Twist-3 Spin Asymmetries for Inclusive Single-Hadron Production in Lepton-Nucleon Scattering

(A. Metz, Temple University, Philadelphia)

- Introduction and Motivation
- A related observable: double-spin asymmetry A_{LT} for $\vec{\ell}\, N^{\uparrow} \to \ell\, X$
- Single-spin asymmetry A_{UT} (A_N) for $\ell\, N^{\uparrow} \to h\, X$
- Single-spin asymmetry A_{UT} (A_N) for $\ell\, N \to \Lambda^{\uparrow}\, X$
- Double-spin asymmetry A_{LT} for $\vec{\ell}\, N^{\uparrow} \to h\, X$
- Summary

talk mainly based on

arXiv:1407.5078, Gamberg, Kang, A.M., Pitonyak, Prokudin

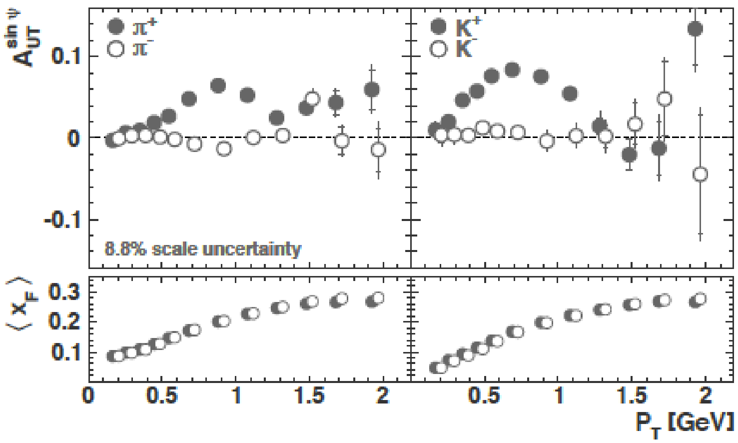
arXiv:1411.6459, Kanazawa, A.M., Pitonyak, Schlegel

arXiv:1503.02003, Kanazawa, A.M., Pitonyak, Schlegel

Introduction and Motivation

1. Data exist for twist-3 spin asymmetries in $\ell\, N \to h\, X$
   - example: A_N for $\ell\, N^{\uparrow} \to h\, X$ (HERMES, 2013 / JLab Hall A, 2013)

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \qquad\qquad x_F = \frac{2P_{hL}}{\sqrt{s}}$$

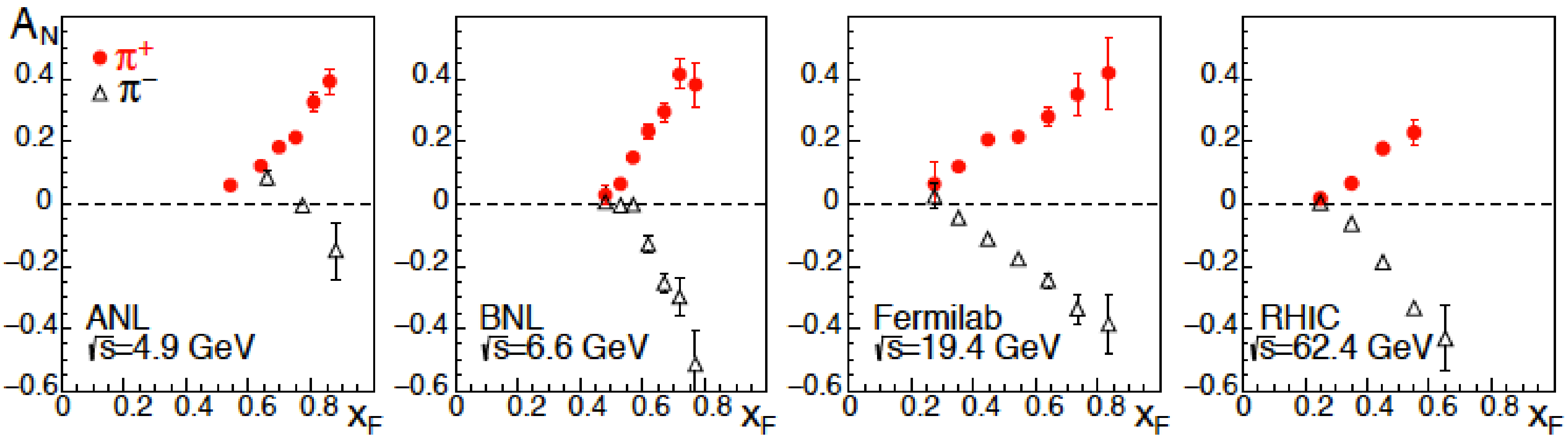


(HERMES, 2013)

   - more data: A_N for $\ell\, N \to \Lambda^{\uparrow}\, X$ (HERMES, 2014)
     A_{LT} for $\vec{\ell}\, N^{\uparrow} \to h\, X$ (JLab Hall A, 2015)
   - can one understand those data, and what can one learn from them ?

2. Related asymmetries in processes like $p\,p \rightarrow h\,X$

- example: A_N for $p\,p^{\uparrow} \rightarrow \pi\,X$ ($\rightarrow$ talks by Pitonyak, ...)



(plot from Aidala, Bass, Hasch, Mallot, 2012)

- data exist for A_N in $p\,p \rightarrow \Lambda^{\uparrow}\,X$ (Bunce et al, 1976 / ...)

- calculation available for A_{LT} for $\vec{p}\,p^{\uparrow} \rightarrow (h, \gamma, \mathrm{jet})\,X$
  Liang, A.M., Pitonyak, Schäfer, Song, Zhou, 2012

- it is challenging to reveal the underlying physics of the available data

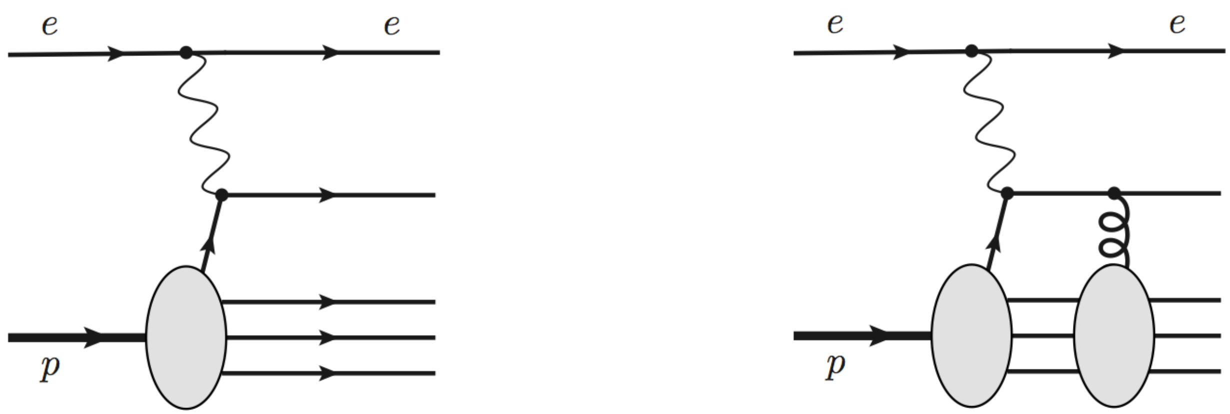
- maybe the asymmetries for $\ell\,N$ help to understand the asymmetries for $p\,p$

3. Playground to solidify and streamline theory tools
   (due to small number of Feynman graphs)
   - gauge-invariance of calculation
   - frame-independence of results
   - higher order corrections

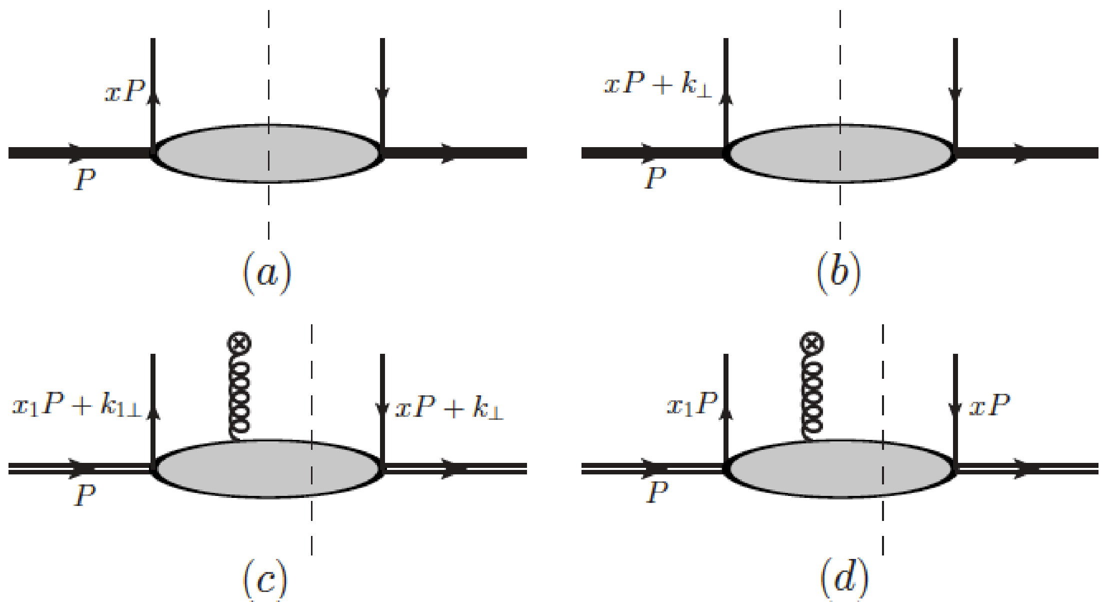
4. Explore potential of those observables for future measurements
   - EIC should be ideal for future studies
     $\longrightarrow$ measurements possible for large transverse momenta $P_{h\perp}$

Reminder: double-spin asymmetry A_{LT} for $\vec{\ell}\, N^{\uparrow} \;\to \ell\, X$

- Re-scattering of struck quark matters at twist-3 (gluon with physical polarization)



- Contributing correlators after factorization



- collinear quark-quark correlator at twist-3 $\longrightarrow$ $g_T(x)$
- $k_\perp$-dependent quark-quark correlator $\longrightarrow$ $\tilde{g}(x) = \int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^2}{2M^2} \, g_{1T}(x, \vec{k}_\perp^2)$
- (collinear) quark-gluon-quark correlator $\longrightarrow$ $F_{FT}(x, x_1) \qquad G_{FT}(x, x_1)$

- Exploit relations between functions
  - relation due to QCD equation of motion

$$x \, g_T(x) = \int dx_1 \Big[G_{DT}(x, x_1) - F_{DT}(x, x_1) \Big]$$

- Final result

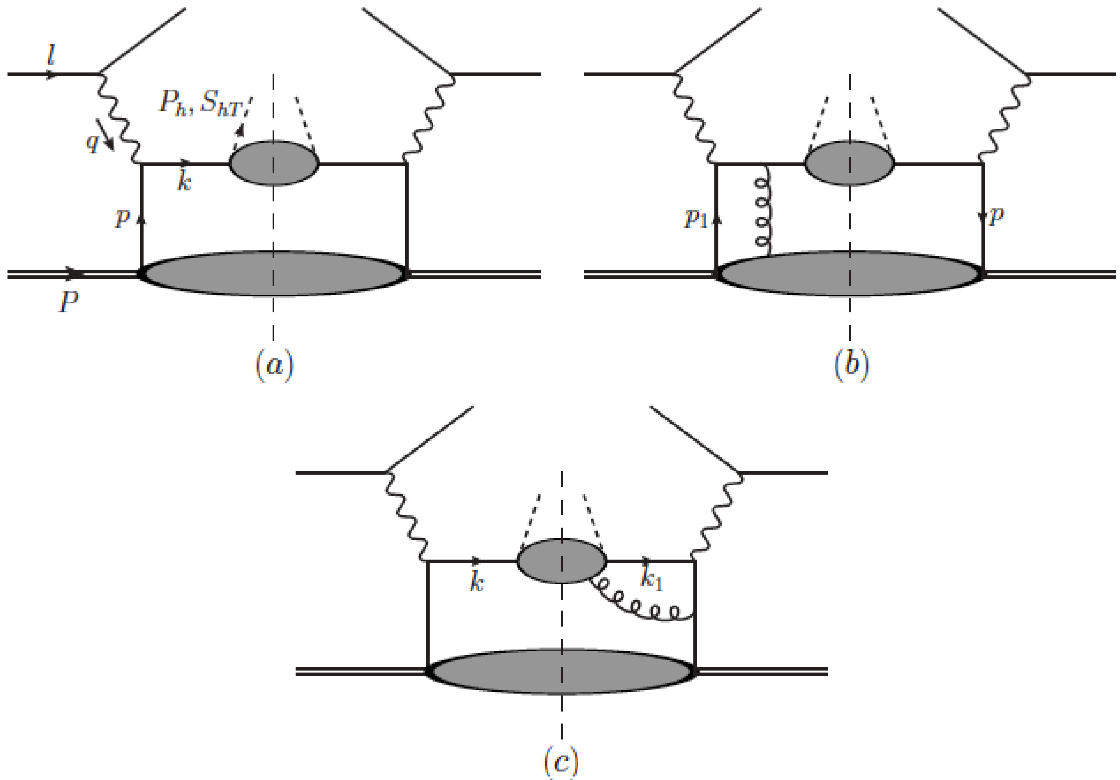
$$\frac{l'^0 d\sigma_{LT}}{d^3\vec{l}'} = -\frac{8 \, \alpha_{em}^2 \, x_B^2 \, \sqrt{1-y} \, M}{Q^5} \, \lambda_\ell \, |\vec{S}_\perp| \cos\phi_S \, \sum_q e_q^2 \, g_T^q(x_B)$$

  - twist-3 effect
  - final result looks rather simple
  - comparable twist-3 observables may have more complicated structure

Single-spin asymmetry A_N for $\ell\, N^{\uparrow} \;\to h\, X$

(Gamberg, Kang, A.M., Pitonyak, Prokudin, 2014)

- Feynman diagrams for LO calculation



  - twist-3 effects associated with nucleon, and with fragmentation process
  - large scale for pQCD calculations provided by $P_{h\perp}$
  - in LO formalism, hadron recoils against (undetected) final state lepton $\to$ Q^2 large

- Analytical result

$$
\begin{aligned}
P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu} \sum_q e_q^2 \int_{z_{\rm min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x} \\
\times\Bigg\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right] \\
+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\Bigg\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right] \\
+\frac{1}{z}H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\Bigg\}\Bigg\}
\end{aligned}
$$

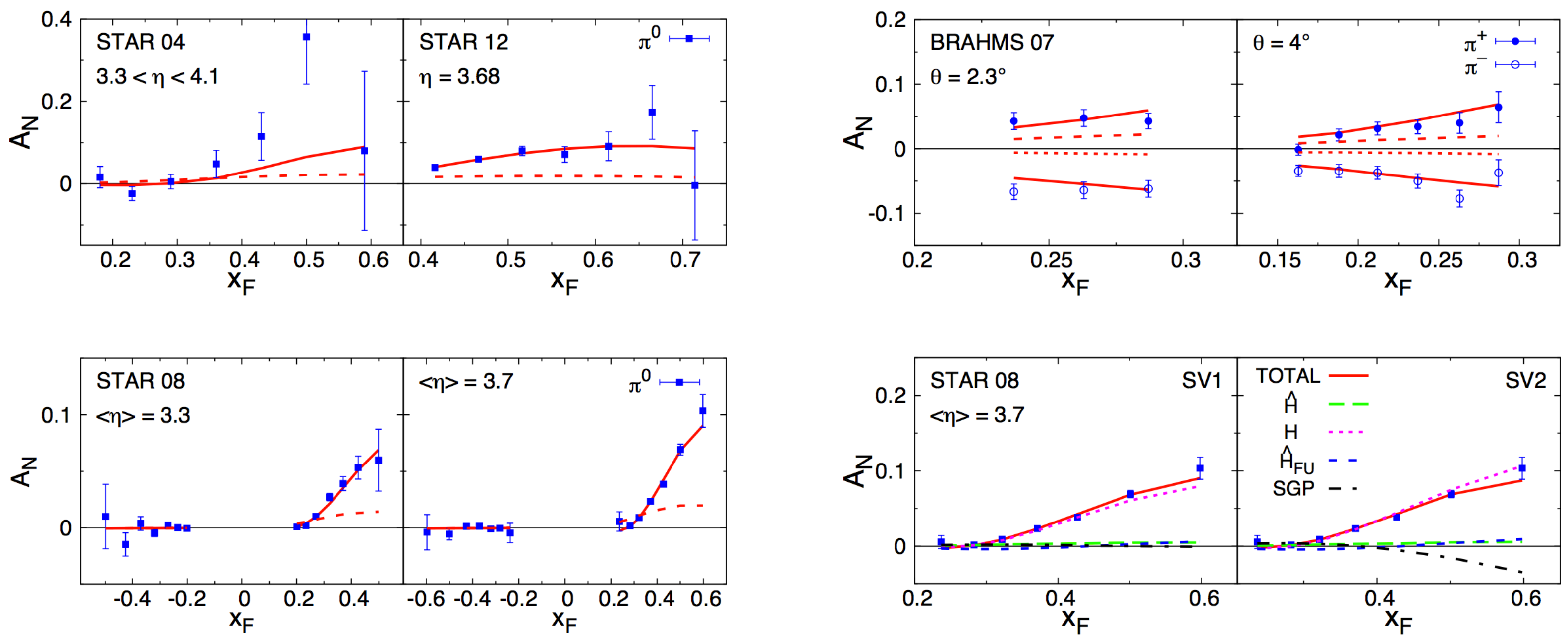
- Ingredients for numerics
  - twist-2 FF D_1 (de Florian, Sassot, Stratmann, 2007)
  - Qiu-Sterman function F_{FT}, from Sivers function $f_{1T}^{\perp}$ (Anselmino et al, 2008)
  - transversity h_1 (Anselmino et al, 2013)
  - twist-3 FF $\hat{H}$, from Collins function $H_1^{\perp}$ (Anselmino et al, 2013)

– twist-3 FF H and $\hat{H}_{FU}^{\Im}$ enter A_N for $p\,p^{\uparrow} \to h\,X$ (A.M., Pitonyak, 2012)

use model-independent relation

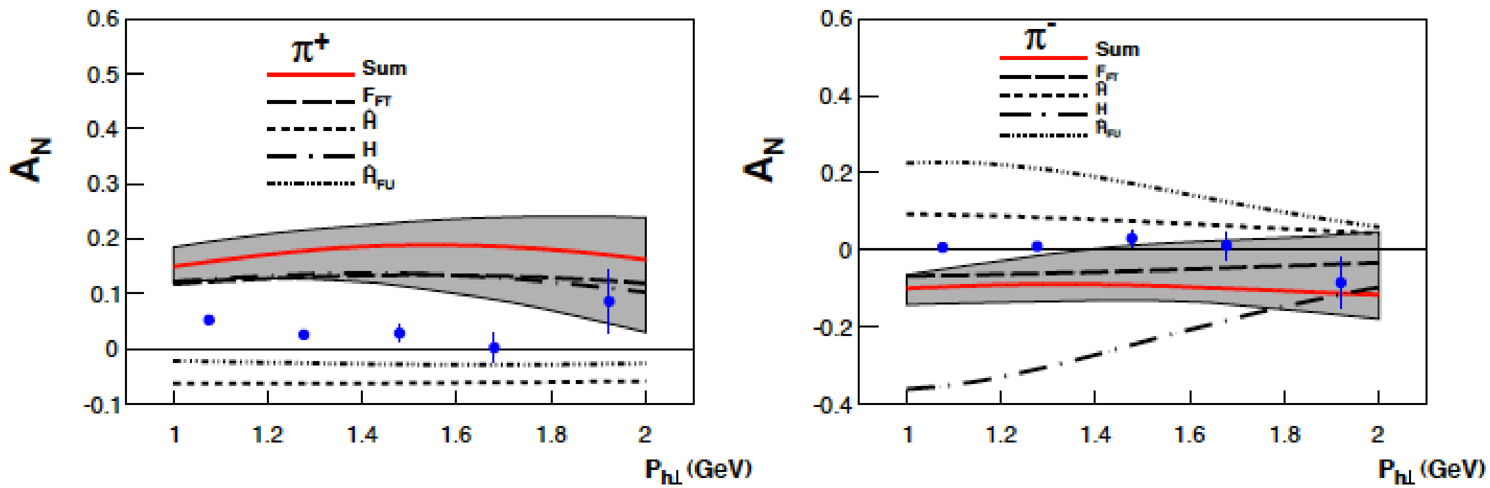
$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1)$$

and fit to RHIC data for A_N (Kanazawa, Koike, A.M., Pitonyak, 2014)



* good fit can be obtained $(\chi^2/\mathrm{d.o.f} = 1.03)$
* A_N dominated by twist-3 FFs (beyond moment of Collins function)
* data on A_N for $\ell\,N^{\uparrow} \to h\,X$ may allow cross check

- Numerical results and discussion
  - comparison with HERMES data (JLab data are at very low $P_{h\perp}$)



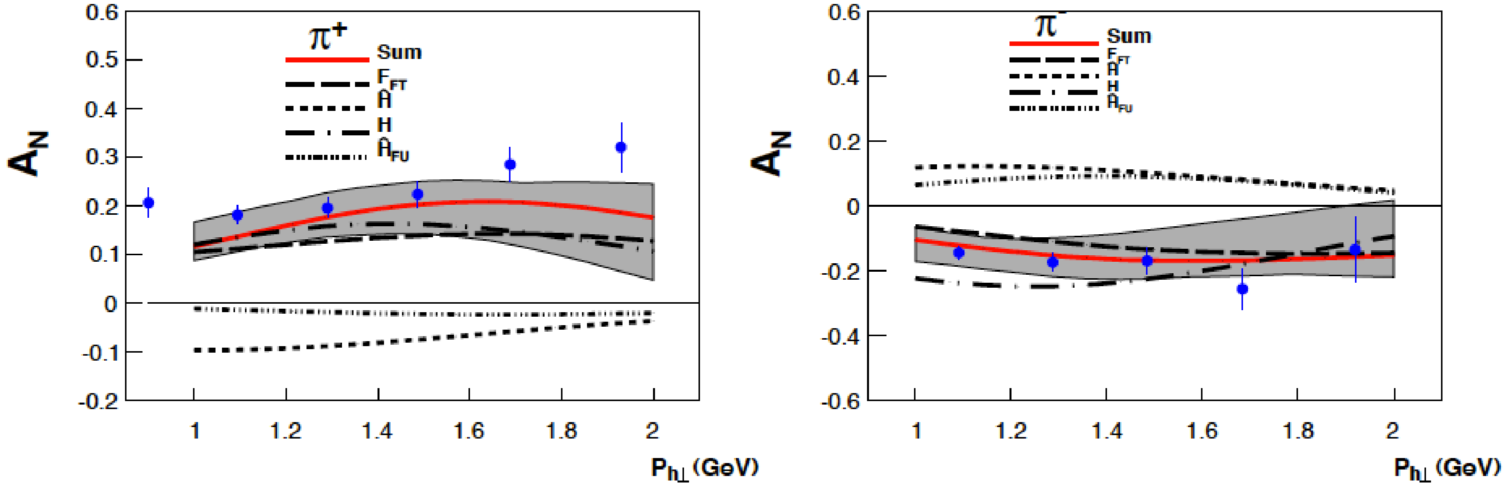
    - error band based on uncertainties of $f_{1T}^{\perp}$, h_1, $H_1^{\perp}$ only
    - relatively poor comparison with data, especially for π^+ production
    - potential reasons for discrepancy:

      (1) no error band for twist-3 FF $\hat{H}_{FU}^{\Im}$ and hence for FF H

      (2) (significant) other source(s) for A_N in $p\, p^{\uparrow} \rightarrow h\, X$

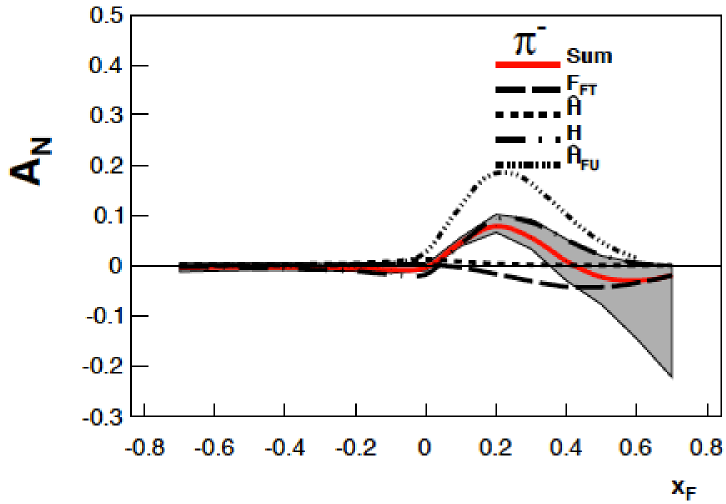
      (3) leading order formalism not appropriate for rather low $P_{h\perp}$ of available data;
      HERMES: even data at highest $P_{h\perp}$ dominated by quasi-real photo-production
      $\rightarrow$ calculation of NLO correction needed

– better comparison with "DIS" sub-set of HERMES data $(Q^2 \geq 1\,\mathrm{GeV}^2)$

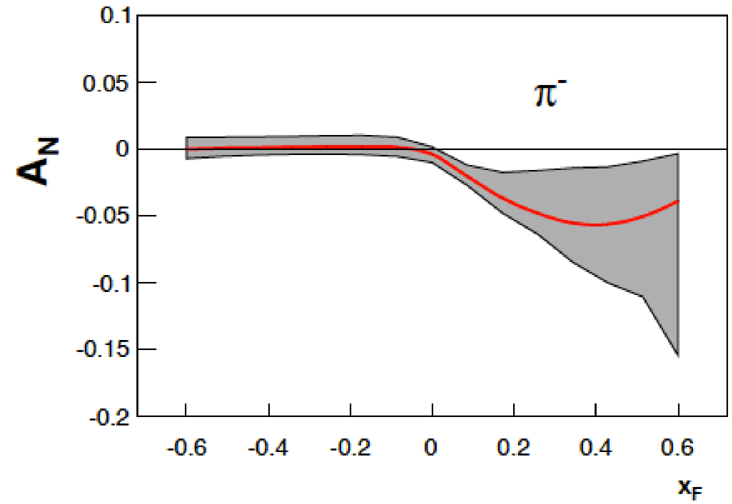


– prospects for measurement at an EIC $(\sqrt{S} = 63\,\mathrm{GeV},\ \ P_{h\perp} = 3\,\mathrm{GeV})$

with $\hat{H}^{\Im}_{FU}$



without $\hat{H}^{\Im}_{FU}$



$*$ description of A_N in $p\,p^{\uparrow} \to h\,X$ through twist-3 fragmentation may be checked

Single-spin asymmetry A_N for $\ell\, N \to \Lambda^{\uparrow}\, X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2015)

- Same Feynman graphs as for A_N in $\ell\, N^{\uparrow} \to h\, X$

- Analytical results
  - twist-3 distribution contribution

$$\frac{P_h^0 d\sigma_{LC}^{Dist}}{d^3\vec{P}_h} = \frac{8\pi M \alpha_{em}^2}{S}\, \epsilon_{\perp}^{P_{h\perp} S_{hT}} \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{xz^3} \frac{1}{S+T/z} \frac{1}{\hat{u}}\, H_1^q(z) \left(x\, \frac{dH_{FU}^q(x,x)}{dx} \right) \left[-\frac{\hat{s}^2 \hat{u}}{\hat{t}^3} \right]$$

    - $*$ no non-derivative term (first observed by Zhou, Yuan, Liang, 2008)

  - twist-3 fragmentation contribution

$$\frac{P_h^0 d\sigma_{LC}^{Frag}}{d^3\vec{P}_h} = \frac{2M_h \alpha_{em}^2}{S}\, \epsilon_{\perp}^{P_{h\perp} S_{hT}} \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{xz^3} \frac{1}{S+T/z} \frac{1}{-\hat{t} - x\hat{u}}\, f_1^q(x)$$
$$\times \left[z \frac{d\hat{D}_T^q(z)}{dz}\, \hat{\sigma}_D + \hat{D}_T^q(z)\, \hat{\sigma}_N + \frac{1}{z} D_T^q(z)\, \hat{\sigma}_2 + \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{1/z - 1/z_1} \frac{1}{\xi}\, \hat{D}_{FT}^{q,\Im}(z, z_1)\, \hat{\sigma}_3 \right]$$

    - $*$ (again) contributions from three sources

- Discussion
  - identified all relevant parton correlation functions and their relations
  - first complete result in twist-3 collinear factorization
  - results obtained in both light-cone gauge and Feynman gauge
    (for the first time for twist-3 fragmentation contribution to transverse SSA)
  - checked fragmentation contribution to A_N for $\ell \, N^{\uparrow} \to h \, X$ in Feynman gauge
  - numerical estimate needed
  - ingredients available for calculation of A_N for process like $p \, p \to \Lambda^{\uparrow} \, X$

Double-spin asymmetry A_{LT} for $\vec{\ell}\, N^{\uparrow} \to h\, X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2014)

- Same Feynman graphs as before, but somewhat different treatment of kinematics
- Analytical results (calculation in lepton-hadron cm frame)
  - results obtained in both light-cone gauge and Feynman gauge
  - twist-3 distribution contribution

$$\frac{P_h^0\, d\sigma_{LT}^{Dist}(\lambda_\ell, \vec{S}_\perp)}{d^3\vec{P}_h} = -\frac{8\alpha_{em}^2}{S}\, M\, \vec{P}_{h\perp}\cdot\vec{S}_\perp\, \lambda_\ell \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{z^3}\, \frac{1}{S+T/z}\, \frac{1}{x\hat{u}}\, D_1^{h/q}(z)$$
$$\times \left\{ \left(\tilde{g}^q(x) - x\frac{d\tilde{g}^q(x)}{dx} \right) \left[\frac{\hat{s}(\hat{s}-\hat{u})}{2\hat{t}^2} \right] + x\, g_T^q(x) \left[\frac{\hat{u}}{2\hat{t}} \right] + \int dx_1\, G_{DT}^q(x, x_1) \left[\frac{\hat{u}(\hat{s}-\hat{u})}{\xi \hat{t}^2} \right] \right\}$$

    * (again) contributions from three sources

  - twist-3 fragmentation contribution

$$\frac{P_h^0\, d\sigma_{LT}^{Frag}(\lambda_\ell, \vec{S}_\perp)}{d^3\vec{P}_h} = -\frac{8\alpha_{em}^2}{S}\, M_h\, \vec{P}_{h\perp}\cdot\vec{S}_\perp\, \lambda_\ell \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{z^3}\, \frac{1}{S+T/z}\, \frac{1}{zx\hat{t}}\, h_1^q(x)\, E^{h/q}(z) \left[-\frac{\hat{s}}{\hat{t}} \right]$$

    * final result becomes rather simple

- Comparison between lepton-hadron cm-frame and nucleon-hadron cm-frame
  - agreement after taking into account the following relation (LIR):
    (Bukhvostov, Kuraev, Lipatov, 1984 / ...)

$$g_T(x) = g_1(x) - \frac{2}{x} \int dx_1 \, \frac{1}{\xi} \, G_{DT}(x, x_1)$$

  - matters here only for twist-3 distribution contribution
  - LIR also allows one to simplify the final result

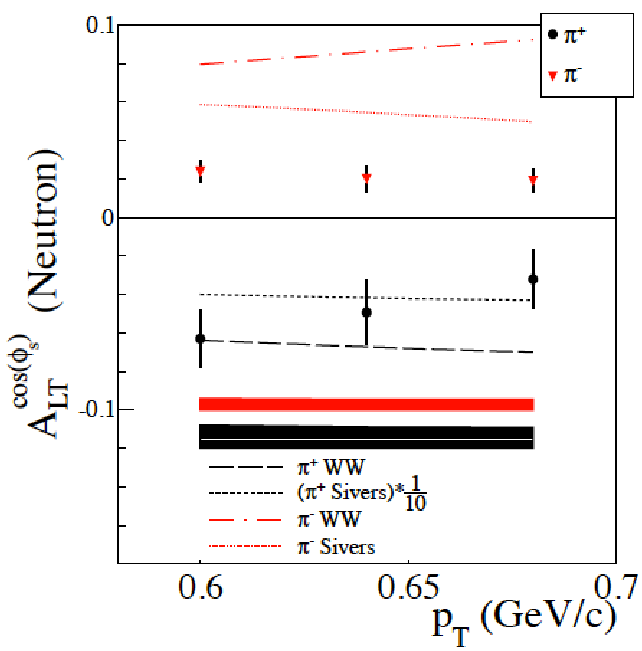
$$\frac{P_h^0 \, d\sigma_{LT}^{Dist}(\lambda_\ell, \vec{S}_\perp)}{d^3 \vec{P}_h} = -\frac{8\alpha_{em}^2}{S} \, M \, \vec{P}_{h\perp} \cdot \vec{S}_\perp \, \lambda_\ell \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{z^3} \, \frac{1}{S + T/z} \, \frac{1}{x\hat{u}} \, D_1^{h/q}(z)$$
$$\times \left\{ \left(\tilde{g}^q(x) - x \frac{d\tilde{g}^q(x)}{dx} \right) \left[\frac{\hat{s}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] + x \, g_T^q(x) \left[\frac{-\hat{s}\hat{u}}{\hat{t}^2} \right] + x \, g_1^q(x) \left[\frac{\hat{u}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] \right\}$$

    * in particular, numerics becomes much easier (g_1 instead of G_{DT})

- Numerical results and discussion (twist-3 distribution term only)
  - use Wandzura-Wilceck approximation for g_T
  - two models for $\tilde{g}$

    (1) WW-type approximation: $\tilde{g}(x) = g_{1T}^{(1)}(x) \approx x \int_x^1 \frac{dy}{y}\, g_1(y)$

    (2) large N_c analysis suggests: $\tilde{g}(x) \approx -f_{1T}^{(1)}(x)$ (Pobylitsa, 2003)
  - comparison with data from JLab Hall A



(JLab Hall A, 2015)

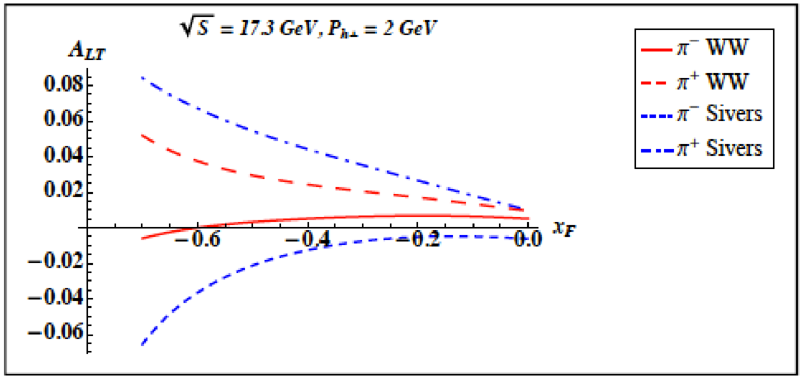
∗ agreement in sign

∗ quantitative description only for π^+ for WW-type approximation

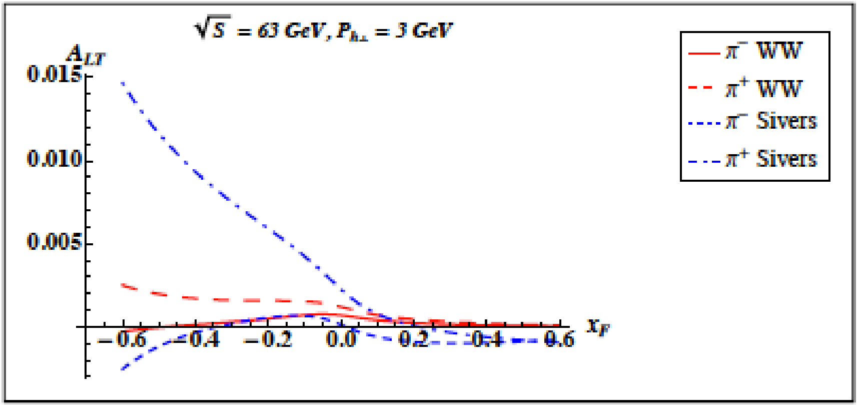
∗ but, keep in mind that $P_{h\perp}$ of data certainly too low

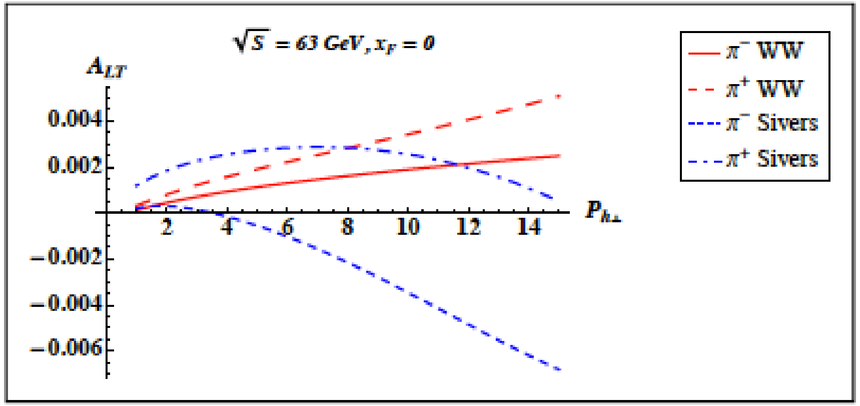
∗ in general, A_{LT} may allow one to study the TMD g_{1T}

– prediction for COMPASS ($\sqrt{S} = 17.3\,\text{GeV},\ P_{h\perp} = 2\,\text{GeV}$)



– prediction for an EIC ($\sqrt{S} = 63\,\text{GeV}$)





– A_{LT} becomes very small at higher energies

Summary

- Twist-3 spin asymmetries for $\ell\, N \to h\, X$ are interesting "new" observables

- They may give new insight into non-perturbative parton correlation functions

- They may give new insight into corresponding asymmetries for hadronic collisions

- First data available for three such asymmetries

- LO calculations available for those three asymmetries
  - helped to solidify theory tools (gauge invariance, frame independence)
  - phenomenology is at exploratory stage

- Calculation of higher order corrections needed