

New Tests of Quarkonium Production Mechanisms using Jet Substructure

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Review of Quarkonium Production Theory

Heavy Quarkonium Fragmenting Jet Functions

New Tests of NRQCD Using Jet Observables

Cross sections for e^+e^- , pp collisions

Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \rightarrow J/\psi + X) = \sum_n \sigma(gg \rightarrow c\bar{c}(n) + X) \langle \mathcal{O}^{J/\psi}(n) \rangle$$

$n = {}^{2S+1}L_J^{(1,8)}$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

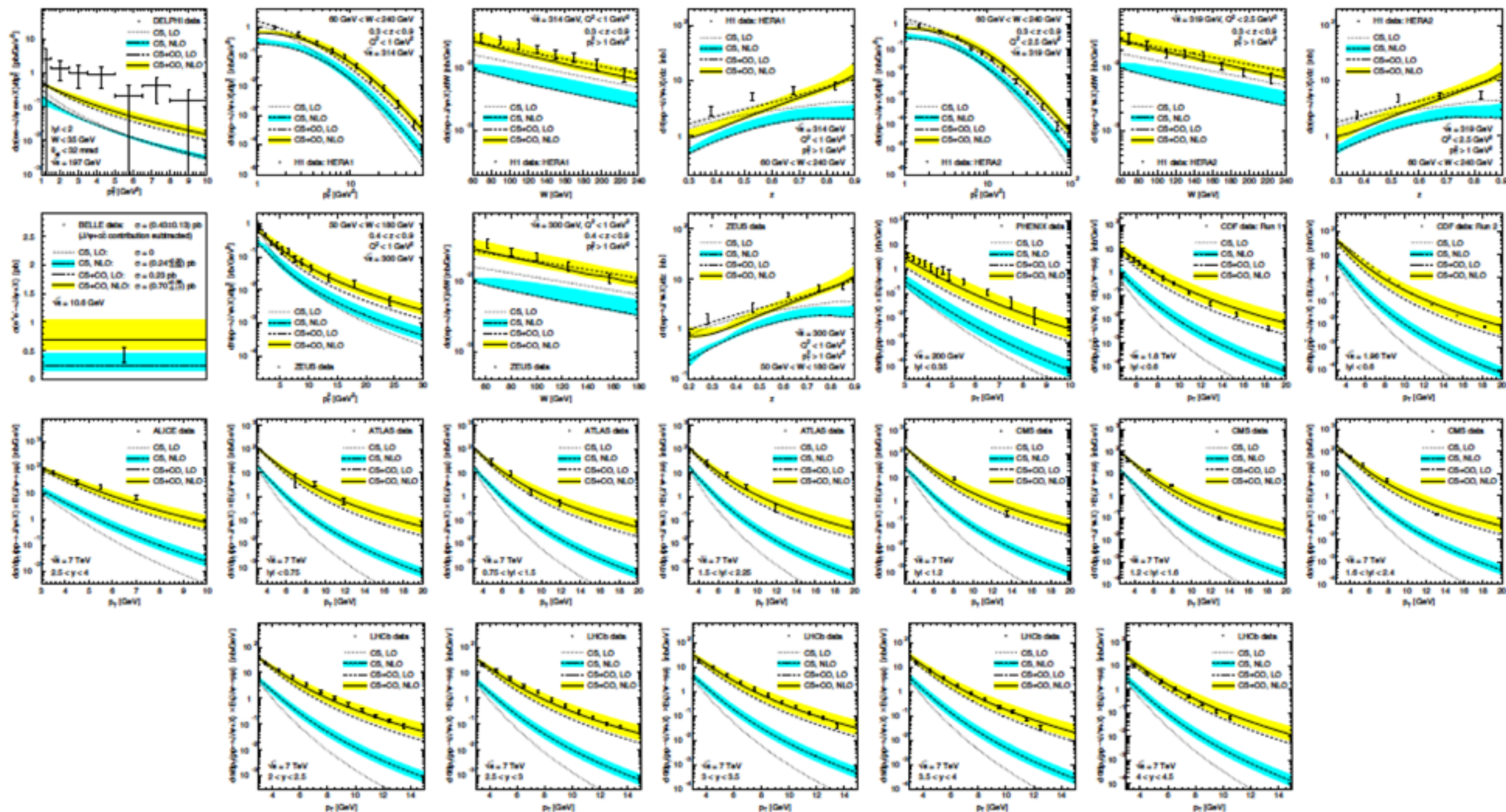
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle \sim v^3$$

CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]}) \rangle \sim v^7$$

color-octet mechanisms

Global Fits with NLO CSM + COM

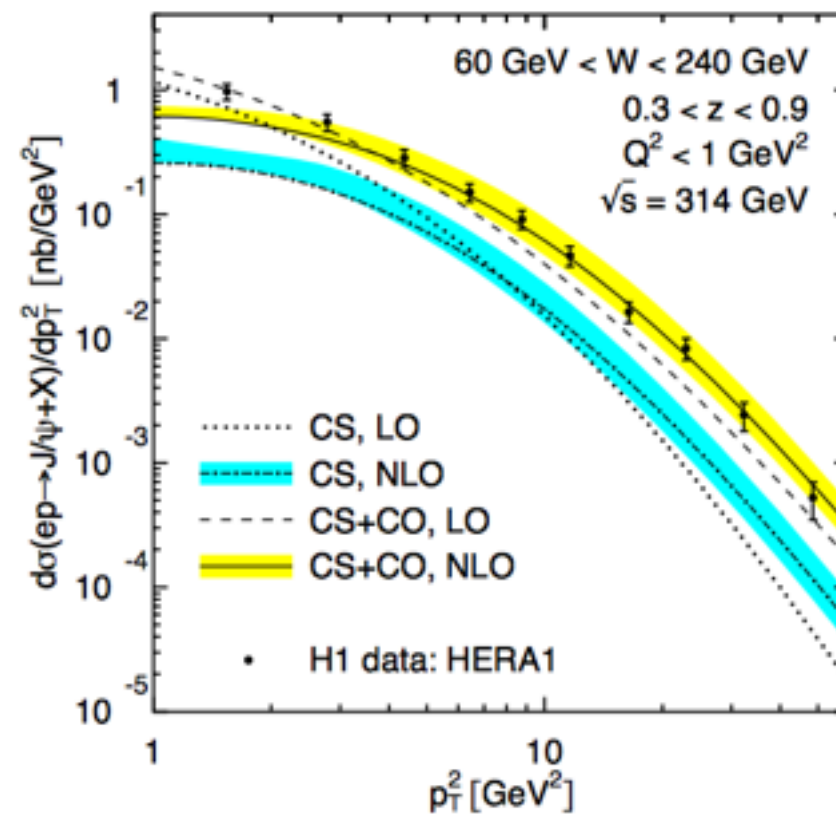
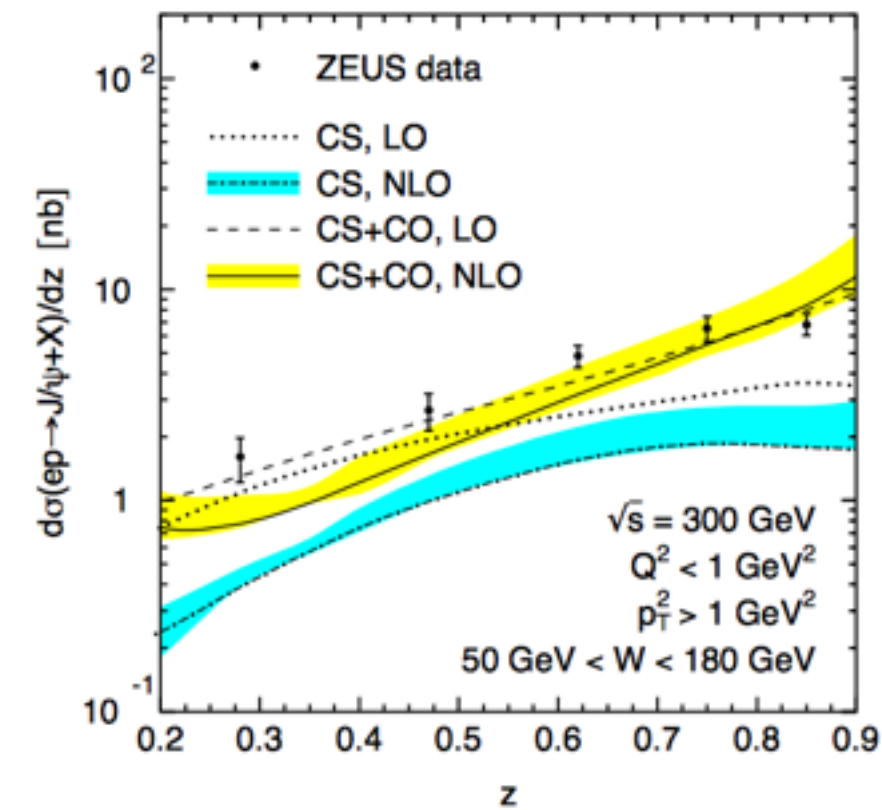


$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \rightarrow J/\psi + X$$

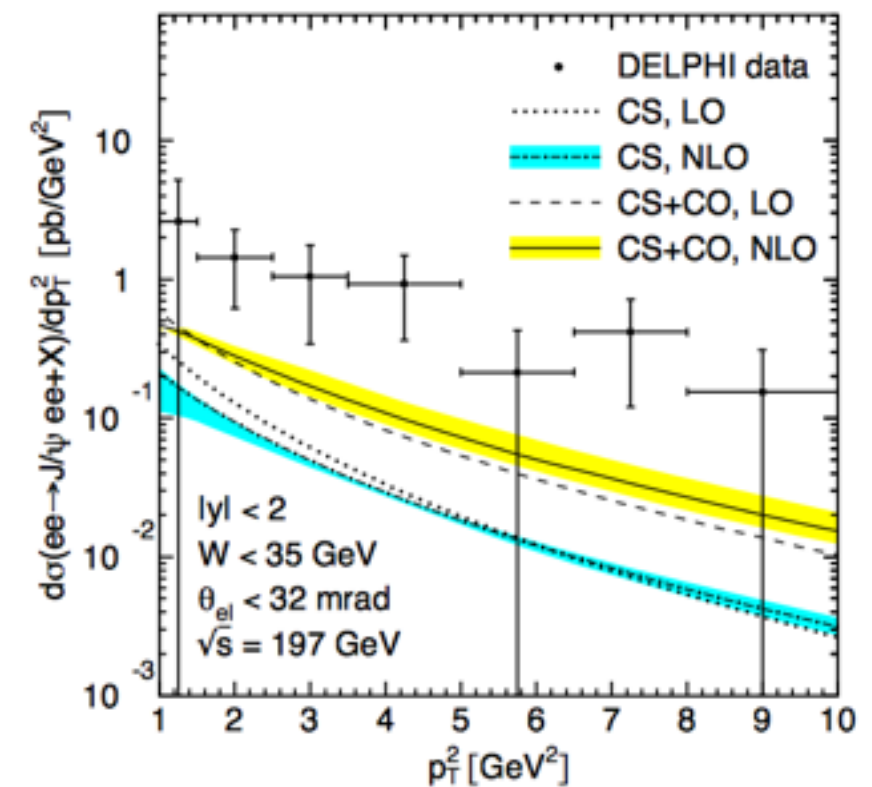
fit to 194 data points, 26 data sets,

Butenschoen and Kniehl, PRD 84 (2011) 051501

NLO: CSM + COM Required to Fit Data



$$ep \rightarrow J/\psi + X$$



$$\gamma^* \gamma^* \rightarrow J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

extracted LDME satisfy NRQCD v-scaling

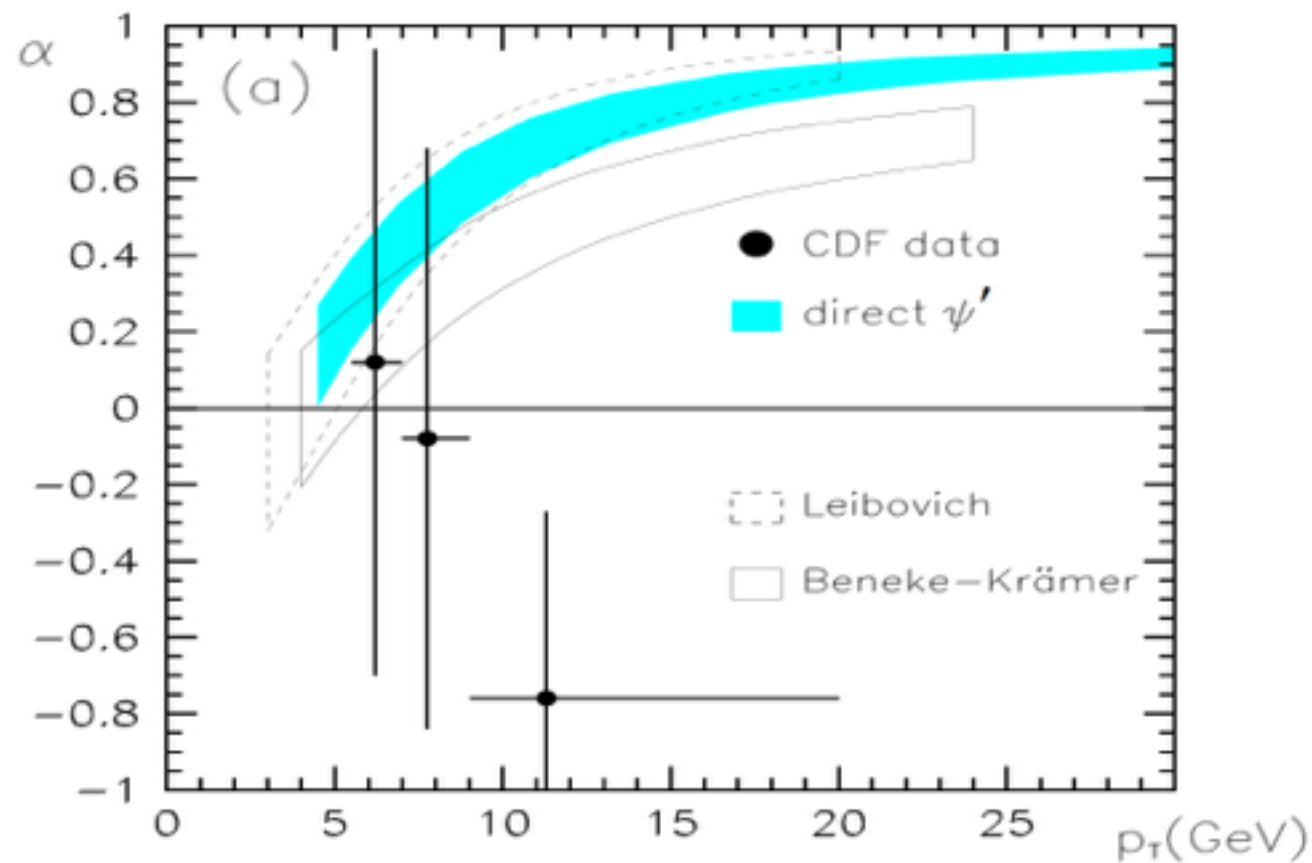
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$

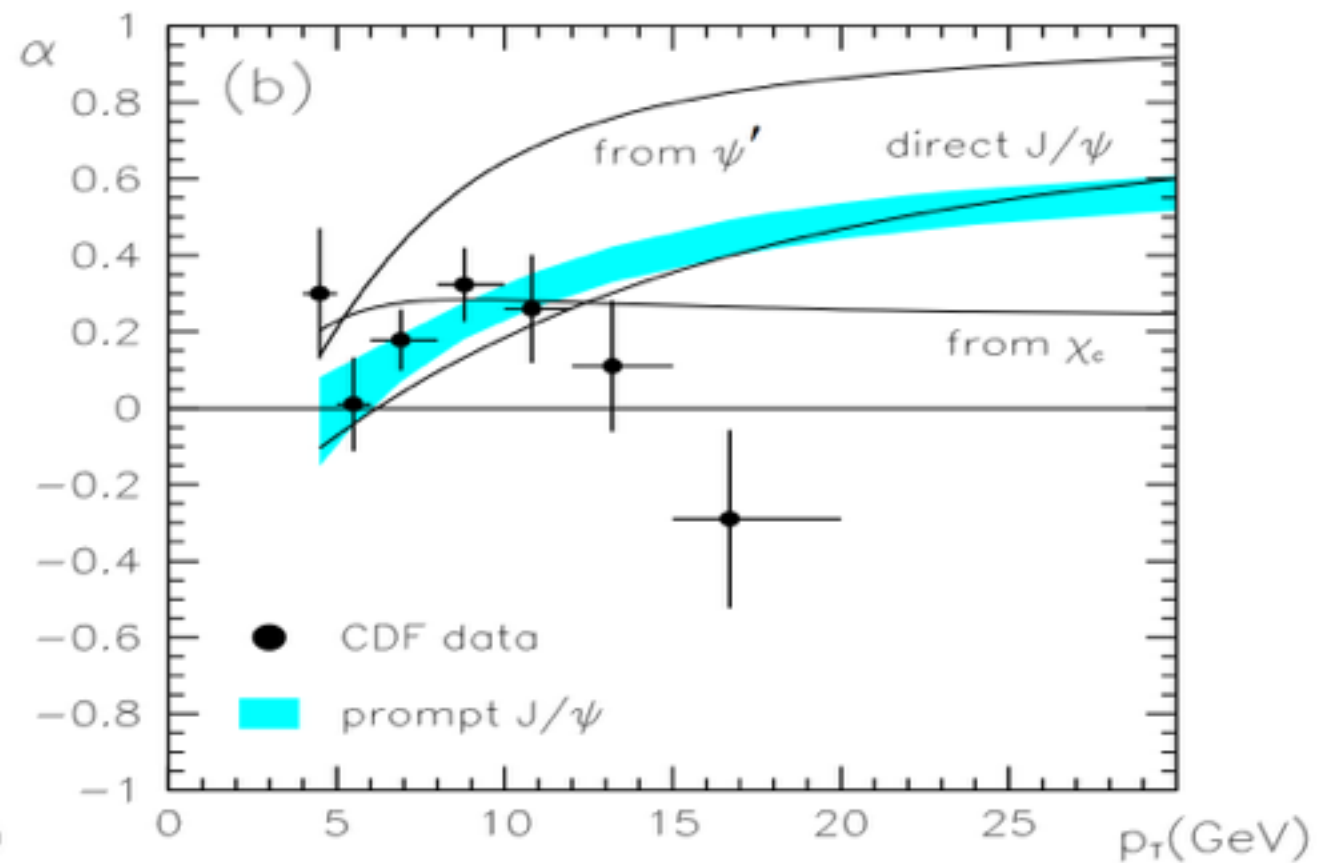
$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

Polarization Puzzle

$^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ , ψ'

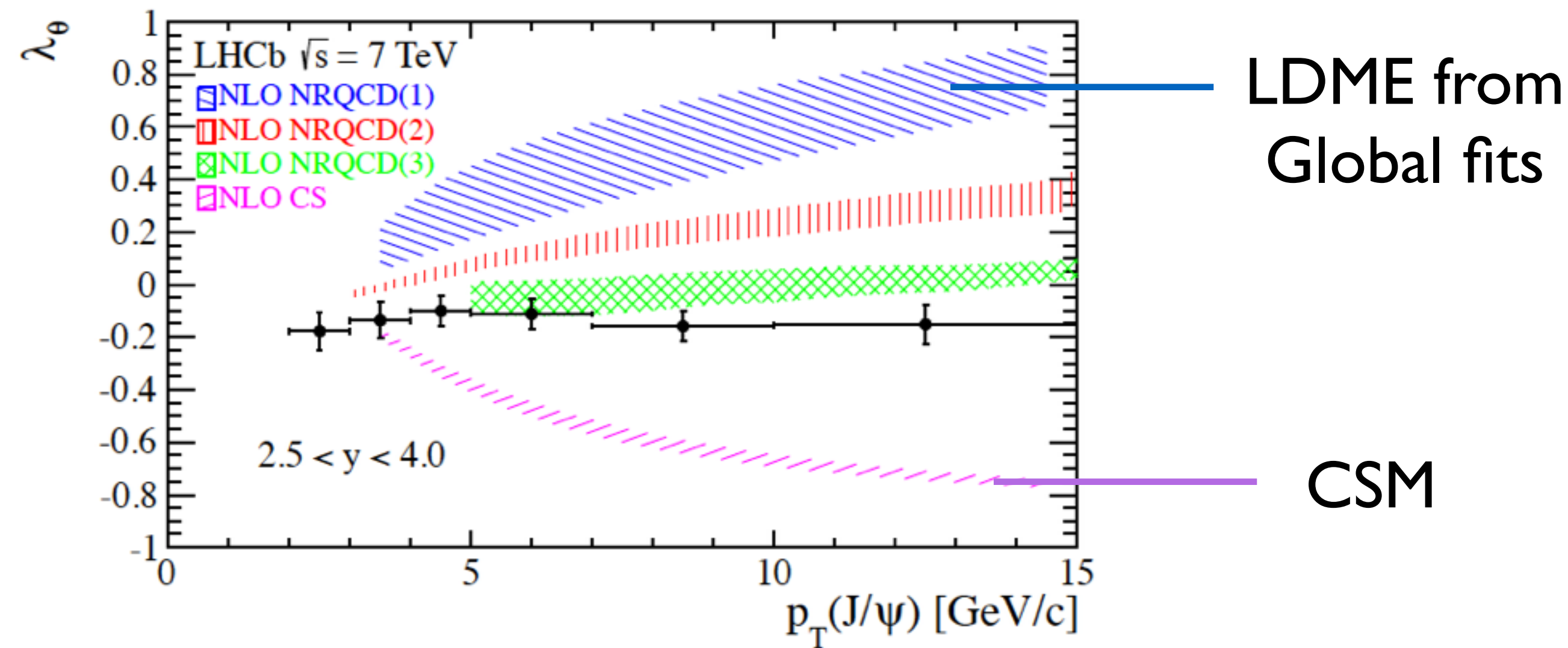


ψ'

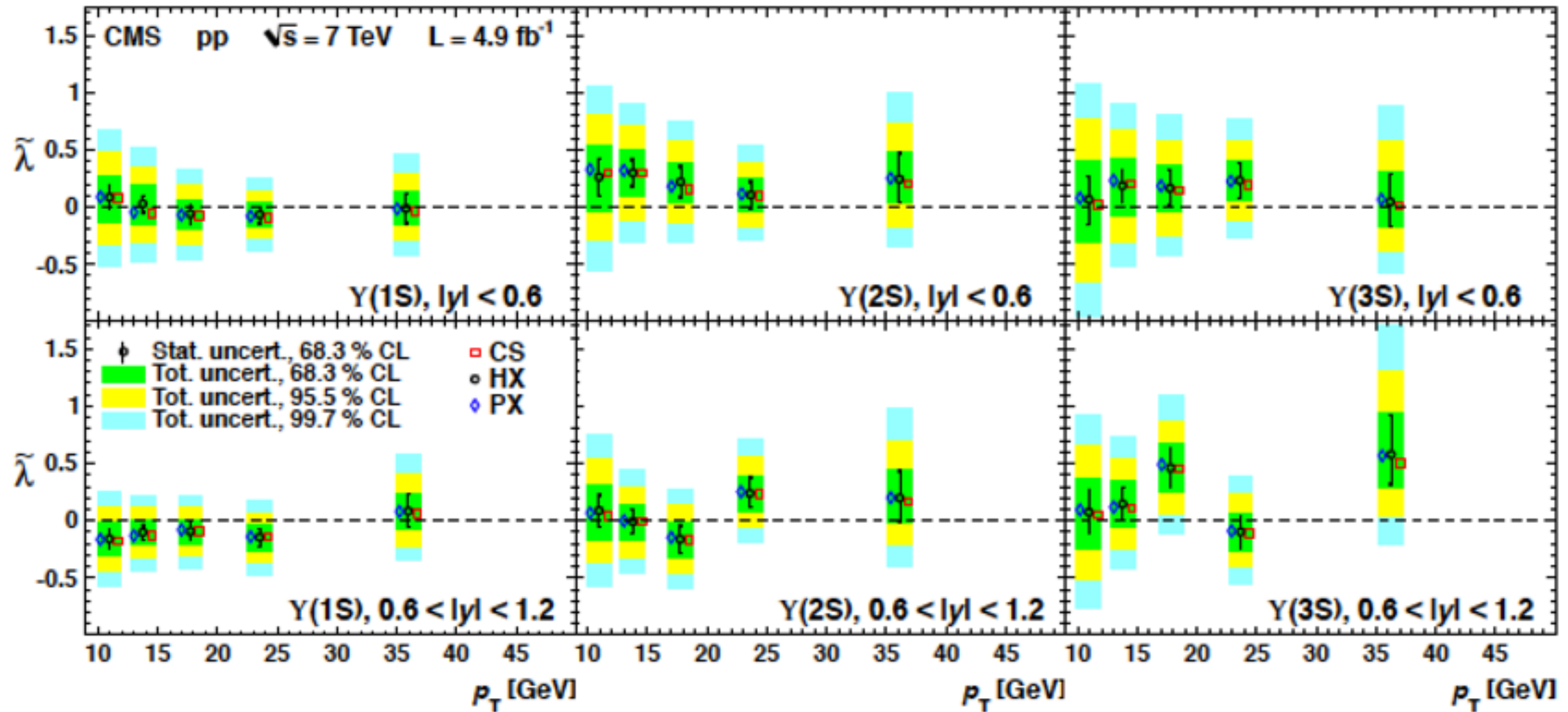


J/ψ

Polarization of J/ψ at LHCb



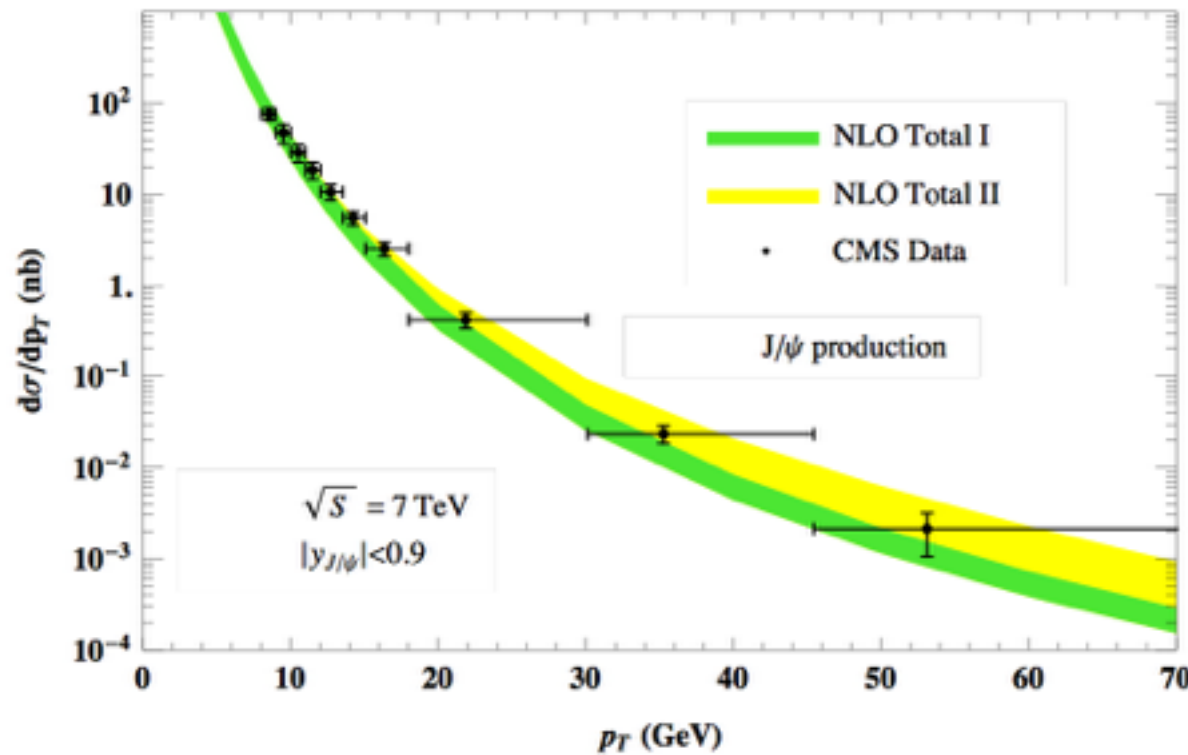
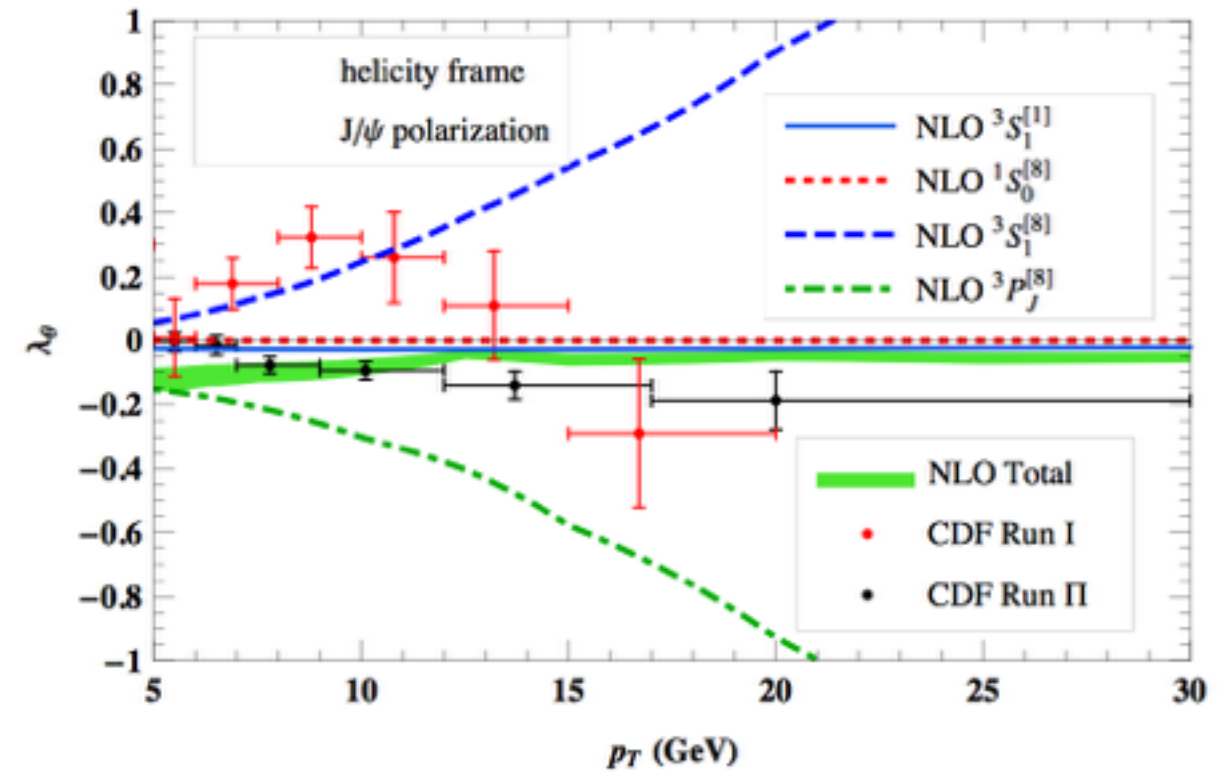
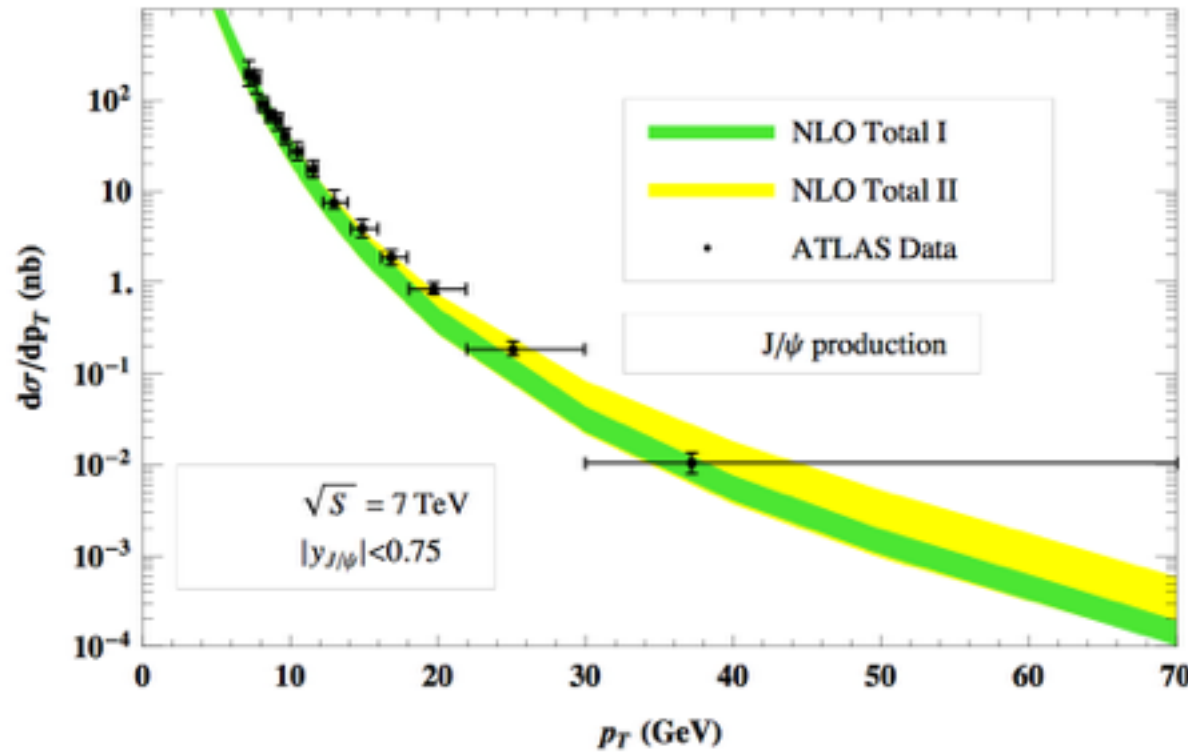
Polarization of $\Upsilon(nS)$ at CMS



Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_T production, polarization

Chao, et. al. PRL 108, 242004 (2012)



$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV ³	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10 ⁻² GeV ³	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10 ⁻² GeV ³	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ 10 ⁻² GeV ³
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

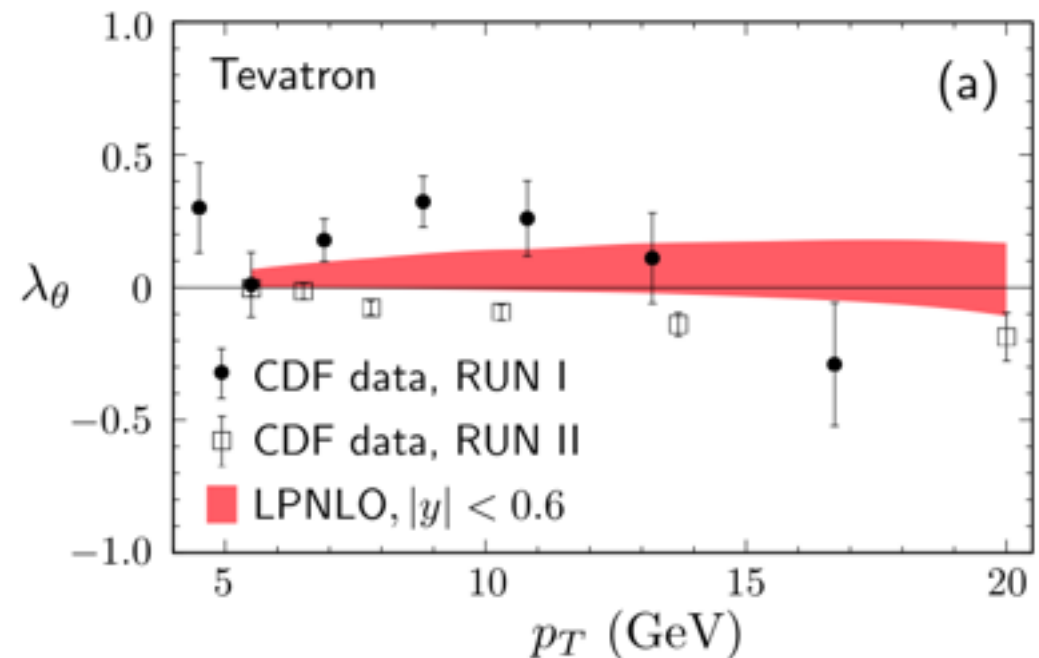
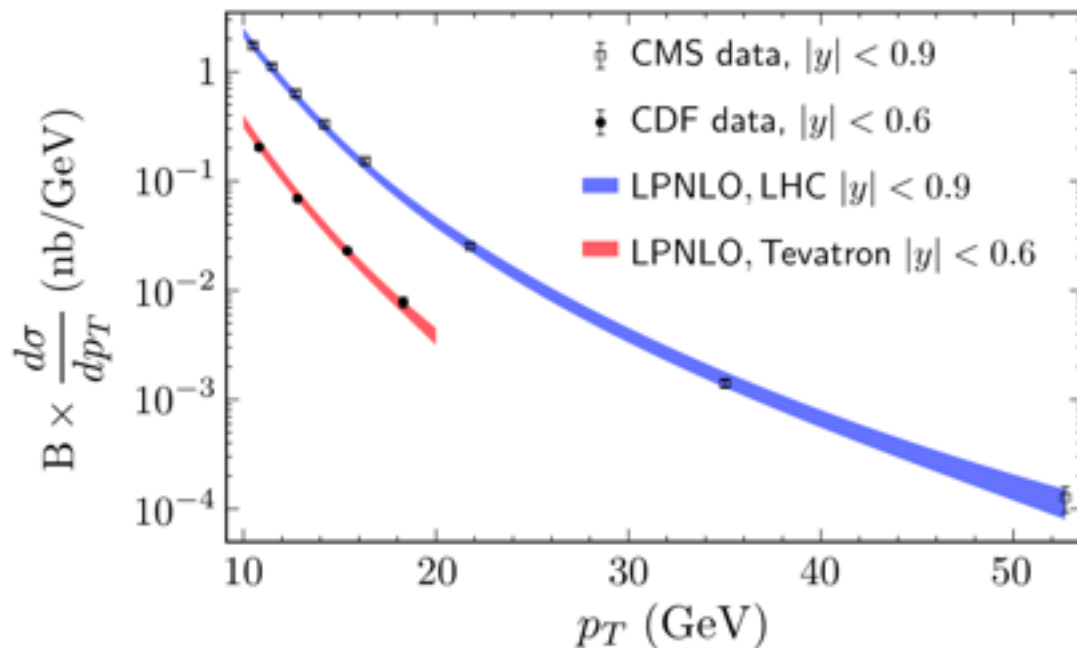
Recent Attempts to Resolve J/ψ Polarization Puzzle

i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001 (2014)

ii) resum logs of p_t/m_c using AP evolution

iii) fit COME to p_t spectrum, predict basically no polarization



Extracted COME inconsistent with global fits

$$\langle \mathcal{O}^{J/\psi} (^1S_0^{(8)}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$$

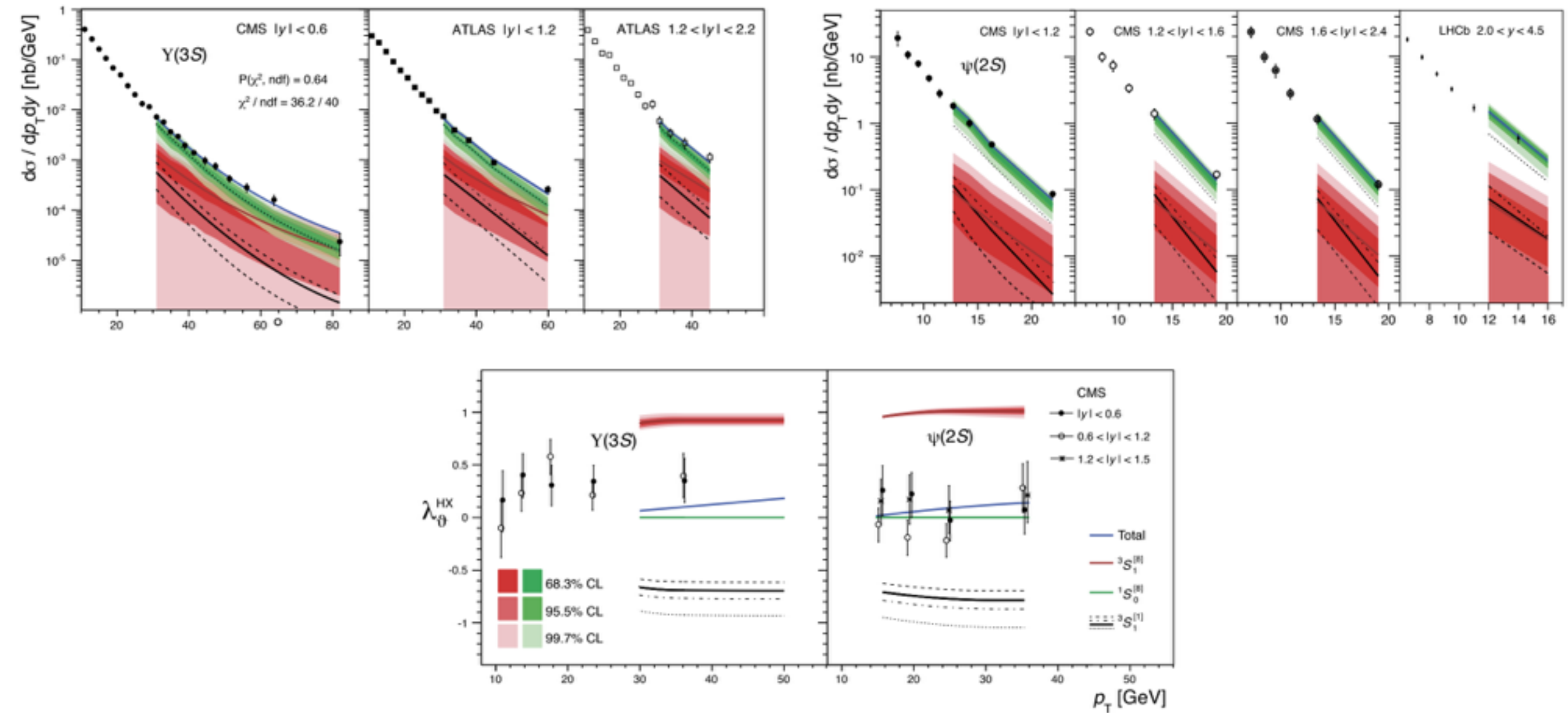
$$\langle \mathcal{O}^{J/\psi} (^3S_1^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi} (^3P_0^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$$

Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press



argue for $^1S_0^{(8)}$ dominance in both $\psi(2S)$ & $Y(3S)$ production

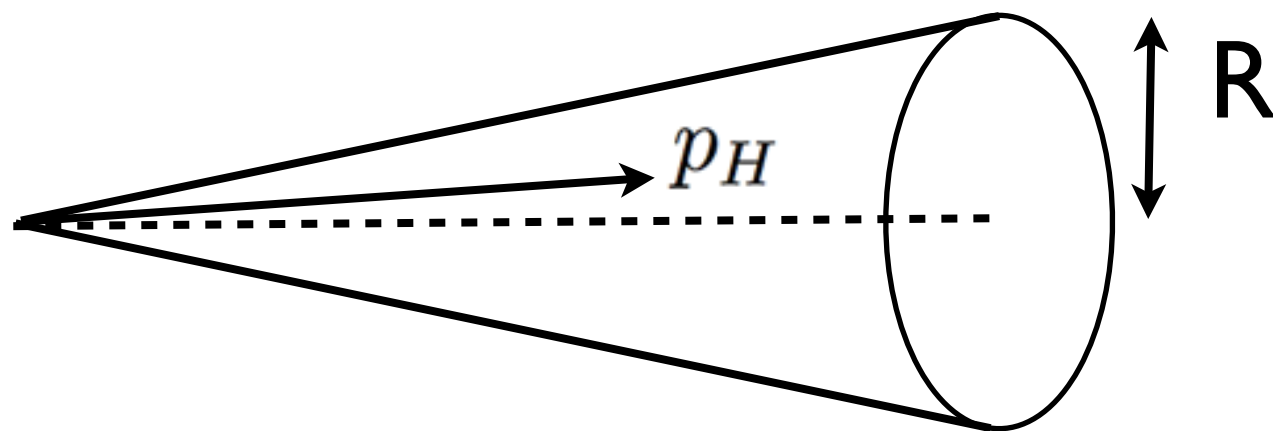
Fragmenting Jet Functions

Procura, Stewart, arXiv:0911.4980

Jain, Procura, Waalewijn, arXiv:1101.4953

Procura, Waalewijn, arXiv:1111.6605

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\text{jet}}^+$$

cross sections determined by **fragmenting jet function (FJF)**:

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dz}(e^+e^- \rightarrow h X) = \sum_i \int_z^1 \frac{dx}{x} C_i(E_{\text{cm}}, x, \mu) D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$\frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell}$$

$$\mathcal{G}_g^h(E, R, \mu, z) \longrightarrow D_i^h(z/x, \mu) , J_{\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 dz z D_j^h(z, \mu) = 1$$

$$\longrightarrow J_i(E, R, z, \mu) = \frac{1}{2} \sum_h \int \frac{dz}{(2\pi)^3} z \mathcal{G}_i^h(E, R, z, \mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell} J_i(E, R, z, \mu)$$

$$\longrightarrow \frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \mathcal{G}_i^h(E, R, z, \mu) \prod_{\ell} J_{\ell} .$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E, R, z, \mu) = \sum_i \int_z^1 \frac{dz'}{z'} \mathcal{J}_{ij}(E, R, z', \mu) D_j^h\left(\frac{z}{z'}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{4E^2 \tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E, R, z, \mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi} \left[\left(L^2 - \frac{\pi^2}{24} \right) \delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z) \right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z) \ln z & z \leq 1/2 \\ \frac{2(1-z+z^2)^2}{z} \left(\frac{\ln(1-z)}{1-z} \right)_+ & z \geq 1/2. \end{cases}$$

$$L = \ln[2E \tan(R/2)/\mu].$$

scale for $\mathcal{J}_{ij}(E, R, z, \mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 dz z \mathcal{J}_{ij}(R, z, \mu) = 2(2\pi)^3 J_i(R, \mu)$$

NRQCD fragmentation functions

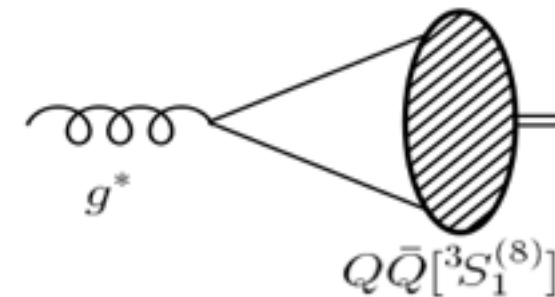
Braaten, Yuan, hep-ph/9302307

Braaten, Chen, hep-ph/9604237

Braaten, Fleming, hep-ph/9411365

Perturbatively calculable **at the scale $2m_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3} \langle O^\psi(^3S_1^{(8)}) \rangle \delta(1-z).$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2} \frac{\langle O^\psi(^3S_1^{(1)}) \rangle}{M_\psi^3} \int_0^z dr \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2} \\ \sum_{i=0}^2 z^i \left(f_i(r, y) + g_i(r, y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}} \right),$$

Altarelli-Parisi evolution: $2m_c$ to $2E \tan(R/2)$

FJF in terms of fragmentation function

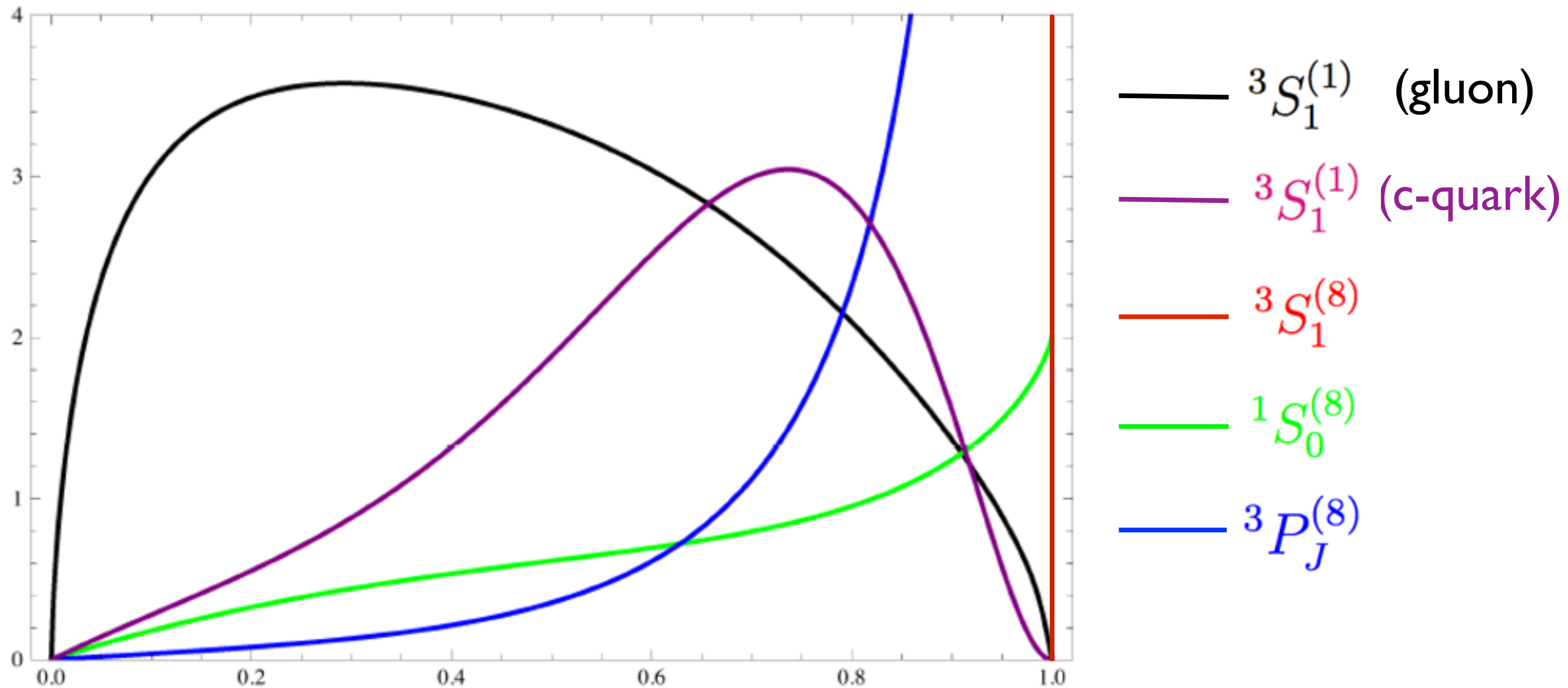
$$\begin{aligned}\mathcal{G}_g^\psi(E, R, z, \mu) = & D_{g \rightarrow \psi}(z, \mu) \left(1 + \frac{C_A \alpha_s}{\pi} \left(L_{1-z}^2 - \frac{\pi^2}{24} \right) \right) \\ & + \frac{C_A \alpha_s}{\pi} \left[\int_z^1 \frac{dy}{y} \tilde{P}_{gg}(y) L_{1-y} D_{g \rightarrow \psi} \left(\frac{z}{y}, \mu \right) \right. \\ & + 2 \int_z^1 dy \frac{D_{g \rightarrow \psi}(z/y, \mu) - D_{g \rightarrow \psi}(z, \mu)}{1-y} L_{1-y} \\ & \left. + \theta \left(\frac{1}{2} - z \right) \int_z^{1/2} \frac{dy}{y} \hat{P}_{gg}(y) \ln \left(\frac{y}{1-y} \right) D_{g \rightarrow \psi} \left(\frac{z}{y}, \mu \right) \right]\end{aligned}$$

$$L_{1-z} = \ln \left(\frac{2E \tan(R/2)(1-z)}{\mu} \right)$$

For large E, FJF $\sim$ NRQCD frag. function (at scale $2E \tan(R/2)$)

$$\mathcal{G}_g^h(E, R, \mu = 2E \tan(R/2), z) \rightarrow D_g^\psi(z, 2E \tan(R/2)) + O(\alpha_s)$$

NRQCD FF's (at scale $2m_c$)

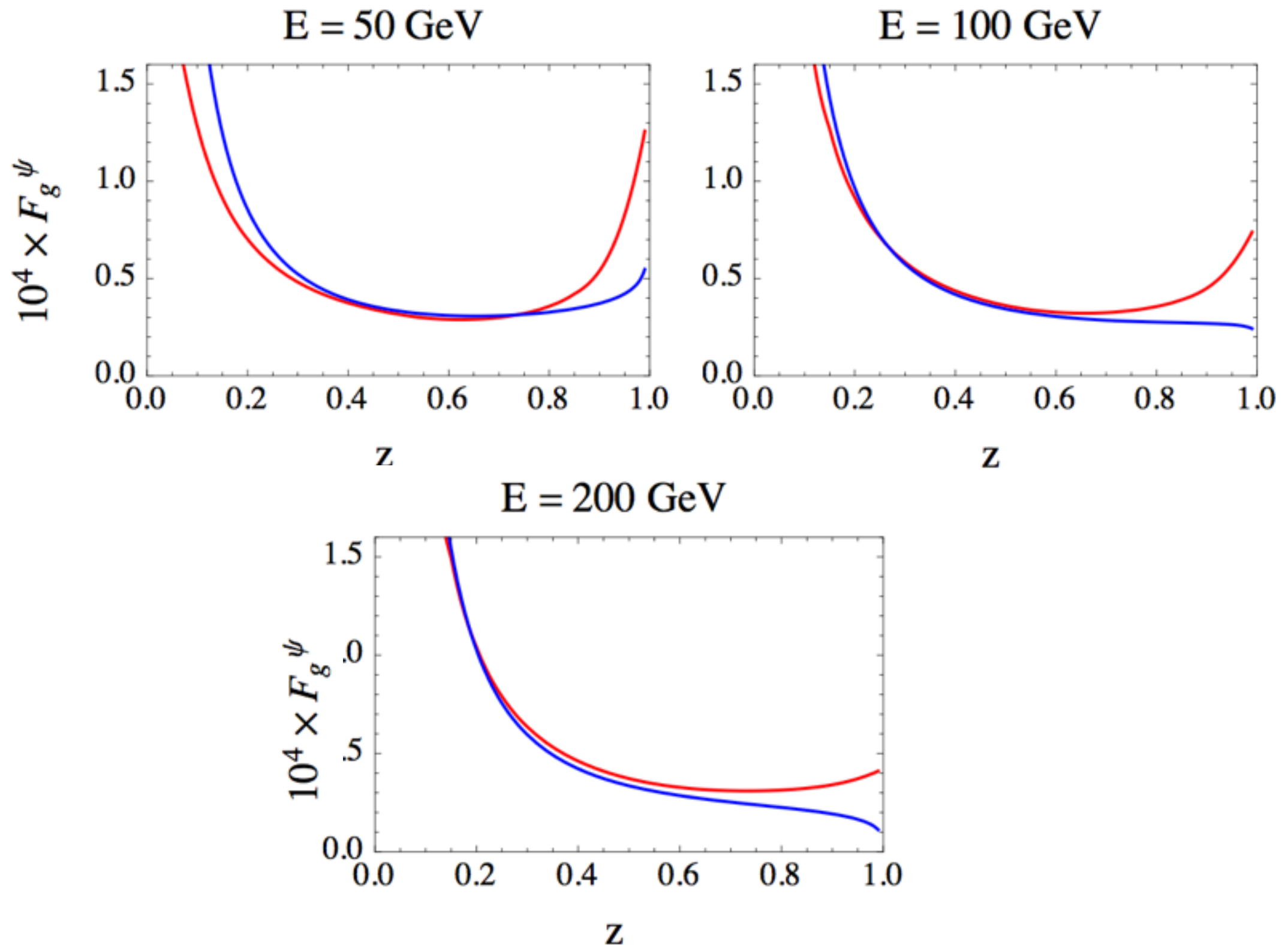


(normalization arbitrary)

Evolution to $2E \tan(R/2)$ will soften discrepancies

Color-Octet 3S_1 fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

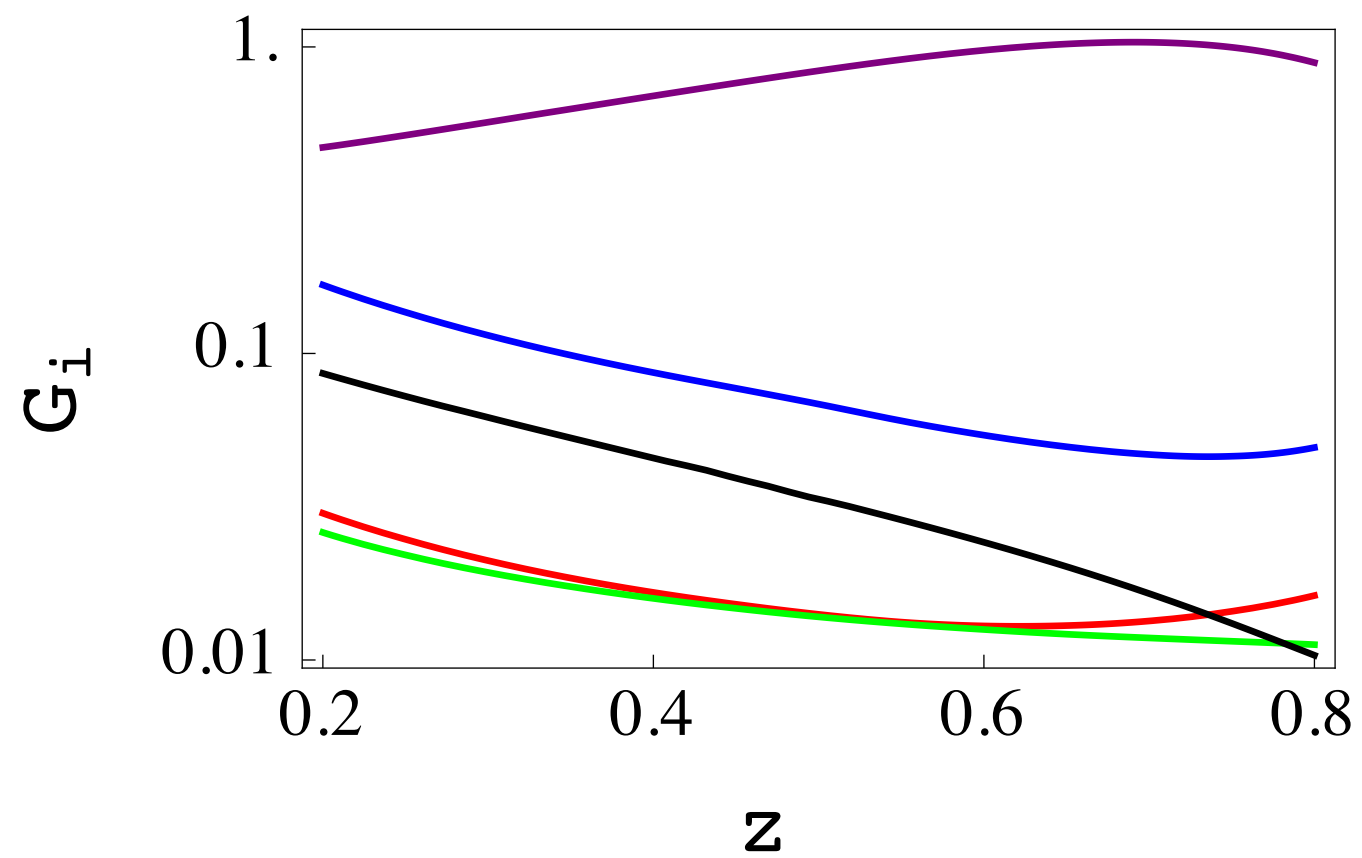


— fragmentation function

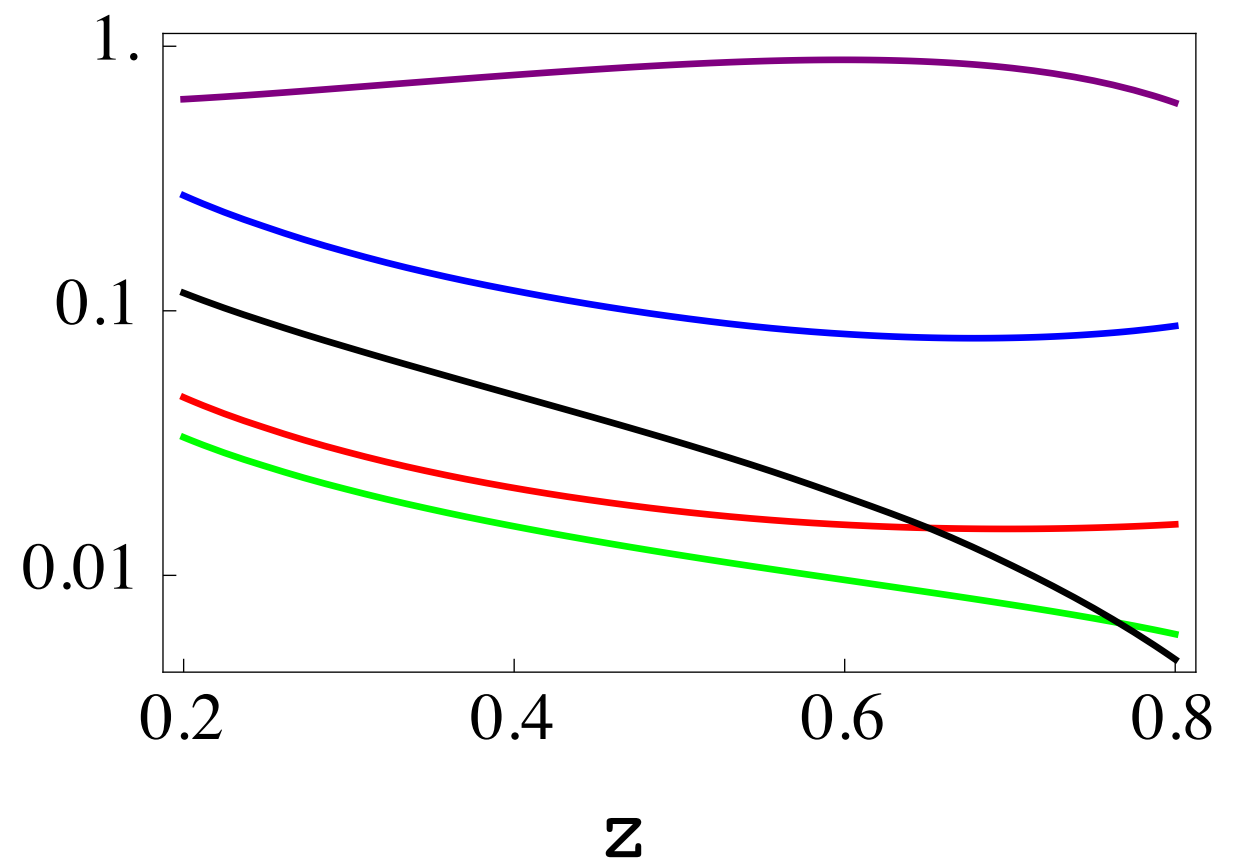
— fragmenting jet function

FJF's at Fixed Energy vs. z

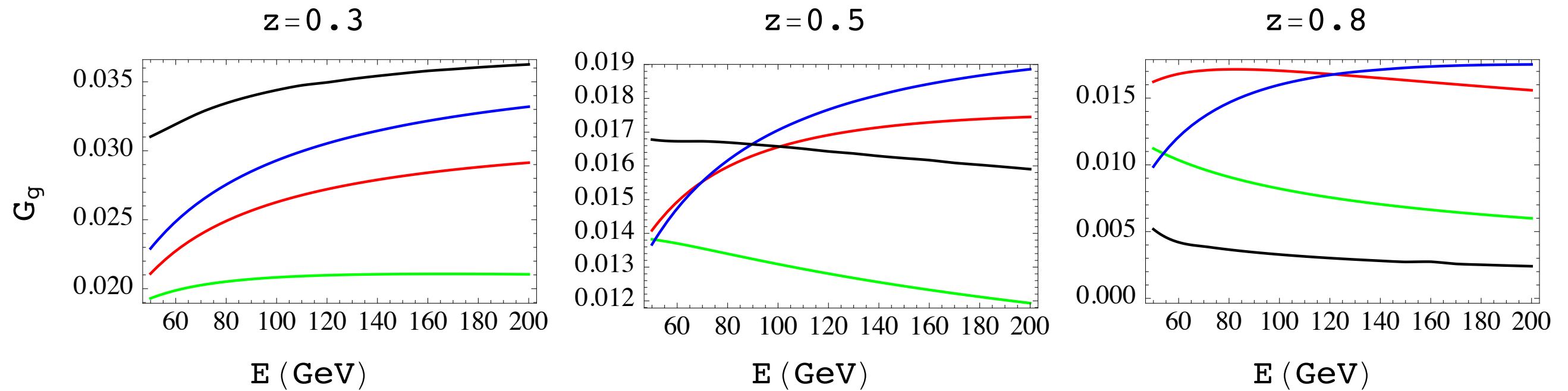
$E = 50 \text{ GeV}$



$E = 200 \text{ GeV}$



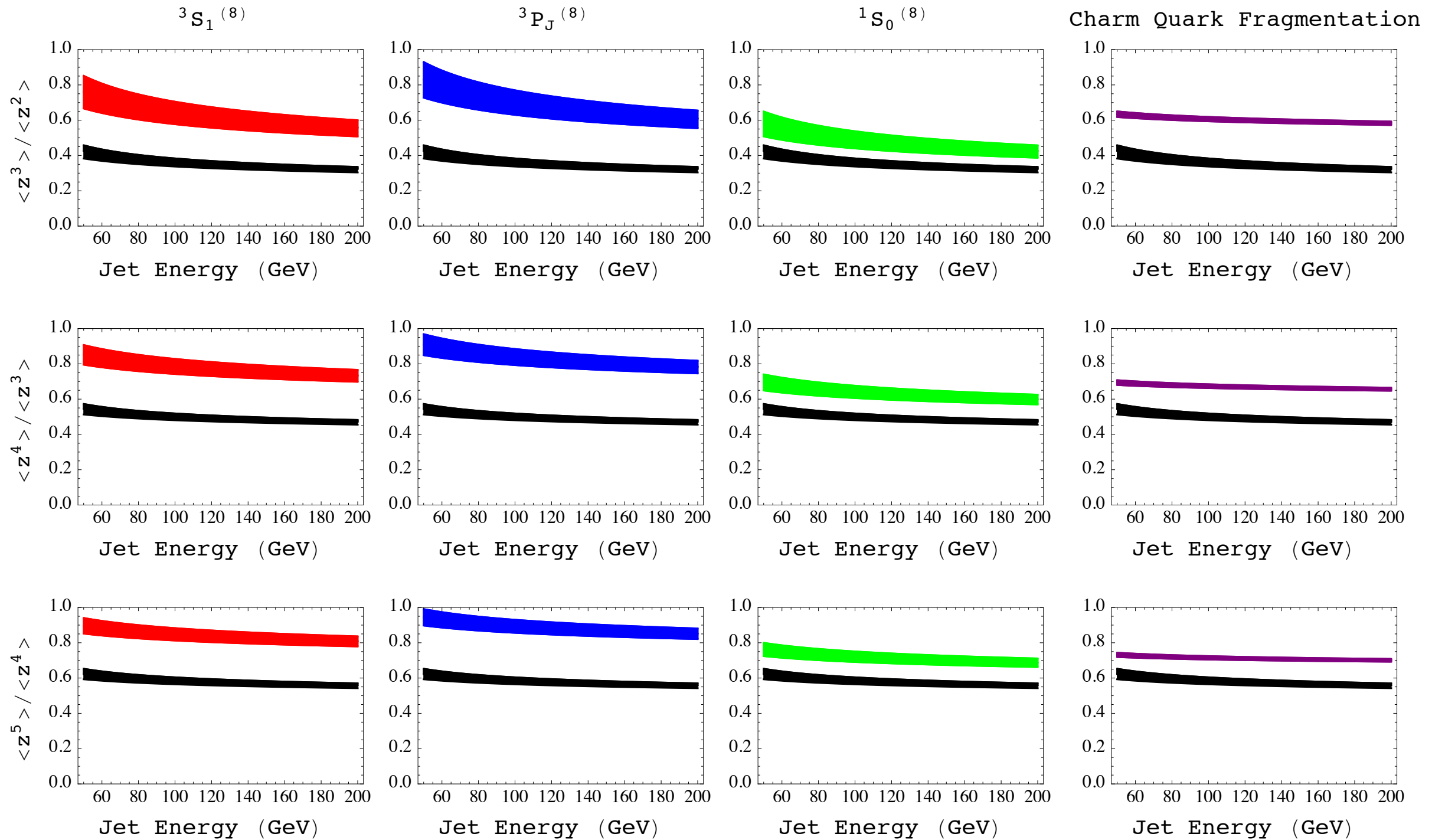
FJF's at Fixed z vs. Energy



$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if $z > 0.5$

Ratios of Moments

$$E \tan(R/2) < \mu < 4E \tan(R/2)$$



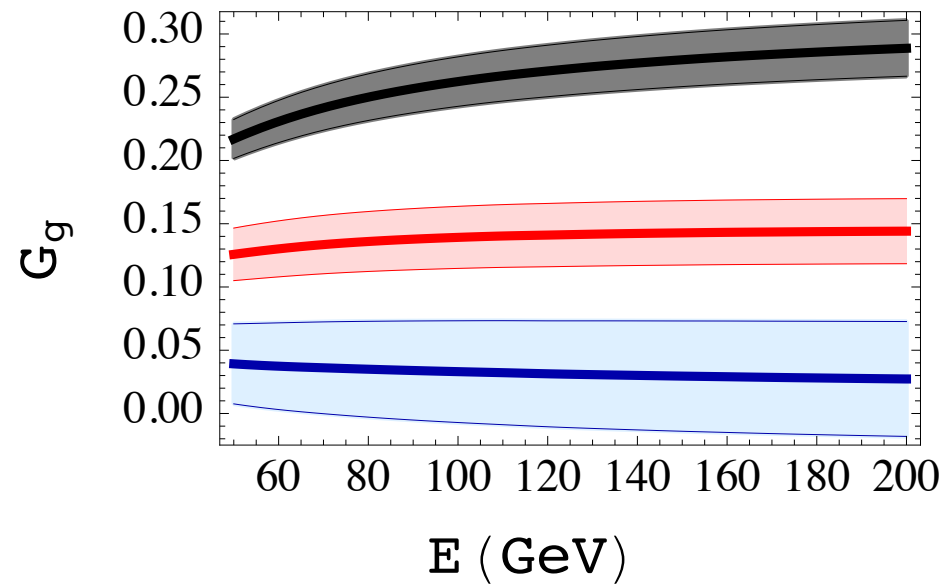
Ratios of Moments

$$\frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3P_J^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(8)}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{1S_0^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{\text{c-quark}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(1)}}$$

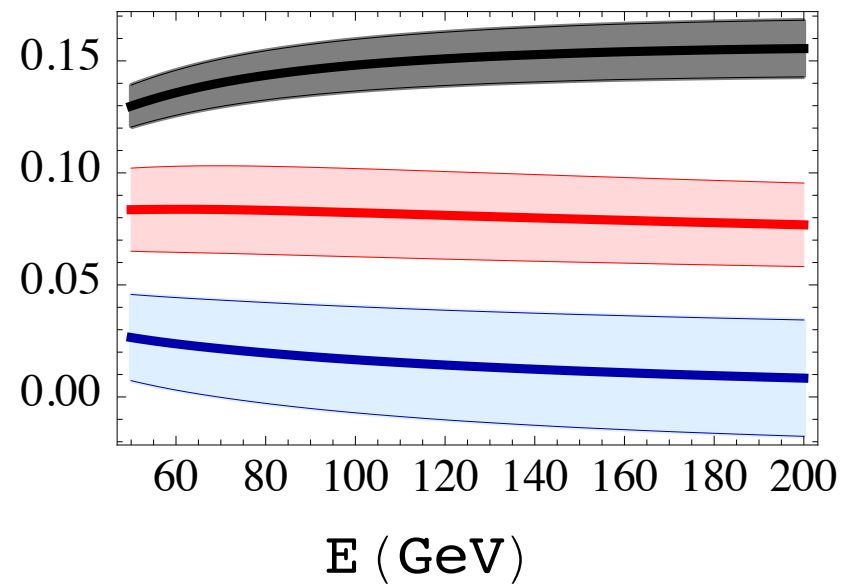
Gluon FJF for different extractions of LDME

fix z , vary energy

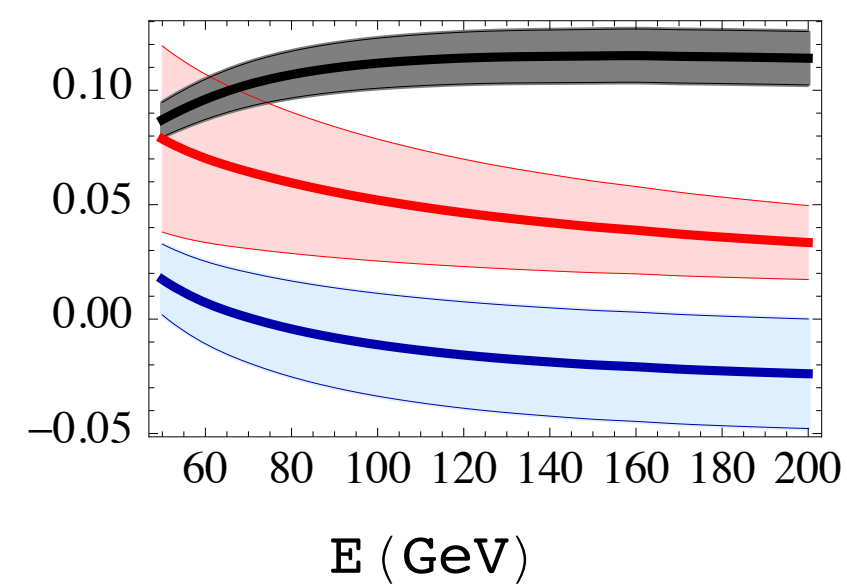
$z = 0.3$



$z = 0.5$



$z = 0.8$

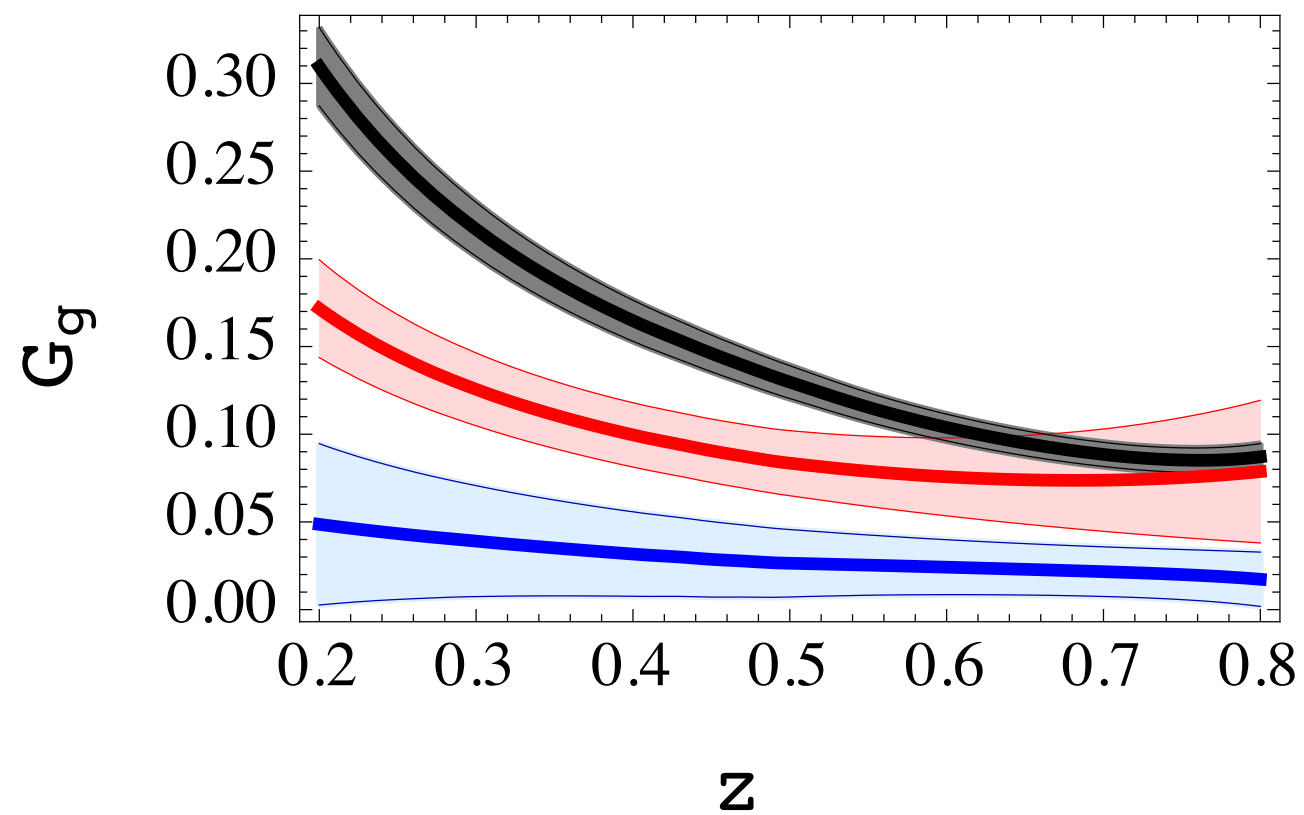


- Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822
- Bodwin, et. al. arXiv:1403.3612
- Chao, et. al. PRL 108, 242004 (2012)

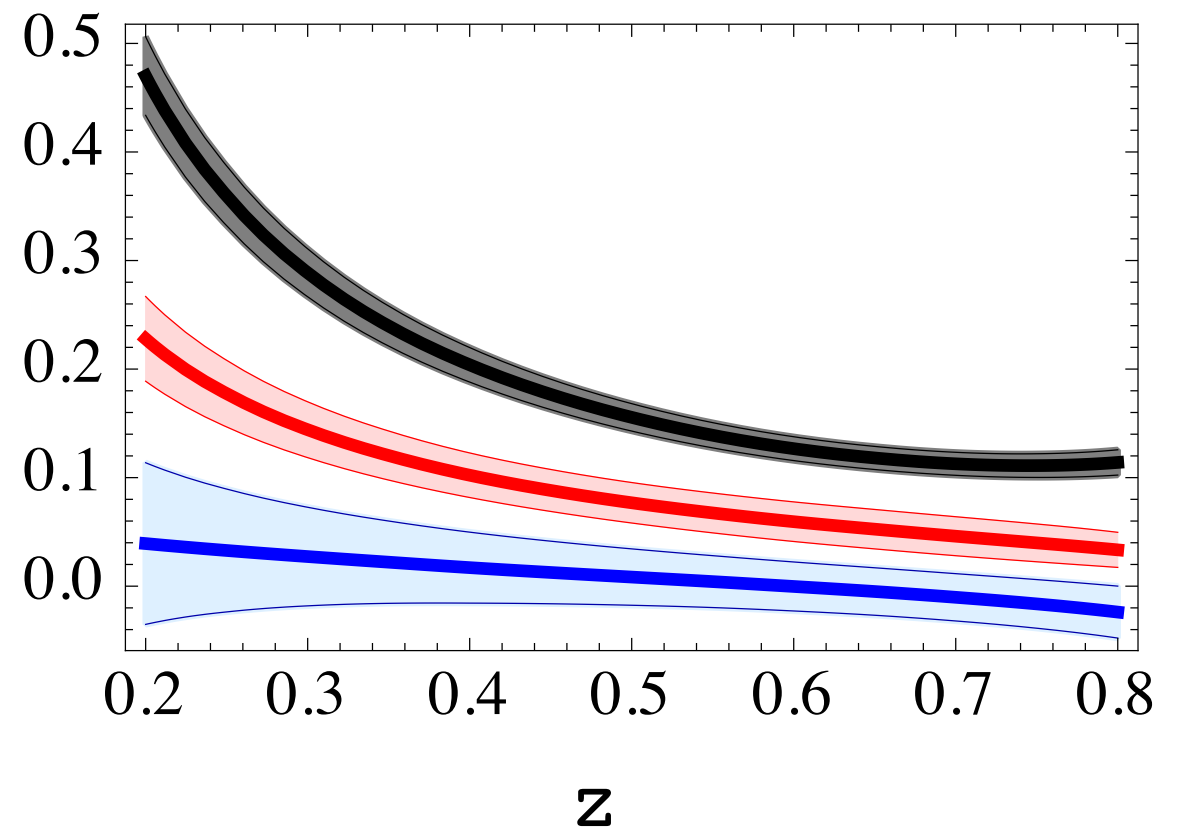
Gluon FJF for different extractions of LDME

fix energy, vary z

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Work in Progress

(w/ R. Bain, A. Hornig, Y. Makris)

$$e^+e^- \rightarrow q\bar{q}g \quad \text{vs. Pythia}$$
$$\quad \quad \quad \hookrightarrow J/\psi \text{ jet}$$

pp collisions

$e^+e^- \rightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010)101

$$d\sigma = H \times J^q \otimes J^{\bar{q}} \otimes J^g \otimes S$$

unmeasured jets:

E, R

measured jets:

angularity: $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

$$\omega = \sum_i p_i^- \quad s = \omega^2 \tau_0$$

$e^+e^- \rightarrow$ Jets Formula (NLL')

$$\begin{aligned}
\frac{1}{\sigma^{(0)}} \frac{d\sigma_N}{d\tau_{a_1}^1 \cdots d\tau_{a_M}^M} &= H(\mu_H) \left(\frac{\mu_H}{\bar{\omega}_H} \right)^{\omega_H(\mu, \mu_H)} \prod_{k=M+1}^N J_{\omega_k}^k(\mu_J^k) \left(\frac{\mu_J^k}{\omega_k \tan \frac{R}{2}} \right)^{\omega_J^k(\mu, \mu_J^k)} S^{\text{unmeas}}(\mu_S^\Lambda) \\
&\times \prod_{i=1}^M \left\{ [1 + f_J^i(\tau_a^i, \mu_J^i) + f_S^i(\tau_a^i, \mu_S^i)] \left(\frac{\mu_S^i \tan^{1-a} \frac{R}{2}}{\omega_i} \right)^{\omega_S^i(\mu, \mu_S^i)} \right. \\
&\times \left. \left(\frac{\mu_J^i}{\omega_i} \right)^{(2-a)\omega_J^i(\mu, \mu_J^i)} \frac{1}{\Gamma[-\omega_J^i(\mu, \mu_J^i) - \omega_S^i(\mu, \mu_S^i)]} \frac{1}{(\tau_a^i)^{1+\omega_J^i(\mu, \mu_J^i) + \omega_S^i(\mu, \mu_S^i)}} \right\}_+ \\
&\times \exp \left[\mathcal{K}(\mu; \mu_H, \mu_J^{1,\dots,N}, \mu_S^{1,\dots,M}, \mu_S^\Lambda) + \gamma_E \Omega(\mu; \mu_J^{1,\dots,M}, \mu_S^{1,\dots,M}) \right],
\end{aligned}$$

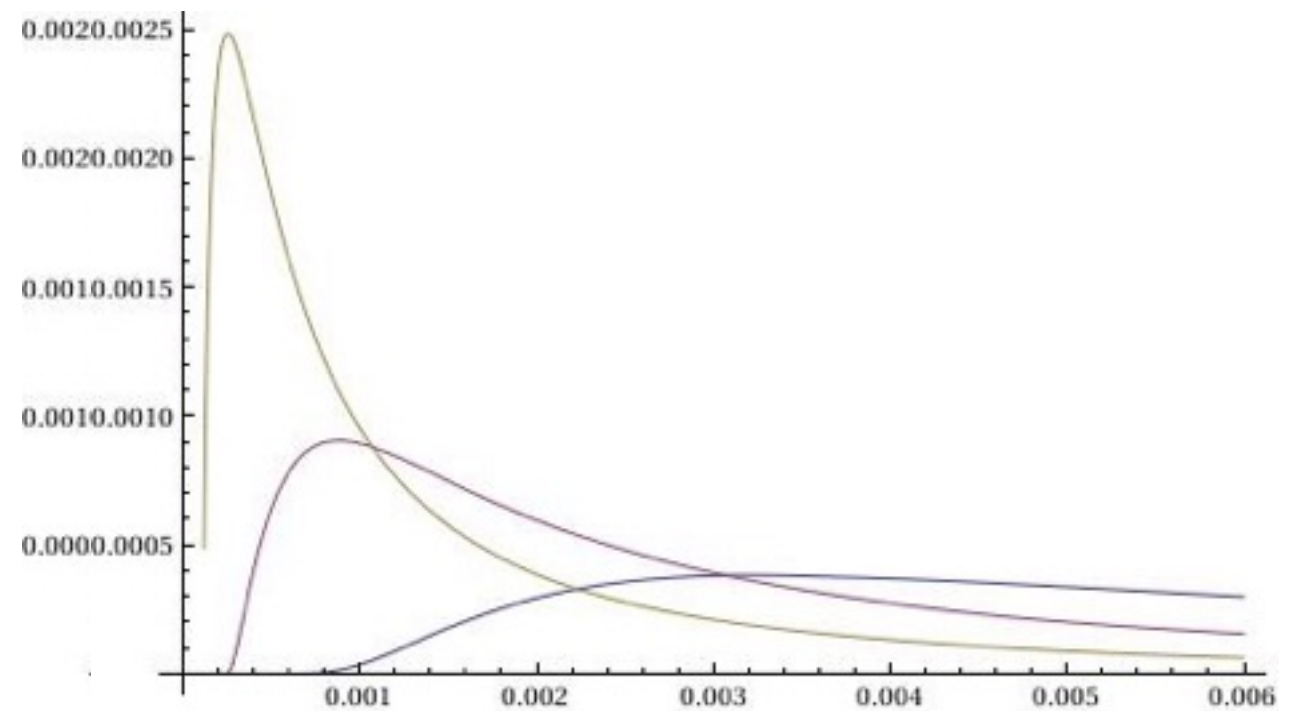
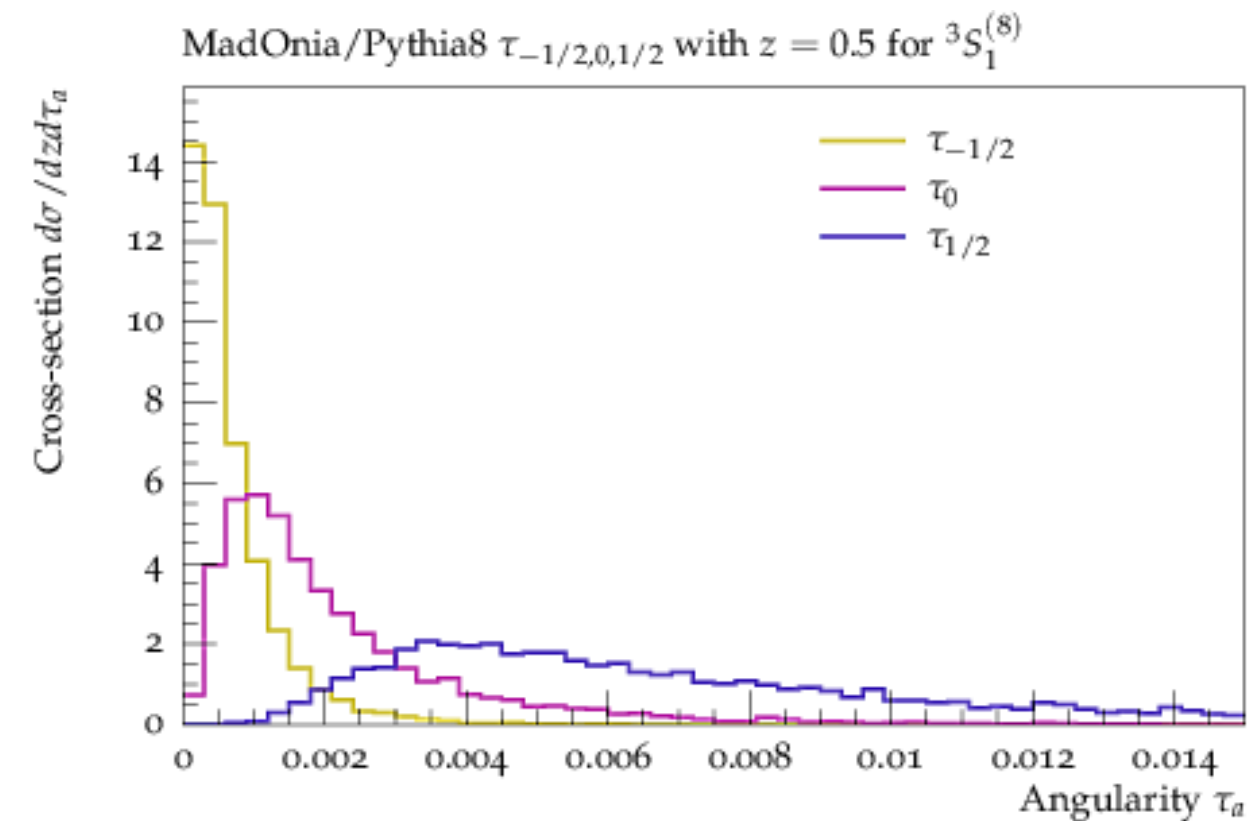
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 &\times \prod_{i=1}^M \left\{ [1 + f_J^i(\tau_a^i, \mu_J^i) + f_S^i(\tau_a^i, \mu_S^i)] \left(\frac{\mu_S^i \tan^{1-a} \frac{R}{2}}{\omega_i} \right)^{\omega_S^i(\mu, \mu_S^i)} \right. \\
 &\times \left. \left(\frac{\mu_J^i}{\omega_i} \right)^{(2-a)\omega_J^i(\mu, \mu_J^i)} \frac{1}{\Gamma[-\omega_J^i(\mu, \mu_J^i) - \omega_S^i(\mu, \mu_S^i)]} \frac{1}{(\tau_a^i)^{1+\omega_J^i(\mu, \mu_J^i) + \omega_S^i(\mu, \mu_S^i)}} \right\}_+ \\
 &\times \exp \left[\mathcal{K}(\mu; \mu_H, \mu_J^{1,\dots,N}, \mu_S^{1,\dots,M}, \mu_S^\Lambda) + \gamma_E \Omega(\mu; \mu_J^{1,\dots,M}, \mu_S^{1,\dots,M}) \right],
 \end{aligned}$$

RGE evolution

Resummed Cross Section vs. Pythia

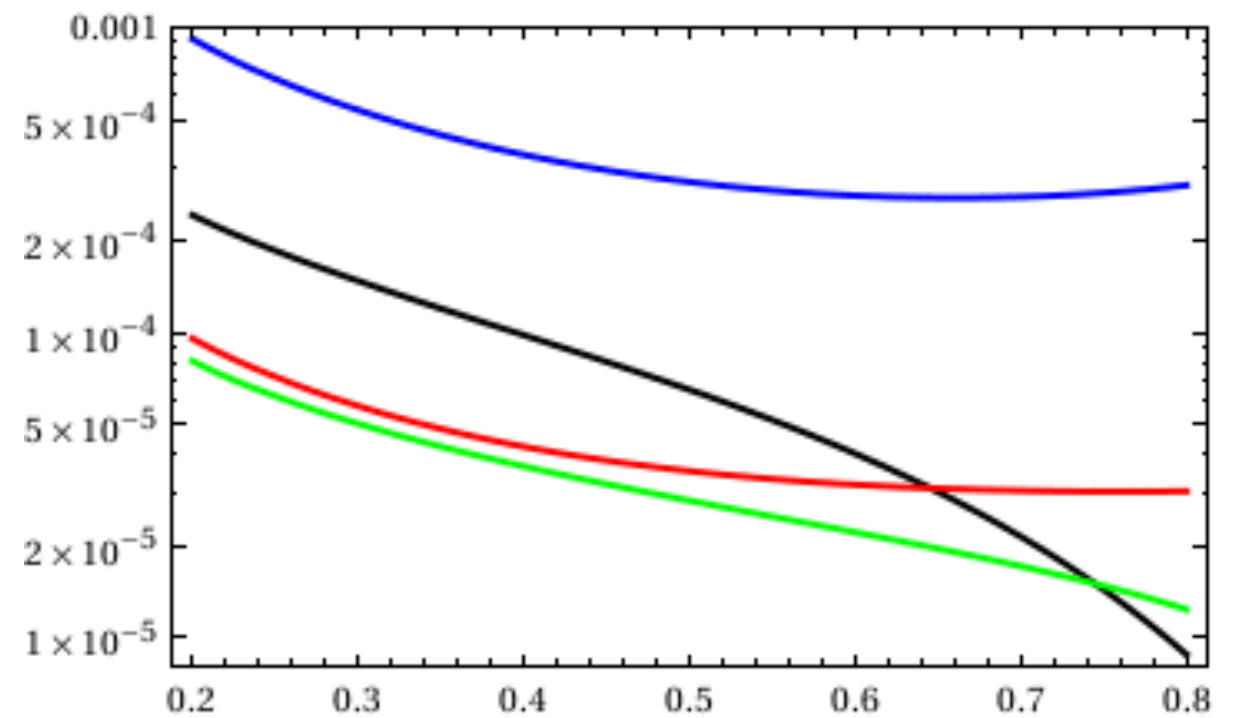
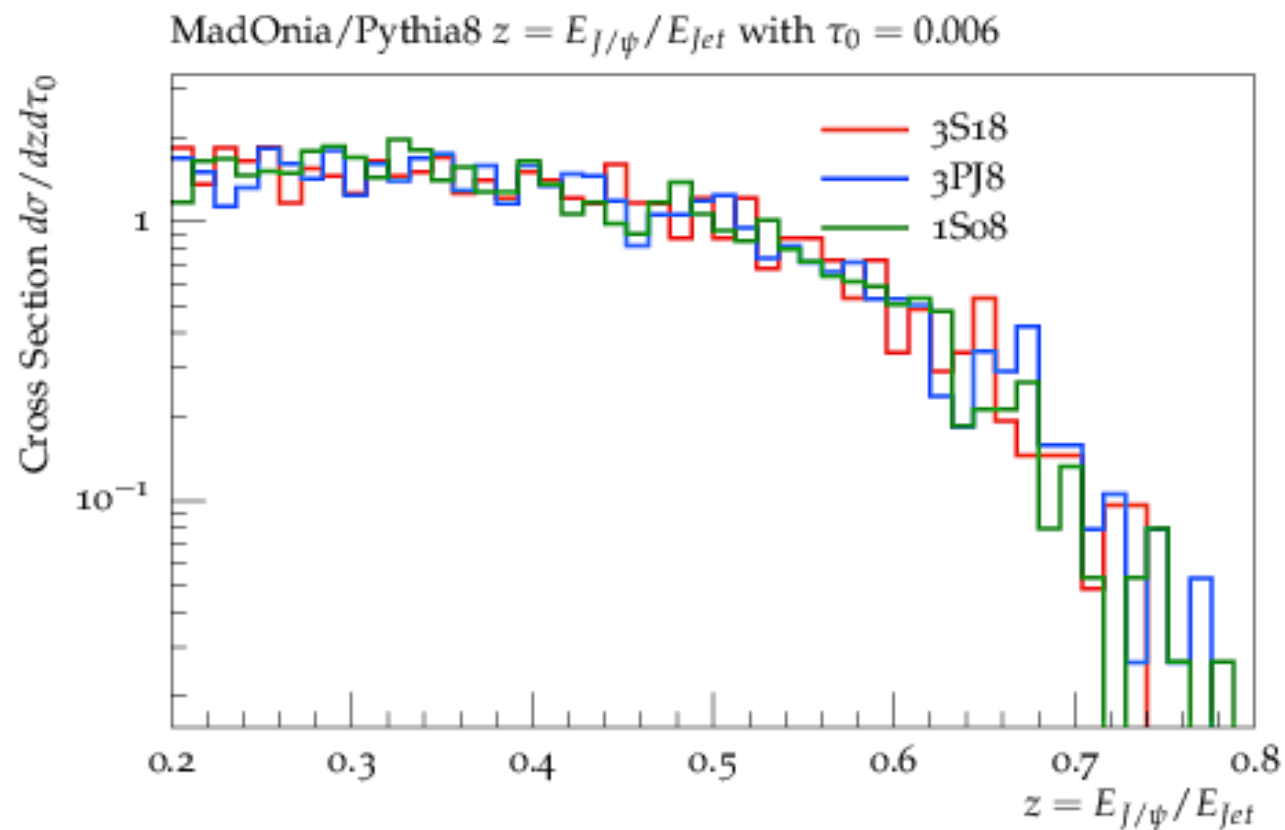
τ_a dependence



to appear, R. Bain, A. Hornig, Y. Makris, T.M.

Resummed Cross Section vs. Pythia

z dependence



Pythia predicts same z dependence for all color-octet mechanisms

Resummed Cross Section vs. Pythia

Agreement: factorized form for cross section

$$\frac{d\sigma}{d\tau_a dz} \propto D(z)F(\tau_a)$$

τ_a dependence

Disagreement: z dependence

differentiating mechanisms

Analogous Formulae for pp collisions

to appear, A. Hornig, Y. Makris, T.M.

boost invariant angularity

$$\begin{aligned}\tau_a^{e^+e^-} &= \frac{1}{2E_J} \sum_i |p_T^{iJ}| e^{-(1-a)|y_{iJ}|} \\ &= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin \theta_J} \right)^{2-a} (1 + \mathcal{O}(\theta_{iJ}^2))\end{aligned}$$

$$\tau_a^{pp} = \left(\frac{2E_J}{p_T} \right)^{2-a} \tau_a^{e^+e^-} + \mathcal{O}(\tau_a^2)$$

modified jet function

$$J_i(\tau_a) = \left(\frac{p_T}{2E_J} \right)^{2-a} J_i^{e^+e^-} \left(\left(\frac{p_T}{2E_J} \right)^{2-a} \tau_a \right)$$

Analogous Formulae for pp collisions

Soft Function in e^+e^-

rotationally invariant cuts: $E < E_{\min}$

Soft Function in pp

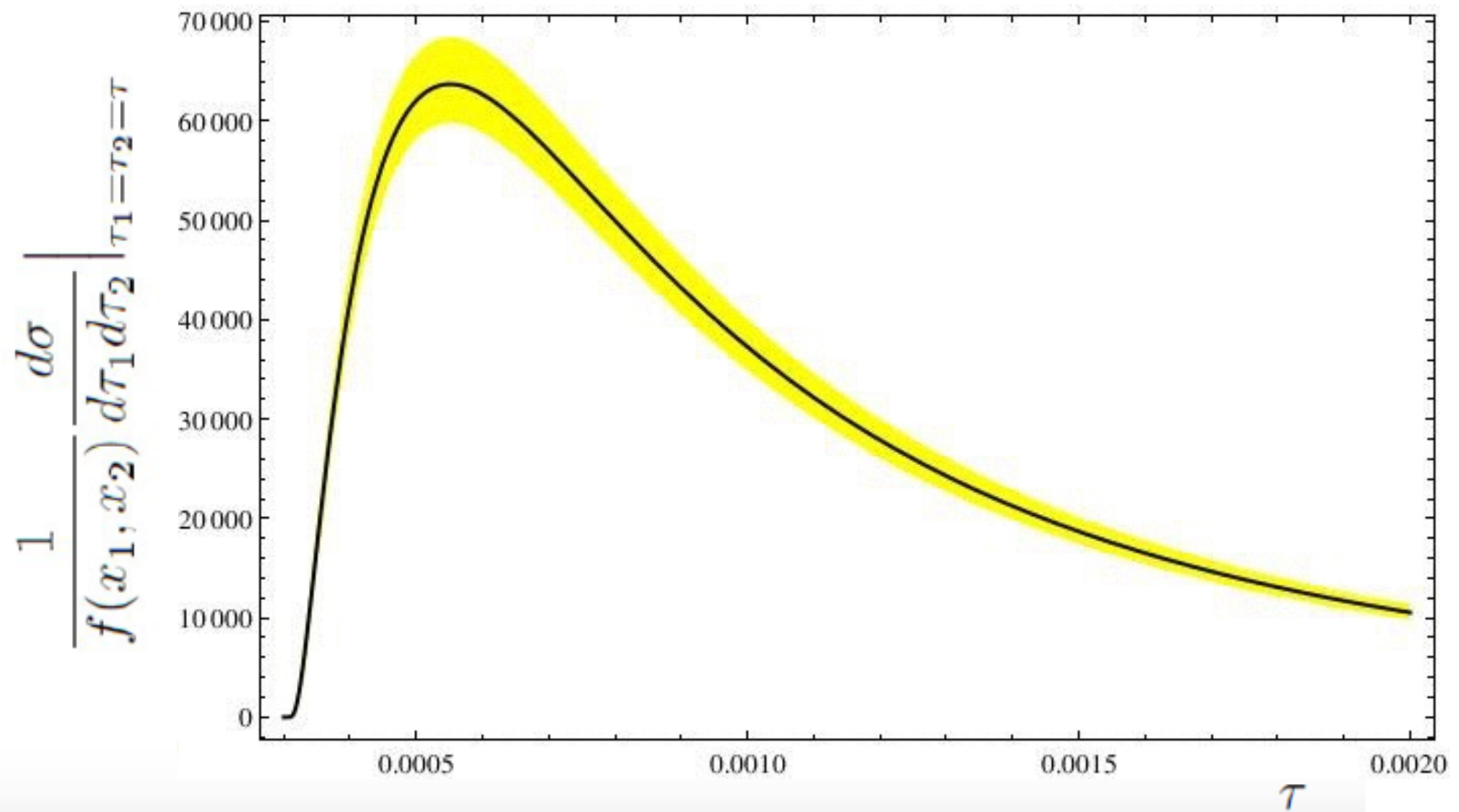
boost invariant cuts, observables: p_T , rapidity

hard, soft functions are matrices in color space

$$d\sigma(\tau_a^1, \tau_a^2) = \frac{p_T x_1 x_2}{8\pi E_{\text{cm}}^4} \frac{1}{N} B(x_1; \mu) \bar{B}(x_2; \mu) \text{Tr}\{\mathbf{H}(\mu) \mathbf{S}(\tau_a^1, \tau_a^2; \mu)\} \otimes [J_1(\tau_a^1; \mu) J_2(\tau_a^2; \mu)]$$

$pp \rightarrow 2$ jets with boost invariant soft function

A. Hornig, Y. Makris, T.M.



$q\bar{q}$ channel only

Conclusions

NRQCD describes much world data on quarkonium data but puzzles,
esp. polarization, remain

existing analyses focus on inclusive p_t spectra, polarization can we find
other observables distinguish various production mechanisms at high p_T ?

**measuring $Q\bar{Q}$ within jets, and using jet observables
should provide insights into $Q\bar{Q}$ production**

quarkonium fragmenting jet functions (FJFs)

**If $^1S_0^{(8)}$ mechanism dominates high p_T production
FJF should have negative slope for $z(E)$, for $z > 0.5$**

Work in Progress:

comparing SCET factorization theorems w/ Monte Carlo
developing analytic formulae for pp collisions

Backup

fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^-x^+/2} \frac{1}{4N_c} \text{Tr} \sum_X \langle 0 | \not{n} \Psi(x^+, 0, 0_\perp) | Xh \rangle \langle Xh | \bar{\Psi}(0) | 0 \rangle \Big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\left(\frac{p_h^-}{\omega}, \mu\right) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \text{Tr} \sum_X \not{n} \langle 0 | [\delta_{\omega, \bar{\mathcal{P}}} \delta_{0, \mathcal{P}_\perp} \chi_n(0)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\omega} \text{Im} \int d^4x e^{ik \cdot x} i \langle 0 | T \bar{\chi}_{n, \omega, 0_\perp}(0) \frac{\not{n}}{4N_c} \chi_n(x) | 0 \rangle$$

fragmentation jet function (SCET)

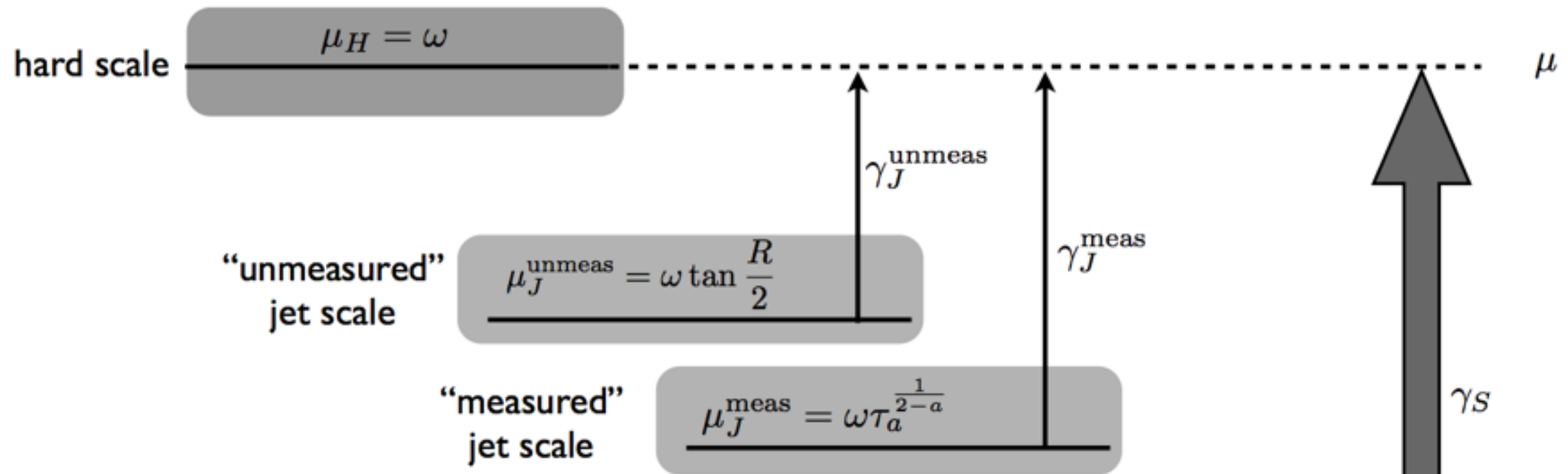
$$\mathcal{G}_{q, \text{bare}}^h(s, z) = \int d^4y e^{ik^+y^-/2} \int dp_h^+ \sum_X \frac{1}{4N_c} \text{tr} \left[\frac{\not{n}}{2} \langle 0 | [\delta_{\omega, \bar{\mathcal{P}}} \delta_{0, \mathcal{P}_\perp} \chi_n(y)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \right]$$

$$\delta(p^+/z - P_H^+) \rightarrow \delta(p^+/z - P_H^+) \delta(p^- - s/p^+)$$

FF

FJF

Scales in Jet Cross section



EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

