Nuclear medium effect on the η decay into 3π: isospin asymmetry v.s. partial restoration of chiral symmetry

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based on S. S. and T. Kunihiro,
Prog. Theor. Exp. Phys., 2015, 013D03.

Contents

- Introduction
  - $\eta \to 3\pi$ decay in free space
- The $\eta \to 3\pi$ decay in nuclear medium with chiral effective field theory
  - η-π^0 mixing angle
  - $\eta \to 3\pi$ decay width

  in nuclear medium
- Summary and future prospects

The $\eta \rightarrow 3\pi(\pi^+\pi^-\pi^0, 3\pi^0)$ decay

- G parity violating process
  - width with one-hundred eV order ← $\Gamma(\eta \rightarrow 2\gamma)$~300eV
- No contribution from the leading order of QED D.G.Sutherland,NPB4(1968)315.

→ The QCD originated isospin-symmetry breaking
(u and d quark mass difference) is essential.

- Current algebra does not work well. J.S. Bell and D.G. Sutherland, NPB4(1968)315.
- The significant final-state-interaction (FSI) effect ↔ the $\pi\pi$ scalar channel
  - Approach with the phenomenological way
    C.Roiesnel, T.N.Truong,NPB187(1981)293.
  - Approach with the perturbative way
    J.Gasser,H.Leutwyler,NPB250(1985)465
    J.Bijnens,K.Ghorbani,JHEP11(2007)030.

⇒ Quantitative understanding of $\eta \rightarrow 3\pi$

How is the $\eta \rightarrow 3\pi$ width modified by nuclear medium?

Isospin symmetry is broken explicitly

$\rho=\rho_n+\rho_p$ and $\delta\rho=\rho_n-\rho_p\neq 0$ in general

Purpose of our work

Investigating the effect of nuclear medium on $\eta \to 3\pi$ decay

■ Method: Chiral effective field theory in nuclear medium

U.G.Meissner, et al.,Ann.Phys.297(2002)27,N.Kaiser, et al.NPA697(2002)255.

- The Fermi momentum of the nuclear medium k_f is treated perturbatively .

  ($k_f \sim 2m_\pi$ @$\rho=\rho_0$)

  - Fermi momentum, spatial momenta of hadrons, NG boson masses are small.
- We include the meson and nucleon one loop diagrams.

  - The nuclear medium is a free Fermi gas at one-loop order ($O(k_f^3)$).

  - The contribution from meson 1 loop containing a part of the FSI is included.

J.Gasser,H.Leutwyler,NPB250(1985)465.

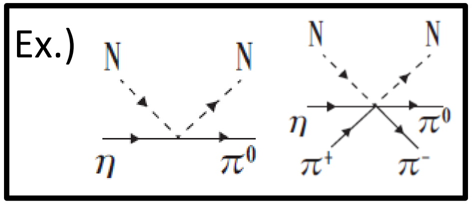
Chiral EFT in nuclear medium

$$\mathcal{L} = \boxed{\mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi}} + \boxed{\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}}$$

Responsible for the nuclear medium effect

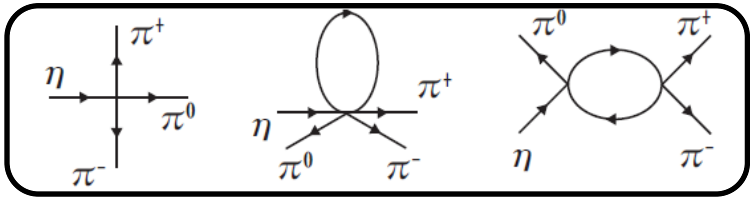
Contribute from the free-space part (containing the effect of FSI)

➡ The meson-baryon vertices are obtained.

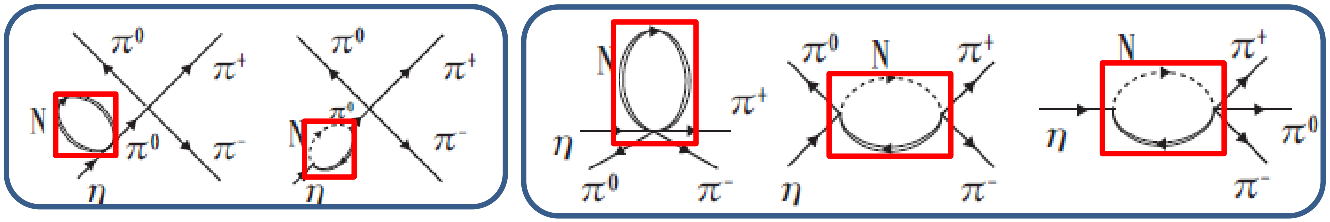


Diagrams

Density-independent part (containing FSI effect) J.Gasser,H.Leutwyler,NPB250(1985)465.



The diagrams providing the medium effect



Modification of η-π^0 mixing angle

Modification of vertex

※Nuclear medium effect comes from the nucleon loops through Pauli blocking

η-π^0 mixing angle in nuclear medium

- ✓ η-π^0 mixing angle

$$\tan 2\theta = -\frac{2m^2_{\eta_8\pi_3}}{m^2_{\eta_8} - m^2_{\pi_3}}$$

$$D^{-1}(p;k_f^{(p,n)}) = \begin{pmatrix} D^{(0)}_{\eta_8}(p) - \Pi_{\eta_8}(k_f^{(p,n)}) & -\Pi_{\eta_8\pi_3}(k_f^{(p,n)}) \\ -\Pi_{\eta_8\pi_3}(k_f^{(p,n)}) & D^{(0)}_{\pi_3}(p) - \Pi_{\pi_3}(k_f^{(p,n)}) \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \pi^0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \pi_3 \end{pmatrix}$$

$m^2_{\eta_8\pi_3}(\rho)$

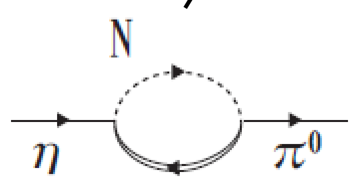
Result of calculation

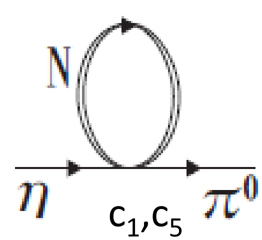
$$\tan 2\theta^{(\rho)} = \frac{2}{m^2_\eta - m^2_{\pi^0}} \left(\frac{m_1^2}{\sqrt{3}} + \left(\frac{g_A^2 m_\eta^2}{4\sqrt{3} f^2 m_N} + \frac{2c_5 m_\pi^2}{\sqrt{3} f^2} \right) \delta\rho + \frac{4c_1 m_1^2}{\sqrt{3} f^2}\rho \right)$$

※η and π^0 masses are free-space one assuming the medium modification of the nuclear medium is small.

Contribution from free space

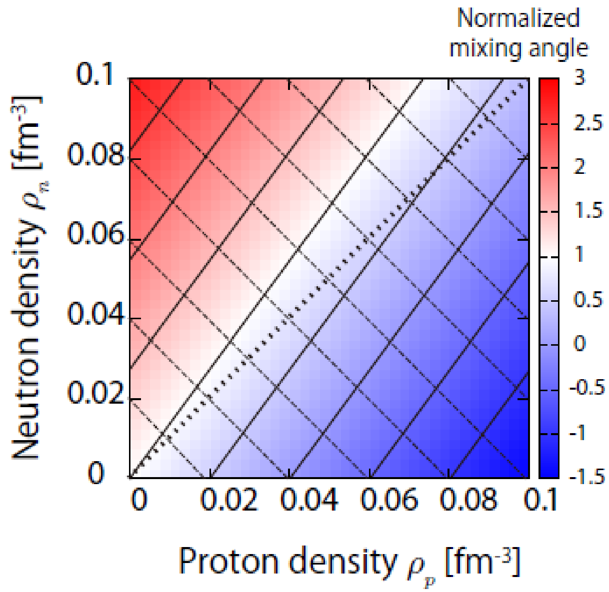
$m_1^2 \propto m_u - m_d$
Reflect the isospin symmetry breaking

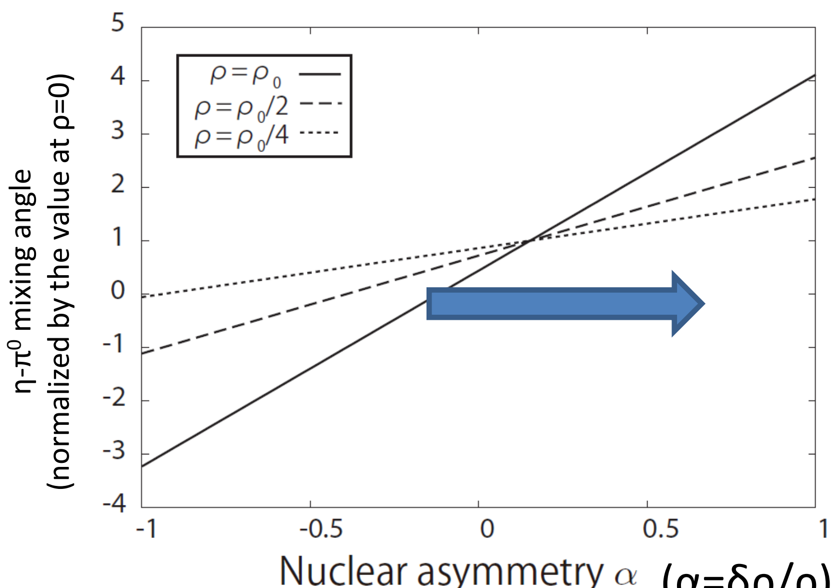




(Both ρ and δρ affect the η-π^0 mixing angle)

■ η-π⁰ mixing angle





(α=δρ/ρ)

※Large α=large asymmetry

Mixing angle is enhanced by large α.

(~3 times larger @$\rho=\rho_0$)

The consistent result with the expectation.

δρ breaks the isospin symmetry breaking.
→Enhancement of the isospin breaking by α.

※Slope of the η-π⁰ mixing angle with fixed ρ

$$\frac{d\theta}{d\alpha} \sim \frac{\rho/\sqrt{3}f^2}{m_\eta^2 - m_{\pi^0}^2}\left(\frac{g_A^2 m_\eta^2}{4m_N} + 2c_5 m_\pi^2\right)$$

η→π+π-π0 decay width in nuclear medium

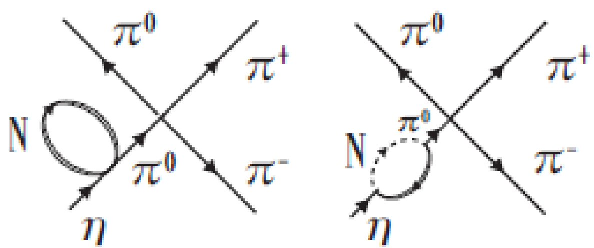
Result of calculation

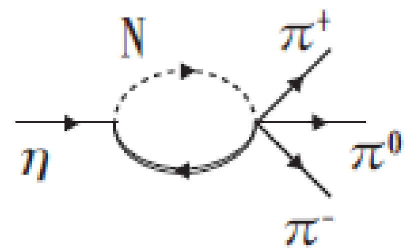
$$\mathcal{M}_{\eta\to\pi^0\pi^+\pi^-} = -\frac{m_1^2}{3\sqrt{3}f^2}\left(1+\frac{3(s-s_0)}{m_\eta^2-m_{\pi^0}^2}\right)+\sin\theta^{(0)}\mathcal{M}^{(4)\mathrm{vac}}_{\eta\to\pi^0\pi^+\pi^-}$$

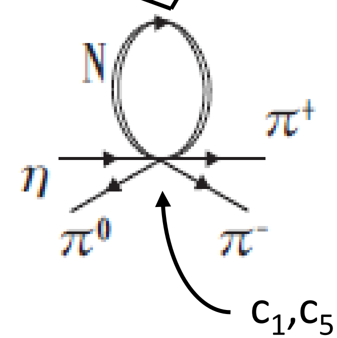
$$+\left\{-\frac{\delta\rho}{f^2}\frac{s-s_0}{m_\eta^2-m_{\pi^0}^2}\left(\frac{g_A^2 m_\eta^2}{4\sqrt{3}f^2}+\frac{2c_5 m_\pi^2}{\sqrt{3}f^2}\right)\right\}-\frac{4c_1{}^2_1}{3\sqrt{3}f^4}\frac{3(s-s_0)}{m_\eta^2-m_{\pi^0}^2}$$

$$+\left(\frac{g_A^2}{48\sqrt{3}f^4}(m_\eta-E_{\pi^0})-\frac{2c_5 m_\pi^2}{3\sqrt{3}f^4}\right)\delta\rho-\frac{4c_1\rho}{f^2}\frac{m_1^2}{3\sqrt{3}f^2}$$

From η-π^0 mixing

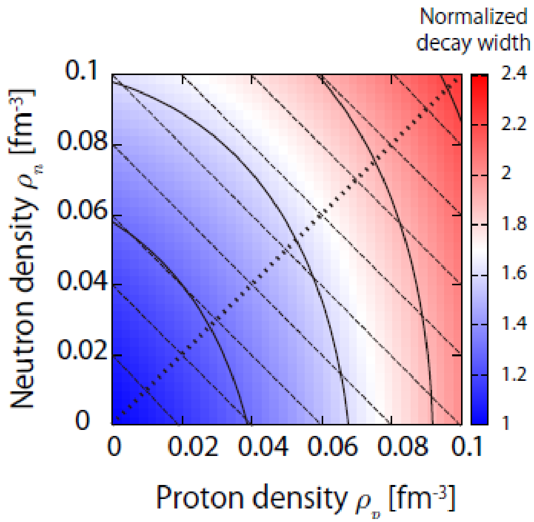


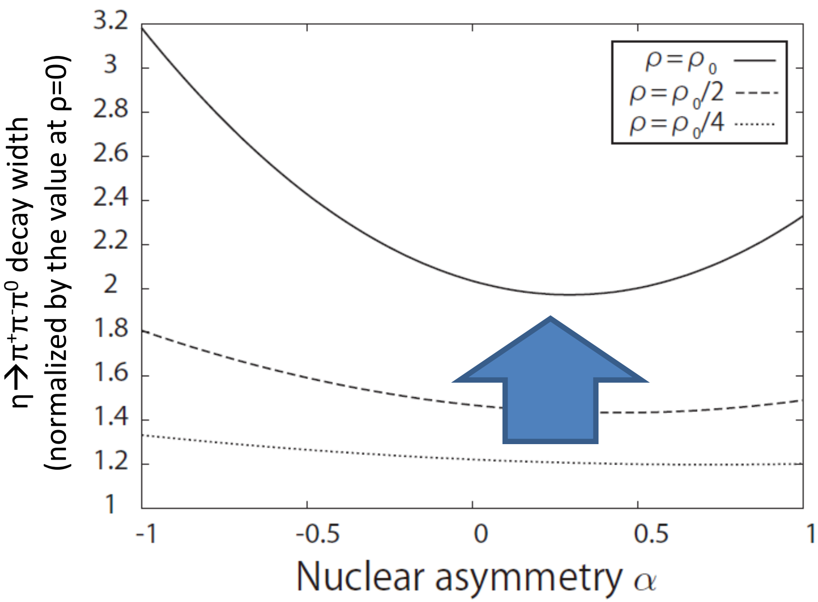




($s_0=m_\eta^2+3m_\pi^2$)

■ $\eta \rightarrow \pi^+\pi^-\pi^0$ decay width $\left(\Gamma \propto \int d\Pi \, |\mathcal{M}|^2\right)$





- ✓ $\Gamma_{\eta\rightarrow\pi+\pi-\pi0}$~160eV@ρ=0
- ✓ Error from LECs is about 10%

- $\Gamma_{\eta\rightarrow\pi+\pi-\pi0}$ is enhanced by large |α|
- Large enhancement of decay width by ρ

2-3 times larger @$\rho=\rho_0$

Effect of δρ is natural.
(same reason as η-π^0 mixing angle)

What is the origin of the enhancement by ρ?

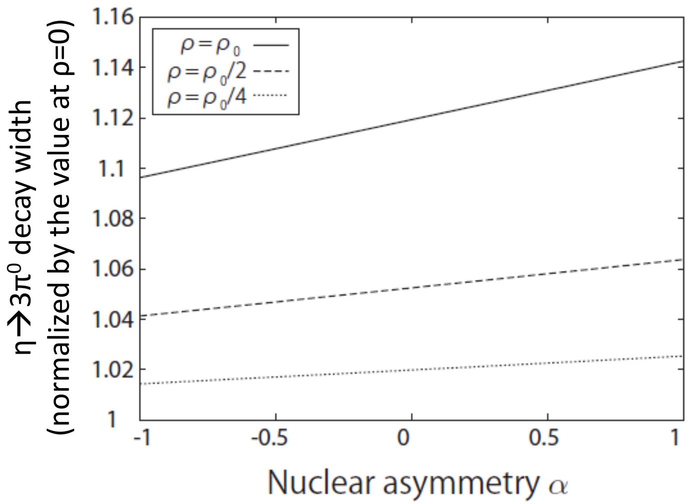
→The further analysis should be done!

η→3π⁰ decay width in nuclear medium

Result of calculation

$$\mathcal{M}_{\eta\to 3\pi^0} = -\frac{m_1^2}{\sqrt{3}f^2} + \mathcal{M}^{(4)\mathrm{vac}}_{\eta\to 3\pi^0} - \frac{2m_1^2 c_1}{3\sqrt{3}f^4}\rho - \frac{c_5 m_\pi^2}{3\sqrt{3}}\delta\rho$$

※The dependence on the mixing angle is cancelled out
by the symmetrization of the identical final state particles at this order.



- ✓ $\Gamma_{\eta\to 3\pi 0}$~300eV@ρ=0
- ✓ Error from LECs is about 10%

- $\Gamma_{\eta\to 3\pi 0}$ is enhanced by the large α.
- The enhancement is larger with larger ρ
- The α dependence is moderate
  compared with the $\eta\to\pi^+\pi^-\pi^0$ case (~×1.1@$\rho=\rho_0$)

Summary

We investigated the nuclear medium effect on the η-π^0 mixing angle and the $\eta \rightarrow 3\pi$ width

- The η-π^0 mixing angle is enhanced by $\delta\rho$. (~3 times larger @$\rho=\rho_0$)
- The total baryon density ρ and asymmetric density $\delta\rho$ enhance the $\eta \rightarrow \pi^+\pi^-\pi^0$ decay width (2-3 times larger @$\rho=\rho_0$)
- The enhancement of the $\eta \rightarrow 3\pi^0$ decay width is relatively small compared with the $\eta \rightarrow \pi^+\pi^-\pi^0$ case (~10% @$\rho=\rho_0$)

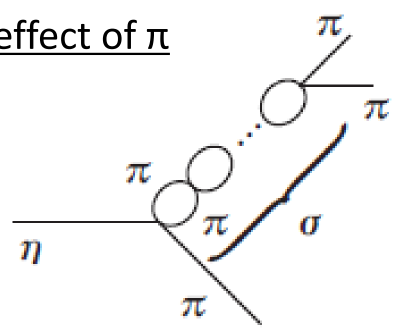
Future prospects

- Contribution from higher order should be estimated.
- The resonances (Δ(in πN), N*(in ηN)) may affect this process.
- Mass reduction of η' with finite ρ→change of the mixing property of η-η'.

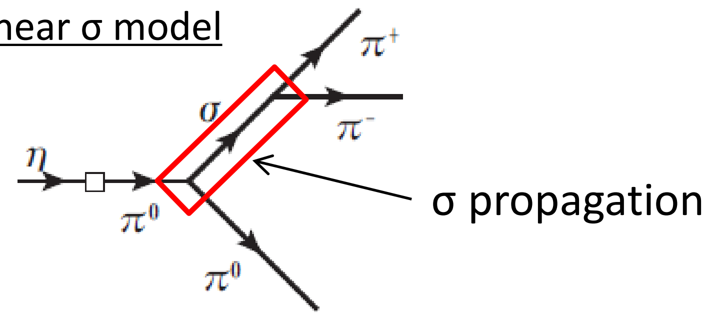
K.Saito,K.Tsushima,A.W.Thomas,Prog.Part.Nucl.Phys.,58(2007)1,
H.Nagahiro, M.Takiazawa, S.Hirenzaki, PRC74(2006)045203.

- Analysis with explicit σ degree of freedom may be interesting.
  (Analysis with linear sigma model is in progress.)

FSI effect of π



Linear σ model



※Relevance of the FSI of π in scalar channel and σ mode

- The effect of the electro-magnetic field
  as an isospin symmetry breaking source in the η→3π decay process
  - Additional mixing with vector meson (ω)

Thank you for your attention!