Subleading Harmonic Flow in Heavy Ion Collisions

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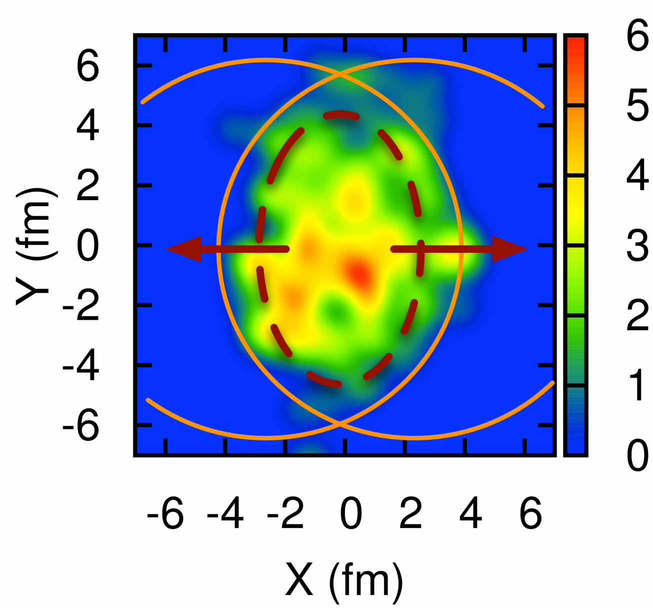
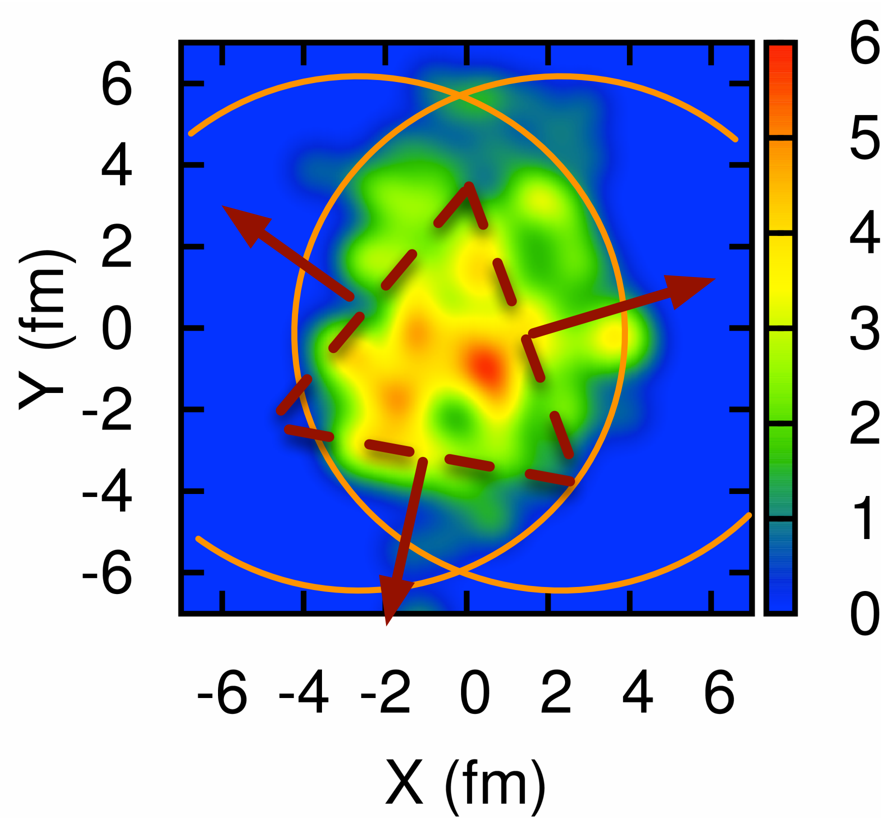


- R. Bhalerao, J.Y. Ollitrault, S. Pal, and DT, arXiv:1410.7739, PRL in press
- Aleksas Mazeliauskas, and DT, arXiv:1501.03138, PRC

Multiple Sources of Triangular Flow and Notation:



Quantify the initial e-dense w. triangularity:

$\langle . \rangle_{\rm ev}$ denotes ave over the entropy dist. in a single event

$$\epsilon_{3,3} = -\frac{\left\langle r^3 e^{i3\phi} \right\rangle_{\rm ev}}{R^3_{\rm rms}}$$

And find to a first approximation:

$$v_3(p_T, \eta) \propto \epsilon_{3,3}$$

But:

1. The fluctuating geometry in radius and rapidity can not be fully characterized by ϵ_3
2. This sub-geometry controls the details of the p_T, η dependence of $v_3(p_T, \eta)$

Want to find additional measurements to understand and quantify

the fluctuating geometry!

The Flow Covariance Matrix and Notation

- For each event expand the particle distribution in a fourier series

$$\frac{\mathrm{d}N}{\mathrm{d}p\,\mathrm{d}\varphi_{\mathbf{p}}} = V_0(p) + \sum_{n=1}^{\infty} V_n(p) e^{in\varphi_{\mathbf{p}}} + \mathrm{h.c.}\,,$$

so $V_n(p)$ is a sum of particles, *not normalized*. (p labels either p_T or η)

- The sample estimate for the complex $V_n(p_T)$ is a sum of all particles

$$V_n(p) = \frac{1}{\Delta p} \sum_{j=1}^{M(p)} e^{in\varphi_j}$$

- The statistics of the flow is given by the covariance matrix:

$$C(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle$$

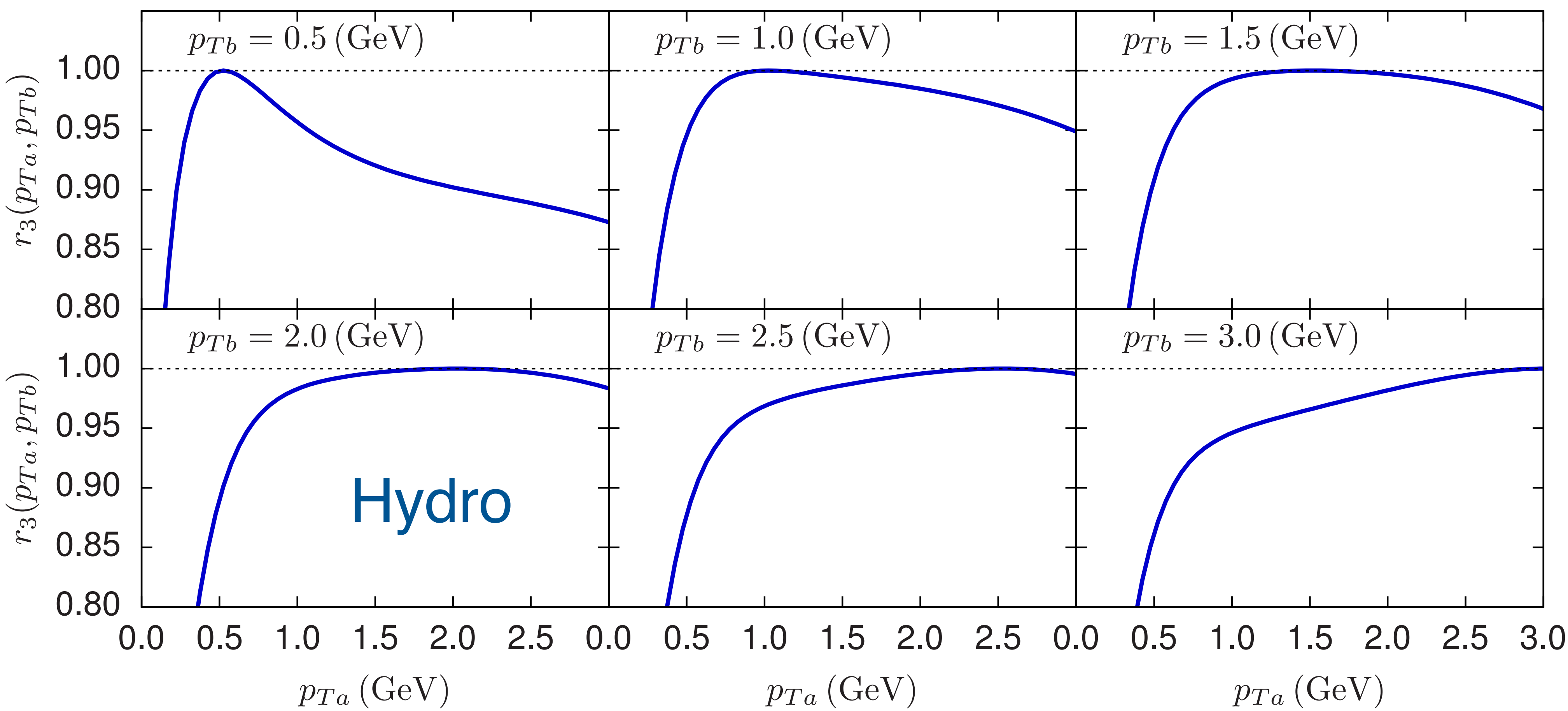
Our goal is to characterize this matrix!

Characterizing the covariance matrix with $r(p_1, p_2)$ and factorization breaking

$$r_3(p_1, p_2) = \frac{\langle V_3(p_1) V_3^*(p_2) \rangle}{\sqrt{\langle |V_3(p_1)|^2 \rangle \langle |V_3(p_2)|^2 \rangle}} < 1$$

If there is are multiple stat. independent harmonic flows in an event (F. Gardim et al, arxiv:1211.0989)

$$r(p_1, p_2) < 1$$



Principal Component Analysis – Warm up

- First assume there is one source of flow in the event $\propto \epsilon_3$:

$$V_3(p) = \underbrace{\xi}_{\text{complex}} \times \underbrace{V_3^{(1)}(p)}_{\text{fixed real func of } p \; V_2\{2\}(p)}$$

where $\xi \propto \epsilon_3$

$$\xi = \frac{\epsilon_3}{\sqrt{\langle|\epsilon_3|^2\rangle}} = \text{a random complex number with } \left\langle|\xi|^2\right\rangle = 1$$

- Then the covariance matrix factorizes

$$\langle V_3(p_1)V_3^*(p_2)\rangle = V_3^{(1)}(p_1)V_3^{(1)}(p_2)$$

and

$$r(p_1, p_2) = \frac{\langle V_3(p_1)V_3^*(p_2)\rangle}{\sqrt{\langle|V_3(p_1)|^2\rangle \langle|V_3(p_2)|^2\rangle}} = 1$$

Principal Component Analysis

- In general, the covariance matrix can be written as a sum of <u>eigenvectors</u>

$$\langle V_3(p_1) V_3^*(p_2) \rangle = \sum_a V_3^{(a)}(p_1) V_3^{(a)}(p_2)$$

where

$$V_3^{(a)}(p) = \underbrace{\sqrt{\lambda_a}}_{\text{flow magnitude}} \times \underbrace{\psi^{(a)}(p)}_{\text{normalized flow e-vector}}$$

- The event-by-event flow is expanded in terms of eigenvectors:

$$V_3(p) = \xi_1 V_3^{(1)}(p) + \xi_2 V_3^{(2)}(p) + \ldots$$

The eigenvectors are the statistically independent flows in the event!

PCA Continued

- So:

$$V_3(p) = \underbrace{\xi_1}_{\text{Random-complex with unit variance}} \times \underbrace{V_3^{(1)}(p)}_{\text{leading flow}}$$

$$+ \underbrace{\xi_2}_{\text{phase records the flow angle}} \times \underbrace{V_3^{(2)}(p)}_{\text{subleading flow}} + \ldots$$

- The flows are uncorrelated

$$\langle \xi_a \xi_b^* \rangle = \delta_{ab}$$

- Thus

$$\langle V_3(p) V_3^*(p) \rangle = \sum_a V_3^{(a)}(p_1) V_3^{(a)}(p_2)$$

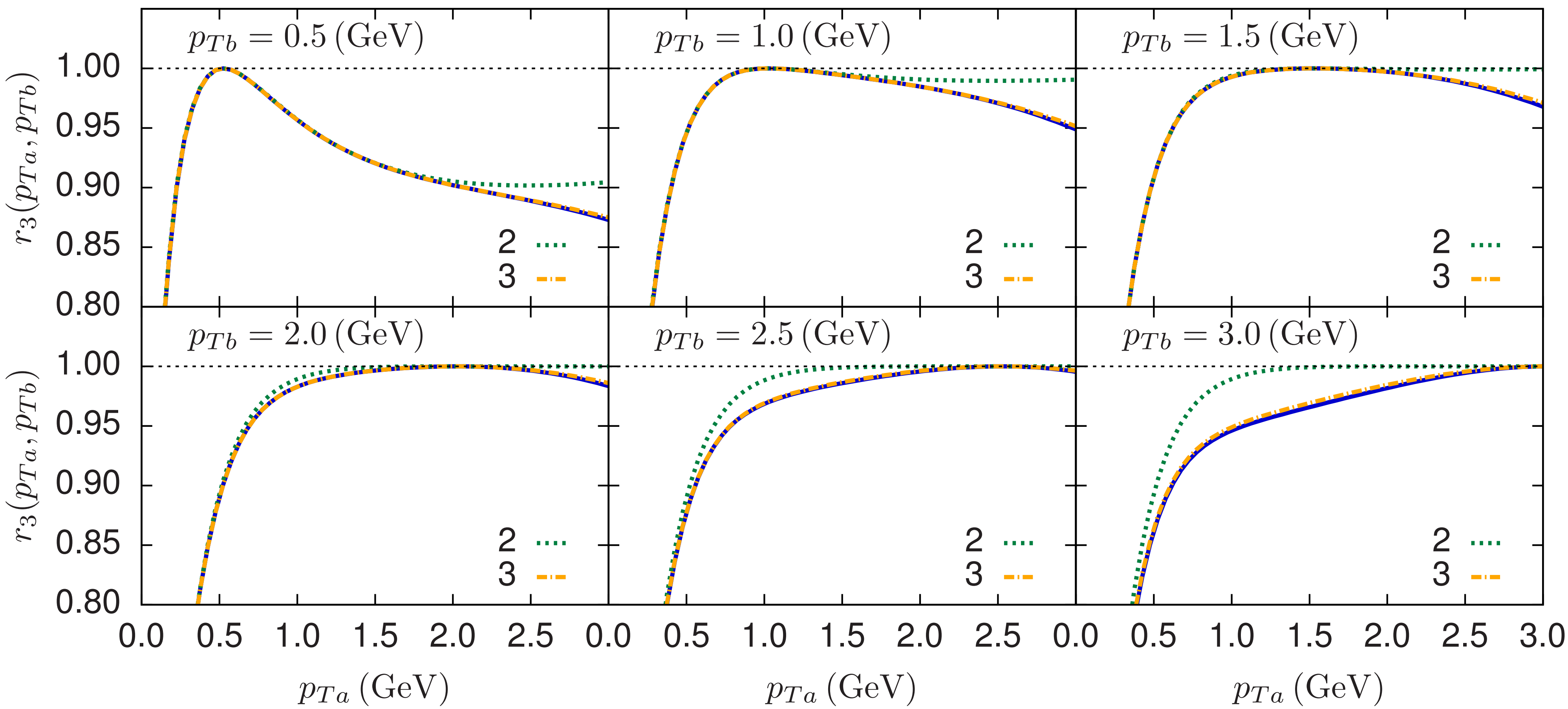
Usually only a few ~ 2 eigenvectors are needed to characterize the covariance matrix

The eigenvectors usually have a physical interpretation.

The covariance matrix in hydro and its eigen-decomposition:

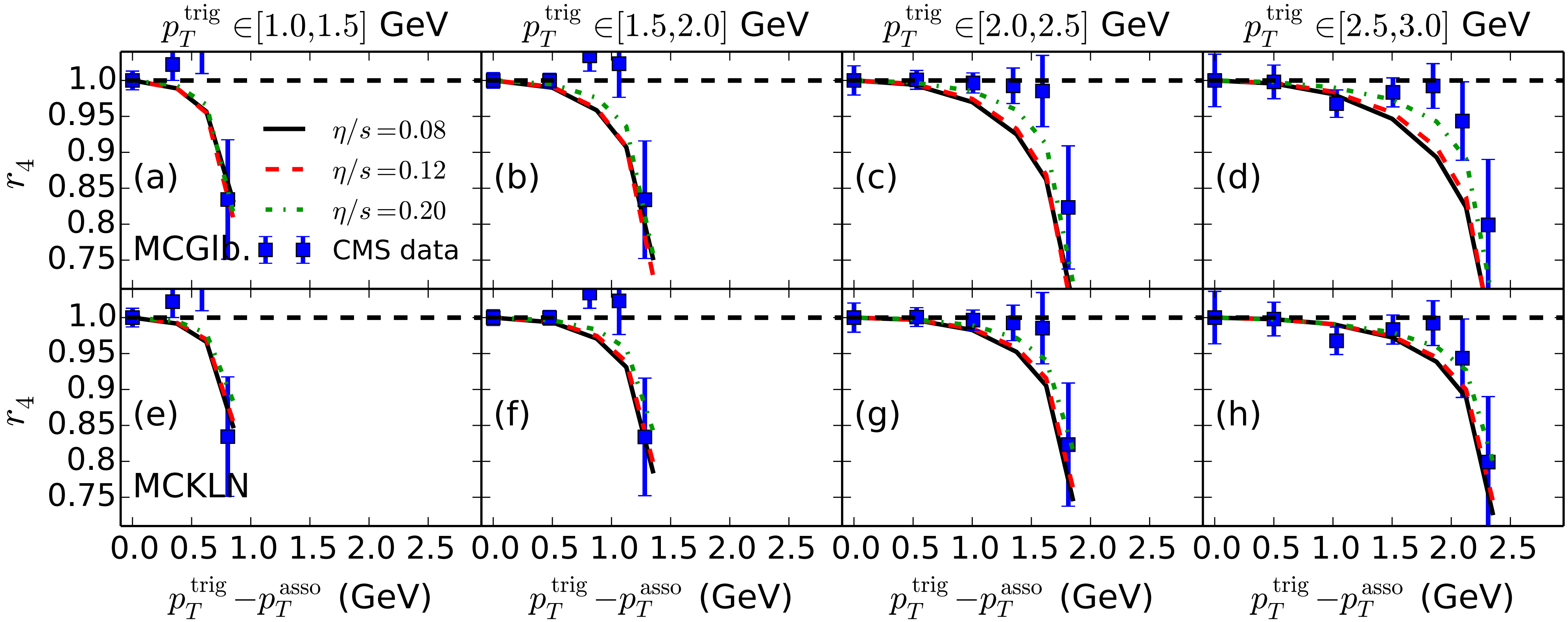
- Only two eigenvectors with $\lambda_2 \ll \lambda_1$:

$$r(p_1, p_2) \simeq 1 - \frac{1}{2}\left(\frac{V_3^{(2)}(p_1)}{V_3^{(1)}(p_1)} - \frac{V_3^{(2)}(p_2)}{V_3^{(1)}(p_2)}\right)^2$$



Two (or three) evectors summarize the covariance matrix up to $p_T = 2\,(\mathrm{or}\ 3)\ \mathrm{GeV}$

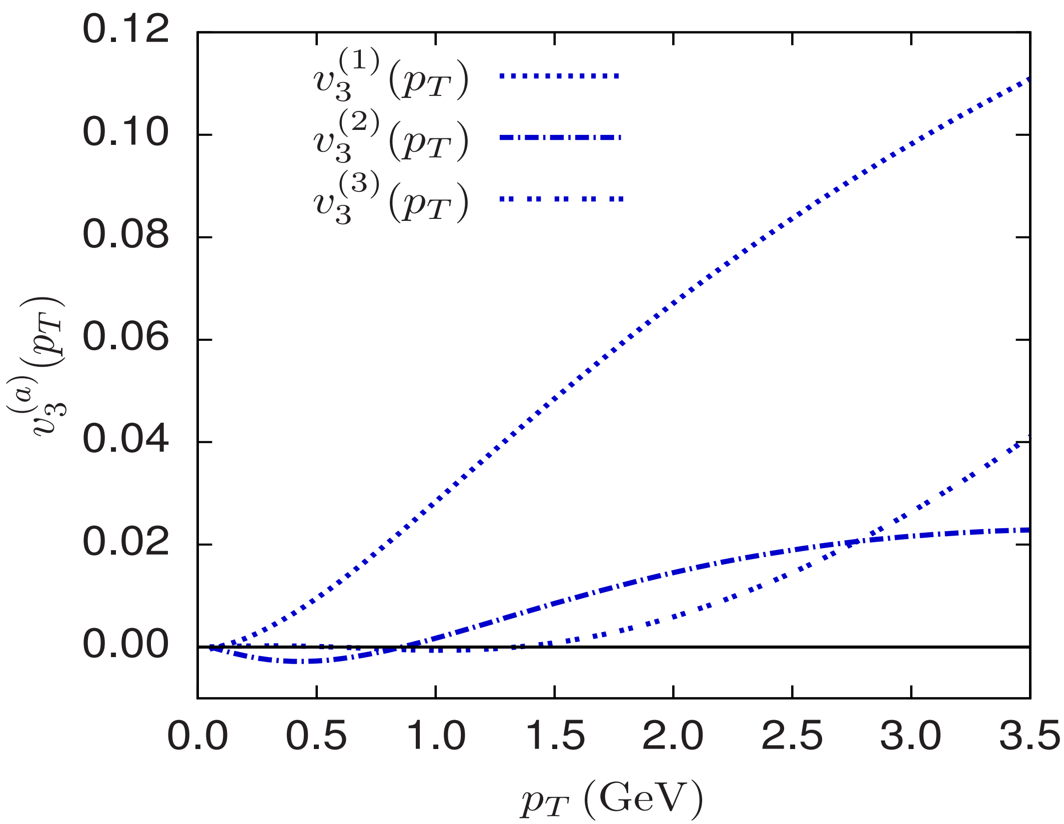
Chun Shen, Zhi Qiu, Heinz, arxiv:1502.0463



Completely summarized by two eigenvectors!

The eigenvectors in central collisions:

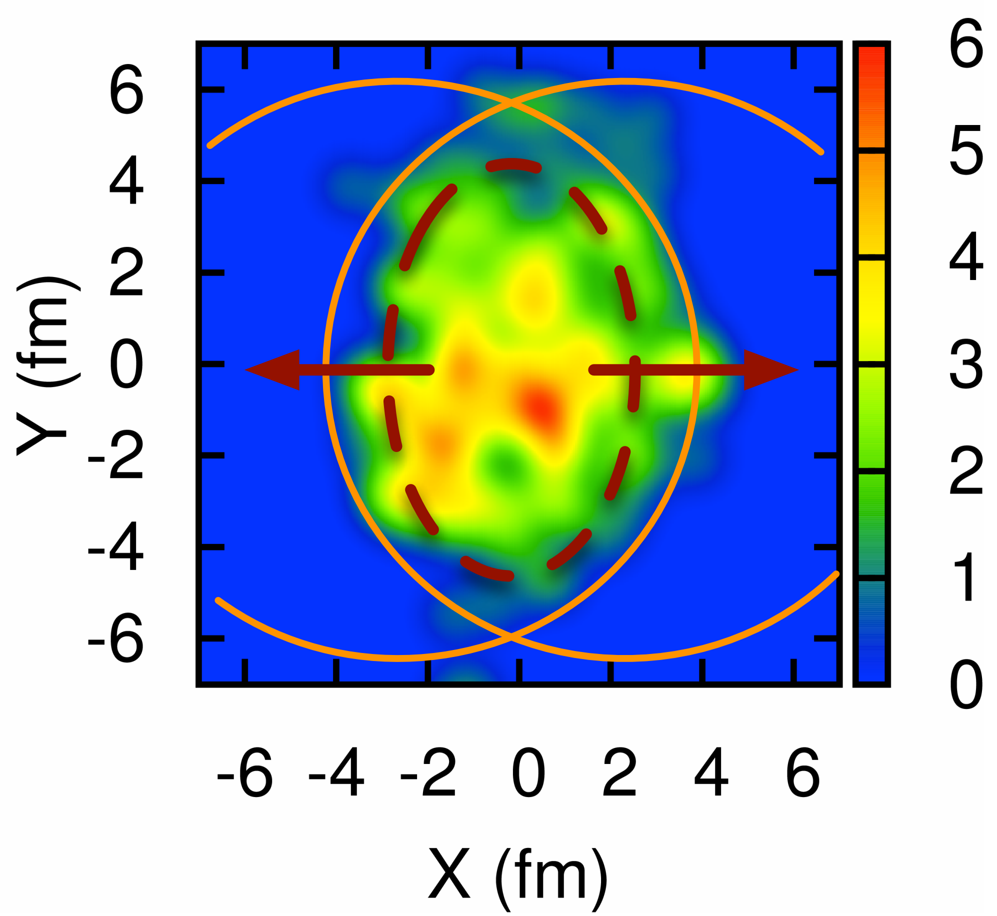
$$v_n^{(a)}(p_T) \equiv V_n^{(a)}(p_T)/\left\langle dN/dp_T \right\rangle$$



- The leading eigenvector $v_3^{(1)}(p_T)$ is essentially $v_3\{2\}(p_T)$
- The sub-leading flow describes the stat.-independent flucts around $v_3\{2\}(p_T)$

Recap and next steps

$S(r, \phi) =$ entropy distribution in transverse plane



This fluctuating initial entropy density drives two principal flows

Correlation between the subleading harmonic flow and the geometry

- The initial entropy density $\equiv S(r,\phi)$ in the transverse plane is written:

$$S(r,\phi) \equiv S_0(r) + \sum_{n=1}^{\infty} S_n(r) e^{in\phi} \quad \text{where} \quad \underbrace{S_3(r)}_{\text{complex}} \equiv \int \frac{d\phi}{2\pi} e^{-i3\phi} S(r,\phi)$$

- The event-by-event flow is described by amplitudes and phases of flow vectors

$$V_3(p) = \xi_1 \, V_3^{(1)}(p) + \xi_2 \, V_3^{(2)}(p) + \ldots$$

$\xi_a = |\xi| e^{i3\phi_a}$ records the amplitude and angular orientation of a-th flow

- Calculate the correlation between the flow and the geometry

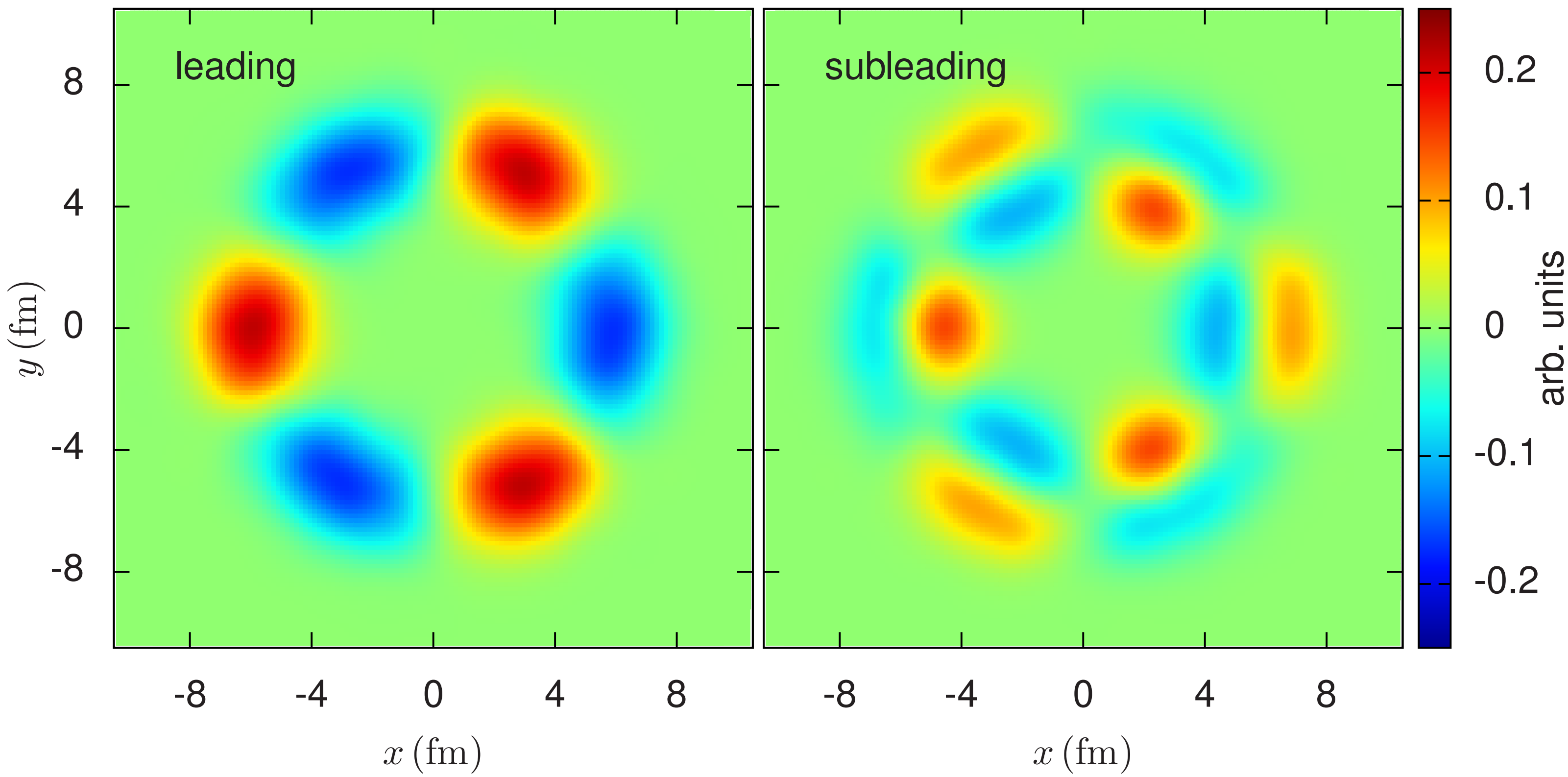
$$\overline{S}_3(r;\xi) = \langle S_3(r) \, \xi_a^* \rangle$$

or the average entropy rotated into event plane of the a-th flow

$$\overline{S}(r,\phi;\xi_a) = \langle |\xi| \, S(r,\phi+\phi_a) \rangle$$

Correlation between the flow and the geometry:

$$r^3 \times \qquad \underbrace{\left[\overline{S}(r,\phi;\xi_a) - \langle|\xi|S(r,\phi)\rangle\,\right]}_{\sim 15\%\text{ triangular perturbation of average}}$$



- For the leading flow, the integral of plot is $\propto \epsilon_3$

The subleading flow is strongly correlated with an excitation of the triangular geometry

Finding a geometrical predictor of the subleading flow

- Consider $\epsilon_{3,3}$ and the leading flow to set notation

$\langle . \rangle_{\rm ev}$ is ave. over entropy dist. in the event

$$\epsilon_3 \propto \Big\langle e^{i3\phi} \underbrace{r^3}_{\text{weight } \rho(r)} \Big\rangle_{\rm ev}$$

choosing $\rho(r) \propto r^3$ correlates well with the leading triangular flow

- Consider choosing a weight $\rho(r)$ for selecting the subleading flow

$$\xi_2 V^{(2)} \propto \epsilon_3^{(2)} \qquad \text{where} \qquad \epsilon_3^{(2)} \equiv \left\langle e^{i3\phi}\, \rho(r) \right\rangle_{\rm ev}$$

- Expand $\rho(r)$ with a fourier transform, or <u>fit</u> with a discrete set of fourier modes:

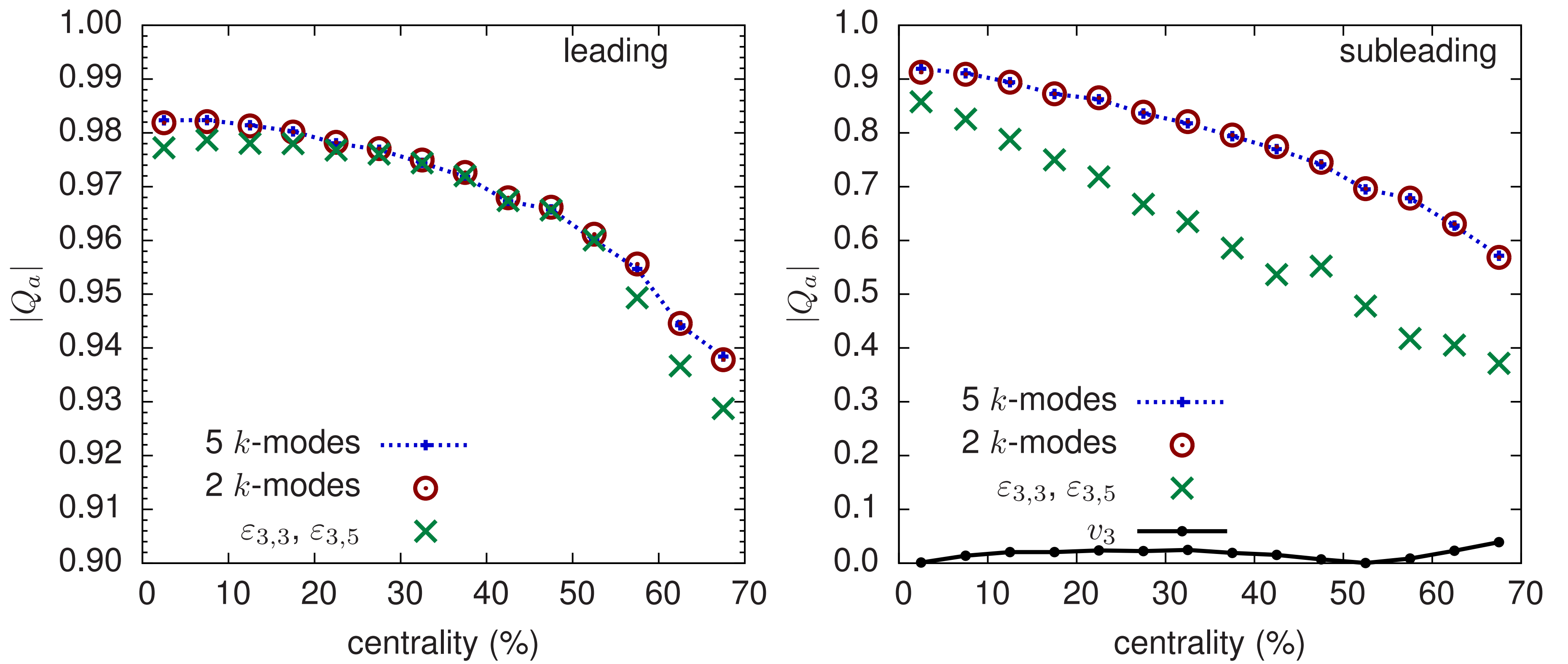
$$\rho(r) = \sum_i c_i J_3(k_i r)$$

Best fit:

$$\text{flow} \propto \langle e^{i3\phi}\rho(r)\rangle_{\rm ev}$$

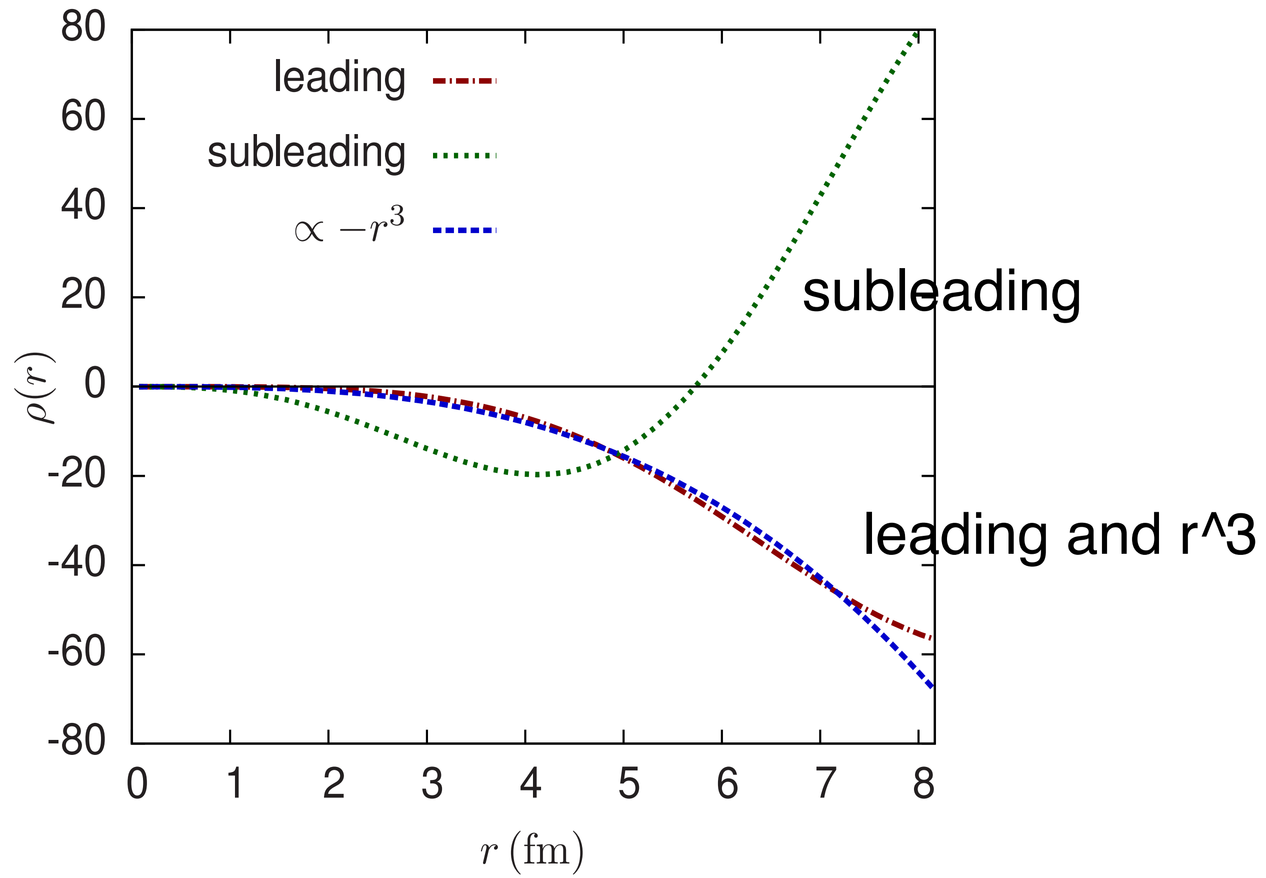
Quantify the correlation coefficient, Q, between the flow and the geometric predictor

$$Q = \begin{cases} 1 & \text{means perfectly correlated} \\ 0 & \text{uncorrelated} \end{cases}$$

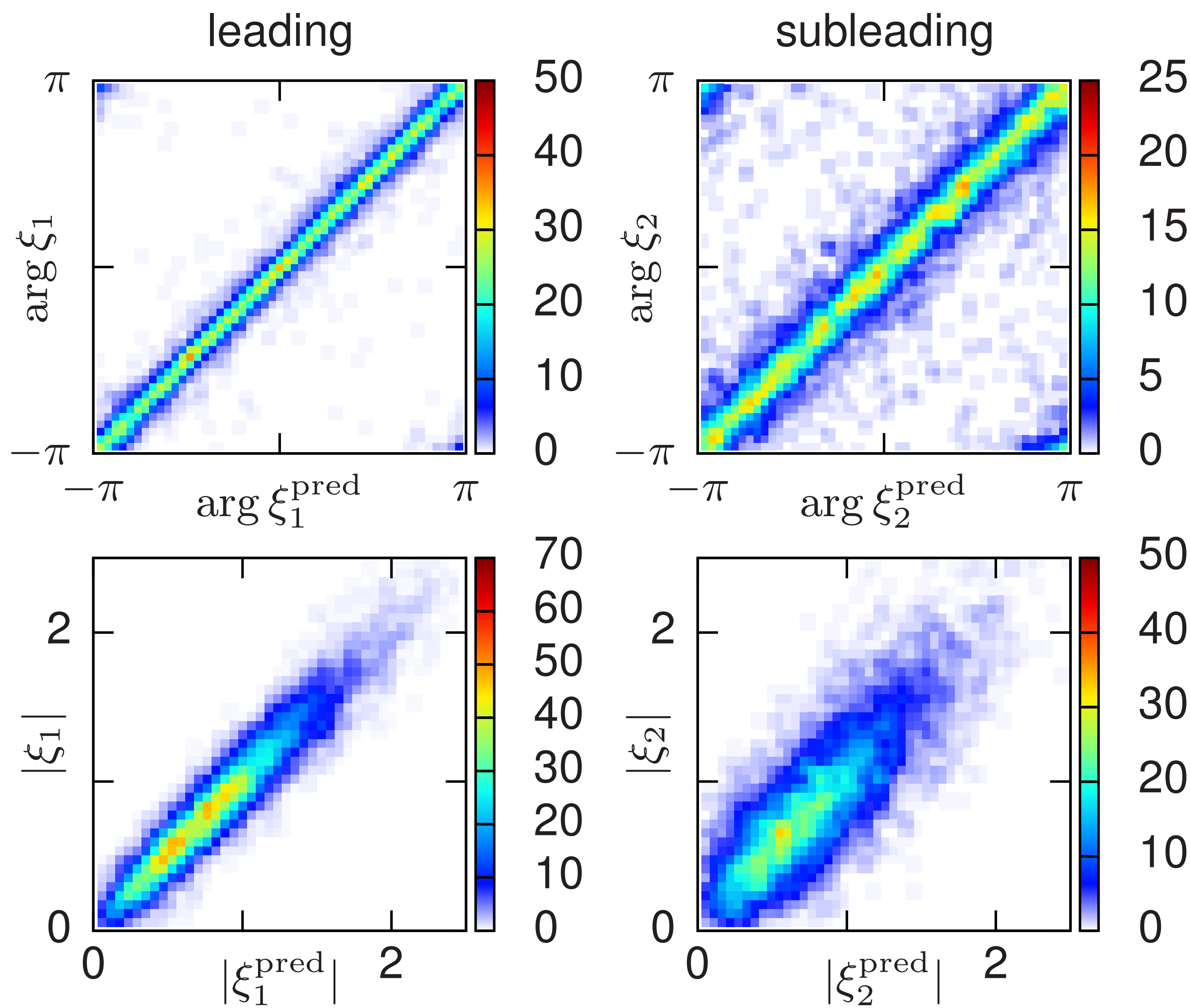


Best fit in central collisions:

$$\xi_2 V_3^{(2)} \propto \epsilon_3^{(2)} \qquad \text{where} \qquad \epsilon_3^{(2)} \equiv \langle e^{i3\phi} \underbrace{\rho(r)}_{\text{fit func}} \rangle_{\rm ev}$$



Correlation between the flow ξ_a and its geometrical predictor $\xi_a^{\rm pred}$:



Summary

1. PCA is an efficient and systematic way to describe the flow correlations in the data

2. The different principal components often have a clear physical meaning

   (a) Radial excitations of the geometry are responsible for factorization breaking

   (b) But in peripheral collisions the correlation is not as strong? Non-linear mixing?

3. Other directions to pursue:

   (a) Rapidity fluctuations

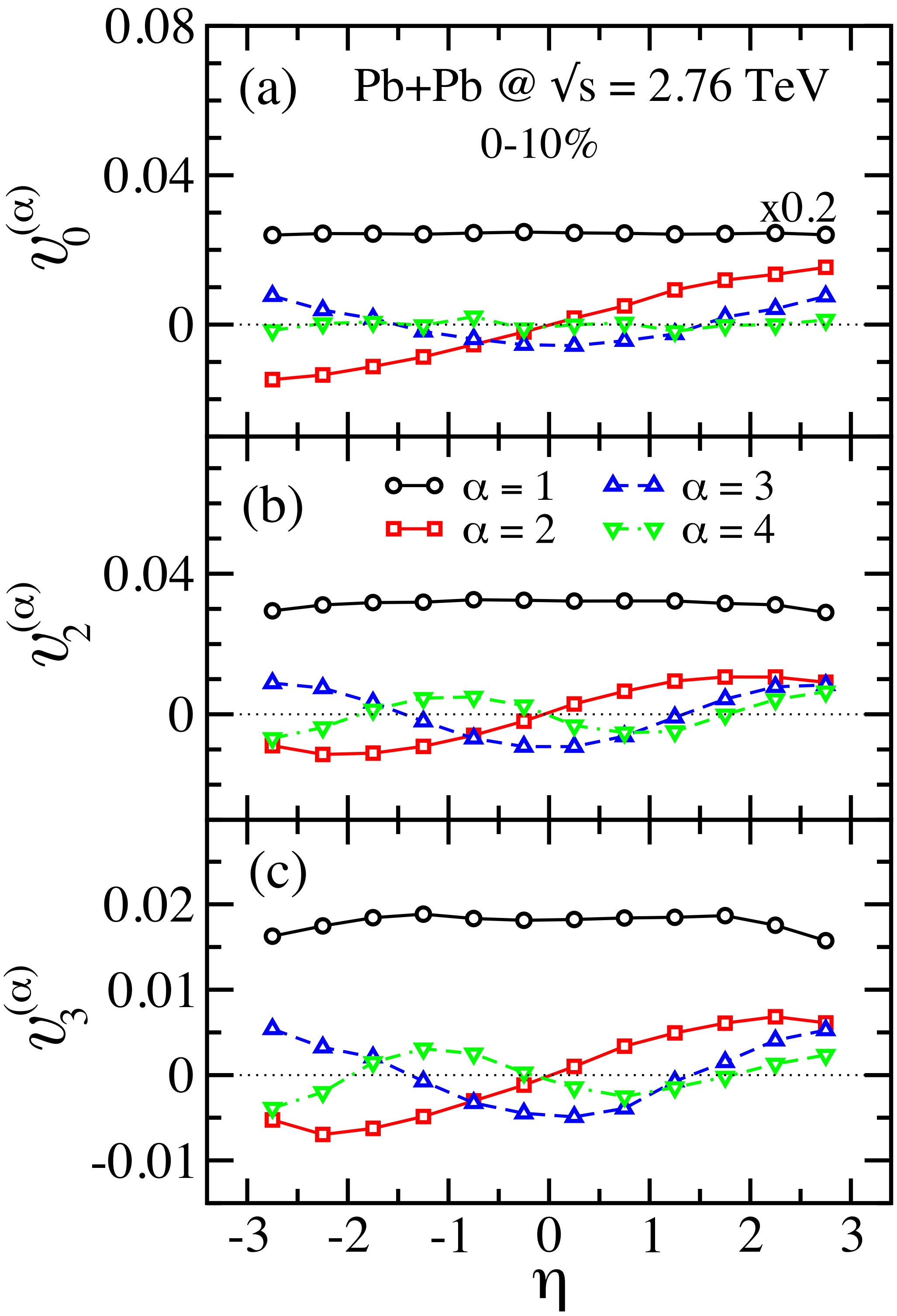
   (b) Rare probes (J/ψ, γ, D) and radial flow

Rapidity Correlations with AMPT (Z.W. Lin et al, nucl-th/0411110)

$$r(\eta_1, \eta_2) = \frac{\langle V_n(\eta_1) V_n^*(\eta_2) \rangle}{\sqrt{\langle |V_n(\eta_1)|^2 \rangle \langle |V_n(\eta_2)|^2 \rangle}} < 1$$

Can study rapidity fluctuations in much the way measuring *torqued* fireballs

(P. Bozek et al, arXiv:1011.3354)



(a)
Pb+Pb @ √s = 2.76 TeV
0-10%
x0.2
(b)
α = 1
α = 2
α = 3
α = 4
(c)
0.08
0.04
0
0.02
0.01
-0.01
-3
-2
-1
1
2
3
η

Summary

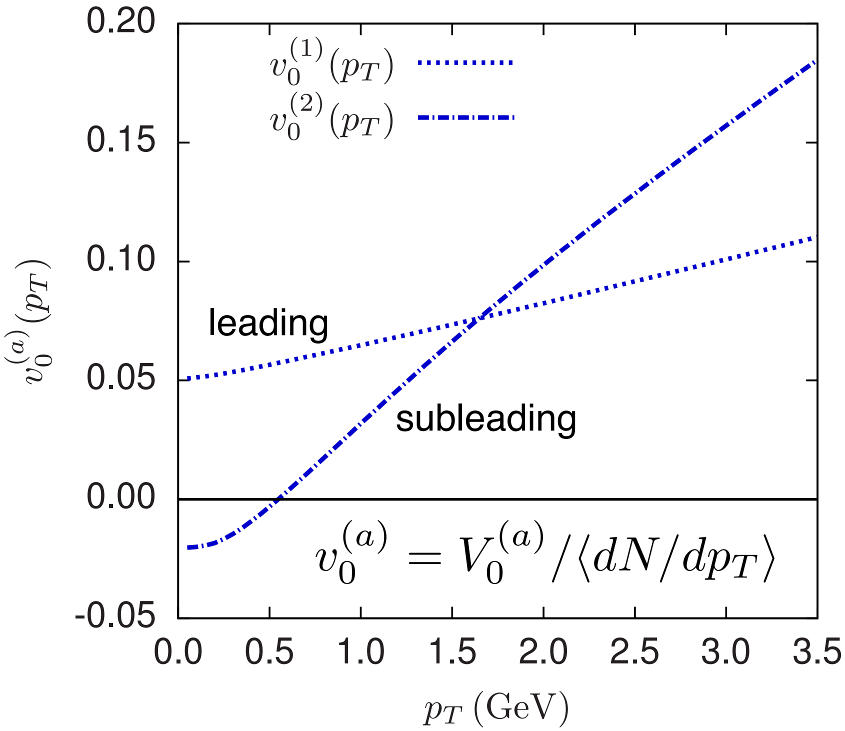
1. PCA is an efficient and systematic way to describe the flow correlations in the data
2. The different principal components often have a clear physical meaning
   (a) Radial excitations of the geometry are responsible for factorization breaking
   (b) But in peripheral collisions the correlation is not as strong? Non-linear mixing?
3. Other directions to pursue:
   (a) Rapidity fluctuations
   (b) Rare probes and radial flow

Thank you!

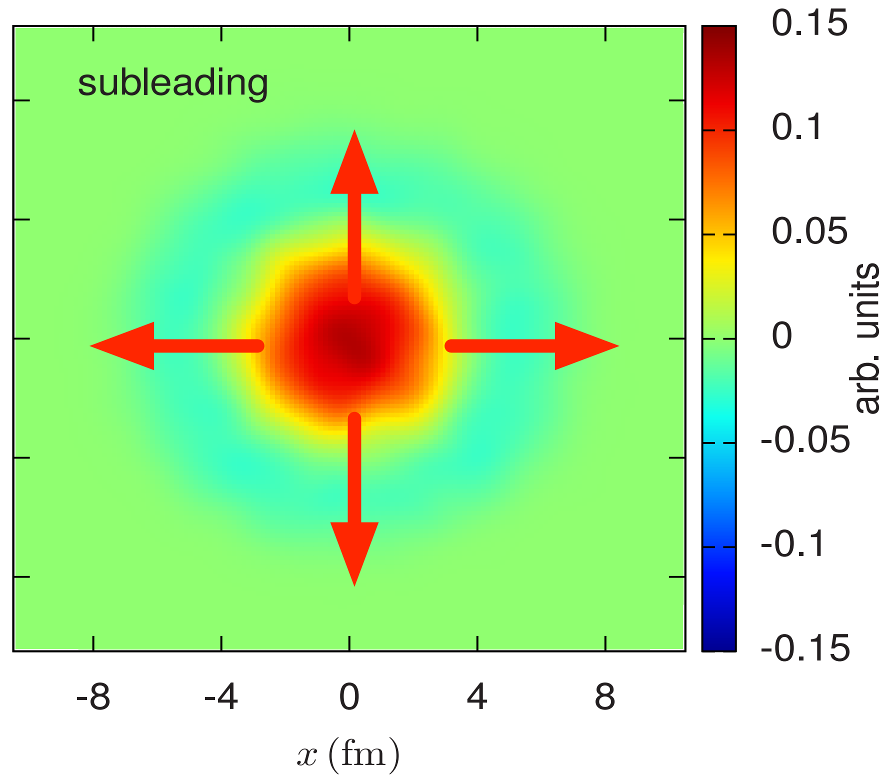
Radial Flow:

$$V_0(p) = \xi_1 V_0^{(1)}(p) + \xi_2 V_0^{(2)}(p) + \ldots$$

Subleading mode



Correlation with geometry



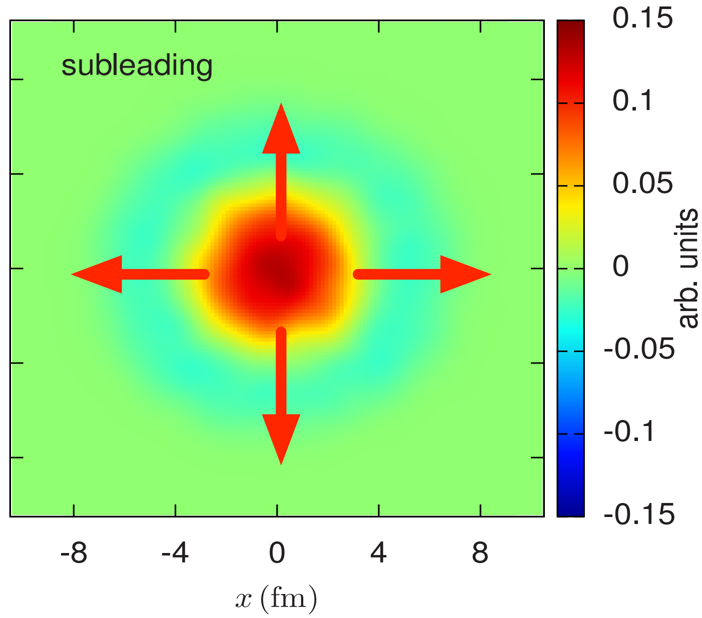
Two eigenmodes:

1. The leading eigenmode is from global multiplicity (impact param) fluctuations.
2. The subleading eigenmode is a *dynamical* response to geometry *like elliptic flow*

Radial flow of rare probes:

$$V_0(p) = \xi_1 V_0^{(1)}(p) + \underbrace{\xi_2 V_0^{(2)}(p)}_{\text{e-by-e radial flow of bulk}} + \ldots$$

Does J/ψ follow?



- We want to know if the rare probes, J/ψ, D, γ, "follow the flow". So measure

$$v_0^D(p_T) = \frac{\langle \xi_2 \, dN_D/dp_T \rangle}{\langle dN_D/dp_T \rangle}$$

  – This equivalent to finding the eigenvectors of the combined probe+bulk system

This measurement has better statistics than v_2, and is a good a measure of collectivity!