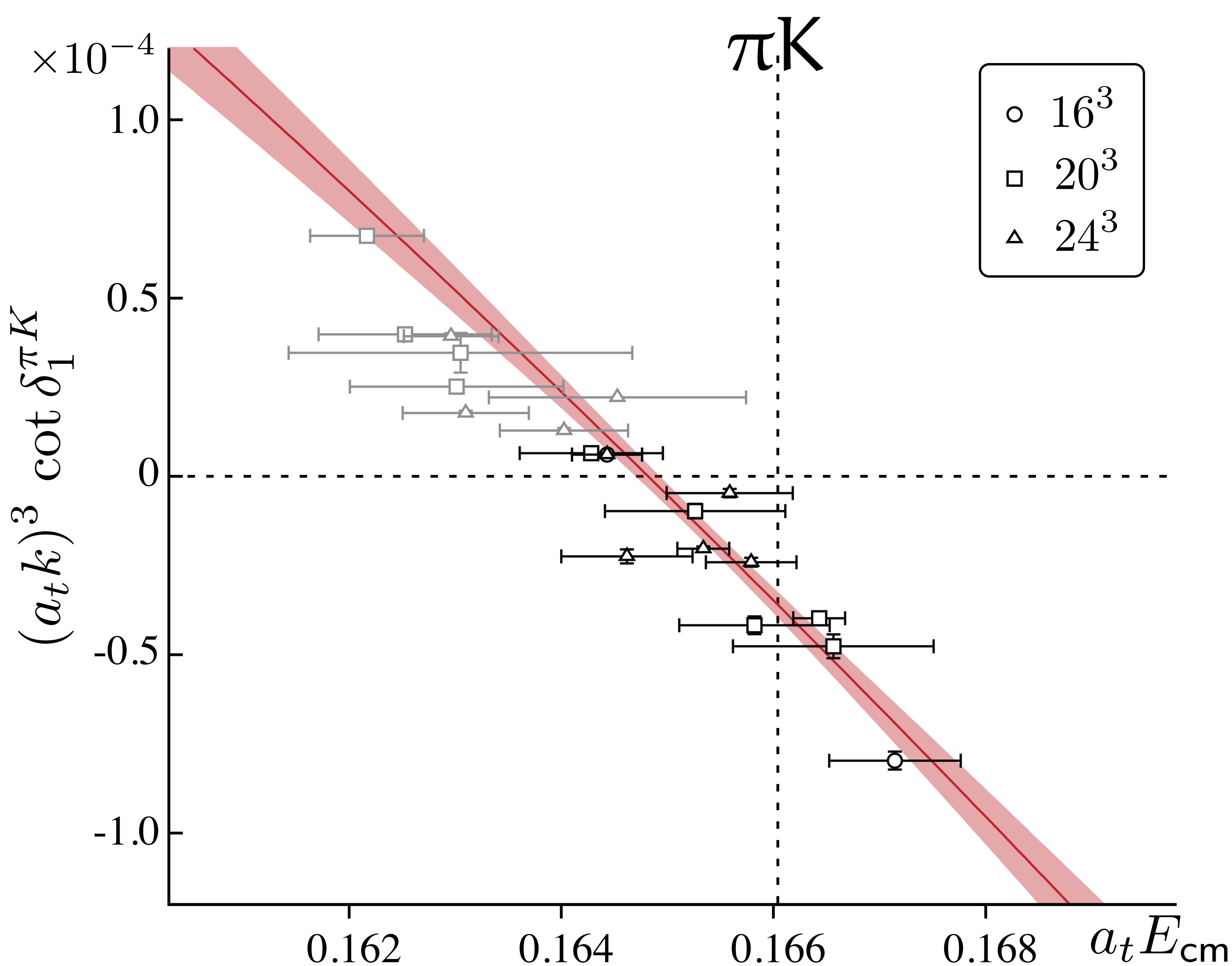
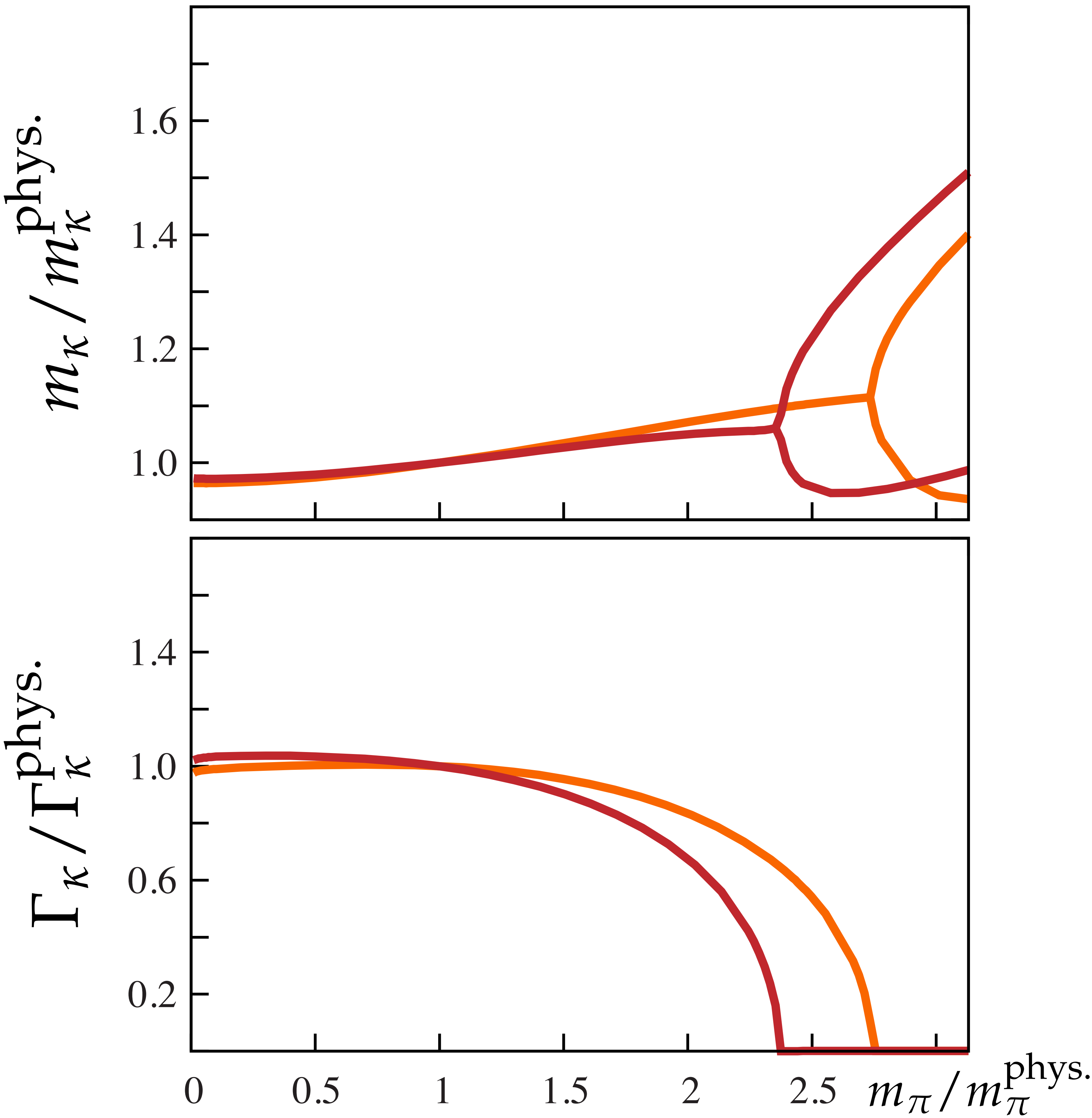


In t there is a pole on the real axis just below πK threshold:

Bound state in $J^P=1^-$

Virtual bound state κ

Pelaez and Nebreda using Unitiarised SU(3) Chiral Perturbation theory



More on virtual bound state

In an effective range parameterisation, strong interactions near threshold lead to a large *a*.

In *S*-wave large *a* automatically leads to a pole near-threshold.

$$k_{cm} \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r k_{cm}^2$$

$$t = \frac{1}{2} \frac{E_{cm}}{k_{cm} - \frac{i}{a}} \qquad k_{cm} = \mp \frac{i}{a}$$

Arguments appear to hold for constant terms in K-matrix (slightly complicated by Chew-Mandelstam).

Appears to break down for *P*-wave and higher.