Factorized power expansion for high-p_T heavy-quarkonium production

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Based on work done with Zhong-Bo Kang, Yan-Qing Ma, Jian-Wei Qiu, and George Sterman

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Outline

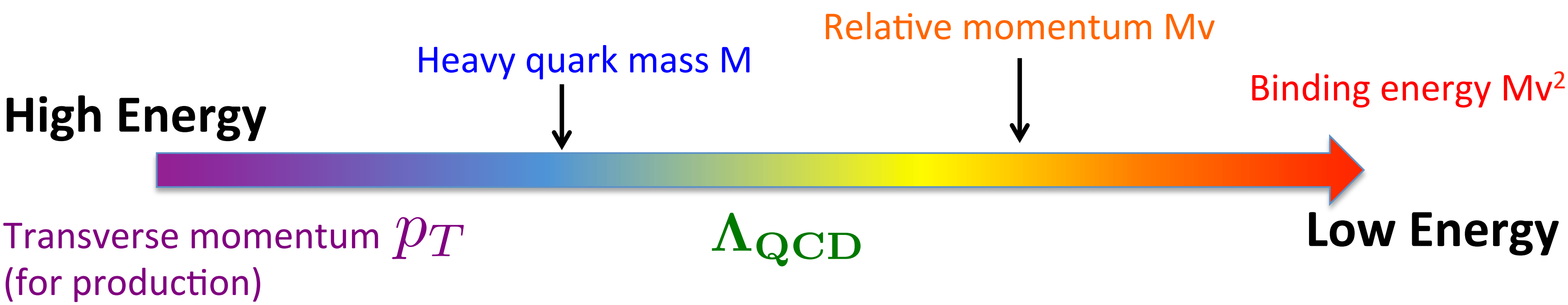
- ✧ Introduction
  - Heavy quarkonium, historical production models,
  - Difficulties of NRQCD model
- ✧ QCD factorization
  - Formalism and predictive power
- ✧ Calculate unpolarized fragmentation functions with NRQCD
  - Cancellation of divergences
- ✧ LO QCD factorization vs. NLO NRQCD model
- ✧ Calculate polarized fragmentation functions with NRQCD
- ✧ Summary and outlook

Heavy quarkonium & models

❖ Non-relativistic QCD system

Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

❖ Multiple separated scales



❖ Historical models for production

- Color Singlet Model (CSM): 1975 — Einhorn and Ellis (1975), Chang (1980), Berger and Jones (1981)…
- Color Evaporation Model (CEM): 1977 — Fritsch (1977), Halzen (1977), …
- NRQCD factorization: 1986, 1994 — Caswell, Lapage (1986); Bodwin, Braaten, Lepage (1995)

NRQCD

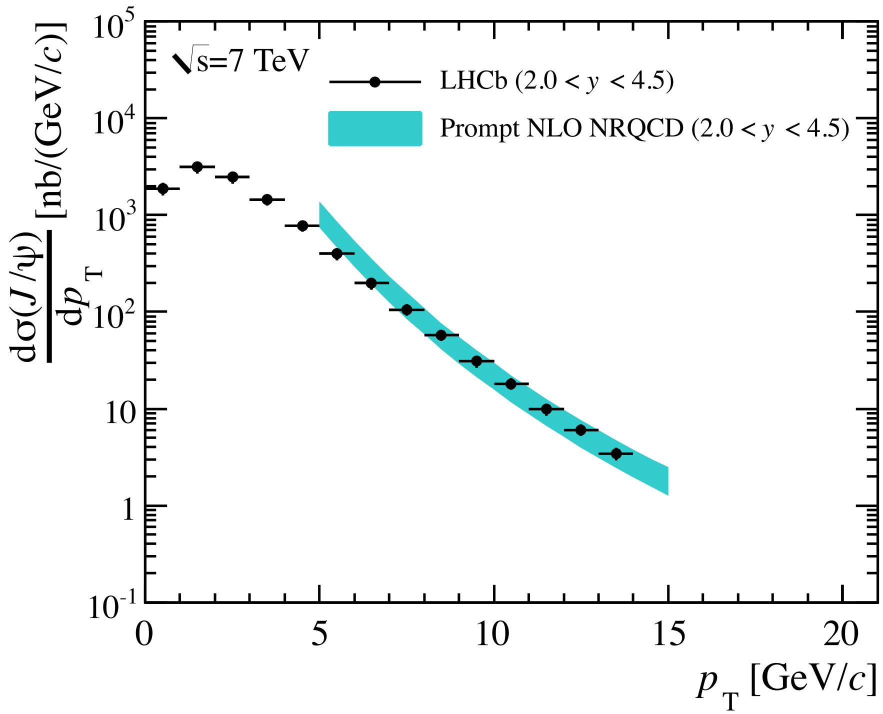
❖ NRQCD factorization for HQ production

Bodwin, Braaten, Lepage (1995)

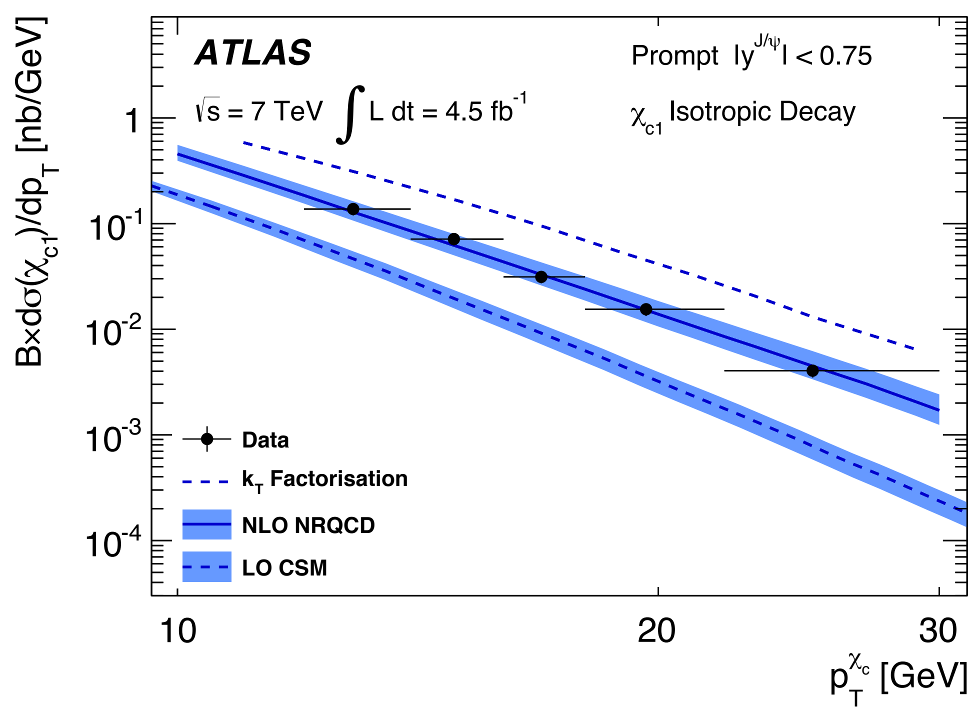
$$d\sigma_{AB\to\psi+X} = \sum_{c\bar{c}[n]} d\hat{\sigma}_{AB\to c\bar{c}[n]+X} \times \langle \mathcal{O}_n^{\psi} \rangle$$

NRQCD long-distance matrix element (LDME)

Ordered by powers of v



LHCb collaboration, 2011



Atlas collaboration, 2014

Puzzles in NRQCD

- Large high-order correction

  Due to power enhancement at high orders of α_s

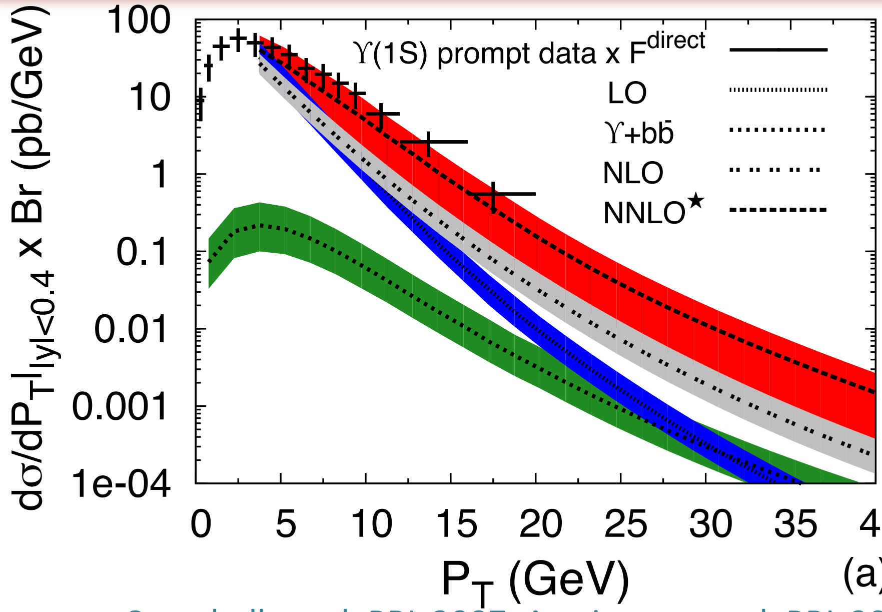
- $\psi(2S)$ & $\Upsilon(3S)$ Polarization puzzle

  NRQCD: with LDMEs extracted from unpolarized data, NRQCD predicts transverse polarization at large pT.

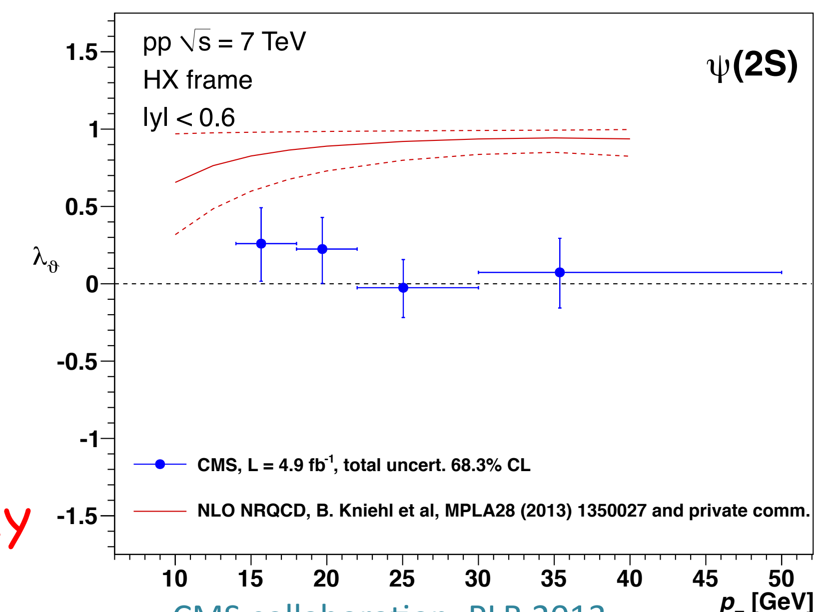
  Data: almost no polarization at current collider energies

- Universality of NRQCD LDMEs

  A global fitting of NRQCD LDMEs on J/ψ production implies the LDMEs may not be universal. Butenschoen PRD 2011



Campbell et, al. PRL 2007. Artoisenet et, al. PRL 2008



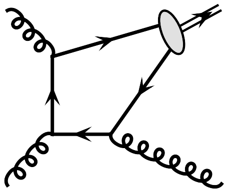
CMS collaboration, PLB 2013

Polarization and universality puzzle may be related to the power enhancement.

Why high-order correction is large?

Example: ${}^3S_1^{[1]}$ channel

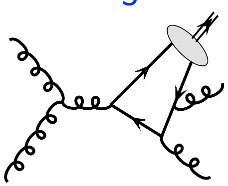
- LO in α_S, but NNLP in $1/p_T$

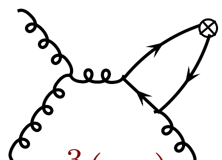


$$\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_T^2} \sim \alpha_s^3(p_T)\frac{m_Q^4}{p_T^8}$$

Sum over 6 relativistic spin states

- NLO in α_S, but dominated by NLP in $1/p_T$





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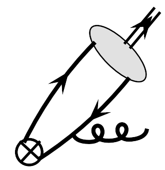
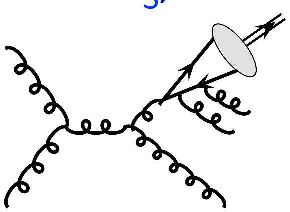


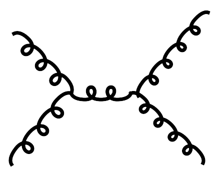
Heavy quark pair fragmentation

$$\frac{d\hat{\sigma}^{\mathrm{NLO}}}{dp_T^2} \sim \alpha_s^4(p_T)\frac{m_Q^2}{p_T^6}\log(\frac{p_T^2}{m_Q^2}) \rightarrow \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)m_Q^2$$

$\mu_0 \gtrsim 2m_Q$

- NNLO in α_S, but dominated by LP in $1/p_T$





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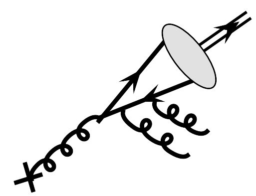


Single parton fragmentation

$$\frac{d\hat{\sigma}^{\mathrm{NNLO}}}{dp_T^2} \sim \alpha_s^4(p_T)\frac{1}{p_T^4}\log^m(\frac{p_T^2}{m_Q^2}) \rightarrow \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu)\log^m(\mu^2/\mu_0^2)$$

At large p_T, more suitable to expand by powers of m_Q^2/p_T^2 before α_S.

NLP QCD Factorization

❖ Factorization formalism

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

LP: Single Parton Fragmentation

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta_1)/2z, p(1\pm\zeta_2)/2z)) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q)$$

NLP: $Q\bar{Q}$ Fragmentation

Spinor:
vector (V),
axial-vector (A),
tensor (T)
Color: 1 or 8

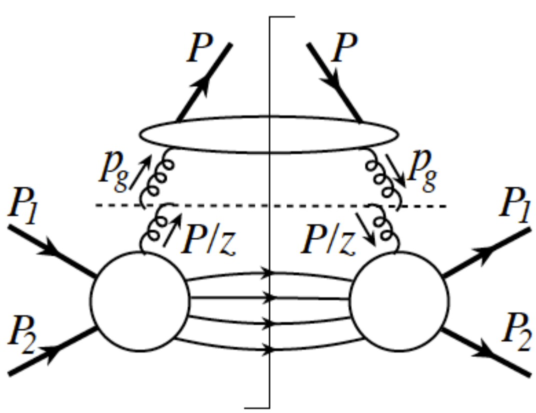
$$+\mathcal{O}(m_Q^4/p_T^8)$$

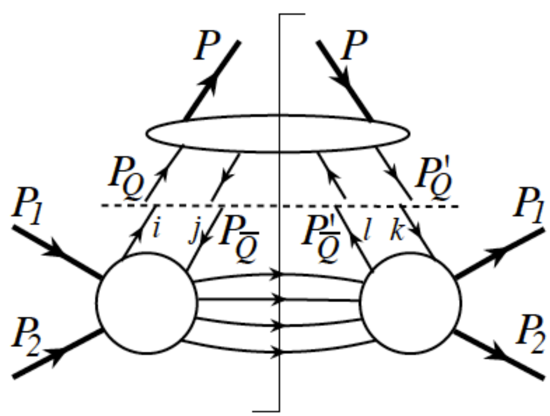
3 variables: z, ζ_1, ζ_2

$\zeta_1 = 1 \Leftrightarrow P_{\bar{Q}} \cdot n = 0$
$\zeta_1 = -1 \Leftrightarrow P_Q \cdot n = 0$
ζ_2 is for the complex conjugate.
ζ_1 and ζ_2 can be different.

Expand cross section first in powers of $1/p_T$, then α_s.

Factorization is proved to all orders in α_s for both LP and NLP





Nayak, Qiu and Sterman, PRD (2005) … ,
Kang, Qiu and Sterman, PRL (2011)…
Kang, Ma, Qiu and Sterman, PRD (2014).
SCET approach: Fleming et.al., PRD (2012)

Predictive Power

❖ Short-distance hard part can be calculated perturbatively

Choose $\mu \sim \mathcal{O}(p_T)$, no large logarithms exist. Perturbation should converge fast.

Hard parts of all channels have been calculated to LO.

Kang, Ma, Qiu, Sterman, PRD 91.014030

❖ Evolution equations determine the scale dependence of FFs

$$\frac{d}{d\ln\mu^2} D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z) \otimes D_{H/j}(z, m_Q, \mu)$$

If only keep LP, same as DGLAP evolution

$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta_1,\zeta_2,m_Q,\mu)$$

NLP contributes to LP via evolution

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta_1,\zeta_2,m_Q,\mu)$$

By solving the evolution equations, $\log(\mu^2/\mu_0^2)$ is resummed.

❖ Evolution kernels can be calculated in perturbative QCD

All evolution kernels have been calculated to LO.

Fleming et.al., PRD (2013)
Kang, Ma, Qiu and Sterman, PRD (2013)

Predictive power of QCD factorization relies on input FFs

Input fragmentation functions

❖ Input FFs include all non-perturbative interaction

- Relative production rate
- Polarization
- Very hard to be extracted from data

Example: unpolarized J/ψ

$D_{H/f}(z, m_Q, \mu_0)$ $\quad f = q, c, b, g$ $\quad$ 4 functions

$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q, \mu_0)$ $\quad [Q\bar{Q}(\kappa)] = v^{[1,8]}, a^{[1,8]}, t^{[1,8]}$ $\quad$ 6 functions

3 variables

20 functions if with polarization

❖ Different from fragmentation to pion or kaon

- Large heavy quark mass $m_Q \gg \Lambda_{QCD}$ → Partially perturbative
- Separated energy scales in heavy quarkonium → Apply NRQCD to calculate the FFs

Ten or twenty unknown functions → **Three** unknown LDMEs

Braaten, Yuan… since 1993

Greatly enhance the predictive power of QCD factorization.

Matching Input FFs to NRQCD

❖ Factorization form in NRQCD factorization

$$D_{f\to H}(z;m_Q,\mu_0)=\sum_{[Q\bar{Q}(n)]}\hat{d}_{f\to[Q\bar{Q}(n)]}(z;m_Q,\mu_0,\mu_\Lambda)\langle\mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle$$

- Short-distance coefficients $\hat{d}_{f\to[Q\bar{Q}(n)]}$ are perturbative, insensitive to the long-distance hadronization process

$$\underline{D_{f\to[Q\bar{Q}(n')]}(z;m_Q,\mu_0)}=\sum_{[Q\bar{Q}(n)]}\hat{d}_{f\to[Q\bar{Q}(n)]}(z;m_Q,\mu_0,\mu_\Lambda)\underline{\langle\mathcal{O}^{[Q\bar{Q}(n')]}_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}$$

Calculated with pQCD

Calculated with NRQCD

Get the coefficient by matching LHS and RHS

- Heavy quark pair FFs are similar.

If NRQCD factorization is valid, short-distance coefficients are IR finite to all orders of α_s.

- Scales: $\mu_0 \gtrsim 2m_Q$, $\mu_\Lambda \sim m_Q$
- With no large logarithms and power corrections, NRQCD factorization should give stable α_s and v expansion.

Special Divergences Cancellation for FFs

❖ UV: Need renormalization for composite operators, in addition to QCD renormalization

❖ IR: due to the parameters z, ζ_1, ζ_2

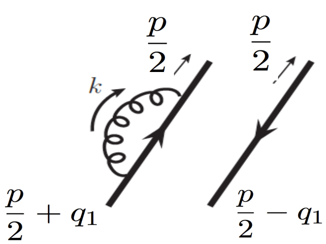
- Single Parton FF: cancellation between virtual & real

Virtual diagrams: $\frac{1}{\epsilon_{IR}}\delta(1-z)$ Real diagrams: $(1-z)^{-1-2\epsilon}$ Diverge at $z \to 1$

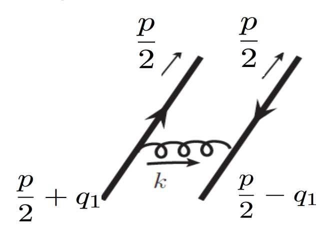
$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon_{IR}}\delta(1-z) + \frac{1}{(1-z)_+} - 2\epsilon\left(\frac{\log(1-z)}{(1-z)}\right)_+$$

$$\int_0^1 dz \frac{1}{(1-z)_+} f(z) = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$ Converge at $z \to 1$

- Heavy quark pair FF: additional amplitude level cancellation



$\frac{\mathbf{1}}{\epsilon_{\mathbf{IR}}}\delta(\zeta_1)$ and $\frac{\mathbf{1}}{\epsilon_{\mathbf{IR}}}\delta'(\zeta_1)$



$\zeta_1^{-1-2\epsilon}$ and $\zeta_1^{-2-2\epsilon}$

Diverge at $\zeta_1 \to 0$

Generalized plus/minus-distributions

$$\int d\zeta_1 \left[\frac{1}{\zeta_1^n}\right]_{n\pm} f(\zeta_1) = \int_{-1}^1 d\zeta_1 \left(\frac{\theta(\zeta_1)}{\zeta_1^n} \pm \frac{\theta(-\zeta_1)}{(-\zeta_1)^n}\right)\left[f(\zeta_1) - \sum_{i=0}^{n-1}\frac{1}{i\,!}f^{(i)}(0)\,\zeta_1^i\right]$$

Similarly for ζ_2 in the complex conjugate of the amplitude.

Example: NLP FFs

Example: $[Q\bar{Q}(a^{[8]}) \to [Q\bar{Q}(^1S_0^{[8]})]$ Both virtual and real contributions

✧ Leading order:

$$\hat{d}^{\,\mathrm{LO}}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}(^1S_0^{[8]})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0)=\frac{1}{N_c^2-1}\frac{1}{2m_Q}\,\delta(1-z)\,\delta(\zeta_1)\,\delta(\zeta_2)$$

✧ Next-to-leading order:

$$\hat{d}^{\,\mathrm{NLO}}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}(^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q)=\frac{\alpha_s}{64\,\pi m_Q(N_c^2-1)}$$

$$\times\left\{\Gamma_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}(a^{[8]})]}\left(z,\frac{1+\zeta_1}{2},\frac{1+\zeta_2}{2};\frac{1}{2},\frac{1}{2}\right)\ln\left[\frac{\mu_0^2}{m_Q^2}\right]\right.$$

$$\left.+R(z,\zeta_1,\zeta_2)+\delta(1-z)[V(\zeta_1)\delta(\zeta_2)+V(\zeta_2)\delta(\zeta_1)]\right\}$$

Heavy quark pair evolution kernel

Remnant of renormalization of composite operator

$$V(\zeta_1)=\frac{1}{N_c}\left\{2\left[-\left(\frac{1}{\zeta_1^2}\right)_{2+}+\left(\frac{1}{\zeta_1}\right)_{1+}+\left(\frac{\ln(\zeta_1^2)}{\zeta_1}\right)_{1+}\right]-[(\zeta_1)_{0+}+1]\ln(\zeta_1^2)-(\zeta_1)_{0+}+1\right\}$$

Remnants of amplitude level IR divergence cancellation

$$R(z,\zeta_1,\zeta_2)=\frac{1}{N_c}\left\{\Delta_+^{[8]}\left[-2z\left(\frac{\ln(2-2z)}{1-z}\right)_+-\frac{z}{(1-z)_+}\right]-8\left[(\ln 2)^2+\ln 2\right]C_A^2\,\delta(\zeta_1)\,\delta(\zeta_2)\,\delta(1-z)\right\}$$

Remnants of IR pole cancellation between real and virtual corrections

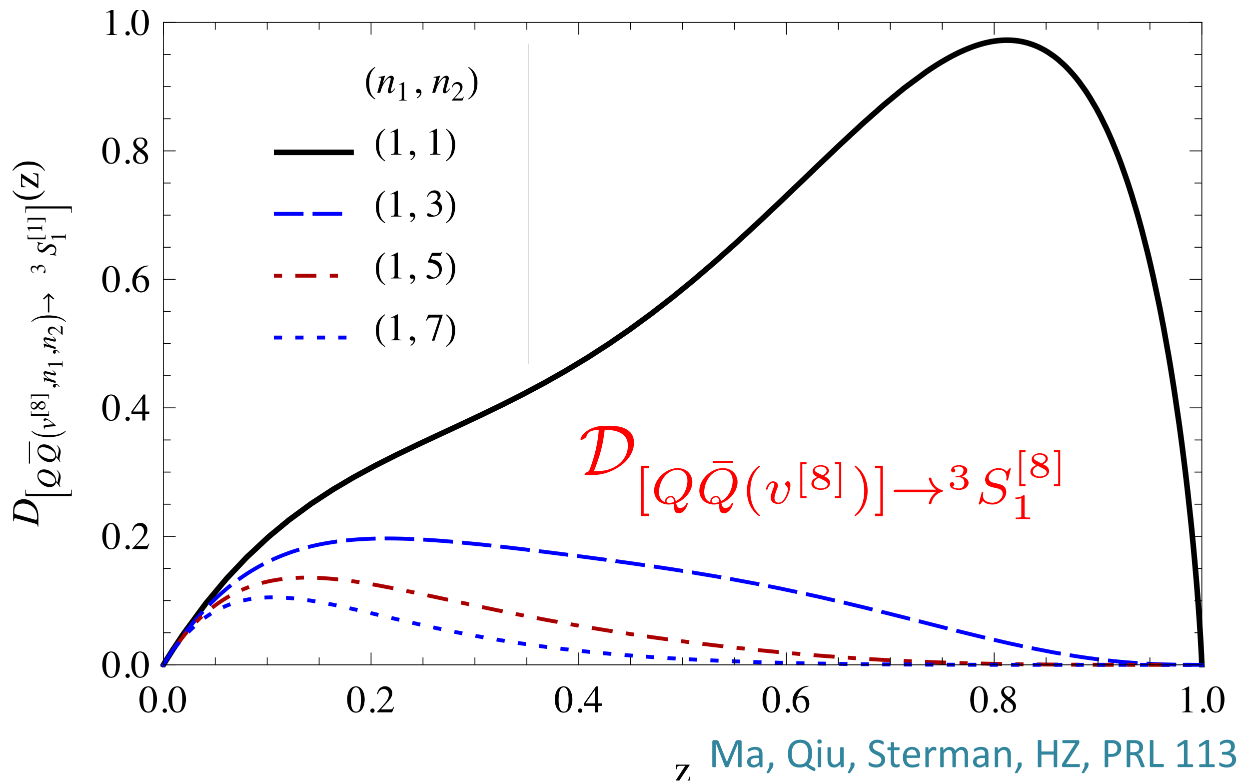
Input NLP FF: Moments

Unlike the FFs of light hadrons extracted from the data, quarkonium FFs calculated in NRQCD are distributions defined under integration (δ, +/-, …)

Moments: $$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$$

Fragmentation contribution dominated by large z region – two steep falling PDFs

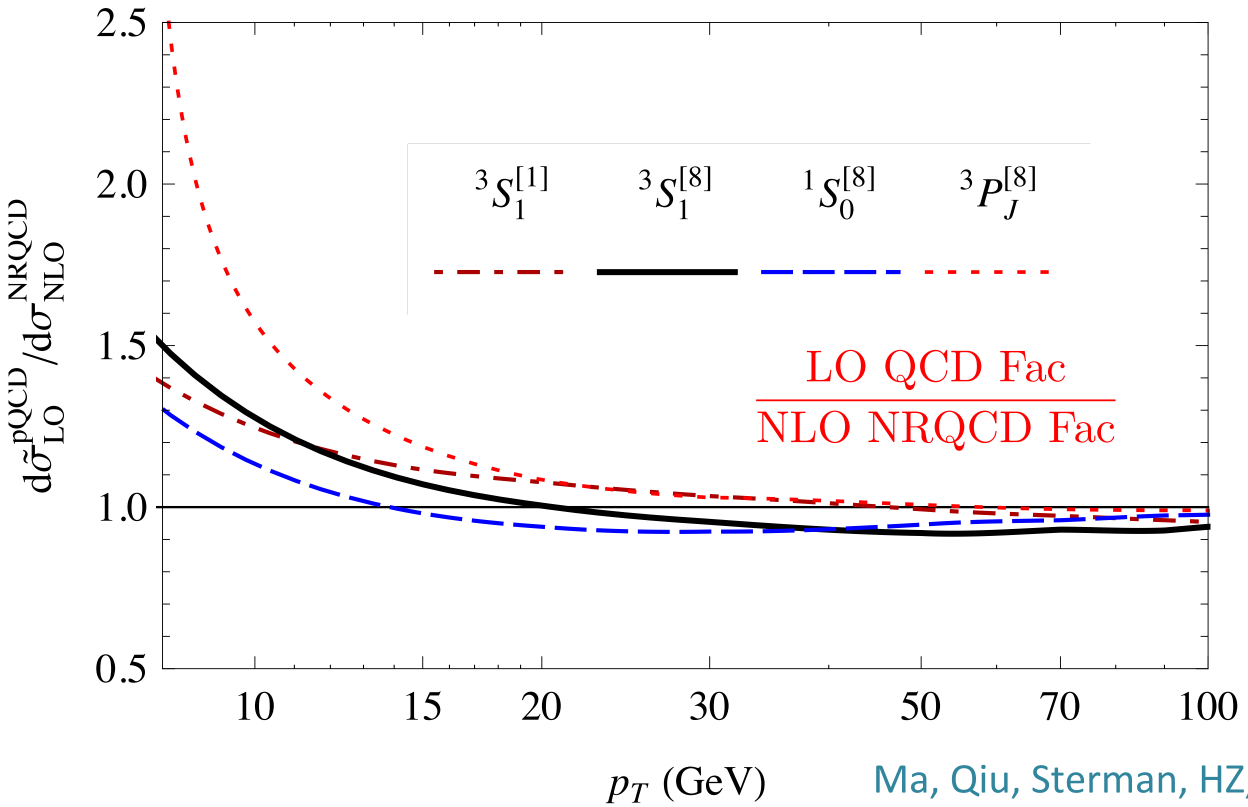
Higher moments decrease quickly for moderate and large z, indicating the probability of a relativistic heavy quark pair with large relative momentum to form quarkonium is small.



Ma, Qiu, Sterman, HZ, PRL 113.042002

Compare with NLO NRQCD

- Inclusive hadron production for polarization-summed J/ψ
- Analytical LO QCD factorization can reproduce the complicated, numerical NLO NRQCD calculation channel by channel at $p_T > 15$ GeV
- The good fit indicates the LO QCD factorization calculation includes the dominant effect of NRQCD at NLO
- QCD factorization approach has better control of high-order corrections



Ma, Qiu, Sterman, HZ, PRL 113.042002

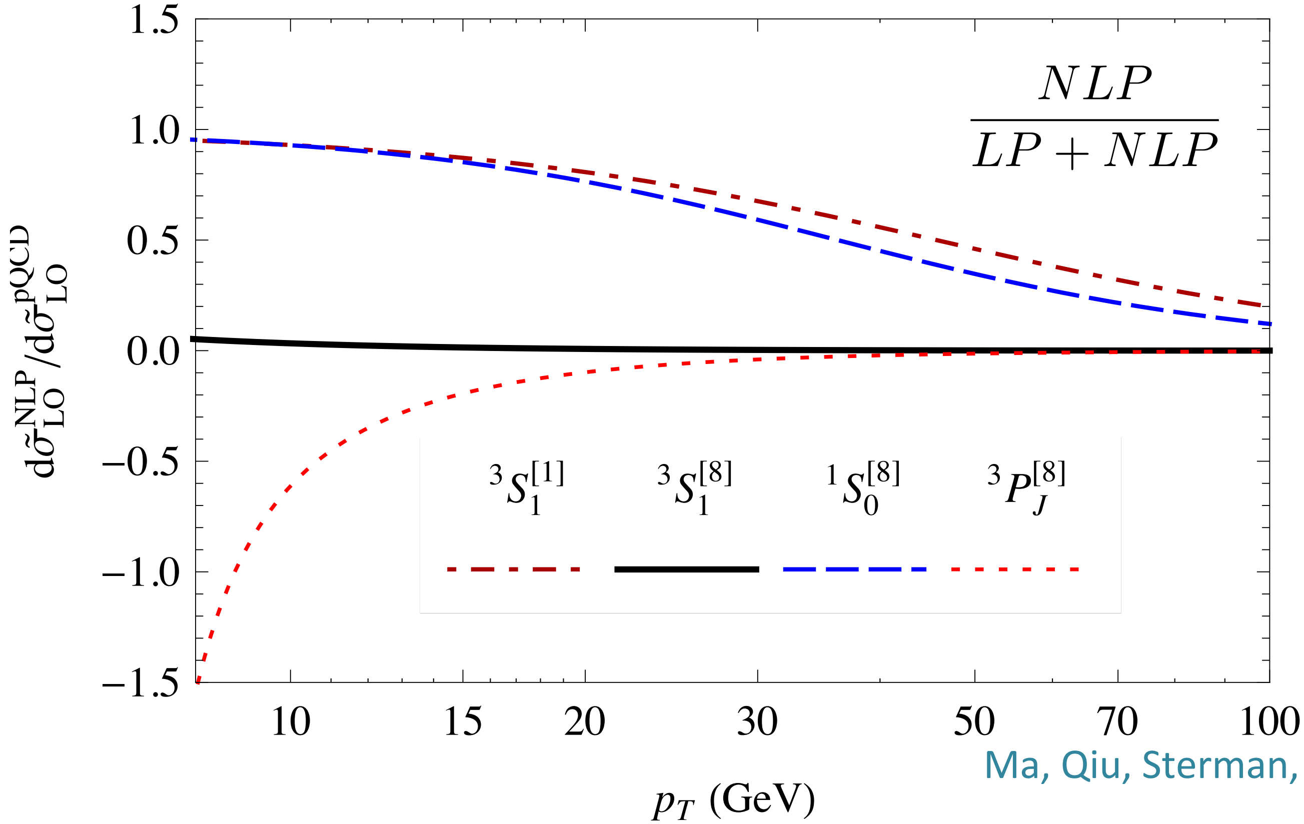
Importance of NLP

❖ Weight of LO NLP contribution in full LO QCD factorization result

NLP ${}^3S_1^{[8]}$: Negligible

NLP ${}^3P_J^{[8]}$: small when p_T > 20 GeV

NLP ${}^3S_1^{[1]}$ and ${}^1S_0^{[8]}$: crucial even at p_T ~ 70 GeV



Ma, Qiu, Sterman, HZ, PRL 113.042002

Use NNLO result for LP fragmentation in ${}^3S_1^{[1]}$ channel

Braaten and Yuan, PRL 1993
Bodwin, Kim and Lee, JHEP 2012

Input FFs to Polarized Quarkonium

- Definitions for polarized NRQCD LDMEs in d dimensions

$$N = d - 1$$ Number of spatial dimensions

- For $^3S_1$ channel

N-dimensional heavy quark pair in $^3S_1$ state: $[^3S_1] = \chi^\dagger \sigma^i \psi$ i = 1,2 … N

3-dimensional spherical harmonics $Y_{1,0} \propto \cos\theta$ Longitudinal

$Y_{1,\pm 1} \propto \sin\theta\, \mathrm{e}^{\pm i\phi}$ Transverse

Observation: rotate around z-axis by π (equivalently, flip the direction of all axes except z), $Y_{1,\pm 1}$ flips sign but $Y_{1,0}$ does not change.

$$[^3S_1]_{\mathrm{T}} = \frac{1}{2}\left\{[^3S_1] - [^3S_1]_{(\text{rotate } \pi)}\right\} = \chi^\dagger \sigma^{i_\perp} \psi$$

$$[^3S_1]_{\mathrm{L}} = \frac{1}{2}\left\{[^3S_1] + [^3S_1]_{(\text{rotate } \pi)}\right\} = \chi^\dagger \sigma^{z} \psi$$

$$\mathcal{O}^{H_\lambda}(^3S_{1,T}^{[8]}) = \frac{1}{(d-2)} \chi^\dagger \sigma^{j_\perp} T^a \psi (a^\dagger_{H_\lambda} a_{H_\lambda}) \psi^\dagger \sigma^{j_\perp} T^a \chi$$

$$\mathcal{O}^{H_\lambda}(^3S_{1,L}^{[8]}) = \chi^\dagger \sigma^{z} T^a \psi (a^\dagger_{H_\lambda} a_{H_\lambda}) \psi^\dagger \sigma^{z} T^a \chi$$

Ma, Qiu, HZ, arXiv: 1501.04556

Input FFs to Polarized Quarkonium

❖ For ^3P_J channel

Ma, Qiu, HZ, arXiv: 1501.04556

N-dimensional heavy quark pair in ^3P_J state: $[^3P_J\ ] = \chi^\dagger(-\frac{i}{2}\overleftrightarrow{D}^j)\sigma^k\psi$ j,k = 1,2 … N

From N=3 L-S coupling

J=0: SO(N) symmetry

J=1: anti-symmetric of L and S

J=2: symmetric of L and S

For example: $^3P_2$

$$[^3P_2\ ]_{J_z=\pm1} = \frac{1}{2}\left\{[^3P_2\ ] - [^3P_2\ ]_{(\text{rotate }\pi)}\right\}$$

$$[^3P_2\ ]_{J_z=0,\pm2} = \frac{1}{2}\left\{[^3P_2\ ] + [^3P_2\ ]_{(\text{rotate }\pi)}\right\}$$

N=3 case:

$$Y_{2,\pm2} \propto \sin^2\theta\, e^{\pm2i\phi}$$

$$Y_{2,0} \propto (3\cos^2\theta - 1)$$

Observation: $Y_{2,0}$ has SO(N-1) symmetry

$$[^3P_2\ ]_{J_z=\pm2} \propto \chi^\dagger\left[-\frac{i}{2}(\frac{1}{2}\overleftrightarrow{D}^{\{j_\perp}\sigma^{k_\perp\}} - \frac{\delta^{j_\perp k_\perp}}{d-2}\overleftrightarrow{\boldsymbol{D}}_\perp\cdot\boldsymbol{\sigma}_\perp)\right]$$

$$[^3P_2\ ]_{J_z=\pm1} \propto \chi^\dagger(-\frac{i}{2}\overleftrightarrow{D}^{\{j_\perp}\sigma^{z\}})\psi$$

$$[^3P_2\ ]_{J_z=0} \propto \chi^\dagger\left[-\frac{i}{2}(\overleftrightarrow{D}^z\sigma^z - \frac{1}{d-2}\overleftrightarrow{\boldsymbol{D}}_\perp\cdot\boldsymbol{\sigma}_\perp)\right]\psi$$

NLP is mainly longitudinal

- All short-distance coefficients of polarized FFs are finite in NLO NRQCD calculation.

  Ma, Qiu, HZ, arXiv: 1501.04556

  - LP: contribute to transversely polarized J/ψ.
  - NLP: contribute mainly to longitudinally polarized J/ψ.

LO results of important channels

	${}^3S_1^{[1]}$	${}^3S_1^{[8]}$	${}^3P_J^{[8]}$	${}^1S_0^{[8]}$
g		T		
$v^{[1]}$	L			
$v^{[8]}$		L	L	
$a^{[1]}$				
$a^{[8]}$			T	Un

- In addition to direct contribution, NLP FFs at input scale $\mu_0 \gtrsim 2m_Q$ also contribute to LP FFs at larger scale via the mixed kernel in the evolution

$$\frac{d}{d\ln\mu^2} D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j} \otimes D_{H/j} + \frac{1}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s^2}{(2\pi)^2} \Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta_1,\zeta_2) \otimes \mathcal{D}_{H/[Q\bar{Q}](\kappa)}(z,\zeta_1,\zeta_2,m_Q,\mu)$$

- At moderate p_T, the LP and NLP contribution compete and result in unpolarized J/ψ.

With the calculated polarized FFs, NLP QCD factorization is very promising to solve the long standing polarization puzzle.

Summary and outlook

- NLP QCD factorization proposes expansion of $1/p_T$ before α_S, and proved the factorization to all orders of α_S for both LP and NLP.
- With the hard part and evolution kernels calculated, the predictive power of QCD factorization relies on input FFs, but, hard to extract all of them from the data.
- Assuming NRQCD factorization at input scale $\mu_0 \gtrsim 2m_Q$, we calculated these input unpolarized and polarized quarkonium FFs up to a few NRQCD LDMEs.
- LO hard parts $\otimes$ LO FFs reproduce "very complicated" NLO NRQCD calculation, clearly showing the $1/p_T$ expansion of cross section is better organized.
- NLO hard parts $\otimes$ NLO FFs, with resummation of $\ln(m_Q^2/p_T^2)$ and the refitted NRQCD LDMEs, is very promising to solve the polarization puzzle.
- For universality puzzle, global analysis needs more data from various processes, and more calculations.

Backup slides

NLP contribution is important

Power expansion of the cross section

$$\frac{\mathrm{d}\,\sigma_{pp\to J/\psi}}{\mathrm{d}\,p_T^2} = \hat{\sigma}^{\mathrm{LP}}_{pp\to a} \otimes D_{a\to J/\psi} + \hat{\sigma}^{\mathrm{NLP}}_{pp\to Q\bar{Q}} \otimes \mathcal{D}_{Q\bar{Q}\to J/\psi} + \mathcal{O}(\frac{1}{p_T^8})$$

$\mathcal{O}(1/p_T^4)$ $\qquad$ $\mathcal{O}(m_Q^2/p_T^6)$

Intuitively, easier to find a quarkonium in a heavy quark pair than in a single parton.

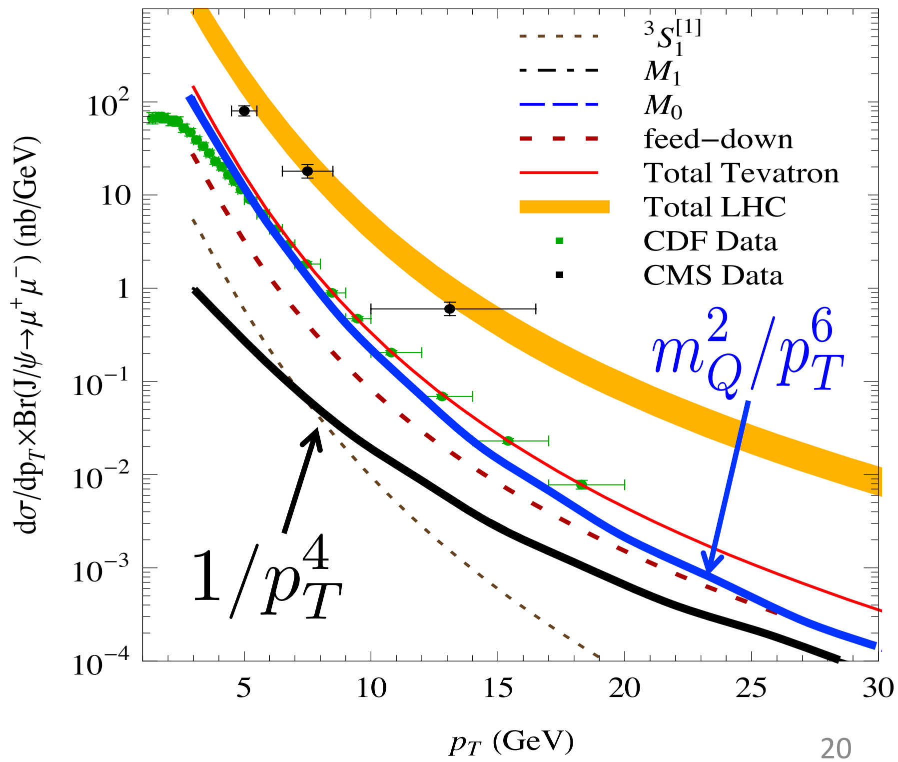
➢ Importance of NLP

- Direct evidence:

$1/p_T^6$ is shown large in the unpolarized J/ψ production in current collider energies.

- Indirect evidence:

J/ψ and ϒ are produced almost unpolarized.



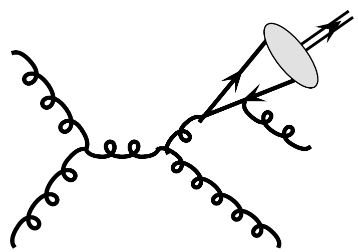
Ma, Wang, Chao PRL (2011)

Compare with NLO NRQCD

Channel	$^3S_1^{[1]}$	$^3S_1^{[1]}$	$^3S_1^{[8]}$	$^3S_1^{[8]}$	$^1S_0^{[8]}$	$^1S_0^{[8]}$	$^3P_J^{[8]}$	$^3P_J^{[8]}$
Power	LP	NLP	LP	NLP	LP	NLP	LP	NLP
PDFs	···	LO	LO	LO	NLO	LO	NLO	LO
FFs	···	α_s^1	α_s^1	α_s^0	α_s^2	α_s^0	α_s^2	α_s^0
SDCs	···	α_s^3	α_s^2	α_s^3	α_s^2	α_s^3	α_s^2	α_s^3

Example: $^1S_0^{[8]}$ channel

- NLO NRQCD factorization

NLO PDF $\otimes$ 

- LO QCD factorization

LO PDF $\otimes$