# Crab cavity effect on beam polarization in EIC

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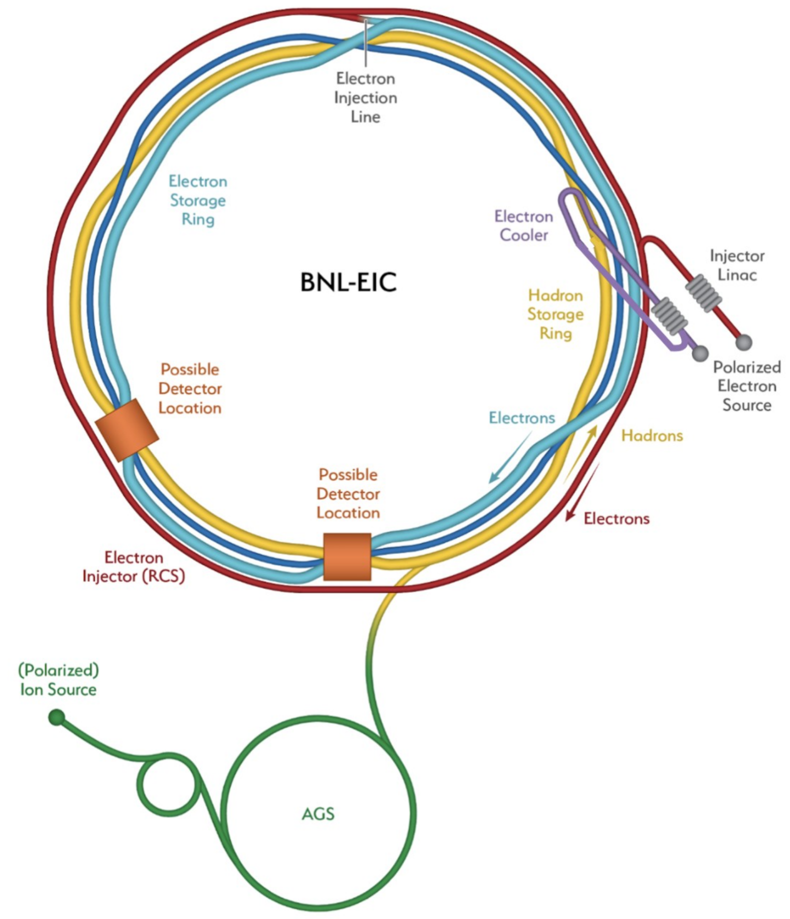
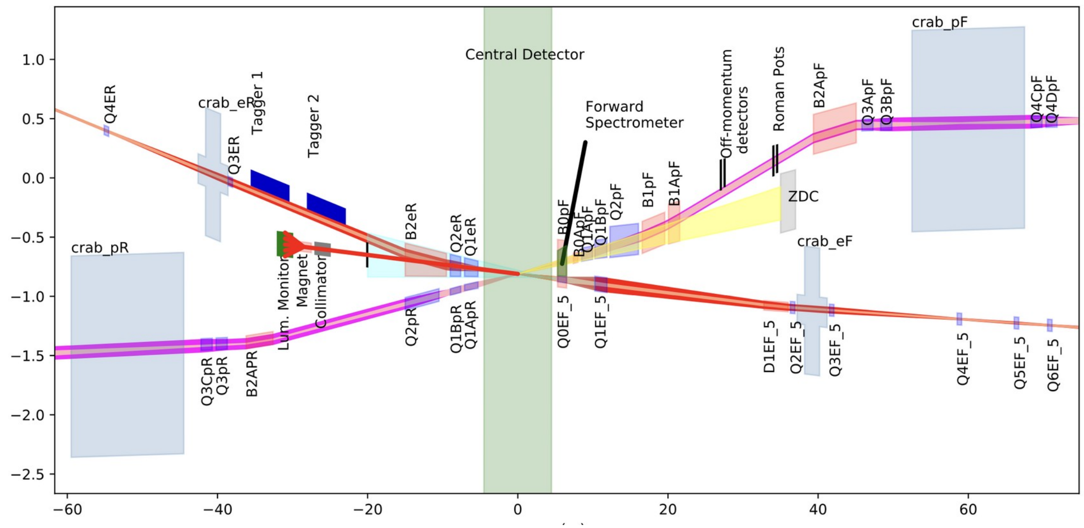
# EIC

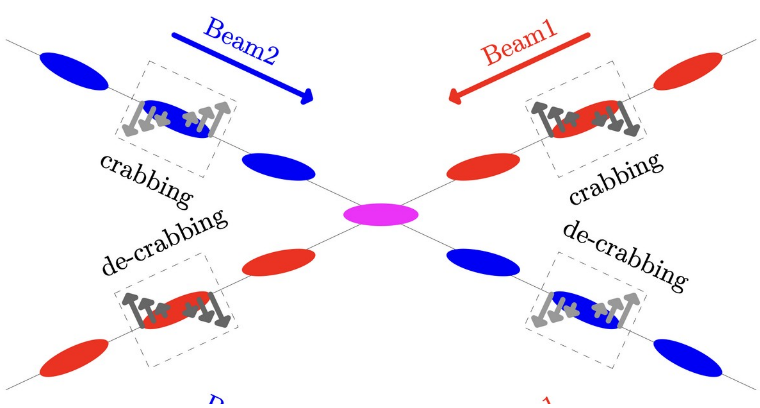
- Future unique machine with highly polarized proton and electron beams, will investigate spin and flavor structure of nucleon and nuclei, test QCD predictions
- Polarized light ions and electron beams
- Goal polarization for proton, $^3$He and electrons is 70%
- Longitudinal polarization in IP



# RF crab cavity

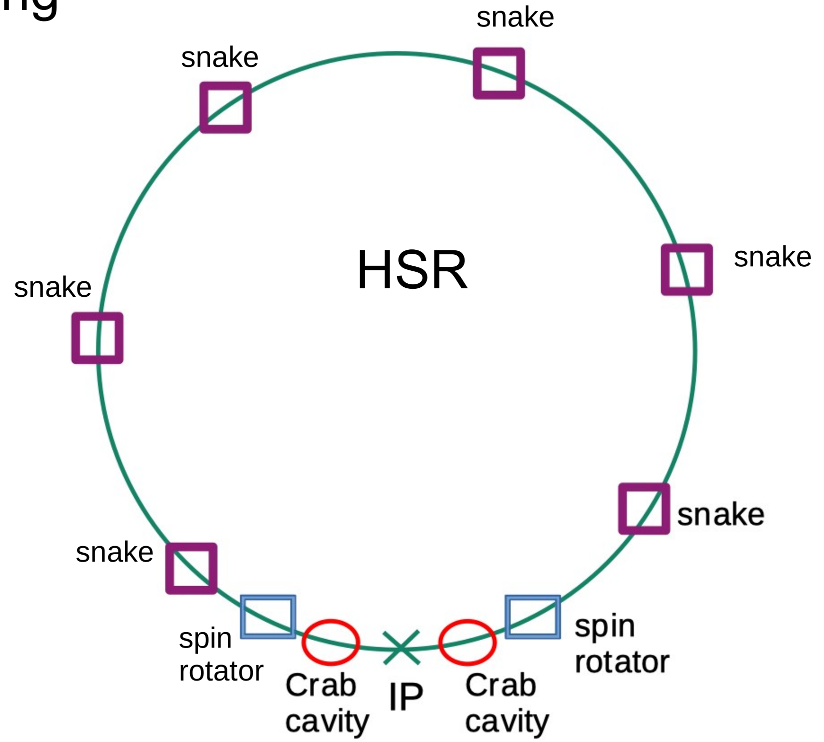
- At IP beams intersect at the crossing angle of 25 mrad. To restore the luminosity the bunches will be rotated by half interaction angle.
- Two RF crab cavities will be used in each electron and proton rings



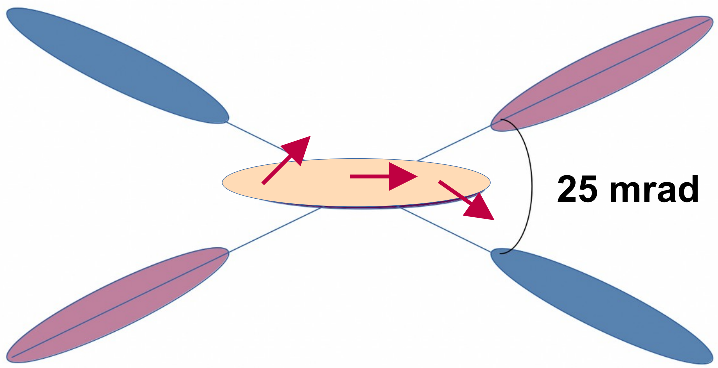
# Spin rotators

- 6 Siberian snakes in HSR and 4 in ESR will be used to overcome spin resonances during acceleration
- 6 Snakes separated with 60$^o$ bending angle
- Produces 180$^o$ rotation around horizontal axis
- Two spin rotators in each ring to rotated polarization in longitudinal position:
    - Helical dipole magnets in HSR
    - Solenoidal magnets in ESR



# Crab cavity influence on the bunch

- Phase advance between crab cavity and IP is close but not exactly 90$^o$
- Head and tail of the bunch are out of the nominal orbit
- That means spins of head/tail particles are getting horizontal kick, which potentially can lead to polarization spreading and decreasing longitudinal spin component




**How large is this effect?**

# Orbit kick of the crab cavity and a corresponding spin kick

Thomas-BMT equation:

$$\frac{d\vec{S}}{dt}=\vec{\Omega}\times\vec{S}$$

$$\vec{\Omega}=-q[G+\frac{1}{\gamma}\vec{B}-\frac{G\gamma}{\gamma-1}(\vec{\beta}\cdot\vec{B})\vec{\beta}-(G+\frac{1}{\gamma-1})(\vec{\beta}\times\vec{E})]$$

$$\gamma\gg 1 \qquad B\|\rightarrow 0$$

$$\vec{\Omega}=-qG[\vec{B}-(\vec{\beta}\times\vec{E})]$$

$$\dot{\alpha}=|\Omega|=qG[B+E]$$

Lorentz force equation:

$$\frac{d\vec{\beta}}{dt}=\frac{q}{\gamma}(\vec{E}-\vec{\beta}\times\vec{B})$$

$$\frac{d\vec{\beta}}{dt}=\vec{W}\times\vec{\beta}$$

$$\frac{d\vec{\beta}}{dt}=\frac{q}{\gamma}\beta\times(\vec{E}\times\vec{\beta}-\vec{B})$$

$$\vec{W}=\frac{q}{\gamma}(\vec{E}\times\vec{\beta}-\vec{B})$$

$$\dot{\theta}=|W|=\frac{q}{\gamma}[E+B]$$

$$\dot{\alpha}=\gamma G\,\dot{\theta}$$

# Stable spin axis calculation

Kick of the Crab cavity corresponding to 12.5 mrad bunch tilt

$$\begin{pmatrix} x_{ip} \\ x'_{ip} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_{c1}}{\beta_{ip}}}\cos(\mu)+\alpha_{ip}\sin(\mu) & \sqrt{\beta_{c1}\beta_{ip}}\sin(\mu) \\ \frac{-(1+\alpha_{c1}\alpha_{ip})\sin(\mu)+(\alpha_{c1}-\alpha_{ip})\cos(\mu)}{\sqrt{\beta_{c1}\beta_{ip}}} & \sqrt{\frac{\beta_{ip}}{\beta_{c1}}}(\cos(\mu)-\alpha_{c1}\sin(\mu)) \end{pmatrix} \begin{pmatrix} 0 \\ \theta \end{pmatrix}$$

# Stable spin axis calculation

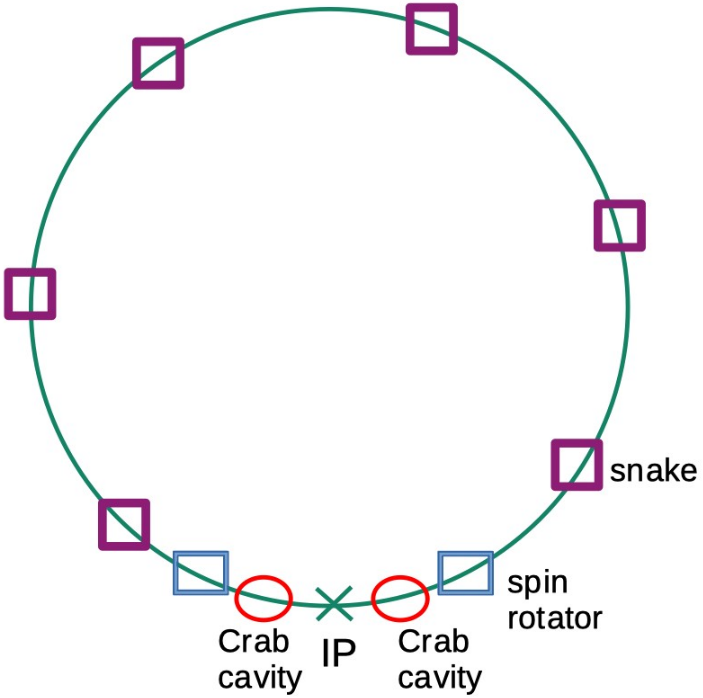
One turn spin rotation matrix:

$$T_{turn}=T_{cc2e-ip}\cdot T_{cc2b-cc2e}\cdot T_{rot2-cc2b}\cdot T_{rot2}\cdot T_{sn6-rot2}\cdot T_{snk-dn}\cdot T_{sn5-sn6}\cdot T_{snk-up}\cdot T_{sn4-sn5}\cdot T_{snk-dn}$$
$$\cdot T_{sn3-sn4}\cdot T_{snk-up}\cdot T_{sn2-sn3}\cdot T_{snk-dn}T_{sn1-sn2}\cdot T_{snk-up}\cdot T_{sn1-rot1}\cdot T_{rot1}\cdot T_{rot1-cc1e}\cdot T_{cc1e-cc1b}T_{cc1b-ip}$$

$$\vec{a}=\frac{1}{2}\sin(\frac{\phi}{2})Tr(\vec{\sigma}\cdot T_{turn}) \qquad \cos(\phi/2)=\frac{1}{2}Tr(T_{turn})$$

Stable spin axis direction on 1 $\sigma$:

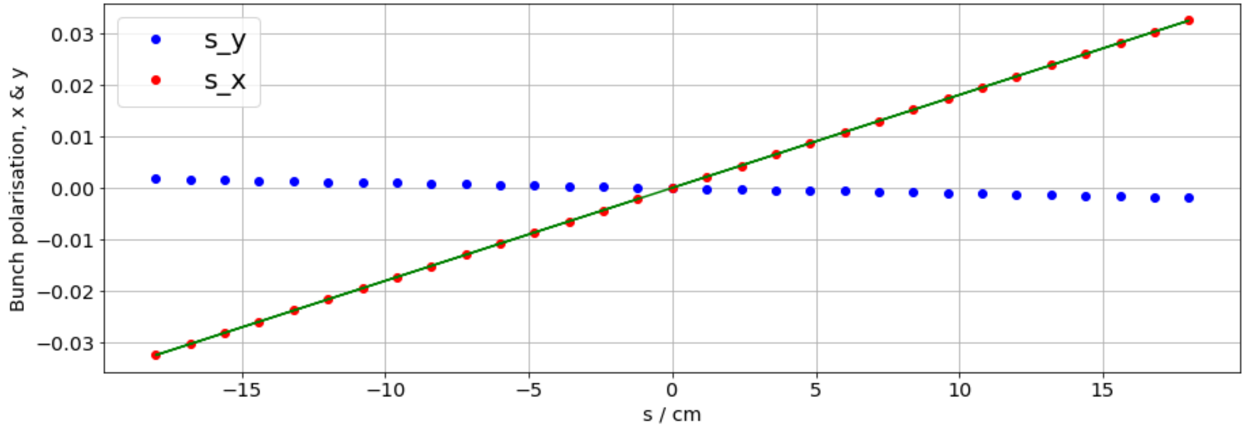
| | $a_s$ | $a_x$ | $a_y$ |
|---|---|---|---|
| protons 275 GeV | 0.99994 | 0.011 | 0.0006 |
| protons 100 GeV | 0.9999975 | 0.0022 | -0.0001 |
| protons 41 GeV | 0.9999995 | 0.00092 | 0.0002 |
| electrons 18 GeV | 0.999998 | 0.0015 | 0.00005 |



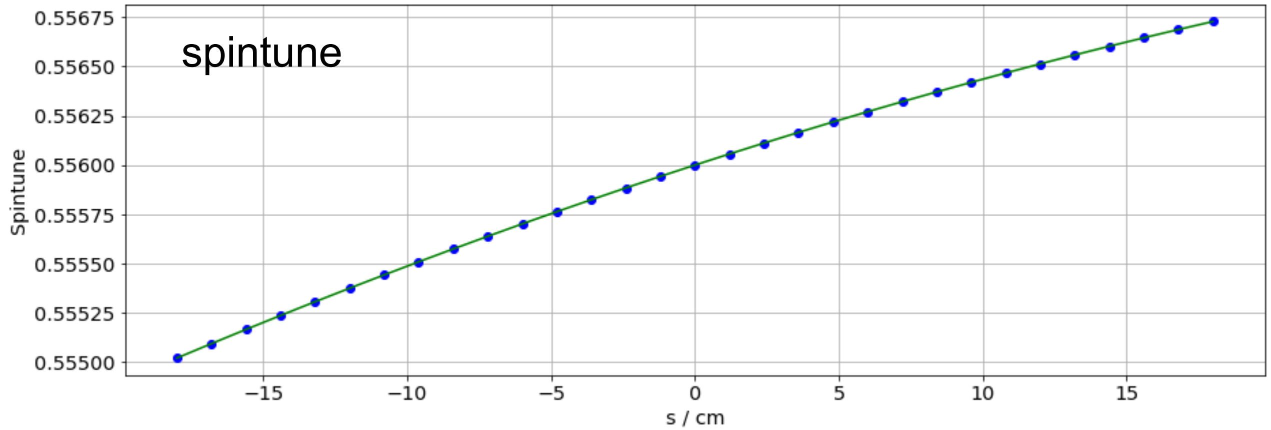
# Stable spin axis depending on s in bunch

protons, 275 GeV

$$\vec{a}=\frac{1}{2}\sin\left(\frac{\phi}{2}\right)Tr\left(\vec{\sigma}\cdot T_{turn}\right)$$



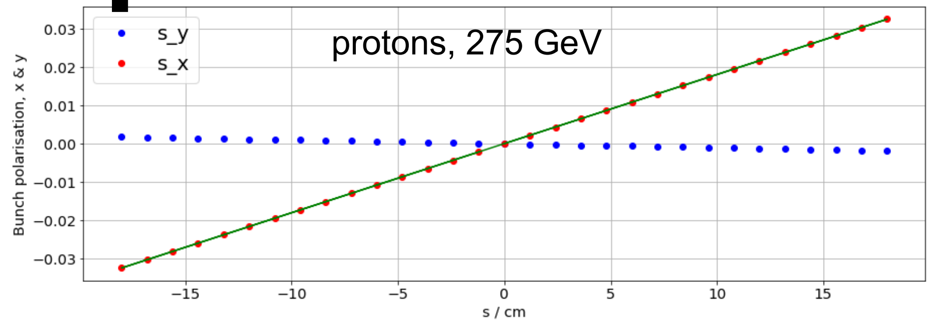

$$\cos\left(\phi/2\right)=\frac{1}{2}Tr\left(T_{turn}\right)$$

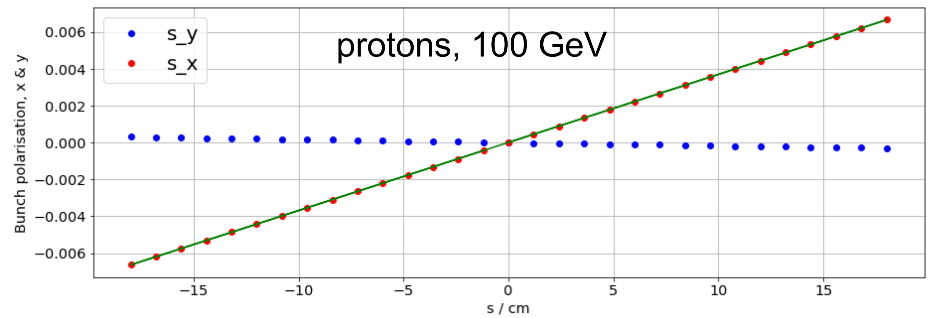


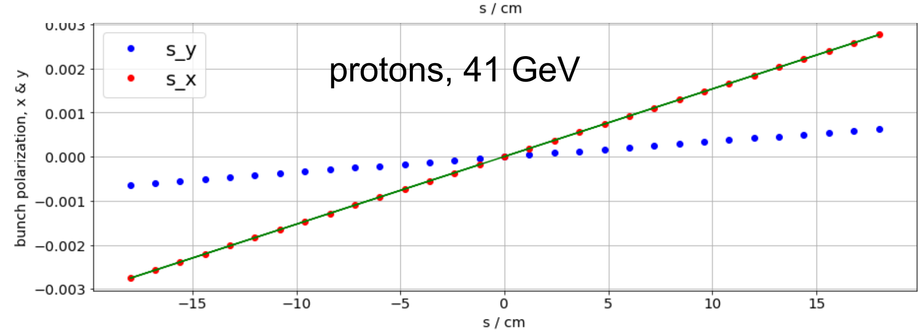
# Average bunch polarization

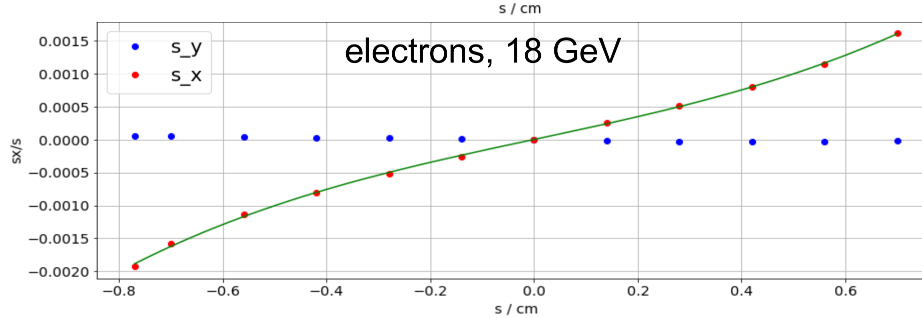
$$\langle P_b \rangle = \frac{1}{\sqrt{2\pi}\,\sigma} \int \cos(ks)\, e^{-\frac{s^2}{2\sigma^2}}\, ds$$

| | Bunch polarization reduced by |
|---|---|
| protons 275 GeV | 6e-5 |
| protons 100 GeV | 3e-6 |
| protons 41 GeV | 4e-7 |
| electrons 18 GeV | 4e-6 |



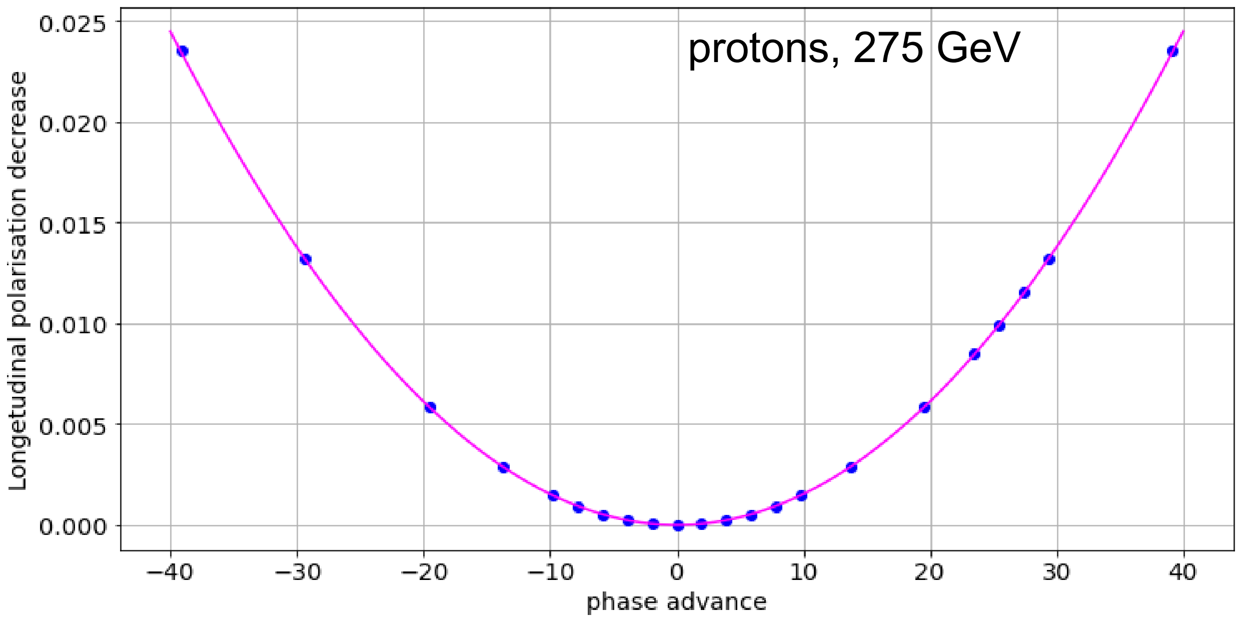



# Tolerance level for phase advance

- Phase advance between the crab cavity and IP is in the order of 5°.
- In case of polarization loss of 1% phase advance is ~25 factor larger



# Outlook

- Crab cavities cause the head and tail of a bunch to deviate from the nominal orbit. The impact of this effect on polarization spreading has been investigated.
- Stable spin axises depending on the position in the bunch and bunch polarization loss were calculated.
- Polarization loss due to crab cavity is in the order of 0.01% for protons of 275 GeV, which is negligible.
- The effects of energy variations were not considered yet.