What is the actual size of the proton?

$$r_p = 0.84087 \pm (.00039)\ fm$$

$$r_p = 0.877 \pm (.005)\ fm$$

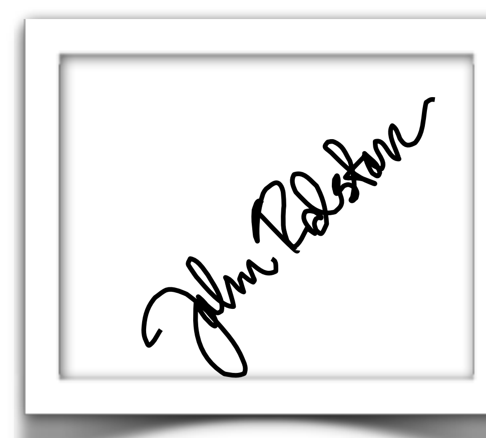
$$r_p = 0.879 \pm (.008)\ fm$$

Do we know the uncertainty?

$$r_p = \qquad \pm\,(.00039)\,fm$$

$$r_p = \qquad \pm\,(.005)\,fm$$

$$r_p = \qquad \pm\,(.008)\,fm$$



Proton Size Puzzle: "charge radius" from muonic hydrogen disagrees with electron data

$7\sigma!$

$$r_p = 0.84087 \pm (.00039)\ fm$$ muonic atom

$$r_p = 0.877 \pm (.005)\ fm$$ spectroscopy

$$r_p = 0.879 \pm (.008)\ fm$$ MAMI, JLAB

- definitions ?
- fitting the form factor ?
- muon atomic physics ?
- new physics of muons ?
- what?

WHY ?

$7\sigma!$

An unusual case needing careful "sensitivity analysis"

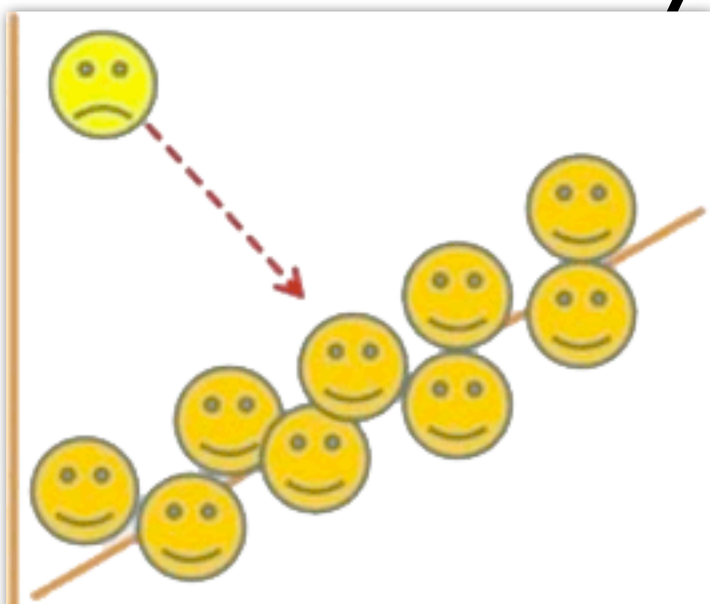
"In statistics a robust confidence interval is a robust modification of confidence intervals, meaning that one modifies the non-robust calculations of the confidence interval so that they are not badly affected by outlying or aberrant observations in a data-set.

There are various definitions of a "robust statistic." Strictly speaking, a robust statistic is resistant to errors in the results, produced by deviations from assumptions ."

what's been reported for the uncertainty of r_p is exquisitely sensistive to procedure

What's new:

previous analysis of
electronic H is unreliable.
Biased by a novel kind of "outlier"



in a scientifically conservative approach, the outlier will be removed

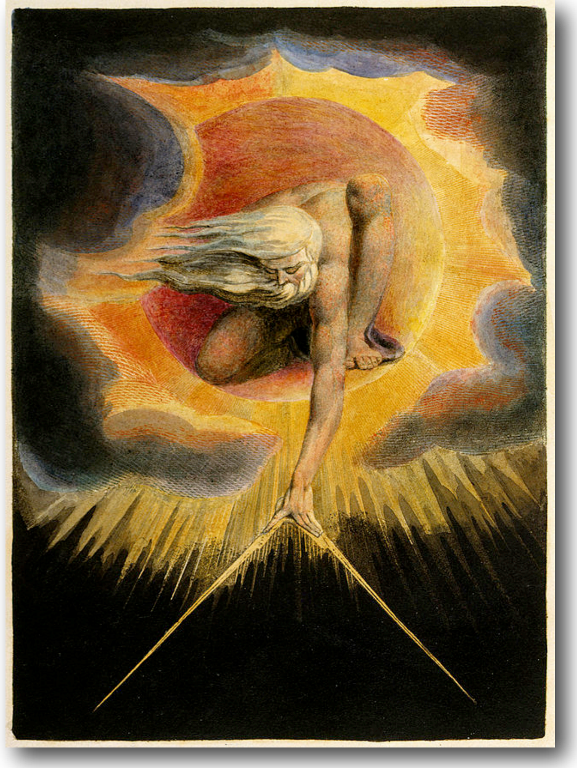
We find the disagreement is about $2.5\sigma - 3.5\sigma$

$$r_p = 0.87 \pm 0.01\,\mathrm{fm};$$
$$R_\infty = 1.09737315685 1 \times 10^7\,\mathrm{m}^{-1}$$
$$\pm\, 8 \times 10^{-5}\,\mathrm{m}^{-1}$$

dramatic effect on the error bars

electron scattering very competitive

What's so sensitive to analysis?



muonic atom? Easy theory, direct experiment. Getting muons in place is real hard. Simple analysis.

electron scattering? Leading order theory, plus work. Long history of experimental consistency. Numerous checks and balances.

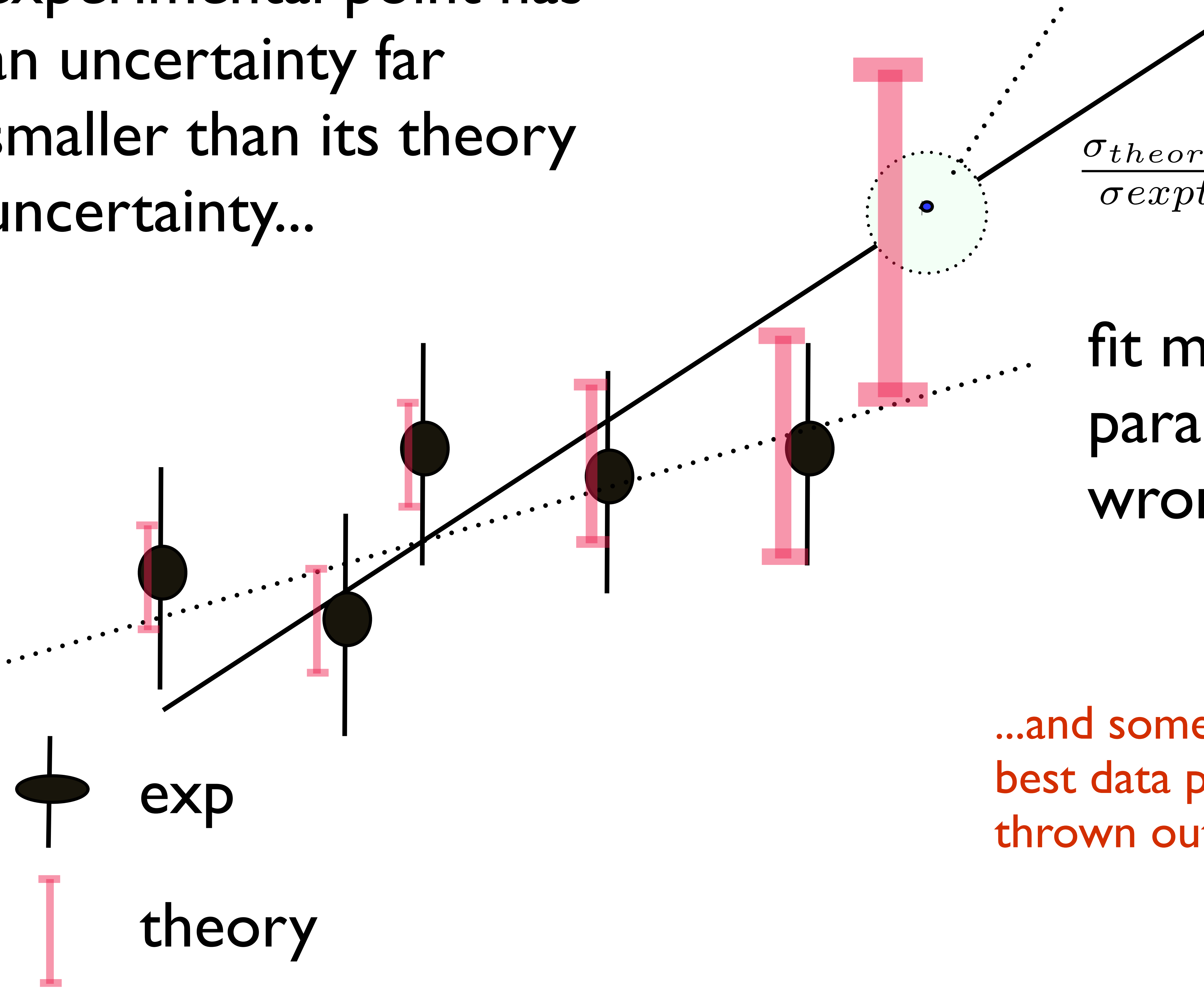
electronic hydrogen? The most difficult theory, and at very high orders. A very small tiny effect is buried under many other very small effects. Superb experimental data.

What checks and balances?

Sensitivity lesson: if an experimental point has an uncertainty far smaller than its theory uncertainty...

outlier in the data space of **experimental uncertainties**

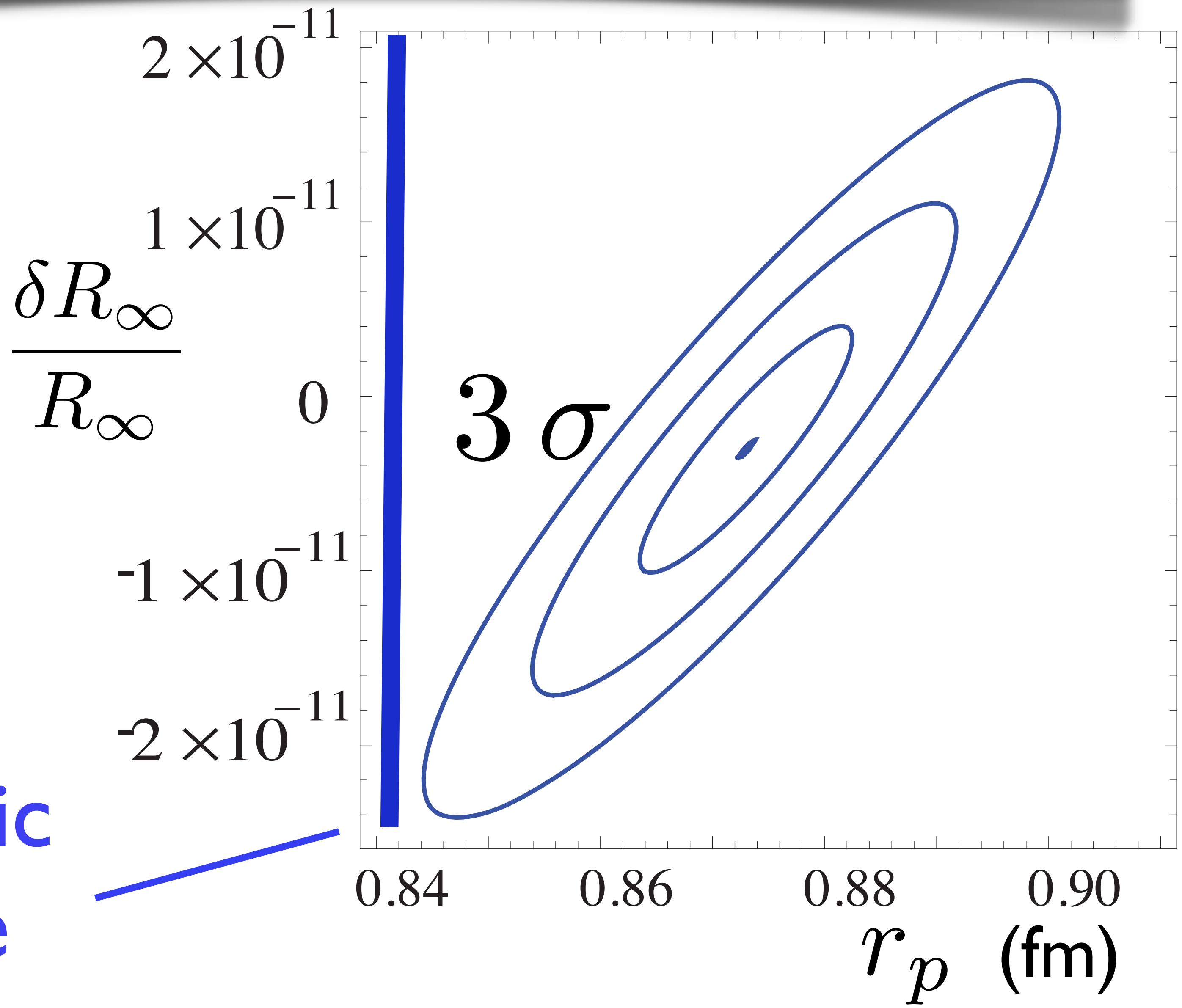
$$\frac{\sigma_{theory}}{\sigma_{expt}} \sim 10^2 - 10^3$$



fit may constrain parameters to a wrong subspace...

...and sometimes the best data point should be thrown out our independent analysis of the spectroscopic basis for the puzzle

work with John Martens, KU



muonic value

electronic H spectroscopy

$\chi^2 = \text{constant}$ contours

1,2,3 sigma CL

previous analysis, highly sensitive to small effects

We find it's a $2.5\sigma - 3.5\sigma$ disagreement, not 7σ

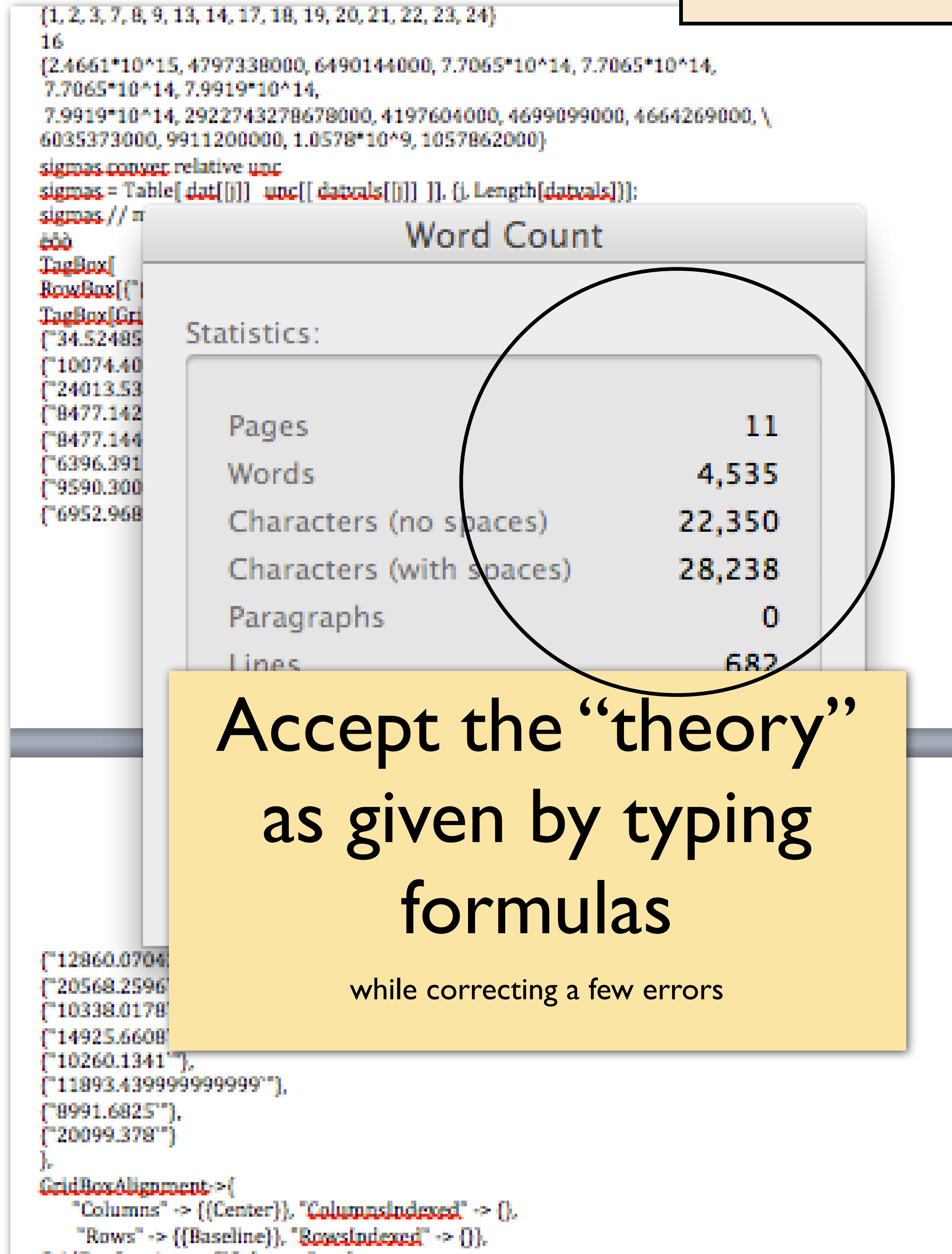
How do *inputs* affect *outputs?*

Review is over.
Our contribution starts here

Theory: 75 years
28000 keystrokes

mathematica! In C++, estimate 260000

Breit, Dirac, Bethe...Yennie, Sapirstein, Ericson,Brodsky...Eides, Grotch, Shelyuto, Borie, Karshenboim, Mohr, Kotochigova, Pachucki,Yerokin et al, Jenstchura...



Accept the “theory” as given by typing formulas

while correcting a few errors

validating 28k keystrokes of theory implementation

data set: 16 H transitions selected by CODATA for 20 years, 2010 includes 1S3S

	σ_{expt} Hz	f_{expt} Hz	$f_{our\ calc}$ Hz
1S2S →	35	$2.46606141319 \times 10^{15}$	$2.46606141319 \times 10^{15}$
	10074	4.797338×10^{9}	$4.79733066539 \times 10^{9}$
	24014	6.490144×10^{9}	$6.49012898284 \times 10^{9}$
	8477	$7.70649350012 \times 10^{14}$	$7.70649350016 \times 10^{14}$
	8477	$7.7064950445 \times 10^{14}$	$7.70649504449 \times 10^{14}$
	6396	$7.70649561584 \times 10^{14}$	$7.70649561578 \times 10^{14}$
	9590	$7.99191710473 \times 10^{14}$	$7.99191710481 \times 10^{14}$
	6953	$7.99191727404 \times 10^{14}$	$7.99191727409 \times 10^{14}$
	12860	$2.92274327868 \times 10^{15}$	$2.92274327867 \times 10^{15}$
	20568	4.197604×10^{9}	$4.19759919778 \times 10^{9}$
	10338	4.699099×10^{9}	$4.6991043085 \times 10^{9}$
	14926	4.664269×10^{9}	$4.66425337748 \times 10^{9}$
	10260	6.035373×10^{9}	$6.03538320383 \times 10^{9}$
	11893	9.9112×10^{9}	$9.91119855042 \times 10^{9}$
	8992	1.057845×10^{9}	$1.05784298986 \times 10^{9}$
	20099	1.057862×10^{9}	$1.05784298986 \times 10^{9}$
		experiment	JM+JPR

no theory errors listed here

compare two versions of theory on two machines; round off error under control

Review is over. Our contribution starts here

We speak Atomic

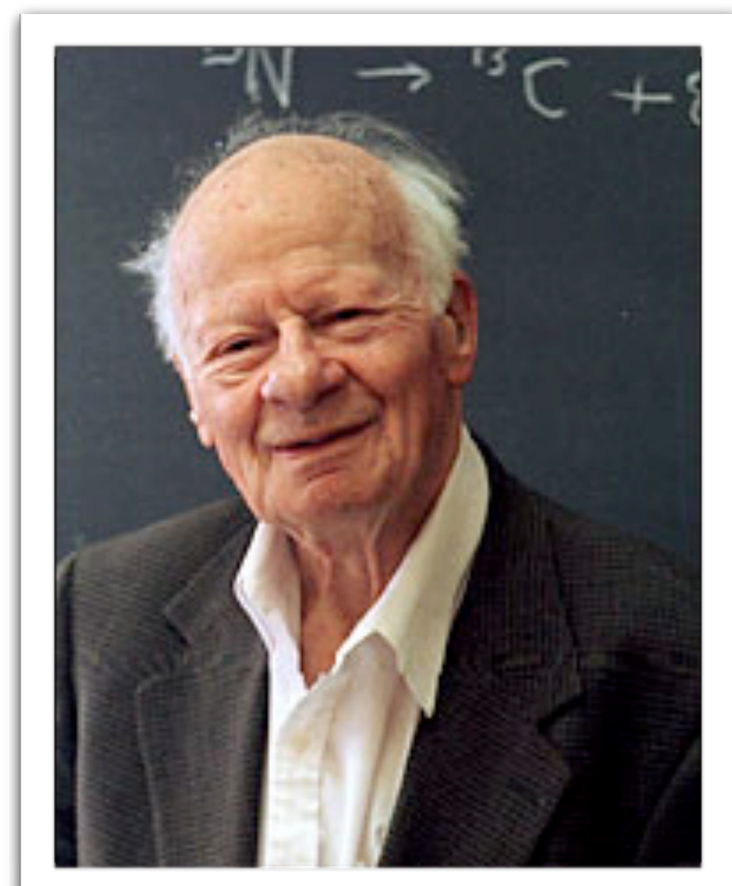
* natural units are frequency. It's what's measured

* planck's constant errors are unacceptably large

* ground state frequency $R_\infty c = 3 \times 10^{15}$ Hz

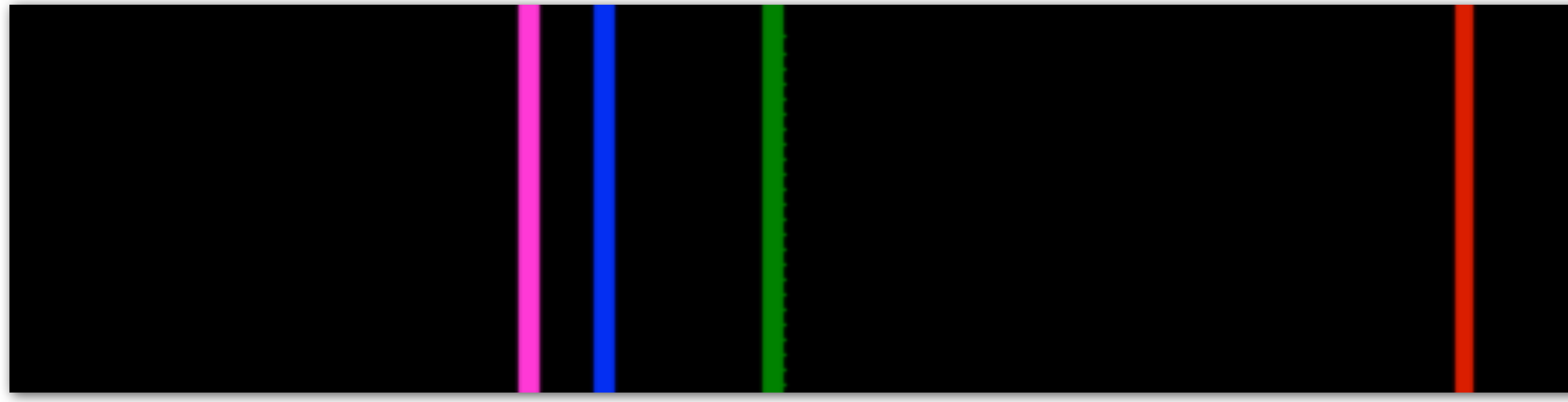
* proton size effect 1.5 Mhz in electronic H

* To measure size to 0.1% in electronic H needs 1 kHz theory errors



the term "Lamb shift" can mean the particular splitting of one transition observed by Willis Lamb in 1945, or it (more often) means everything beyond the bound state prediction of the Dirac equation as relativistic quantum mechanics...*not quantum field theory*

Hydrogen spectrum: two (2) parameters



$\longleftarrow \quad m_r \quad \longrightarrow$

- the Rydberg constant

given

$$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar} \equiv R_\infty^\bullet\left(1+\frac{\delta R_\infty}{R_\infty}\right)$$

- the proton charge radius

r_p

BUT THESE TWO ARE HIGHLY CORRELATED

2-parameter theory

Far better determined by other experiments:

- the fine structure constant
- the proton/electron mass ratio

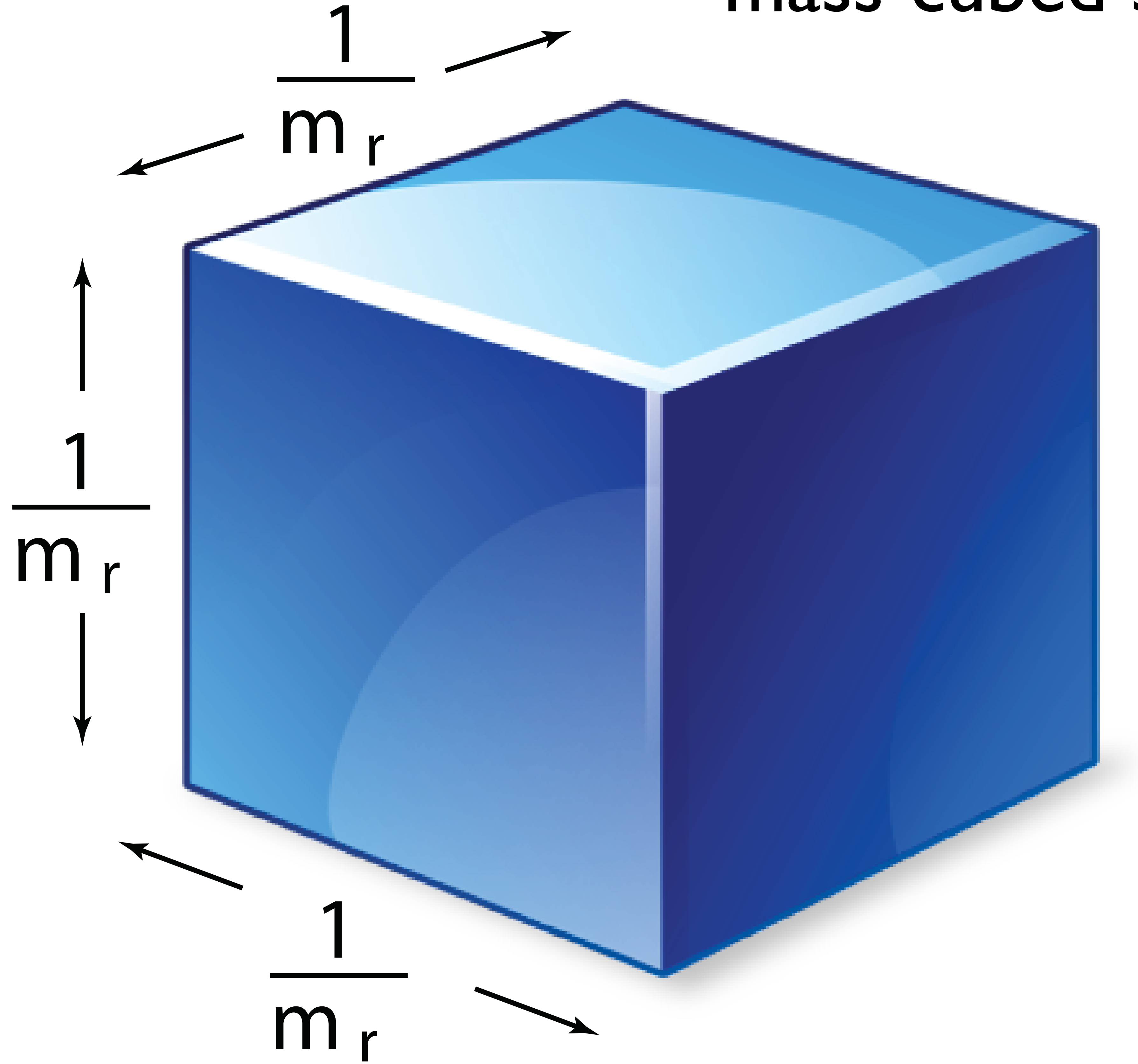
$\alpha^\bullet = 0.0072973525698$ given

$(m_p/m_e)^\bullet = 1836.152672$ given

The superscript $^\bullet$ indicates a reference value not to be fit.

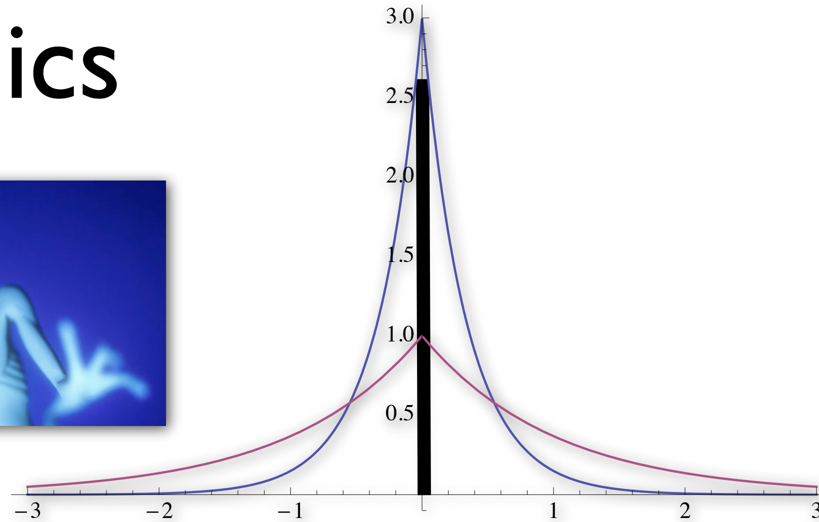
We speak Atomic

mass-cubed size effect



assuming a cubic atom

Review: 90 second course in atomic physics



$$\Delta E_n \sim < \psi_n | e\Delta V | \psi_n >;$$

$$eV(q) \sim \frac{e^2 F(\vec{q}^2)}{\vec{q}^2} \sim \alpha \left(\frac{1}{\vec{q}^2} + \frac{< r_p^2 > \vec{q}^2}{\vec{q}^2} \right);$$

$$eV_0(r) + \Delta V(r) \sim \frac{\alpha}{r} + \alpha < r_p^2 > \delta^3(r);$$

$$\Delta E_n \sim \alpha < r_p^2 > \psi_n^*(0)\psi_n(0);$$

$$a_n^3 \psi_n^*(0)\psi_n(0) \sim 1; \quad \psi_n^*(0)\psi_n(0) \sim \frac{1}{a_n^3} \sim \frac{\alpha^3 m_r^3}{n^3};$$

$$\Delta E_n \sim \frac{\alpha^4 < r_p^2 > m_r^3}{n^3}$$

smaller size
wave function
bigger
proton size
effect

$$(\frac{m_\mu}{m_e})^3 = 207^3 \sim 10^7$$

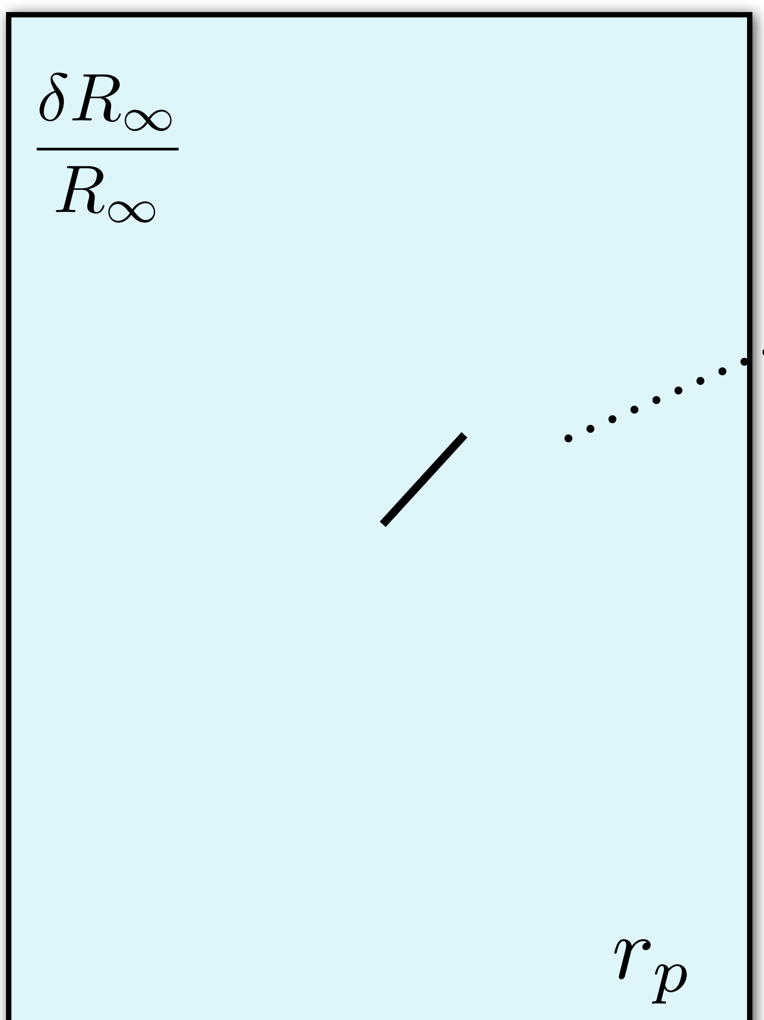
$$\Delta E_{n\ell}^{size} = \frac{2(Z\alpha)^4 m_r^3 < r_p^2 > c^4}{3\hbar^2 n^3} \delta_{\ell 0}$$

a puzzle
inside a puzzle

Data Analysis
Mysteriously "Stiff"

Extreme sensitivity,
disgusting resolution.

1S2S makes super skinny
chi^2 contour plots
defy machine accuracy

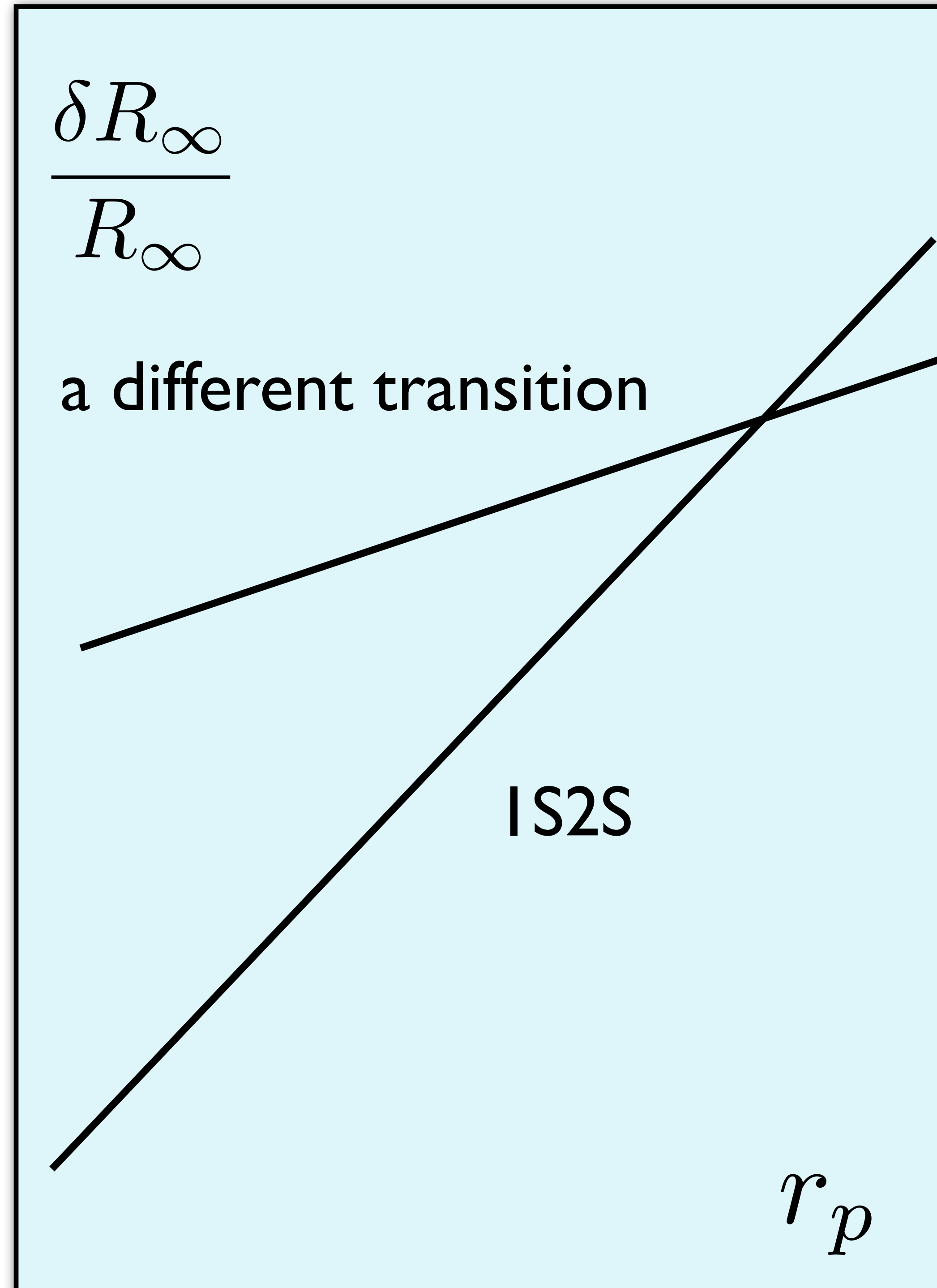


what's going on?

"Concept of Counting"

Fact:
any ONE transition gives ONE datum
for two parameters $(r_p,\, R_\infty)$

...the result is a particular
line of degeneracy
from a one-point fit



You'll then fit
the whole data set...

$$\chi^2 = \sum_i \left(f_i^{theory}(r_p,\, R_\infty) - f_i^{experiment}\right)^2 / \sigma_i^2 .$$

...no single datum should matter that much...

“Concept Slide”

ENTER, the experimental uncertainties

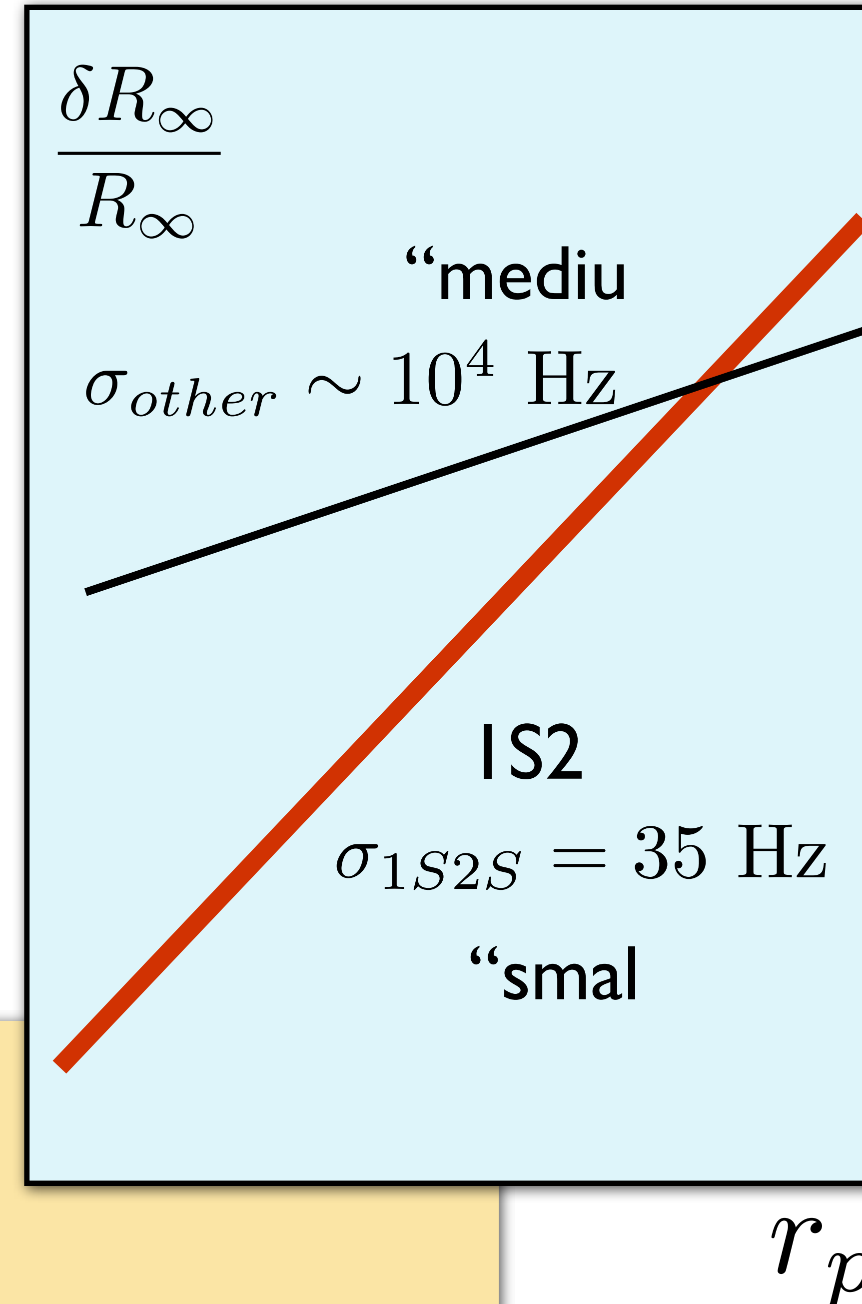
ONE ultra-precise point dominates

1S2S

$$\sigma_i = (35, 6396,\ 6953,\ \ldots\ 20568,\ 24014)\ \text{Hz}$$

$$\chi^2 \sim 1/\sigma^2$$

The mean value of $\sigma_j^2/\sigma_{1S2S}^2 = 148,400.$



Prediction of Concept Slide

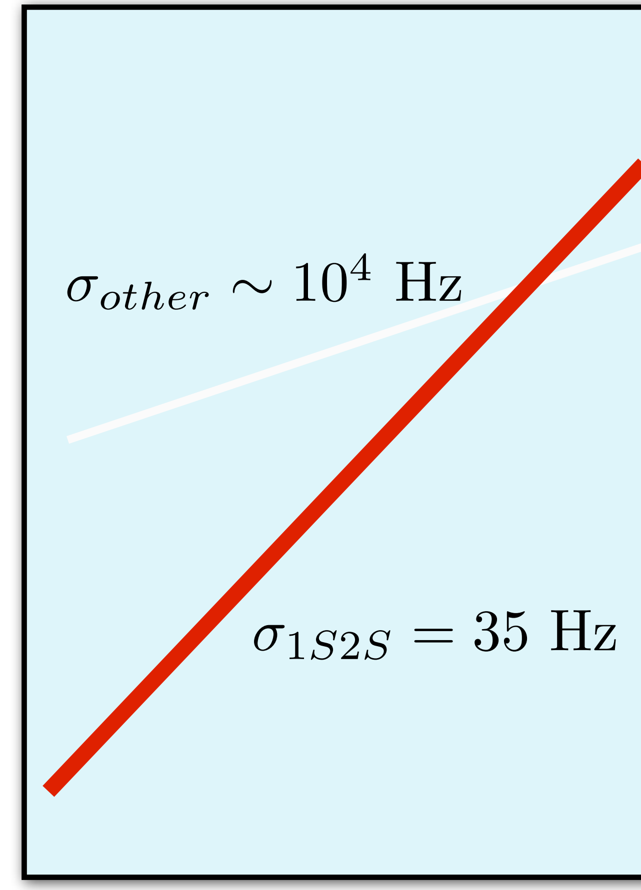
Given $< \sigma_j^2 > / \sigma_{1S2S}^2 = 148,400...$

$$\downarrow$$

$$\chi^2 \sim 1/\sigma^2$$

$$\downarrow$$

ultra-precise
1S2S => "exact" constraint
on *fitted* parameters r_p R_∞



insert
800 lb
gorilla
graphic
here

Yet the theory is not exact.
Theory errors >> 35 Hz

The result: extreme sensitivity to
theory errors of 1S2S

Theoretical uncertainties: Not well controlled

1S uncertainty *estimated* the largest,
maybe 3kHz - 30 kHz

a part of the 2-loop self energy:

Leading log expansion breaks down

$$\Delta E_{1S;\,(6)} = \frac{\alpha^2 (Z\alpha)^6 m_e c^2}{8\pi^2} (B_{63} log^3((Z\alpha)^{-2}) + B_{62} log^2((Z\alpha)^{-2}) + B_{61} log^1((Z\alpha)^{-2}) + B_{60}),$$
$$= \frac{\alpha^2 (Z\alpha)^6 m_e c^2}{8\pi^2} (282 - 62 + 476 - 61.6) \sim 728\, kHz.$$

jenschura pachucki 2003
eides et al 2007, 2000

$\Delta \mathcal{E} \Delta t \gtrsim 1$

3-digit accuracy
Yet different calculations
differ by 100%
(yerokin et al)

Meanwhile:

σ_{1S2S} is by far the smallest *experimental* uncertainty

$$\chi^2 = \frac{(f^{expt}_{1S2S} - f^{theory}_{1S2S})}{(35\, Hz)^2})^2 + \frac{(f^{expt}_{1S3S} - f^{theory}_{1S3S})}{(13000\, Hz)^2})^2 + \ldots$$

Paradox : ☆ 1S has the largest theory uncertainty,
estimated 3kHz - 30 kHz error, or more

☆ σ_{1S2S} = 35 Hz is by far the *smallest* experimental uncertainty

$$\chi^2 = \frac{(f_{1S2S}^{expt} - f_{1S2S}^{theory})}{(35\,Hz)^2})^2 + \frac{(f_{1S3S}^{expt} - f_{1S3S}^{theory})}{(13000\,Hz)^2})^2 + \ldots$$

$$\text{why is this not order } \frac{3500 Hz}{35\,Hz} \sim 10^4 \text{ ?}$$

the answer is known, but buried:

"However, one thing can be stated with certainty: the exact agreement of those two ultra-precise 1S2S measurements with the QED calculations cannot be considered as a confirmation of the QED theory, because it is the result of the fitting of the fundamental constants based on these (and other) transitions." A. Kramida, Atomic Data and Nuclear Data Tables, 96, 586 (2010)

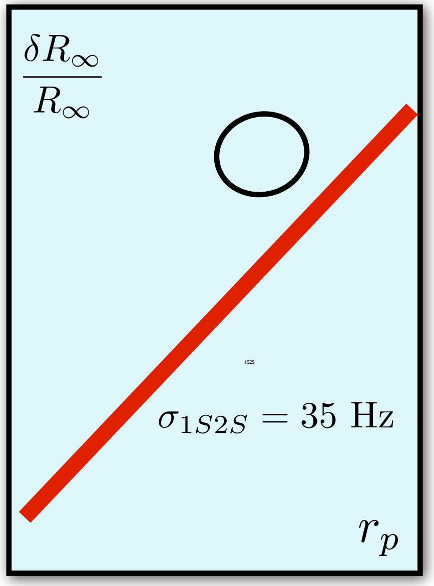
ONE EXACT FIT HAPPENS TRIVIALLY

1S2S **one-point trivial fit** predicts everything... when included

At the best fit value,

$$\frac{\partial \chi^2}{\partial R_\infty} = 2c \sum_i (R_\infty c \Delta \hat{f}_i^{theory} - \Delta f_i^{exp})/\sigma_i^2 \to 0;$$

$$\frac{\partial^2 \chi^2}{\partial R_\infty^2} = 2c^2 \sum_i \Delta \hat{f}_i/\sigma_i^2 \sim 2c^2 \sum_i \Delta \hat{f}_i^{expt}/\sigma_i^2$$



with...

$$\Delta f_i^{expt}/\sigma_i^2 = (2.1 \times 10^{12}, 1.9 \times 10^7, 1.8 \times 10^7 \ldots$$

Estimate with first non-trivial point:

$$\Delta R_\infty \sim \sqrt{(\frac{1}{2}\partial^2\chi^2/\partial R_\infty^2)^{-1}} \sim \sqrt{\frac{\sigma_2^2 R_\infty}{f_2^{expt} c}}$$

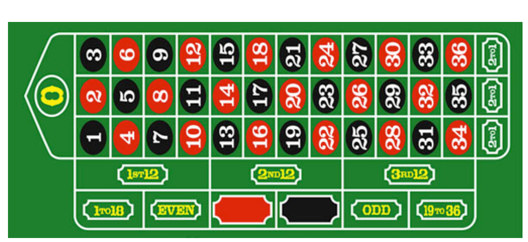
$$= \sqrt{\frac{1.07 \times 10^7}{1.9 \times 10^7 \times 3 \times 10^8}} = 4.4 \times 10^{-5};$$

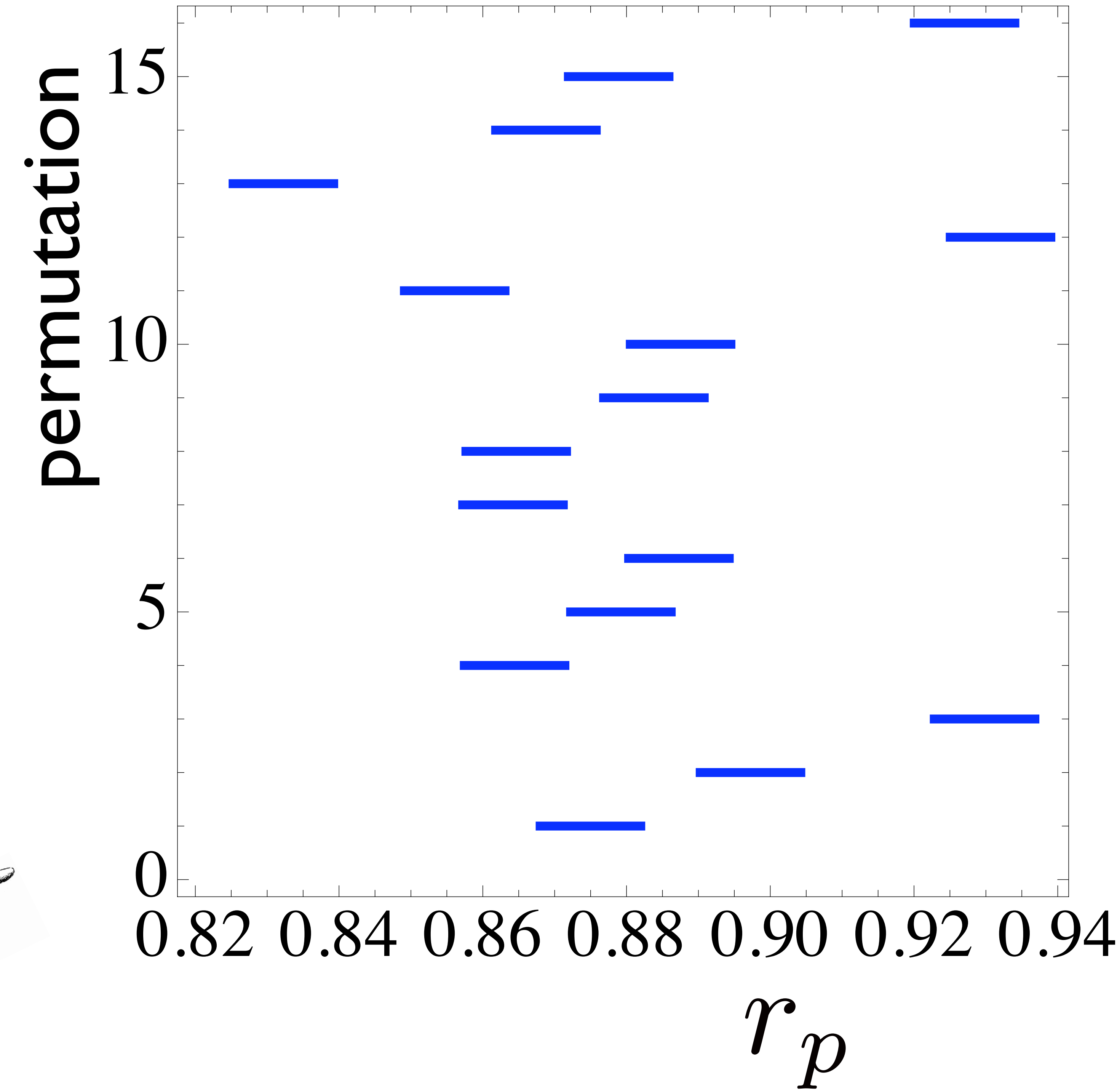
0.8 * CODATA2010
using 82 (28) parameters

$$\frac{\Delta R_\infty}{R_\infty} \sim 4 \times 10^{-12}.$$

In case you missed the point: The ultra-precise datum forces a perfect fit by circular procedure

data fitting roulette: cyclically permute sigmas. cycle 35 Hz through all



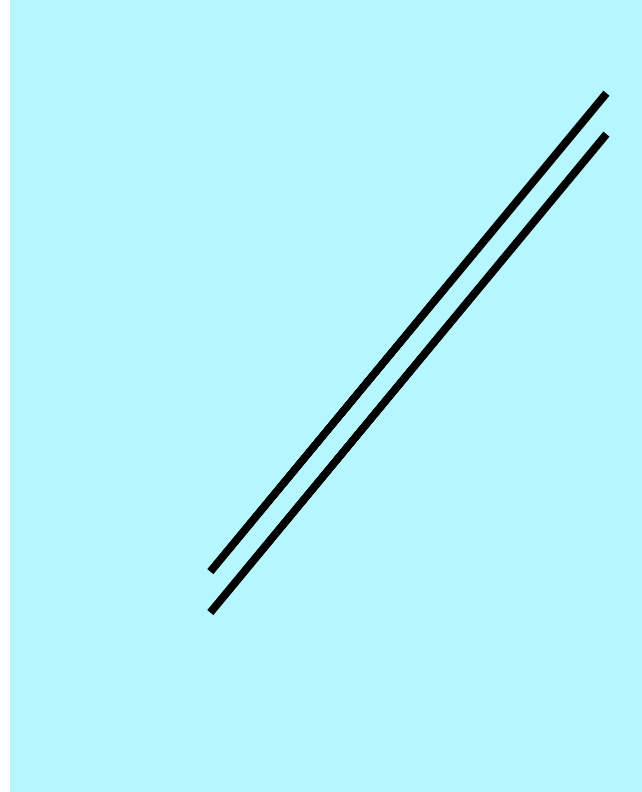


each permutation yields tiny error bars

the set lies far outside the error bars !

There is a certain confidence region in the r_p R_∞ plane

With the 1S2S extreme sensitivity,



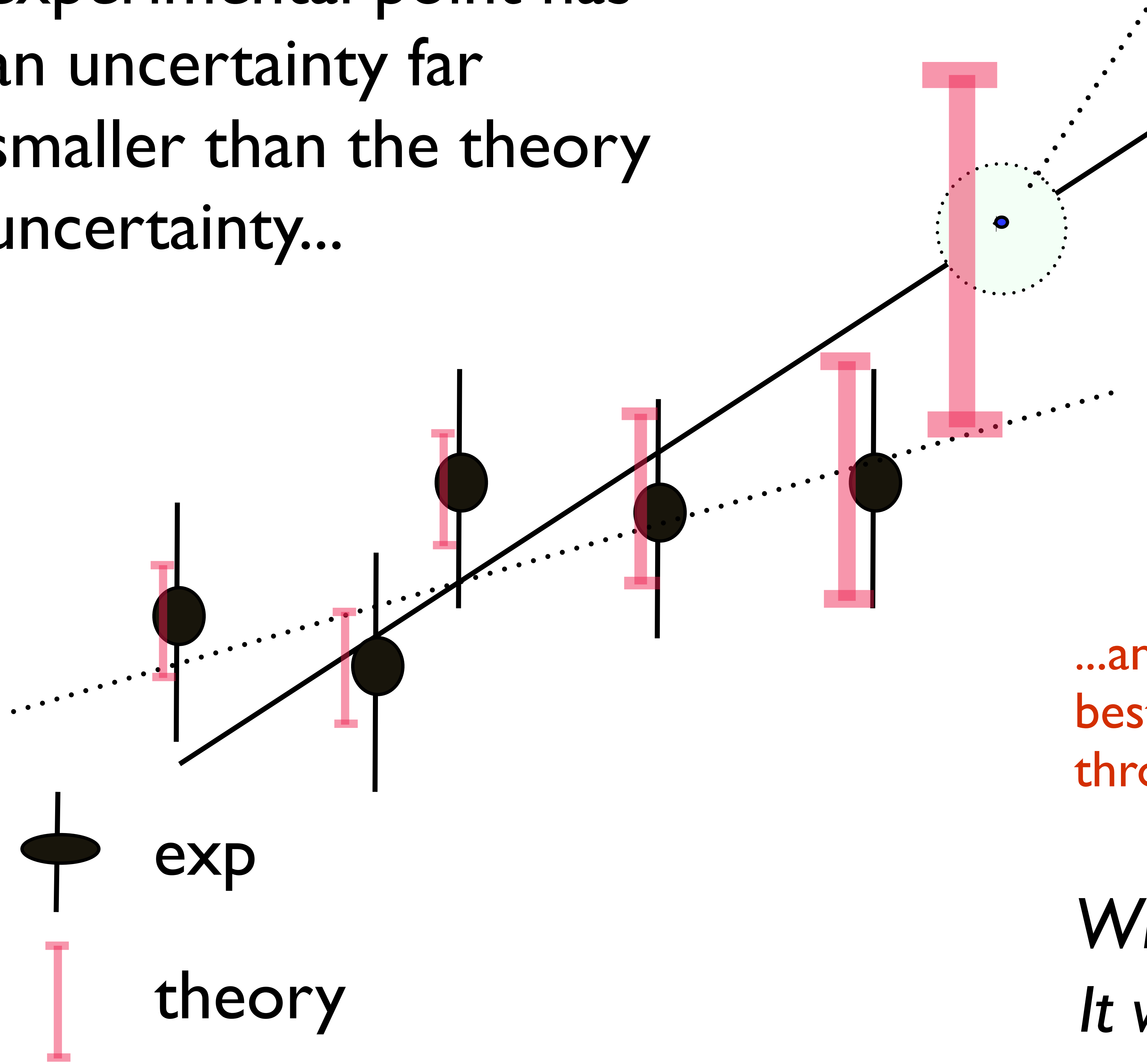
...all the points in one region are more than 10,000 units of chi-squared different from the other

Why are people citing raw “uncertainties”?

Sensitivity lesson: if an experimental point has an uncertainty far smaller than the theory uncertainty...

outlier in the data space of experimental uncertainties

$$\frac{\sigma_{theory}}{\sigma_{expt}} \sim 10^2 - 10^3$$



it may constrain parameters to a wrong subspace...

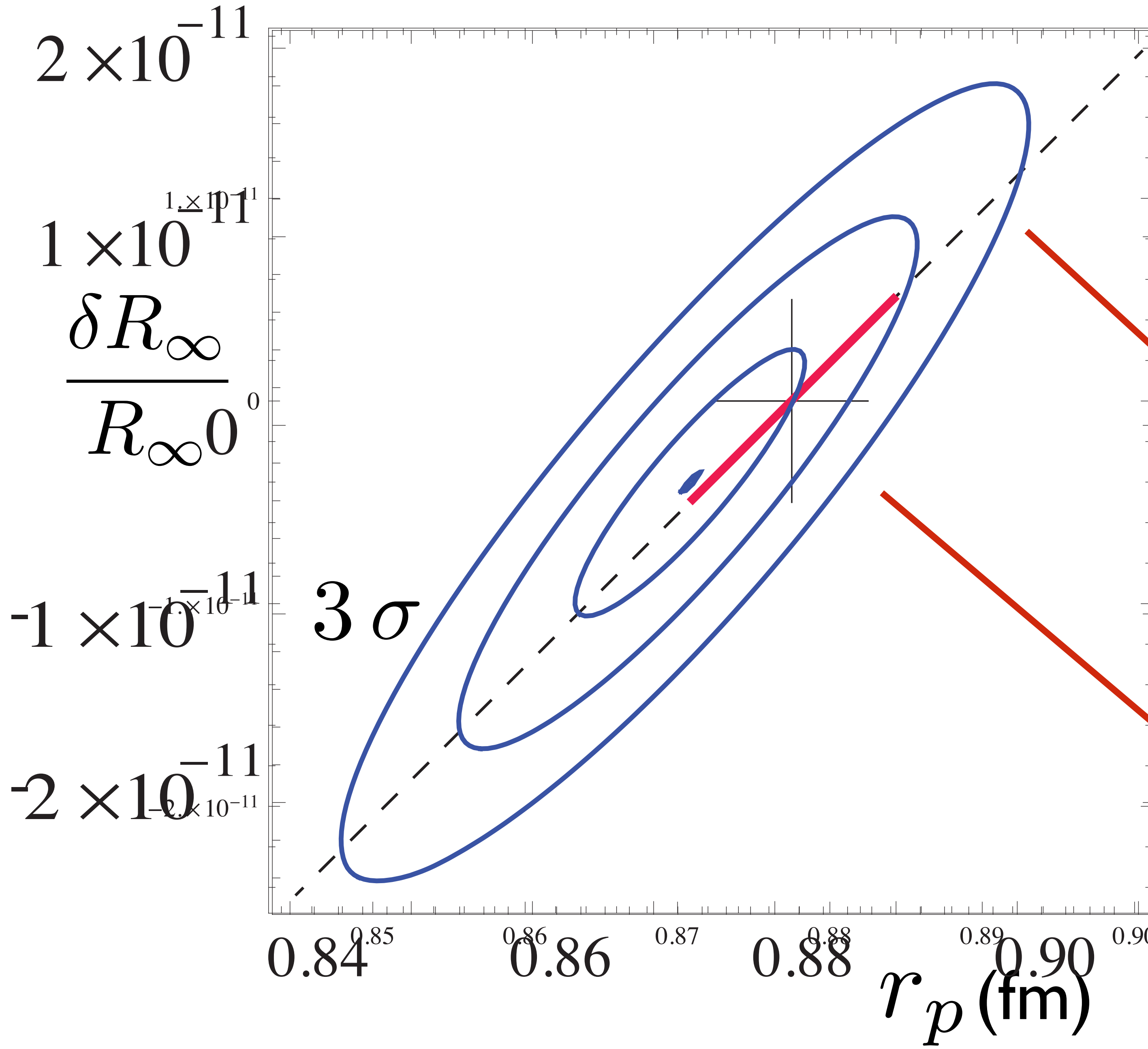
...and sometimes the best data point should be thrown out

Why fret?

It was just one point!

Regardless of the rest of the data
...or correctness of theory...
try to use what's reliable

ellipses:
no 1S2S



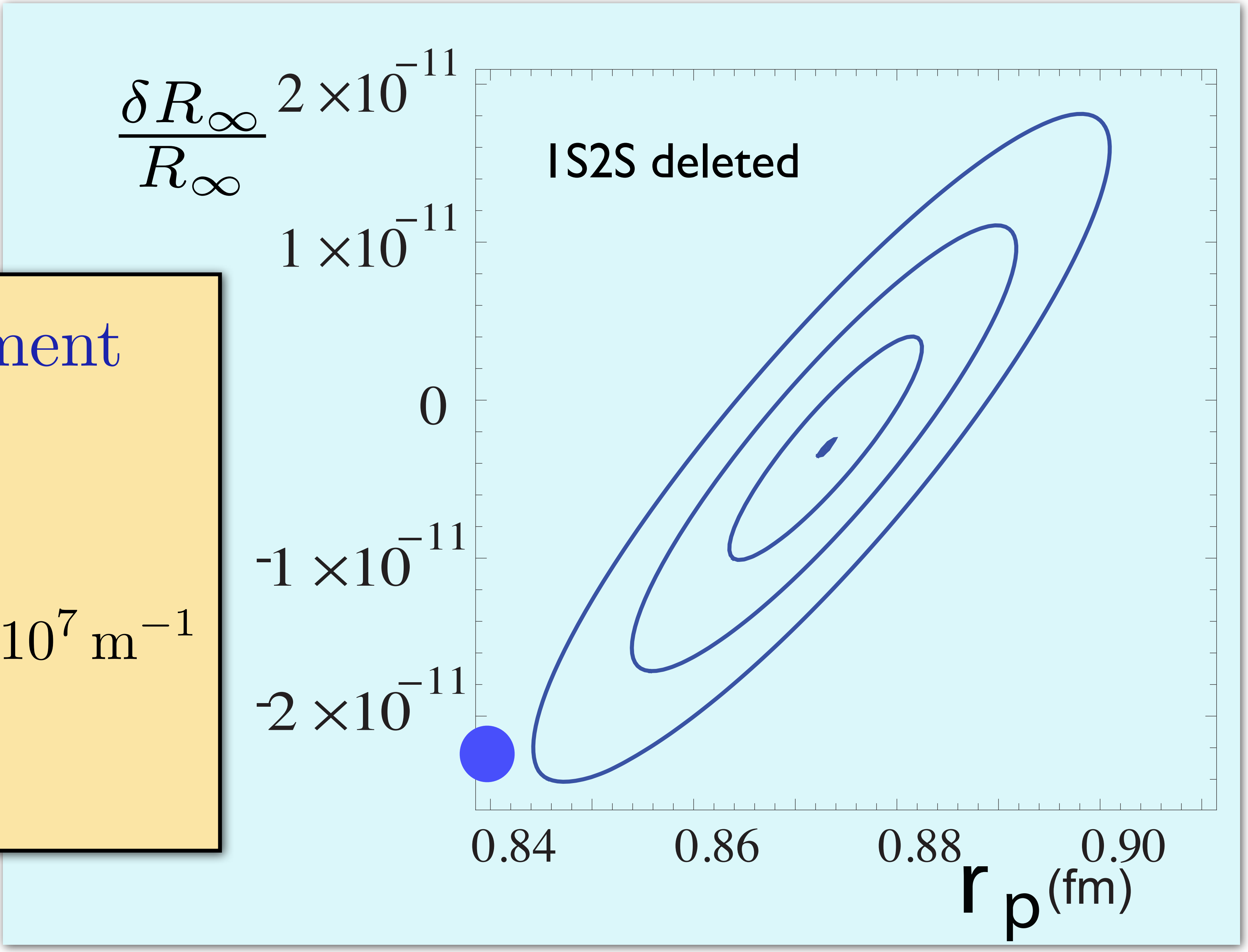
$(r_p,\ \delta R_\infty / R_\infty)$
will lie on this line
whenever
1S2S included

predicted
uncertainties

Thin bars: CODATA2010 (includes 1S2S) (reports no confidence region!)
Red Segment: our confidence region including 1S2S (thick for visibility)

We recommend assessing the proton size problem on the basis of:

- comparing protons to protons
- simple robust analysis
- minimal sensitivity to uncertain quantities



We find the disagreement is about $2.5\sigma - 3.5\sigma$

$$r_p = 0.87 \pm 0.01\,\mathrm{fm};$$

$$R_\infty = 1.097373156851 \times 10^7\,\mathrm{m}^{-1} \pm 8\times10^{-5}\,\mathrm{m}^{-1}$$

1S2S deleted


thanks!

why bother with muonic atom ?

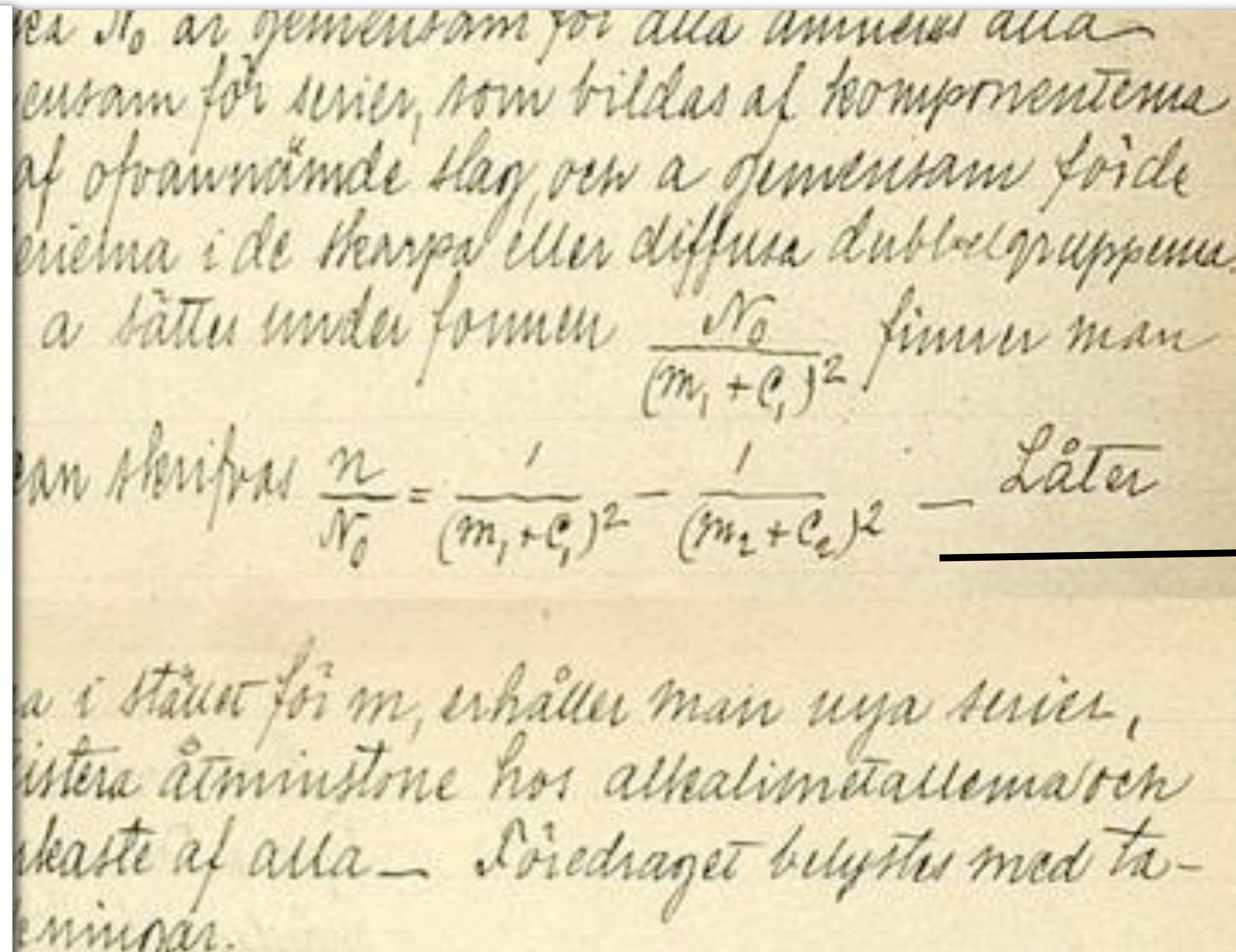
"to improve measurement of the Rydberg constant"

"13.6 eV".

$$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$$

finite size causes annoying uncertainty of R_∞

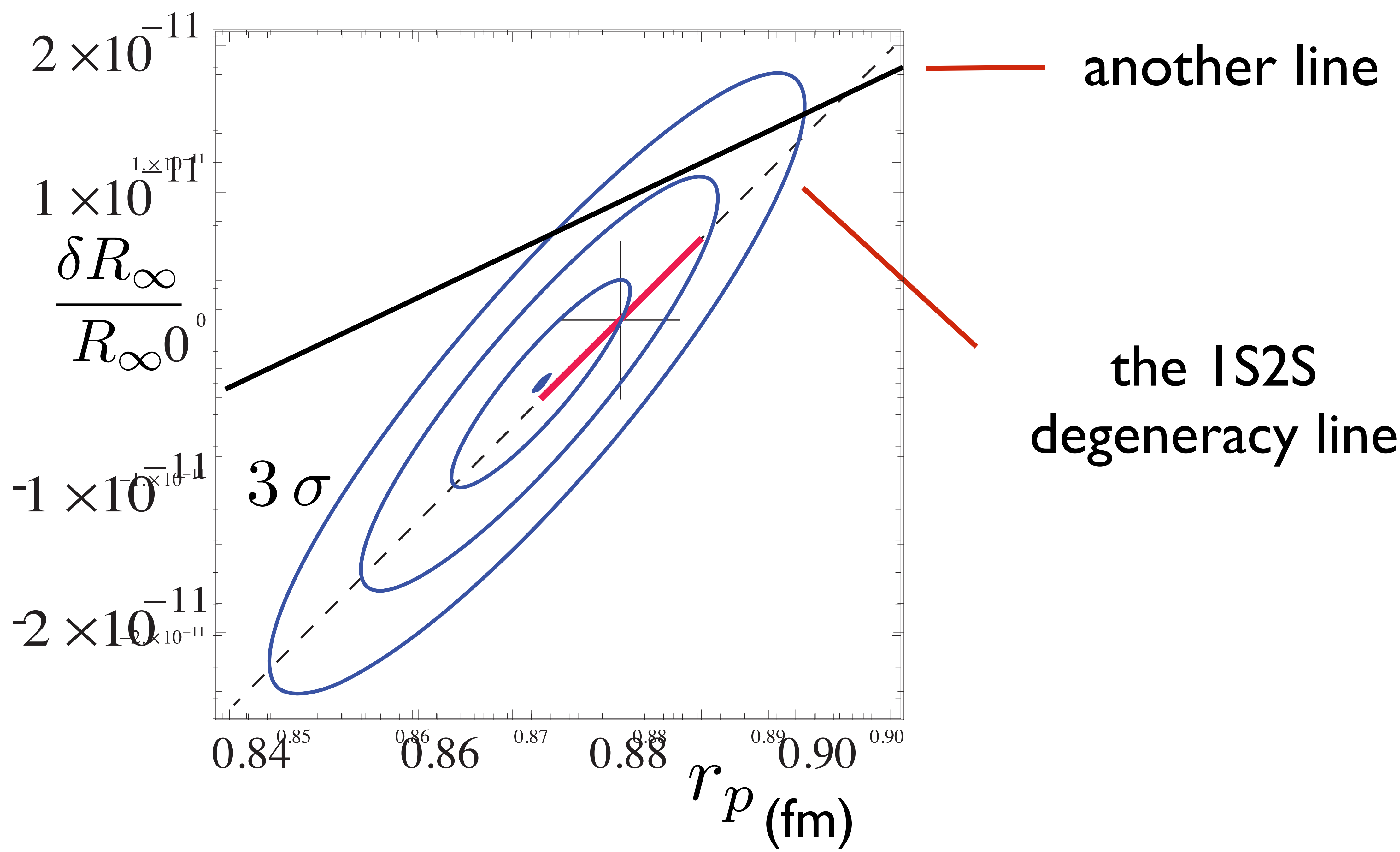
J. Rydberg



er N_0 är gemensam för alla ämnen och alla
ensam för serier, som bildas af komponenterna
af ofvannämnde slag och a gemensam för de
erierna i de skarpa eller diffusa dubbelgrupperna.
a sätter under formen $\frac{N_0}{(m_1+c_1)^2}$ finner man
an skrifvas $\frac{n}{N_0} = \frac{1}{(m_1+c_1)^2} - \frac{1}{(m_2+c_2)^2}$ — Låter
a i stället för m, erhåller man nya serier,
istera åtminstone hos alkalimetallerna och
kaste af alla — Förhållandet belyses med ta-
ningar.

"quantum defect" ignored by Bohr; re-appears in Dirac spectrum

Any transition might have dominated... they can't all be correct



the "other" line shown is barely within 2 -sigma

Proton size has previous been quantified relative to *world's smallest-ever* sigma

REVIEWS OF MODERN PHYSICS, VOLUME 84, OCTOBER–DECEMBER 2012

CODATA recommended values of the fundamental physical constants: 2010*

Peter J. Mohr,† Barry N. Taylor,‡ and David B. Newell§

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

(published 13 November 2012)

This paper gives the 2010 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. The 2010 adjustment takes into account the data considered in the 2006 adjustment as well as the data that became available from 1 January 2007, after the closing date of that adjustment, until 31 December 2010, the closing date of the new adjustment. Further, it describes in detail the adjustment of the values of the constants, including the selection of the final set of input data based on the results of least-squares analyses. The 2010 set replaces the previously recommended 2006 CODATA set and may also be found on the World Wide Web at physics.nist.gov/constants.

DOI: 10.1103/RevModPhys.84.1527 PACS numbers: 06.20.Jr, 12.20.−m

purpose is "to periodically provide the international scientific and technological communities with an internationally accepted set of values of the fundamental physical constants and closely related conversion factors for use worldwide."

REVIEWS OF MODERN PHYSICS, VOLUME 84, OCTOBER–DECEMBER 2012

CODATA recommended values of the fundamental physical constants: 2010*

Peter J. Mohr,[†] Barry N. Taylor,[‡] and David B. Newell[§]
National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

global fit to all constants

149 input data
82 parameters

sector most relevant
to proton radius:

25 experimental input data
28 adjustable constants

free parameters = # data+3

TABLE XXX. The 28 adjusted constants (variables) used in the least-squares multivariate analysis of the Rydberg-constant data given in Table XVIII. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table XXXI.

Adjusted constant	Symbol
Rydberg constant	R_∞
Bound-state proton rms charge radius	r_p
Bound-state deuteron rms charge radius	r_d
Additive correction to $E_H(1S_{1/2})/h$	$\delta_H(1S_{1/2})$
Additive correction to $E_H(2S_{1/2})/h$	$\delta_H(2S_{1/2})$
Additive correction to $E_H(3S_{1/2})/h$	$\delta_H(3S_{1/2})$
Additive correction to $E_H(4S_{1/2})/h$	$\delta_H(4S_{1/2})$
Additive correction to $E_H(6S_{1/2})/h$	$\delta_H(6S_{1/2})$
Additive correction to $E_H(8S_{1/2})/h$	$\delta_H(8S_{1/2})$
Additive correction to $E_H(2P_{1/2})/h$	$\delta_H(2P_{1/2})$
Additive correction to $E_H(4P_{1/2})/h$	$\delta_H(4P_{1/2})$
Additive correction to $E_H(2P_{3/2})/h$	$\delta_H(2P_{3/2})$
Additive correction to $E_H(4P_{3/2})/h$	$\delta_H(4P_{3/2})$
Additive correction to $E_H(8D_{3/2})/h$	$\delta_H(8D_{3/2})$
Additive correction to $E_H(12D_{3/2})/h$	$\delta_H(12D_{3/2})$
Additive correction to $E_H(4D_{5/2})/h$	$\delta_H(4D_{5/2})$
Additive correction to $E_H(6D_{5/2})/h$	$\delta_H(6D_{5/2})$
Additive correction to $E_H(8D_{5/2})/h$	$\delta_H(8D_{5/2})$
Additive correction to $E_H(12D_{5/2})/h$	$\delta_H(12D_{5/2})$
Additive correction to $E_D(1S_{1/2})/h$	$\delta_D(1S_{1/2})$
Additive correction to $E_D(2S_{1/2})/h$	$\delta_D(2S_{1/2})$
Additive correction to $E_D(4S_{1/2})/h$	$\delta_D(4S_{1/2})$
Additive correction to $E_D(8S_{1/2})/h$	$\delta_D(8S_{1/2})$
Additive correction to $E_D(8D_{3/2})/h$	$\delta_D(8D_{3/2})$
Additive correction to $E_D(12D_{3/2})/h$	$\delta_D(12D_{3/2})$
Additive correction to $E_D(4D_{5/2})/h$	$\delta_D(4D_{5/2})$
Additive correction to $E_D(8D_{5/2})/h$	$\delta_D(8D_{5/2})$
Additive correction to $E_D(12D_{5/2})/h$	$\delta_D(12D_{5/2})$

adjusted in fit

Table XVIII shows 50 ``principal input data for the determination of the 2010 recommended value of the Rydberg constant R_{∞}".

However 25 of the 50 are theory parameters treated as adjustable constants
That makes one "additive correction" per energy level

Actually, more than100 externally chosen parameters are introduced to fit three (3) physical constants

Suppose the muonic data and theory are correct

What size of electronic theory error is needed?

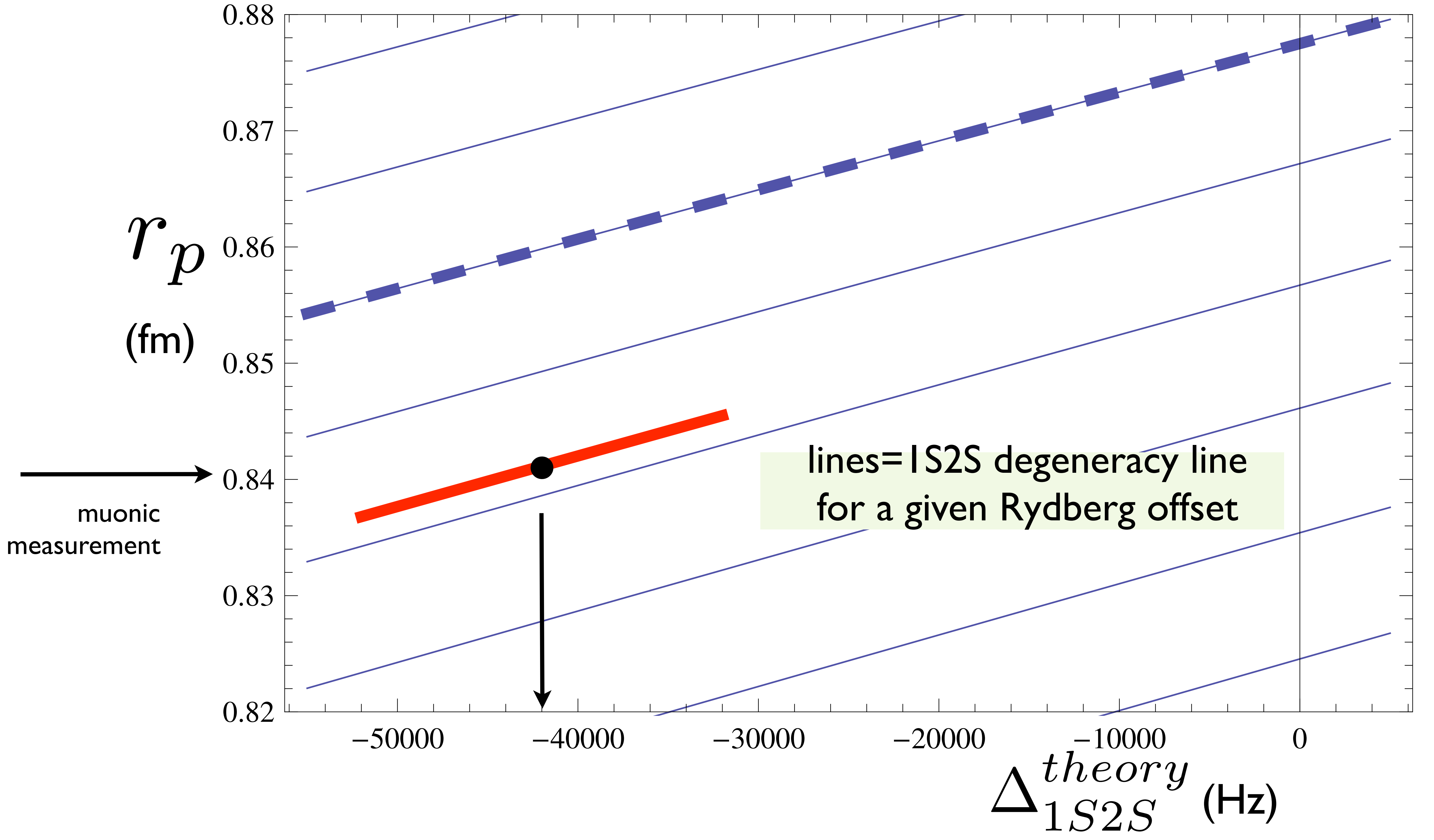


TABLE XVIII. Summary of principal input data for the determination of the 2010 recommended value of the Rydberg constant R_∞.

Item No.	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
$A1$	$\delta_H(1S_{1/2})$	0.0(2.5) kHz	$[7.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A2$	$\delta_H(2S_{1/2})$	0.00(31) kHz	$[3.8 \times 10^{-13}]$	Theory	IV.A.1.l
$A3$	$\delta_H(3S_{1/2})$	0.000(91) kHz	$[2.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A4$	$\delta_H(4S_{1/2})$	0.000(39) kHz	$[1.9 \times 10^{-13}]$	Theory	IV.A.1.l
$A5$	$\delta_H(6S_{1/2})$	0.000(15) kHz	$[1.6 \times 10^{-13}]$	Theory	IV.A.1.l
$A6$	$\delta_H(8S_{1/2})$	0.0000(63) kHz	$[1.2 \times 10^{-13}]$	Theory	IV.A.1.l
$A7$	$\delta_H(2P_{1/2})$	0.000(28) kHz	$[3.5 \times 10^{-14}]$	Theory	IV.A.1.l
$A8$	$\delta_H(4P_{1/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	Theory	IV.A.1.l
$A9$	$\delta_H(2P_{3/2})$	0.000(28) kHz	$[3.5 \times 10^{-14}]$	Theory	IV.A.1.l
$A10$	$\delta_H(4P_{3/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	Theory	IV.A.1.l
$A11$	$\delta_H(8D_{3/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A12$	$\delta_H(12D_{3/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A13$	$\delta_H(4D_{5/2})$	0.0000(35) kHz	$[1.7 \times 10^{-14}]$	Theory	IV.A.1.l
$A14$	$\delta_H(6D_{5/2})$	0.0000(10) kHz	$[1.1 \times 10^{-14}]$	Theory	IV.A.1.l
$A15$	$\delta_H(8D_{5/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A16$	$\delta_H(12D_{5/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A17$	$\delta_D(1S_{1/2})$	0.0(2.3) kHz	$[6.9 \times 10^{-13}]$	Theory	IV.A.1.l
$A18$	$\delta_D(2S_{1/2})$	0.00(29) kHz	$[3.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A19$	$\delta_D(4S_{1/2})$	0.000(36) kHz	$[1.7 \times 10^{-13}]$	Theory	IV.A.1.l
$A20$	$\delta_D(8S_{1/2})$	0.0000(60) kHz	$[1.2 \times 10^{-13}]$	Theory	IV.A.1.l
$A21$	$\delta_D(8D_{3/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A22$	$\delta_D(12D_{3/2})$	0.000 00(13) kHz	$[5.6 \times 10^{-15}]$	Theory	IV.A.1.l
$A23$	$\delta_D(4D_{5/2})$	0.0000(35) kHz	$[1.7 \times 10^{-14}]$	Theory	IV.A.1.l
$A24$	$\delta_D(8D_{5/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A25$	$\delta_D(12D_{5/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A26$	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.080(34) kHz	1.4×10^{-14}	MPQ-04	IV.A.2
$A27$	$\nu_H(1S_{1/2} - 3S_{1/2})$	2 922 743 278 678(13) kHz	4.4×10^{-12}	LKB-10	IV.A.2
$A28$	$\nu_H(2S_{1/2} - 8S_{1/2})$	770 649 350 012.0(8.6) kHz	1.1×10^{-11}	LK/SY-97	IV.A.2
$A29$	$\nu_H(2S_{1/2} - 8D_{3/2})$	770 649 504 450.0(8.3) kHz	1.1×10^{-11}	LK/SY-97	IV.A.2
$A30$	$\nu_H(2S_{1/2} - 8D_{5/2})$	770 649 561 584.2(6.4) kHz	8.3×10^{-12}	LK/SY-97	IV.A.2
$A31$	$\nu_H(2S_{1/2} - 12D_{3/2})$	799 191 710 472.7(9.4) kHz	1.2×10^{-11}	LK/SY-98	IV.A.2
$A32$	$\nu_H(2S_{1/2} - 12D_{5/2})$	799 191 727 403.7(7.0) kHz	8.7×10^{-12}	LK/SY-98	IV.A.2

One astonishing QED prediction now explained

Jentschura, Kotochigova, LeBigot, Mohr, Taylor

PRL **95,** 163003 (2005) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2005

TABLE I. Transition frequencies in hydrogen ν_H and in deuterium ν_D used in the 2002 CODATA least-squares adjustment of the values of the fundamental constants and the calculated values. Hyperfine effects are not included in these values.

Experiment	Frequency interval(s)	Reported value ν/kHz	Calculated value ν/kHz
Niering *et al.* [1]	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.103(46)	2 466 061 413 187.103(46)
Weitz *et al.* [2]	$\nu_H(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	4 797 338(10)	4 797 331.8(2.0)
	$\nu_H(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	6 490 144(24)	6 490 129.9(1.7)
	$\nu_D(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	4 801 693(20)	4 801 710.2(2.0)
	$\nu_D(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	6 494 841(41)	6 494 831.5(1.7)

$$\sigma_{theory} << \sigma_{expt}$$

1S2S *exact agreement* experiment v calculated

`` the values of the constants... are correlated, particularly those for R_{∞} and r_{p}... The uncertainty of the calculated value for the $1s-2s$ frequency in hydrogen is increased by a factor of about 500 if such correlations are neglected." Jentschura, Kotochigova, LeBigot, Mohr, Taylor

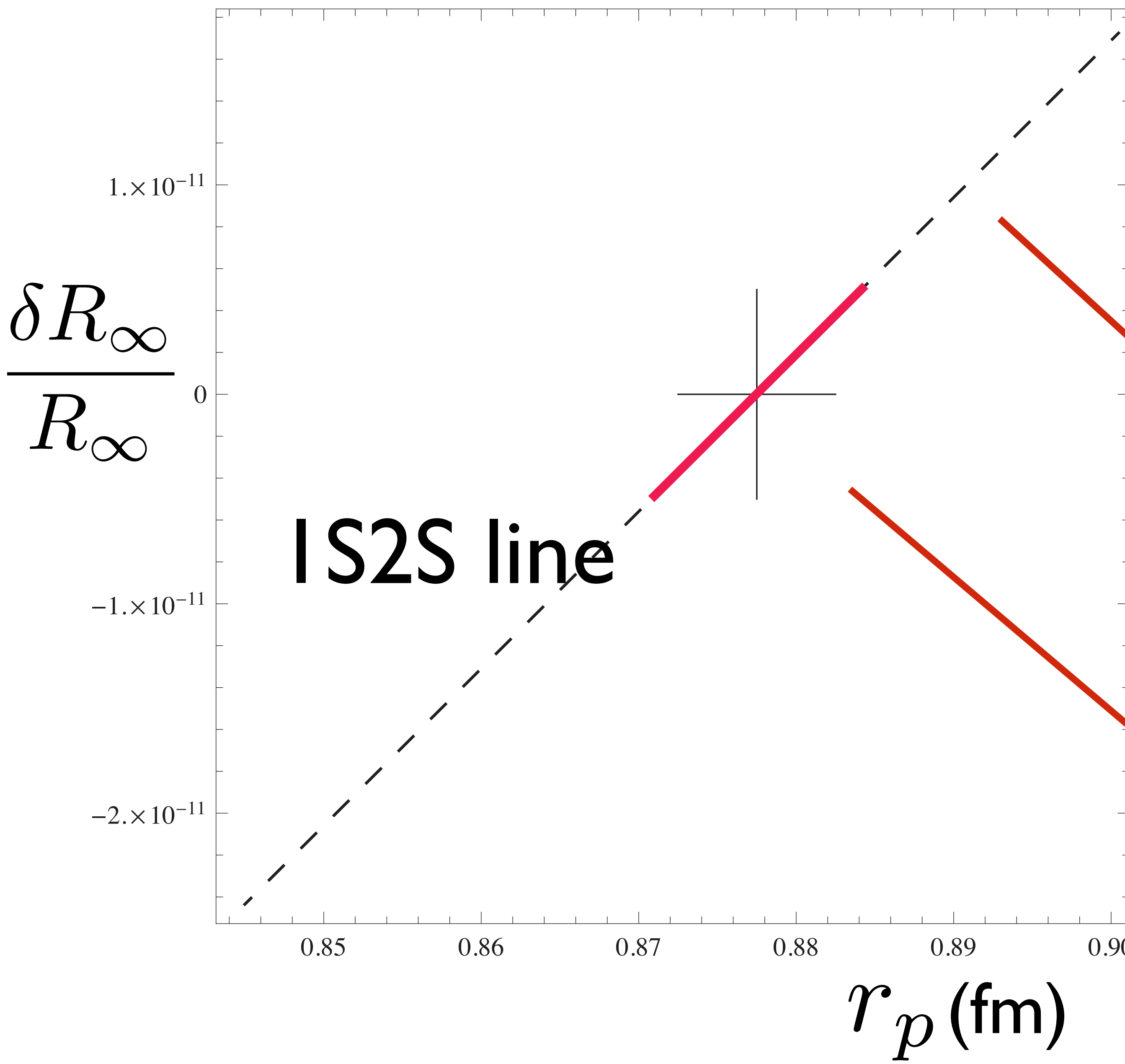
Okay. 500 x 46 Hz = 23000 Hz theory uncertainty

"However, one thing can be stated with certainty: the exact agreement of those two ultra-precise 1S2S measurements with the QED calculations cannot be considered as a confirmation of the QED theory, because it is the result of the fitting of the fundamental constants based on these (and other) transitions." A. Kramida, Atomic Data and Nuclear Data Tables, 96, 586 (2010)

TABLE XXX. The 28 adjusted constants (variables) used in the least-squares multivariate analysis of the Rydberg-constant data given in Table XVIII. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table XXXI.

Adjusted constant	Symbol
Rydberg constant	R_∞
Bound-state proton rms charge radius	r_p
Bound-state deuteron rms charge radius	r_d
Additive correction to $E_\mathrm{H}(1\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(1\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(2\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(2\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(3\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(3\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(4\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(4\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(6\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(6\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(8\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(8\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{H}(2\mathrm{P}_{1/2})/h$	$\delta_\mathrm{H}(2\mathrm{P}_{1/2})$
Additive correction to $E_\mathrm{H}(4\mathrm{P}_{1/2})/h$	$\delta_\mathrm{H}(4\mathrm{P}_{1/2})$
Additive correction to $E_\mathrm{H}(2\mathrm{P}_{3/2})/h$	$\delta_\mathrm{H}(2\mathrm{P}_{3/2})$
Additive correction to $E_\mathrm{H}(4\mathrm{P}_{3/2})/h$	$\delta_\mathrm{H}(4\mathrm{P}_{3/2})$
Additive correction to $E_\mathrm{H}(8\mathrm{D}_{3/2})/h$	$\delta_\mathrm{H}(8\mathrm{D}_{3/2})$
Additive correction to $E_\mathrm{H}(12\mathrm{D}_{3/2})/h$	$\delta_\mathrm{H}(12\mathrm{D}_{3/2})$
Additive correction to $E_\mathrm{H}(4\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(4\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{H}(6\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(6\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{H}(8\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(8\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{H}(12\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(12\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{D}(1\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(1\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{D}(2\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(2\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{D}(4\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(4\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{D}(8\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(8\mathrm{S}_{1/2})$
Additive correction to $E_\mathrm{D}(8\mathrm{D}_{3/2})/h$	$\delta_\mathrm{D}(8\mathrm{D}_{3/2})$
Additive correction to $E_\mathrm{D}(12\mathrm{D}_{3/2})/h$	$\delta_\mathrm{D}(12\mathrm{D}_{3/2})$
Additive correction to $E_\mathrm{D}(4\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(4\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{D}(8\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(8\mathrm{D}_{5/2})$
Additive correction to $E_\mathrm{D}(12\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(12\mathrm{D}_{5/2})$

Regardless of the rest of the data
...or correctness of theory...
the 800 pound datum-gorilla gets his way



"one-point fit"

$(r_p,\, \delta R_\infty / R_\infty)$

will lie on this line
whenever
1S2S included
no matter what else

predicted
uncertainties