6th Workshop of the APS Topical Group on Hadronic Physics,

Baltimore, April 8-10, 2015

The free energy and the entropy of a heavy quark pair in the quark-gluon plasma

D. Kharzeev

Stony Brook University

BROOKHAVEN NATIONAL LABORATORY

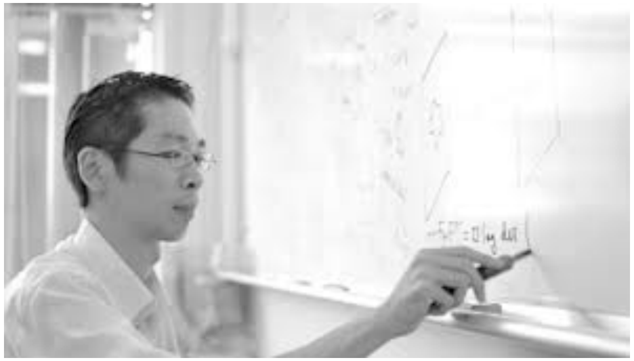
Outline

- The quarkonium suppression puzzle
- Entropy and the entropic force
- Entropy and deconfinement in holography
- Entropic destruction of quarkonium in the QGP

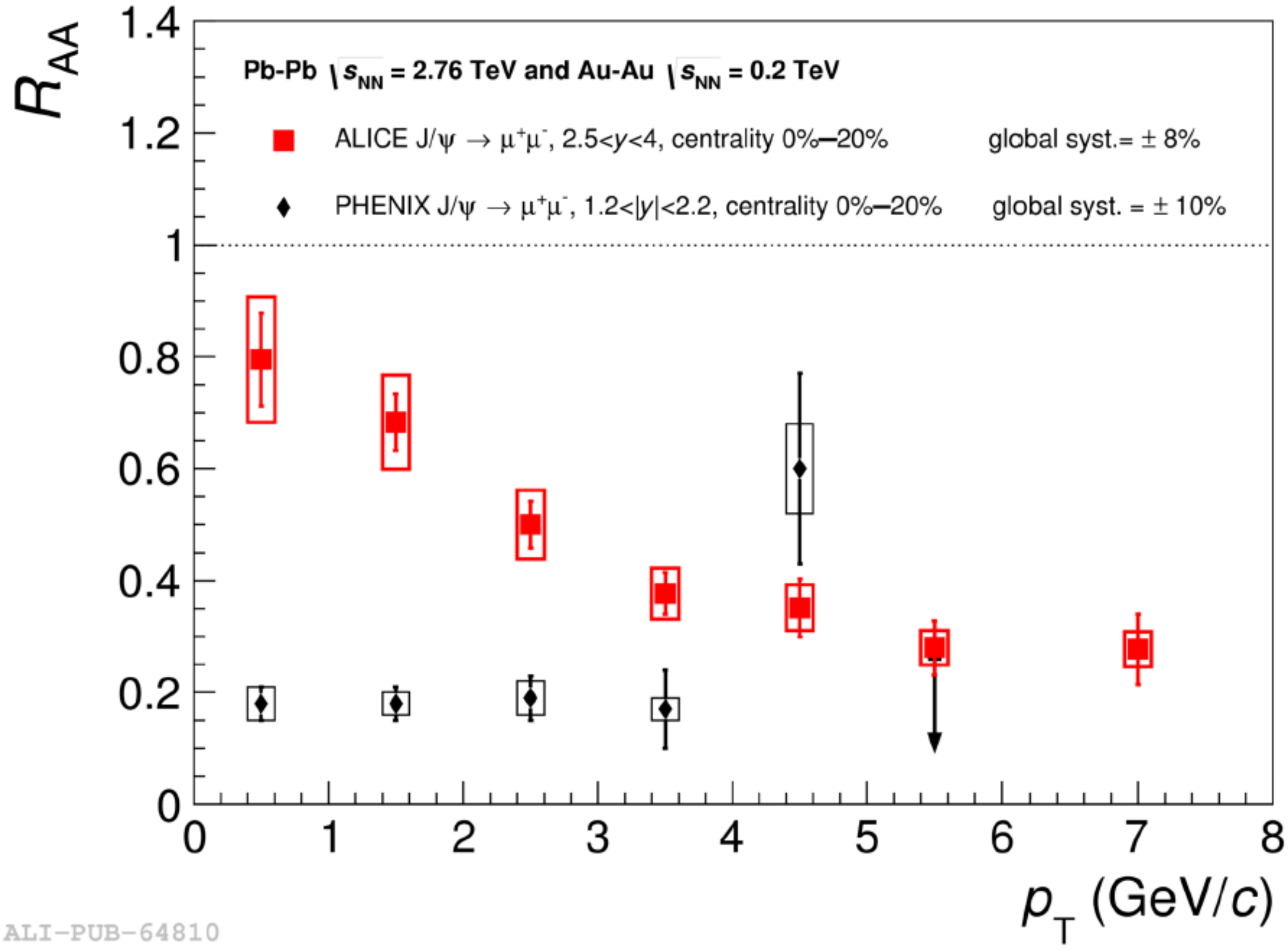
Based on:

DK, Phys. Rev. D 90 (2014) 074007

K. Hashimoto and DK,
Phys. Rev. D 90 (2014) 125012

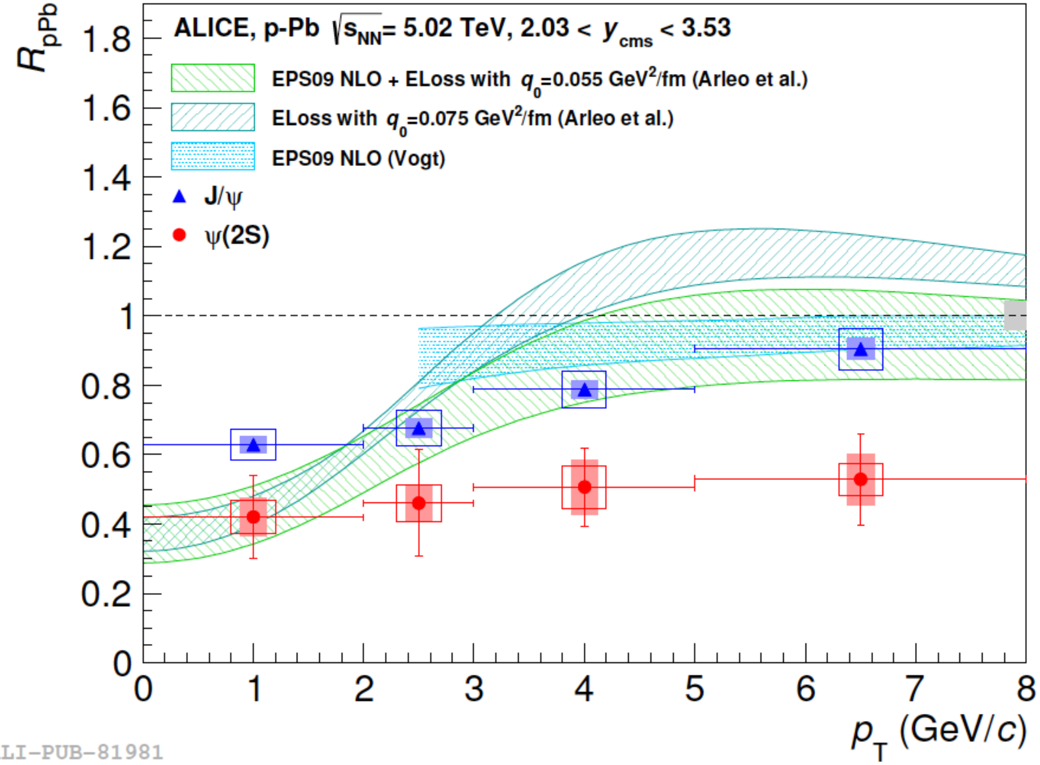


Quarkonium suppression puzzle



Why is quarkonium suppression stronger at RHIC (lower energy density) than at LHC (higher energy density)?

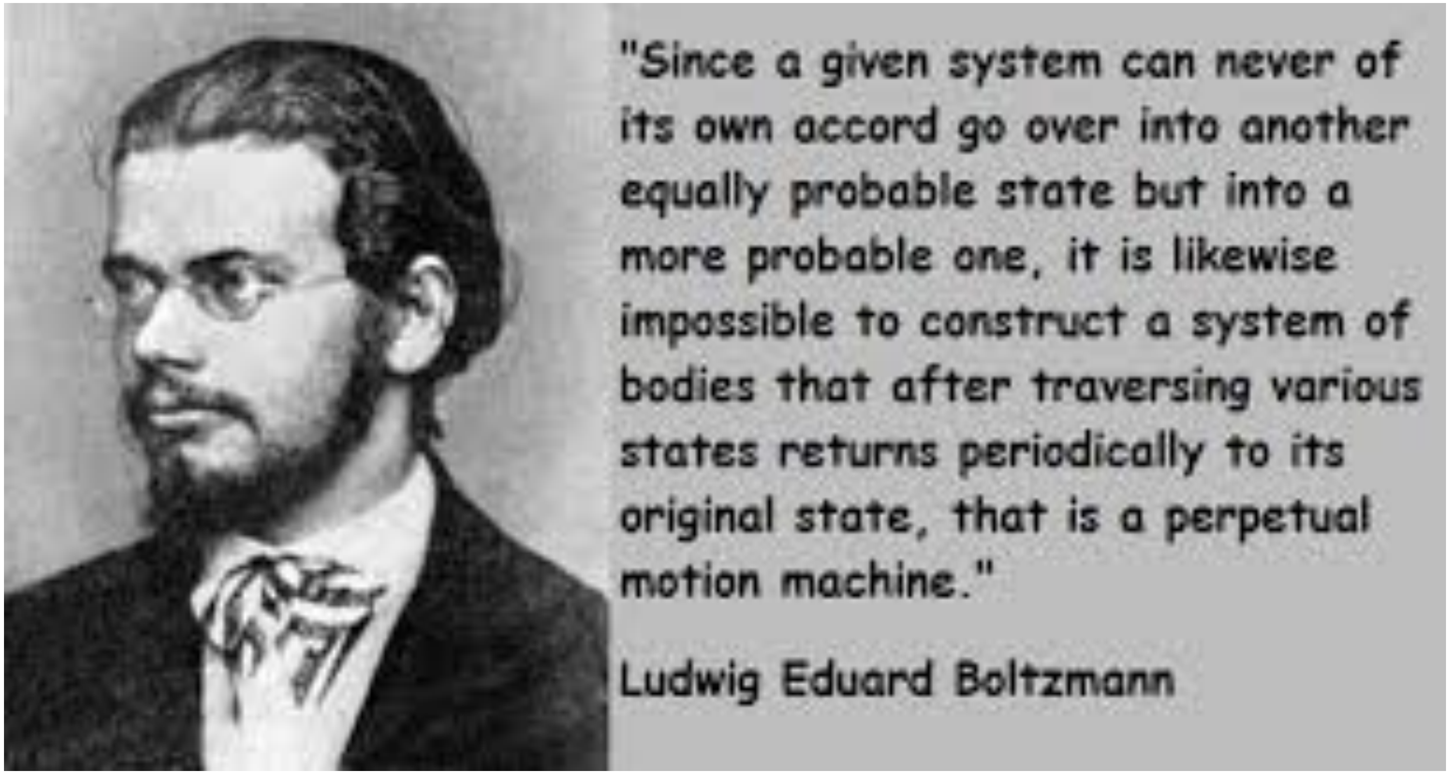
Quarkonium suppression puzzle



Not an initial state effect. Recombination?
Are other explanations possible?

Is suppression a monotonic function of energy density?

The approach in this talk will be based on Boltzmann’s notion of entropy:



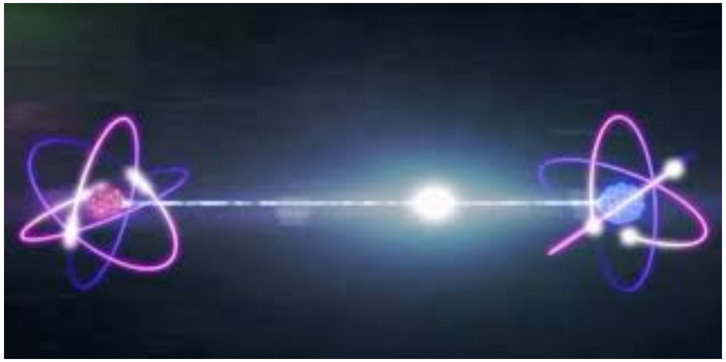
"Since a given system can never of its own accord go over into another equally probable state but into a more probable one, it is likewise impossible to construct a system of bodies that after traversing various states returns periodically to its original state, that is a perpetual motion machine."

Ludwig Eduard Boltzmann

the system is driven to the most probable state with the largest entropy

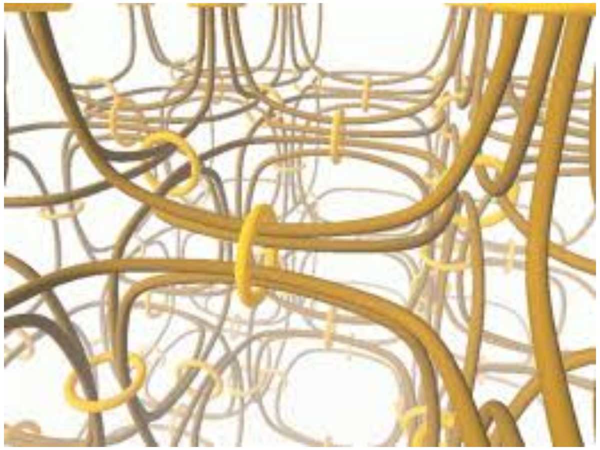
Entropy and quantum theory:

entanglement



black holes

topological phases of matter



Helmholtz free energy

$A = U - T\,S$

what if $U=U(x)$ and $S=S(x)$?

in equilibrium, $dA/dx = 0$, so

the Newton force $F = - dU/dx$

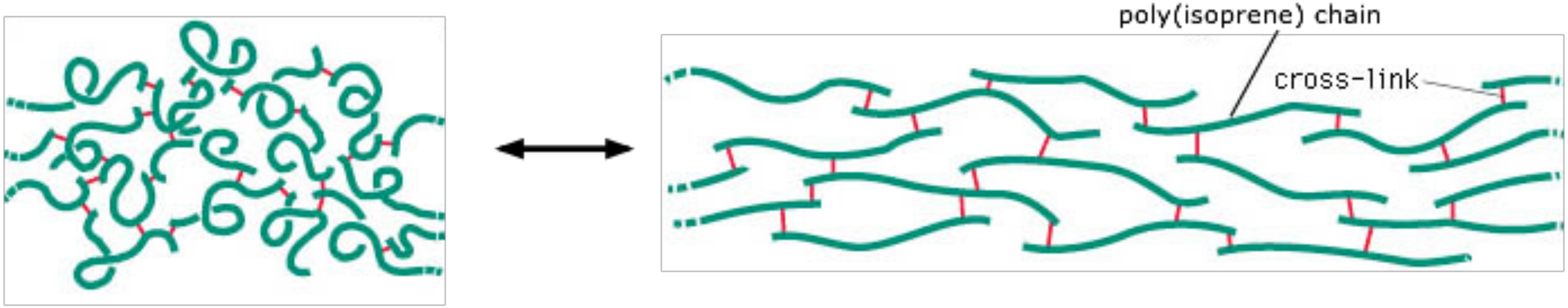
is balanced by the “**entropic force**”

$$F_{ent} = T\,dS/dx$$

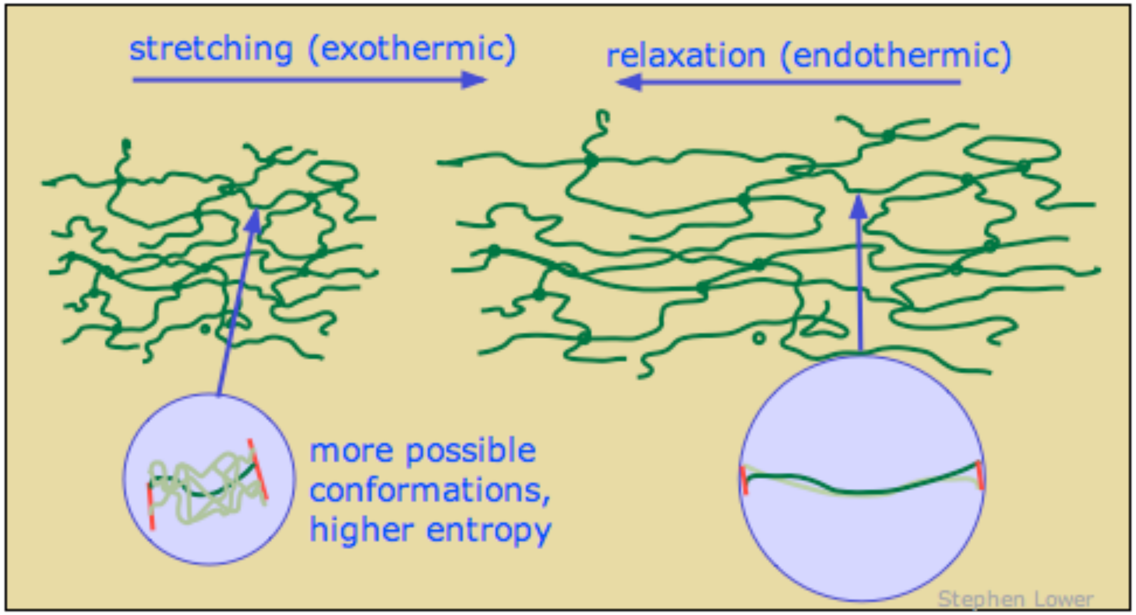
R. Neumann, 1980

A familiar example of the entropic force: rubber bands

A familiar example of the entropic force: rubber bands



Rubber-like protein: elastin



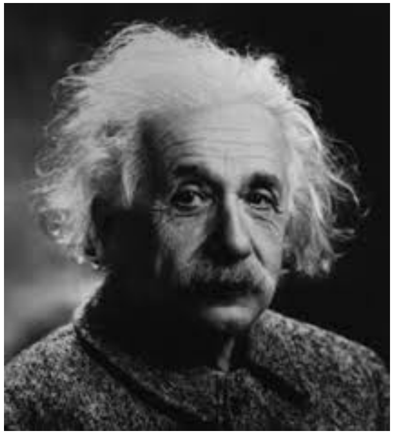
$dS/dx < 0$

Entropic force leads to the rubber band **contraction**

http://chemwiki.ucdavis.edu/

Boltzmann’s entropy

“The equation S = k log W + const appears without an elementary theory —or however one wants to say it— devoid of any meaning from a phenomenological point of view.”



Albert Einstein (1910)

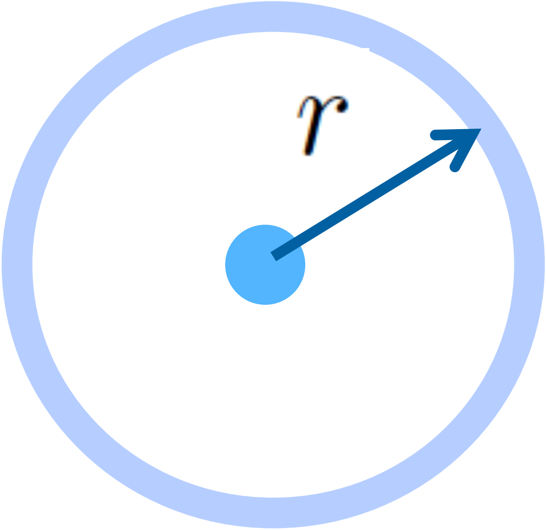
Entropic force and Einstein’s diffusion

Entropic force provides a simple way to derive the equations of kinetic theory

Diffusion

Entropic force and Einstein's diffusion

Entropic force provides a simple way to derive the equations of kinetic theory:



number of states proportional to

$$dV(r) = 4\pi r^2 dr \equiv \Omega(r)dr$$

therefore, Boltzmann's entropy is

$$S(r) = k\ln\Omega(r) = 2k\ln r + const$$

and there is an outward **entropic force**

$$F(r) = T\frac{\partial S}{\partial r} = \frac{2kT}{r}.$$

Entropic force and Einstein’s diffusion

The entropic force is balanced by the Stokes force:

$$\langle F(r)\rangle = c\left\langle \frac{dr}{dt}\right\rangle \qquad c = 6\pi R\eta$$

Performing an average over Gaussian distribution (large number of diffusion steps),

$$P(r) = \frac{4\, r^2}{\sqrt{\pi}\, q(t)^3}\, \exp\left(-\frac{r^2}{q(t)^2}\right)$$

we get the Einstein formula:

$$\boxed{\langle x^2(t)\rangle = 2Dt}$$

$$D = \frac{kT}{c} = \frac{kT}{6\pi R\eta}$$

$$\langle x^2\rangle = \langle r^2\rangle/3$$

Chandrasekhar's law

Particle bound by a quadratic potential $U(r) = ar^2/2$

$\Rightarrow$ confining Hooke's force $F_H = -\frac{\partial U}{\partial r} = -ar$

balanced by the same entropic force;

We thus get

$$a\langle r \rangle = 2kT \left\langle \frac{1}{r} \right\rangle$$

or

$$\boxed{\langle x^2 \rangle = \frac{kT}{a}}$$

- thermal expansion!

Linear confinement

Confining force $F_c = -\partial U/\partial r \overset{\smile}{=} -\sigma$

balanced by the same entropic force;

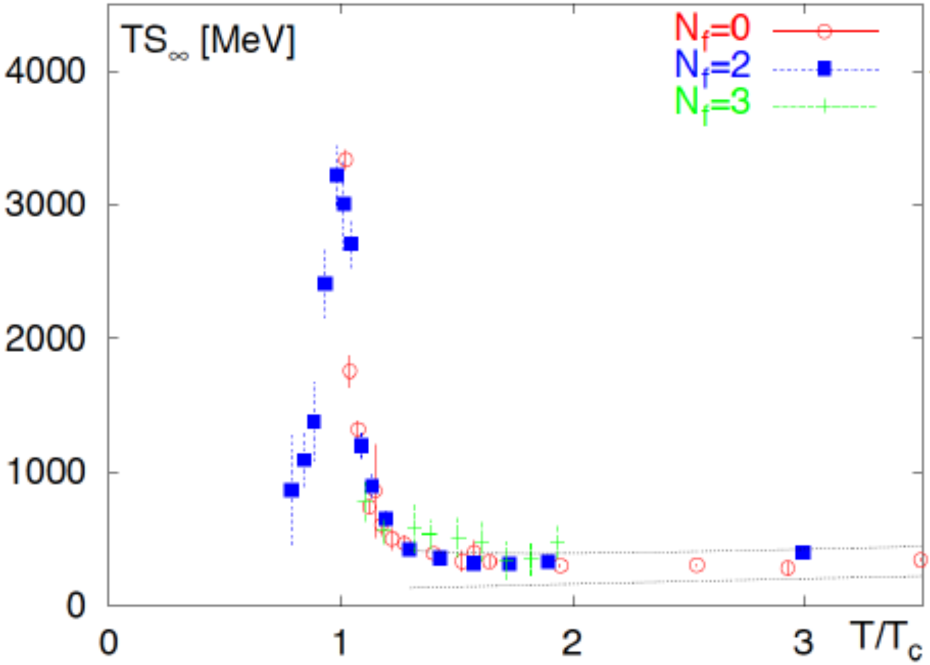
We get $\boxed{\langle x^2 \rangle = \frac{8}{\pi}\left(\frac{kT}{\sigma}\right)^2}$ quadratic dependence, faster thermal expansion

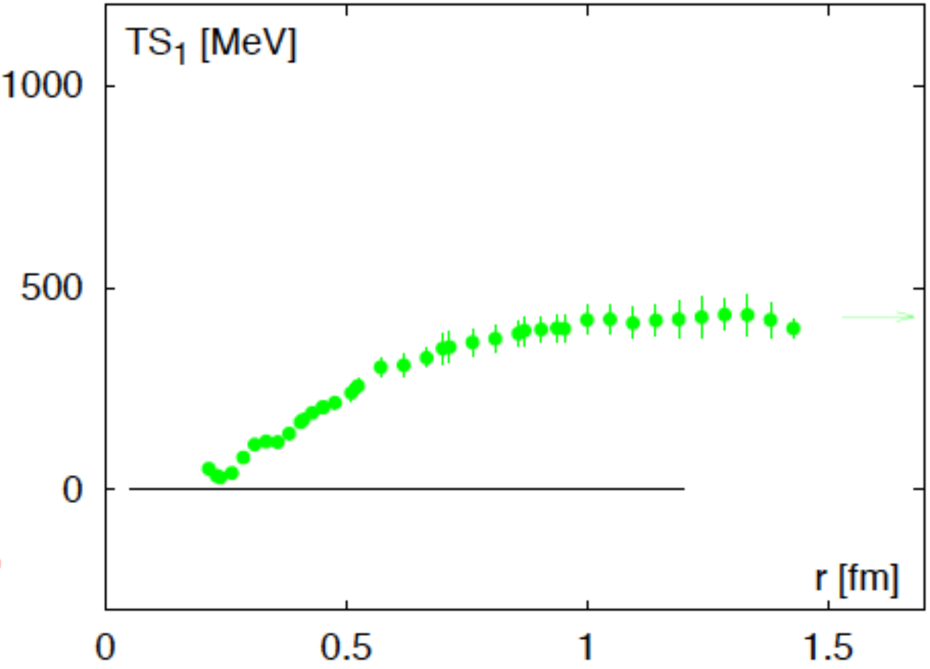
But this is not the only entropic effect!

The entropy of heavy quarkonium in the QGP

$S_{peak} = 16.5$
$\exp(S_{peak}) \sim$ **10^7 states!**
what are they?

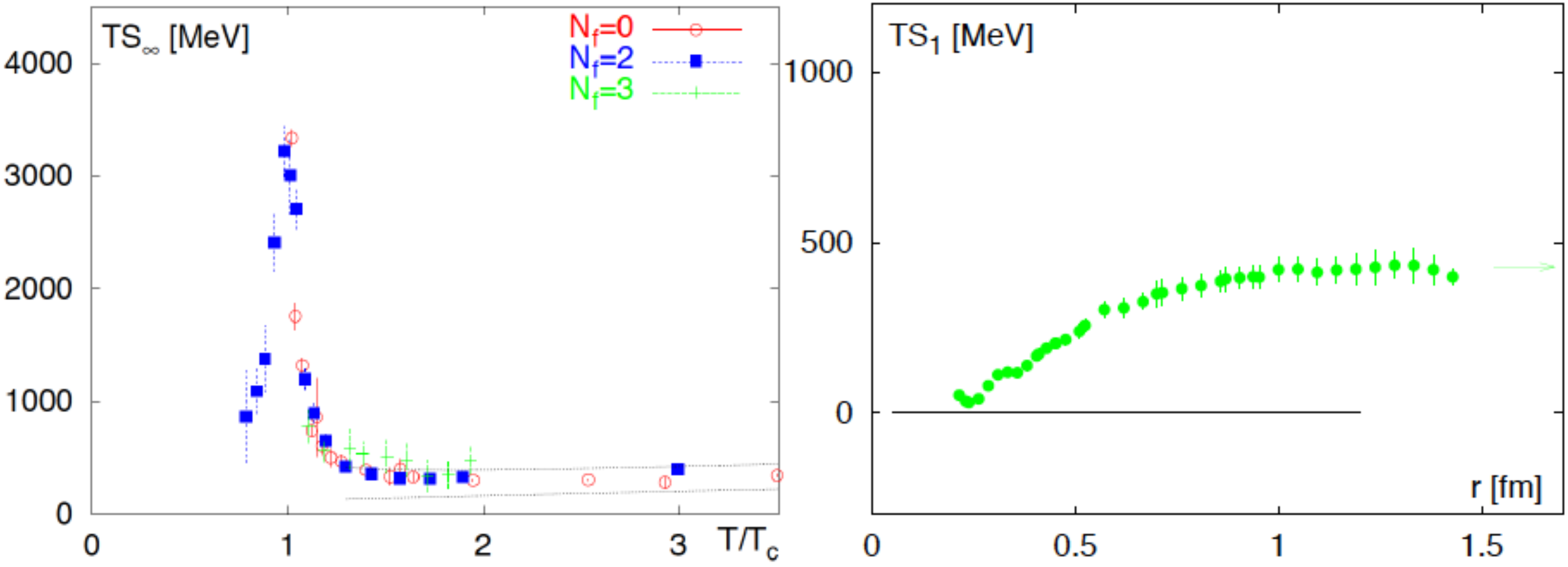
$dS/dx > 0$
repulsive entropic force
(unlike rubber bands)





Kaczmarek, Zantow ‘05

The entropy of heavy quarkonium in the QGP



Kaczmarek, Zantow ‘05

S_{peak} = 16.5

Can this entropy be described by Debye screening?

NO: $S_D \equiv 2 \times s_c \; 4\pi L_D^3/3 \simeq 1.2$

(the entropy density from HotQCD, arxiv: 1407.6387)

Large entropy of the heavy quark pair leads to a strong repulsive entropic force

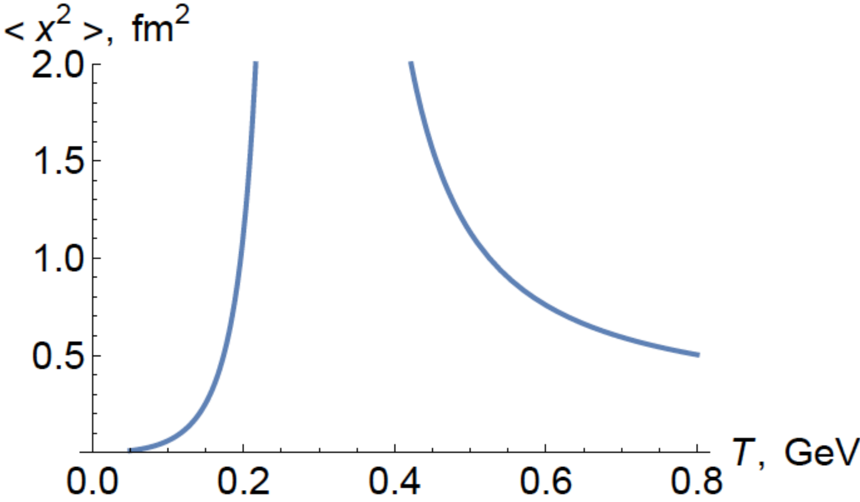
Taking this force into account, we get

$$T\frac{\partial S_{lat}(r,T)}{\partial r} + \frac{4kT}{\sqrt{\pi}q} = \frac{\partial U(r,T)}{\partial r}$$

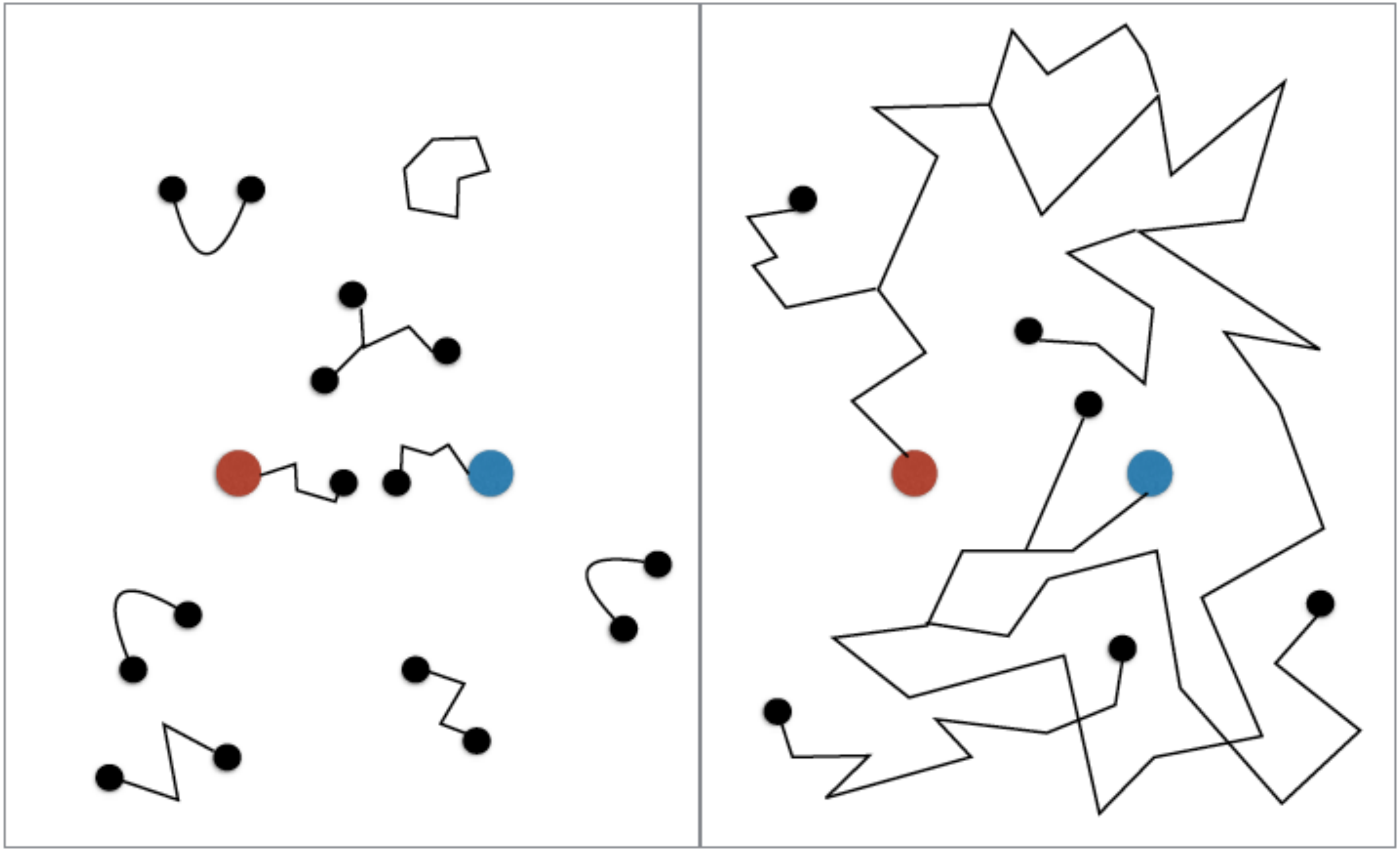
which yields $\langle x^2 \rangle = \frac{8}{\pi}\left(\frac{\sigma(T)}{kT} - s'(T)\right)^{-2}$

divergence of the bound state size (destruction): a very strong suppression near T_c

(has to be quantified)



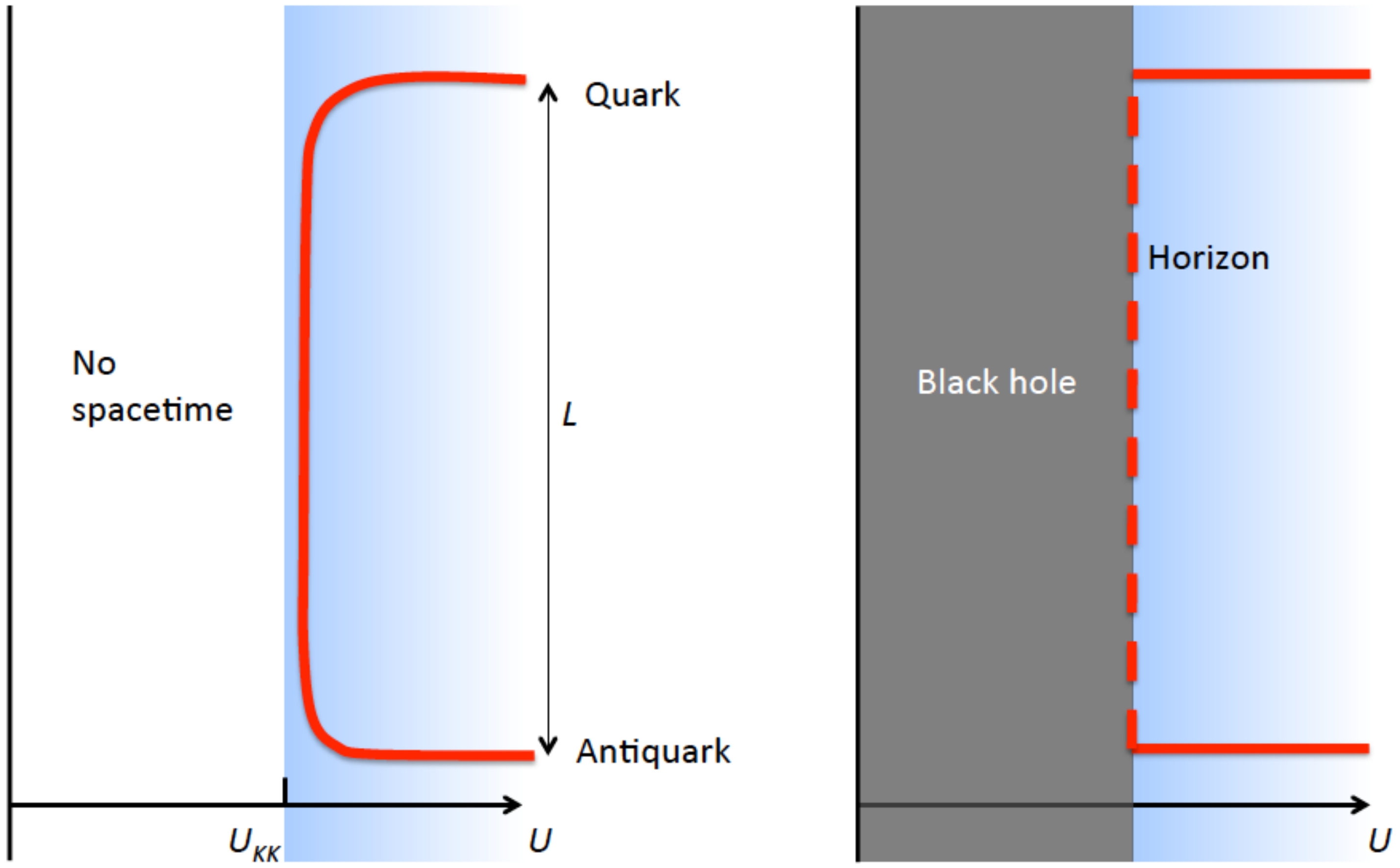
Possible physical picture: the “long string”



DK, Phys. Rev. D 90 (2014) 074007

$T < T_c$ $T \sim T_c$

Physical picture: a holographic description



$T < T_c$

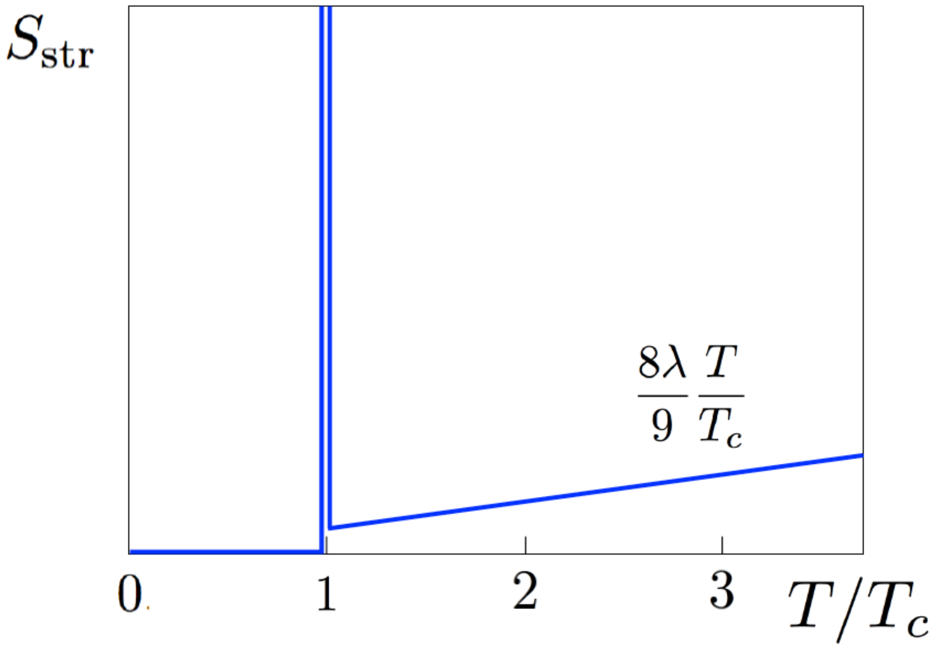
$T > T_c$

“Witten QCD” :
confining, not conformal

K. Hashimoto and DK,
Phys. Rev. D 90 (2014) 125012

Physical picture:
the formation of the black hole at T_c, and the associated entropy, give rise to a sharp peak in the entropy of the heavy quark pair

the presence of this entropy invalidates the potential approach (coupling to on-shell d.o.f. => no instantaneous potential)



K. Hashimoto and DK,
Phys. Rev. D 90 (2014) 125012

Outlook

1. What is the microscopic origin of the huge entropy associated with the heavy quark pair at T_c?
2. Need to develop a quantitative theory of entropic destruction