## Polarised Two Pion Photoproduction For MesonEx

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Decay process can be studied in a number of related ways :
```

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Moments of spherical harmonic distributions
    Fourier analysis up to some truncation in L
    CLAS results (Battaglieri et al, 2000)
    still being analysed by JPAC
Spin Density Matrix Elements
    Classic Schilling, Seybouth and Wolf paper for Vector mesons
    Extended to electroproduction
    Assumes only P-wave contributions
    Recent GlueX results
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## Partial Wave Amplitudes

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        Allows multitude of contributing resonances of all l
        Mass dependence allows pole extraction
    GOAL for spectroscopy
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When analysing final states with CLAS12 that have $>2$ particles (+ e') We must consider the two-body decays when measuring cross sections and beam spin asymmetries if we integrate over decay angle. or else they are not reliable (they are a product of detector*physics)

## Polarised Photoproduction - Amplitudes and Moments

The vector $\boldsymbol{P}_{\gamma}$ encodes the information about the polarization of the beam [10]. Similarly, one defines

$$
\begin{equation*}
I(\Omega, \Phi)=I^{0}(\Omega)+\boldsymbol{I}(\Omega) \cdot \boldsymbol{P}_{\gamma}(\Phi), \tag{A7}
\end{equation*}
$$

with the vector of polarized intensities $\boldsymbol{I}=\left(I^{1}, I^{2}, I^{3}\right)$. The angular distribution can be expanded in unpolarized moment $H^{0}$ and polarized moments $\boldsymbol{H}=\left(H^{1}, H^{2}, H^{3}\right)$ via

$$
\begin{align*}
I^{0}(\Omega) & =\sum_{L M}\left(\frac{2 L+1}{4 \pi}\right) H^{0}(L M) D_{M 0}^{L *}(\phi, \theta, 0), \\
\boldsymbol{I}(\Omega) & =-\sum_{L M}\left(\frac{2 L+1}{4 \pi}\right) \boldsymbol{H}(L M) D_{M 0}^{L *}(\phi, \theta, 0) . \tag{A8b}
\end{align*}
$$

The extra minus sign in the definition of $\boldsymbol{H}$ ensures that $H^{1}(00)$ is positive for positive reflectivity waves, cf. Sect. D. The moments are expressed in terms of the $\eta \pi^{0}$ SDME:

$$
\begin{aligned}
H^{0}(L M) & =\sum_{\substack{\ell \ell^{\prime} \\
m m^{\prime}}}\left(\frac{2 \ell^{\prime}+1}{2 \ell+1}\right)^{1 / 2} C_{\ell^{\prime} 0 L 0}^{\ell 0} C_{\ell^{\prime} m^{\prime} L M}^{\ell m} \rho_{m m^{\prime}}^{\alpha, \ell \ell^{\prime}} \\
\boldsymbol{H}(L M) & =-\sum_{\substack{\ell \ell^{\prime} \\
m m^{\prime}}}\left(\frac{2 \ell^{\prime}+1}{2 \ell+1}\right)^{1 / 2} C_{\ell^{\prime} 0 L 0}^{\ell 0} C_{\ell^{\prime} m^{\prime} L M}^{\ell m} \rho_{m m^{\prime}}^{\ell \ell^{\prime}}
\end{aligned}
$$

$$
\begin{aligned}
& { }^{(\epsilon)} \rho_{m m^{\prime}}^{0, \ell \ell^{\prime}}=\kappa \sum_{k}\left([\ell]_{m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{m^{\prime} ; k}^{(\epsilon) *}+(-1)^{m-m^{\prime}}[\ell]_{-m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{-m^{\prime} ; k}^{(\epsilon) *}\right) \\
& { }^{(\epsilon)} \rho_{m m^{\prime}}^{1, \ell \ell^{\prime}}=-\epsilon \kappa \sum_{k}\left((-1)^{m}[\ell]_{-m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{m^{\prime} ; k}^{(\epsilon) *}+(-1)^{m^{\prime}}[\ell]_{m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{-m^{\prime} ; k}^{(\epsilon) *}\right) \\
& { }^{(\epsilon)} \rho_{m m^{\prime}}^{2, \ell \ell^{\prime}}=-i \epsilon \kappa \sum_{k}\left((-1)^{m}[\ell]_{-m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{m^{\prime} ; k}^{(\epsilon) *}-(-1)^{m^{\prime}}[\ell]_{m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{-m^{\prime} ; k}^{(\epsilon) *}\right)
\end{aligned}
$$

$$
{ }^{(\epsilon)} \rho_{m m^{\prime}}^{3, \ell \ell^{\prime}}=\kappa \sum_{k}\left([\ell]_{m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{m^{\prime} ; k}^{(\epsilon) *}-(-1)^{m-m^{\prime}}[\ell]_{-m ; k}^{(\epsilon)}\left[\ell^{\prime}\right]_{-m^{\prime} ; k}^{(\epsilon) *}\right)
$$

## Ambiguities in Linear Polarised PWA

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Ambiguities in partial wave analysis of two spinless meson photoproduction

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In the general linear polarised case there
is only a single "trivial" complex conjugate ambiguous solution
-Specific cases with 0 magnitude waves, can complicate this
-Should limit 1 wave per reflectivity to have +ve imaginery part
=> Remove ambiguity from MaxLikelihood search
*our plots removed the trivial ambiguity
**but so does the electron polarisation !

## Moments in terms of S,P waves

## Parameters are normalised : <br> $\mathrm{H}^{\bullet}(0,0)=2$ and PW magnitudes $<1$

$$
\begin{aligned}
H^{0}(0,0)= & 2\left({ }^{+} S^{+} S\right)+2\left({ }^{-} S^{-} S\right)+2\left({ }^{+} P_{-1}^{+} P_{-1}\right)+2\left({ }^{+} P_{+1}^{+} P_{+1}\right) \\
& +2\left({ }^{-} P_{-1}^{-} P_{-1}\right)+2\left({ }^{-} P_{+1}^{-} P_{+1}\right)+2\left({ }^{+} P_{0}^{+} P_{0}\right)+2\left({ }^{-} P_{0}^{-} P_{0}\right) \\
H^{1}(0,0)= & \left.2\left({ }^{+} S^{+} S\right)+-2\left({ }^{-} S^{-} S\right)+-4\left({ }^{+} P_{+1}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P+1}-^{+} \phi_{P-1}\right)\right) \\
& \left.+4\left({ }^{-} P_{+1}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P+1}-{ }^{-} \phi_{P-1}\right)\right)+2\left({ }^{+} P_{0}^{+} P_{0}\right)+-2\left({ }^{-} P_{0}^{-} P_{0}\right) \\
H^{0}(1,0)= & 2.3\left({ }^{+} S^{+} P_{0}\right) \cos \left(\left(^{+} \phi_{S}-^{+} \phi_{P 0}\right)\right) \\
& \left.+2.3\left({ }^{-} S^{-} P_{0}\right) \cos \left({ }^{-} \phi_{S}-^{-} \phi_{P 0}\right)\right) \\
H^{1}(1,0)= & \left.2.3\left({ }^{+} S^{+} P_{0}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P 0}\right)\right) \\
& \left.-2.3\left({ }^{-} S^{-} P_{0}\right) \cos \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P 0}\right)\right) \\
H^{0}(1,1)= & \left.\left.-1.2\left({ }^{+} S^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P-1}\right)\right)+1.2\left({ }^{+} S^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P+1}\right)\right) \\
& \left.\left.-1.2\left({ }^{-} S^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{S}-^{-} \phi_{P-1}\right)\right)+1.2\left({ }^{-} S^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{S}-^{-} \phi_{P+1}\right)\right) \\
H^{1}(1,1)= & \left.\left.-1.2\left({ }^{+} S^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P-1}\right)\right)+1.2\left({ }^{+} S^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+1.2\left({ }^{-} S^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P-1}\right)\right)+-1.2\left({ }^{-} S^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P+1}\right)\right) \\
H^{2}(1,1)= & \left.\left.-1.2\left({ }^{+} S^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{S}-^{+} \phi_{P-1}\right)\right)+-1.2\left({ }^{+} S^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{S}-{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+1.2\left({ }^{-} S^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P-1}\right)\right)+1.2\left({ }^{-} S^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P+1}\right)\right) \\
H^{3}(1,1)= & \left.\left.1.2\left({ }^{+} S^{+} P_{-1}\right) \sin \left({ }^{+} \phi_{S}-{ }^{+} \phi_{P-1}\right)\right)+1.2\left({ }^{+} S^{+} P_{+1}\right) \sin \left({ }^{+} \phi_{S}-{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+1.2\left({ }^{-} S^{-} P_{-1}\right) \sin \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P-1}\right)\right)+1.2\left({ }^{-} S^{-} P_{+1}\right) \sin \left({ }^{-} \phi_{S}-{ }^{-} \phi_{P+1}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
H^{0}(2,0)= & -0.4\left({ }^{+} P_{-1}^{+} P_{-1}\right)+-0.4\left({ }^{+} P_{+1}^{+} P_{+1}\right)+-0.4\left({ }^{-} P_{-1}^{-} P_{-1}\right) \\
& -0.4\left({ }^{-} P_{+1}^{-} P_{+1}\right)+0.8\left({ }^{+} P_{0}^{+} P_{0}\right)+0.8\left({ }^{-} P_{0}^{-} P_{0}\right) \\
H^{1}(2,0)= & \left.\left.0.8\left({ }^{+} P_{+1}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P+1}-{ }^{+} \phi_{P-1}\right)\right)-0.8\left({ }^{-} P_{+1}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P+1}-{ }^{-} \phi_{P-1}\right)\right) \\
& +0.8\left({ }^{+} P_{0}^{+} P_{0}\right)-0.8\left({ }^{-} P_{0}^{-} P_{0}\right) \\
H^{0}(2,1)= & \left.\left.-0.7\left({ }^{+} P_{0}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P 0}-^{+} \phi_{P-1}\right)\right)+0.7\left({ }^{+} P_{0}^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{P 0}-^{+}{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.-0.7\left({ }^{-} P_{0}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P 0}-^{-} \phi_{P-1}\right)\right)+0.7\left({ }^{-} P_{0}^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{P 0}-{ }^{-}{ }^{-} \phi_{P+1}\right)\right) \\
H^{1}(2,1)= & \left.\left.-0.7\left({ }^{+} P_{0}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P 0}-^{+} \phi_{P-1}\right)\right)+0.7\left({ }^{+} P_{0}^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{P 0}-{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+0.7\left({ }^{-} P_{0}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P 0}-{ }^{-} \phi_{P-1}\right)\right)-0.7\left({ }^{-} P_{0}^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{P 0}-{ }^{-} \phi_{P+1}\right)\right) \\
H^{2}(2,1)= & \left.\left.-0.7\left({ }^{+} P_{0}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P 0}-^{+} \phi_{P-1}\right)\right)-0.7\left({ }^{+} P_{0}^{+} P_{+1}\right) \cos \left({ }^{+} \phi_{P 0}-{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+0.7\left({ }^{-} P_{0}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P 0}-^{-} \phi_{P-1}\right)\right)+0.7\left({ }^{-} P_{0}^{-} P_{+1}\right) \cos \left({ }^{-} \phi_{P 0}-{ }^{-} \phi_{P+1}\right)\right) \\
H^{3}(2,1)= & \left.\left.0.7\left({ }^{+} P_{0}^{+} P_{-1}\right) \sin \left({ }^{+} \phi_{P 0}-^{+} \phi_{P-1}\right)\right)+0.7\left({ }^{+} P_{0}^{+} P_{+1}\right) \sin \left({ }^{+} \phi_{P 0}-{ }^{+} \phi_{P+1}\right)\right) \\
& \left.\left.+0.7\left({ }^{-} P_{0}^{-} P_{-1}\right) \sin \left({ }^{-} \phi_{P 0}-{ }^{-} \phi_{P-1}\right)\right)+0.7\left({ }^{-} P_{0}^{-} P_{+1}\right) \sin \left({ }^{-} \phi_{P 0}-{ }^{-} \phi_{P+1}\right)\right) \\
H^{0}(2,2)= & \left.-0.98\left({ }^{+} P_{+1}^{+} P_{-1}\right) \cos \left({ }^{+} \phi_{P+1}-{ }^{+} \phi_{P-1}\right)\right) \\
& \left.-0.98\left({ }^{-} P_{+1}^{-} P_{-1}\right) \cos \left({ }^{-} \phi_{P+1}-{ }^{-} \phi_{P-1}\right)\right) \\
H^{1}(2,2)= & 0.49\left({ }^{+} P_{-1}^{+} P_{-1}\right)+0.49\left({ }^{+} P_{+1}^{+} P_{+1}\right) \\
& -0.49\left({ }^{-} P_{-1}^{-} P_{-1}\right)-0.49\left({ }^{-} P_{+1}^{-} P_{+1}\right) \\
H^{2}(2,2)= & 0.49\left({ }^{+} P_{-1}^{+} P_{-1}\right)-0.49\left({ }^{+} P_{+1}^{+} P_{+1}\right) \\
& -0.49\left({ }^{-} P_{-1}^{-} P_{-1}\right)+0.49\left({ }^{-} P_{+1}^{-} P_{+1}\right) \\
H^{3}(2,2)= & \left.0.98\left({ }^{+} P_{+1}^{+} P_{-1}\right) \sin \left({ }^{+} \phi_{P+1}-{ }^{+} \phi_{P-1}\right)\right) \\
& \left.+0.98\left({ }^{-} P_{+1}^{-} P_{-1}\right) \sin \left({ }^{-} \phi_{P+1}-{ }^{-} \phi_{P-1}\right)\right)
\end{aligned}
$$

For vector mesons these moments $=0$ as $S$-wave $=0$

## Spin Density Matrix Elements, <br> photoproduction

## GlueX results



$\rho_{00}^{1}$

$\operatorname{Imp}_{10}^{2}$


Measurement of Spin-Density Matrix Elements in $\rho(770)$ Production with a Linearly Polarized

## Photon Beam at $E_{\gamma}=8.2-8.8 \mathrm{GeV}$

$$
\begin{align*}
W^{0}(\cos \vartheta, \varphi) & =\frac{3}{4 \pi}\left(\frac{1}{2}\left(1-\rho_{00}^{0}\right)+\frac{1}{2}\left(3 \rho_{00}^{0}-1\right) \cos ^{2} \vartheta\right.  \tag{10}\\
& \left.-\sqrt{2} \operatorname{Re} \rho_{10}^{0} \sin 2 \vartheta \cos \varphi-\rho_{1-1}^{0} \sin ^{2} \vartheta \cos 2 \varphi\right) \\
W^{1}(\cos \vartheta, \varphi) & =\frac{3}{4 \pi}\left(\rho_{11}^{1} \sin ^{2} \vartheta+\rho_{00}^{1} \cos ^{2} \vartheta\right.  \tag{11}\\
& \left.-\sqrt{2} \operatorname{Re} \rho_{10}^{1} \sin 2 \vartheta \cos \varphi-\rho_{1-1}^{1} \sin ^{2} \vartheta \cos 2 \varphi\right) \\
W^{2}(\cos \vartheta, \varphi) & =\frac{3}{4 \pi}\left(\sqrt{2} \operatorname{Im} \rho_{10}^{2} \sin 2 \vartheta \sin \varphi\right. \\
& \left.+\operatorname{Im} \rho_{1-1}^{2} \sin ^{2} \vartheta \sin 2 \varphi\right) \tag{12}
\end{align*}
$$

$$
\begin{aligned}
\rho_{00}^{0} & =\frac{1}{3}\left(5 H^{0}(20)+1\right) \\
\Re \rho_{10}^{0} & =\frac{5}{\sqrt{12}} H^{0}(21) \\
\rho_{1-1}^{0} & =-\frac{5}{\sqrt{6}} H^{0}(22) \\
\rho_{11}^{1} & =-\frac{1}{3} H^{1}(00) \\
\rho_{00}^{1} & =-\frac{5}{2} H^{1}(20)-\frac{1}{3} H^{1}(00) \\
\Re \rho_{10}^{1} & =-\frac{5}{\sqrt{12}} H^{1}(21) \\
\rho_{1-1}^{1} & =\frac{5}{\sqrt{6}} H^{1}(22) \\
\Im \rho_{10}^{2} & =-\frac{5}{\sqrt{12}} H^{2}(21) \\
\Im \rho_{1-1}^{2} & =\frac{5}{\sqrt{6}} H^{2}(22) \\
\Im \rho_{10}^{3} & =-\frac{5}{\sqrt{12}} H^{3}(21) \\
\Im \rho_{1-1}^{3} & =\frac{5}{\sqrt{6}} H^{3}(22)
\end{aligned}
$$

## Extracting Partial Waves from GlueX SDMEs

$$
\mathcal{I}(\Omega, \Phi)=\mathcal{I}_{0}(\Omega)-\mathcal{I}_{1}(\Omega) P_{\gamma L} \cos 2 \Phi-\mathcal{I}_{2}(\Omega) P_{\gamma L} \sin 2 \Phi-\mathcal{I}_{3}(\Omega) P_{\gamma C}
$$

Generate events from SDME intensities
$\mathcal{I}_{0}(\Omega)=\frac{3}{4 \pi}\left\{\frac{1}{2}\left(1-\rho_{00}^{0}\right)+\frac{1}{2}\left(3 \rho_{00}^{0}-1\right) \cos ^{2} \theta-\sqrt{2} \Re\left[\rho_{10}^{0}\right] \sin 2 \theta \cos \phi-\rho_{1-1}^{0} \sin ^{2} \theta \cos 2 \phi\right\}$

$$
\mathcal{I}_{1}(\Omega)=\frac{3}{4 \pi}\left\{\rho_{11}^{1} \sin ^{2} \theta+\rho_{00}^{1} \cos ^{2} \theta-\sqrt{2} \rho_{10}^{1} \sin 2 \theta \cos \phi-\rho_{1-1}^{1} \sin ^{2} \theta \cos 2 \phi\right\}
$$

$\mathcal{I}_{2}(\Omega)=\frac{3}{4 \pi}\left\{\sqrt{2} \Im \rho_{10}^{2} \sin 2 \theta \sin \phi+\Im \rho_{1-1}^{2} \sin ^{2} \theta \sin 2 \phi\right\}$
Then fit to extract partial waves for $\rho$ production, $S, P_{0}, P_{1}, P_{-1}$

$$
\begin{aligned}
I^{0}(\Omega) & =\kappa \sum_{\epsilon, k}\left|U_{k}^{(\epsilon)}(\Omega)\right|^{2}+\left|\widetilde{U}_{k}^{(\epsilon)}(\Omega)\right|^{2}, \\
I^{1}(\Omega) & =-\kappa \sum_{\epsilon, k} 2 \epsilon \operatorname{Re}\left(U_{k}^{(\epsilon)}(\Omega)\left[\widetilde{U}_{k}^{(\epsilon)}(\Omega)\right]^{*}\right), \\
I^{2}(\Omega) & =-\kappa \sum^{2} 2 \epsilon \operatorname{Im}\left(U_{k}^{(\epsilon)}(\Omega)\left[\widetilde{U}_{k}^{(\epsilon)}(\Omega)\right]^{*}\right),
\end{aligned}
$$



$$
I^{3}(\Omega)=\kappa \sum_{\epsilon, k}\left|U_{k}^{(\epsilon)}(\Omega)\right|^{2}-\left|\widetilde{U}_{k}^{(\epsilon)}(\Omega)\right|^{2}
$$

i.e. in principle helicity SDMEs already constrained => Can perform electron beam polarimetry

## Polarisation asymmetries for



## Quasi-real Vector Meson Electroproduction

$$
\begin{equation*}
W\left(\cos \theta, \phi, \Phi, \alpha_{2}=0, \pi\right)=W^{\mathrm{unpol}}(\cos \theta, \phi, \Phi) \pm W^{\mathrm{long} \mathrm{pol}}(\cos \theta, \phi, \Phi) \tag{88}
\end{equation*}
$$

HOW TO ANALYSE VECTOR-MESON PRODUCTION
IN INELASTIC LEPTON SCATTERING
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## G. WOLF

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$W^{\mathrm{unpol}}(\cos \theta, \phi, \Phi)=\frac{1}{1+(\epsilon+\delta) R} \frac{3}{4 \pi}$
$\times\left[\frac{1}{2}\left(1-\rho_{00}^{0}\right)+\frac{1}{2}\left(3 \rho_{00}^{0}-1\right) \cos ^{2} \theta-\sqrt{2} \operatorname{Re} \rho_{10}^{0} \sin 2 \theta \cos \phi-\rho_{1-1}^{0} \sin ^{2} \theta \cos 2 \phi\right.$
$-\epsilon \cos 2 \Phi\left\{\rho_{11}^{1} \sin ^{2} \theta+\rho_{00}^{1} \cos ^{2} \theta-\sqrt{2} \operatorname{Re} \rho_{10}^{1} \sin 2 \theta \cos \phi-\rho_{1-1}^{1} \sin ^{2} \theta \cos 2 \phi\right\}$
$-\epsilon \sin 2 \Phi\left\{\sqrt{2} \operatorname{Im} \rho_{10}^{2} \sin 2 \theta \sin \phi+\operatorname{Im} \rho_{1-1}^{2} \sin ^{2} \theta \sin 2 \phi\right\}$
$W^{\text {long pol }}(\cos \theta, \phi, \Phi)=\frac{1}{1+(\epsilon+\delta) R} \frac{3}{4 \pi}$

$$
X P\left[\sqrt{1-\epsilon^{2}}\left\{\sqrt{2} \operatorname{Im} \rho_{10}^{3} \sin 2 \theta \sin \phi+\operatorname{Im} \rho_{1-1}^{3} \sin ^{2} \theta \sin 2 \phi\right\}+\right.
$$

For low $Q^{2}$ we assume,

$$
\rho_{\gamma}(\Phi)=\frac{1}{2}\left(1-\epsilon \cos 2 \Phi \sigma_{x}-\epsilon \sin 2 \Phi \sigma_{y}-P_{b e a m} \sqrt{1-\epsilon^{2}} \sigma_{z}\right)
$$

With $\varepsilon$ the virtual photon polarisation

Analyse both missing pion topologies
In general these should not pass the mesonex trigger ( $2 *$ FD tracks) ? But actually most events seem to have this trigger bit

There is a prescaled (32) FT*FD*CD trigger, cut on this trigger bit ! currently trigger is not part of simulation

The $\pi$ - may be detected in the $F T$ and assumed as the e-. This peaks close to missing pion events. Cut on $\theta_{\pi}>\theta_{e}$ : removes almost all, but effects acceptance

Additional cut on $70<\theta_{\mathrm{p}}<130^{\circ}$, removes some background but not signal
Isolate exclusive signal with sPlots fits to missing mass squared - split data in $W$, $t$, and $\cos \theta$ to reduce dependencies on $M^{2}$

## Splot Background Subtraction fits

Fit with Simulation template for signal , polynomial for background Need to add 3 pion simulation template for background





$$
\begin{aligned}
& t=-0.5 \\
& \cos \theta=0.7
\end{aligned}
$$

$$
\cos ^{00}
$$



Fit components for Pi2MissMass2





Exclusive Topology 1.8M



Missing pion Topology





Due to large (8 degree) forward hole, acceptance is low at low mass (opening angle) Acceptance recovered by reconstructing 1 pion, but prescaled in the trigger


Additional Cuts :
3. $0<W<4.2$
$0.6<M(2 \pi)<0.9$
Correct for pol.

Shape OK
Wrong sign

Actual Data From GlueX results
"Fit" to Data

Prediction esults

$\cos \theta$

"Fit" to Data

Prediction From GlueX results

Shape OK
Wrong sign


Additional Cuts :
3. $0<W<4.2$
$0.6<M(2 \pi)<0.9$
Correct for pol.
$-0.5<\mathrm{t}^{\prime}<-0.1$

$\cos \boldsymbol{\theta}$



## Additional Cuts :

3. $0<W<4.2$
$0.6<M(2 \pi)<0.9$
Correct for pol.

Data has larger H(22) moment.
=> Larger $\mathrm{P}_{-1}$
Smaller $P_{0}$

Not good consistency at the moment with GlueX and between W bins

Particular problem with rho000 due to $\cos \theta$
distribution





Rho1 10



## Fit Projections

| Black points - data | Top - low t (-0.1) |
| :--- | :--- |
| Red line Fit - result Bottom - high t (-0.6) |  |



Fit components for Pi2MesonCosThHel



Fit components for Pi2MesonPhiHel


Top - low t (-0.1)
Bottom - high t (-0.6)





Fit components for PolPhi





## Proper Fits : Moments

H_A_L_M
A=> polarisation
$L=$ Legendre
$M=\phi$ modulation





## Proper Fits :Partial Waves

Results more or less follow expectations :

Large ${ }^{+} \mathbf{P}_{+1}(S-c h a n n e l$ hel. Cons.)
Other 'P grow with t'
-ve reflectivity should be smaller
S-waves should be small ~0

s
 +S


S


## 

${ }^{-1}$

$+P_{-1}$

$\mathrm{P}_{-1}$


$\mathrm{P}_{0}$

$+P_{0}$

$-\mathrm{P}_{0}$


$P_{1}$

$+P_{1}$

$-P_{1}$


## Beam Polarimetry

When performing PWA we leave the "circular" pol. As a free parameter, thereby extracting it.

From this the beam polarisation may be calculated.

To get correct absolute polarisation we require excellent acceptance correction.
Not there yet!!!

Derived photon circular Polarisation


## Relative Beam Polarisation Monitor

In principle we could use the Helicity dependent Spherical

Helicity Moments Harmonics to monitor beam polarisation

Here we take all events and Fit $H^{0}$ and $H^{1}$ moments for each run in Spring 2019 data

Fractional uncertainty 5-10\%/run


## Conclusions

MesonEx aims to extract Partial Wave Amplitudes for a number of reactions

Currently we are using $\rho$ photoproduction as a validation of method and Experimental effects (backgrounds, acceptances)

We also measure SDMEs and Spherical Harmonic moments for this
Currently we see significant discrepancies as a result we need to :
Analyse exclusive final state - low background,low acceptance for $\rho$
Improve background subtractions - use more simulated models
Apply momentum and efficiency corrections
Apply trigger effects in simulations
In addition this reaction may potentially be used as an absolute and relative beam polarimeter


Exclusive Topology $\mathrm{Q}^{2}>1.5$ \&\& $\mathrm{W}>3$ 0.3M





Larger $\mathrm{N}^{*}$ contribution than for Quasi-real photoproduction

- possibly just due to acceptance (larger transerver momentum in final state)
- how to analyse?


## Missing pion Topology $\mathrm{Q}^{2}>1.5$ \&\& W > 3 0.6M









## Example Moments

Considering only +ve reflectivity S,D-,D0,D+ waves The expressions for $\mathrm{H}^{\alpha}(4,2)$ are relatively straightforward (minus Clebsh Gordan coeffs)

```
H
H
H
H}\mp@subsup{\mathbf{H}}{}{(4,2)}=\mathbf{2((D+)(D-)}\operatorname{sin}(\mp@subsup{\varphi}{\mp@subsup{\textrm{D}}{+}{+}}{}-\mp@subsup{\varphi}{\textrm{D}.}{})
```

Here we have 4 equations with 4 unknowns and it is clear we can extract
The magnitudes (sum and difference of $\mathrm{H}^{1}(42)$ and $\mathrm{H}^{2}(42)$
The phases (from ratio of $\mathrm{H}^{9}(42)$ and $\mathrm{H}^{3}(42)$ ).
Without $\mathrm{H}^{3}(42)$ we could just extract $\cos \left(\varphi_{\mathrm{D}_{+}-\varphi_{\mathrm{D}}}\right)$ leaving a sign ambiguity in ( $\varphi_{\mathrm{D}+}-\varphi_{\mathrm{D}}$ ).


Improve ~25\% Red to blue


Phases relative to $\mathrm{P}_{+1}$


Improve ~200\% Red to blue
Linear Pol Elliptical Pol EllipticalFit Pol Truth (== SDME fit results)



Improve ~15\% Red to blue



Linear Pol
Less accurate results for Elliptical Pol
EllipticalFit Pol Truth -ve reflectivity

Circular polarisation makes no impact


Perhaps because magnitudes are Small (|+ve| $\left.{ }^{2}=0.97,|-v e|^{2}=0.03\right)$ consistent with truth

Or small true phase diff(P0-P1)



Linear Pol Elliptical Pol


## Actual 0.34

Phases relative to $P_{+1}$


Linear Fit unknown Pol Truth (== SDME fit results)


Comparing red to black little loss in accuracy


Linear Pol
Elliptical Pol
Elliptical unknown Lin. and circ. Pol Truth


Phases relative to $\mathbf{P}_{+1}$


## Actual 0.34

Not knowing polarisations does not effect results! (Blue and black)


## Photon SDM Schilling,

For linearly polarized photons eq. (11) reads:

$$
|\gamma\rangle=-\frac{1}{\sqrt{2}}\left(\mathrm{e}^{-i \Phi}\left|\lambda_{\gamma}=+1\right\rangle-\mathrm{e}^{i \Phi}\left|\lambda_{\gamma}=-1\right\rangle\right)
$$

where $\Phi$ is the angle between the polarization vector of the photon, $\varepsilon=$ $(\cos \Phi, \sin \Phi, 0)$, and the production plane $(x, z$ plane) (note: our definition of $\Phi$ differs by a sign from that of ref. [4]). The density matrix is

$$
\rho^{\text {pure }}(\gamma)=\frac{1}{2}\left(\begin{array}{cc}
1 & -\mathrm{e}^{-2 i \Phi} \\
-\mathrm{e}^{2 i \Phi} & 1
\end{array}\right)
$$

For elliptically polarized photons, eq. (11) reads:

$$
|\gamma\rangle=\frac{1}{\sqrt{2\left(a^{2}+b^{2}\right)}}\left\{-(a+b) \mathrm{e}^{-i \Phi}\left|\lambda_{\gamma}=+1\right\rangle+(a-b) \mathrm{e}^{i \Phi}\left|\lambda_{\gamma}=-1\right\rangle\right\},
$$

(14) We generalize these results to the case of partially polarized photons and put them into a standard form by writing $\rho(\gamma)$ as a linear combination of the matrices $I, \sigma_{i}(i=1,2,3)$, which from a complete set in the space of $2 \times 2$ hermitian matrices

$$
\begin{equation*}
\rho(\gamma)=\frac{1}{2} I+\frac{1}{2} \boldsymbol{P}_{\gamma} \cdot \boldsymbol{\sigma}, \tag{18}
\end{equation*}
$$

where $I$ is the $2 \times 2$ unit matrix, $\sigma_{i}$ are the three Pauli matrices. The length $P_{\gamma}$ of the three-vector $\boldsymbol{P}_{\gamma}$ is equal to the degree of polarization. The direction of $\boldsymbol{P}_{\gamma}$ depends on the kind of polarization, e.g. (from eqs. (13)
(15) and (15)):

$$
\begin{align*}
& \boldsymbol{P}_{\gamma}=P_{\gamma}(0,0, \pm 1) \\
& \boldsymbol{P}_{\gamma}=P_{\gamma}(-\cos 2 \Phi,-\sin 2 \Phi, 0) \tag{19}
\end{align*}
$$

(16) for circular polarization with $\lambda_{\gamma}= \pm 1$ and for linear polarization respectively with $0 \leqslant P_{\gamma} \leqslant 1$.
where $a$ and $b$ are the lengths of the principal axes of the ellipse and $\Phi$ is the azimuthal angle of the principle axis $a$. The corresponding density matrix is given by:

$$
\rho^{\text {pure }}(\gamma)=\frac{1}{2}\left|\begin{array}{ll}
1+2 a \sqrt{1-a^{2}} & \mathrm{e}^{-2 i \Phi}\left(1-2 a^{2}\right)  \tag{17}\\
\mathrm{e}^{2 i \Phi}\left(1-2 a^{2}\right) & 1-2 a \sqrt{1-a^{2}}
\end{array}\right|,
$$

with $a, b$ normalized to $a^{2}+b^{2}=1$. Obviously the cases of circularly or linearly polarized photons can be obtained by specializing eq. (17) to $a=$ $\pm 1 / \sqrt{2}$ or $a=1$ respectively.

## Quasi-real electroproduction

HOW TO ANALYSE VECTOR-MESON PRODUCTION IN INELASTIC LEPTON SCATTERING
K. SCHILLING

Fakultàt Physik der Universitat Bielefeld, Bielefeld

## G. WOLF

Deutsches Elektronen-Synchrotron DESY, Hamburg

$$
\rho(\gamma)=\frac{1}{2} \sum_{\alpha=0}^{8} \tilde{\Pi}_{\alpha} \Sigma^{\alpha} ;
$$

$$
\widetilde{\Pi}=\left\{1,-\epsilon \cos 2 \Phi,-\epsilon \sin 2 \Phi, \frac{2 m}{Q}(1-\epsilon) P_{0}, \epsilon+\delta\right.
$$

$$
\sqrt{2 \epsilon(1+\epsilon+2 \delta)} \cos \Phi, \sqrt{2 \epsilon(1+\epsilon+2 \delta)} \sin \Phi
$$

$$
\begin{equation*}
\left.\frac{2 m}{Q}(1-\epsilon)\left(P_{1} \cos \Phi+P_{2} \sin \Phi\right), \frac{2 m}{Q}(1-\epsilon)\left(P_{1} \sin \Phi-P_{2} \cos \Phi\right)\right\} \tag{65}
\end{equation*}
$$

$$
\begin{equation*}
W\left(\cos \theta, \phi, \Phi, \alpha_{2}=0, \pi\right)=W^{\mathrm{unpol}}(\cos \theta, \phi, \Phi) \pm W^{\mathrm{long} \mathrm{pol}}(\cos \theta, \phi, \Phi) \tag{88}
\end{equation*}
$$

$W^{\mathrm{unpol}}(\cos \theta, \phi, \Phi)=\frac{1}{1+(\epsilon+\delta) R} \frac{3}{4 \pi}$
$\times\left[\frac{1}{2}\left(1-\rho_{00}^{0}\right)+\frac{1}{2}\left(3 \rho_{00}^{0}-1\right) \cos ^{2} \theta-\sqrt{2} \operatorname{Re} \rho_{10}^{0} \sin 2 \theta \cos \phi-\rho_{1-1}^{0} \sin ^{2} \theta \cos 2 \phi\right.$
$-\epsilon \cos 2 \Phi\left\{\rho_{11}^{1} \sin ^{2} \theta+\rho_{00}^{1} \cos ^{2} \theta-\sqrt{2} \operatorname{Re} \rho_{10}^{1} \sin 2 \theta \cos \phi-\rho_{1-1}^{1} \sin ^{2} \theta \cos 2 \phi\right\}$
$-\epsilon \sin 2 \Phi\left\{\sqrt{2} \operatorname{Im} \rho_{10}^{2} \sin 2 \theta \sin \phi+\operatorname{Im} \rho_{1-1}^{2} \sin ^{2} \theta \sin 2 \phi\right\}$

$$
W^{\text {long pol }}(\cos \theta, \phi, \Phi)=\frac{1}{1+(\epsilon+\delta) R} \frac{3}{4 \pi}
$$

$$
\times P\left[\sqrt{1-\epsilon^{2}}\left\{\sqrt{2} \operatorname{Im} \rho_{10}^{3} \sin 2 \theta \sin \phi+\operatorname{Im} \rho_{1-1}^{3} \sin ^{2} \theta \sin 2 \phi\right\}+\right.
$$

For low $\mathrm{Q}^{2}$ we assume,

$$
\rho_{\gamma}(\Phi)=\frac{1}{2}\left(1-\epsilon \cos 2 \Phi \sigma_{x}-\epsilon \sin 2 \Phi \sigma_{y}-P_{b e a m} \sqrt{1-\epsilon^{2}} \sigma_{z}\right)
$$



## Quasi-real photoproduction:

- Detection of multiparticle final state from meson decay in the large acceptance spectrometer CLAS
- Detection of the scattered electron for the tagging of the quasi-real photon in the CLAS12 FT
- High-intensity and high linear-polarization tagged "photon" beam; degree of polarization determined event-by-event from the electron kinematics
- Longitudinal e- polarisation transferred to virtual photon as "circular polarisation"
- In FT acceptance $P_{\text {lin }}$ and $P_{\text {circ }} \sim 0.65$

Start with the same waveset as in ambiguity paper

| $[\ell]_{m}$ | Magnitude | Phase |
| :--- | :---: | :---: |
| $S_{0}$ | 0.499 | $0^{\circ}$ |
| $D_{-1}$ | 0.201 | $15.4^{\circ}$ |
| $D_{0}$ | 0.567 | $174^{\circ}$ |
| $D_{1}$ | 0.624 | $-81.6^{\circ}$ |

> No -ve reflectivity (for now)
$\mathcal{I}(\Omega, \Phi)=\mathcal{I}_{0}(\Omega)-\mathcal{I}_{1}(\Omega) P_{\gamma L} \cos 2 \Phi-\mathcal{I}_{2}(\Omega) P_{\gamma L} \sin 2 \Phi-\mathcal{I}_{3}(\Omega) P_{\gamma C}$.
Generate data 10 k events with full $\alpha=0,1,2,3$ intensities $P_{y c}, P_{y L}$ uniform in range 0-0.5

Perform 50 fits with different polarisation information Only fit for the non-zero generated waves, as in the paper.

Negative Log Likelihood is shown on y-axis, amplitude components on x axis
Solution => highest likelihood

## Simulations - Unpolarised





Ambiguous solutions (for D+ and D-) and Complex conjugates

But low chance of local max.

## Simulations - Unpolarised



Ambiguous solutions (for D+ and D-) and Complex conjugates

Low chance of local max. Uncertainties large.

## Simulations - Linearly Polarised (as paper)





Single solution with Complex conjugate Smaller uncertainties

## Simulations - Circular polarised




Ambiguous solutions (D+ and D-)
But no complex conjugate Smaller uncertainties

## Simulations - Elliptically Polarised

Fit results for 10 k events



NLL


Single Solution no Complex conjugate

## Potential polarimetry ?

From linear polarised fit we have fully determined the partial waves - We can already calculate the $I^{3}$ intensity for both complex conjugates



Complex conjugate solutions H3 moments

- So we are overconstrained in our fits
=> should result in smaller uncertainties
=> Or we can introduce an additional unknown parameter
- the photon circular polarisation degree
- Next I just redo the elliptically polarised fit with $P_{y c}$ as a parameter not an observable.


Single Solution no Complex conjugate


Best solution yields Correct $\mathrm{P}_{\text {circ }}=0.25$


NLL:parPcirc


Fit I , I, I


Fit $I, I, I, I$



Final uncertanties are similar apart from D- which : $0.17 \rightarrow 0.10$

