# The Spin Asymmetries of the Nucleon Experiment - SANE

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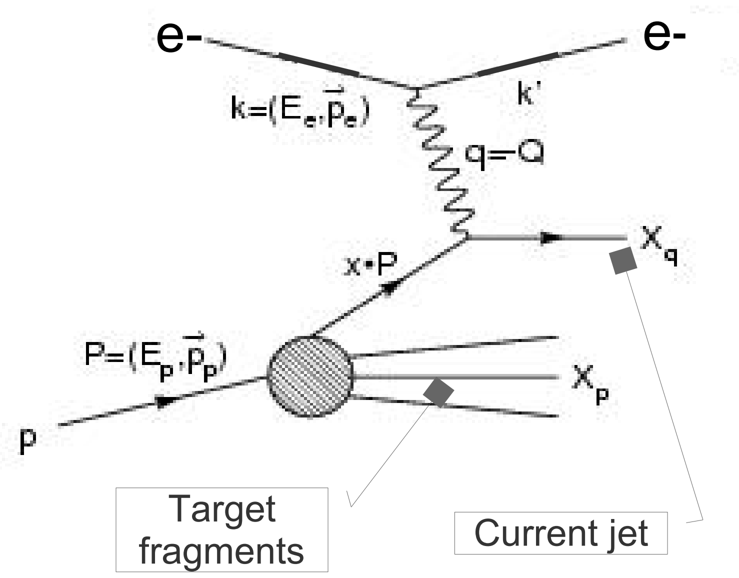
6th Workshop of the APS Topical
Group on Hadronic Physics
Baltimore, April 10, 2015

# Probing the Nucleon with Polarized Electromagnetic Scattering

# Inelastic *e-nucleon* Scattering

- Inclusive EM scattering is described by hadronic and leptonic tensors
- Symmetries reduce hadronic tensor to four structure functions (SF's):
  - Symmetric part: unpolarized $W_1$, $W_2$
  - Anti-symmetric part: spin-dependent $G_1$, $G_2$




(http://www.desy.de/~gbrandt/feyn/)

$$W^A_{\mu\nu} = 2\epsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ M^2 S^\sigma \boldsymbol{G_1}(\nu, Q^2) + \left[ M\nu S^\sigma - p^\sigma S\cdot q \right] \boldsymbol{G_2}(\nu, Q^2) \right\}$$

Inclusive scattering: undetected final state

- $G_1$, $G_2$, $W_1$ and $W_2$, contain all the information on nucleon structure that can be extracted from inclusive data

# Structure Functions in Inclusive DIS

- In high energy DIS, $\boldsymbol{G_1}$, $\boldsymbol{G_2}$, $\boldsymbol{W_1}$ and $\boldsymbol{W_2}$, become scaling functions of only one variable, up to log violations.

$$\lim_{Q^2,\nu\to\infty} M\,W_1(\nu,Q^2)=F_1(x) \qquad \lim_{Q^2,\nu\to\infty} M^2\nu\,G_1(\nu,Q^2)=g_1(x)$$

$$\lim_{Q^2,\nu\to\infty} \nu\,W_2(\nu,Q^2)=F_2(x) \qquad \lim_{Q^2,\nu\to\infty} M\,\nu^2\,G_2(\nu,Q^2)=g_2(x)$$

$$Bjorken \quad x=Q^2/(2M\nu)$$

- In the quark parton model, $\boldsymbol{g_1}$ and $\boldsymbol{F_1}$ can be related to parton distribution functions - PDF's:

$$F_1(x)=\frac{1}{2}\sum e_f^2\left(q_f^{\uparrow}(x)+q_f^{\downarrow}(x)\right) \qquad g_1(x)=\frac{1}{2}\sum e_f^2\left(q_f^{\uparrow}(x)-q_f^{\downarrow}(x)\right)$$

(Index *f* runs over active flavors)

# Virtual Compton Asymmetries

- The spin SF's are also related to virtual photon absorption cross-sections and spin asymmetries (SA)
  - the helicity of the virtual photon-nucleon system is 3/2 or ½ for transverse photons, ½ for longitudinal ones

- SA $A_1$ is defined in terms of the difference for 3/2 and ½ helicity cross sections

$$A_1 = \frac{\sigma_T^{(3/2)} - \sigma_T^{(1/2)}}{\sigma_T^{(3/2)} + \sigma_T^{(1/2)}}$$

$$A_1 = \frac{1}{F_1}\left(g_1 - \gamma^2 g_2\right); \quad \gamma = \frac{2xM}{\sqrt{Q^2}}$$

- $A_2$ represents the interference between initial transverse and final longitudinal amplitudes

$$A_2 = \frac{\sigma_{TL}^{(1/2)}}{\sigma_T^{(3/2)} + \sigma_T^{(1/2)}} \leq \sqrt{\frac{A_1 + 1}{2} R} \leq \boldsymbol{R} = \frac{\sigma_L}{\sigma_T}$$

$$A_2 = \frac{\gamma}{F_1}\left(g_1 + g_2\right) = \frac{\gamma}{F_1} \boldsymbol{g_T}$$

# Model Independent Extraction of Spin Structure Functions

- $G_1$ and $G_2$ can be separated by measuring cross section differences for opposite beam helicities with target spins *parallel* and *transverse* to the beam

$$\Delta\sigma(\theta,\theta_N,\phi)=\frac{4\alpha^2 E'}{Q^2 E}\left[(E\cos\theta_N+E'\cos\alpha)M\,G_1+2EE'(\cos\alpha-\cos\theta_N)G_2\right]$$

$$\cos\alpha=\sin\theta_N\sin\theta\cos\phi+\cos\theta_N\cos\theta,\quad(\theta,\phi: \textit{final lepton angles})$$

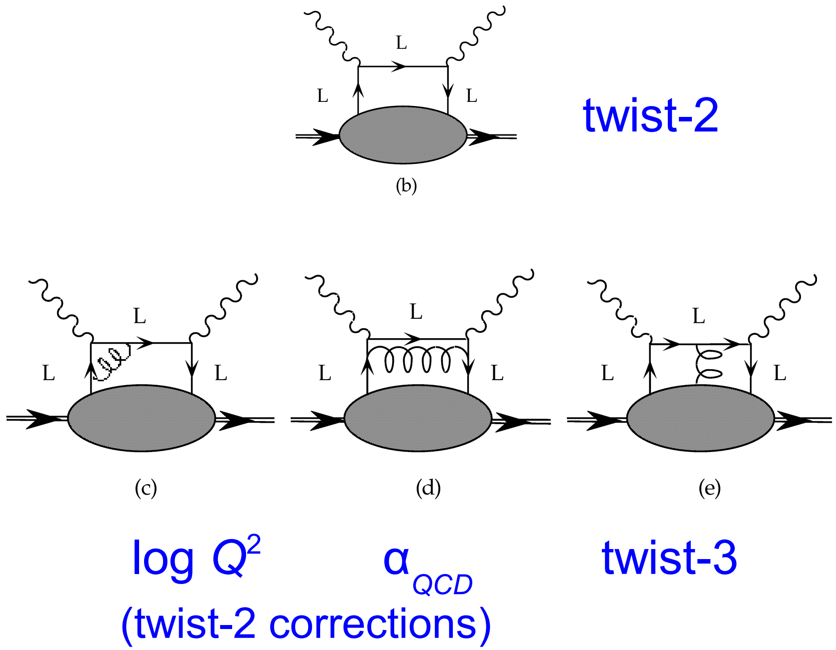
- *transverse* target spin $\theta_N$ : comparable $G_1$, $G_2$ terms

$$\frac{d^2\sigma^{(\uparrow\rightarrow)}}{d\Omega\,dE'}-\frac{d^2\sigma^{(\downarrow\rightarrow)}}{d\Omega\,dE'}=\frac{4\alpha^2 E'}{Q^2 E}E'\sin\theta\cos\phi\left[M\,G_1(\nu,Q^2)+2E\,G_2(\nu,Q^2)\right]$$

  - $G_1$ is twist-2 (plus corrections)
  - $G_2$ has both twist-2 and twist-3 contributions

# Transverse Polarized Scattering: Unlocking Twist-3

- Twist-2 and twist-3 operators contribute at same order in transverse polarized scattering
  - twist-2: handbag diagram
  - twist-3: *qgq* correlations
- direct access to twist-3 via $g_2$:
  - interacting *qgq* is first step to understanding confinement
  - "*Unique feature of spin-dependent scattering*" (R. Jaffe)




(Comments NPP 19, 239 (1990))

# Why is $\boldsymbol{g}_2$ interesting?

- tests <u>twist-3</u> effects = *quark-gluon* correlations
- higher twist corrections to $\boldsymbol{g}_1$ with 3rd moment's $\boldsymbol{d}_2$ matrix element
- also test lattice QCD, QCD sum rules, quark models
- $\boldsymbol{d}_2$ related to color Lorentz force on transverse polarized quark
  (M. Burkardt, AIP Conf. Proc. 1155 (2009) 26)
  - sign of $\boldsymbol{d}_2$ related to sign of transverse deformation (anomalous $\kappa^q$)
- polarizabilities of color fields (with twist-4 matrix element $f_2$)
  - magnetic $\chi_B = (4\boldsymbol{d}_2 + f_2)/3$ and electric $\chi_E = (4\boldsymbol{d}_2 - 2 f_2)/3$.
- contains chiral odd twist-2 = quark transverse spin (mass term)
  - test quark masses (covariant parton models)

# $g_2$ and $g_T$ Spin Structure Functions

Experimentally measured quantities

$$g_T(x) = g_1(x) + g_2(x) = \frac{1}{2}\sum e_q^2 g_T^q(x)$$

$g_T^q$ in terms of Transverse Momentum Dependent distributions [1]

$$g_T(x) = \int d^2\vec{k}_t \frac{\vec{k}_t^2}{2M^2}\frac{g_{1T}^q(x,\vec{k}_t^2)}{x} + \frac{m}{M}\frac{h_1(x)}{x} + \tilde{g}_T(x)$$

twist-3 TMD quark mass term *qgq* interaction

Applying twist-2 Wandzura-Wilczek approximation of $g_2$

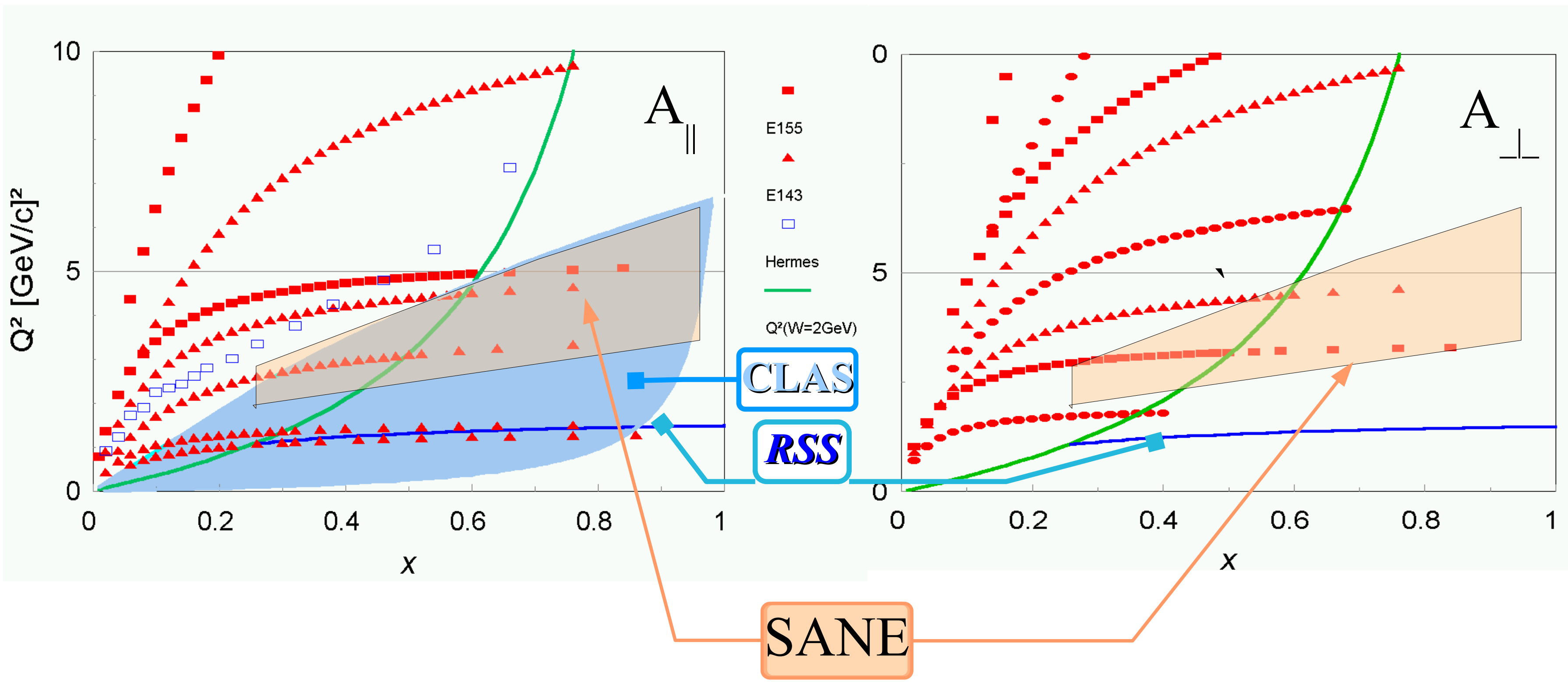
$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

Twist-3 for the nucleon (neglecting quark mass)

$$\bar{g}_2 = \frac{1}{2}\sum e_q^2\left[\tilde{g}_T^q - \int_x^1 \frac{dy}{y}\left(\tilde{g}_T^q(y) - \hat{g}_T^q(y)\right)\right];$$ $\tilde{g}_T = qg$ term, $\hat{g}_T$ = Lorentz invariance [2]

[1] hep-ph/9408305v1
[2] JHEP 0911 (2009) 093

# Proton world $A_{\parallel}$, $A_{\perp}$ data before SANE




- Two beam energies: 5.9 GeV, 4.7 GeV
- Very good high ***x*** coverage with detector at 40°

# Experiment

# Spin Asymmetries of the Nucleon Experiment

(TJNAF E07-003)


SANE Collaboration

Argonne National Lab., Christopher Newport U., Florida International U., Hampton U., Jefferson Lab., U. of New Hampshire, Norfolk S. U., North Carolina A&T S. U., Mississippi S. U., Ohio U., IHEP - Protvino, U. of Regina, Rensselaer Polytechnic I., Rutgers U., Seoul National U., Southern U. New Orleans, Temple U., Tohoku U., U. of Virginia , Yerevan Physics I., Xavier U.

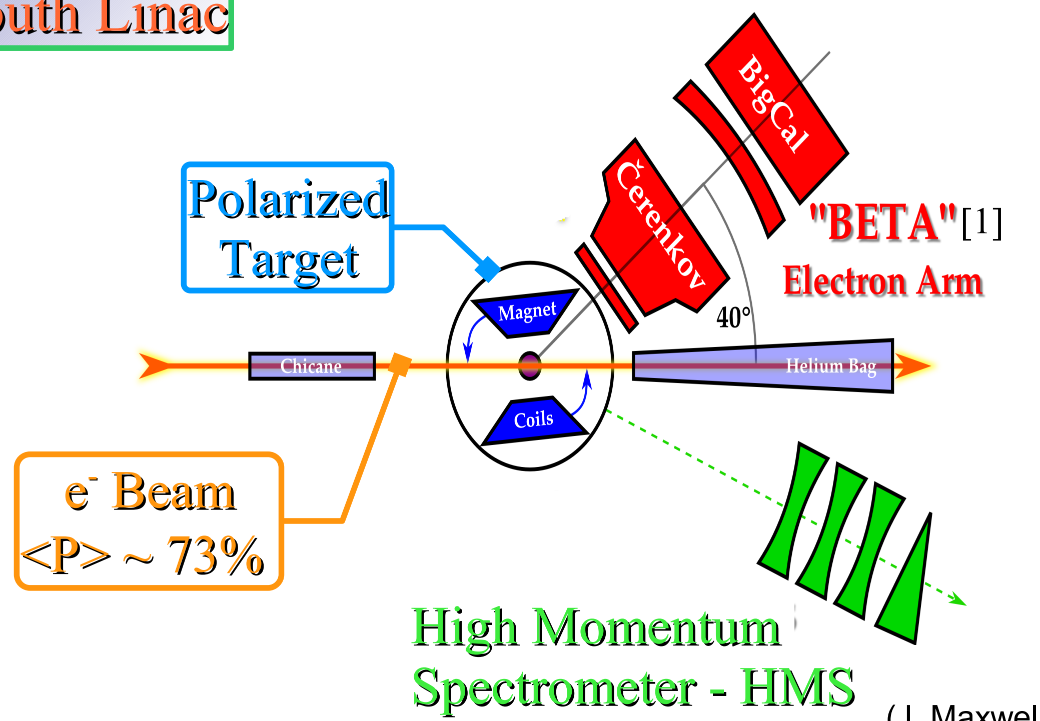
Spokespersons:

S. Choi (Seoul), M. Jones (JLab), Z-E. Meziani (Temple), O. A. Rondon (U. of Virginia)


Goal: Measure the proton spin structure function $\boldsymbol{g_2}(x, Q^2)$ and spin asymmetry $\mathbf{A_1}(x, Q^2)$ for $2.5 \leq \boldsymbol{Q^2} \leq 6.5$ GeV² and $0.3 \leq \boldsymbol{x} \leq 0.8$

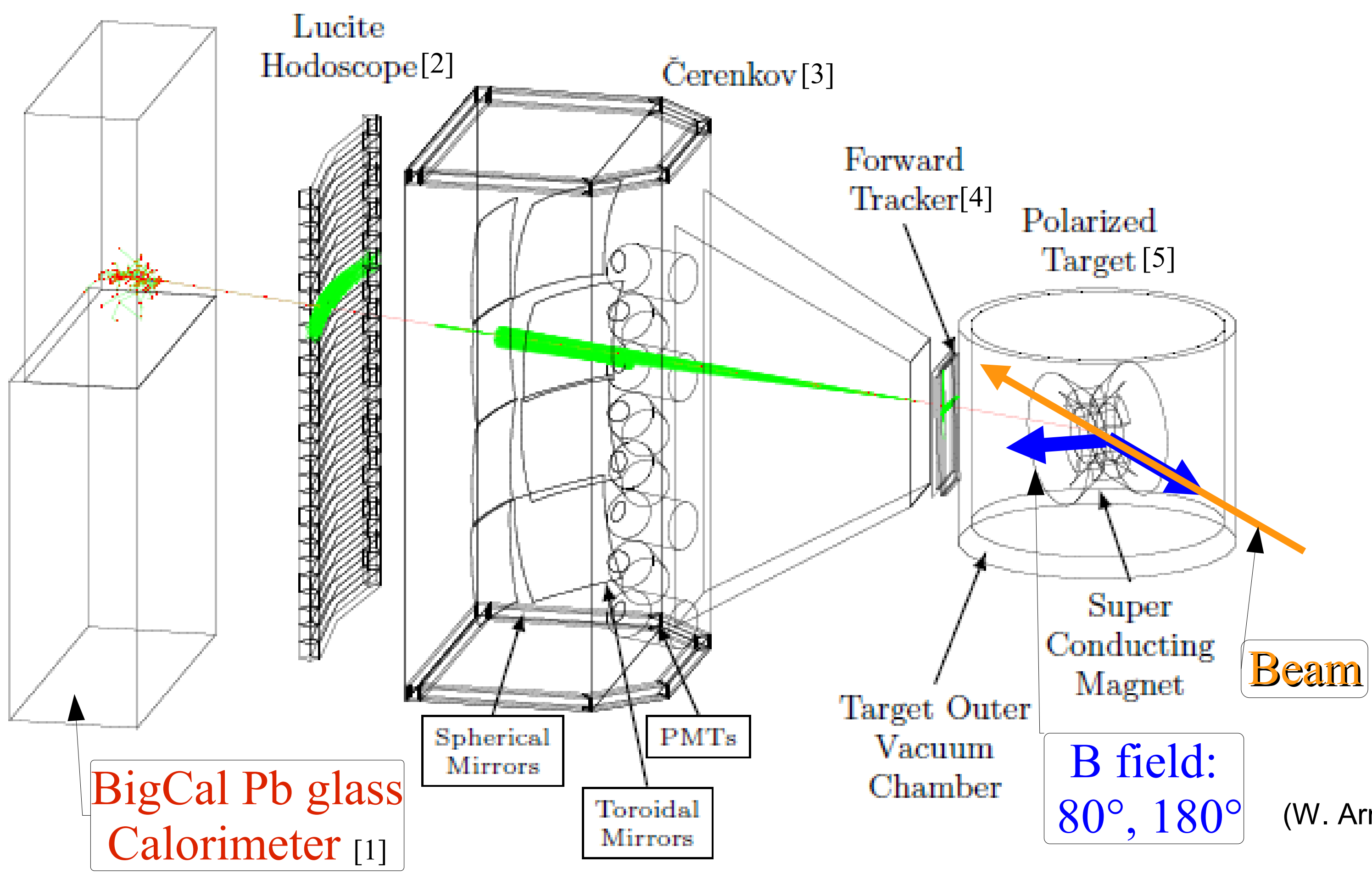
Method: Measure parallel and near-transverse inclusive double spin asymmetries, detecting the electrons with novel non-magnetic large solid angle telescope **BETA**

# SANE Layout in JLab's Hall C


[1] Big Electron Telescope Array
$\Delta\Omega \sim 190$ msr; $\Delta\theta = \pm 10°$

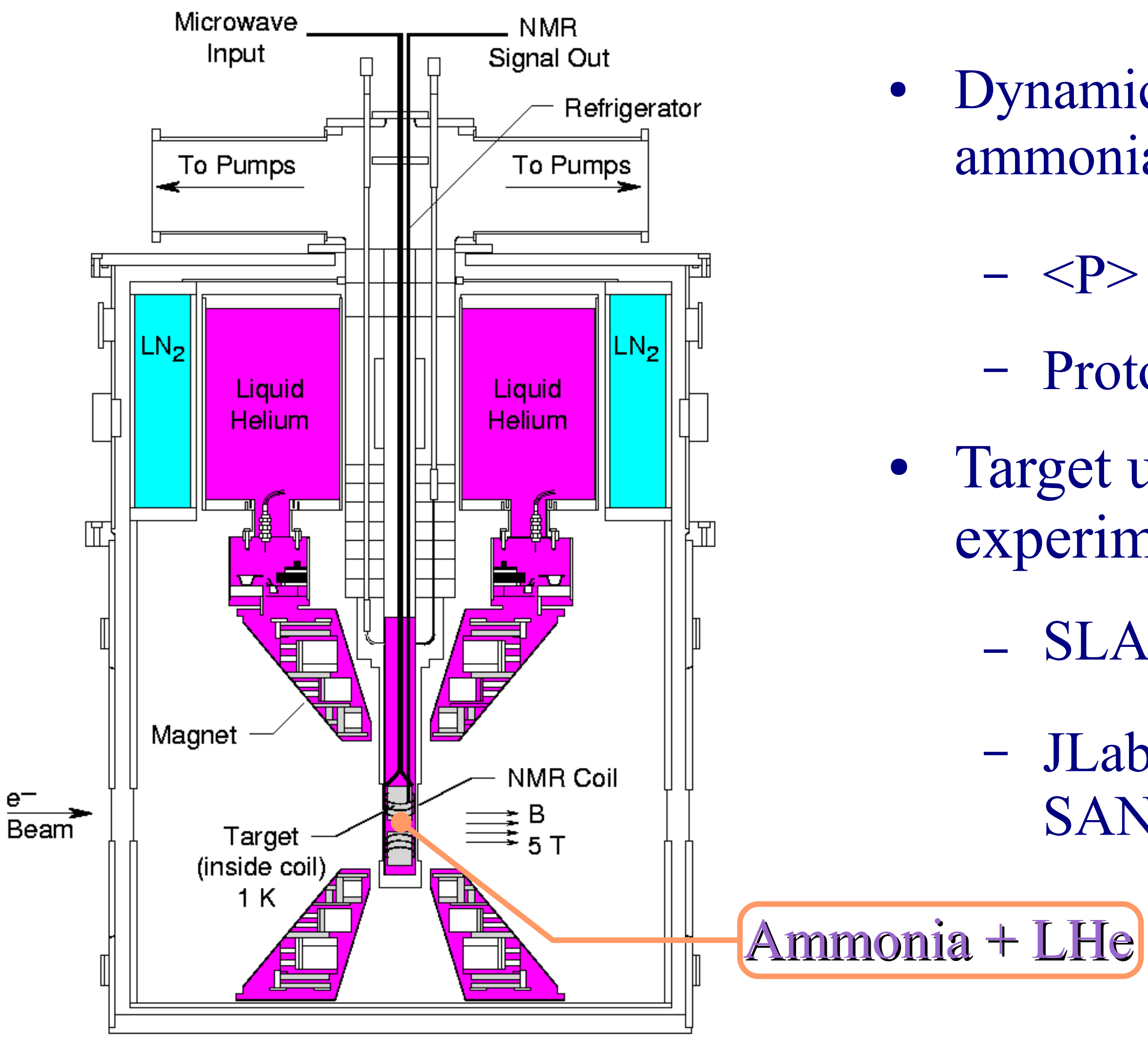
# BETA with DIS electron simulation




(W. Armstrong)

[1] BigCal Collaboration
[2] North Carolina A&T U.
[3] Temple U
[4] Norfolk State U. and U. of Regina
[5] UVA- JLab

# Polarized Target




- Dynamic Nuclear Polarized ammonia $^{14}NH_3$ at 5T and 1K
  - <P> ~ 70% in beam
  - Proton luminosity ~$10^{35}$ Hz cm$^{-2}$
- Target used in multiple experiments:
  - SLAC: E143, E155, E155x ($g_2$)
  - JLab: GEn98, GEn01, *RSS*, SANE

# Data

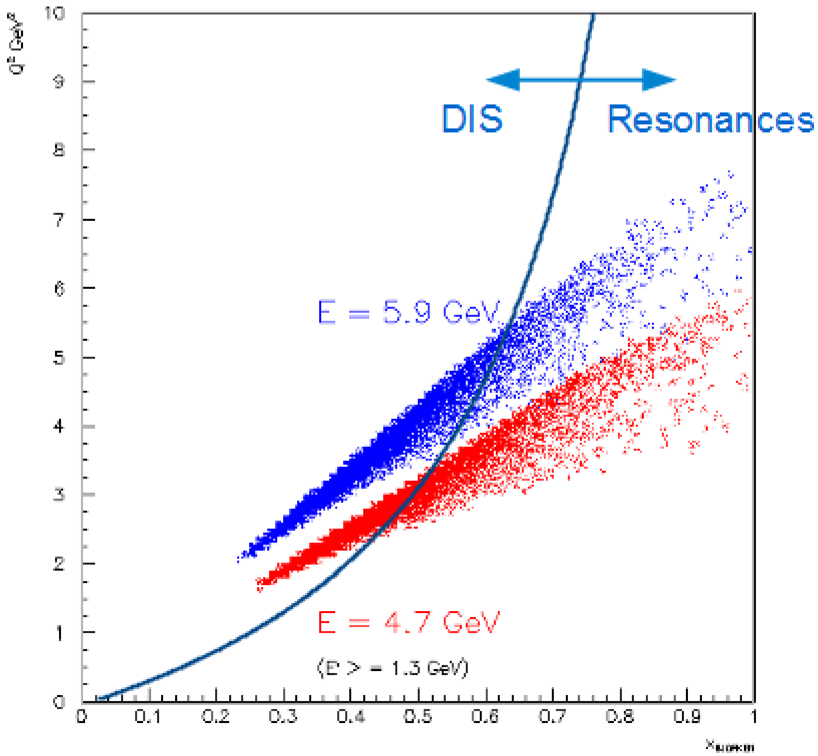
# DATA

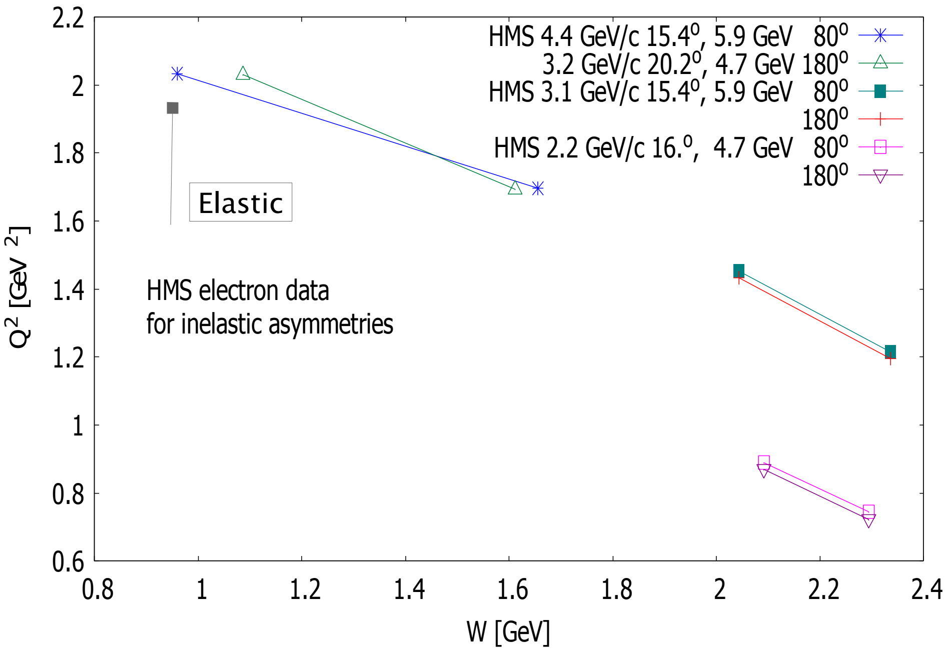
| Detector | Detected particle | Scattering Type | Beam Energy [GeV] | Field Direction | Target |
|---|---|---|---|---|---|
| BETA | *e*, $\pi^0$ | Inclusive inelastic | 5.9, 4.7 | 180°, 80° | NH3 |
| HMS | *e* | Inclusive inelastic | 5.9, 4.7 | 180°, 80° | NH3<br>C, LHe [1] |
| | | Inclusive elastic | 5.9 | 80° | NH3 |
| BETA - HMS | *e - p* | Coincidence elastic | 5.9 | 80° | NH3 |

[1] Unpolarized, for dilution factor

- Data taken in January - March 2009

# BETA and HMS data



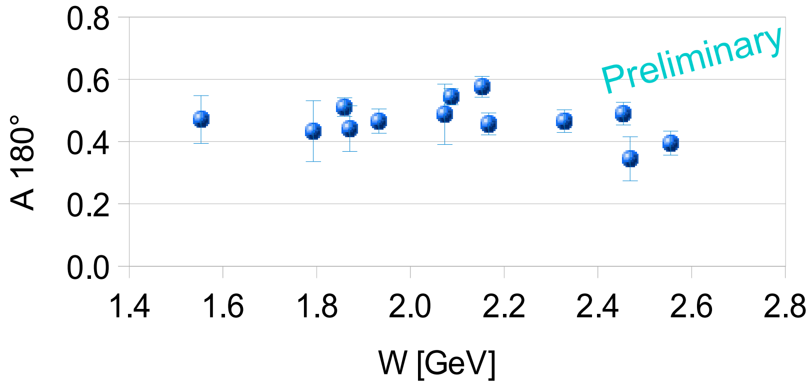




- $Q^2 - x$ phase space of BETA's 80° data
  - cut on E' ≥ 1.3 GeV
- Central kinematics of HMS inclusive asymmetry data

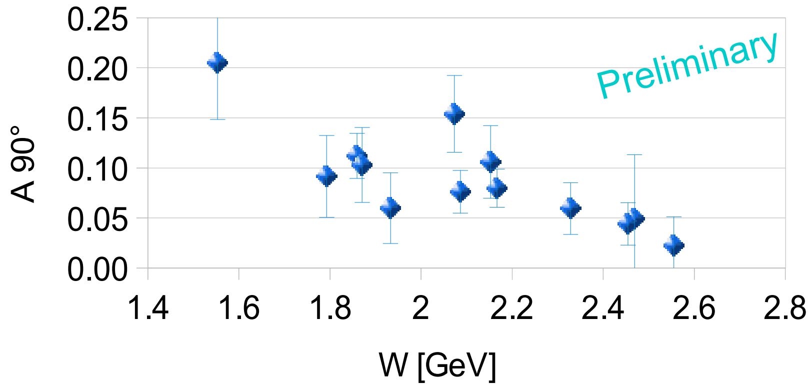
# Measured Asymmetries A(80°), A(180°)

$$A_m = \frac{\epsilon}{f\,P_b P_t C_N}\,;\ \epsilon = \frac{N^- - N^+}{N^- + N^+}$$

$$A_{phys} = \frac{1}{f_{rc}}\left(\frac{A_m - f_b A_b}{1 - f_b}\right) + A_{rc}$$




- $N^{+,-}$ = charge normalized, dead time corrected yields
- $P_b$, $P_t$ = beam, target polarizations
- $f$ = polarized dilution factor
- $C_N$ = $^{14}$N polarization correction
- $A_b$, $f_b$ = background corrections
- $A_{rc}$, $f_{rc}$ = radiative corrections




$$A_{\perp} = (A_{180}\cos 80 + A_{80})/\sin 80$$

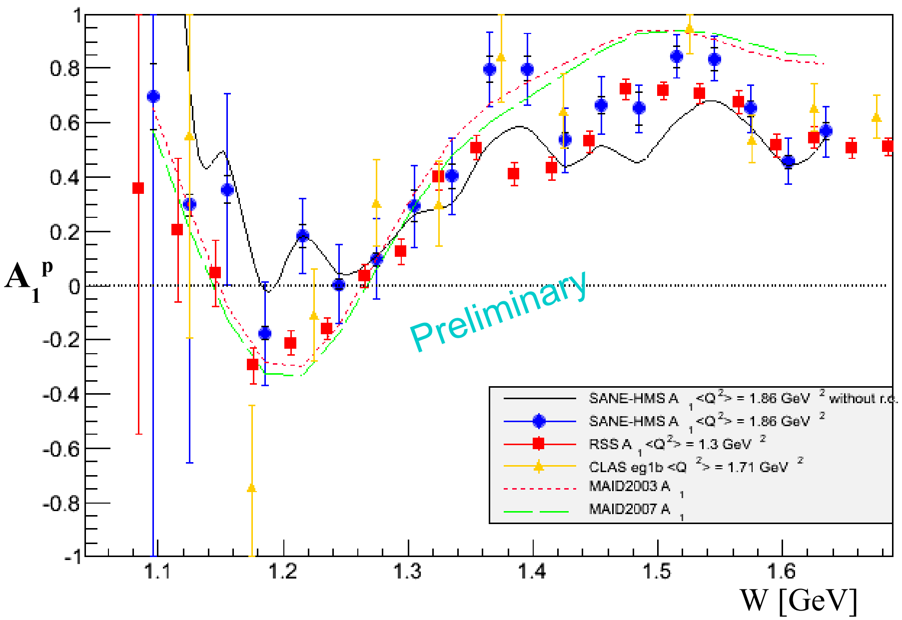
# Preliminary Results

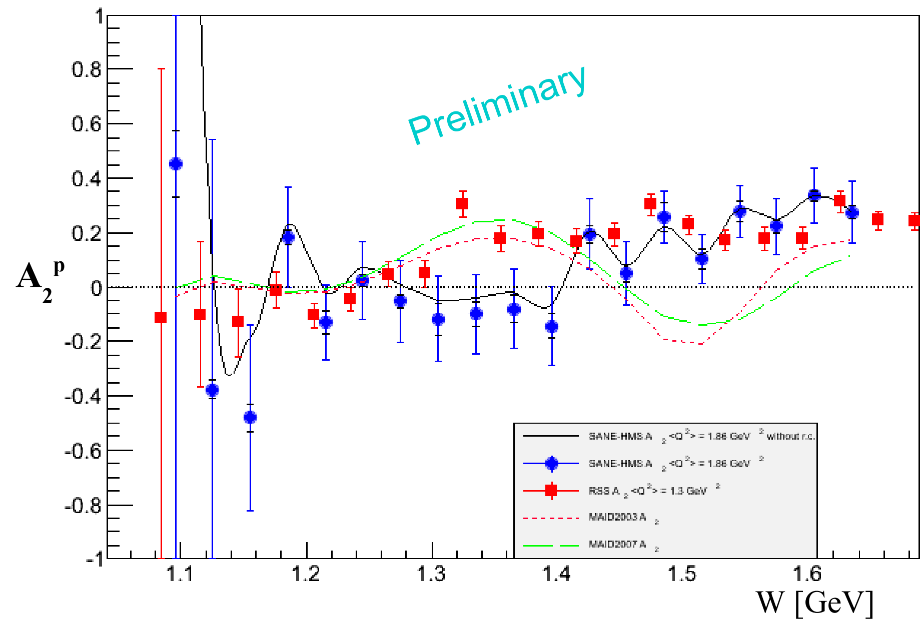
# Spin Asymmetries $A_1$ and $A_2$

- HMS single arm data in the resonances, $\langle Q^2 \rangle \sim \underline{1.8}$ GeV²
  - Model independent separation from measured asymmetries

$$A_1 = \frac{1}{D'}\left(\frac{E - E'\cos\theta}{E + E'} A_{180} + \frac{E'\sin\theta}{(E+E')\cos\phi}\frac{A_{180}\cos 80^\circ + A_{80}}{\sin 80^\circ}\right)$$

$$A_2 = \frac{1}{D'}\frac{1}{2E}\left(\sqrt{Q^2}A_{180} - \sqrt{Q^2}\frac{E - E'\cos\theta}{E'\sin\theta\cos\phi}\frac{A_{180}\cos 80^\circ + A_{80}}{\sin 80^\circ}\right)$$




(H-y. Kang)

# Spin Asymmetries $A_1$ and $A_2$

- HMS single arm data in the resonances, $\langle Q^2 \rangle \sim$ <u>1.3</u> GeV²
  - Model independent separation from measured asymmetries

$$A_1 = \frac{1}{D'}\left(\frac{E-E'\cos\theta}{E+E'}A_{180} + \frac{E'\sin\theta}{(E+E')\cos\phi}\,\frac{A_{180}\cos 80^\circ + A_{80}}{\sin 80^\circ}\right)$$

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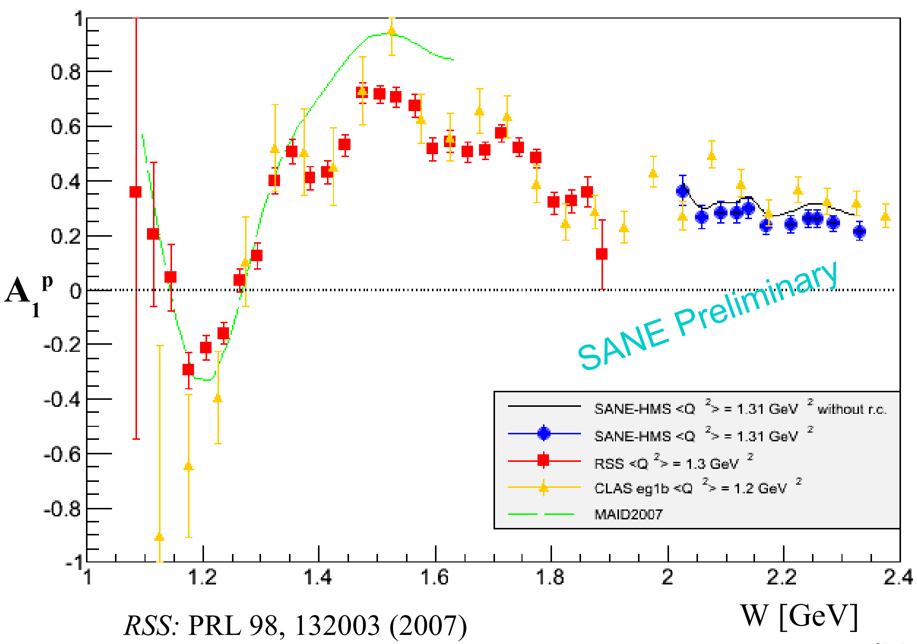
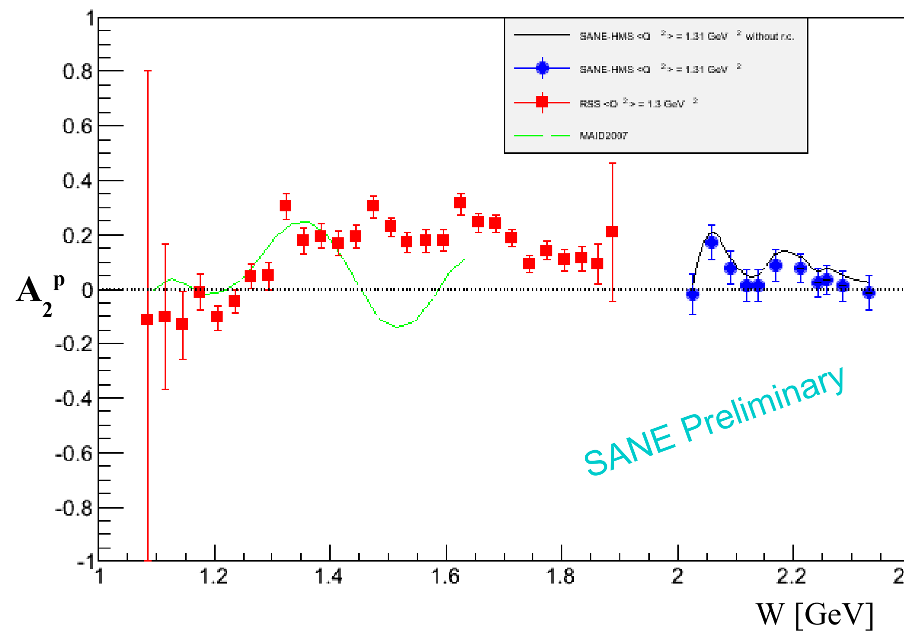


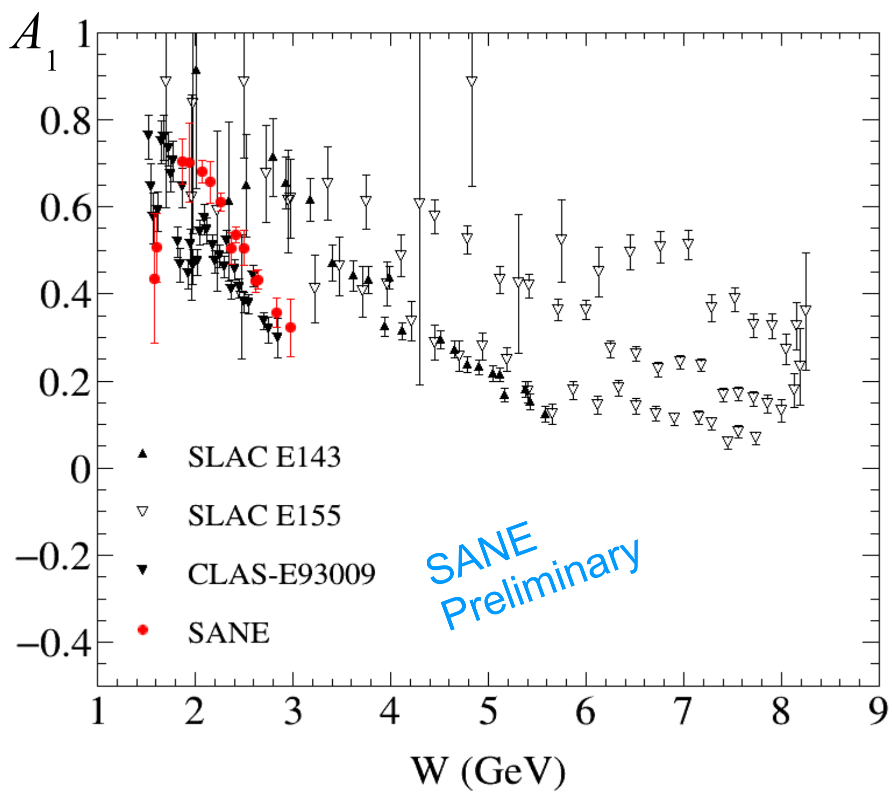




*RSS:* PRL 98, 132003 (2007)

(H-y. Kang)

# DIS Spin Asymmetry $A_1$

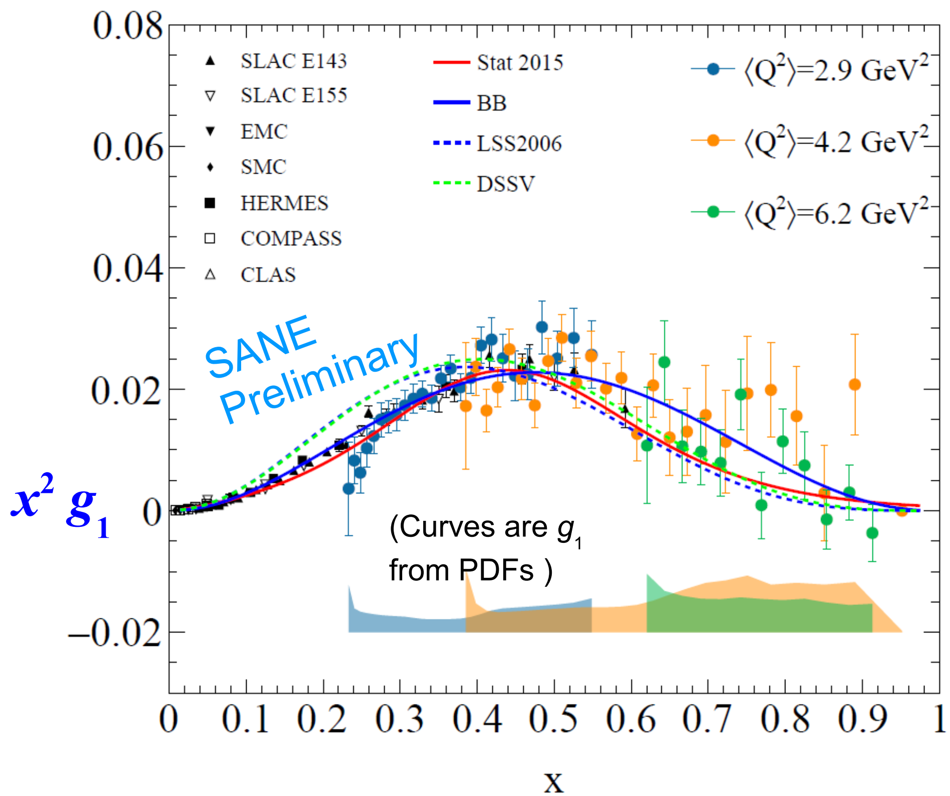
- $A_1(W)$ shows clear decreasing trend
- SANE BETA data
  - statistical errors only
- SLAC data plotted for individual spectrometers
  - very broad $Q^2$ range
- CLAS data at same $W$ but different $Q^2$ merged for clarity




(W. Armstrong)

# $g_1$ and $g_2$ in DIS and Resonances

- BETA proton data
  - DIS and Resonances
  - $g_1$, $g_2^{WW}$ curves from PDF's at 4 GeV²
  - $E' \geq$ 0.6 GeV
  - more data at 1.6 GeV² coming
- SLAC E143, E155, E155x, SMC and HERMES DIS data

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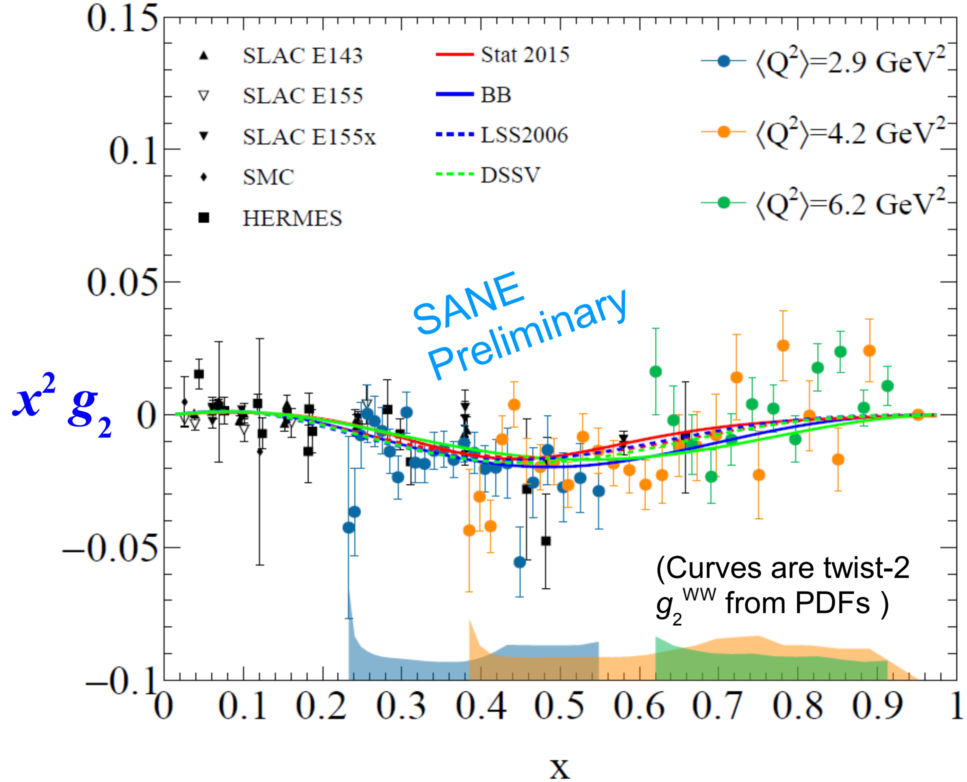



(W. Armstrong)

# Operator Product Expansion for Spin SF's

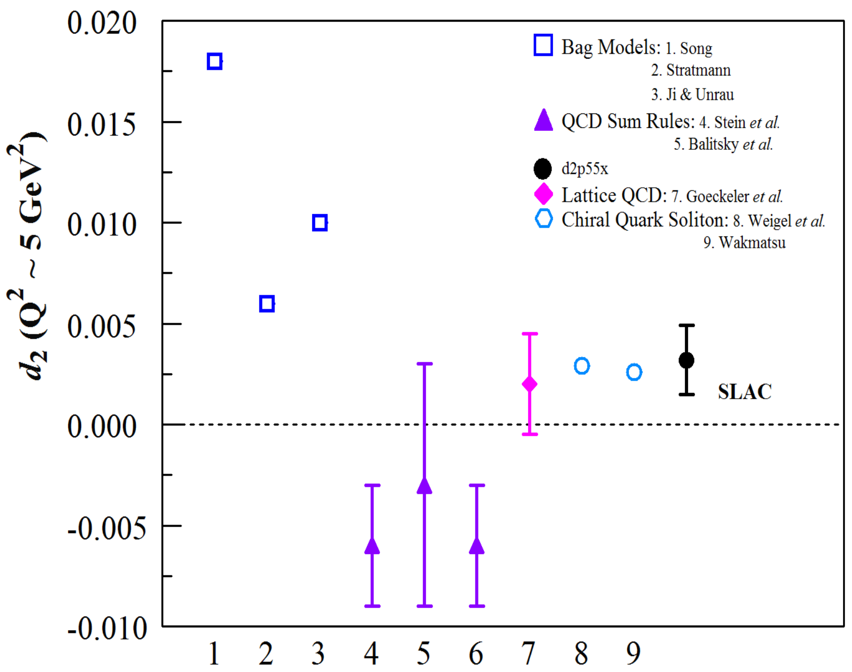
- OPE relates Cornwall-Norton moments to matrix elements of twist-2 $\boldsymbol{a}_N$ and twist-3 $\boldsymbol{d}_N$

$$\int_0^1 x^N g_1(x,Q^2)\,dx = \frac{\boldsymbol{a}_N}{2} + tmc, \qquad N=0,2,4,..$$

$$\int_0^1 x^N g_2(x,Q^2)\,dx = \frac{N(\boldsymbol{d}_N - \boldsymbol{a}_N)}{2(N+1)} + tmc, N=2,4,..$$

$(tmc: target\ mass\ corrections)$

  - $\boldsymbol{d}_2$ is mean color-magnetic field along spin




- Nachtmann moments needed to get twist-3 free of *tmc*

$$\boldsymbol{d}_2(\boldsymbol{Q}^2) = \int_0^1 dx\, \xi^2 \left( 2\frac{\xi}{x} g_1 + 3\left(1 - \frac{\xi^2 M^2}{2Q^2}\right) g_2 \right) \Rightarrow_{Q^2 \to \infty} \int_0^1 dx\, x^2 (2 g_1 + 3 g_2)$$

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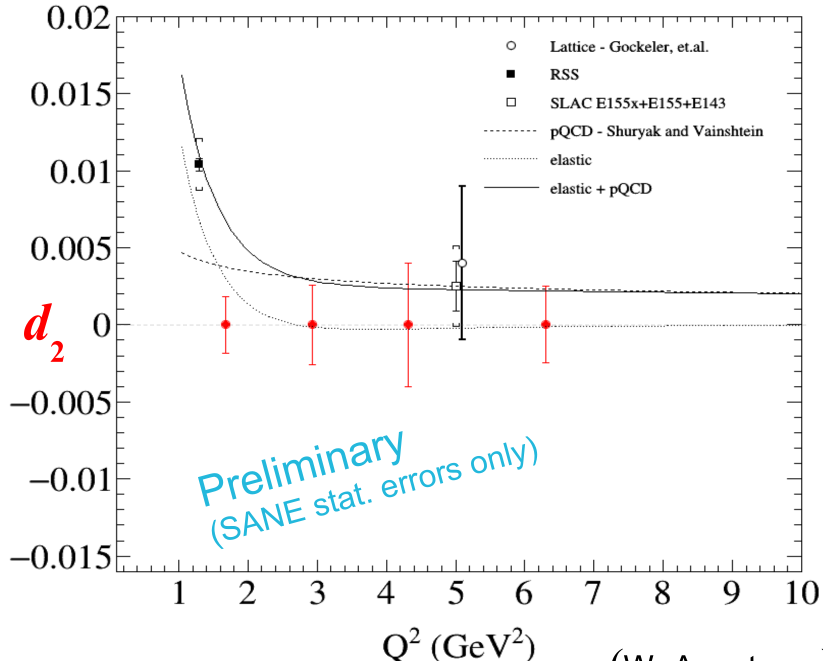


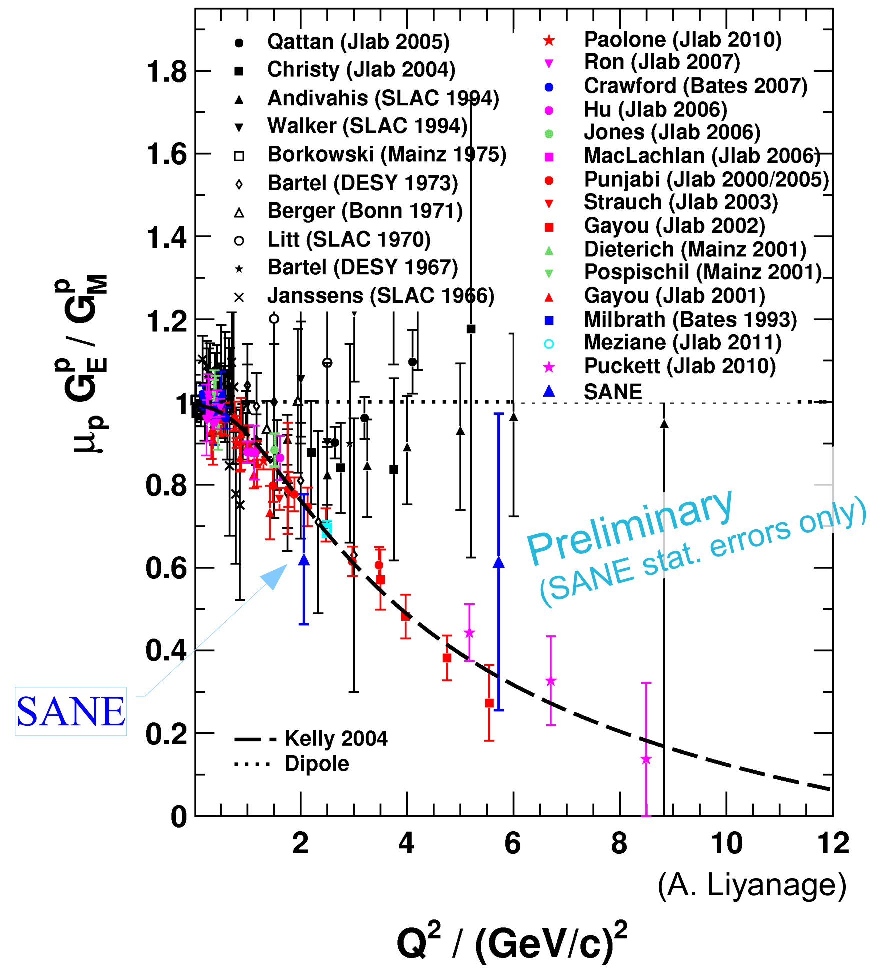

(W. Armstrong)

- SANE analysis final version
- Publications in preparation

# $G_E^p/G_M^p$ from inclusive and coincidence data

Ratio from:

- SANE inclusive HMS data at $Q^2 = 2.06$ GeV²
  - $A_{el}^p = -0.20 \pm 0.02$
  - $G_E^p/G_M^p = 0.60 \pm 0.18 \pm 0.06$ (statistical + systematic error)
- BETA–HMS *e–p* coincidences at $Q^2 = 5.66$ GeV²
  - $G_E^p/G_M^p = 0.67 \pm 0.36$ (statistical error only)



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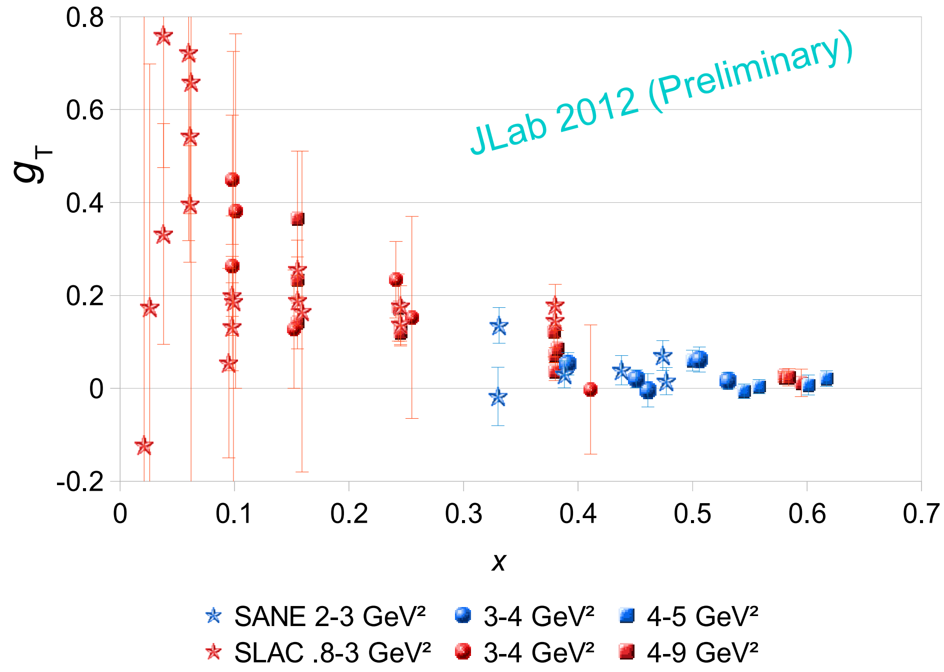
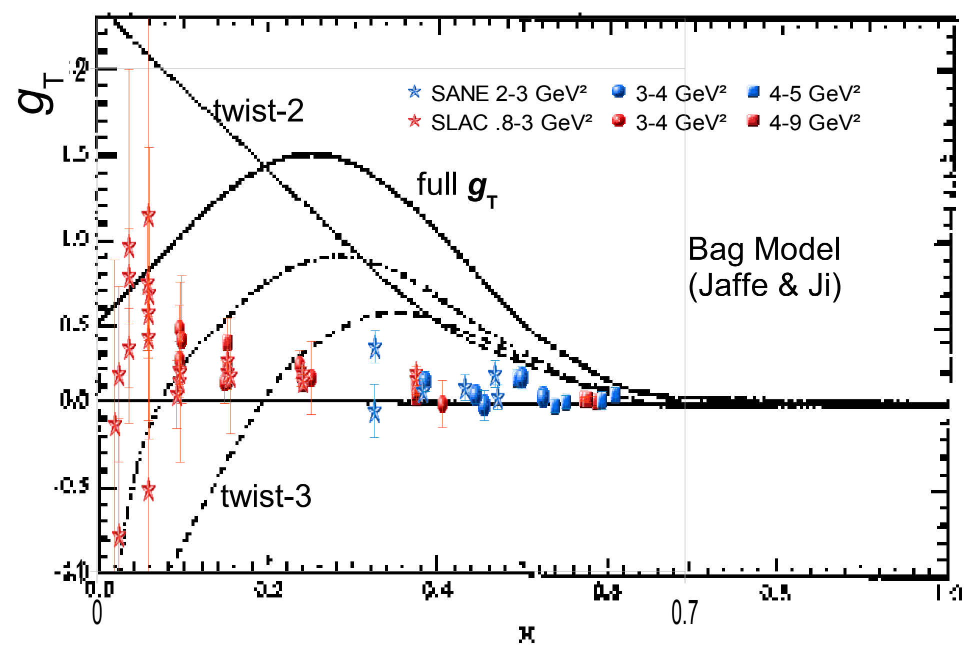
(Affiliations as of end of run - March 2009)

# Extras

# DIS Transverse Spin SF $g_T^p$



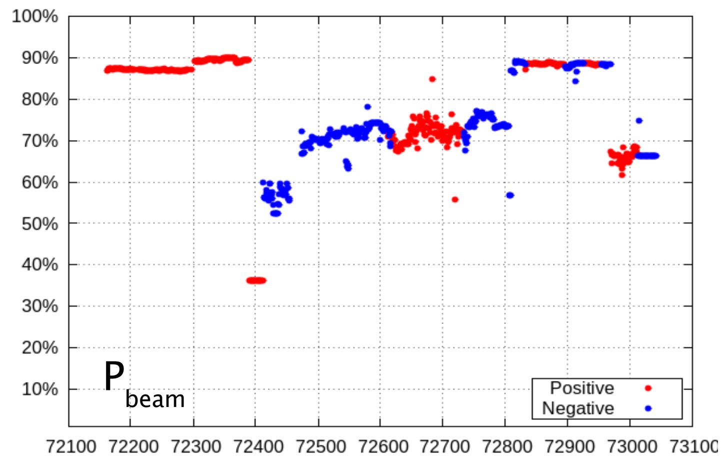



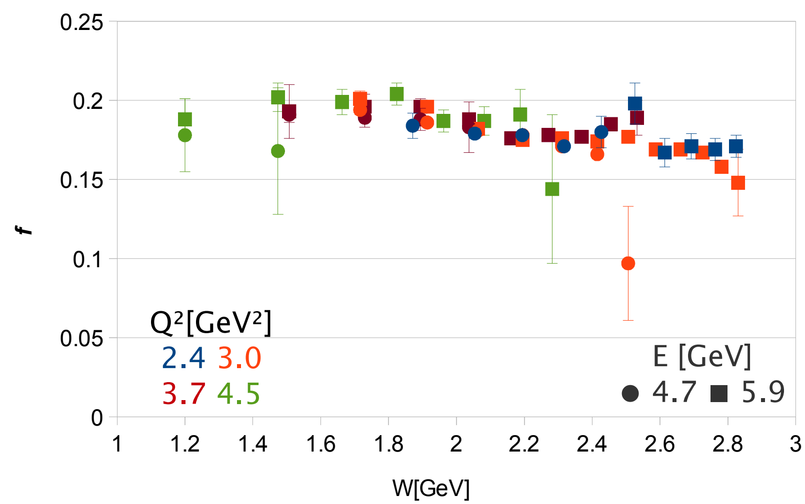

Tangerman & Mulders hep-ph/9408305v1

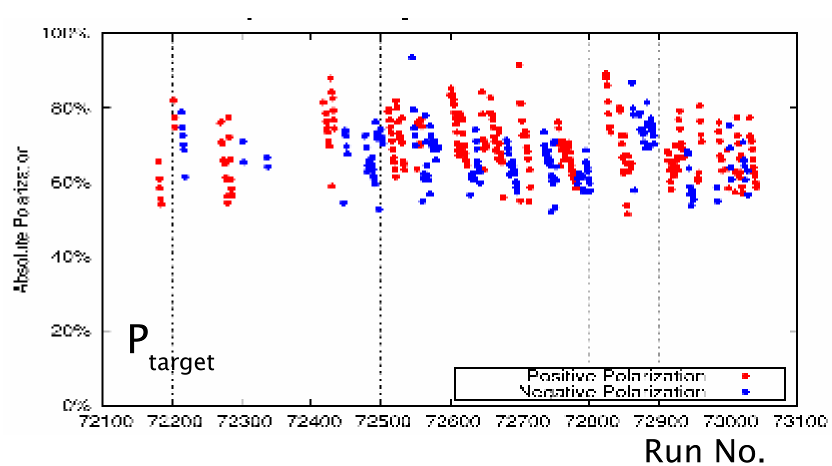
- $g_T^p = F_1 A_2 / \gamma$ measures spin distribution normal to $\gamma^*$
- SANE $\langle g_T^p(x > .3)\rangle = 0.023 \pm 0.006$

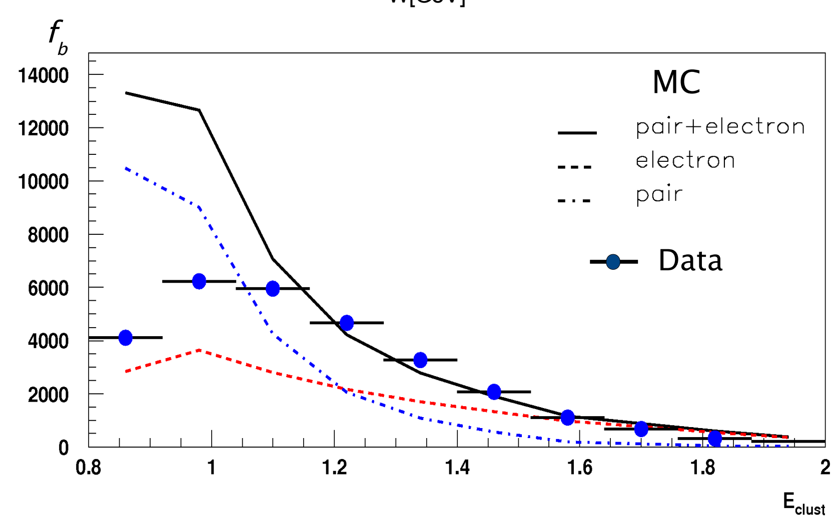
- Bag Model (1990's)
  - Data scaled × 2.5
  - Model updates needed

# Sample of Normalizations and corrections



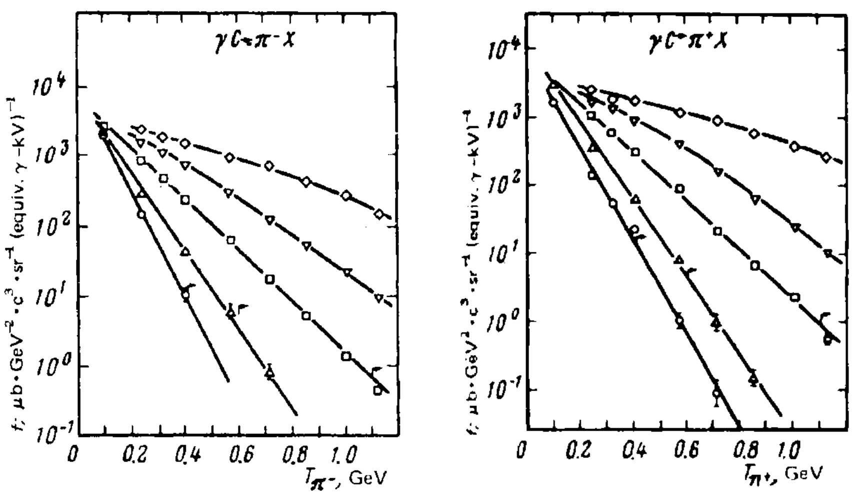



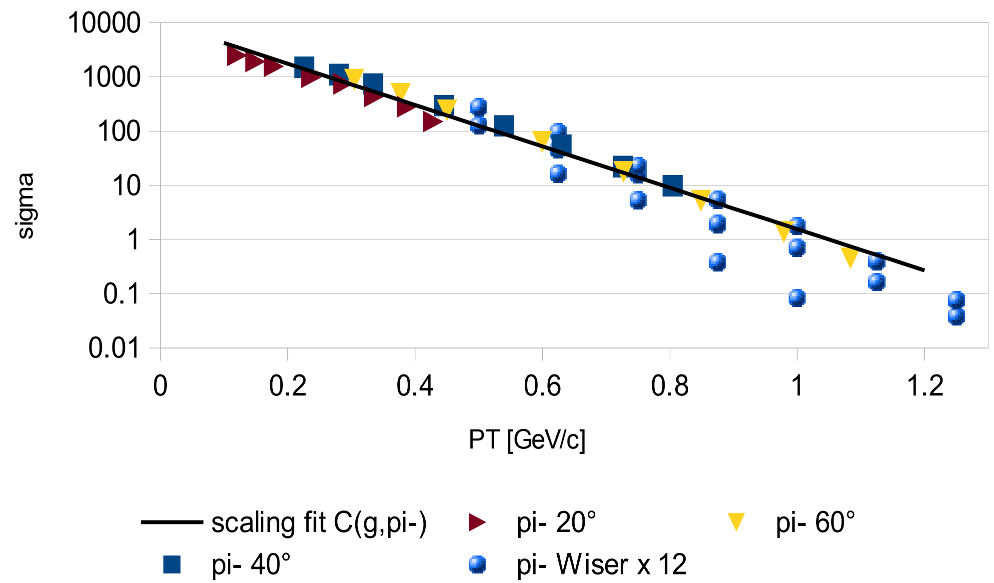
# Pair-symmetric background - I

- BigCal detects both charge signs
  - Significant background from $e^+e^-$ from $\pi^0$ decays
  - Partial control with cut on $E' \geq 1.3$ GeV; worst dilution $< \sim 0.2$
  - Estimate with GEANT simulation of $\pi^0$ production
  - Need inclusive pion photo- and electro-production cross sections
  - Existing D. Wiser parametrization only for H, D targets
  - Parameterized Yerevan pion photoproduction data on C at 4.5 GeV
    - Cross section scales with pion $P_T$: use simple exponential scaling fit
    - Included fit in J. O'Connell EPC code for single arm hadron photo and electroproduction
    - Compared with DESY electroproduction on C at 5 GeV

# Pair-symmetric background - II



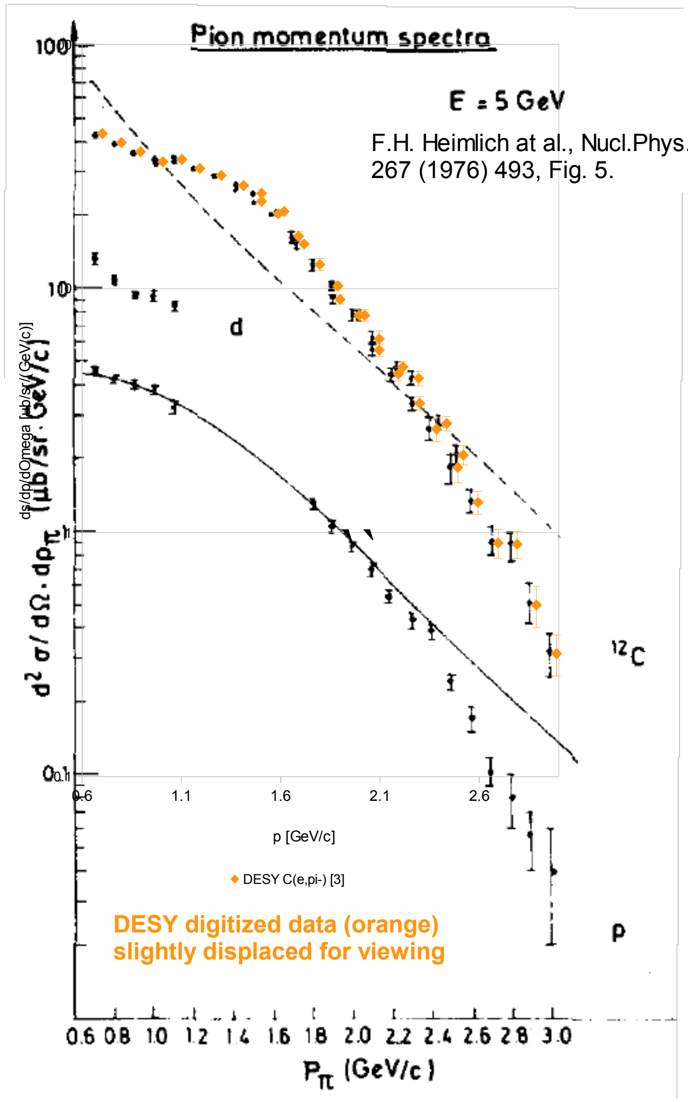

K. Alanakian et al., JETP Lett. 32 (1980) 652

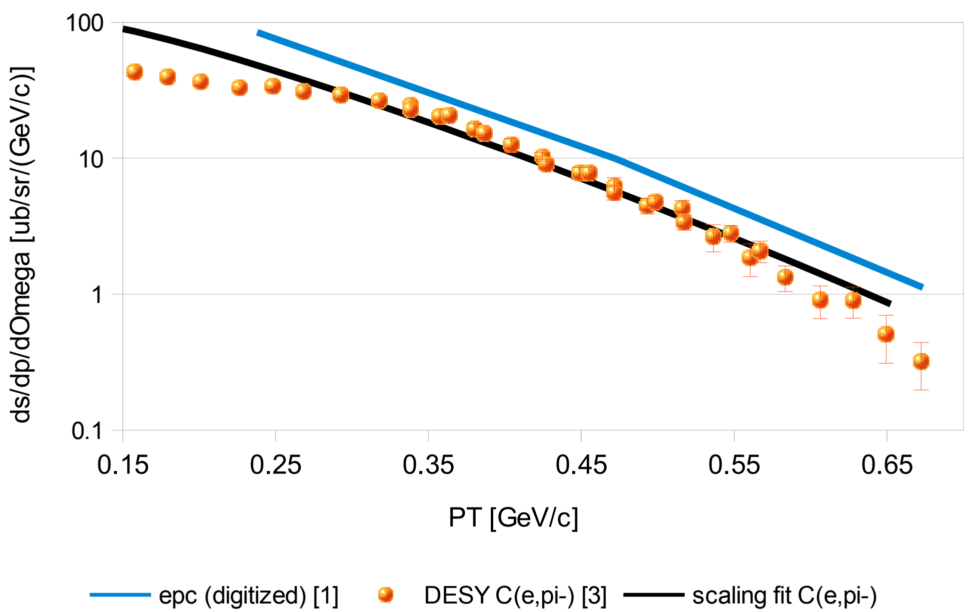



- Fitted $\pi^+$, $\pi^-$ data at 20°, 40°, 60° to $\boldsymbol{\sigma(P_T) = a\, e^{-bPT}}$
- $\pi^0$ fit from average of $\pi^+$ and $\pi^-$

- Wiser $\pi^-$ data on H, scaled 12×, along with $\pi^-$ data on C and scaling fit to C data.

# Pair-symmetric background - III

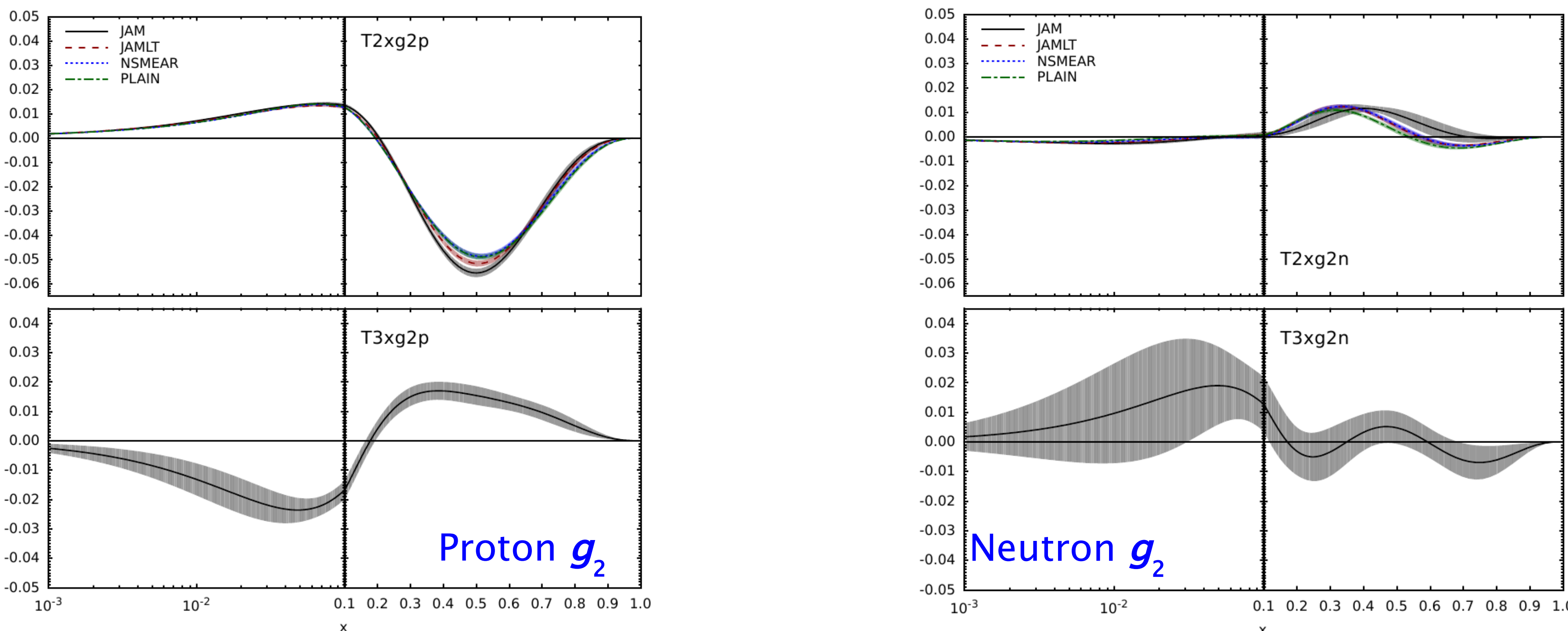



- Test of scaling fit with DESY $C(e,\pi^-)$ data at 5 GeV, 13°


[1] J. O'Connell CEBAF Summer Workshop, F. Gross and J. Lightbody, ed., Newport News, 1988, p. 345

# Jefferson Angular Momentum – JAM Collaboration

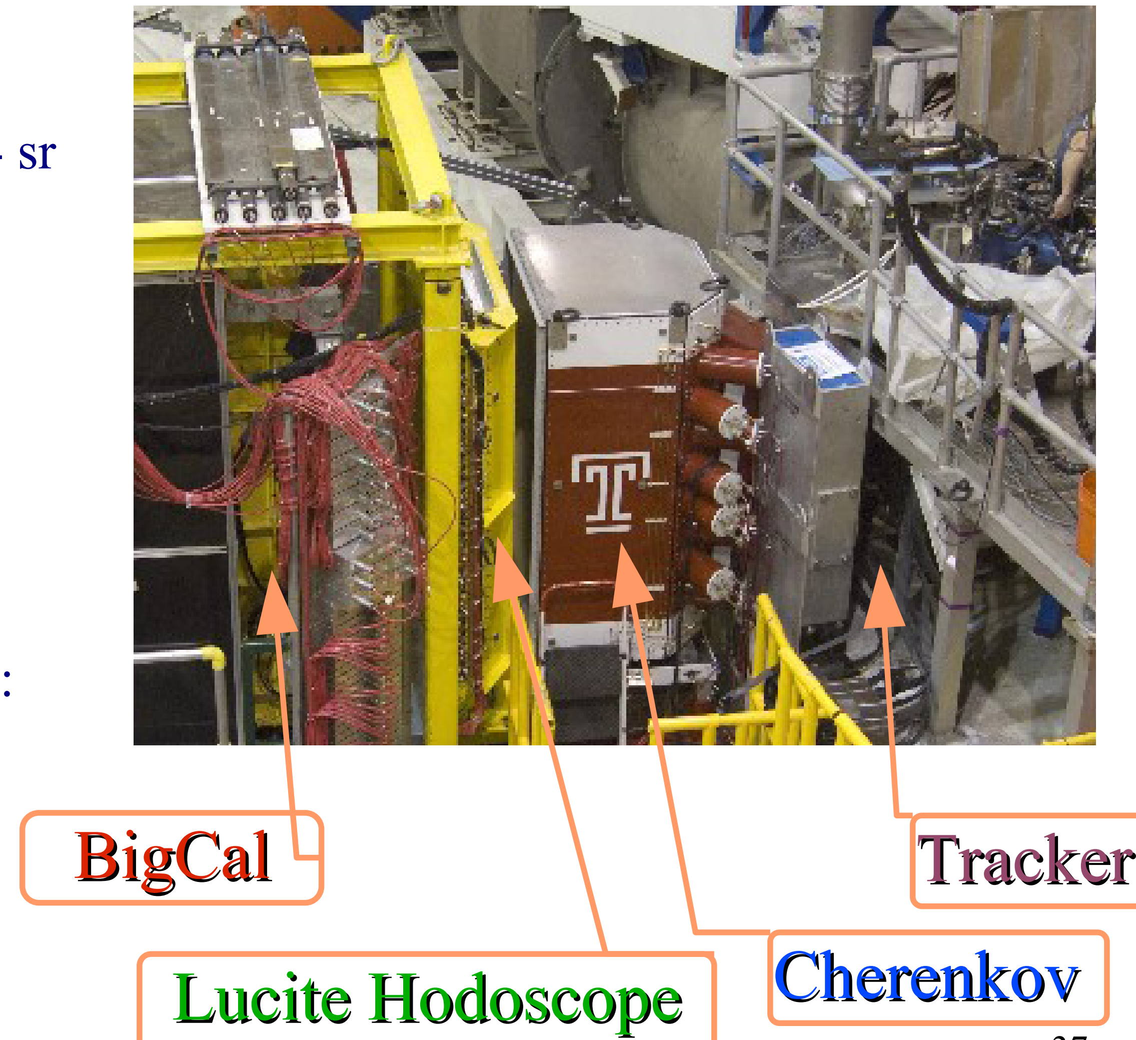



P. Jimenez- Delgado APS Spring 2013

- Joint theorists and experimentalists effort to “*study the quark and gluon spin structure of the nucleon by performing global fits of PDFs*”.
- JAM's spin PDFs are tailored for studies at large Bjorken *x*, as well as the resonance-DIS transition region at low and intermediate ***W*** and $\boldsymbol{Q^2}$.
  http://wwwold.jlab.org/theory/jam/

# Big Electron Telescope Array – BETA

- BETA specs
  - Effective solid angle 0.194 sr
  - Energy resolution $10\%/\sqrt{E(\mathrm{GeV})}$
  - 1000:1 pion rejection
  - angular resolution ~ 1 mr
- Non-magnetic detector
  - detects DIS $e$ and $e^+e^-$ pairs: need to cut on minimum $E'$
  - Target field helps sweep lowest $E$ background (180 MeV/c cutoff)



# Nucleon Spin beyond $G_1$ and $G_2$

- Need to go beyond $a_0$ to understand nucleon spin
  - Orbital angular momentum (OAM) ***L*** is needed.
- Partons have transverse momentum, implies OAM
  - Mulders *et al.*, Transverse Momentum dependent Distributions – TMDs
  - functions of *x* and $k_t$
  - Semi-inclusive scattering (detect final *e*, one hadron)

| Transverse Momentum Distributions by Polarization | | | |
|---|---|---|---|
| ***Target* ↓ \ *quark* →** | ***U*** | ***L*** | ***T*** |
| ***U*** | $f_1(x, k_t)$ | | $h_1^{\perp}$ |
| ***L*** | | $g_1$ | $h_{1L}^{\perp}$ |
| ***T*** | $f_{1T}^{\perp}$ | $g_{1T}^{\perp}$ | $h_1$ $h_{1T}^{\perp}$ |

Longitudinal SSF (leading twist)

$$g_1(x)=\sum g_1^q(x)=\sum\int d^2\vec{k}_t g_{1L}(x,\vec{k}_t^2)$$

Transverse SSF (twist-3)

$$g_{1T}^{(1)}(x)=\sum g_{1T}^{q(1)}(x)=\sum\int d^2\ \vec{k}_t \frac{\vec{k}_t^2}{2M^2} g_{1T}^q(x,\vec{k}_t^2)$$

$$g_T(x)=g_1(x)+\frac{d}{dx}g_{1T}^{(1)}=g_1(x)+g_2(x)$$