The LPM effect in sequential bremsstrahlung

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Outline

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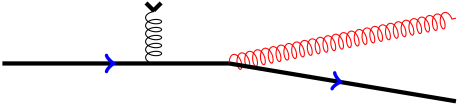
Introduction

- gluon bremsstrahlung is the dominant process through which high energy particles lose energy when moving through the quark-gluon plasma.
- Naively, the bremsstrahlung rate will be roughly,

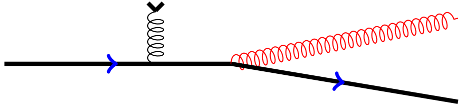


$$\Rightarrow \Gamma = n\sigma v$$

where n is the density of particles in the medium; σ is the cross section for bremsstrahlung arising from a single collision; and v is the relative velocity.

gluon bremsstrahlung

- At high enough energies the formation time of the bremsstrahlung gluon becomes longer than the mean free time between collisions.
- Due to the Landau-Pomeranchuk-Migdal(LPM) effect, consecutive splittings can then no longer be treated as independent, reducing the bremsstrahlung rate from the naive expectation.
- Note: For later diagrammatic convenience, I will be drawing



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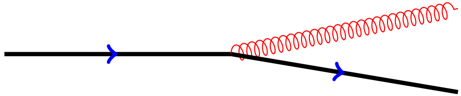
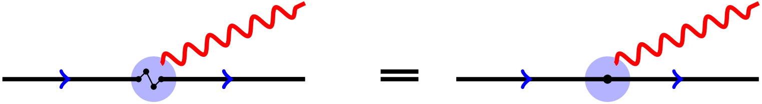


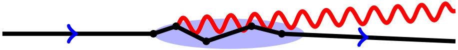
The LPM effect

- To Qualitatively understand the LPM effect, consider an *electron* scattering multiple times from the medium and radiating a *photon*.
- The photon cannot resolve details that are smaller than its wavelength. This will create a region of fuzziness, depicted as the blue shaded region above.
- We won't be able to know from which particular scattering the photon originated, nor can we know if it originated from a single scattering or many scatterings.



The LPM effect

- Now imagine Lorentz boosting this process. Due to time dilation , the circular region of fuzziness will become elongated like an ellipse.
- The length (duration) of this ellipse, is called the formation length (time) of the bremsstrahlung photon.
- The same photon can originate from different scatterings within the formation time,



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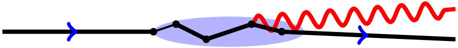
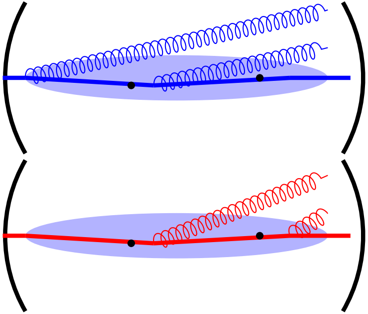


- These different possibilities then interfere quantum mechanically and reduce the bremsstrahlung rate from the naive expectation.
- This reduction in bremsstrahlung rate is called the Landau-Pomeranchuk-Migdal (LPM) effect, which was figured out in the 1950's for electrons and photons, and its quark-gluon generalization was developed in the 1990's [1–3].

What we calculate?

- An important correction to the above picture: what happens if two gluons are radiated within a single formation time.
- That is, generalizing the LPM effect to include interferences between diagrams of the type,



- We are calculating the gluon bremsstrahlung rate, *including* the possibility of double gluon bremsstrahlung.

What we calculate?

- Previous authors have analyzed this problem in the limiting cases where the radiated gluons have very small energies [4,5].
- Our calculation goes beyond that approximation and will be valid for general energies of radiated gluons.

Notation

- In the terms of Feynman diagrams, the LPM effect represents interferences between splittings happening before and after a sequence of scatterings with medium.
- Such interferences can be graphically represented as,

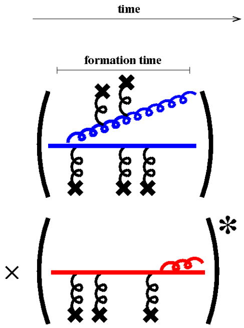
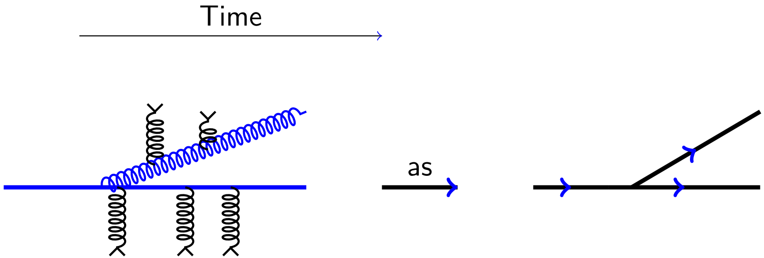


Figure:

Notation

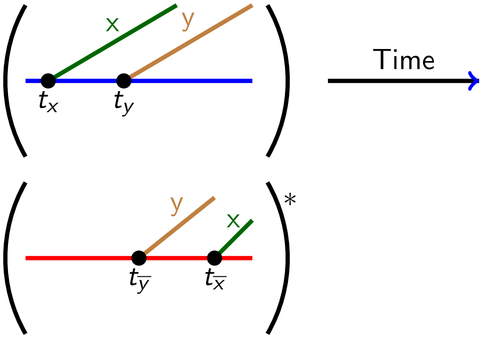
But for diagrammatic convenience, we will draw



with interactions with the medium being implicit.

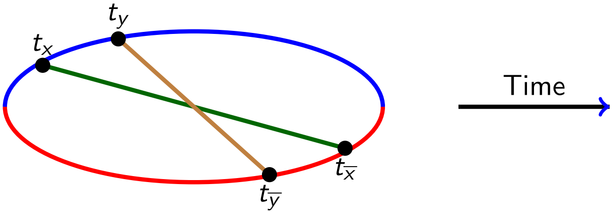
The Calculation

- Then, an interference from double gluon bremsstrahlung can be drawn as,



As a single "process"...

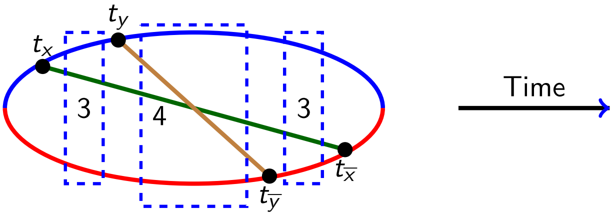
- For calculations like these, it is helpful to think of these terms as being a single "process". We can graphically represent the above interference as,



- The top blue arc represents the amplitude, while the bottom red represents the conjugate amplitude. The green and brown lines represent the x and y gluons as before.
- This is an example of what we called a "crossed" diagram, since the lines cross above.

N-particle evolution

- The problem can now be interpreted as the propagation of 3 particles through the medium between times t_x and t_y, 4 particles between times t_y and $t_{\overline{y}}$ and finally 3 particle propagation from times $t_{\overline{y}}$ and $t_{\overline{x}}$.



- One then just needs to calculate the evolution of in these different regions and glue the results together.

Large p_z!

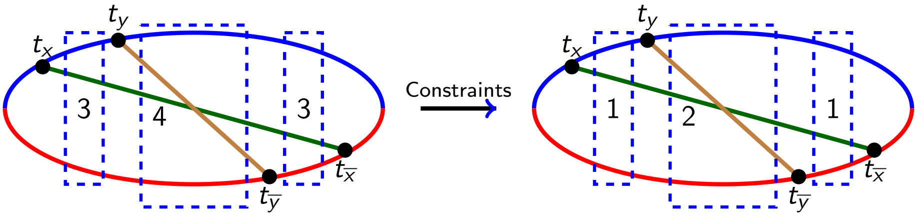
- The process is highly collinear, with the z-component of momentum being extremely large compared to the transverse components.
- In this large p_z approximation, it is possible to reduce the problem to a calculation in 2D non-relativistic quantum mechanics with p_z acting as a "mass" for the two transverse degrees of freedom:

$$E \approx \sqrt{p_\perp^2 + p_z^2}$$
$$\approx \frac{p_\perp^2}{2p_z} + Constant \approx \frac{p_\perp^2}{2mass} + Constant$$

- But these "masses" have the special property that they must add up to zero for all particles in amplitude and conjugate amplitude, at each point in time.

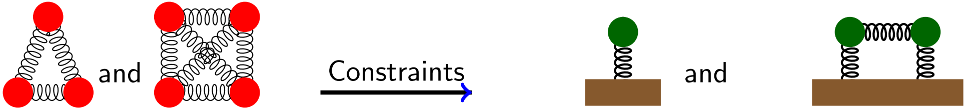
An additional constraint

- Using the constraint that all "masses" must add up to zero and using the center of mass frame, gives us a set of "natural" coordinates which further simplify the problem by reducing it to an effective N-2 particle evolution.



Coupled Harmonic Oscillators

- Finally, in the multiple scattering (harmonic) approximation, the effective Hamiltonian governing the evolution of these particles, becomes that of coupled harmonic oscillators with complex frequencies.



Results

- Using these constraints we can write the contribution from this diagram as,

$$\frac{dI}{dxdy} = \Sigma_{pol} \int_{\text{all t's, B's and C's}} \langle|\delta H|B^x\rangle \langle B^x;t_x|B^y;t_y\rangle \langle B^y|\delta H|C_{12}^y, C_{34}^y\rangle$$
$$\left\langle C_{12}^y, C_{34}^y; t_y \middle| C_{14}^{\overline{y}}, C_{23}^{\overline{y}}; t_{\overline{y}} \right\rangle \left\langle C_{14}^{\overline{y}}, C_{23}^{\overline{y}} \middle| \delta H \middle| B^{\overline{y}} \right\rangle$$
$$\left\langle B^{\overline{y}}; t_{\overline{y}} \middle| B^{\overline{x}}; t_{\overline{x}} \right\rangle \left\langle B^{\overline{x}} \middle| \delta H \middle| \right\rangle$$

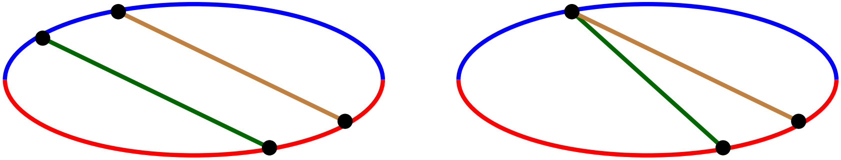
  where the different B's and C's are our "natural" transverse position coordinates.
- In this form, all one needs are the formulas for different elements in the above expression.

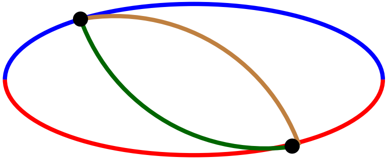
Results

- $\langle |\delta H| B^x \rangle$ and $\left\langle B^y |\delta H| C^y_{12}, C^y_{34} \right\rangle$ can be related to the spin dependent Alterali-Parisi splitting functions.
- The "propagators" $\langle B^x; t_x | B^y; t_y \rangle$ are already well known and are just the QM harmonic oscillator propagators with complex frequencies.
- A major hurdle was calculating the propagator $\left\langle C^y_{12}, C^y_{34}; t_y \middle| C^{\overline{y}}_{14}, C^{\overline{y}}_{23}; t_{\overline{y}} \right\rangle$ which describes evolution in 4-particle (effectively 2-particle) region. We have calculated it in the large N_c limit, and can be expressed as a product of two QM harmonic oscillator propagators with complex frequencies.
- Finally, the rate can be reduced to a one dimensional integral, which can be solved numerically.

Results

- There are three other types of interference terms;





- i.e the uncrossed, and the ones involving the 4 gluon vertex respectively.
- Contributions from all the above types of diagrams can be calculated similarly.

What we have done

- We have completed our calculation of the contribution from the 18 different "crossed" diagrams and we have published our results.
  P. Arnold and Shahin Iqbal, "The LPM effect in sequential bremsstrahlung", arXiv:1501.04964v2 [hep-ph]
- We have shown that the results of previous authors (Bin Wu for example,) can be reproduced in the relevant kinematic limits, i.e $y \ll x \ll 1$.
- We are working to complete this analysis by including the contributions from the "uncrossed" and 4 gluon vertex diagrams.

Thank You!

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