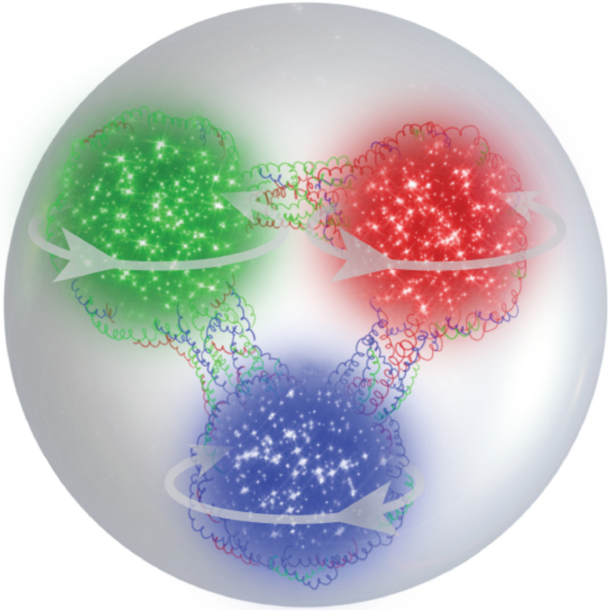
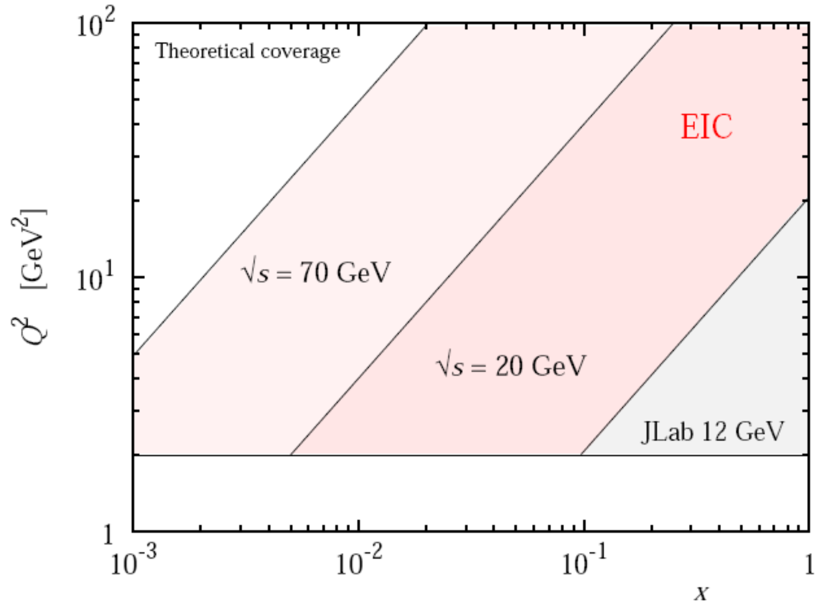
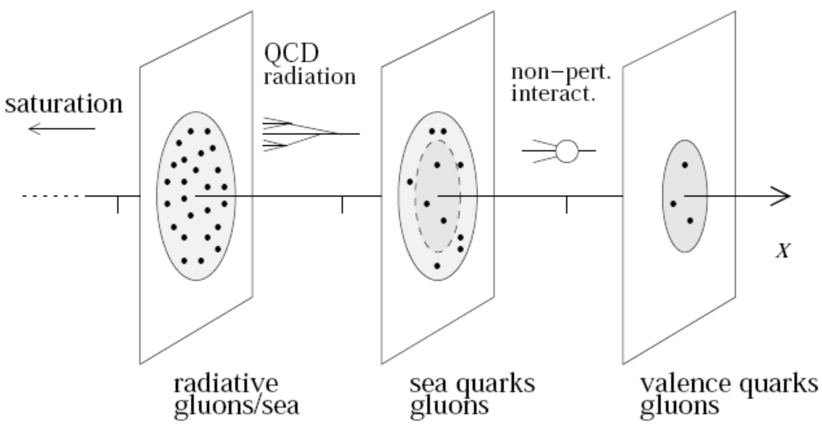
Present status of Phenomenology of the Transverse Spin

Alexei Prokudin



Nucleon landscape





Nucleon is a many body dynamical system of quarks and gluons

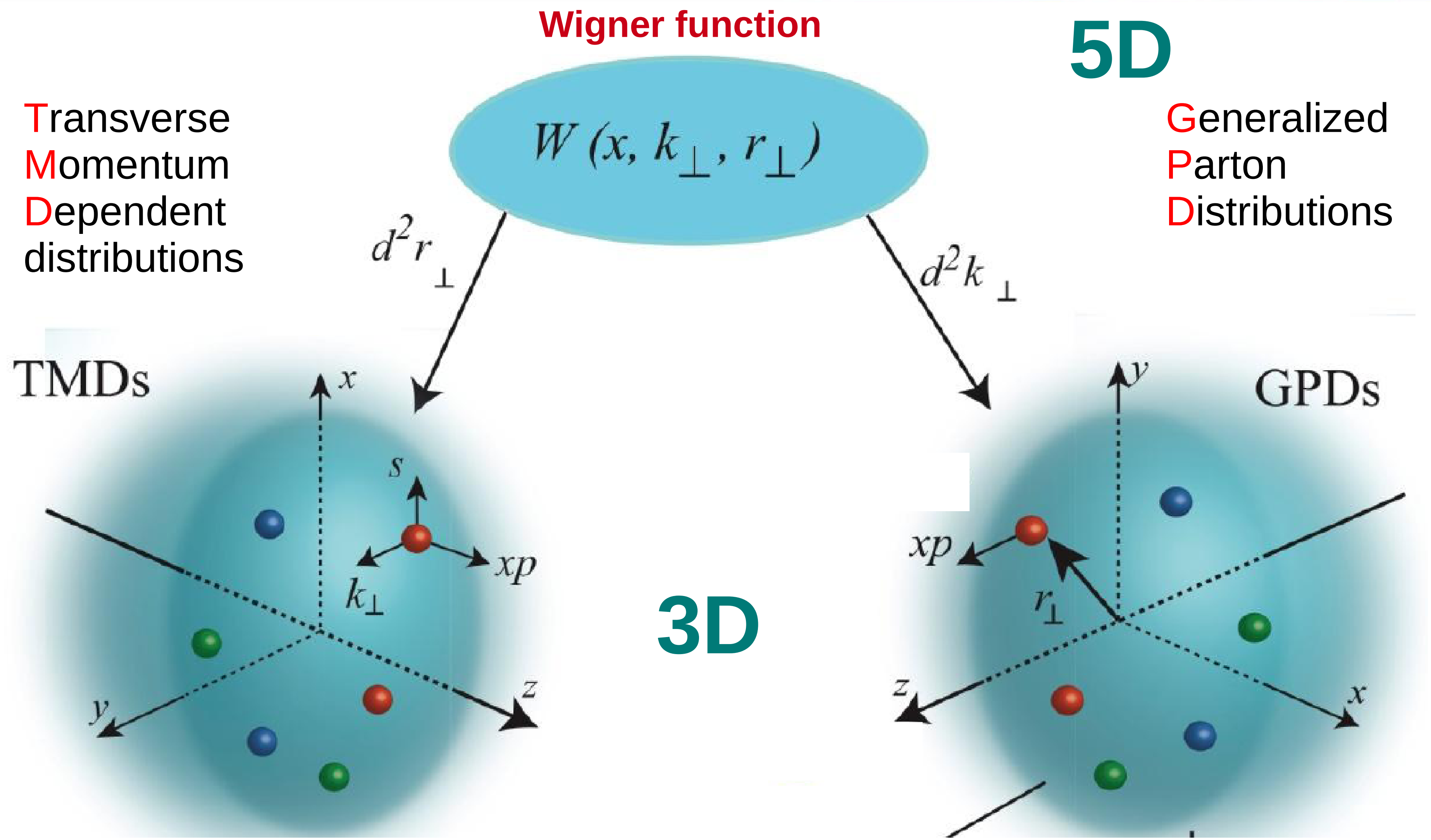
Changing x we probe different aspects of nucleon wave function

How **partons move** and how they are distributed in **space** is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

These distributions are also referred to as **3D (three-dimensional) distributions**

Unified View of Nucleon Structure

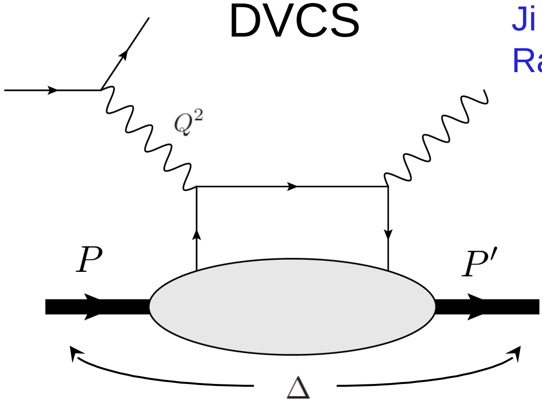


GPDs

TMDs

DVCS

Ji (1997)
Radyushkin (1997)



Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

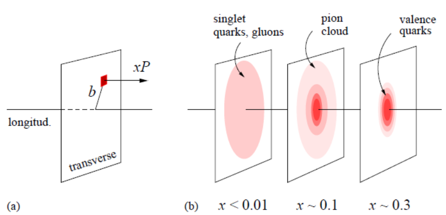
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

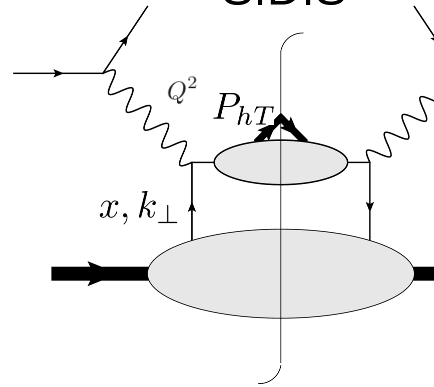
$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



SIDIS



Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

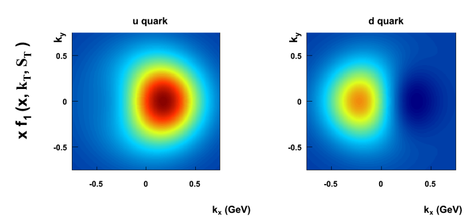
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

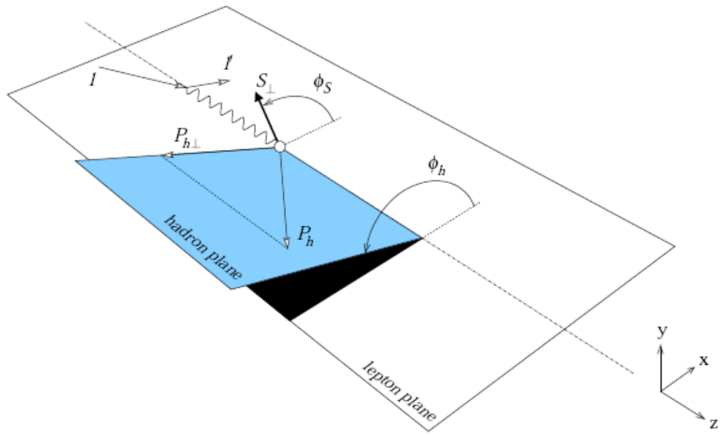
Ji, Ma, Yuan (2004)
Collins (2011)

$$\tilde{f}(x, \vec{b}_T) = \int d^2k_\perp e^{-i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$$

AP (2012)



Semi Inclusive Deep Inelastic Scattering



One can rewrite the cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

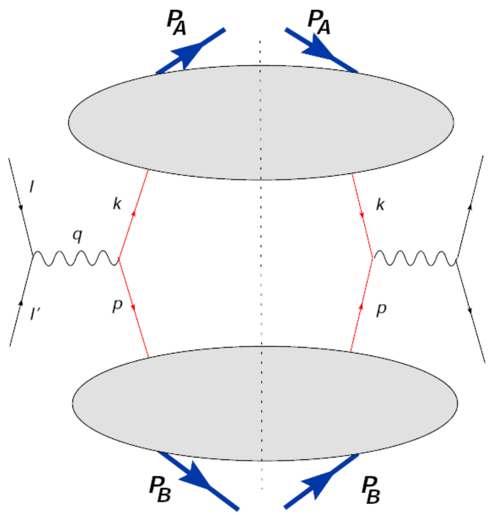
Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

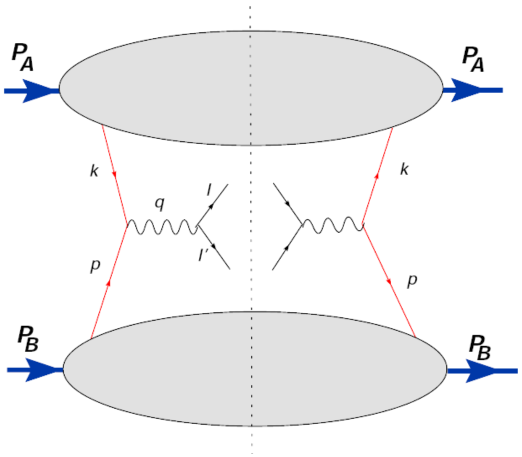
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\;+\ldots$$

e+e- and Drell-Yan





One can rewrite the cross-section of e+e- in terms of **72** structure functions

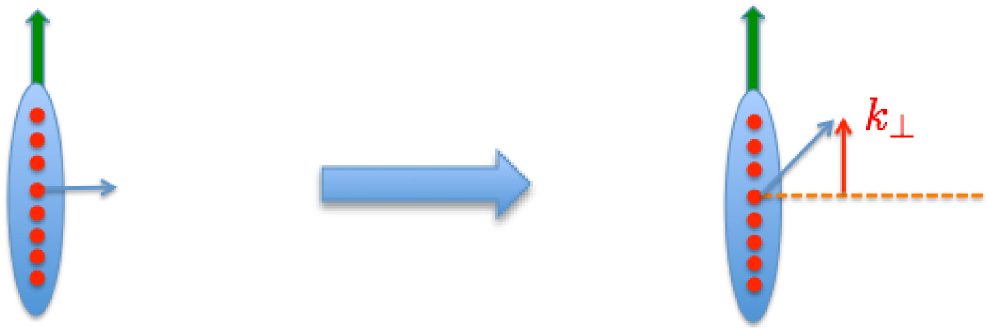
Boer, Jacob, Mulders (1997),
Pitonyak, Schlegel, Metz (2013)

One can rewrite the cross-section of Drell-Yan in terms of **48** structure functions

Tangerman, Mulders (1995),
Boer (1999),
Arnold, Metz, Schlegel (2009)

From 1-D to 3-D

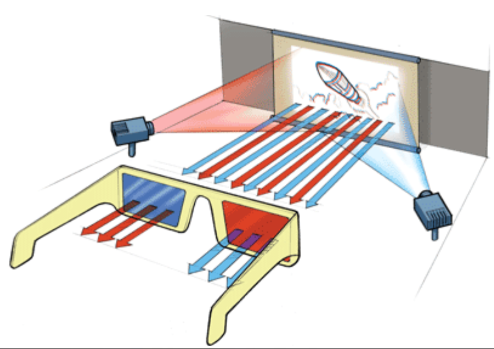
Hadron structure: one-dimensional picture to three-dimensional tomography



$f(x)$ Collinear PDFs

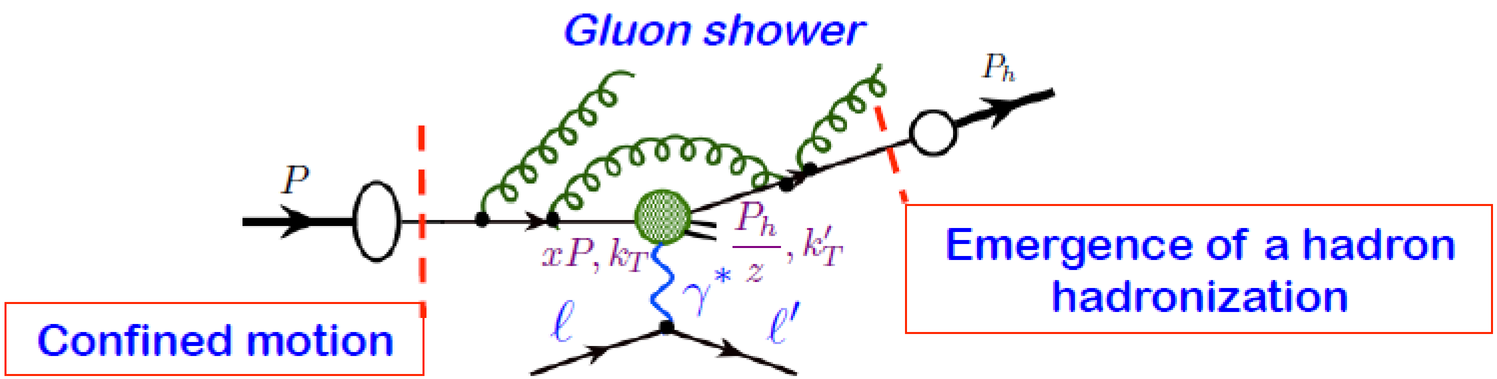
$f(x, k_\perp)$ Transverse momentum distributions (TMDs)

Very interesting and non-trivial consequences:
rich QCD dynamics and new insight on hadron structure



Why QCD evolution is interesting?

Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



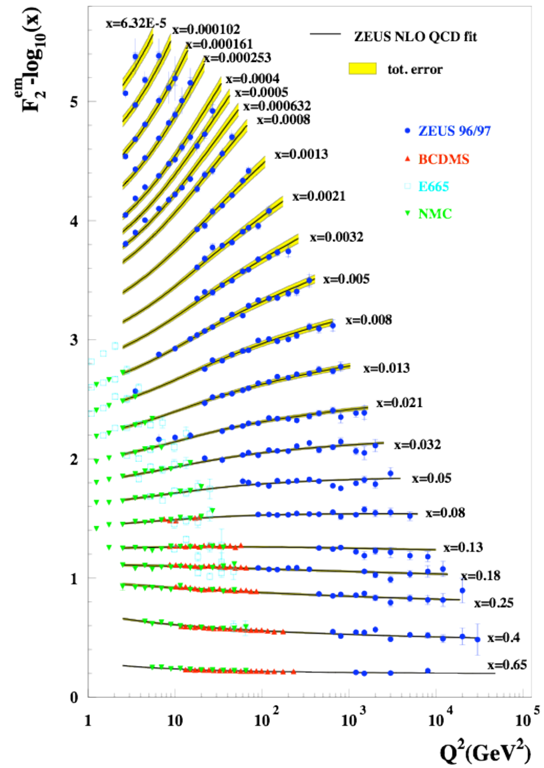
Evolution allows to connect measurements at very different scales

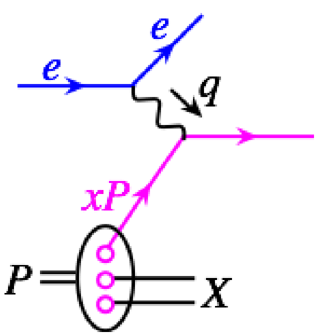
What do we mean by QCD evolution?

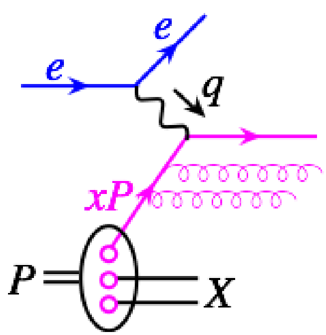
Very well known example:
DGLAP evolution of collinear parton distributions

Take into account perturbative corrections

Single logarithms are resummed order by order in perturbative calculations







$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

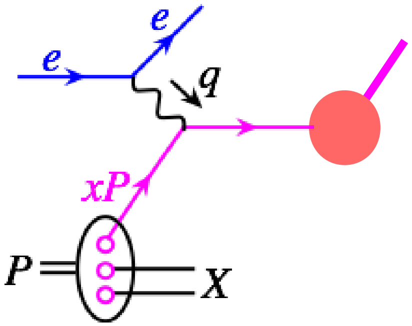
What do we mean by QCD evolution?

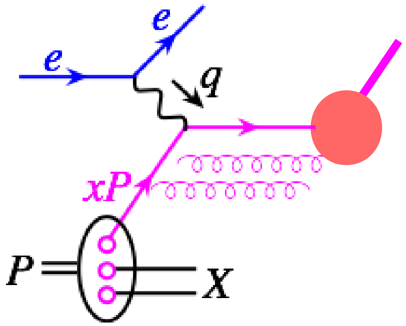
TMD factorization is applicable in case two different scales are observed in processes such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions. Kinematical regime: $Q_T \ll Q$

For SIDIS Q_T is transverse momentum of final hadron

Again we need to take into account perturbative corrections

Double logarithms are resummed order by order in perturbative calculations





$$\left(\alpha_s \ln^2 \frac{Q^2}{Q_T^2}\right)^n$$

Approaches to TMD evolution

Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985
ResBos: C.P. Yuan, P. Nadolsky
Qiu-Zhang 1999, Vogelsang tetc...
Kang-Xiao-Yuan 2011, Sun-Yuan 2013
Prokudin-Kang-Sun-Yuan 2014

"New" Collins approach

Collins 2011
Aybat-Rogers 2011,
Aybat-Collins-Rogers-Qiu, 2012
Aybat-Prokudin-Rogers 2012
Anselmino-Boglione-Melis 2012
Prokudin-Bacchetta 2013
Echevarria-Idilbi-Kang-Vitev 2014

Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012
D'Alesio-Echevarria-Melis-Scimemi 2014

Approaches to TMD evolution

Different approaches are essentially identical

Phenomenological results vary however due to different treatment of initial conditions

TMD evolution in a nut shell

TMD functions are measured at scale Q $\qquad f(x, k_\perp; Q)$

Evolution is performed in Fourier space

$$\tilde{f}(x, b; Q) = \int d^2 k_\perp e^{-ik_\perp b} f(x, k_\perp; Q)$$

Standard CSS formalism, evolution starts from $\mu_b = c/b,\ c = 2e^{-\gamma_E}$

$$\tilde{f}(x, b; Q) = \tilde{f}(x, b; \mu_b) e^{-S_{pert}(b)}$$

$$S^{PT}(b) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right)$$

Perturbative Sudakov factor

$$A = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n A^{(n)} \qquad B = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n B^{(n)}$$

Jefferson Lab

TMD evolution in a nut shell

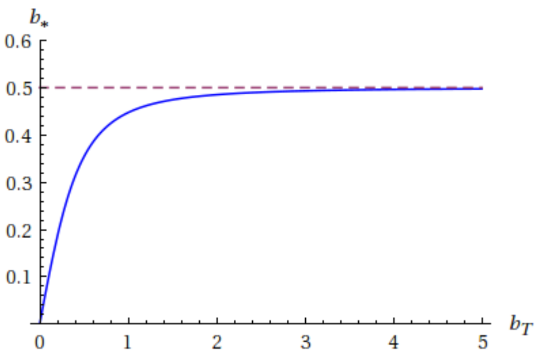
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x, k_\perp; Q) = \int_0^\infty \frac{bdb}{2\pi} J_0(k_\perp b) \tilde{f}(x, b; Q)$$

Non perturbative region needs to be treated.
Common method b* prescription

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$



$$\tilde{f}(x, b; Q) = \tilde{f}(x, b_*; c/b_*) e^{-S_{pert}(b_*)} e^{-S_{NP}(b)}$$

Non perturbative Sudakov factor

TMD evolution in a nut shell

Relation to collinear functions at small values of b:

$$\tilde{f}^{j}(x, b_*; c/b_*) = \sum_{j'=q,g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'}\left(\frac{x}{\hat{x}}, b_*; c/b_*\right) f^{j'}(x; c/b_*)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$

Wilson coefficient

Collinear PDF

For transversity and helicity TMDs: Bacchetta-Prokudin 2013

For Collins function (relation to twist-3 function): Yuan-Zhou 2009, Kang 2011

In future also gluon functions will be important

For gluon twist-3 function: Dai-Kang-Prokudin-Vitev 2014

Taking into account Wilson coefficients is very important!
Large K factors of collinear computations between LO and NLO!

TMD evolution in a nut shell

Precision of extraction depends on precision of calculations

Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Precision is important!

$C^{(1)}$ means that one should use NLO collinear distributions

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

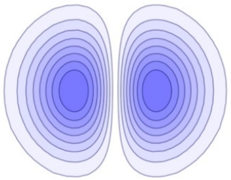
TMD distributions

Three types of modulations

$$f(x, \mathbf{k}_\perp^2)$$

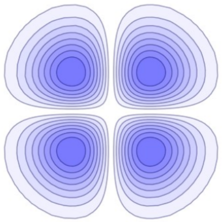
Monopole

$$\frac{\mathbf{k}_{\perp i} S_{Ti}}{M} f(x, \mathbf{k}_\perp^2)$$



Dipole

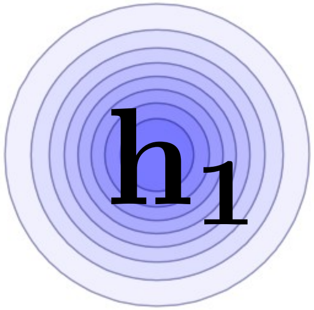
$$\frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j - \frac{1}{2}\mathbf{k}_\perp^2 g_T^{ij}}{M^2} f(x, \mathbf{k}_\perp^2)$$



Quadrupole

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Transversity



TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Transversity

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

$$\Phi(x;P,S) = \frac{1}{2}\left\{ f_1(x)\ \not{P} + S_L g_1(x)\gamma_5\ \not{P} + \frac{1}{2} h_1(x)\gamma_5 [\not{S}_T, \not{P}] \right\}$$

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Unpolarised PDF

Twist-2 collinear PDFs

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Unpolarised PDF

Helicity distribution

Jefferson Lab

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
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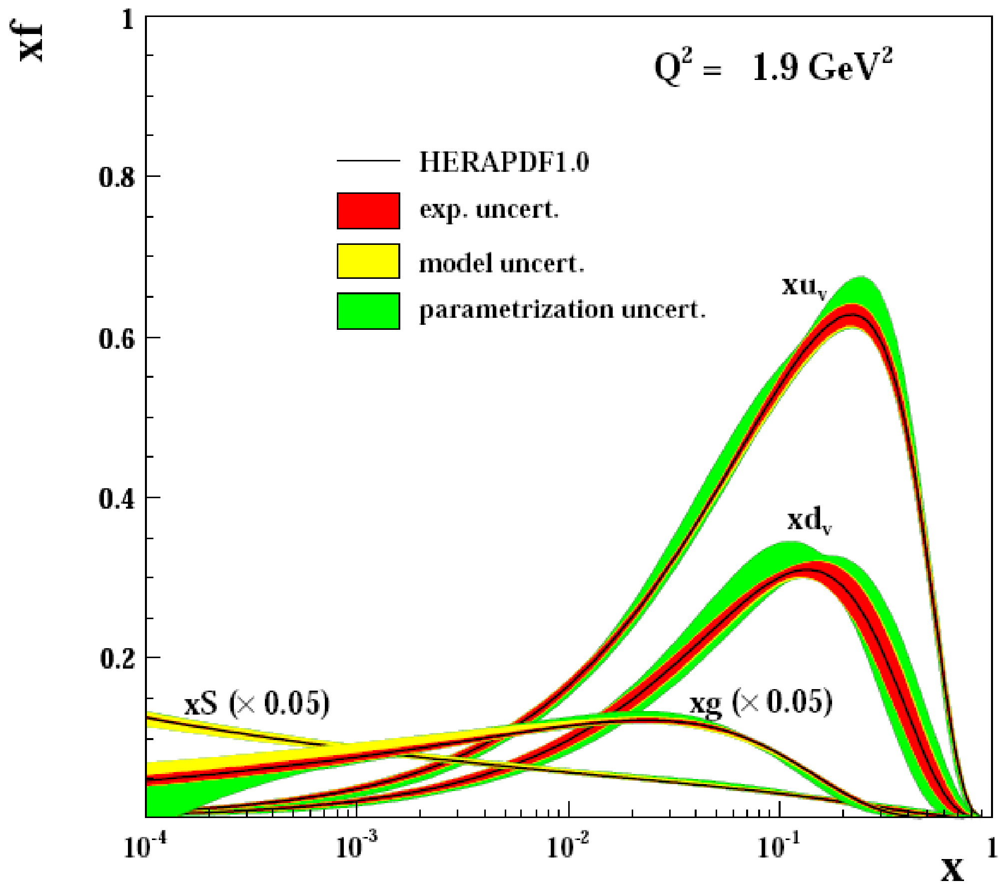
Unpolarised PDF

Helicity distribution

Transversity distribution

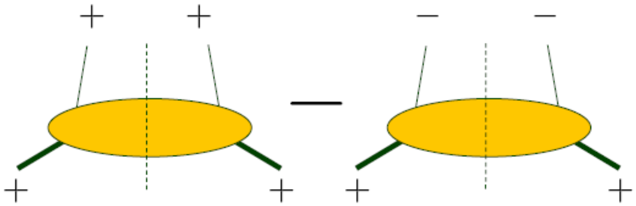
Unpolarised PDFs

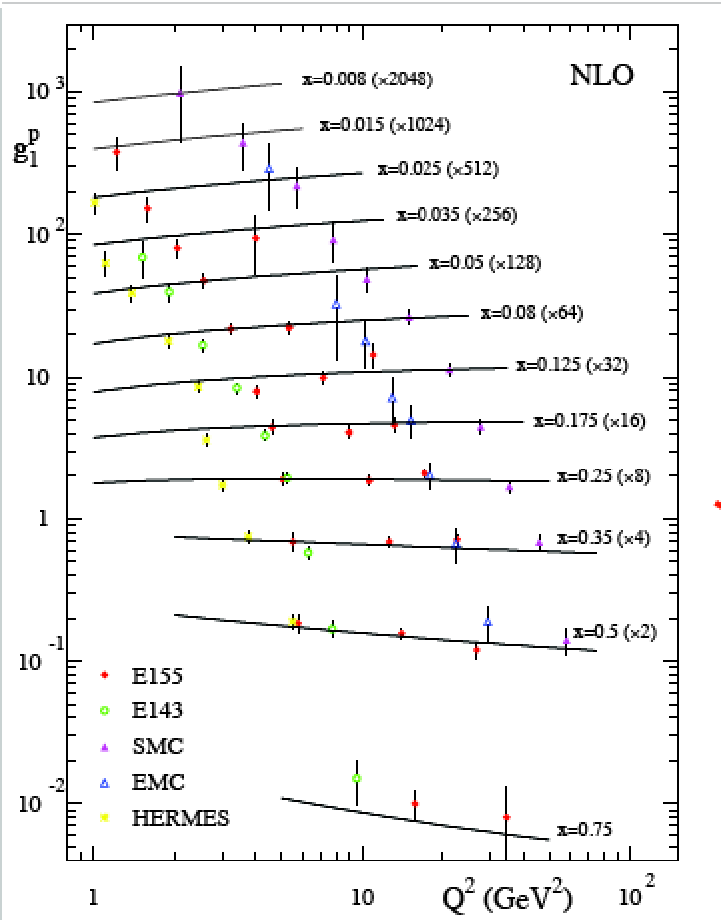
Good knowledge of unpolarised
Parton
Distribution
Functions
is acquired

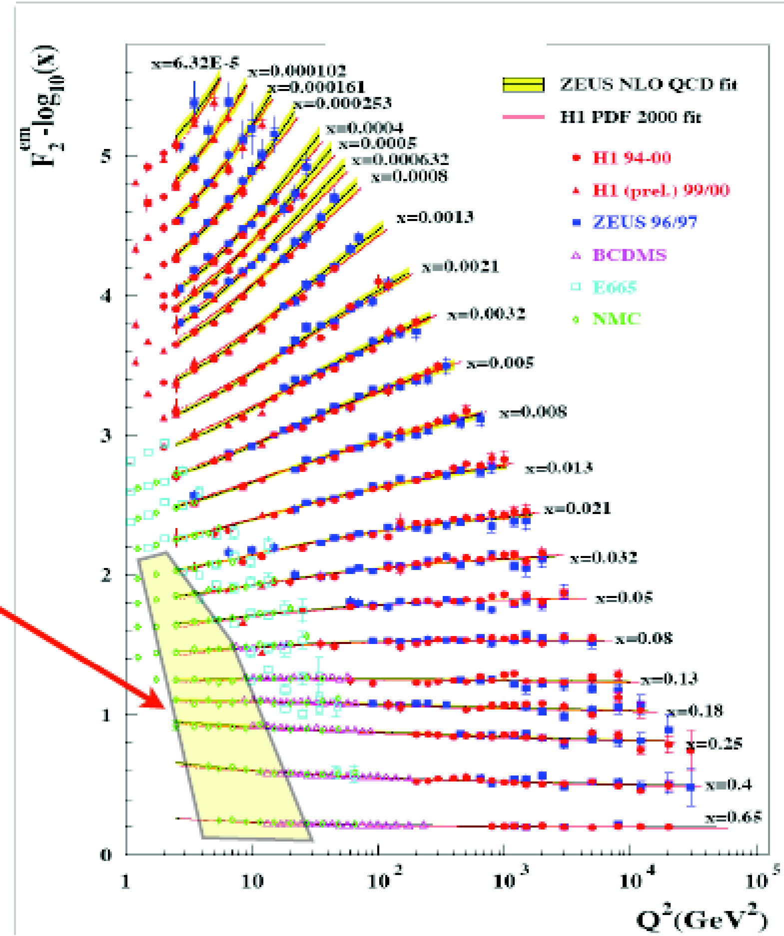


Helicity distributions

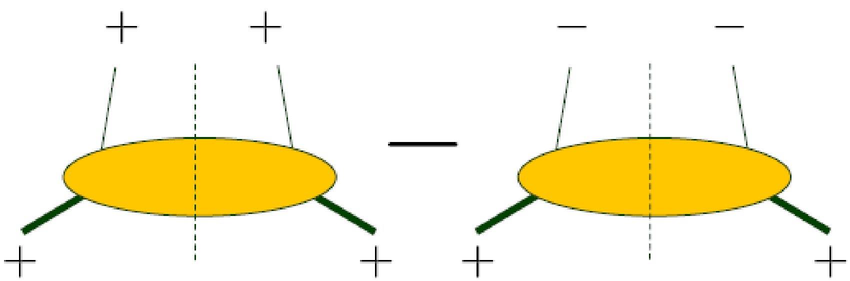
Helicity distributions
are relatively well known



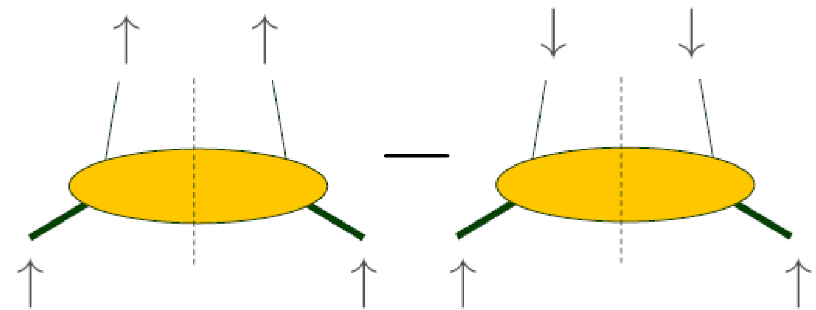




Helicity distribution



Transversity distribution

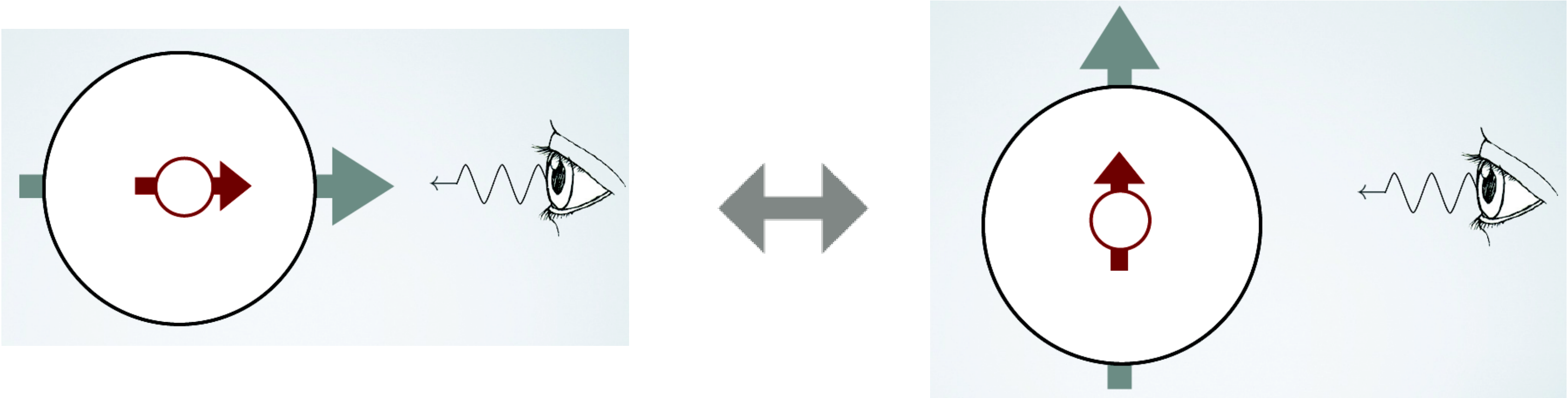


Distribution of transversely polarised quarks inside transversely polarised nucleon

h_1

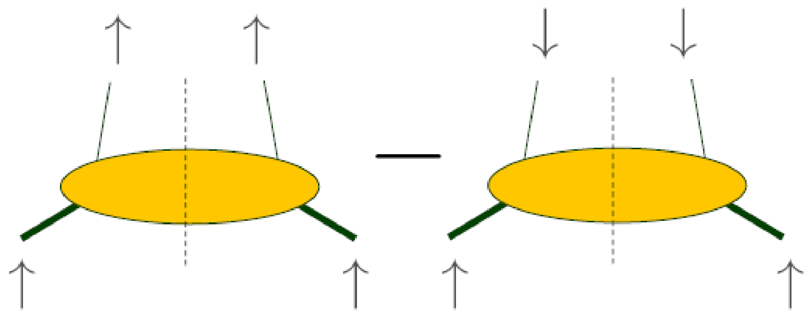
Helicity distribution

Transversity distribution



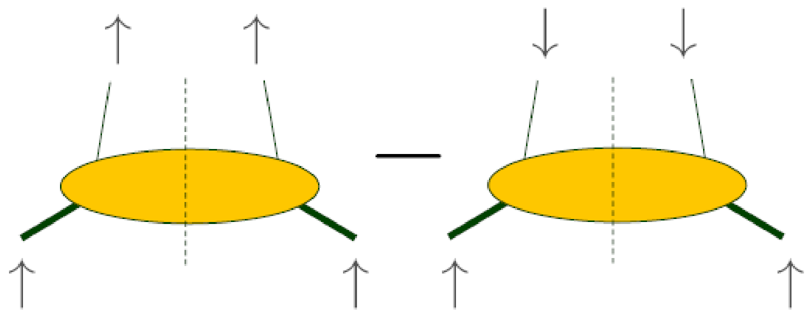
Boost and rotation do not commute → helicity and transversity are different!

Transversity distribution



2005: first data on transversity

Transversity distribution

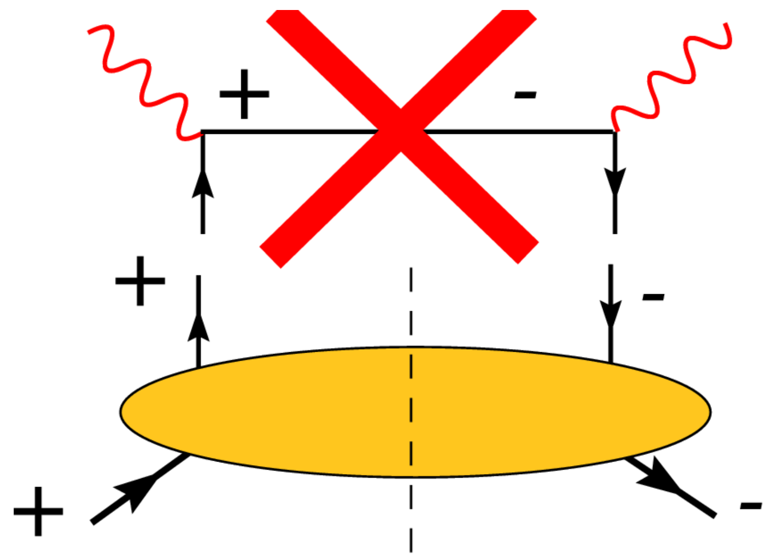


2005: first data on transversity

2012: hundred points from HERMES , COMPASS and JLAB

Transversity distribution

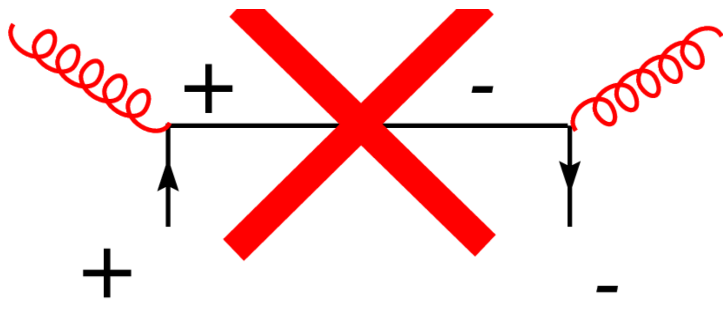
Chiral Odd: it cannot be measured in Deep Inelastic Scattering process



Needs another chiral odd function to be measured

Transversity distribution

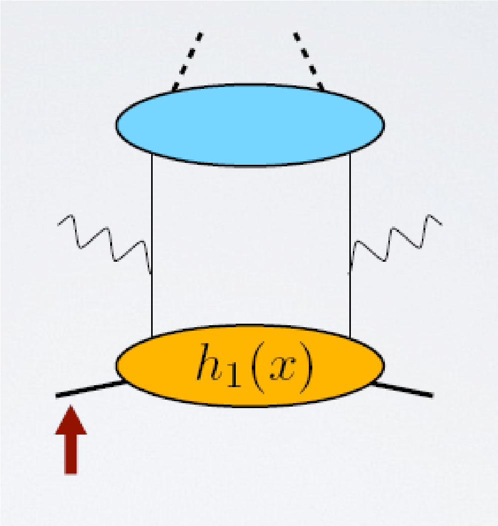
QCD Evolution: no gluon contribution in the evolution



$h_1(x, Q^2)$ is suppressed at low x

JLab 12 is an ideal place to measure transversity → as JLab explores high x region

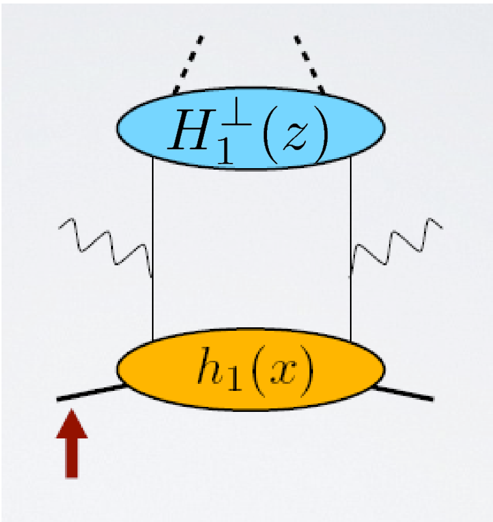
Transversity from SIDIS



First extraction in 2007, Anselmino et al 07

$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

Transversity from SIDIS



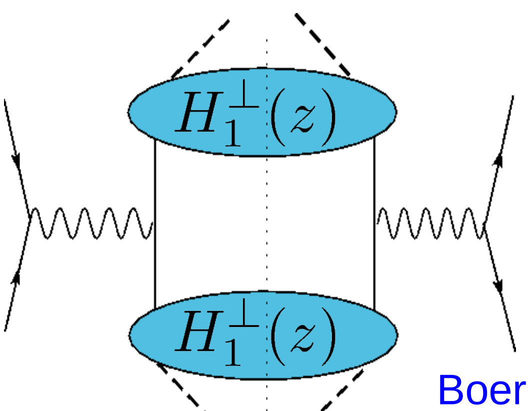
First extraction in 2007, Anselmino et al 07

$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

Two unknowns, transversity $h_1(x)$

Collins Fragmentation Function $H_1^\perp(z)$

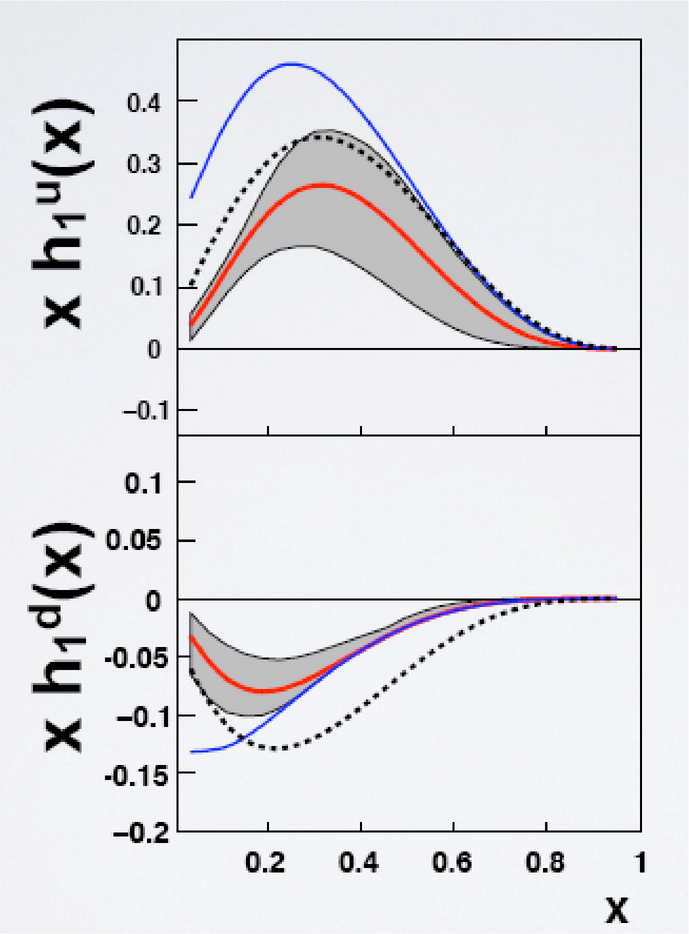
Information on $H_1^\perp(z)$ is available from e^+e^-



Boer (1999)

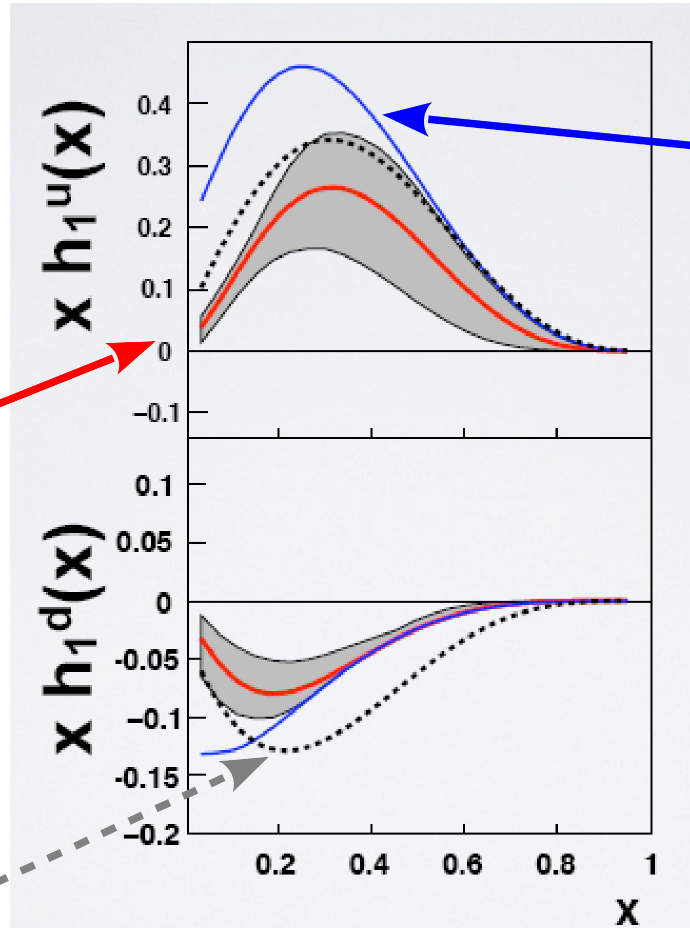
$$A_{e^+e^-} \propto \frac{\sum e_q^2 H_1^{\perp q} \otimes H_1^{\perp \bar{q}}}{\sum e_q^2 D_1^q \otimes D_1^{\bar{q}}}$$

Transversity



Anselmino et al 09

Transversity



Soffer Bound

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

Valid at LO QCD,
Barone 97, Bourelly et al 98

Valid at NLO QCD, Vogelsang 98

Transversity

Helicity

Anselmino et al 09

Tensor charge

Transversity is the source of information on tensor charge

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

Fundamental quantity

Caveat: no sum rules

Transversity at NLL

Kang-Prokudin-Sun-Yuan 2014

First NLL' extraction from the data

$$A^{(1,2)} \quad B^{(1)} \qquad C^{(1)}$$

Collins function is related to twist-3 function

$$\tilde{H}_1^{\perp,\alpha}(z_h, b; \mu_b) \sim \left(\frac{-ib^\alpha}{2z_h}\right) H^{(3)}(z_h; \mu_b)$$

We solve also DGLAP equations for transversity and (diagonal) Collins FF

Yuan-Zhou 2009, Kang 2011

Diagonal part for twist-3 Collins function is:

$$\frac{d}{d\ln\mu^2} H_q^{(3)}(z_h, \mu) = \frac{\alpha_s}{2\pi} P_{i\to q}^H \otimes H_i^{(3)}$$

$$P_{i\to q}^{H_1}(\hat{x}) = \delta_{iq} C_F \left(\frac{2\hat{z}}{(1-\hat{z})_+} + \frac{3}{2}\delta(1-\hat{z})\right)$$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including PT dependence – 122 points

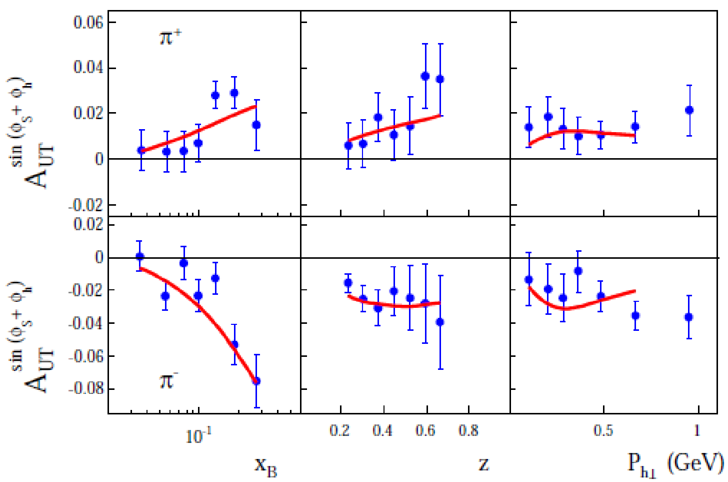
$$\chi^2/\mathrm{d.o.f.} \simeq 0.88$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

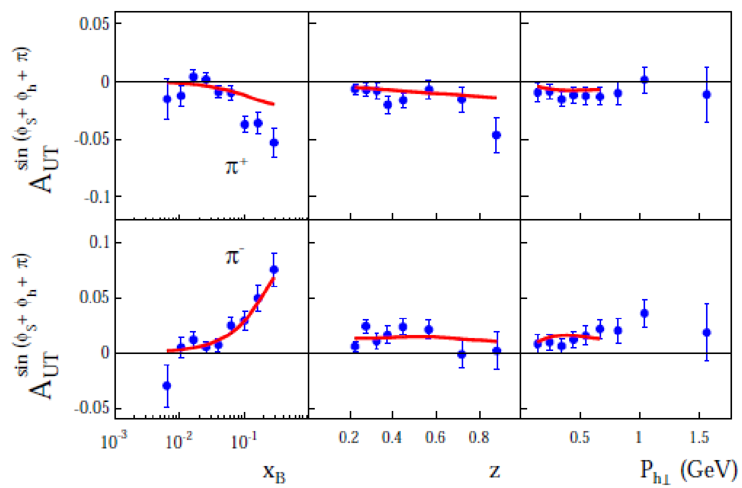
$$\ell P \to \pi^{\pm} X$$

HERMES



$$1 \lesssim \langle Q^2 \rangle \lesssim \ 6 \ \mathrm{GeV}^2$$

COMPASS



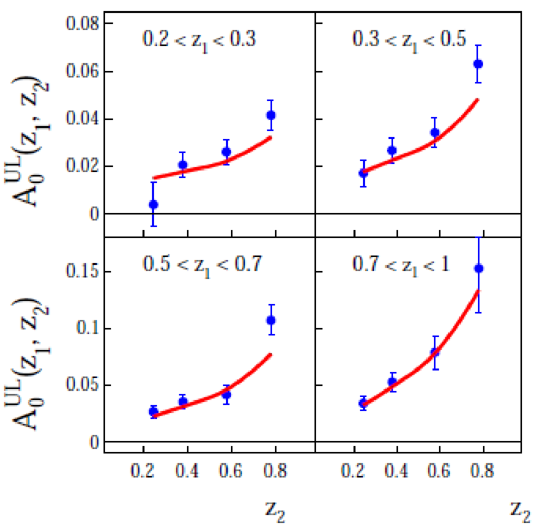
$$1 \lesssim \langle Q^2 \rangle \lesssim \ 21 \ \mathrm{GeV}^2$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

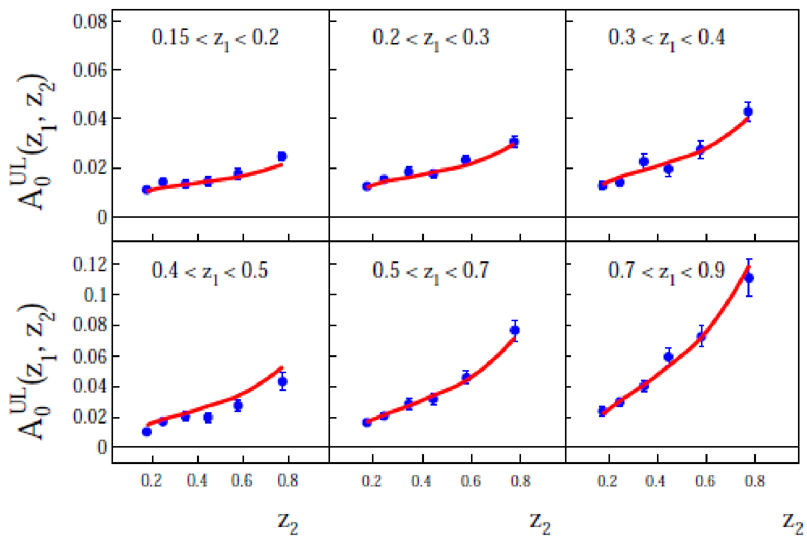
$$e^+e^- \to \pi\pi X$$

BELLE



$$Q^2 = 110 \text{ GeV}^2$$

BABAR

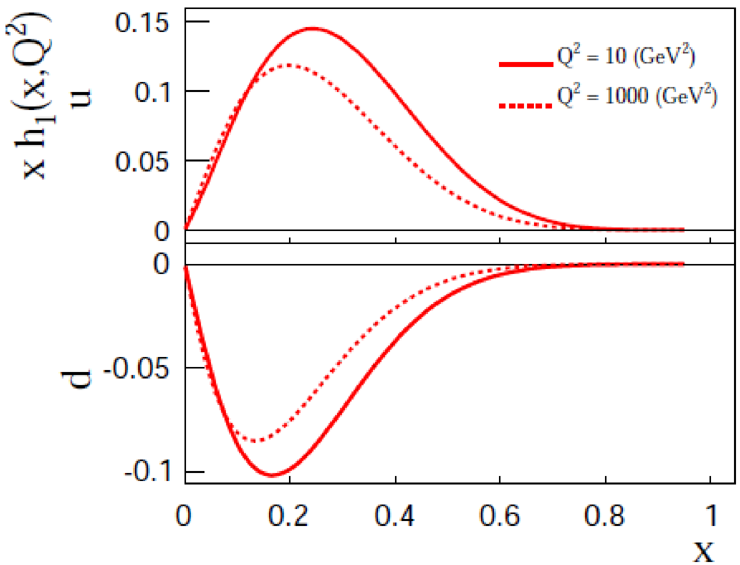


$$Q^2 = 110 \text{ GeV}^2$$

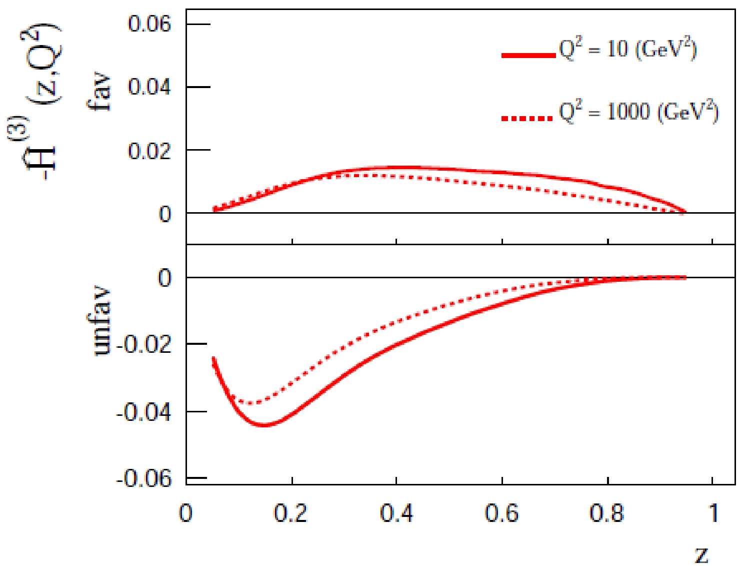
Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

Transversity



Collins



Positive u and negative d transversity

Positive favoured and negative unfavoured Collins FF

Compatible with LO extraction Anselmino et al 2009

Transversity and Collins FF

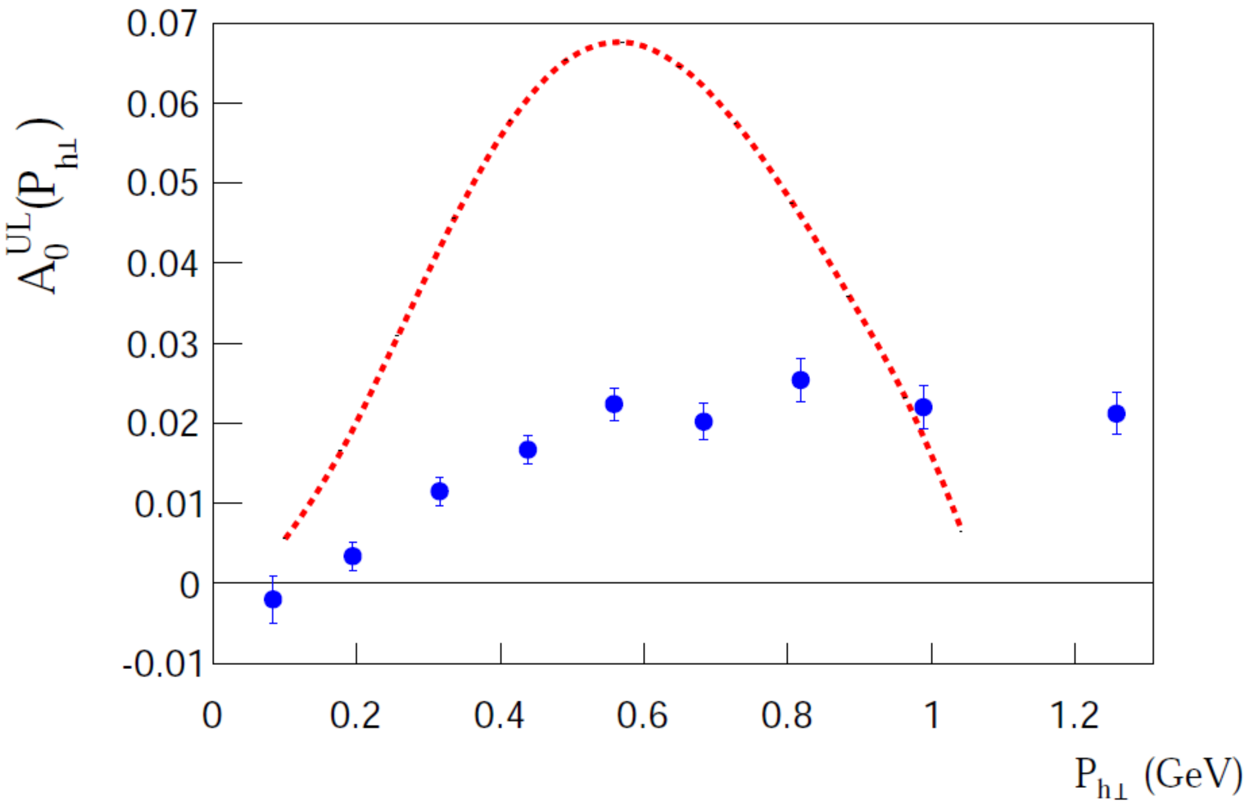
Kang-Prokudin-Sun-Yuan 2014

What are evolution effects?

$$e^+e^- \to \pi\pi X$$

No evolution:

$$Q^2 = 2.4\ \mathrm{GeV}^2$$



Transversity and Collins FF

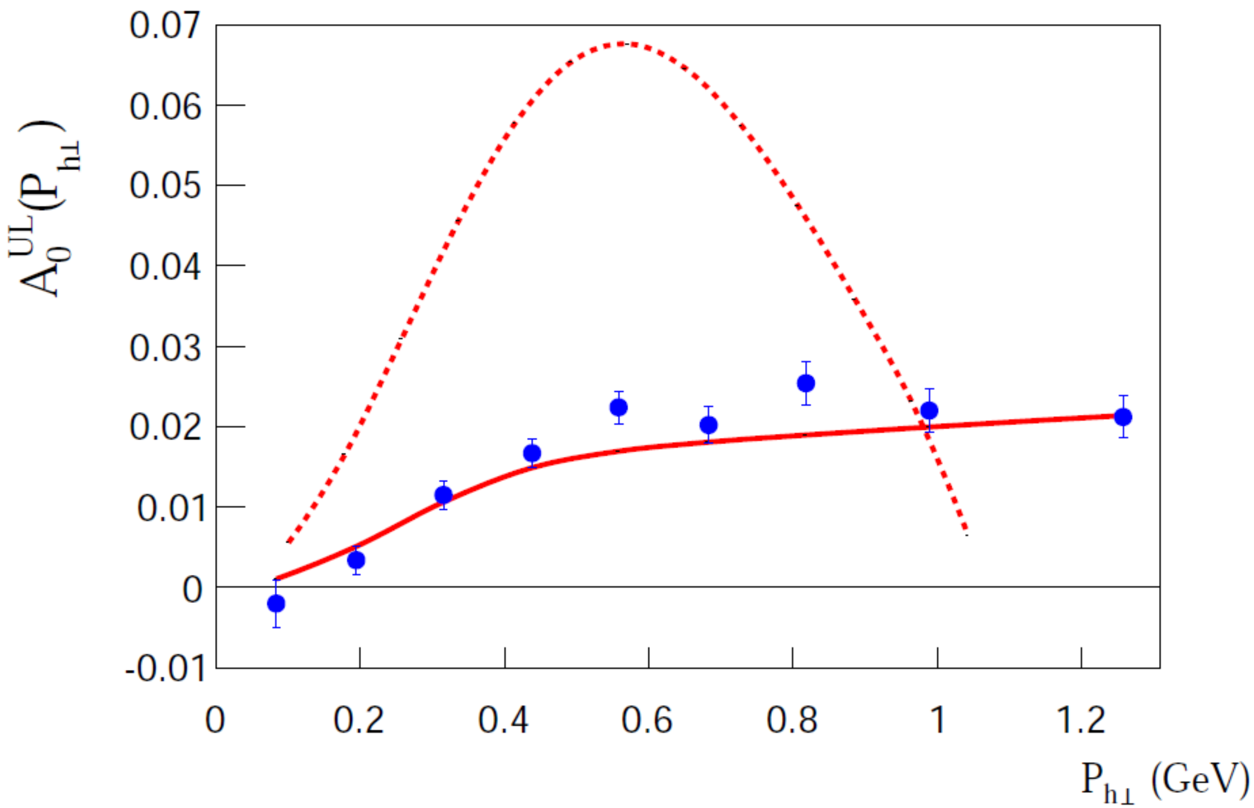
Kang-Prokudin-Sun-Yuan 2014

What are evolution effects?

$$e^+e^- \to \pi\pi X$$

No evolution: $Q^2 = 2.4\ \mathrm{GeV}^2$

NLL' evolution: $Q^2 = 110\ \mathrm{GeV}^2$



$$Q_1^2/Q_2^2 \simeq 50$$

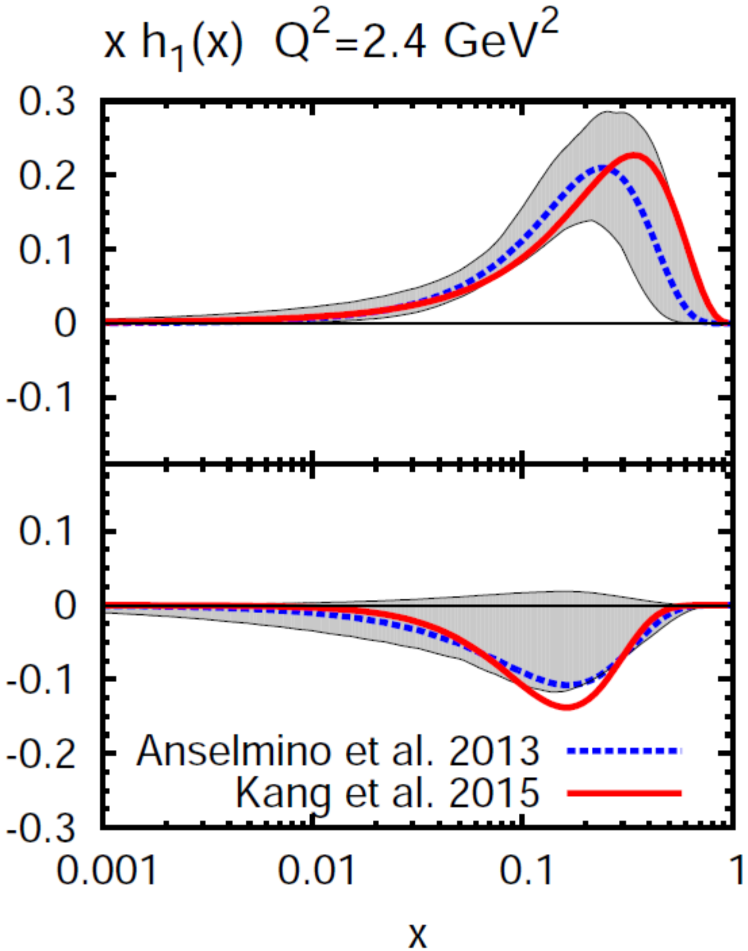
Asymmetry ratio ~ 3.5

Comparison to extractions

Kang-Prokudin-Sun-Yuan 2014

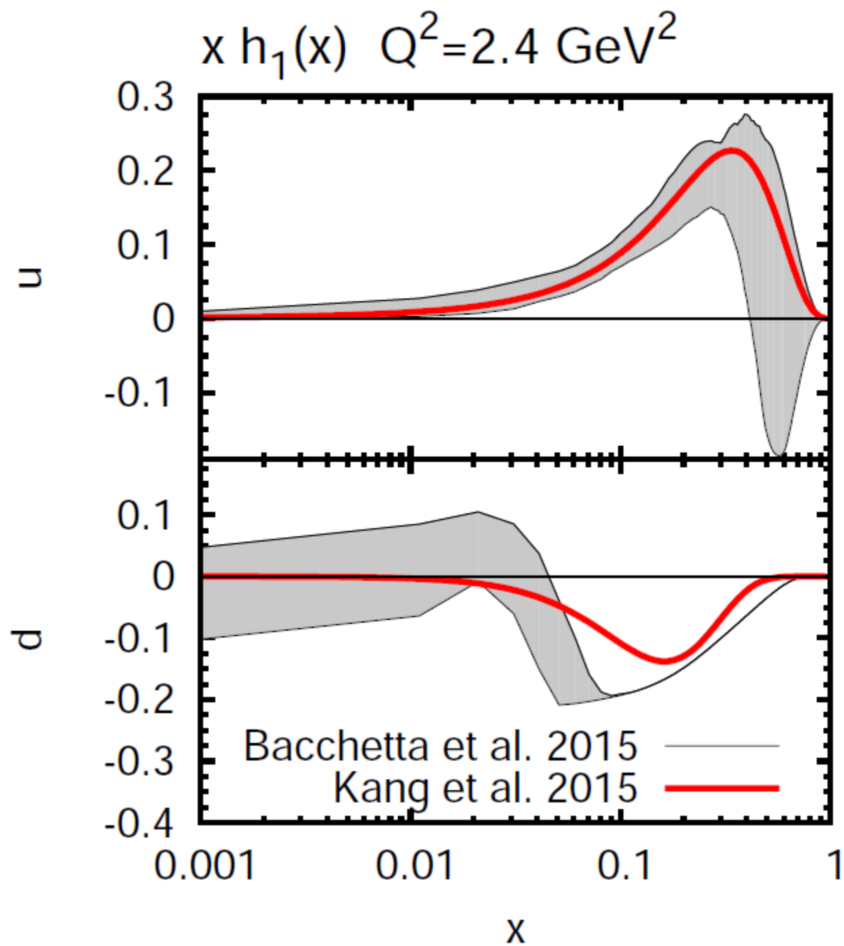
Torino 2013

Anselmino et al 2013



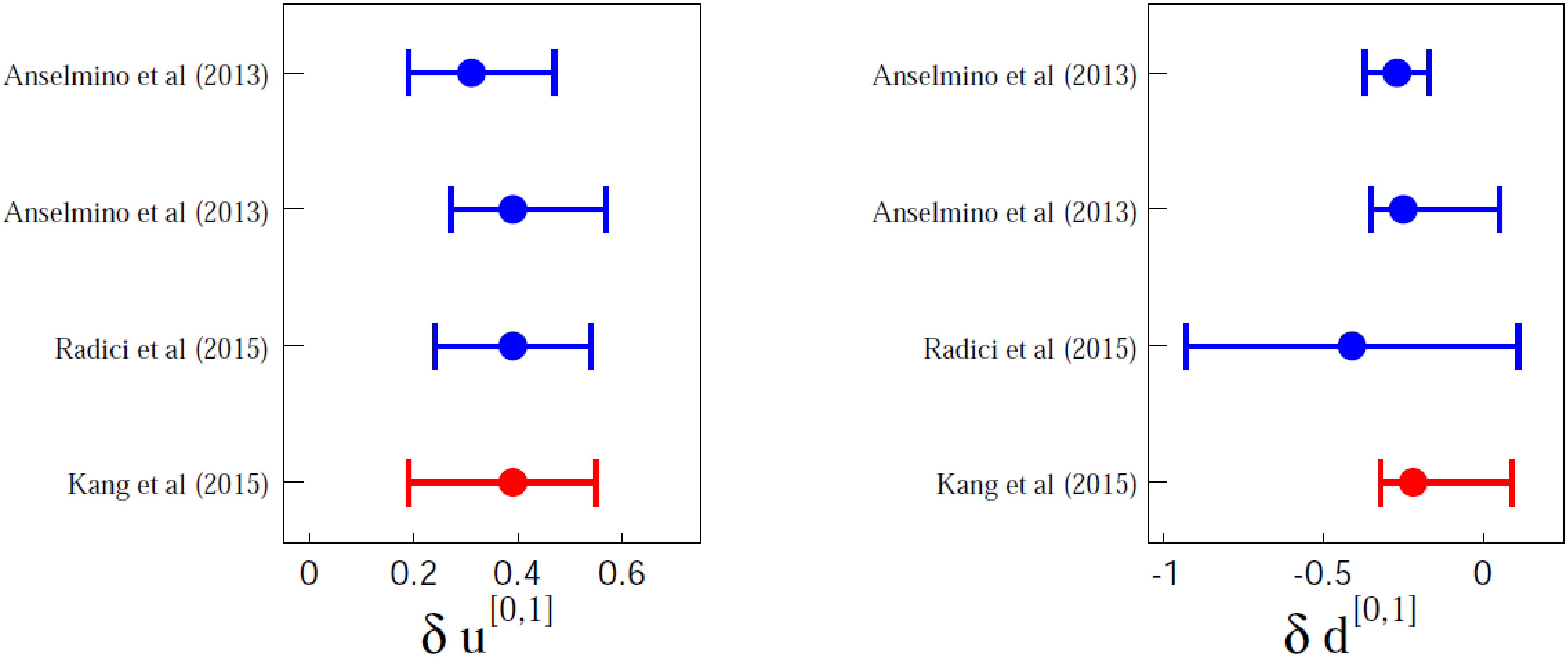
Pavia 2015

Radici et al 2015



Tensor charge

Kang-Prokudin-Sun-Yuan 2014

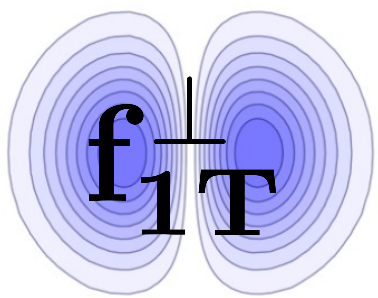


$$\delta u^{[0,1]} = +0.39^{+0.11}_{-0.07}$$

$$\delta d^{[0,1]} = -0.22^{+0.14}_{-0.08}$$

The most accurate modern extraction!

Sivers function



TMD distributions

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

Sivers function

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

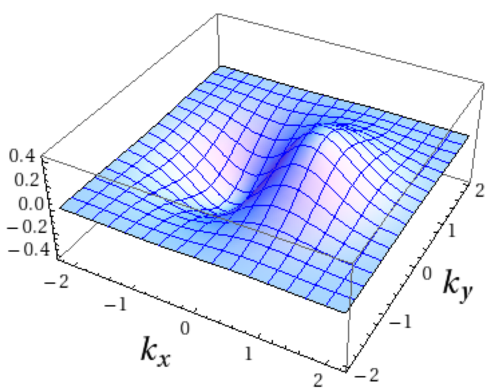
$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\epsilon_T^{ij} \mathbf{k_{Ti}} \mathbf{S_{Tj}}}{M}$$

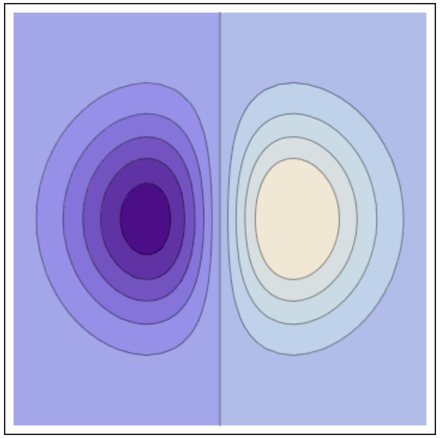
Correlation of the transverse spin and motion of the quarks

Suspect the spin is along Y direction: $S_T = (0, 1)$

Deformation in momentum space is: $k_x \cdot f(k_x^2 + k_y^2)$

This is called “dipole” deformation.





Sivers function

Echevarria-Idilbi-Kang-Vitev 2014

First NLL extraction from the data $A^{(1,2)}$ $B^{(1)}$

Sivers function is related to Qiu-Sterman function

Kang-Xiao-Yuan, 2011
Aybat-Collins-Rogers-Qiu, 2012

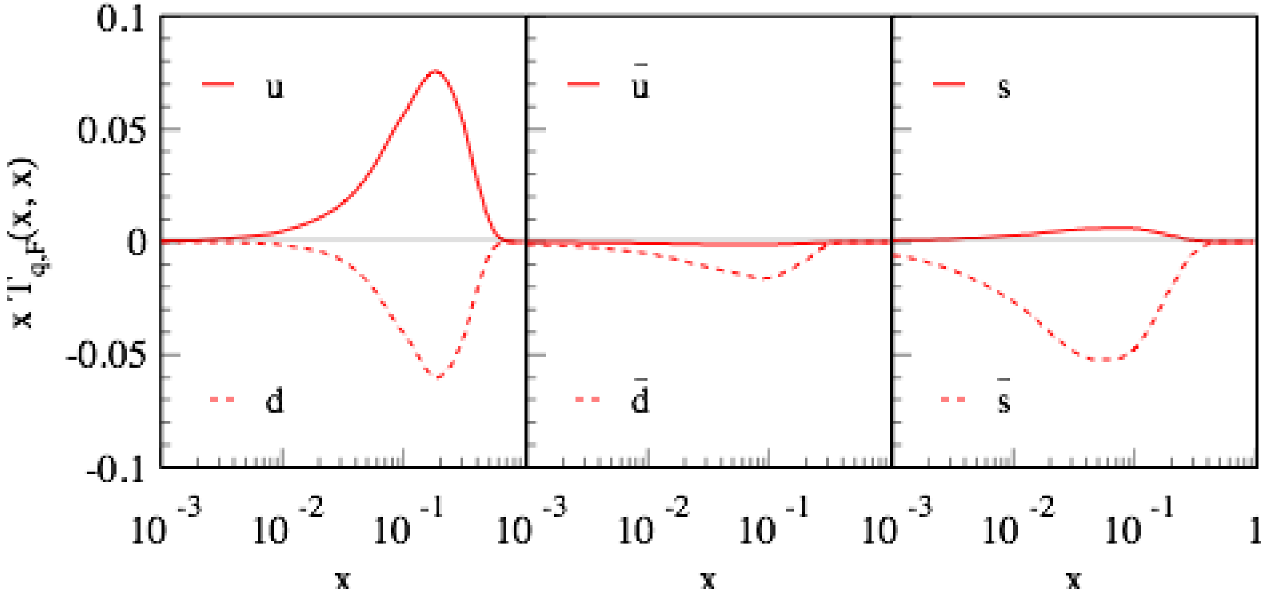
$$\tilde{f}_{1T}^{\perp,\alpha}(x,b;\mu_b) = -\frac{1}{M}\int d^2k_\perp e^{-ik_\perp b} k_\perp^\alpha f_{1T}^\perp(x,k_\perp;\mu_b)$$

$$\tilde{f}_{1T}^{\perp,\alpha}(x,b;\mu_b) = \left(\frac{-ib^\alpha}{2}\right) T(x,x;\mu_b)$$

Sivers function

Echevarria-Idilbi-Kang-Vitev 2014

First NLL extraction from the data



Compatible with previous extractions Anselmino et al 2007

Only u and d Sivers functions are constrained by existing data

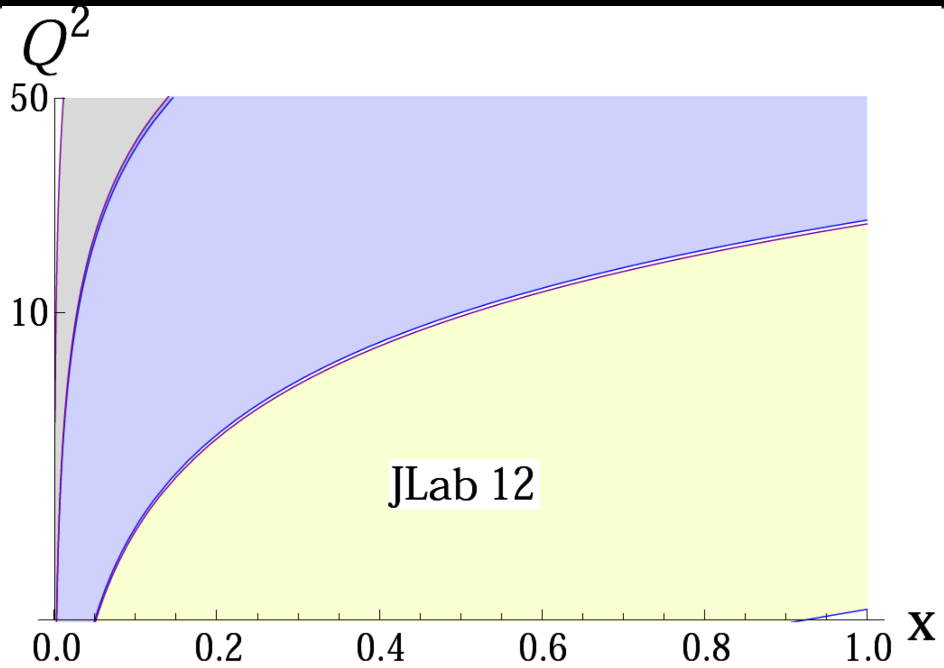
Sea quarks functions are largely unknown – RHIC data and EIC data are needed

Conclusions

~~Conclusions~~

FUTURE

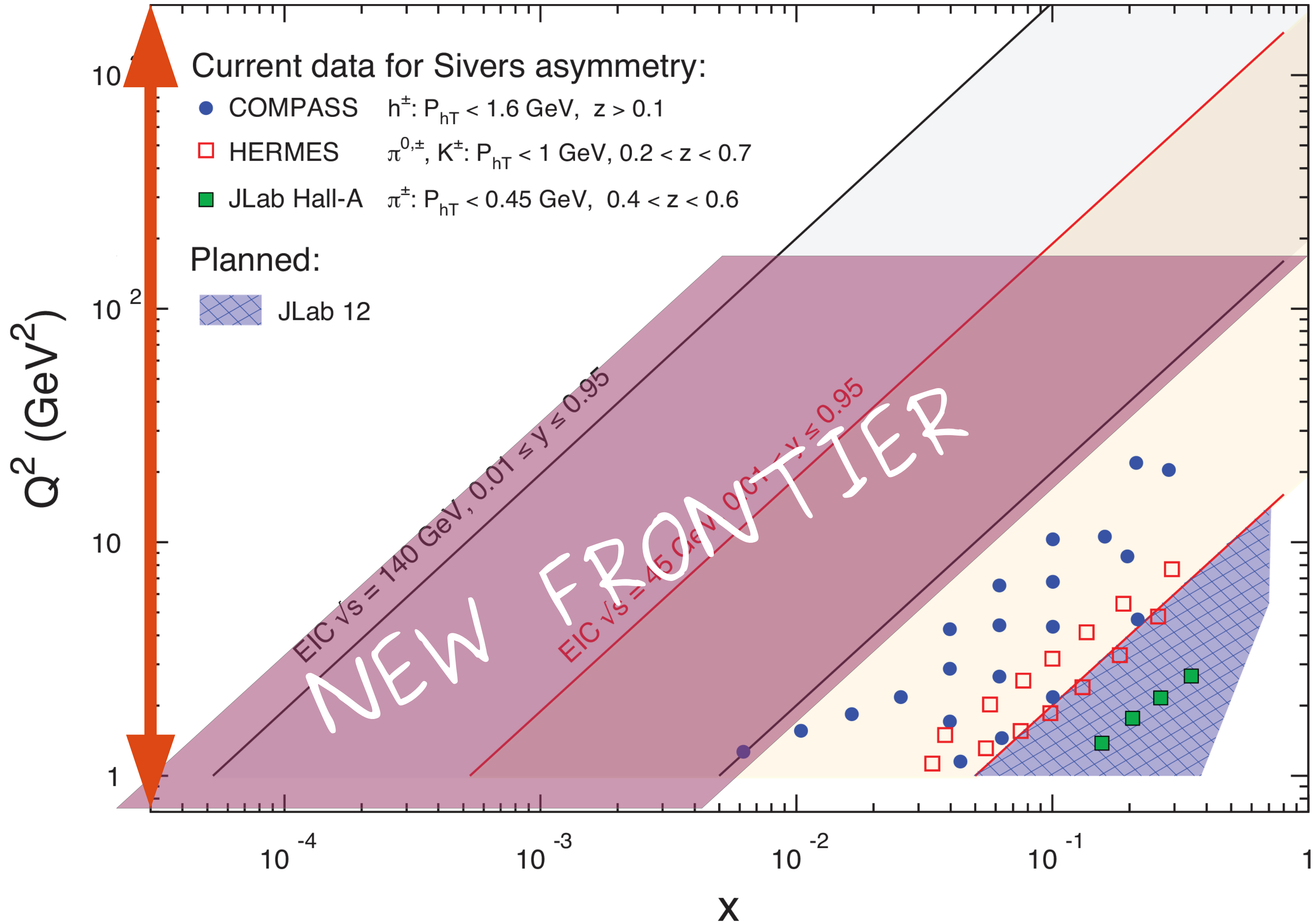
Jefferson Lab 12



Data in large-x region will become available from Jefferson Lab 12

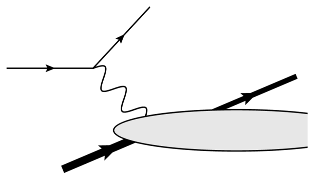
Precision is important – NLL and higher precision in perturbative calculations

Electron Ion Collider



Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

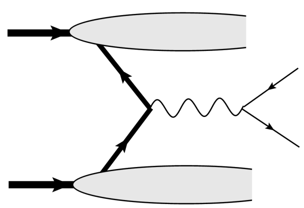
Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them



$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

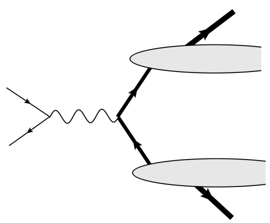
$$\ell + P \to \ell' + h + X$$



$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

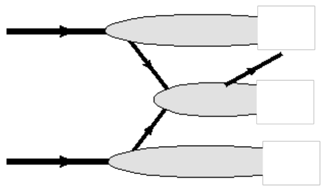
$$P_1 + P_2 \to \bar{\ell}\ell + X$$



$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$



$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Combining measurements from all above is important

THANK YOU!