The Pion Transition Form Factor

Khépani Raya-Montaño

Adnan Bashir - Javier Cobos

Workshop of the APS topical group on hadronic physics.
Baltimore, MD. Apr. 8-10

Outline

- Introduction (Why pion and SDEs?)
- The tools: SDEs
- Pion Transition Form Factor
- Conclusions (hits and misses)

Introduction

- Understanding strong interactions are still being a challenge for physicists; although scientists have developed a powerful theory for studying them, which is Quantum Chromodynamics (QCD).
- Quarks and gluons are the fundamental degrees of freedom of the theory, however, they are not found free, instead they form composite states called hadrons.
- Hadronic form factors are intimately related to its internal structure. But, due to the nonperturbative nature of QCD, unraveling hadronic form factors from first principles is an outstanding problem.
- Then we have Schwinger-Dyson Equations (SDEs). SDE are the equations of QCD and they combine the IR and UV behavior of the theory at once, therefore, SDE are an ideal platform to study quarks and hadrons.

Why pion and SDEs ?

Trivial reasons:

- Pion is the lightest and simplest particle made by quarks. Simple means: two-particle bound state (not three as in baryons), spin 0, pseudoscalar.
- Pion is good for testing: If your model gives you the right pion mass and leptonic decay constant, you are pointing in the right direction.

But, more important:

- Pion is the archetype for meson exchange forces. Also, it came to be considered as an ordinary quark-antiquark quantum mechanical state when constituent quark model were created. Then, it occupies a special place for both nuclear and particle physics.
- Quarks inside hadrons acquire mass through a mechanism called Dynamical Chiral Symmetry Breaking (DCSB). Pions exist (as Goldstone bosons) due to the breaking of the same symmetry. Therefore, **one can relate the one-body problem** (quark propagator) **with the two-body problem** (quark-antiquark bound state).
- Pion has been widely studied (experiment, SDE, lattice) in the last years: Elastic and transition form factors, valence quark distributions, PDA's, PDF's, etc.

Pion and SDEs studies

- **Elastic and transition form factor:**

[1] **Phys.Rev. C65 045211 (2002)** by P. Tandy et al. (SDE)

[2] **Phys.Rev. C81 065202 (2010)** by Xiomara et al. (SDE)

[3] **Phys. Rev. D80 052002 (2009)** BaBar (Experiment)

[4] **Phys. Rev. D86 092007 (2012)** Belle (Experiment).

- **Valence quark distributions:**

[5] **Phys.Rev. C83 062201 (2011)** by Adnan et al. (SDE)

- **PDAs and PDFs:**

[6] **Phys.Rev.Lett. 110 13, 132001 (2013)** by Cobos et al. (SDE).

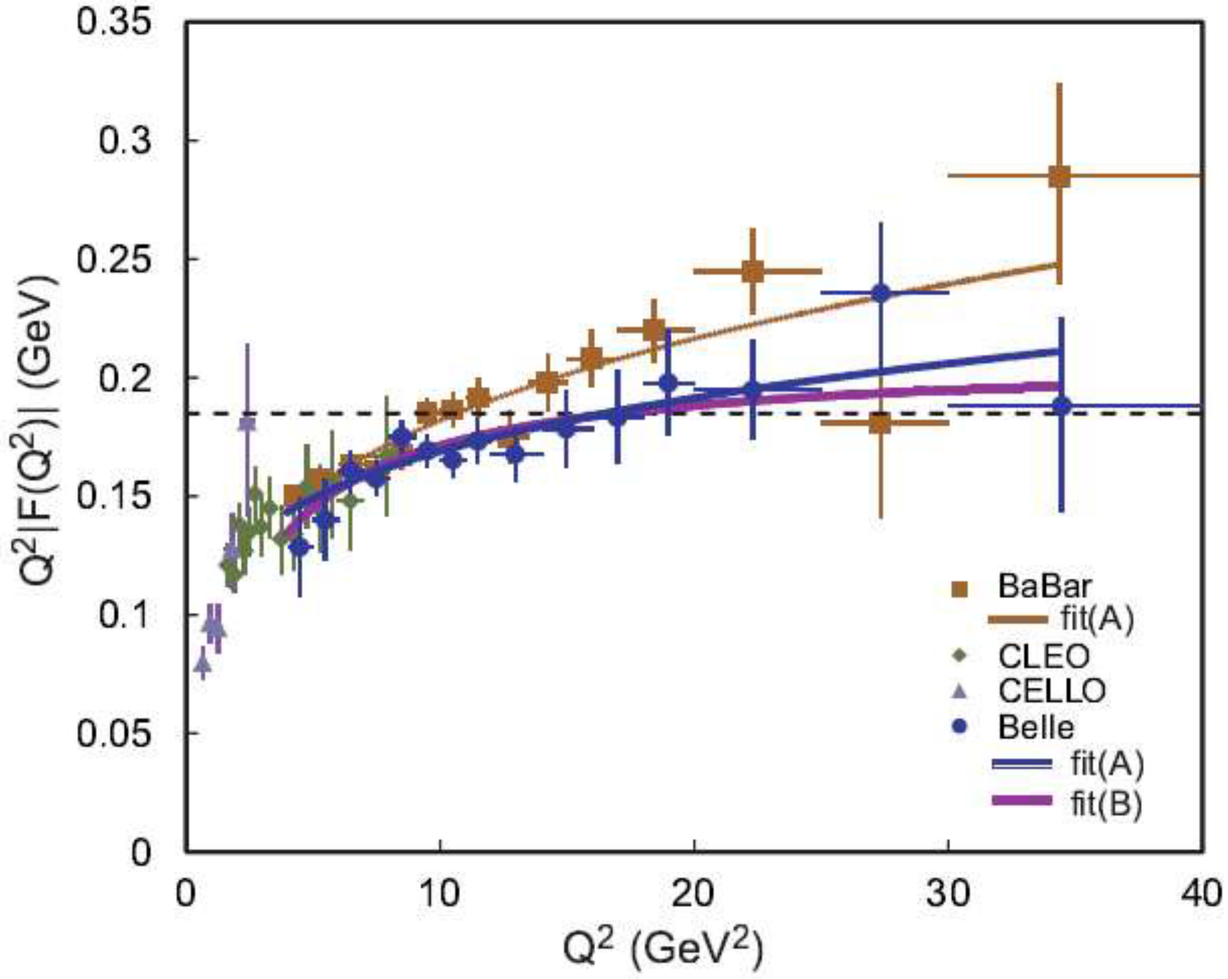
[7] **Phys.Rev. D68 034025 (2003)** by A.W. Thomas et al (Lattice).

- Elastic form factors, valence quark distributions, masses, charge radii and decay constants have been also studied for other hadrons (such as Kaon) through SDEs. **SDEs are a very powerful tool.**

"Collective Perspective on advances in DSE QCD"

Commun.Theor.Phys. 58 79 (2012) by Adnan et plures.

Pion transition form factor - $\gamma\gamma^* \rightarrow \pi^0$

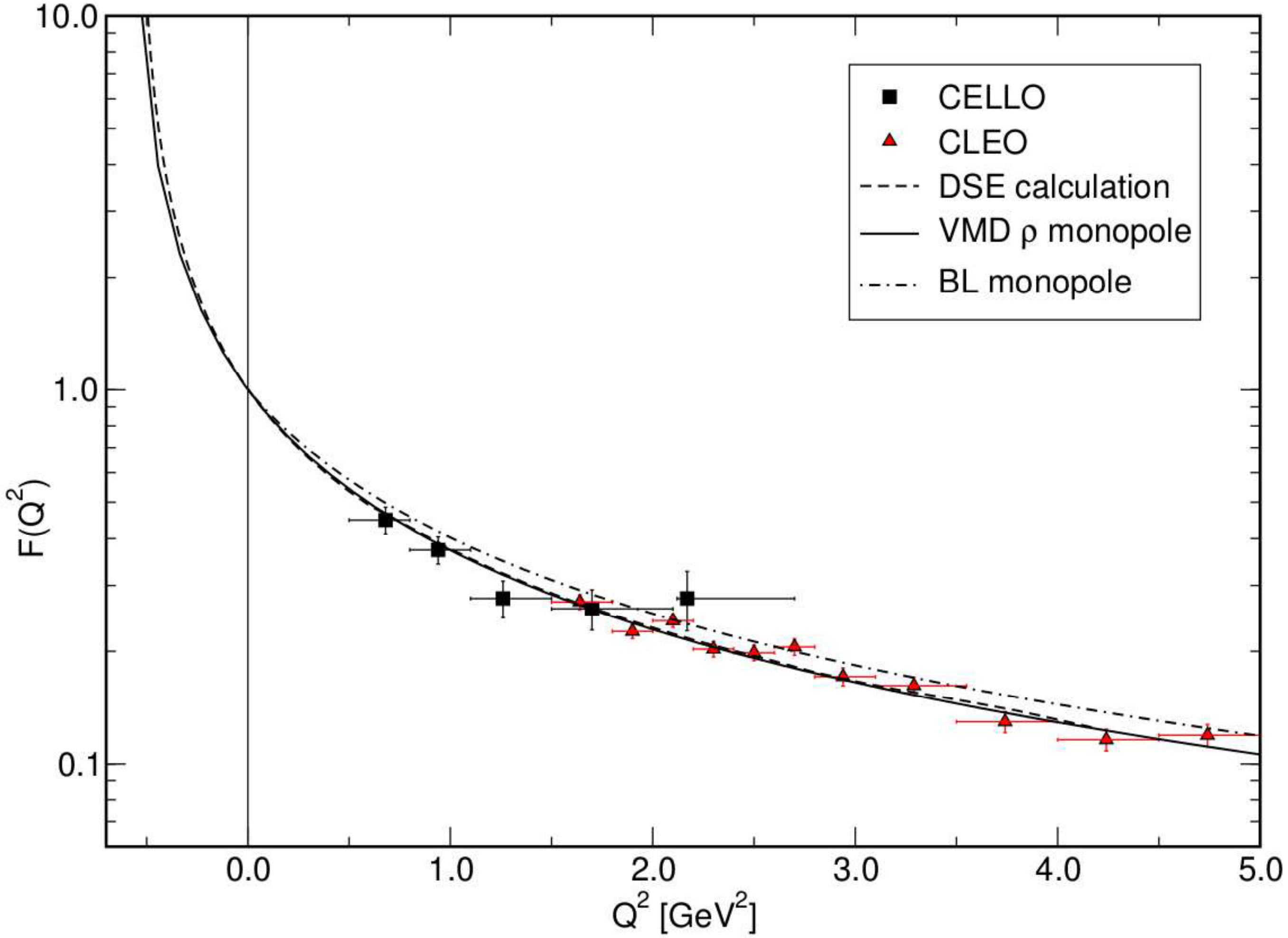


- The pion transition form factor is measured through the process $e^+e^- \rightarrow e^+e^-\pi^0$.
- Many experiments have been done so far; however, at large Q^2, there is not agreement between the experimental data.
- High Q^2 measurements correspond to BaBar [3] and Belle [4] experiments.
- Dashed line is the well known asymptotic limit $2f_\pi$:

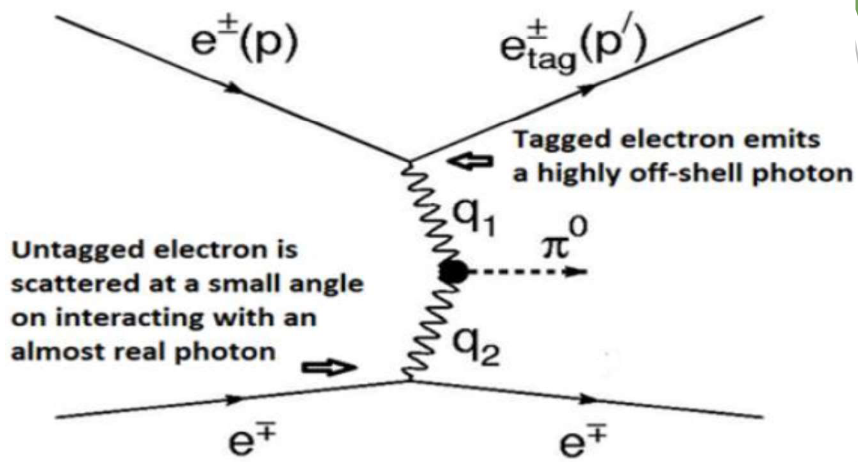
[8] G.P . Lepage and S.J. Brodsky, Phys.Rev. D22, 2157 (1980)

Pion transition form factor

- We have theoretical input by P. Maris and P. Tandy [1] :



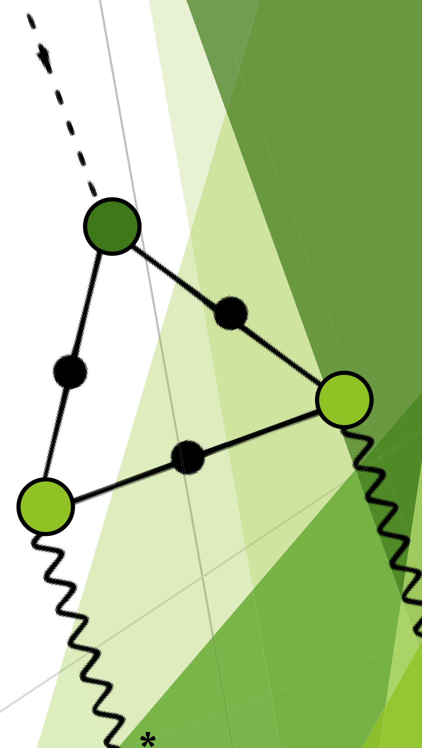
The tools: SDEs



1. Quark Propagator

2. Bethe-Salpeter Amplitudes

3. Quark-Photon Vertex



The tools: Quark propagator

- The renormalized SDE for the quark propagator is written as:

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F}(i\gamma\cdot p) + \mathcal{Z}_4 m(\mu) + \mathcal{Z}_{1F}\int_q^{\Lambda} g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\Gamma^a_\nu(p,q,\mu)\ .$$

- In the rainbow-ladder truncation:

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F}(i\gamma\cdot p) + \mathcal{Z}_4 m(\mu) + \mathcal{Z}_{1F}\int_q^{\Lambda} G(p-q) D^0_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\frac{\lambda^a}{2}\gamma_\nu\ .$$

- Where G(p-q) is an effective coupling.

The tools: Quark propagator

- For G(p-q) we choose the form given at **Phys.Rev. C60 055214 (1999)** [9] by P. Maris and M. Tandy:

$$\frac{G(k^2)}{k^2} = \frac{4\pi^2}{\omega^6} D\, k^2 e^{-k^2/\omega^2} + (2\pi)^2 \frac{\gamma_m}{1/2 \ln[\tau + (1 + k^2/\Lambda_{QCD}^2)^2]} F(k^2) \, ,$$

$$F(k^2) = [1 - \exp(-k^2/(4m_t^2))]/k^2 \, .$$

- The first term fixes the condensate and the pion/kaon mass (once we fix the mass of the light quarks).

- The second term reproduces the strong coupling at 1-loop.

The tools: Quark propagator

- The quark propagator can be written in many ways:

$$S^{-1}(p,\mu) = i\,\gamma\cdot p\;A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{i\,\gamma\cdot p + M(p^2,\mu^2)}{Z(p^2,\mu^2)}\,.$$

$$S(p,\mu) = -i\,\gamma\cdot p\;\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)\,.$$

- The renormalization condition below ensures that we get the correct perturbative behavior:

$$S^{-1}(p,\mu)|_{p=\mu} = i\,\gamma\cdot p + m(\mu) \Rightarrow Z(\mu^2,\mu^2) = 1,\; M(\mu^2,\mu^2) = m(\mu)$$

- The second definition resembles the form of the free propagator; then, we call M(p) ***Mass Function*** and Z(p) ***Wavefunction renormalization***.

The tools: N-ccp parametrization

- The quark propagator is written as:

$$S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)\,.$$

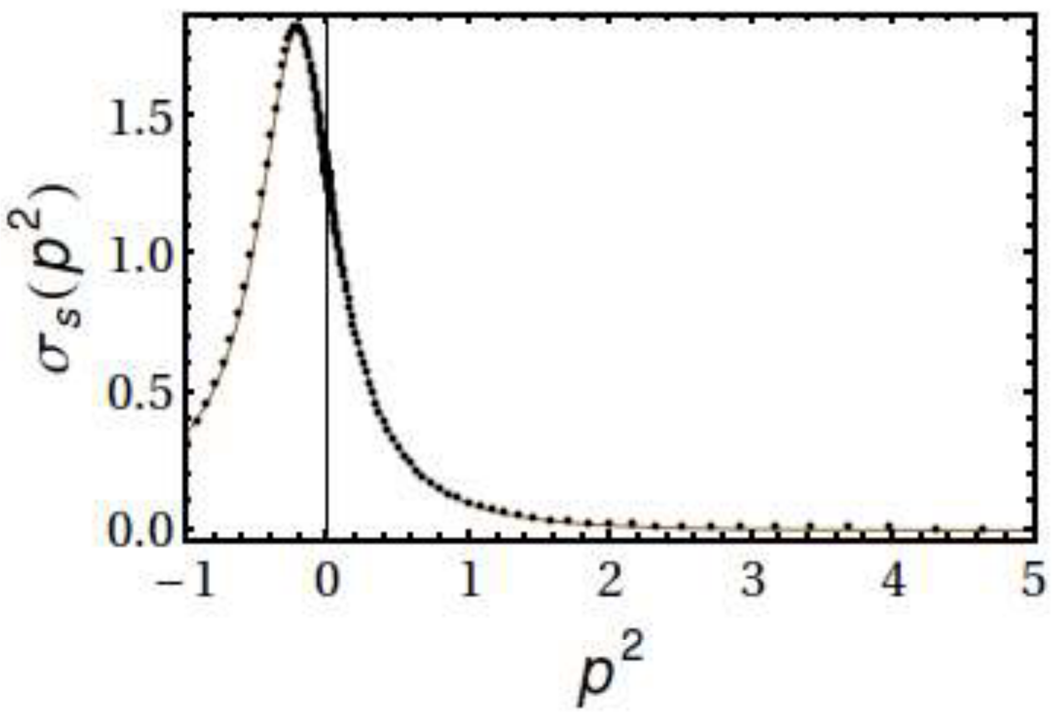
- It can be written in terms of N pairs of complex conjugate poles:

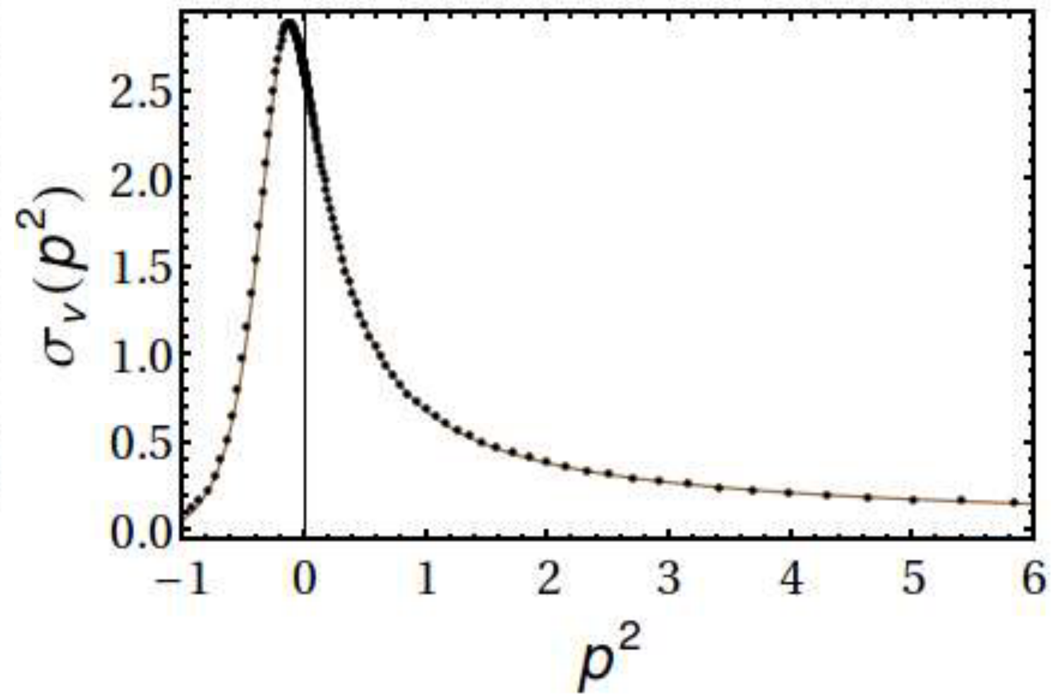
$$\sigma_v(q) = \sum_{k=1}^{N}\left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}}\right)\,,\quad \sigma_s(q) = \sum_{k=1}^{N}\left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}}\right)\,.$$

- Constrained to the UV conditions of the free propagator form.

N. Souchlas [P. Tandy] (2009). *Quark Dynamics and Constituent Masses in Heavy Quarks Systems*. **PhD. Thesis.**

The tools: Nccp-parametrization





- Scalar and vectorial parts of the quark propagator: [Line] 3ccp-parametrization, [Dots] SDE solution.

The tools: Bethe-Salpeter equation

- The Bethe-Salpeter equation (BSE) is written as:

$$\Gamma^{ab}_{M}(p;P) = \int_q^{\Lambda} K(p,q;P) S^a(q+\eta P)\Gamma^{ab}_{M}(q;P)S^b(q-(1-\eta)P)$$

- Where the Bethe-Salpeter amplitude (BSA) for the pion is:

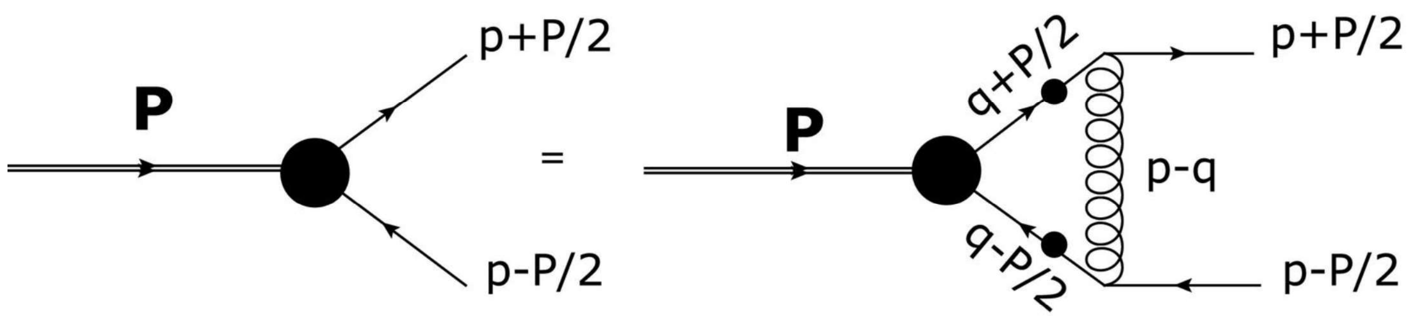
$$\Gamma^{qq}_{\pi}(p;P) = i\gamma_5\, E_\pi(p;P) + \gamma_5\gamma\cdot P\; F_\pi(p;P) \\ +\gamma_5(\gamma\cdot p)(p\cdot P)\; G_\pi(p;P) + \gamma_5\; p_\alpha \sigma_{\alpha\beta} P_\beta\; H_\pi(q;P)$$

The tools: Bethe-Salpeter equation

- In the rainbow-ladder truncation:

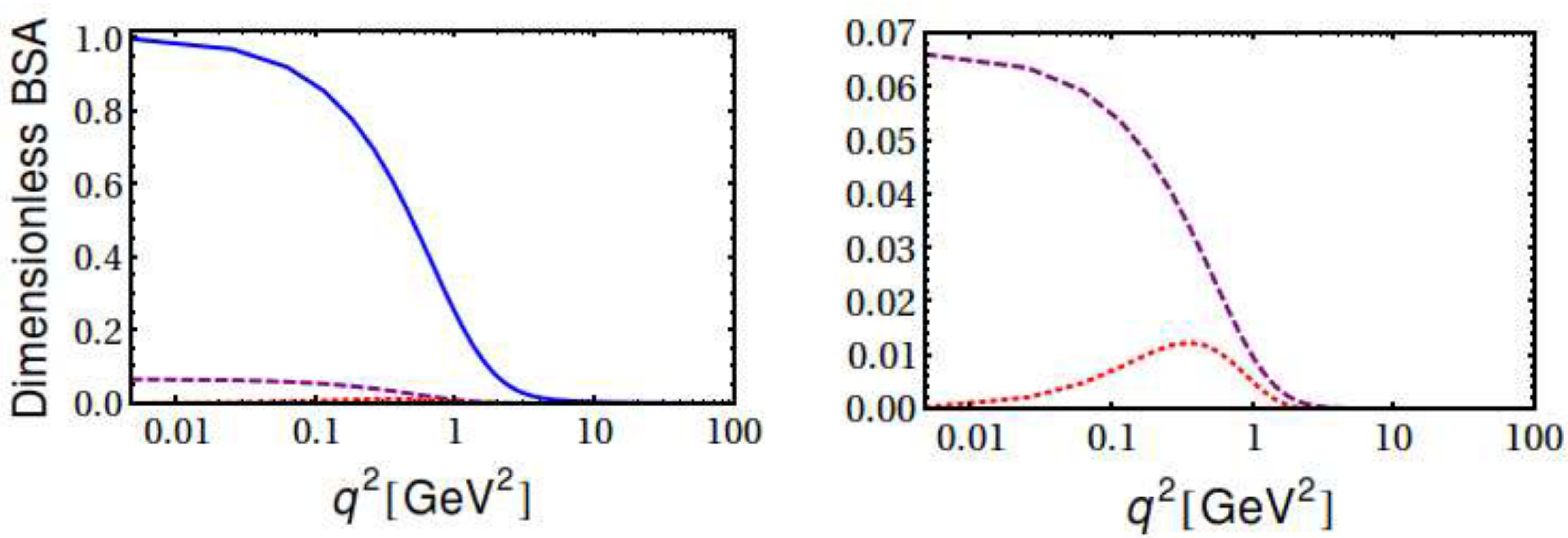
$$\Gamma_M^{ab}(p;P) = -\int_q^\Lambda \frac{G(k^2)}{k^2} D^0_{\mu\nu}(k) \frac{\lambda^c}{2} \gamma_\mu S^a(q+\eta P) \Gamma_M^{ab}(q;P_i) S^b(q-(1-\eta)P) \frac{\lambda^c}{2} \gamma_\nu \, .$$

- Which corresponds to:



when the quarks share equal amount of momentum.

The tools: Bethe-Salpeter equation



- **Dimensionless Bethe-Salpeter amplitudes.** [Blue] E(q,P), [Purple] m_π F(q,P), [Red] m_π q^2 G(q,P).

The tools: Nakanishi representation

- We parametrize the BSA using a Nakanishi-like representation (**Phys.Rev. 130 1230-1235 (1963)).** Which consists in splitting the BSA into IR and UV parts and writting them as follows:

$$A(q,P) = \int_{-1}^{1} dz \int_{0}^{\infty} d\Lambda \left[\frac{\rho^i(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{m+n}} + \frac{\rho^u(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{n}} \right] .$$

- Where the spectral density is written as:

$$\rho^{i,u}(z,\Lambda) = \rho_1(z)\delta(\Lambda - \Lambda^{i,u}) + \cdots$$

- Our form of A is slightly different. First, we choose:

$$\rho_1(z) = \rho_\nu(z) \sim (1-z^2)^\nu .$$

The tools: Nakanishi representation

- Then choose the parametrization explained in [6]. Therefore:

$$A^i(k,P) = c_A^i \int_{-1}^{1} dz \rho_{\nu_A^i}(z)[b_A \hat{\Delta}^4_{\Lambda_A^i}(k_z^2) + \bar{b}_A \hat{\Delta}^5_{\Lambda_A^i}(k_z^2)] \,. \quad E^u(k,P) = c_E^u \int_{-1}^{1} dz \rho_{\nu_E^i}(z) \hat{\Delta}_{\Lambda_E^u}(k_z^2) \,.$$

$$F^u(k,P) = c_F^u \int_{-1}^{1} dz \rho_{\nu_F^u}(z) \Lambda_F^u k^2 \hat{\Delta}^2_{\Lambda_F^u}(k_z^2) \,. \quad G^u(k,P) = c_G^u \int_{-1}^{1} dz \rho_{\nu_G^u}(z) \Lambda_G^u \hat{\Delta}^2_{\Lambda_G^u}(k_z^2) \,.$$

- A stands for amplitude (E,F,G); *i*, *u* for IR and UV. H(k,P) is negligible. Λ, ν, c, b are parameters fitted to the numerical Data. The rest is defined as:

$$\hat{\Delta}_\Lambda(s) = \Lambda \, \Delta_\Lambda(s) \,, \; \Delta_\Lambda(s) = (s + \Lambda^2)^{-1} \,, \; k_z^2 = k^2 + z \, k \cdot P \,.$$

The tools: Quark-Photon vertex.

- At first, we chose bare quark-photon vertex. But we found some problems:

1. Chiral anomaly says that G(0) = 0.5 [2], however, with bare vertex G(0) << 0.5.
2. Then, there is an underestimation at low momentum, and
3. Three summations over ccp => Higher computational time.

- There are many other choices in the market:

1. **Phys.Rev. C85 044205 (2012).** Adnan, Rocío et al.
2. **Phys.Rev. D79 125020 (2009).** A. Kizilersü, M.R. Pennington.
3. **Phys.Rev.Lett. 103, 081601 (2009).** L. Chang and C.D. Roberts.

- However, we model one which satisfies the WI and, at the same time, it results useful for the numerical calculation.

The tools: Quark-Photon vertex.

- We employ unamputated vertex ansatz:

$$S\Gamma_\mu S \to \chi(k_f, k_i) = \sum_{i=1}^{3} T_{\mu i} X_i(k_f, k_i)$$

- Where the tensor structures are:

$$T_{1\mu} = \gamma_\mu$$
$$T_{2\mu} = \beta\gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{\beta}\gamma \cdot k_i \gamma_\mu \gamma \cdot k_f$$
$$T_{3\mu} = i\,\beta(\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + i\bar{\beta}(\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f)$$

- And, the dressing functions:

$$X_1(k_f, k_i) = \Delta_{k^2\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_2(k_f, k_i) = \Delta_{\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_3(k_f, k_i) = \Delta_{\sigma_S}(k_f^2, k_i^2)\,.$$

The tools: Quark-Photon vertex.

- The following definitions apply:

$$\Delta_F(k_f, k_i) = \frac{F(k_f) - F(k_i)}{k_f - k_i}$$

$$\beta = 1 + \alpha e^{-Q^2/(2M^2)}$$

$$\bar{\beta} = 1 - \beta$$

- Where M = M(p^2 = 0) and alpha is set such that G(0) = 1/2.

- Unamputated vertex allows us to remove one of the summations over the 2ccp representation, and then, we need to employ less computational time.

Pion Transition Form Factor

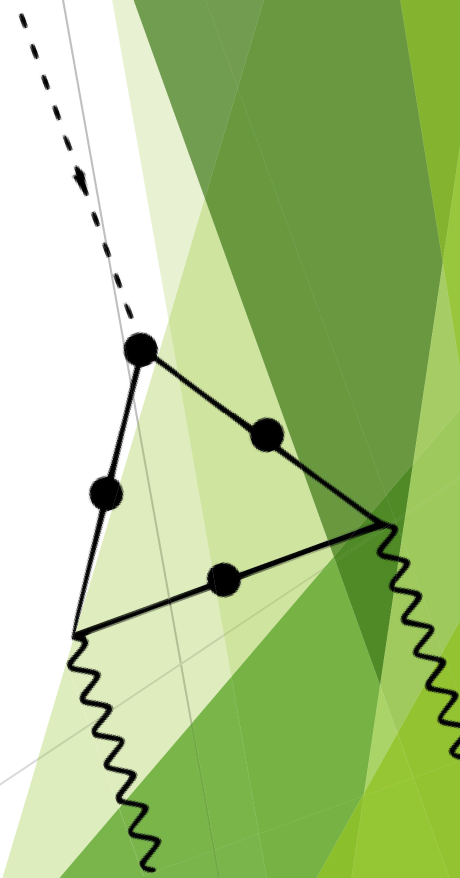
- The pion transition form factor (TFF) is written as:

$$\mathcal{T}_{\mu\nu}(k_1, k_2) = T_{\mu\nu}(k_1, k_2) + T_{\nu\mu}(k_2, k_1) \ ,$$

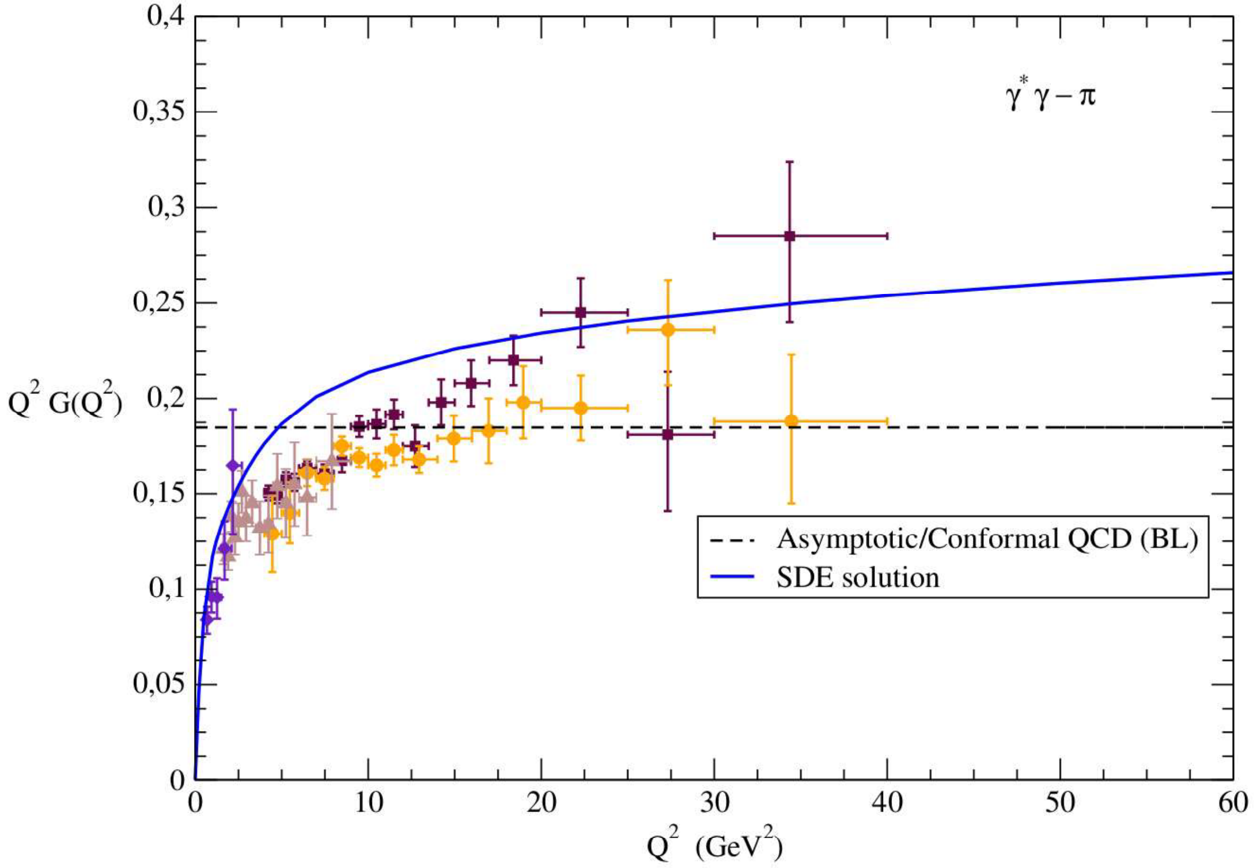
$$\begin{aligned} T_{\mu\nu}(k_1, k_2) &= \frac{\alpha_{em}}{\pi f_\pi} \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G(k_1^2, k_2^2, k_1 \cdot k_2) \\ &= \mathrm{tr} \int \frac{d^4 l}{(2\pi)^4} \chi_\pi(l_1, l_2) i\Gamma_\mu(l_2, l_{12}) S(l_{12}) i\Gamma_\nu(l_{12}, l_1) \ . \end{aligned}$$

- The parametrizations of quark propagator and BSA allows us to solve analytically the integrations over momentum after Feynman Parametrization.

- And then, numerically integrate over Feynman Parameters.



Transition Form Factor

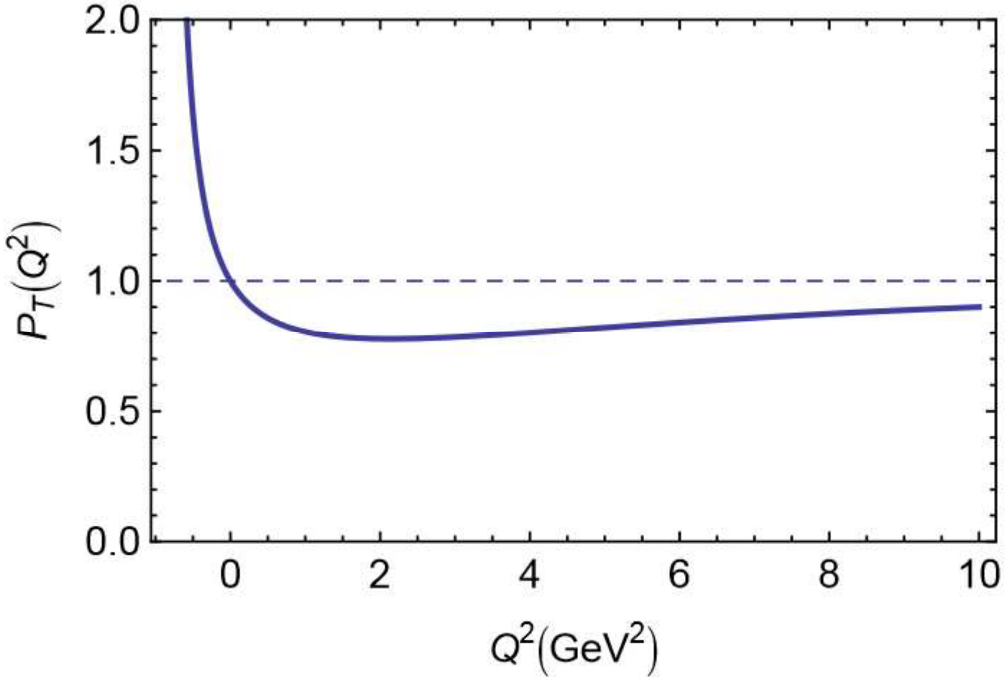


- **Transition Form Factor:** SDE calculation and comparison with experiment. At this point, there is an overestimation of the experimental data.

The tools: Rho-pole effect.

- In the present calculation, we only rely on the WTI solution of vertex ansatz that does not contain any meson poles contribution. One can add a transverse part to the vertex and try a simple contact model [2]:

$$\gamma_\mu^T P_T(Q^2) = \frac{\gamma_\mu^T}{1+K_\gamma(Q^2)}\,, \quad K_\gamma(Q^2) = \frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\alpha(1-\alpha)Q^2\,\overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,Q^2))\,.$$



The tools: Rho-pole effect.

- Or, we could try the relative momentum dependent model, proposed by Tandy et al. at **Phys.Rev. C61 (2000) 045202.**:

$$\Gamma_\mu(q;Q) = \Gamma_\mu^{\mathrm{BC}}(q;Q) - \gamma_\mu^T \frac{N_\rho}{1+q^4/\omega^4} \frac{f_\rho\, Q^2}{m_\rho(Q^2+m_\rho^2)} e^{-\alpha(Q^2+m_\rho^2)} \,.$$

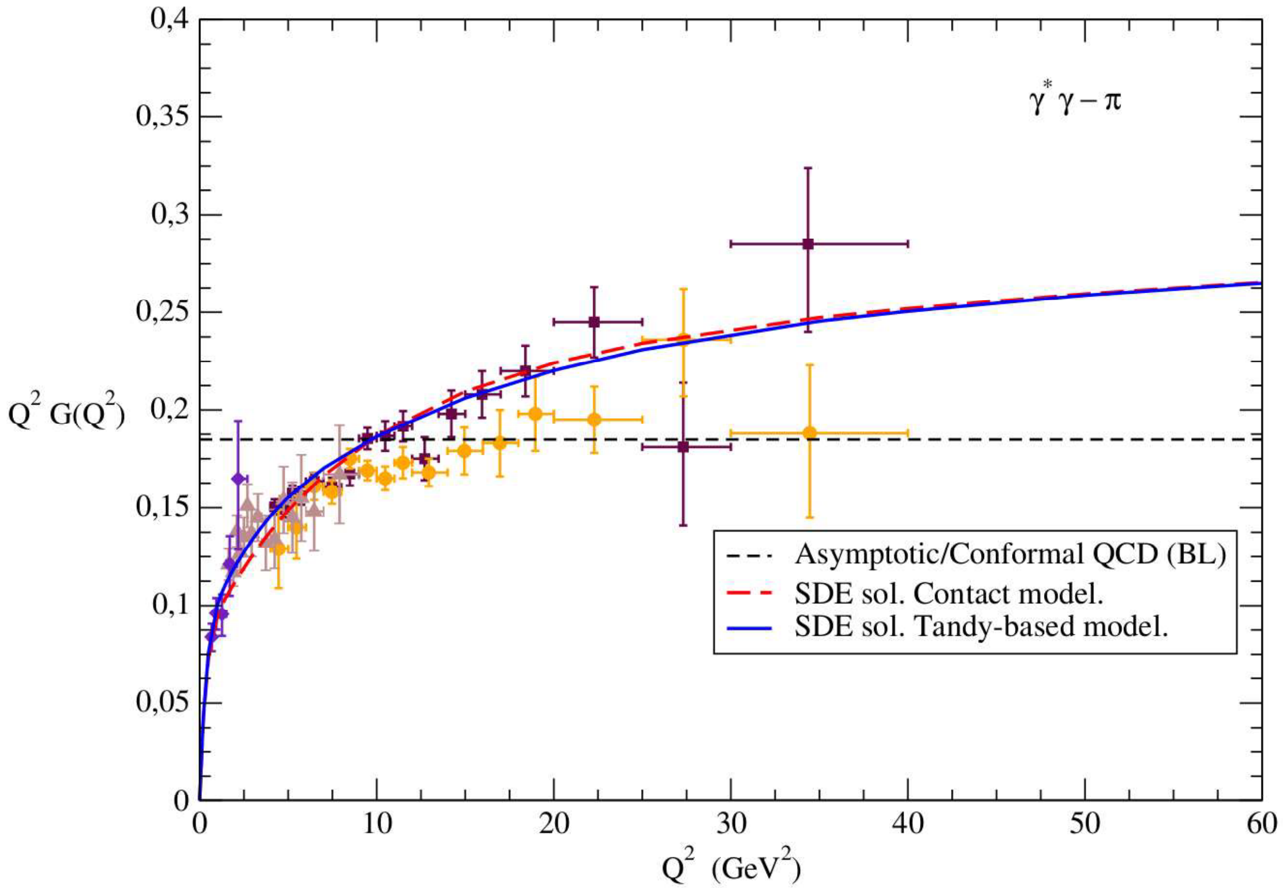
- Where the parameters omega, alpha and Np are chosen to reproduce the pion charge radii.

- Instead, we leave the longitudinal part as shown before and model the transverse part as follows:

$$f(q;Q)\gamma_\mu^T \to -\gamma_\mu^T \frac{1}{a_0 + a_2 Q^2 + a_3 Q^4} \frac{f_\rho Q^2}{m_\rho(Q^2+m_\rho^2)} e^{-\alpha(Q^2+m_\rho^2)} \,.$$

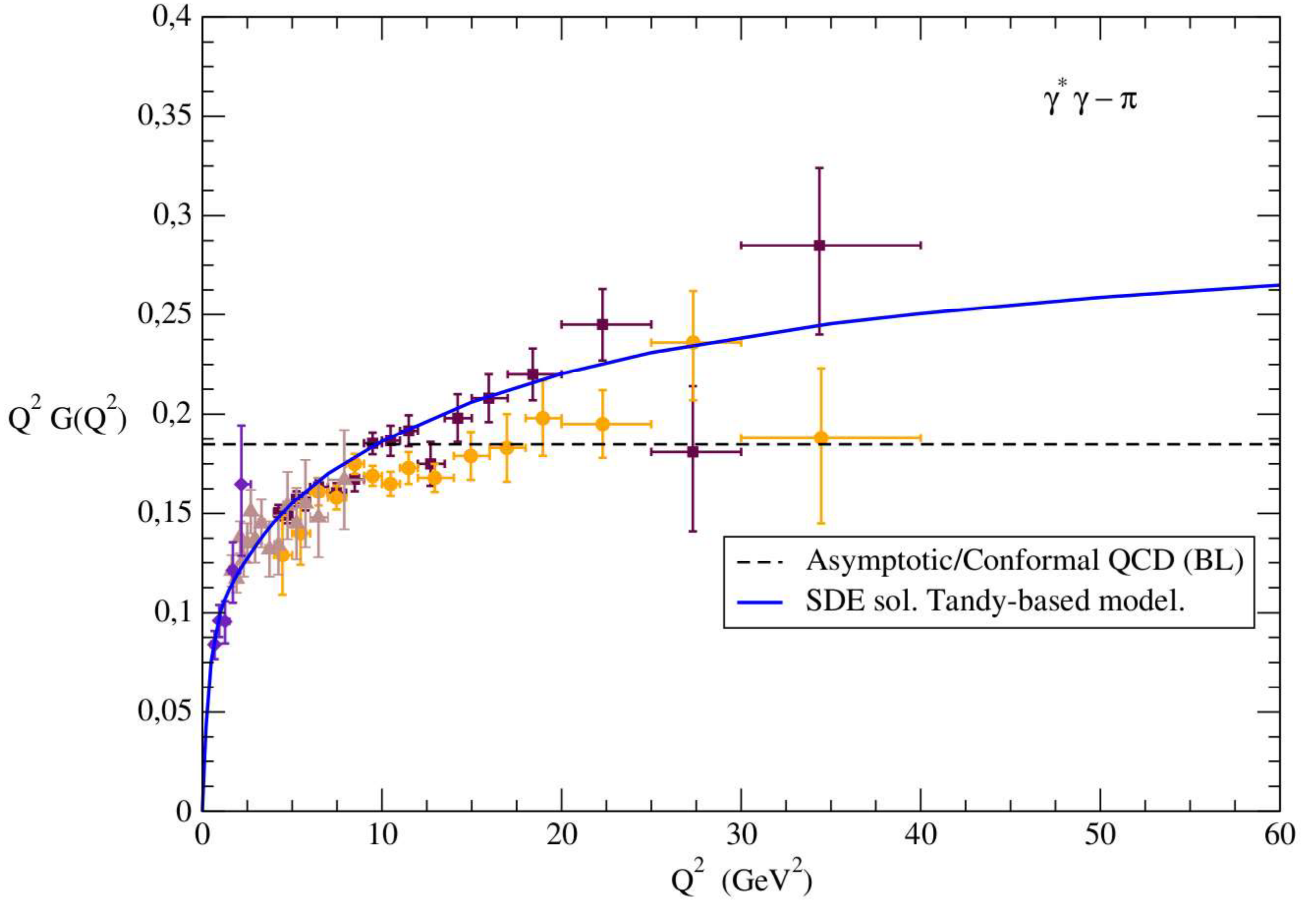
- We fit the coefficients *a* to the experimental data.

TFF: Rho Pole Models



- **Transition Form Factor:** SDE calculation and comparison with experiment. [Red, dashed] Contact model. [Blue, solid] Tandy-based model.

TFF: Rho Pole Model



- **Transition Form Factor:** SDE calculation and comparison with experiment. Tandy-based model.

TFF: Asymptotic limit

- According to [1], the asymptotic limit is:

$$G(Q_1^2, Q_2^2; \mu) \to f_\pi \left\{ \frac{J_\omega(\mu)}{Q_1^2 + Q_2^2} + O\left(\frac{\alpha_s}{\pi}, \frac{1}{(Q_1^2 + Q_2^2)^2} \right) \right\} ,$$

where:

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} \, , \quad J_\omega(\mu) = \frac{4}{3} \int_0^1 dx \frac{\phi_\pi(x, \mu)}{1 + \omega^2 (2x - 1)} \, .$$

- G(Q) has been divided by $2\pi^2 f_\pi$, in order to match the experimental convention. We call this normalization ***"Constant weighing"***.

TFF: Asymptotic limit

- For asymptotic QCD, we have:

  $$\phi_\pi^{asym}(x, \mu \to \infty) = 6x(1-x) \ .$$

- Therefore:

  $$Q^2 G(Q^2) \to 2 f_\pi \ .$$

- This is called the *conformal* limit, or Brodsky-Lepage (BL) Limit [8].

- However, since we choose a finite renormalization point (μ = 2 GeV) for quark propagator and BSA, we expect a different PDA; therefore, we expect a different limit too.

TFF: Asymptotic limit

- The PDA at finite renormalization point μ = 2 GeVis:

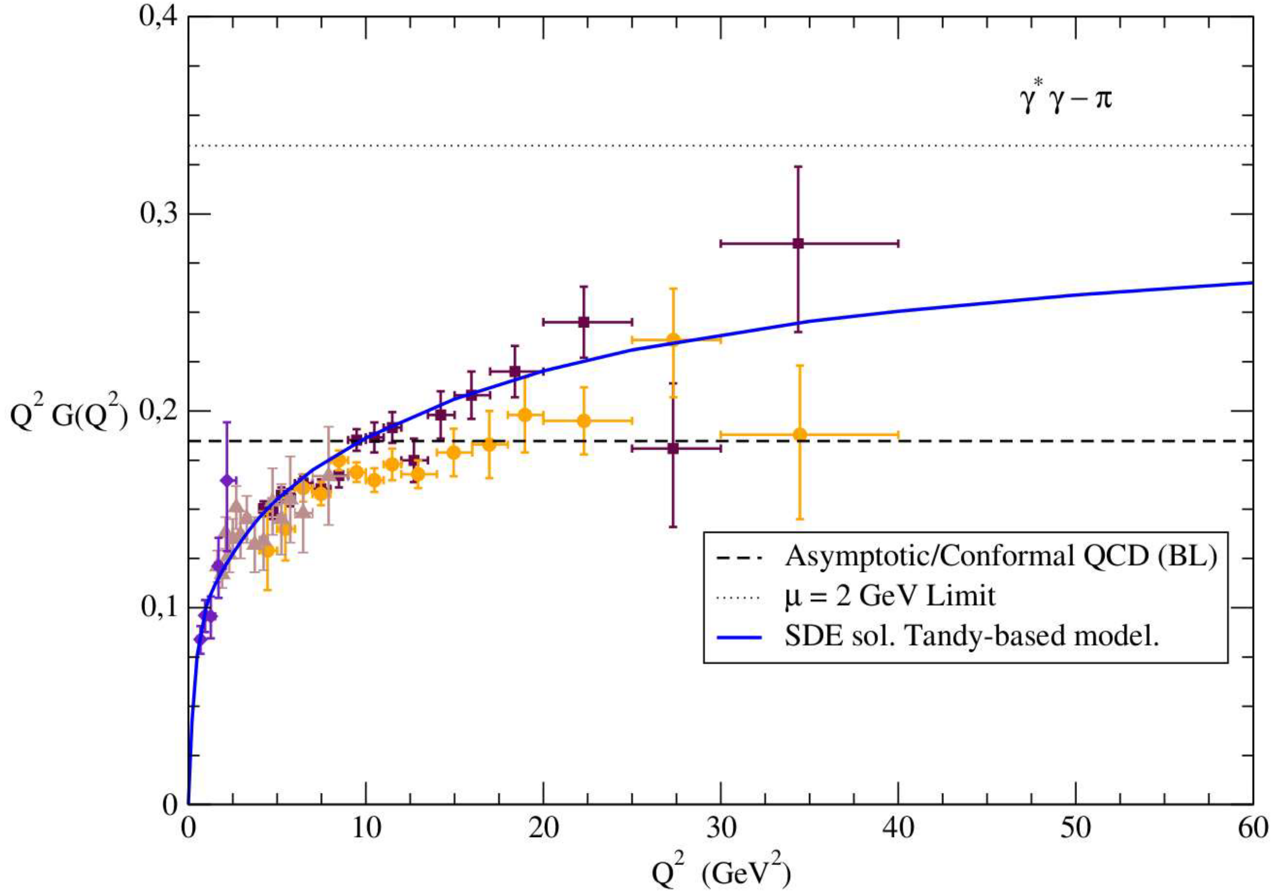
$$\phi_\pi(x,\mu) = 1.71[x(1-x)]^{\alpha_-}[1 + a_2 C_2^{\alpha_+}(2x-1)] \, .$$

- Where the parameters *a* and α were taken from [6]. C(z) are Gegenbauer polynomials.

- Thus we expect, at μ = 2 GeV:

$$Q^2 G(Q^2) \to f_\pi J_1(\mu = 2\ \mathrm{GeV}) = 3.62\ f_\pi \, .$$

TFF: Asymptotic limit



- **Transition Form Factor:** SDE calculation and comparison with experiment. [Blue, solid] SDE solution . [Black, dotted] Asymptotic limit at μ = 2GeV .

TFF: Asymptotic limit - Q^2 Evolution

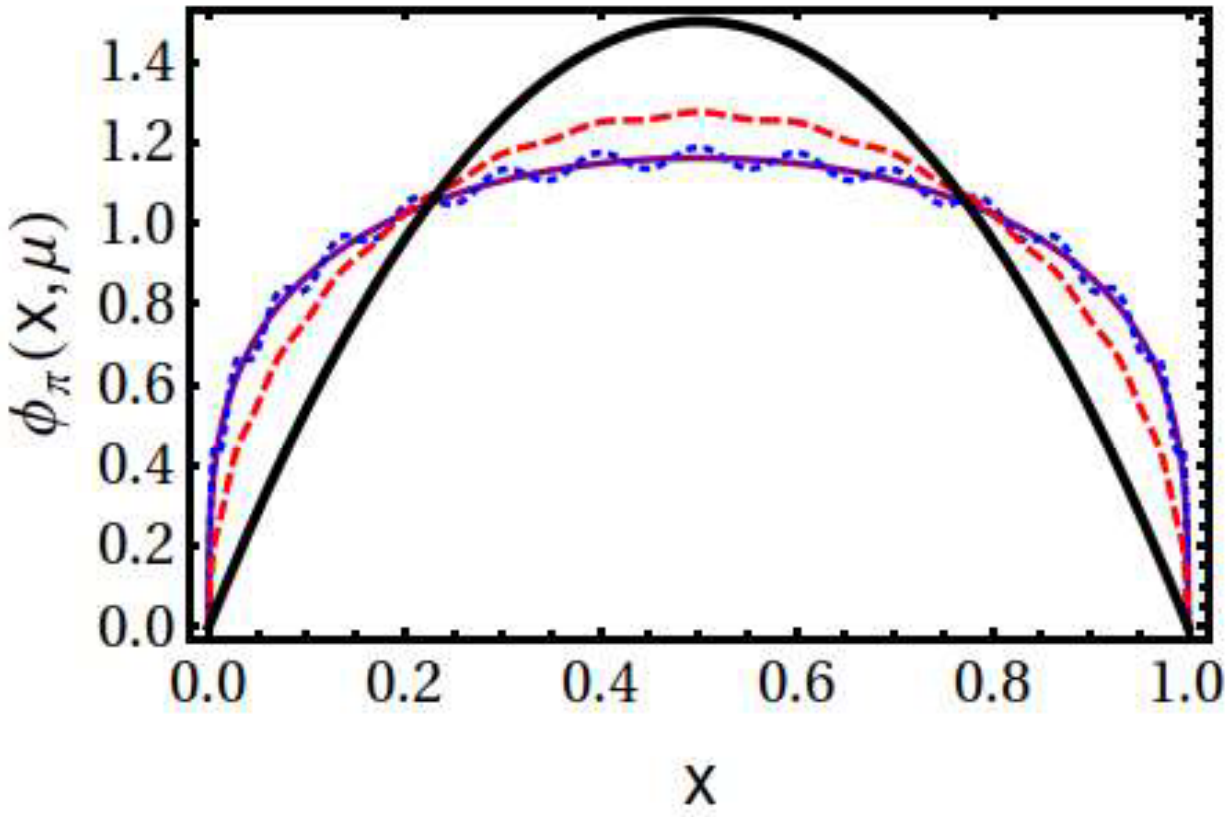
- We expect that the finite μ limit evolve with $\mu^2 = Q^2$; then, for $Q^2 \to \infty$, we arrive at the asymptotic limit.

- This translates into: $Q^2 G(Q^2) \to f_\pi J_1(\mu^2 = Q^2)$. Which implies that we approximate to the conformal limit as Q^2 grows.

- If one knows the 3/2-Gegenbauer expansion at some point mu, we could know the 3/2 expansion at a different mu through the ERBL evolution equations, **Phys.Rev.Lett. 11 092001 (2013)**.

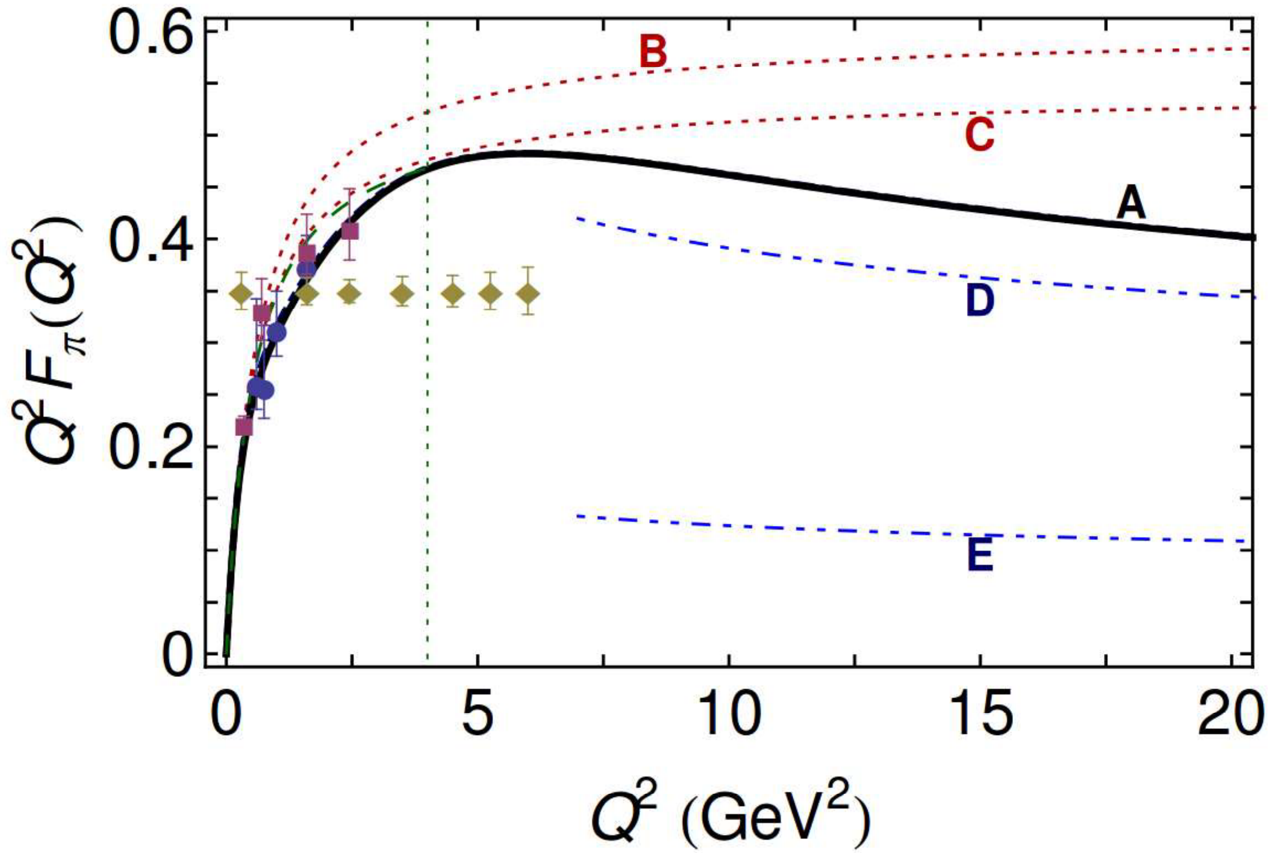
- This idea was already used for the Pion Elastic Form Factor: **Phys.Rev.Lett. 111 14 141802 (2013) by Chang et al.**

Pion Distribution Amplitude



- **PDA:** [Purple] SDE calculation at μ = 2 GeV. [Dotted, Blue] Gegenbauer-3/2 expansion at μ = 2GeV. [Dashed, Red] Gegenbauer-3/2 expansion at μ = 6.6 GeV. [Black] Asymptotic PDA 6x(x-1).

Pion Elastic Form Factor



- **Pion Elastic Form Factor.** [A] SDE calculation. [B-C] Monopole expansion fits. [D] Finite μ limit. [E] Asymptotic limit. We see that the curve is much more closer to the finite mu limit than the asymptotic limit.

TFF: Asymptotic limit - Q^2 Evolution

- That is for the asymptotic limit. What about G(Q)?

- We start with:

$$Q^2 G(Q^2; \mu = 2\ \mathrm{GeV}) \to f_\pi J_1(\mu = 2\ \mathrm{GeV}),$$

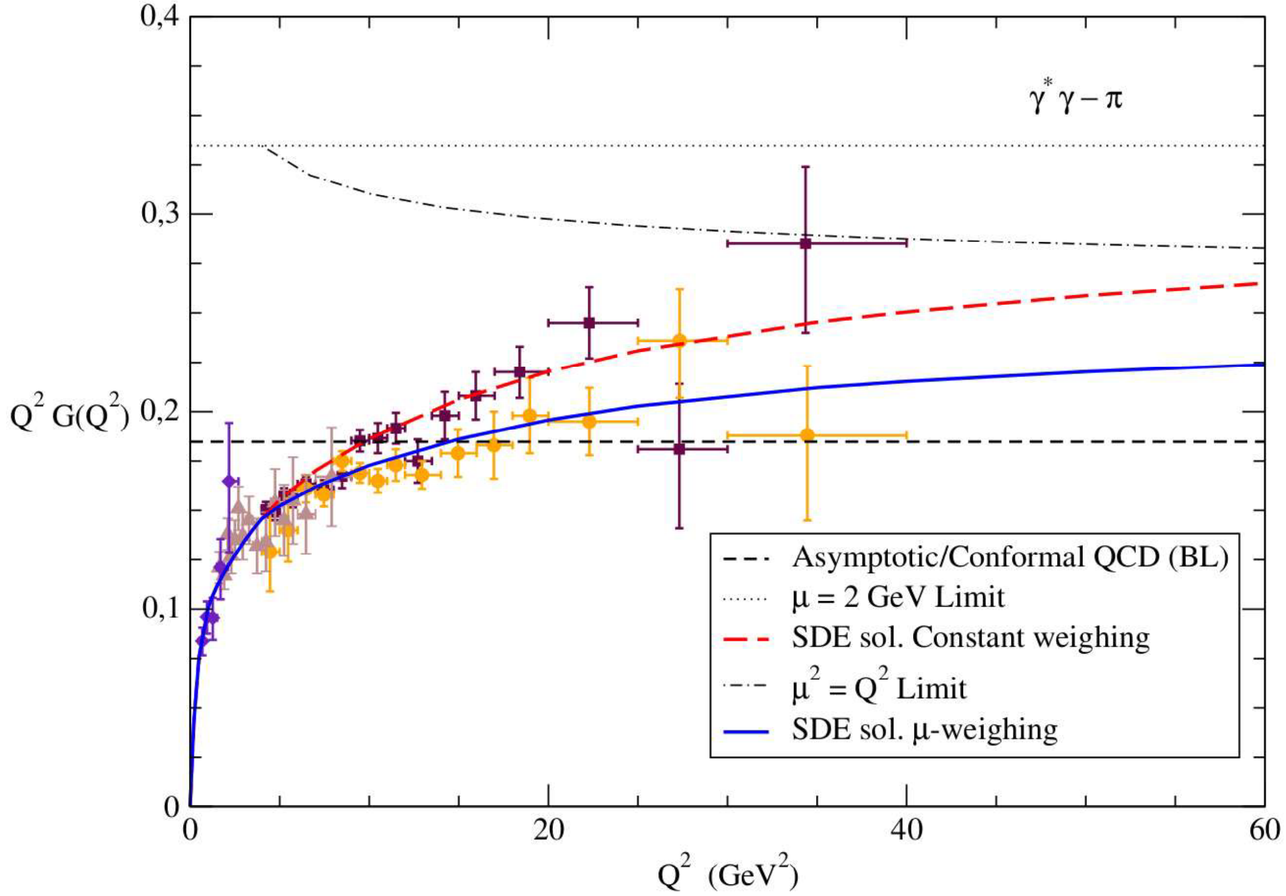
- Multiplying in both sides by $J_1(\mu^2 = Q^2)/J_1(\mu = 2\ \mathrm{GeV})$, we arrive at:

$$Q^2 \frac{J_1(\mu^2 = Q^2)}{J(\mu = 2)} G(Q^2; \mu = 2) \to f_\pi J_1(\mu^2 = Q^2)\,,$$

$$Q^2 \mathcal{G}(Q^2; \mu^2 = Q^2) \to f_\pi J_1(\mu^2 = Q^2).$$

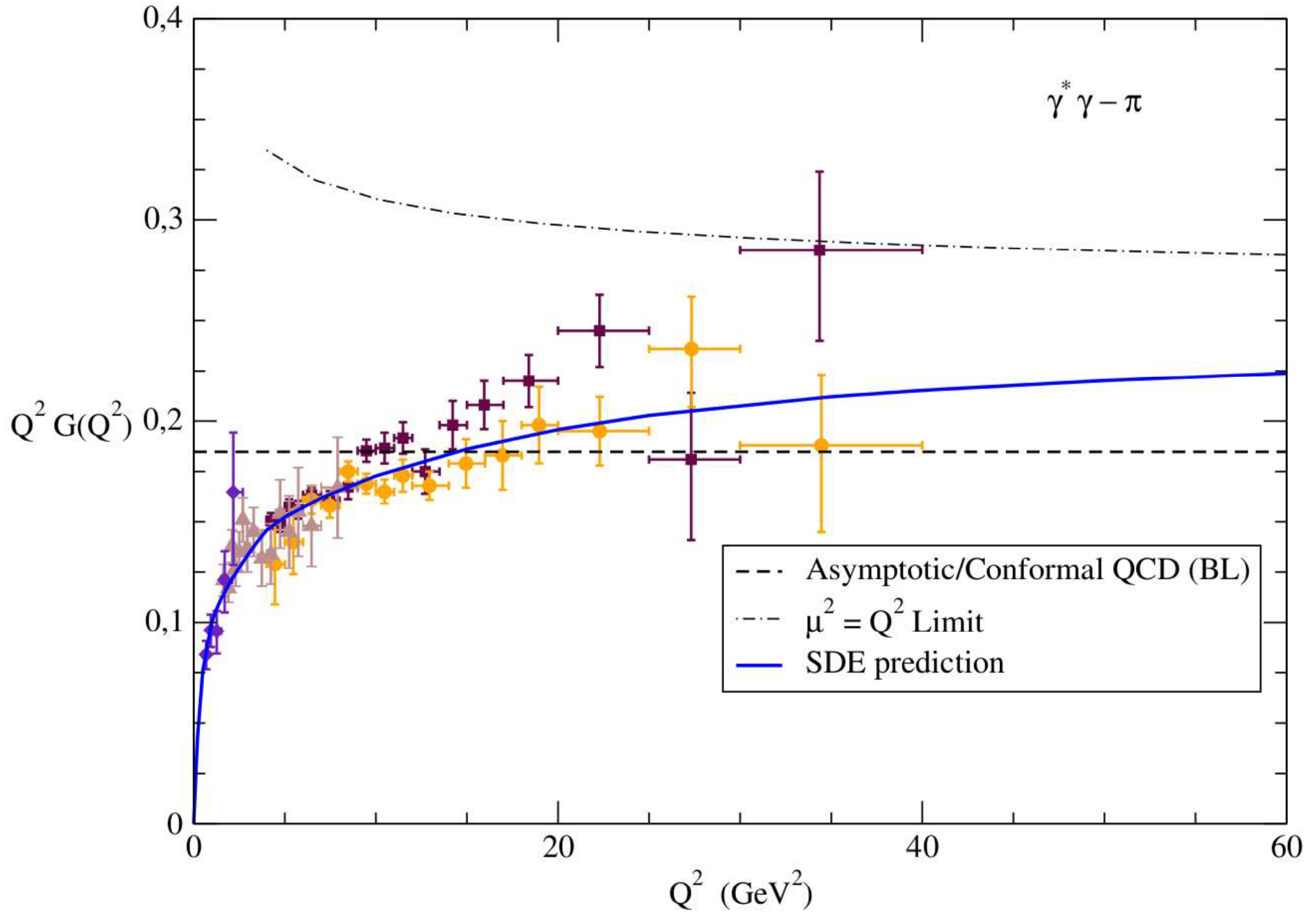
- This procedure is called µ-weighing.

TFF: Asymptotic limit - Q^2 Evolution



- **Transition Form Factor:** SDE calculation and comparison with experiment. [Red, dashed] Constant weighing (μ = 2 GeV). [Blue, solid] μ-weighing (μ^2 = Q^2). **Limits:** [Dotted] μ = 2 GeV, [Dot-dashed] μ^2 = Q^2.

TFF: Asymptotic limit - Q^2 Evolution



- **Transition Form Factor:** SDE prediction (so far) and comparison with experiment.

Conclusions: Hits and misses

- **Hit:** Pion mass and leptonic decay constant agree with experiment.
- **Hit:** We calculated the TFF using complex and realistic parametrizations of the quark propagator and BSA.
- **Hit:** The tools (programs and parametrizations) are ready to be used. Just plug them wherever you need them.
- **Hit:** We model a vertex which satisfices the WTI as well as the Abelian anomaly, G(0). Also, rho pole effects were included.
- **Miss:** One must employ a relative momentum dependent transverse part of the vertex, or, check if our model does not affect the pion charge radii.
- **Miss:** Power law behavior is not clear. G(Q^2) behaves slightly different than 1/Q^2. (Maybe we still below the momentum region where it happens).
- **Miss:** Twisting effects not taken into account yet.

Thanks…