

# Charmonium spectroscopy from Lattice QCD

Daniel Mohler

Baltimore,  
April 9, 2015



- 1 Introduction
- 2 Charmonia from single-hadron operators
  - Low lying charmonia
  - New results on the 1S hyperfine splitting
  - New results on charmonium 1P states and 2S states
  - Exited state energy levels from  $\bar{q}q$
  - Gluonic excitations
- 3 Charmonia including meson-meson states
  - The  $\psi(2S)$  and  $\Psi(3770)$
  - $\chi'_{c0}$  and  $X/Y(3915)$
  - Lattice results for the  $X(3872)$
  - Searches for  $Z_c$  and  $Y(4140)$
- 4 Conclusions and Outlook

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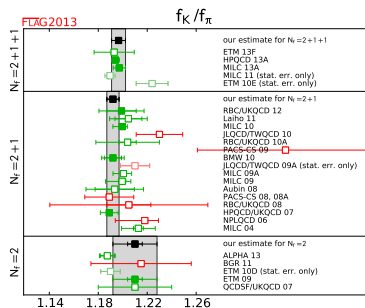
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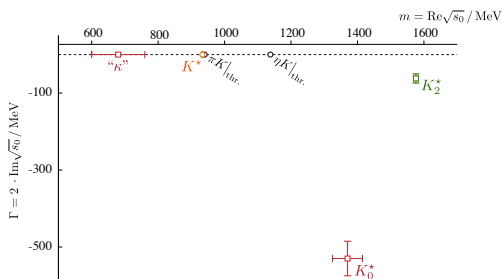
## 4 Conclusions and Outlook

# Two kinds of progress...

## Precision results:



## Exploratory studies:



## Example: FLAG review

See <http://itpwiki.unibe.ch/flag/>

## Example: $\pi K$ - $\eta K$ -scattering

Dudek et al. PRL 113 182001 (2014)

See talk by David Wilson Fri

- I will report on both kind of progress with regard to charmonium
- There will be preliminary data - use with caution

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# Low-lying charmonium: A precision benchmark

- Well understood from potential models
- Well determined in experiment
- **Well separated from open-charm threshold(s)**
- Spin-dependent mass splittings extremely sensitive to the charm-quark mass and heavy-quark discretization

meson	mass	width
$\eta_c$	2983.7(7)	32.0(9) MeV
$J/\Psi$	3096.916(11)	92.9(2.8) keV
$\chi_{c0}$	3414.75(31)	10.3(6) MeV
$\chi_{c1}$	3510.66(3)	0.86(5) MeV
$\chi_{c2}$	3556.20(9)	1.97(11) MeV
$h_c$	3525.38(11)	0.7(4) MeV
$\eta_c(2S)$	3639.4(1.3)	$11.3^{(+3.2)}_{(-2.9)}$ MeV
$\Psi(2S)$	$3686.109^{(+12)}_{(-14)}$	299(8) keV

# Preliminary results from Fermilab-MILC

Fermilab Lattice and MILC collaborations - to be published

We use the *Fermilab method* for the charm quark

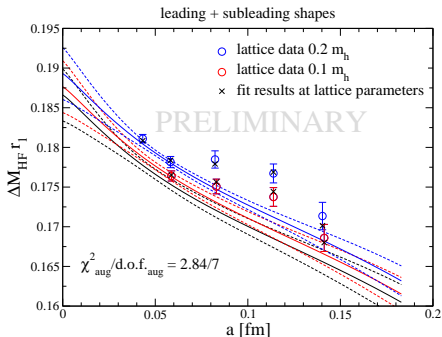
El-Khadra et al., PRD 55, 3933 (1997)

- $m_c$  tuned by demanding the  $D_s$  meson kinetic mass to be physical
- We quote splittings among charmonium states
- 5 lattice spacings with two different light sea-quark masses  
→ Controlled extrapolation to the chiral-continuum limit
- 2+1 flavor simulation with high statistics
- Follows previous efforts

T. Burch et al. PRD 81 034508, 2010

- Charm annihilation contributions are omitted

# Fermilab-MILC results for the 1S hyperfine splitting



- Curves for physical (black),  $0.1 m_s$ , and  $0.2 m_s$  light-quark masses
- This corresponds to  $M_{J/\psi} - M_{\eta_c} = 118.1(2.1) \begin{pmatrix} -1.5 \\ -4.0 \end{pmatrix}$
- Errors include statistics, chiral and continuum extrapolations
- Contribution from disconnected diagrams expected  $\begin{pmatrix} -1.5 \\ -4.0 \end{pmatrix}$

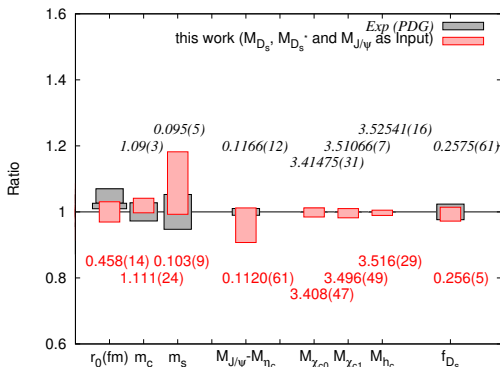
Levkova and DeTar, PRD 83 074504, 2011





# Results from the $\chi$ QCD Collaboration

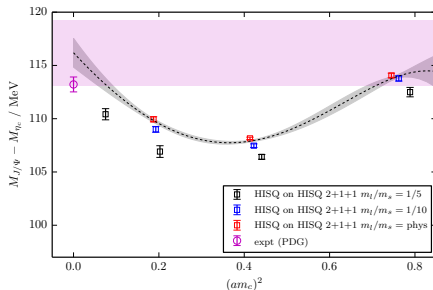
$\chi$ QCD Collaboration, Yang *et al.* arXiv:1410.3343



- Results from Overlap fermions on 2+1 flavor domain wall gauge configurations
- Uncertainty do not include charm-quark annihilation effects

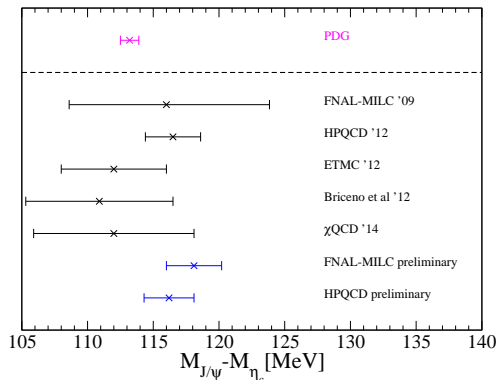
# Preliminary results from the HPQCD Collaboration

HPQCD, Galloway *et al.* PoS LATTICE2014 (2014) 092



- Uses MILC 2+1+1 flavor HISQ (Highly Improved Staggered Quarks) ensembles
- Systematics on 1S hyperfine dominated by estimate of annihilation effects

# A comparison of hyperfine splittings

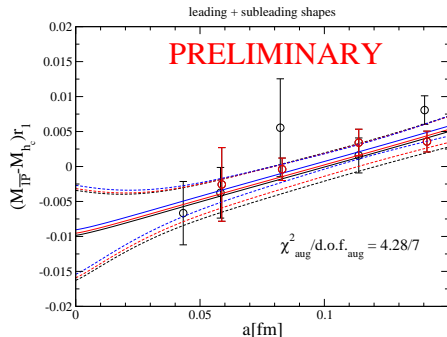
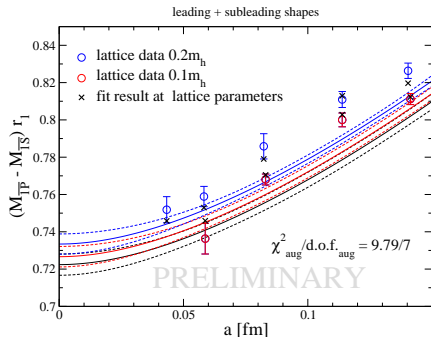


- All results at physical quark masses and in the continuum limit
- Lattice numbers exclude (now dominant?) annihilation effects
- Estimate from data expects a shift of -1.5..-4.5 MeV

Levkova and DeTar, PRD 83 074504, 2011



# Fermilab-MILC 1P-1S and 1P hyperfine splittings

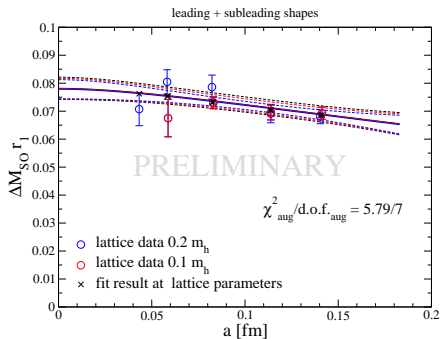


- Not yet included: Scale-setting uncertainty

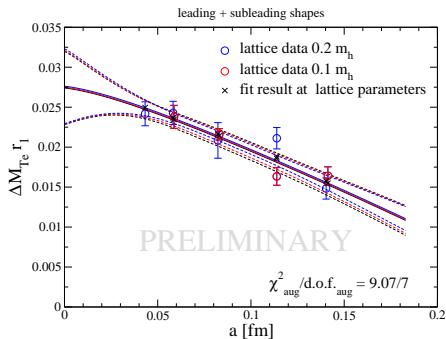
Mass difference	This analysis [MeV]	Experiment [MeV]
1P1S	$457.3 \pm 3.6$	$457.5 \pm 0.3$
1P hyperfine	$-6.2 \pm 4.1$	$-0.10 \pm 0.22$

# Fermilab-MILC P-wave spin-orbit and tensor splittings

$$\Delta M_{\text{Spin-Orbit}} = (5M_{\chi_{c2}} - 3M_{\chi_{c1}} - 2M_{\chi_{c0}})/9$$



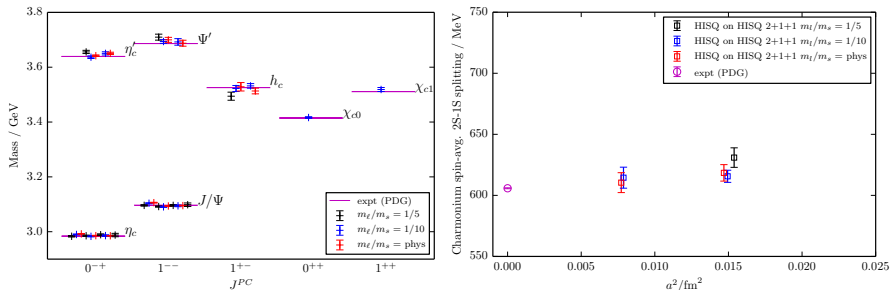
$$\Delta M_{\text{Tensor}} = (3M_{\chi_{c1}} - M_{\chi_{c2}} - 2M_{\chi_{c0}})/9$$



Mass difference	This analysis [MeV]	Experiment [MeV]
1P spin-orbit	$49.5 \pm 2.5$	$46.6 \pm 0.1$
1P tensor	$17.3 \pm 2.9$	$16.25 \pm 0.07$

# Preliminary results from the HPQCD Collaboration

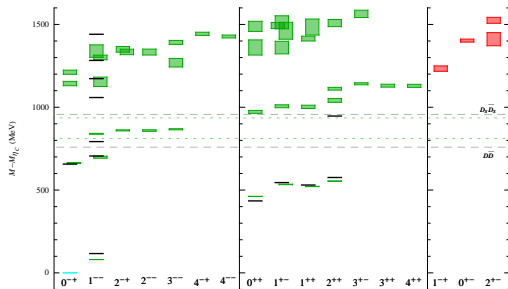
HPQCD, Galloway *et al.* PoS LATTICE2014 (2014) 092



- Uses MILC 2+1+1 flavor HISQ (Highly Improved Staggered Quarks) ensembles
- Shaded region shows the systematic uncertainty

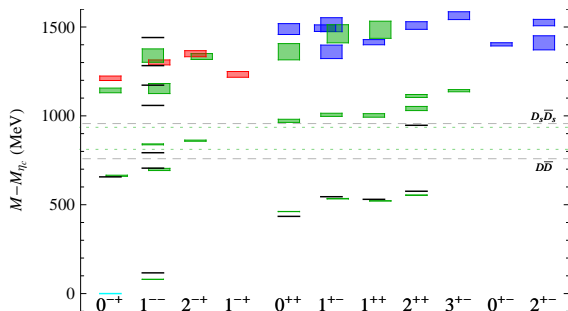
# Excited state energy levels from $\bar{q}q$ operators

HSC, L. Liu et al. JHEP 1207 126 (2012)



- A large number of energy levels including higher spin states can be identified
- Mesons with spin-exotic quantum numbers!
- Relation of energy levels to resonances not straight forward

HSC, L. Liu et al. JHEP 1207 126 (2012)



- Hybrid mesons with both regular and spin-exotic quantum numbers
- Relation of energy levels to resonances not straight forward



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# Single hadron interpolators: what do we see?

- In practical calculations  $\bar{q}q$  and  $qqq$  interpolators couple very weakly to multi-hadron states

McNeile & Michael, Phys. Lett. B 556, 177 (2003); Engel et al. PRD 82, 034505 (2010);  
Bulava et al. PRD 82, 014507(2010); Dudek et al. PRD 82, 034508(2010);

- This is not unlike observations in QCD string-breaking studies

Pennanen & Michael hep-lat/0001015; Bernard et al. PRD 64 074509 2001;

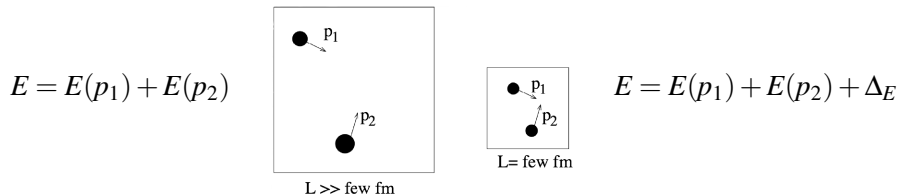
- Necessitates the inclusion of hadron-hadron interpolators



- We know: Energy levels  $\neq$  resonance masses  
Naïve expectation: Correct up to  $\mathcal{O}(\Gamma_R(m_\pi))$
- Was good enough for heavy pion masses where one would deal with bound states or very narrow resonances.

# The Lüscher method

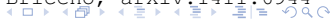
M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.



$$E_n(L) \xrightarrow{(2)} \delta_l \xrightarrow{(3)} m_R; \Gamma_R \text{ or coupling } g$$

- (1) Extract energy levels  $E_n(L)$  in a finite box
- (2) Lüscher formula  $\rightarrow$  phase shift of the continuum scattering amplitude
- (3) Extract resonance parameters (similar to experiment)
  - 2-hadron scattering and transitions well understood; progress for 3 (or more) hadrons but difficult

See LATTICE2014 plenary by Raúl A. Briceño, arXiv:1411.6944



# Technicalities: Lattices used

ID	$N_L^3 \times N_T$	$N_f$	$a[\text{fm}]$	$L[\text{fm}]$	#configs	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

- Ensemble (1) has 2 flavors of nHYP-smearred quarks

Gauge ensemble from Hasenfratz et al. PRD 78 054511 (2008)

Hasenfratz et al. PRD 78 014515 (2008)

- Ensemble (2) has 2+1 flavors of Wilson-Clover quarks

PACS-CS, Aoki et al. PRD 79 034503 (2009)

- On the small volume we use distillation

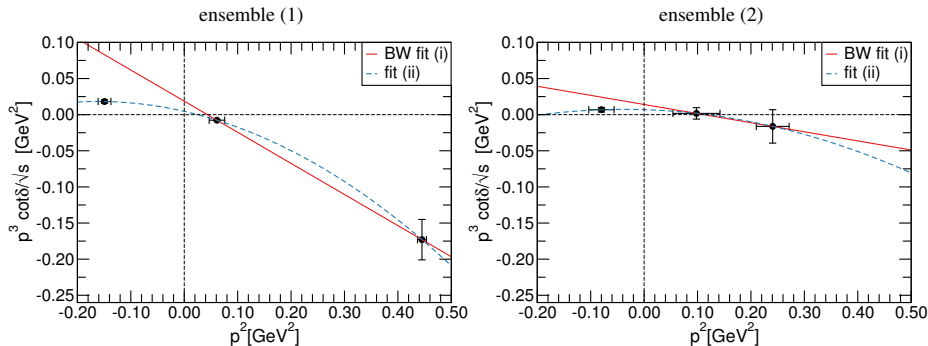
On the larger volume we use stochastic distillation

Peardon et al. PRD 80, 054506 (2009);

Morningstar et al. PRD 83, 114505 (2011)

# $\Psi(3770)$ resonance from Lattice QCD

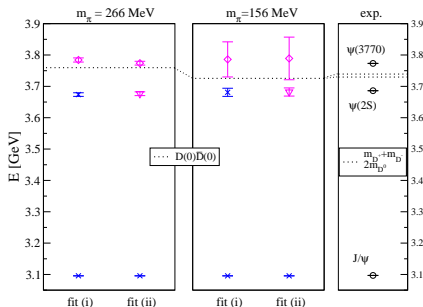
Lang, Leskovec, DM, Prelovsek, arXiv:1503.05363



- Proof of principle - many improvements possible

# $\Psi(3770)$ resonance: Results

Lang, Leskovec, DM, Prelovsek, arXiv:1503.05363



	Mass [MeV]	$g_{\Psi(3770)DD}$
Ensemble(1)	3784(7)(8)	13.2 (1.2)
Ensemble(2)	3786(56)(10)	24(19)
Experiment	3773.15(33)	18.7(1.4)

- First resonance determination of a charmonium state
- Proof of principle - many improvements possible

# $\chi'_{c0}$ and $X/Y(3915)$

- PDG is identifying the  $X(3915)$  with the  $\chi'_{c0}$
- Based on BaBar determination of its quantum numbers
- Some of the reasons to doubt this assignment:

Guo, Meissner Phys. Rev. **D86**, 091501 (2012)

Olsen, arxiv 1410.6534

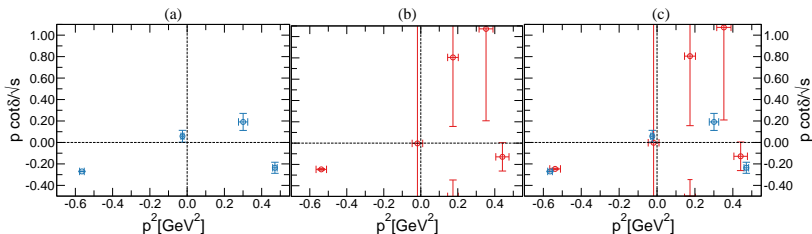
- No evidence for fall-apart mode  $X(3915) \rightarrow \bar{D}D$
- Spin splitting  $m_{\chi_{c2}(2P)} - m_{\chi_{c0}(2P)}$  too small
- Large OZI suppressed  $X(3915) \rightarrow \omega J/\psi$
- Width should be significantly larger than  $\Gamma_{\chi_{c2}(2P)}$
- Investigate  $\bar{D}D$  scattering in S-wave on the lattice!
- Candidates:

$$m = 3837.6 \pm 11.5 \text{ MeV}, \quad \Gamma = 221 \pm 19 \text{ MeV} \quad \text{Guo\&Meissner}$$

$$m = 3878 \pm 48 \text{ MeV}, \quad \Gamma = 347^{+316}_{-143} \text{ MeV} \quad \text{Olsen}$$

# $\chi'_{c0}$ : Exploratory lattice calculation

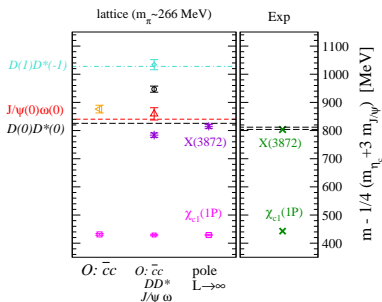
Lang, Leskovec, DM, Prelovsek, arXiv:1503.05363



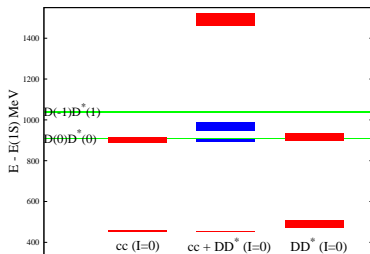
- Assumes only  $\bar{D}D$  is relevant
- Lattice data suggests a fairly narrow resonance with  $3.9\text{GeV} < M < 4.0\text{GeV}$  and  $\Gamma < 100\text{MeV}$
- Future experiment and lattice QCD results needed to clarify the situation



# An $X(3872)$ candidate from Lattice QCD



Prelovsek, Leskovec, PRL 111  
192001 (2013)

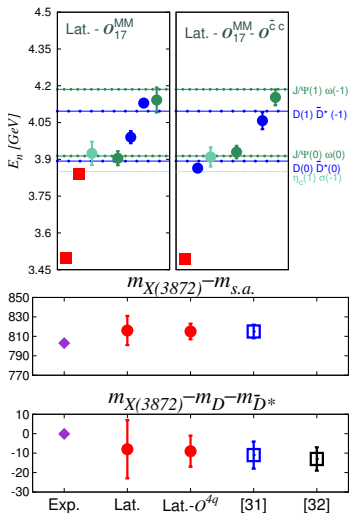


Lee, DeTar, DM, Na,  
arXiv:1411.1389

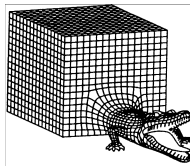
- Neglects charm annihilation and  $J/\psi\omega$
- Seen only when  $\bar{q}q$  and  $\bar{D}^*D$  are used
- The two simulations have vastly different systematics (yet results are similar)

# An $X(3872)$ candidate from Lattice QCD II

Padmanath, Lang, Prelovsek, arXiv:1503.03257

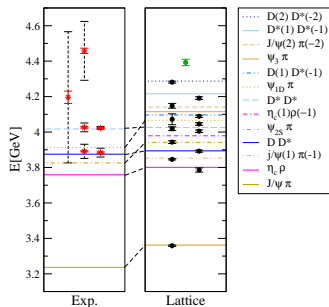


- Without  $\bar{q}q$  interpolators signal vanishes
- Simulations still unphysical in many ways
- Discretization and finite volume effects sizable!



- Makes interpretation as pure molecule or pure tetraquark unlikely

# Search for $Z_c^+$ with $I^G J^{PC} = 1+1^{+-}$

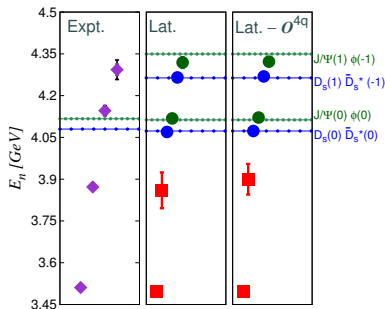


Prelovsek, Lang, Leskovec, DM, Phys.Rev. D91 014504 (2015)

- Simple level counting approach
- We find 13 two meson states as expected
- We find no extra energy level that could point to a  $Z_c$  candidate

# Search for $Y(4140)$ with $J^{PC} = 1^{++}$

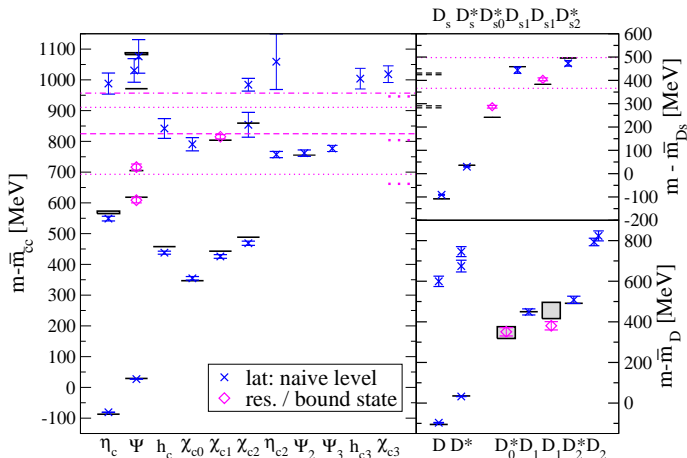
Padmanath, Lang, Prelovsek, arXiv:1503.03257



- Considered only  $J/\psi\phi$  and  $D_s\bar{D}_s^*$
- No additional energy level seen
- Further simulations necessary

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# Snapshot of spectrum calculations at $m_\pi = 266\text{MeV}$



- First principles determination of resonances and bound states just starting
- Program will have to be done with a number of lattice spacings/volumes

# Conclusions & Outlook

- Most current charmonium and charmed meson results are of an exploratory nature
- Low-lying charmonium states are under good control (full error budgets)
- New ideas might be needed for charm-annihilation contributions
- Progress with regard to hybrids and higher spin states
- Bound states and resonances close to  $D\bar{D}$ ,  $D\bar{D}^*$  and  $D^*\bar{D}^*$  can be investigated
- No signal yet for manifestly exotic four quark states
- For states well established on the lattice: calculation of radiative transitions, etc. possible (and will be done)
- A lot of progress but still much to do for excited charmonium

# Thank you!

... also to my collaborators Carleton DeTar, Andreas Kronfeld, Christian Lang, Song-Haeng Lee, Luka Leskovec, Ludmila Levkova, Sasa Prelovsek and Jim Simone



• • •

# Heavy quarks using the *Fermilab method*

El-Khadra et al., PRD 55, 3933

- We tune  $\kappa$  for the spin averaged **kinetic mass**  $(M_{\eta_c} + 3M_{J/\Psi})/4$  to assume its physical value
- General form for the dispersion relation

Bernard et al. PRD83:034503, 2011

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \dots$$

- We compare results from three different fit strategies
- Energy splittings are expected to be close to physical
- For MeV values of masses

$$M = \Delta M + M_{sa,phys}$$

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Hasenfratz et al. PRD 78 014515 (2008)

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PACS-CS, Aoki et al. PRD 79 034503 (2009)

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Peardon et al. PRD 80, 054506 (2009);

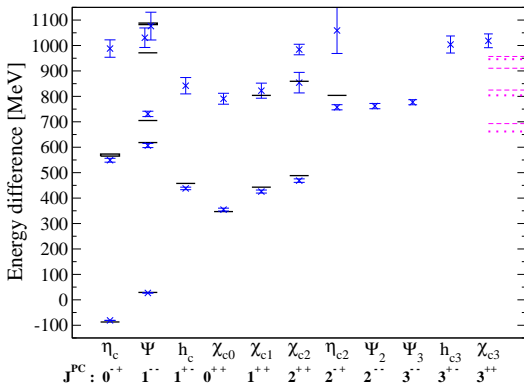
Morningstar et al. PRD 83, 114505 (2011)

# Testing our tuning: charm and light

	Ensemble (1)	Ensemble (2)	Experiment
$m_\pi$	266(3)(3)	156(7)(2)	139.5702(4)
$m_K$	552(1)(6)	504(1)(7)	493.677(16)
$m_\phi$	1015.8(1.8)(10.7)	1018.4(2.8)(14.6)	1019.455(20)
$m_{\eta_s}$	732.3(0.9)(7.7)	692.9(0.5)(9.9)	688.5(2.2)*
$m_{J/\Psi} - m_{\eta_c}$	107.9(0.3)(1.1)	107.1(0.2)(1.5)	113.2(0.7)
$m_{D_s^*} - m_{D_s}$	120.4(0.6)(1.3)	142.1(0.7)(2.0)	143.8(0.4)
$m_{D^*} - m_D$	129.4(1.8)(1.4)	148.4(5.2)(2.1)	140.66(10)
$2m_{\bar{D}} - m_{\bar{c}c}$	890.9(3.3)(9.3)	882.0(6.5)(12.6)	882.4(0.3)
$2M_{\bar{D}_s} - m_{\bar{c}c}$	1065.5(1.4)(11.2)	1060.7(1.1)(15.2)	1084.8(0.6)
$m_{D_s} - m_D$	96.6(0.9)(1.0)	94.0(4.6)(1.3)	98.87(29)

- A single ensemble: **Discrepancies** due to discretization and unphysical light-quark masses **expected**

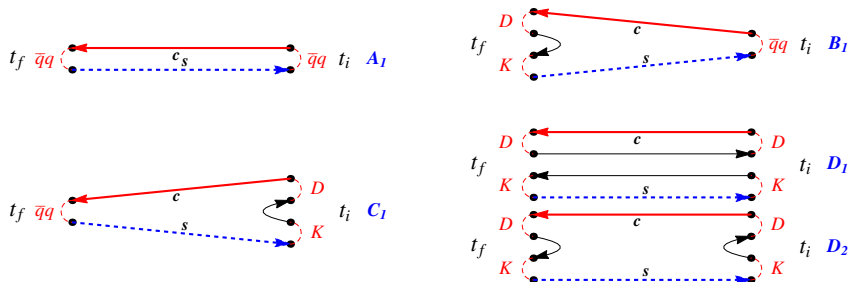
# Low-lying charmonium spectrum



DM, S. Prelovsek, R. M. Woloshyn, PRD 87 034501 (2013);

- Serves as further confirmation of our heavy-quark approach
- Data from 1 ensemble; Errors statistical + scale setting

# Contractions



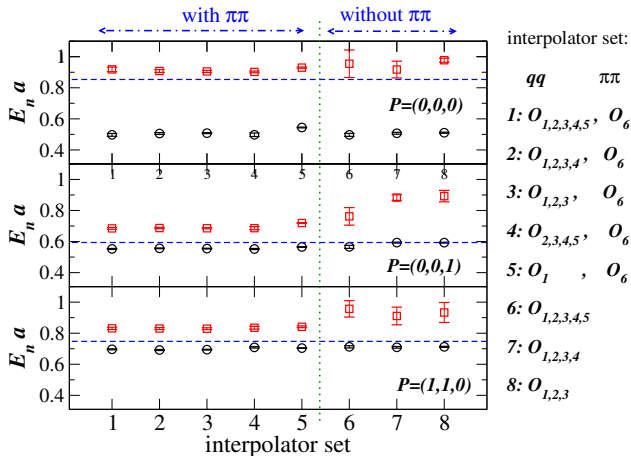
- Handled efficiently within the distillation method

Peardon et al. PRD 80, 054506 (2009)

Morningstar et al. PRD 83, 114505 (2011)

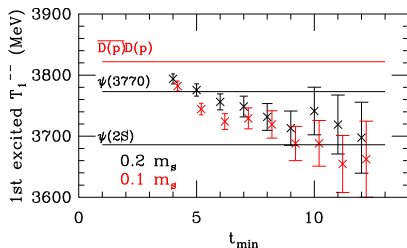
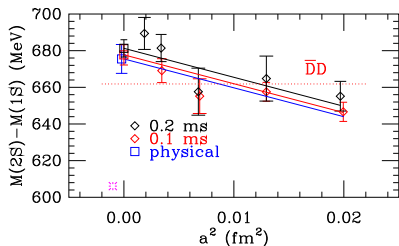
# An example: Different rho momentum frames

Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011) & erratum ibid



# 2S states

Fermilab/MILC, DeTar et al. arXiv:1410.3343



- Simple analysis of 2S states lead to splittings much larger than in experiment
- Strong dependence on fit range and very noisy data