

Electromagnetic Physics I and II

Special topic: Multiple Coulomb scattering (MSC)

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Geant4 Tutorial at Jefferson Lab
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- Electromagnetic (EM) physics overview
 - Introduction, structure of Geant4 EM physics
 - Standard EM physics constructors
- Special EM topics:
 - Secondary production thresholds:
 - ✦ in Geant4
 - ✦ energy loss fluctuation
 - ✦ continuous step limit to energy loss
 - ✦ v.s. tracking cut, `G4UserLimit`, step function
 - EM models per detector region
 - Atomic de-excitation
 - **Multiple Coulomb scattering**
 - ✦ dedicated material

Electromagnetic Physics: special topics

MULTIPLE COULOMB SCATTERING (MSC)

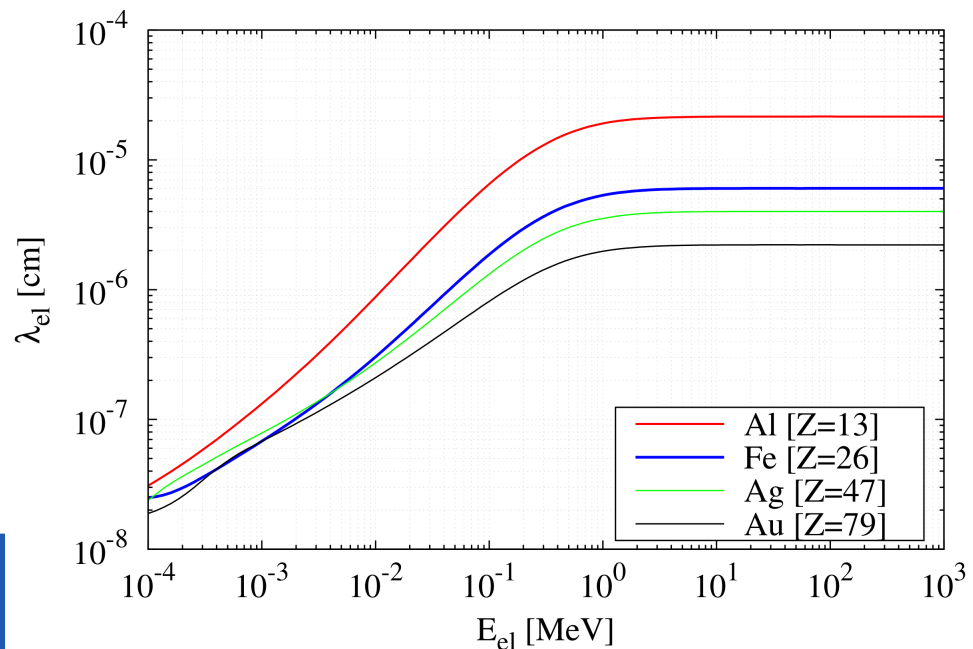
- Multiple Coulomb scattering (MSC)
 - Why should we use MCS models?
 - What the MSC model provides?
 - What makes it challenging?
 - Some notes on the **Geant4** MSC models and step limits.
 - Some notes on the **Geant4 EMZ** v.s. **default** EM physics options.

Multiple coulomb scattering (MSC)

WHY SHOULD WE USE MSC MODELS?

Coulomb scattering: elastic scattering of charged particles (e^-/e^+) on a Coulomb potential (central field of the nucleus screened by the atomic electrons)

- detailed simulation**: event-by-event modelling of elastic scattering is **feasible only if** the mean number of interactions per track is below few hundred
 - this limits the applicability of the **detailed simulation** model **only** for e^-/e^+ with relatively **low kinetic energies** (becomes very inefficient otherwise):
 - **$E_{kin} < \sim 100-200$ keV**: low mean number of elastic scattering along the path
 - **thin targets**: short path resulting again low mean number of elastic scattering

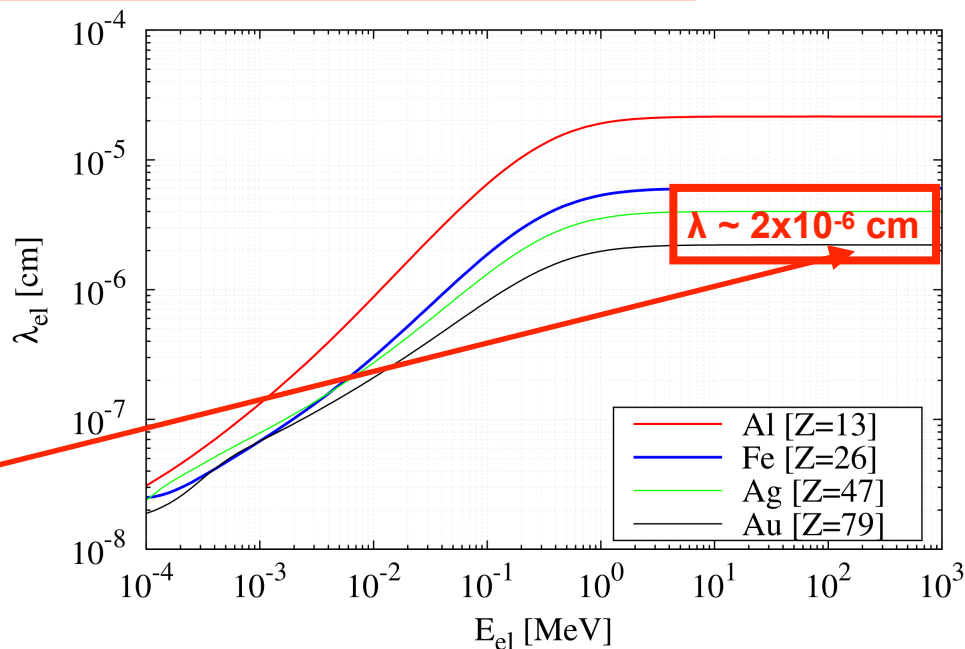
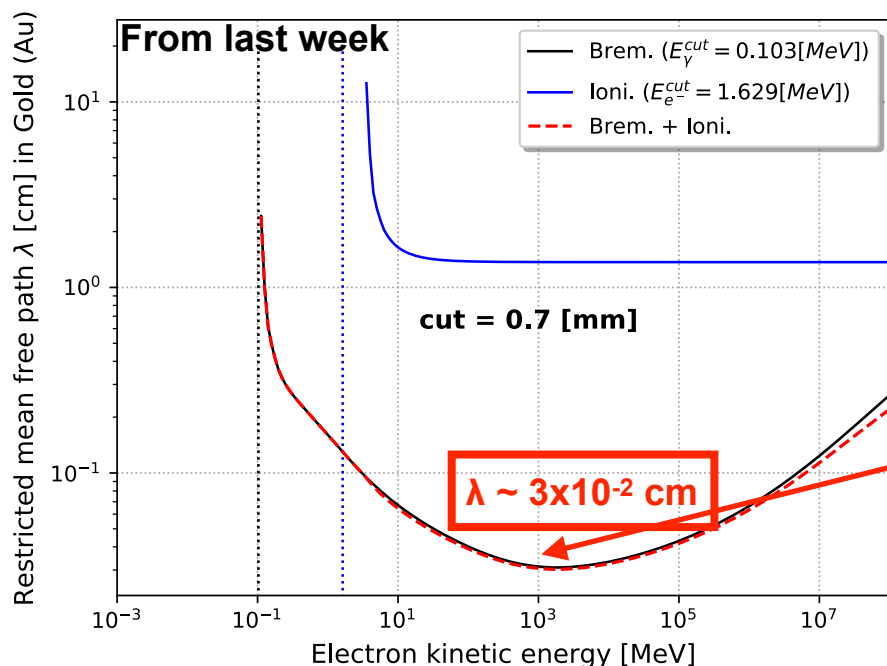


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15 000 x more elastic scattering than anything else at 1 [GeV]!

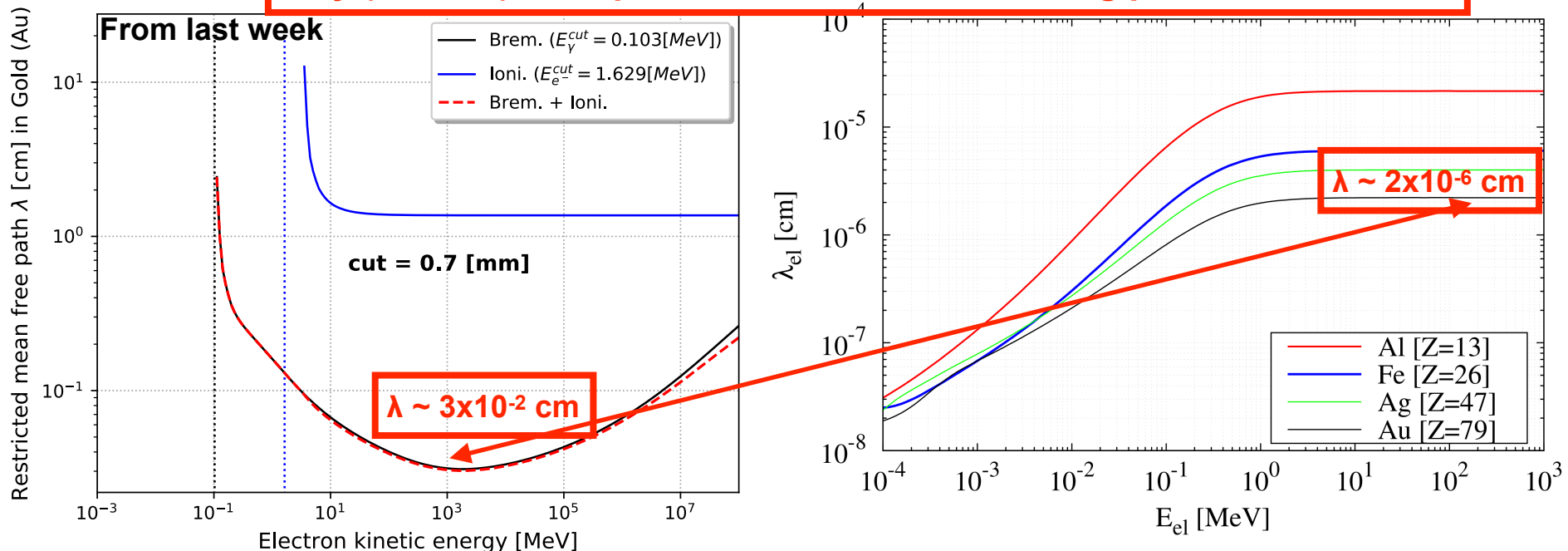


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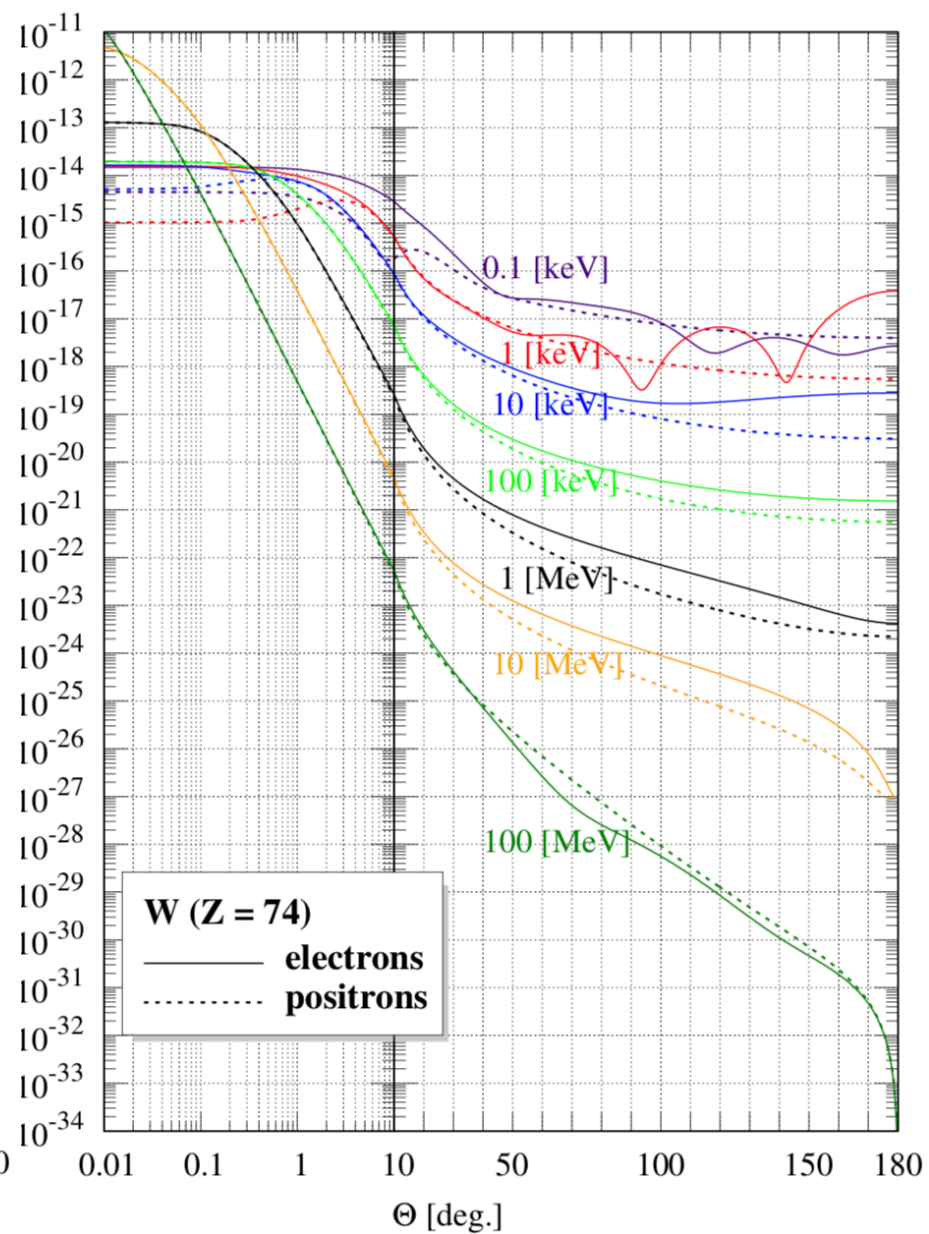
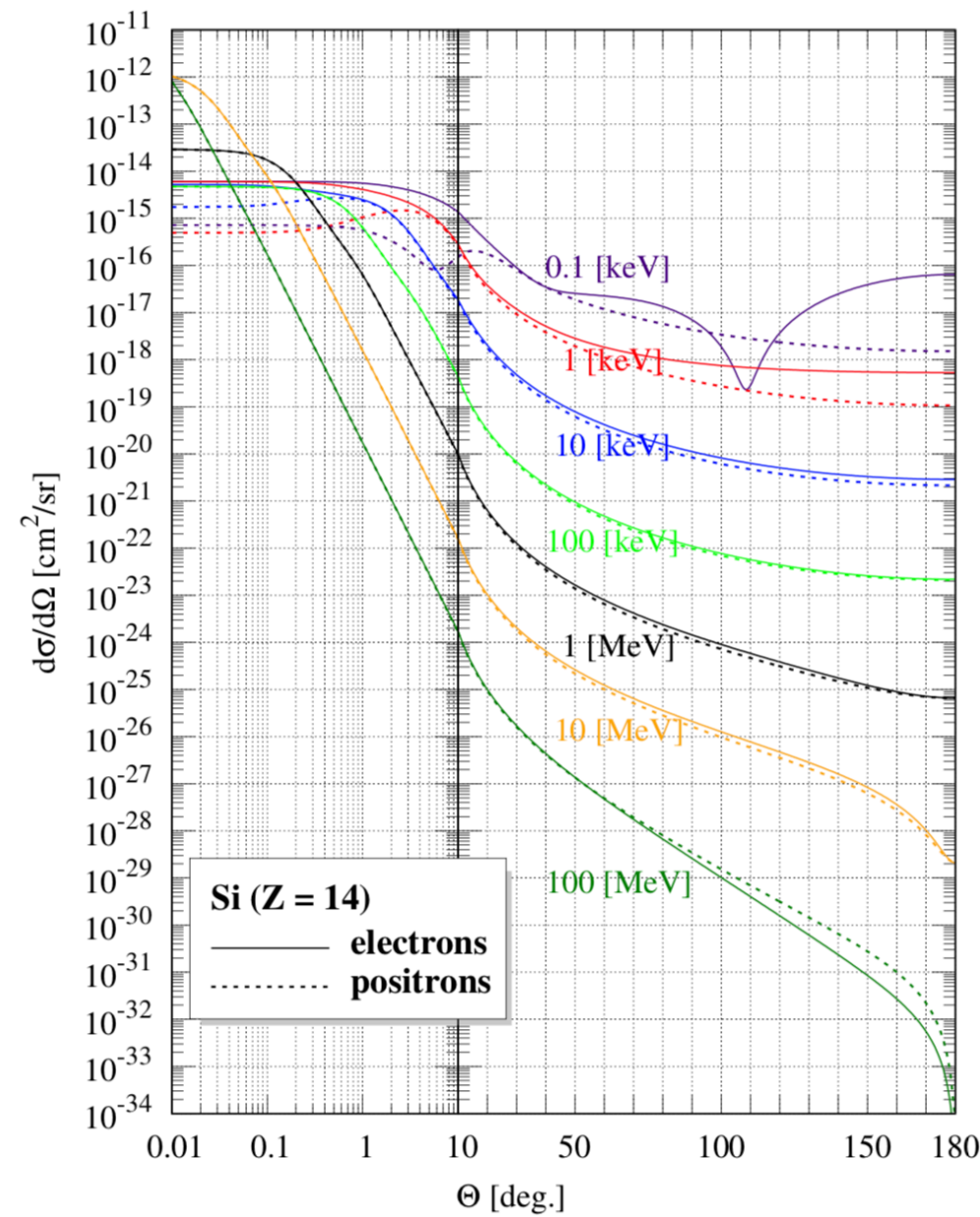
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2. **condensed history simulation:** relies on a **multiple scattering (MSC)** model that accounts the **net effects of many elastic scattering in one step**
 - each particle track is simulated by allowing to make individual **steps** that are **much larger than the average step length between two successive elastic interactions**
 - the step length is computed *“without considering”* the many elastic events along
 - then the net **effects of these high number of elastic interactions**, such as angular deflection and spacial displacement, are **given by an MSC model at each individual** (condensed history) simulation **step**
 - MSC models are **stochastic models** used within the stochastic simulation itself

3. **mixed simulation**: a fusion of the two previous approaches

- **using special MSC model** (with a limited scattering angle approximation) to describe ***small angle scatterings*** along a step by exploiting:
 - the DCS for elastic scattering is strongly peaked to the forward directions
 - many things become simpler in a “*small angle approximation*”

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3. **mixed simulation**: a fusion of the two previous approaches

- **using special MSC model** (with a limited scattering angle approximation) to describe **small angle scatterings** along a step by exploiting:
 - the DCS for elastic scattering is strongly peaked to the forward directions
 - many things become simpler in a “*small angle approximation*”
- **using detailed, event by event simulation to describe high(er) angle scattering events**
- a similar idea to that used in case of ionisation and bremsstrahlung
- while a great idea that makes possible accurate simulation, significantly less efficient than using an MSC model: high angle scattering events (i.e. detailed simulation) starts to dominate very quickly with decreasing e-/e⁺ kinetic energies

Multiple coulomb scattering (MSC)

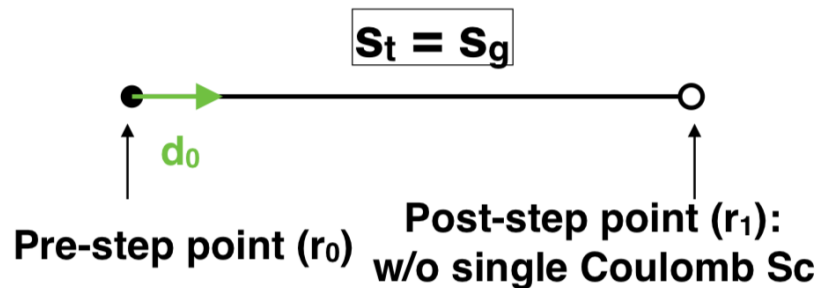
WHAT THE MSC MODEL PROVIDES?

A step with a particle (assuming no field): in an infinite volume !

- Particles **without Coulomb** scattering process (**A**):
 - moves from the pre- to the post step point **along straight line**
- Particles **with Coulomb** scattering described by **single-scattering** model (**B**):
 - the corresponding cross section participated in the (discrete) step limit
 - moves from the pre- to the post step point **along straight line**
 - where the single Coulomb scattering event might or might not happen (depending if that limited the step or not)

A. and B.

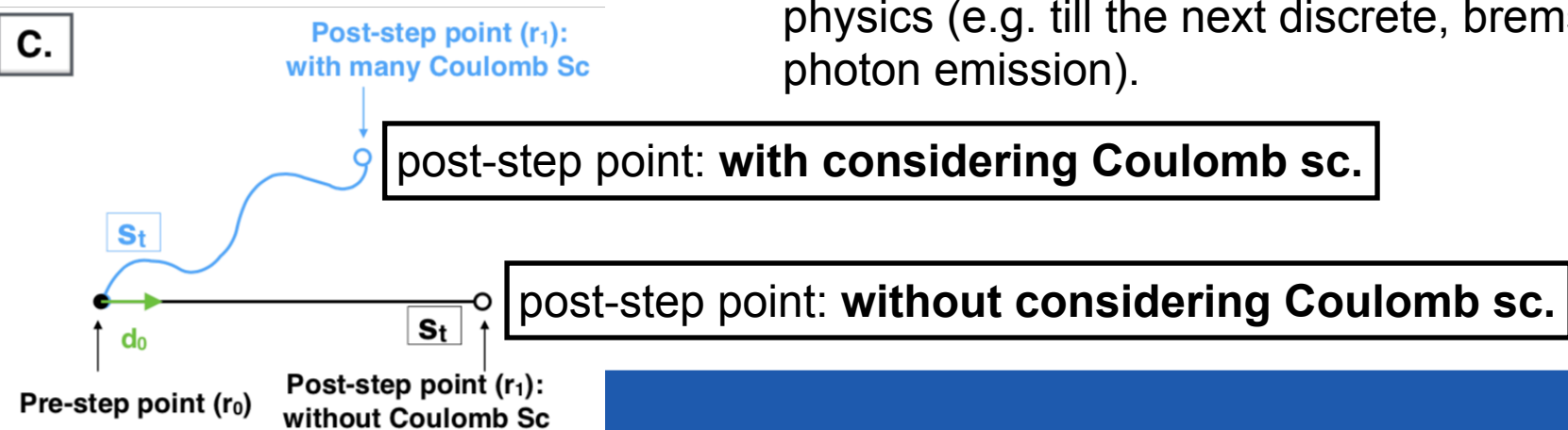
$$\mathbf{r}_1 = \mathbf{r}_0 + \mathbf{d}_0 S_t$$



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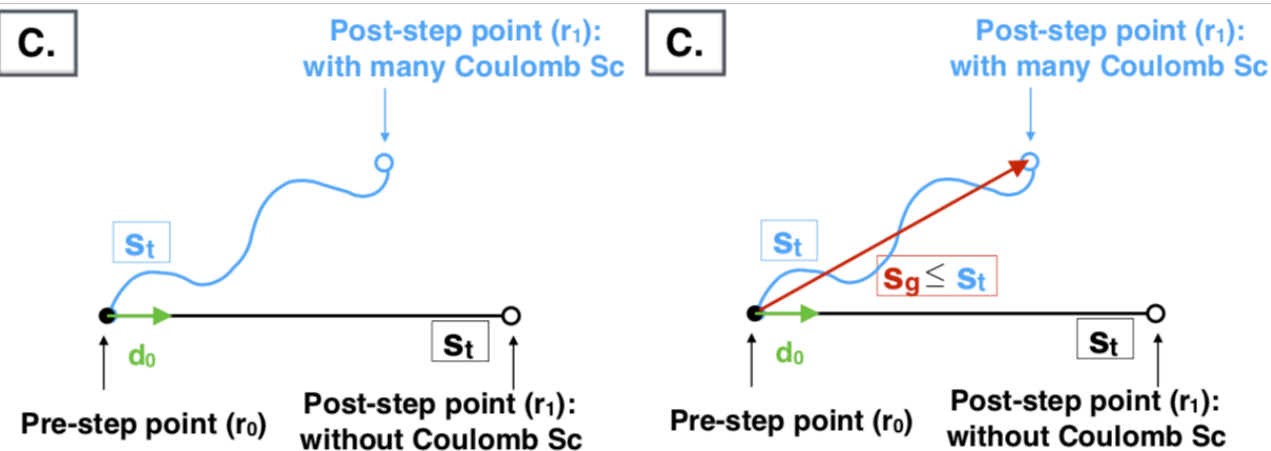
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- Particles **with Coulomb** scattering described by **multiple-scattering** model (C):

Note: s_t is the step length determined by all possible physics (e.g. till the next discrete, bremsstrahlung photon emission).



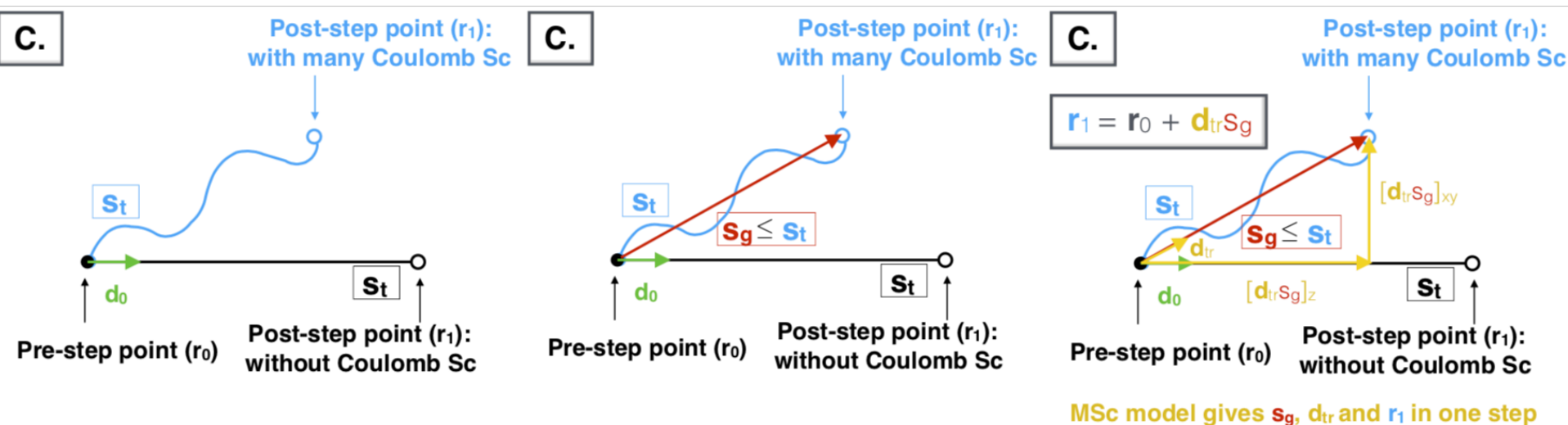
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- Particles **with Coulomb** scattering described by **multiple-scattering** model (**C**):
 - MSC provides:
 - ◆ the projection of the transport vector along the original direction ($[d_{tr}S_g]_z$)
 - ◆ the vector of displacement along the perpendicular plane ($[d_{tr}S_g]_{xy}$)
 - ◆ the final direction at the real post-step point (results of many angular deflections)
 - ◆ all these in one step (i.e. without computing the individual interactions)

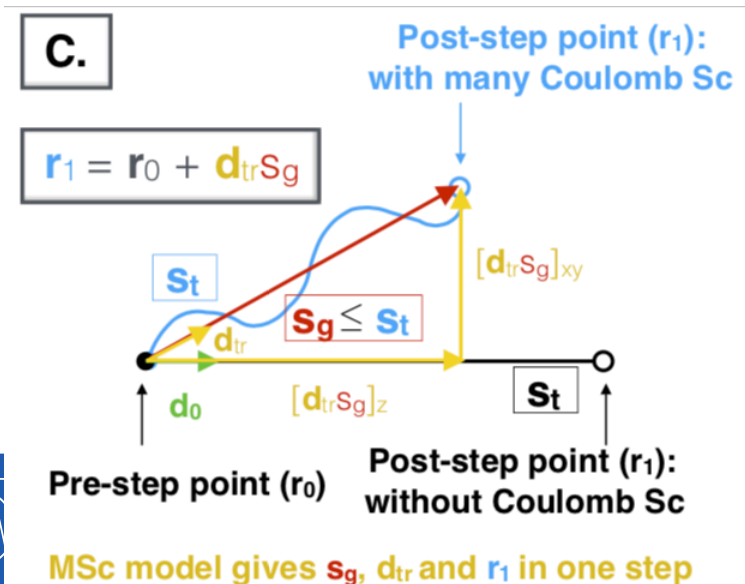
Multiple coulomb scattering (MSC)

WHAT MAKES IT CHALLENGING?

What makes it challenging?

- **Finite volumes:**

- **at the pre-step point**, the MSC model needs to compute **everything**, including the **S_g geometrical step length** (transport distance), **based on the s_t true step length**: from s_t to S_g



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- **at the pre-step point**, the MSC model needs to compute **everything**, including the **s_g geometrical step length** (transport distance), **based on the s_t true step length**: from s_t to s_g
 - however, eventually **the particle might travel a shorter distance** (than $[d_{tr} s_g]_z$) along the original direction **due to hitting a volume boundary**: $s_g' < s_g$ not consistent anymore with s_t :
 - ◆ MSC needs to (re-)estimate the corresponding true step length s_t' (inaccuracy)
 - ◆ the **post-step point is on the volume boundary**: no displacement that results **incorrect final position**
 - each such case is a **clear mistake** in the particle transport: more such mistakes \Rightarrow **less accurate EM shower**
 - **MSC processes** (the models) **impose an additional limit on the step** in order to minimise/avoid these mistakes
 - **different type of MSC step limits**: tries to relax this step limit (\Rightarrow **faster simulation**) \Leftrightarrow **maintain accuracy**
 - ◆ **very strong effects** both to **the speed and accuracy of the EM shower simulation**
 - ◆ the main difference, between **the Geant4 EM physics options** (EMY, EMZ, etc.), is the **MSC model and its step limit type** (and the corresponding parameter values)
- There are several additional challenges that affects the accuracy, complexity of the MSC model and/or the speed of the simulation:
 - accuracy of the MSC angular distributions and the underlying DCS for elastic scattering
 - accuracy of the displacement algorithm, energy loss correction, etc.
 - stepping must be robust in case of field, large and small volumes, etc.

Multiple coulomb scattering (MSC)

SOME NOTES ON THE GEANT4 MSC MODELS AND THE CORRESPONDING STEP LIMITS

- **MSC models:**

- the two main models for describing e-/e⁺ multiple Coulomb scattering: **G4UrbanMscModel** and **G4GoudsmitSaundersonMscModel**
- while **G4UrbanMscModel** is an empirical model the **G4GoudsmitSaundersonMscModel** is based on solid theoretical foundations with several higher order corrections providing the most accurate description
- **G4UrbanMscModel** is the model used in most cases of EM physics constructors (default, EMV, EMX, EMY)
- **G4GoudsmitSaundersonMscModel** is used in EM physics constructors (EMZ, LIV, PEN, _GS)
- the MSC stepping algorithm and its parameters are the main differences in these options

- the **EMZ**, **LIV**, **PEN** EM physics constructors: **the most accurate EM physics simulation (EMZ)**

- the **G4GoudsmitSaundersonMscModel** is used **with its most accurate settings**
- the MSC **step limit** is the **UseSafetyPlus** (special version for the GS model):
 - ◆ the MSC step limit is computed based on the safety value
 - ◆ the algorithm switches to single scattering near the volume boundary (**Skin**, measured in elastic MFP units)
 - ◆ makes possible a stepping and **tracking of e-/e⁺ that is free from any errors**
 - ◆ **can be very slow** in case of several boundary crossing
- **UseMottCorrection**: **all the higher order corrections** (Mott correction to the angular distributions, scattering power correction, etc.) **are active** (only for the GS MSC model)
- the corresponding UI commands (the default values in these EM constructors):

- /process/msc/StepLimit UseSafetyPlus
- /process/msc/Skin 3
- /process/msc/UseMottCorrection true

- the **EMY** EM physics constructors: **less accurate but a bit faster**
 - the **G4UrbanMscModel** is used **with the UseDistanceToBoundary MSC step limit type**:
 - ◆ the MSC step limit is computed by using the distance to the volume boundary along the current direction
 - ◆ this distance is shortened in order to try to avoid hitting the boundary
 - ◆ the algorithm switches to single scattering near the volume boundary (**Skin**, measured in elastic MFP units)
 - additional step limit components are also used, among which the most important is a kind of heuristic:
 - ◆ the **maximum of a fraction of the initial range (RangeFactor)** of the particle (when entered in the current volume) and **a fraction of the safety (SafetyFactor)**
 - the faster than the previous but can lead to errors (especially in case of magnetic field)
 - one can use exactly the same step limit and settings even with the GS MSC model
 - the corresponding UI commands (the default values in this EM constructors):
 - `/process/msc/StepLimit UseDistanceToBoundary`
 - `/process/msc/Skin 1`
 - `/process/msc/RangeFactor 0.04`
 - `/process/msc/SafetyFactor 0.6`

- the default EM physics constructors: **less accurate but faster** (a stronger compromise on accuracy for gaining speed)
 - the **G4UrbanMscModel** is used with the **UseSafety** MSC step limit type:
 - ◆ there is no single scattering steps near the volume boundary (less accurate boundary crossing)
 - the heuristic is relaxed further (in case of e-/e⁺):
 - ◆ a function of the fraction of the initial range (**RangeFactor**) is used letting the particles to go for a longer step (when it is though to be possible)
 - the **_GS** EM physics constructor provides the same physics setting, including the MSC stepping algorithm and parameters but using the **G4GoudsmitSaundersonMscModel** instead of the **G4UrbanMscModel** model for e-/e⁺
 - the corresponding UI commands (the default values in this EM constructors):
 - /process/msc/StepLimit UseSafety
 - /process/msc/RangeFactor 0.04
 - /process/msc/SafetyFactor 0.6
- the **EMV**, **EMX** physics constructors: should be used only in case of very large volumes (a kind of “don’t case” setting)
 - large range factor value (**RangeFactor 0.2**) very relaxed MSC step limit (**Minimal**)
 - on the top of this, displacement is turned OFF in case of **EMX**
 - (while **ApplyCuts** is turned ON in case of **EMV**)

Multiple coulomb scattering (MSC)

SOME NOTES ON THE GEANT4 EMZ V.S. DEFAULT EM PHYSICS OPTIONS

- as mentioned, main difference between the EM options is the model used for describing the MSC of e^-/e^+ , its settings, especially the MSC step limit algorithm
- there are of course additional differences when using moving from the **default** to the direction of **EMZ**: more accurate (more complex) models are used in case of several interactions, different step function parameters in the energy loss related step limit, atomic relaxation is activated, etc.
- when **tuning the EM physics** settings:
 - use **EMZ** to see the most accurate physics simulation results that can be achieved
 - perform the same simulation with the **default** or other relaxed EM physics options to see the gap between the physics accuracy and computing time
 - start to refine the relaxed EM physics constructor settings in order to increase/relax the accuracy and/or the computing time of the simulation
- if you are **happy with the accuracy given by the default EM settings**, you might **try to relax even further that settings** in order to **gain speed** while the accuracy do not change much:
 - e.g. **increase the MSC range factor** from 0.04 to 0.06 relaxing further the MSC step limits, leading to **smaller number of simulation step** with e^-/e^+ which can **significantly reduce the computing time**
 - keep doing this **as long as** the effects of the corresponding increased rate of MSC related errors (hitting boundary) **do not change much your results**
 - keep in mind that **different EM models can be used in different detector regions**, that provides the possibility for **refining the stepping according to e.g. the granularity** of the detector

- if one would like to **decrease the accuracy gap between** the results given by **EMZ** and the **default EM physics** constructors **while keeping the computing time under control**:
 - first the origin(s) of the main difference(s) need to be identified
 - keeping in mind, that:
 - ◆ the **main difference** between the EM options is the **MSC model for e-/e+ and its settings**
 - ◆ the **_gs** EM option has the same physics as the **default**, except the MSC model for e-/e+ that is the same as in **EMZ**
 - ◆ but keep in mind, that **_gs** option has **as relaxed MSC stepping algorithm as in the default** (actually a bit even more relaxed with a range factor of 0.06 instead of the 0.04)
 - one can easily confirm that the main source of the difference between the results obtain with the **default** and **EMZ** options is indeed the description of the e-/e+ MSC by:
 - ◆ starting from the original **_gs** EM option, **gradually apply more accurate settings** of the underlying **G4GoudsmitSaundersonMscModel** in order to **bring closer** and closer to its usage in **EMZ**:
 1. first use the error free stepping: `/process/msc/StepLimit UseSafetyPlus`
 2. if the results are still deviate from that of **EMZ**: `/process/msc/Skin 3`
 3. if the results are still deviate from that of **EMZ**: `/process/msc/UseMottCorrection true`
 - ◆ the above settings brings accuracy of **_gs** EM option from the **default** to that of similar to **EMZ** in most cases !
 - there are different possibilities depending on the results of the above: if those, especially 1. brings the required accuracy:
 - ◆ one might try the **UseDistanceToBoundary** that is a bit relaxed MSC step limit with even decreased values of the **RangeFactor** (< 0.06 - 0.04) and/or higher value of the **Skin** (e.g. 2, 3)
 - ◆ or even the more relaxed **UseSafety** MSC step limit combined with a smaller value of the **RangeFactor** (< 0.06 - 0.04) and/or **SafetyFactor** (< 0.6)

THAT'S IT FOR TODAY