Correction of partial snake resonances with betatron coupling at the Brookhaven AGS

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Polarized Sources, Targets and Polarimetry (PSTP '24), Newport News, VA, USA

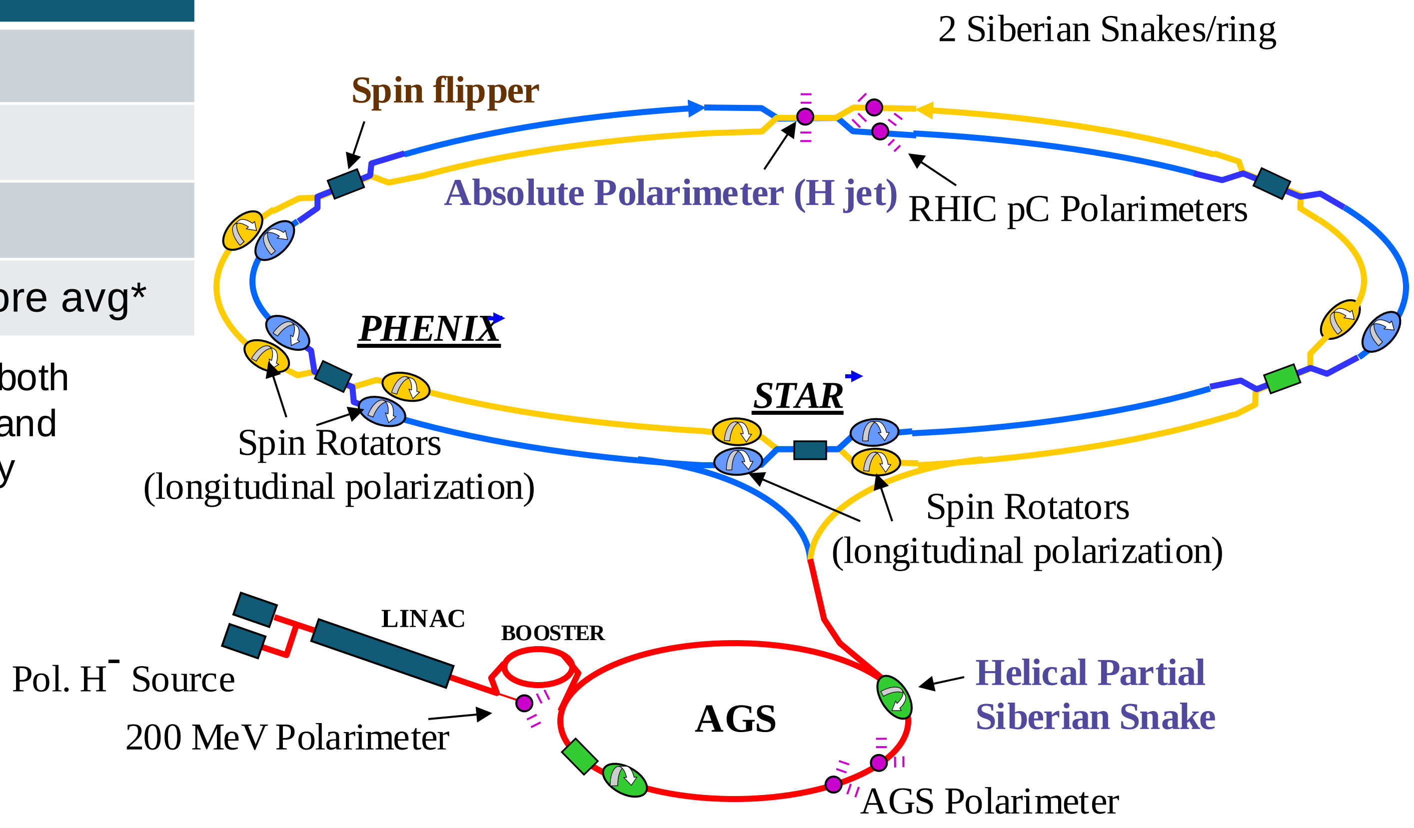
RHIC Polarized Beam Complex

	Max Energy [GeV]	Pol. At Max Energy [%]	
Source+Linac	1.1	82-84	
Booster	2.5	~80-84	
AGS	23.8	67-70	p-Carbon
RHIC	255	55-60	Jet, full store avg*

For intensity ~2.2x10[11] at AGS extraction

* Includes both ramp loss and store decay

	Relative Ramp Polarization Loss (Run 17, full run avg)
AGS	17 %
RHIC	8 %



Polarized Protons in AGS

Polarization is preserved in the AGS with two partial helical dipole snakes (10% and 6% rotation)

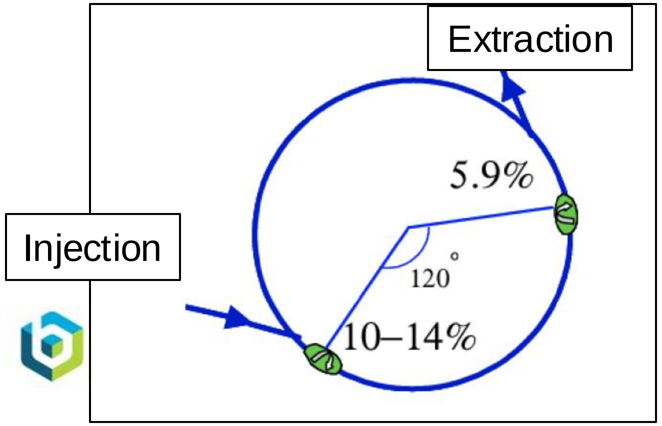
Provides spin tune 'gap' where imperfection and vertical intrinsic resonance condition are never met

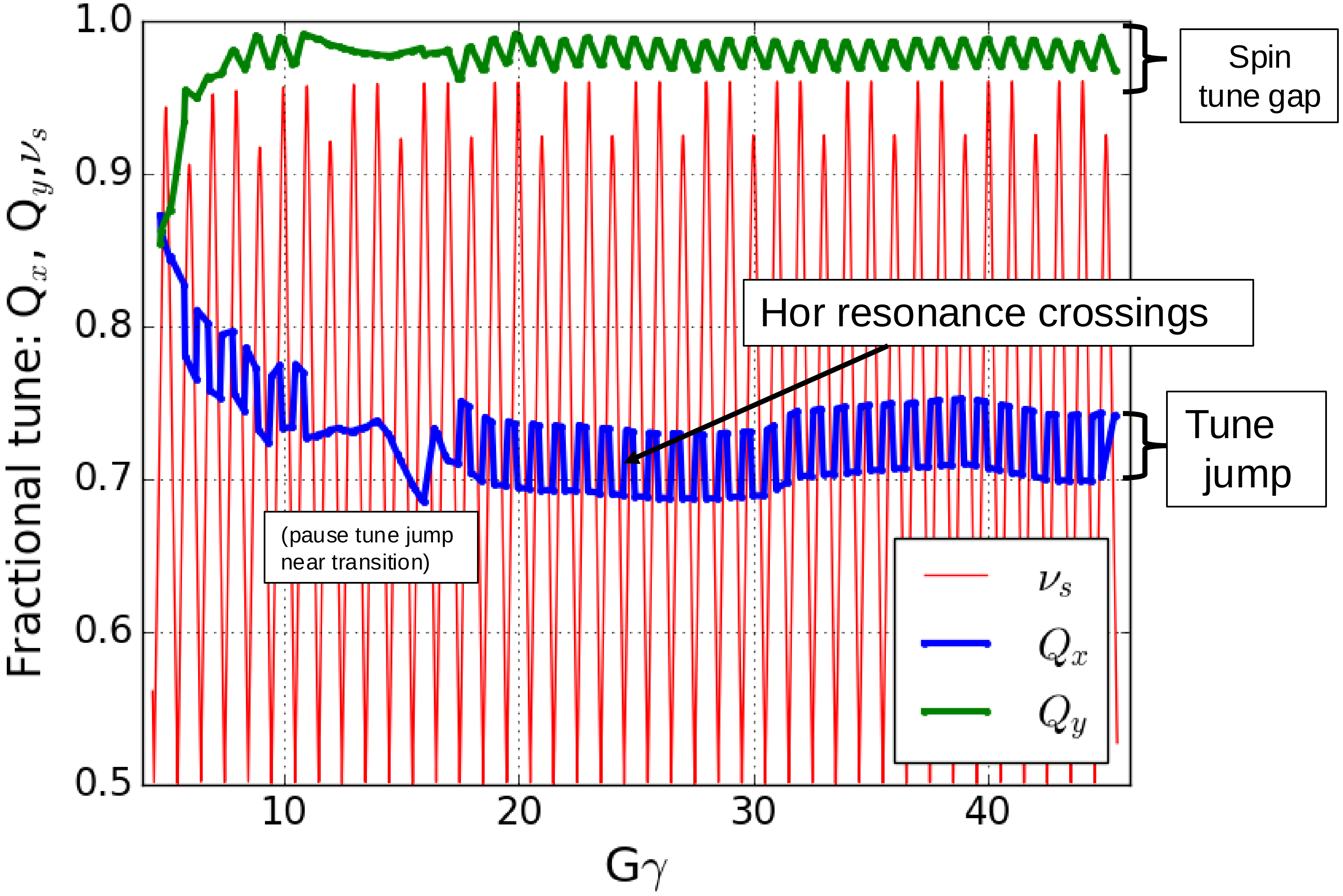
- $\nu_s \neq N$ (full spin flips)
- $\nu_s \neq N \pm Q_y$

Horizontal resonance condition still met

- $\nu_s = N \pm Q_x$
- Horizontal resonance are weak, but many (82 crossings)
- Currently handled with fast tune jump

$\Delta Q_x = 0.04$, 100 μs





Partial snakes drive horizontal resonances

Simple case (one partial snake)

Spin motion consists of

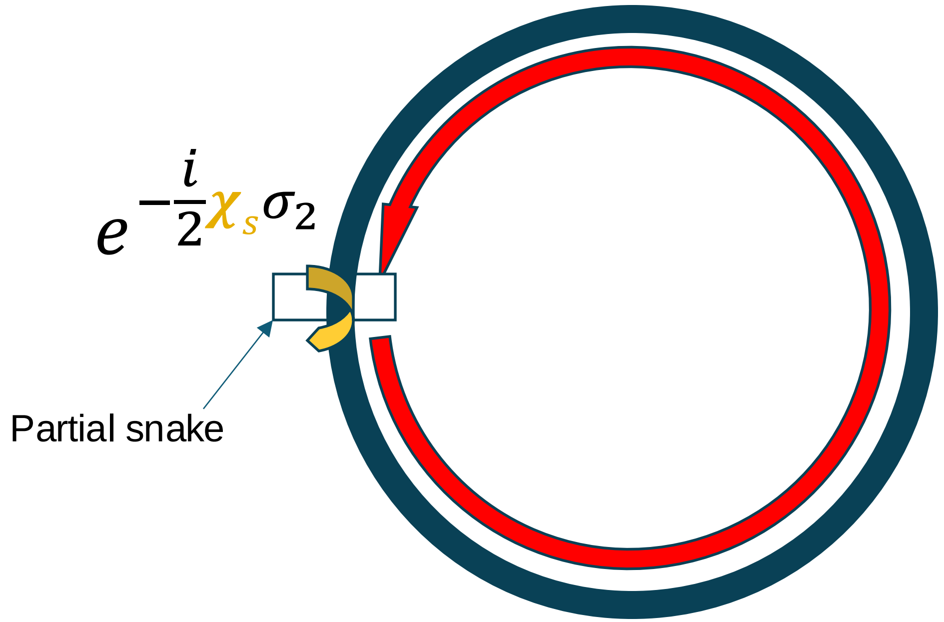
1. Spin rotation about longitudinal by angle χ_s
2. Design particle spin precesses about vertical by $2\pi G\gamma$ from main bend
3. From horizontal betatron motion, extra precession angle $(1 + G\gamma)\Delta x'$

The horizontal betatron motion modulates the spin precession phase at every snake transit at Q_x producing sideband resonances at

$$\boxed{\nu_s \cong G\gamma = N +/- Q_x}$$

"Toy AGS ring"

$$e^{-\frac{i}{2}\chi_s\sigma_2}$$



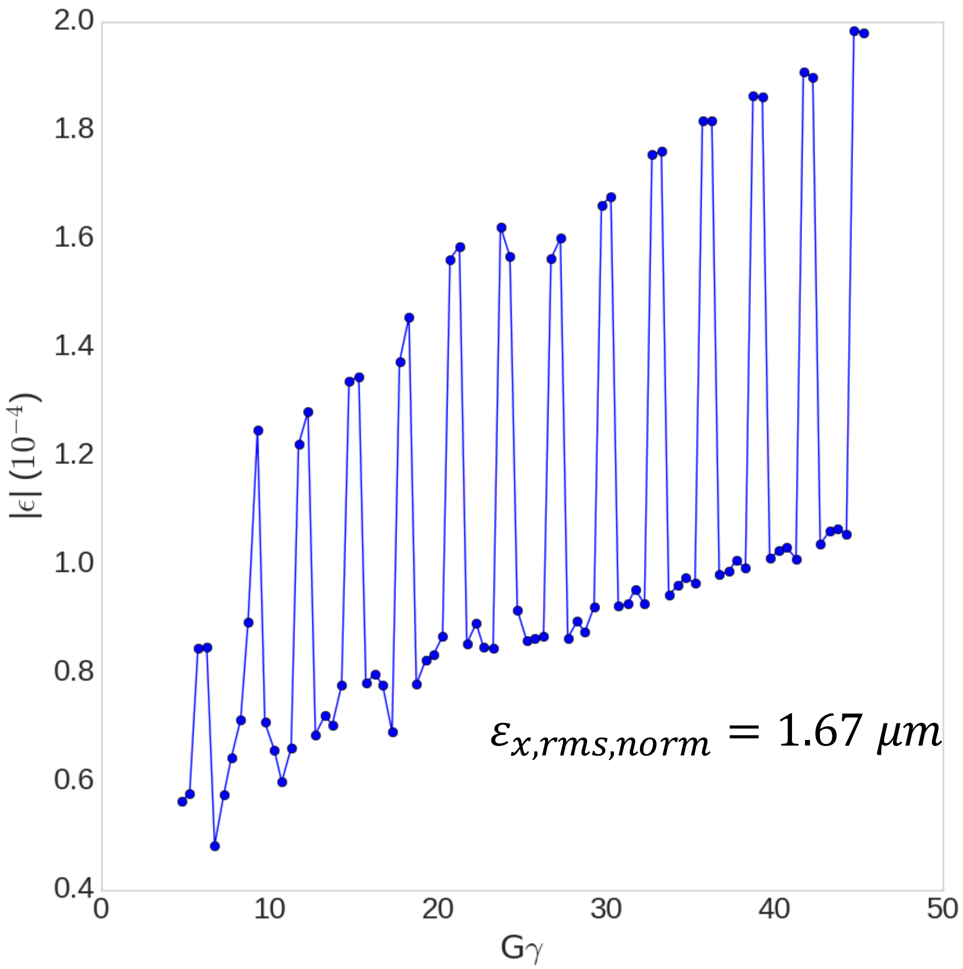
$$e^{-\frac{i}{2}\left(2\pi G\gamma-(1+G\gamma)\Delta x'\right)\sigma_3}$$

$\Delta x'$ = the one-turn change in betatron angle

Horizontal resonances in the AGS

- Two snakes, separated by 1/3 circumference
    - Modulated resonance amplitude highest near Gɣ = 3N (when snakes add constructively)
- Horizontal resonances occur every 4-5 ms at the standard AGS acceleration rate

Horizontal Resonance Amplitudes in AGS

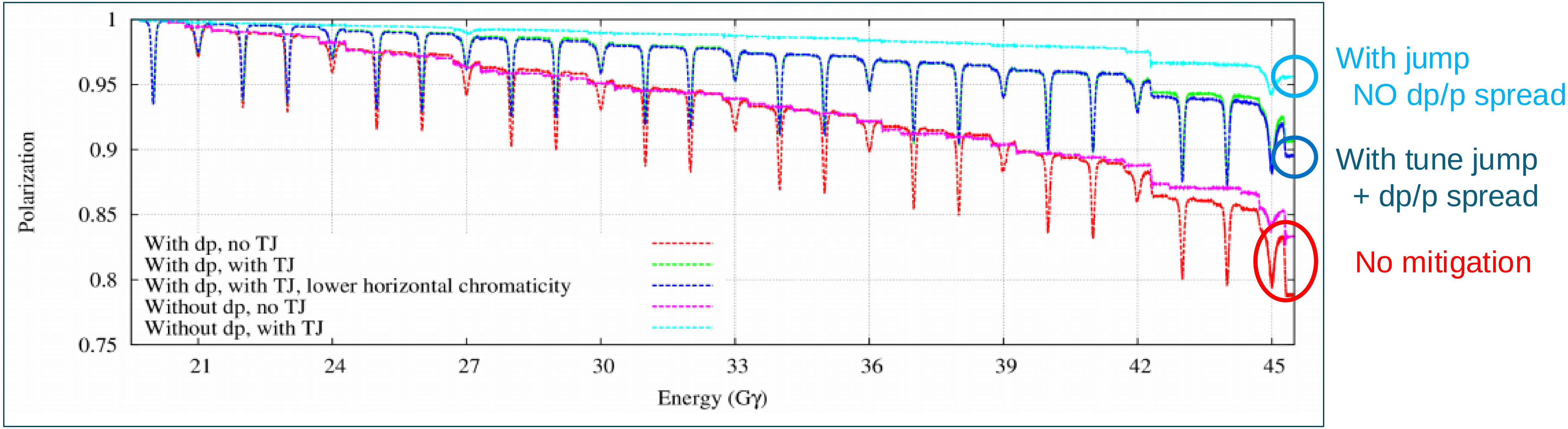


Polarization loss from horizontal resonances

With no mitigation: ~15-20% relative polarization loss
Horizontal tune jump (used since 2011): Avoids half the loss, ~10% relative polarization loss remains
Remaining polarization loss from residual crossing rate, spin tune spread comparable to tune jump size

Zgoubi simulation of polarization loss with and without tune jump, agrees well with measured tune jump performance

Goal of resonance compensation is to reduce the polarization loss from horizontal snake resonances to 0%



Plot courtesy of Y. Dutheil

Resonance compensation via coupling

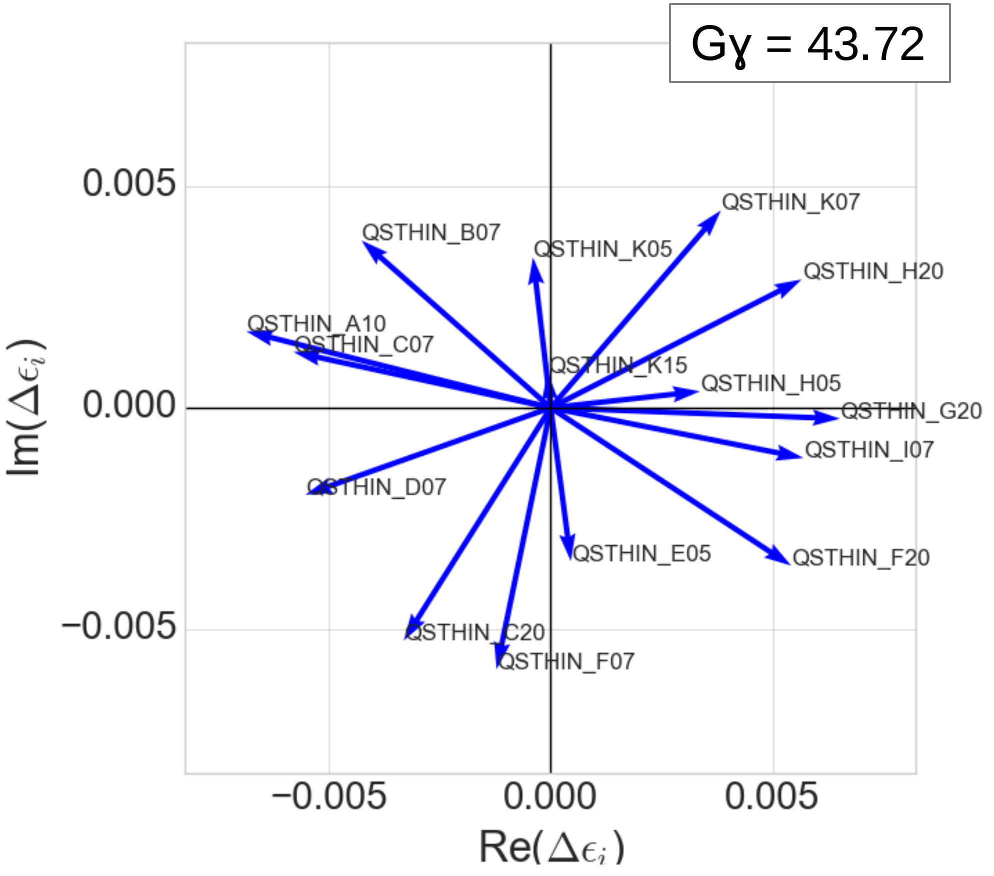
Depolarizing resonances from partial snakes and betatron coupling have the same frequencies

$$\nu_s \cong G\gamma = N \pm Q_x$$

Every skew quadrupole represents a complex resonance driving term (calculated using SPRINT algorithm)

Place a collection of skew quadrupoles in the ring to cancel the partial snake driving term.
"Just" linear algebra

Spin resonance terms from skew quads in AGS



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Partial snake resonance suppression with betatron coupling

In principle minimizing resonance strength (calculated with SPRINT) and coupling (two complex numbers) takes 4 skew quads

BUT:
There are 82 such linear algebra problems to solve
AND:
Phasing between skew quadrupole terms and snake terms varies wildly resonance to resonance
AND:
Correction vectors are frequently parallel, especially near strong vertical intrinsic resonances

Solution strategy: search for a solution with a distributed number of weaker, fast ramping skew quadrupoles

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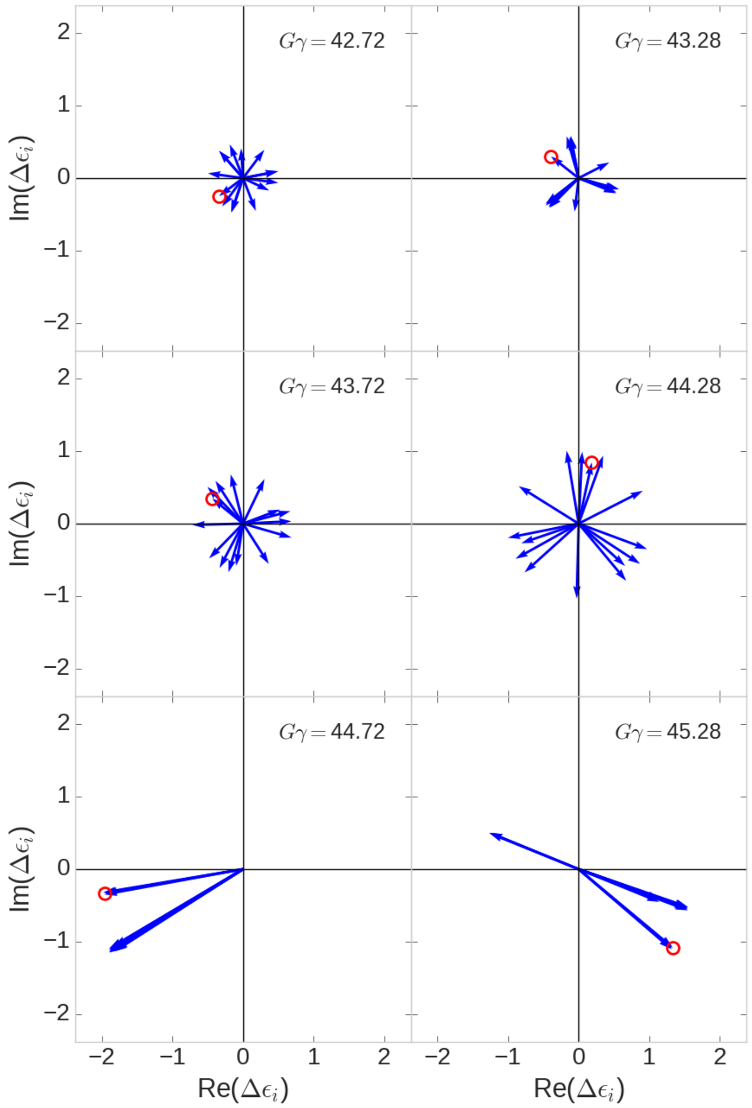
Skew quad correction vectors (last six hor resonances)

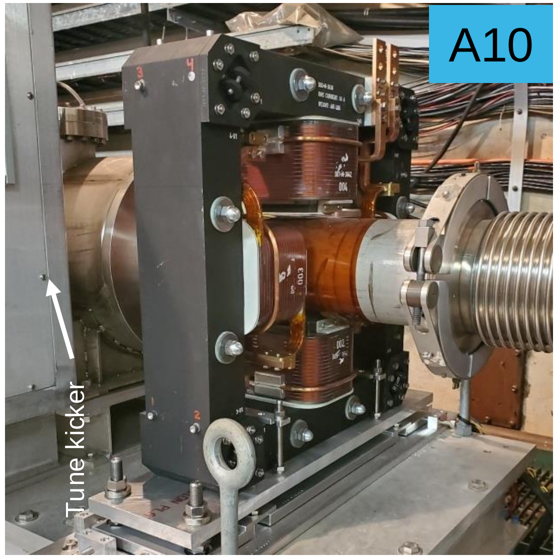


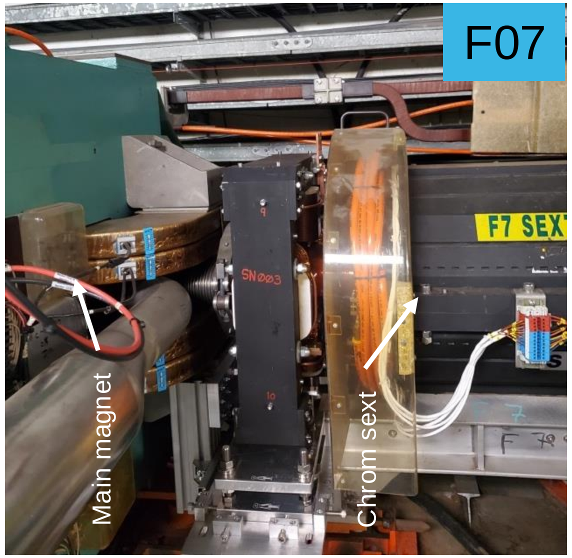
Phase offset chosen so that snake drive term is purely real
Red circle marks the same skew quad in each frame, phase changes rapidly from resonance to resonance

Magnets and locations

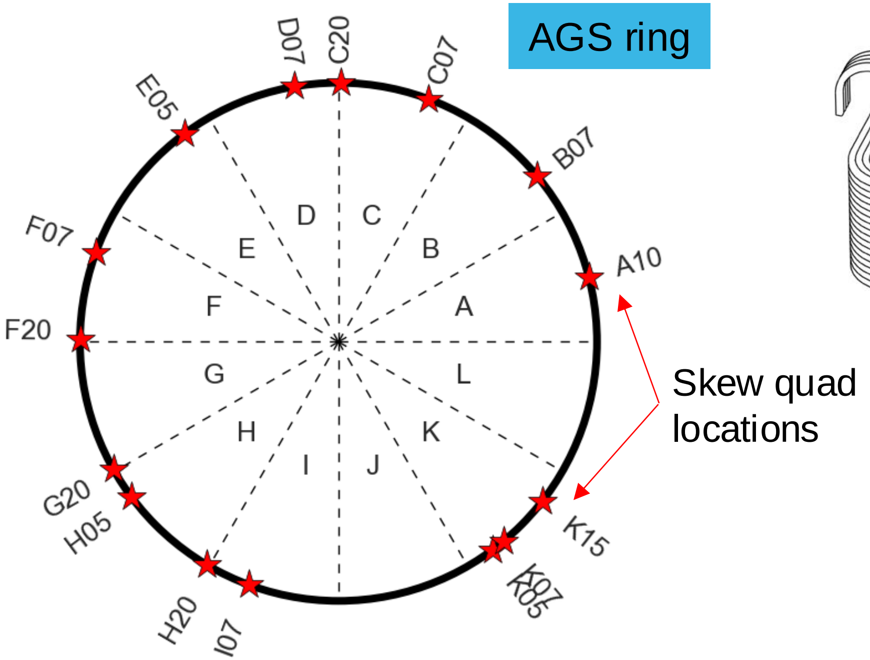
- A set of 15 skew quadrupoles with integrated skew quadrupole gradient at least 0.2 T meets the physics requirements
  - 9 placed at narrow locations in the AGS adjacent to sextupoles
  - 6 locations in longer (mostly empty) straight sections
  - Locations determined largely by brute force optimization from the available ~30 locations

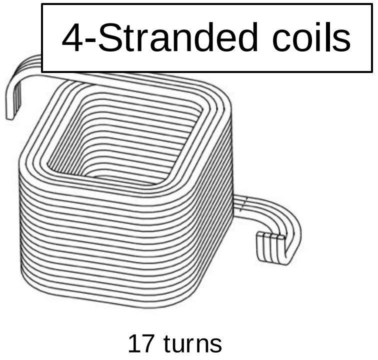
  Note: At top energy one AGS main magnet causes ~70° of spin precession
    - Geographically close != close in phase





Magnet Param		unit
Length (mech.)	0.17	m
Bore diameter	0.16	m
Pole tip field (max)	0.15	T
Int grad (max)	0.32	T
Current (max)	275	A
Current (rms,max)	60	A
Lamination thick.	0.635	mm





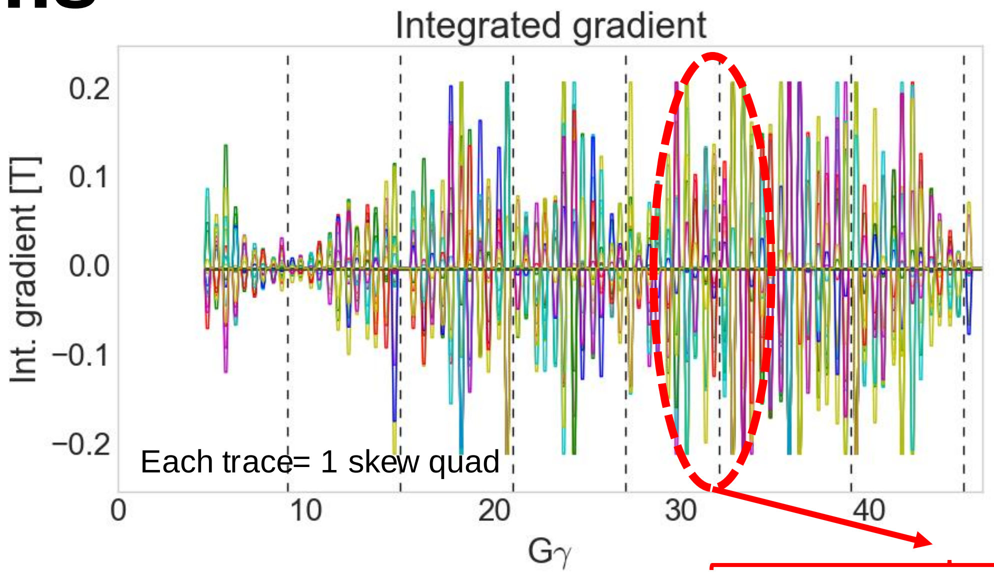
Calculated correction waveforms

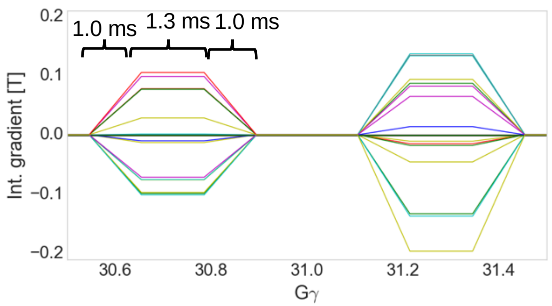
- Pulsing the skew quads with 1 ms rise, 1.3 ms flattop and 1 ms fall allows changing currents quickly to accommodate new correction vectors every resonance

  Also avoids having skew fields during strong vertical intrinsics (tracking showed small polarization losses at these with skew quads powered)

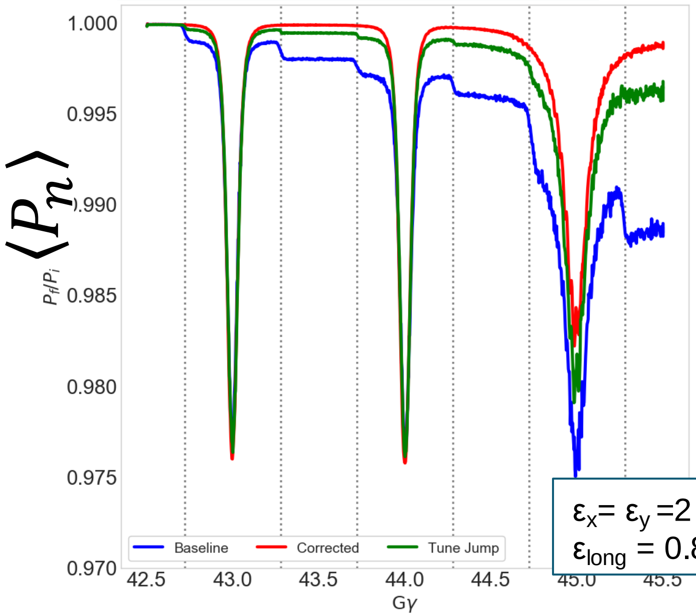
- Calculated resonance strength reducible to zero at almost every resonance
  - Resonance strength, |ε| = 0
  - Tune shift from coupling, $\Delta Q_y < 0.005$

- Correction active 10x longer than tune jump
  - Covers whole spin tune spread
  - Less sensitive to 'drift' in beam energy timing
  - No residual 'crossing rate': full compensation





Zgoubi Spin tracking

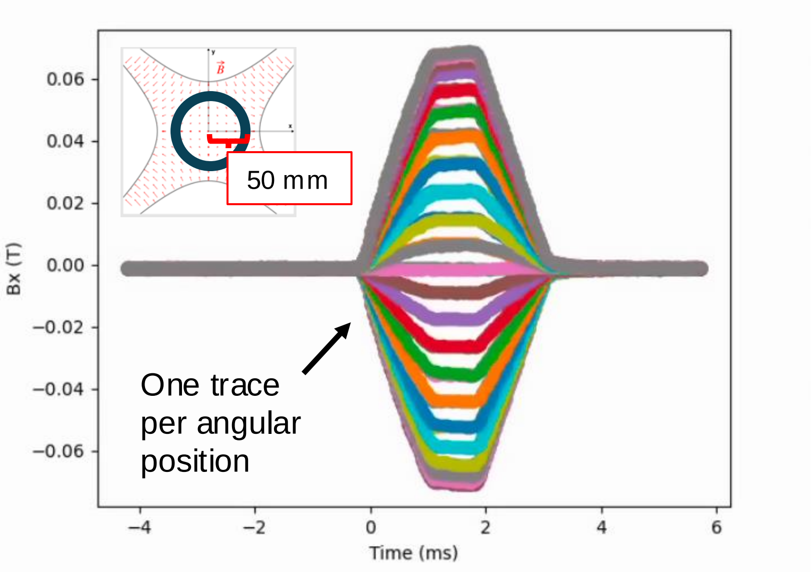


$\varepsilon_x = \varepsilon_y = 2\ \mu m$ (rms norm)
$\varepsilon_{long} = 0.8$ eV-s

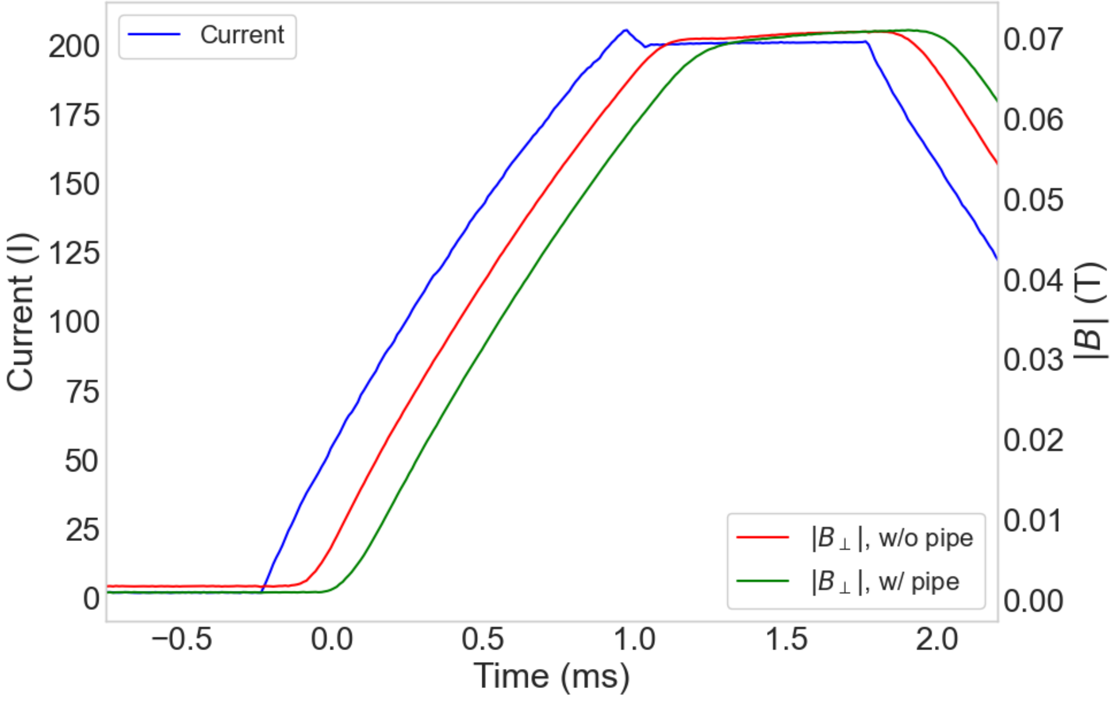
Magnetic measurements, field delay

- Pulsed Hall probe measurements (200 A, 1.2 ms rise time)
  - Hall probe measurement during pulse(bandwidth 0-2.5 kHz)
  - 49 points around a 50 mm reference circle, longitudinally centered in magnet
- Measured with and without AGS straight section beam pipe
  - 1.5 mm thick stainless steel, round cross section
  - Field delay (relative to applied current) is ~200 us, accounted for in pulse timing request

Hor Field Measurement



Hall probe field delay measurement



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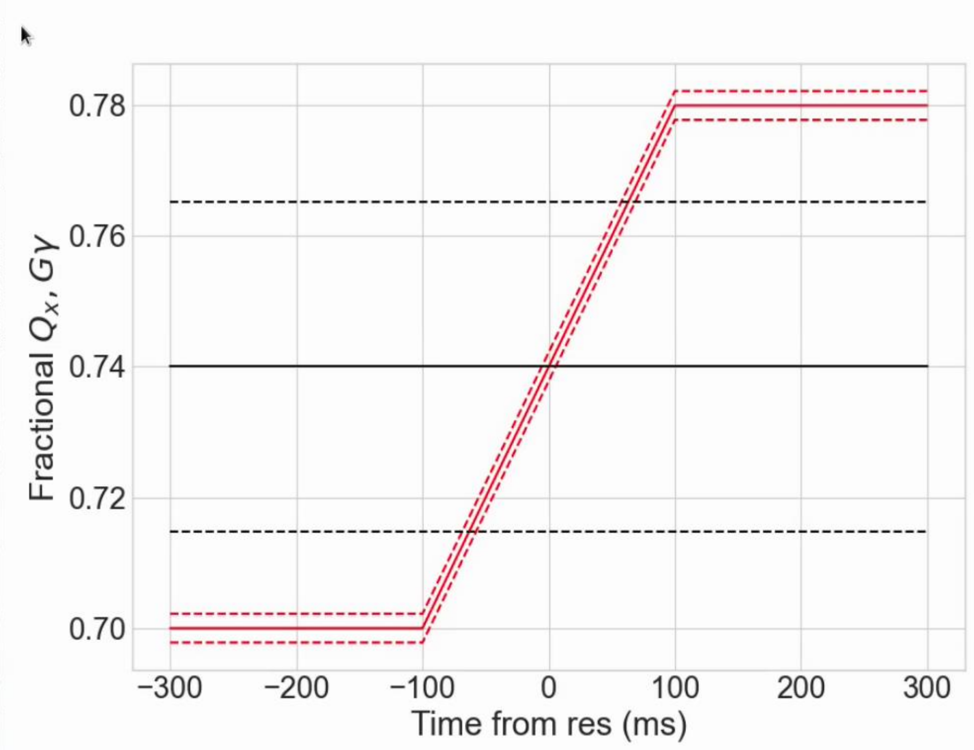
Commissioning: Proof of principle single resonance crossing

- At nominal acceleration rate ($dG\gamma/d\theta = 4.7 \times 10^{-5}$), max polarization loss from a single resonance is 0.1-0.5%
  - too small to to measure individually
- Configure a crossing at fixed energy: just above nominal extraction, with ramped horizontal tune and *very slow ramp rate* (>100x longer)

Parameter	Value
$G\gamma$	45.74
dp/p (full base)	1×10^{-3}
Chrom ξ_x	4
ΔQx	0.08
Tune ramp length [ms]	200
Crossing rate (α)	$\boxed{1.7 \times 10^{-7}}$

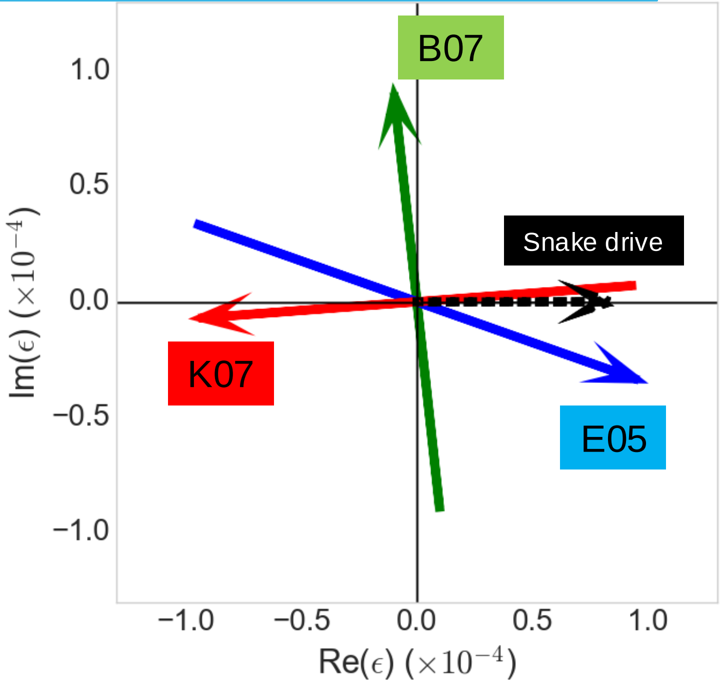
Slow crossing gives measurable 20-25% relative polarization loss

Resonance crossing



Commissioning: Proof of principle single resonance crossing

Resonance 'portrait' at Gɣ = 45.74

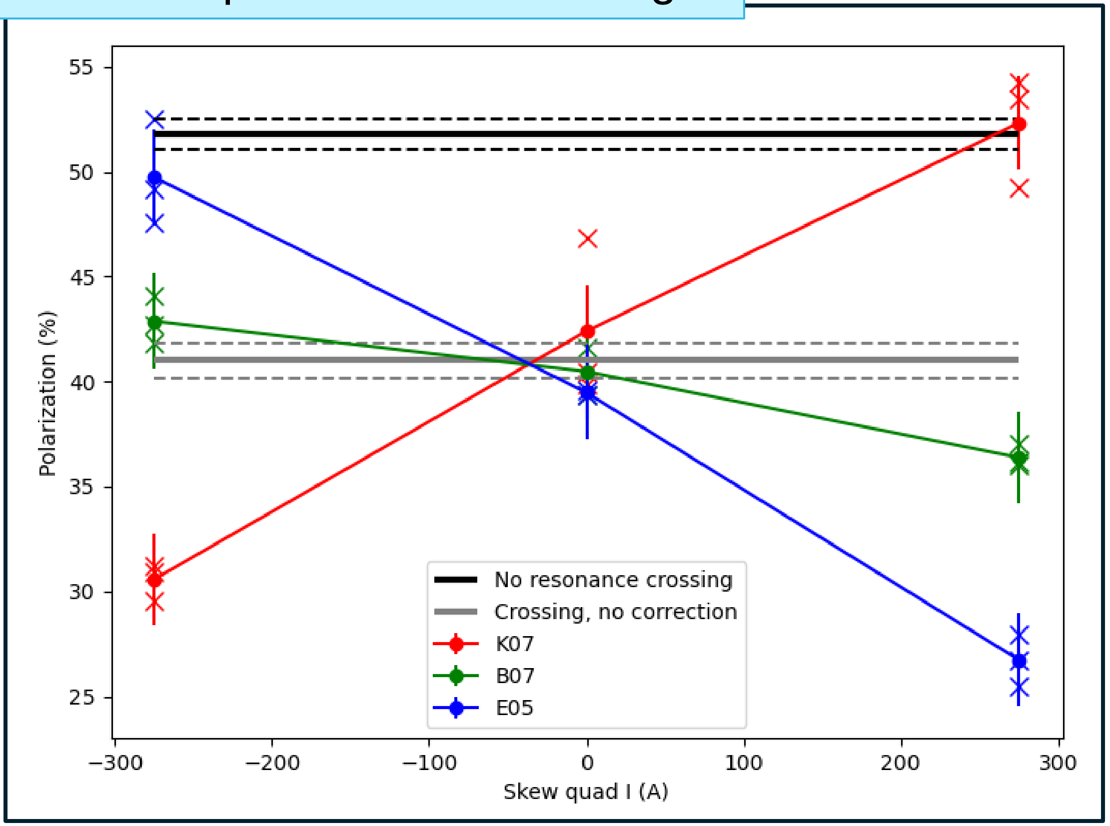


Select three skew quads with good relative phasing

- K07 in phase with snakes
- E05 180º from K07
- B07 orthogonal to snake drives

Skew quad arrow length is full current range of supply (arrow head is positive)

Polarization response to skew strength

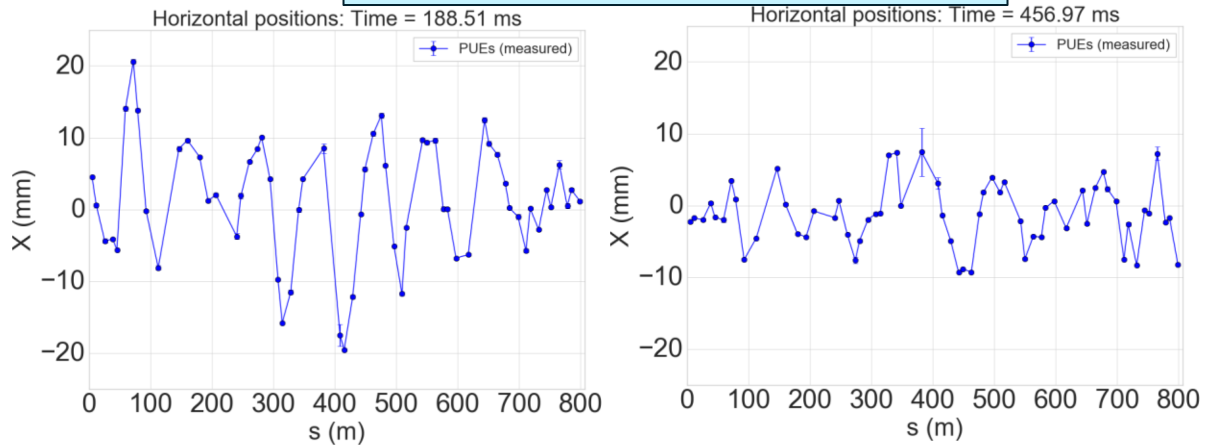


- Phasing of skew quads is as expected
- Demonstration of total correction
- In anti-correcting phase, expect *more* loss from simple Froissart-Stora estimate
    - May be multiple crossings from synchtron motion during long crossing
    - To be investigated in simulation

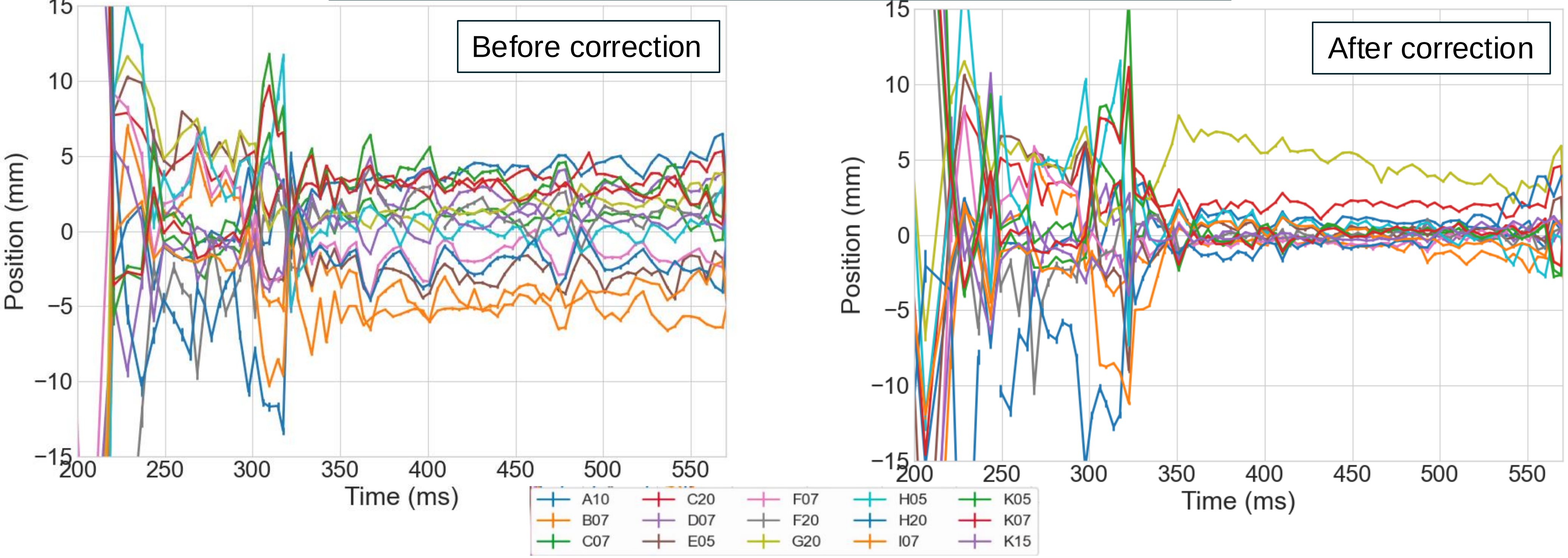
Commissioning: Orbit effects

- Large horizontal orbit excursions in AGS
- High vertical tune (8.985 – 8.991)
- Horizontal off-centering in skew quads leads to large vertical orbit changes and beam loss.
- Beam-based orbit offsets measured and corrected
  - Skew quads pulsed, infer offset from vertical orbit change + model
  - Correction limited by weak steering dipoles

Sample horizontal orbits (at BPMs)

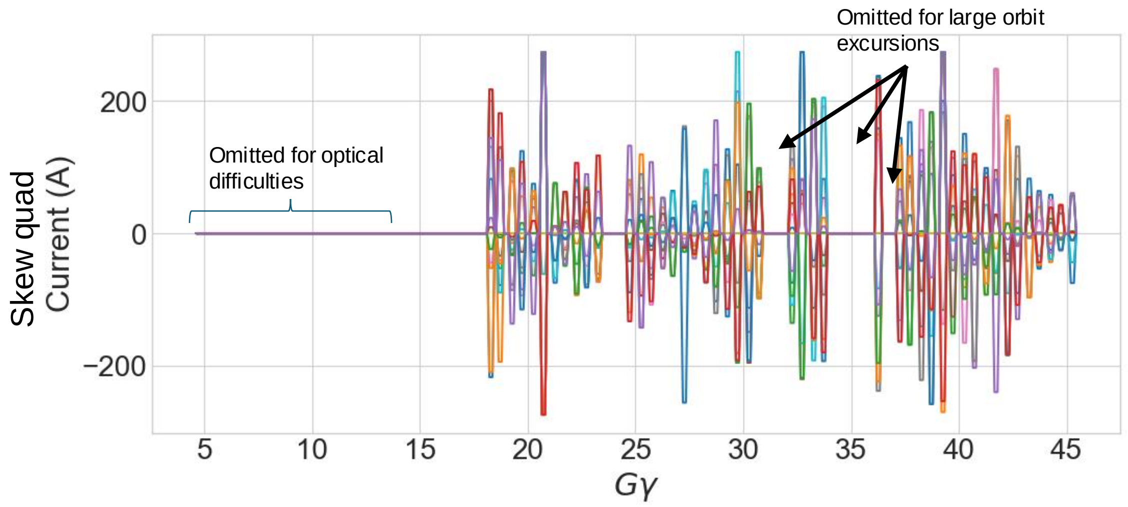


Horizontal position at skew quads, inferred from orbit differences

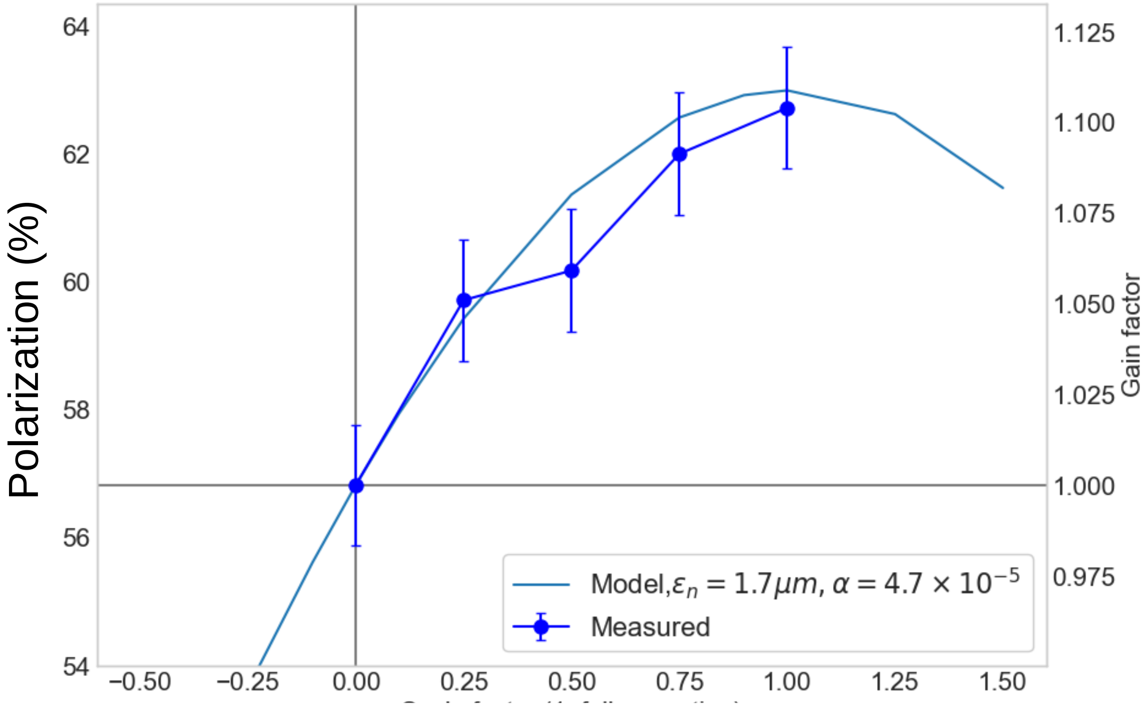


Polarization increase from skew quad resonance correction

- Initially, only about 2/3 of the pulses active
- Omitting
  - Pulses for resonances where orbit excursions still caused beam loss
  - Early resonances where optics distorted by snakes (requires special care)
  - Leaving out two skew quads (C20, G20) which have difficult to correct for horizontal orbit mis-centering



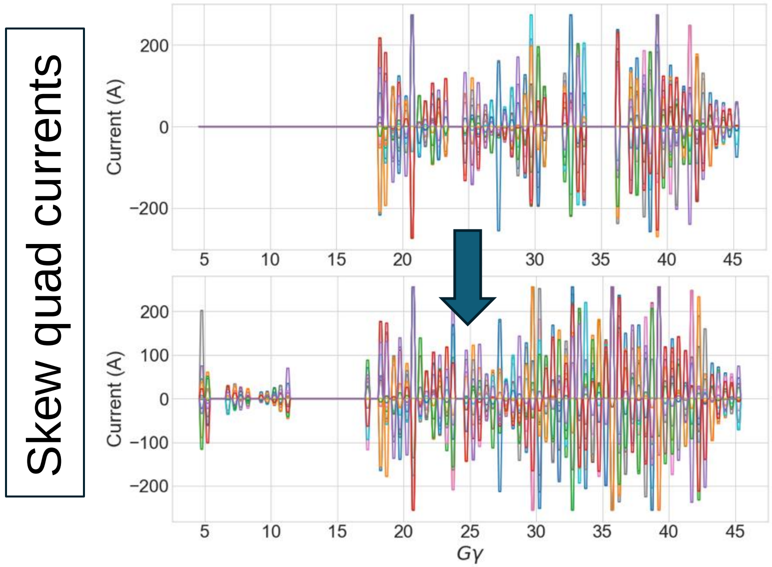
- A gain of 10% relative polarization is equivalent to gain from tune jump scheme
- Good agreement with model expectation
  - Implies that for resonances that we can pulse the quads for, we get what we expect

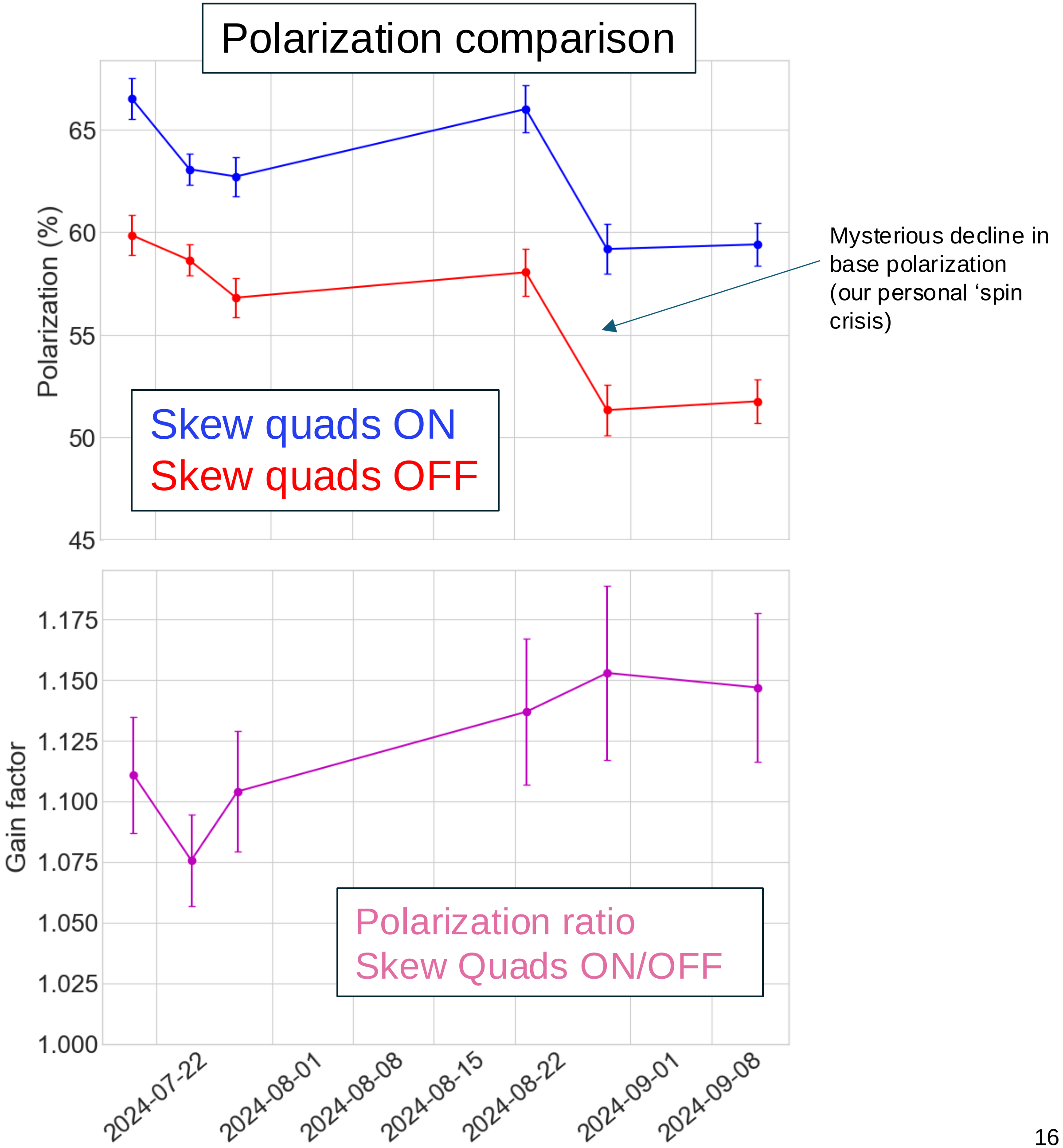


Correction amplitude (1 = full correction,0 = no correction)

Polarization increase from skew quad resonance correction

- Number of enabled pulses limited mostly by orbit effects
- Included model-predicted orbit response of the skew quads in the optimization to minimize resulting vertical rms
  - Resonance strength, $|\varepsilon| = 0$
  - Tune shift from coupling, $\Delta Q_y < 0.005$
  - Vertical $|M_{orm}*(k_{skew}*x_{skew})|_{max} < 1$ mm
- Permits enabling more pulses
- Up to 15% relative gain from skew quads (compared with 8-10% from tune jump)





State of the project

- 15% relative gain factor improves over the 10% gain factor of the tune jump
- Skew quad resonance correction configuration began commissioning in March 2024, mostly behind RHIC operations
  - March: polarity, orbit effect checks with unpolarized beam
  - Apr: polarized beam checks, single resonance tests
  - May-June: RHIC startup and development (limited AGS time)
  - July-Aug: Ramp tests and commissioning, orbit correction and compensation
  - Sep 6th to present: RHIC fills with AGS in skew quad resonance

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State of the project: looking forward

Relative loss from horizontal resonances might be as high as 20% (only 15% accounted for)

Only one week left in Run 24 polarized proton operation in the RHIC (and injectors).
Might be last week of RHIC polarized proton operation ever

No more currently scheduled before RHIC shutdown after Run 25

Development runs planned in the AGS behind gold operation in 2025 and ~1 month/year during the "Middle Ages" between RHIC and EIC.

What can we do?

- Heavily accelerator model-driven effort
  - Individual resonances too weak to tune individually empirically
- Low energy corrections might be incomplete due to optical errors from
  - Large effects of complicated helical dipole field
  - Feed-down from orbit offsets/misalignments

- Potentially ~2% from resonances near transition
  - Not currently corrected. Needs particular attention

Thanks for your attention!

Partial snake resonances are 'hybrid' resonances

M.Bai showed ***horizontal closed orbit motion*** **can interact with a vertical intrinsic to produce a set of sideband resonances*.**

Similar procedure shows ***horizontal betatron motion*** **can interact with the snake spin kick in the same way**

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} F & -\xi \\ -\xi^* & -F \end{pmatrix}\Psi$$

$$\xi = i\,Xs\,\delta(\theta-\theta_s)$$

$$F = G\gamma - (1+G\gamma)x''(\theta)\rho$$

$$\Psi = \exp[\frac{i}{2}\sigma 3(G\gamma\theta - (1+G\gamma)x'(\theta)\rho\,]\tilde{\Psi} \xrightarrow[\textit{spin precessing frame}]{}$$

$$\frac{d\tilde{\Psi}}{d\theta} = -\frac{i}{2}\begin{pmatrix} 0 & -\tilde{\xi} \\ -\tilde{\xi}^* & -0 \end{pmatrix}\tilde{\Psi}$$

$$\boxed{\tilde{\xi} = i\,Xs\,\delta(\theta-\theta_s)\;e^{-i(G\gamma\theta-(1+G\gamma)x'(\theta))}}$$

$$\tilde{\xi} = i\,Xs\,\delta(\theta-\theta_s)\;e^{-i(G\gamma\theta-(1+G\gamma)x'(\theta))}$$

$$x'(\theta) = \widetilde{x'}\sin(\nu_x\theta)$$

Expansion to get trig out of the exponent. Keep linear terms and Fourier transform:

$$\varepsilon_K = \frac{iX_s}{2\pi}\int \delta(\theta-\theta_s)\,e^{-iG\gamma\theta+iK\theta}\left[1-\widetilde{x'}\,\frac{1+G\gamma}{2}\;e^{\pm\nu_x\phi(\theta)}\right]d\theta$$

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*Phys Rev Lett, Vol 84, 6, 2000

Partial snake resonances are ‘hybrid’ resonances

$$\varepsilon_K = \frac{iX_s}{2\pi}\int \delta(\theta-\theta_s)\, e^{-iG\gamma\theta+iK\theta}\left[\underline{1} - \underline{\widetilde{x}'\,\frac{1+G\gamma}{2}\; e^{\pm\nu_x\phi(\theta)}}\right] d\theta$$

Term 1: Amplitude $X_s/2\pi$, non-zero when $G\gamma = K$, snake imperfections

Term 2: Amplitude ~ $\widetilde{x}'$, non-zero when $G\gamma \pm \nu_x = K$, intrinsic resonance driven by the phase modulation of the snake imperfections

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What if another intrinsic resonance is present at frequency v_x: $\xi = i\, Xs\; \delta(\theta-\theta_s) + \varepsilon_x e^{\pm\nu_x\phi(\theta)}$

$$\varepsilon_K = \frac{iX_s}{2\pi}\int \delta(\theta-\theta_s)\, e^{-iG\gamma\theta+iK\theta}\left[1 - \boxed{\left(\widetilde{x}'\,\frac{1+G\gamma}{2} + \varepsilon_x\right)}\; e^{\pm\nu_x\phi(\theta)}\right] d\theta$$

A second intrinsic resonance at the horizontal betatron tune can be used to cancel the partial snake resonance term.

Easiest way to make one is with betatron coupling

Error tolerance

What if the calculated correction has the wrong phase or amplitude? Difficult to tune individual resonances, signal very small.

Froissart-Stora polarization loss for small residual resonance $\Delta\varepsilon$

$$\frac{P_f}{P_i} = \frac{1 - \frac{\pi|\Delta\varepsilon|^2}{\alpha}}{1 + \frac{\pi|\Delta\varepsilon|^2}{\alpha}} \approx 1 - \frac{2\pi|\Delta\varepsilon|^2}{\alpha}$$

$\Delta\varepsilon = \varepsilon_{snk} - \varepsilon_{correction}$

$\varepsilon_{snk,max} = 2 \times 10^{-4}$

$\alpha = 4.7 \times 10^{-5}$

$\boxed{1-(P_f/P_i)_{max} = 0.5\%}$ (single uncorrected res)

Phase error

- Worst case is global phase error
- Dominated by determination of Gɣ
- Error on Gɣ bounded by observation that tune jump is effective:
  - ΔGɣ <= 0.02 (~8° phase error)

$$\frac{|\Delta\varepsilon|}{|\varepsilon_{snk,max}|} \approx |\sin(2\pi\Delta G\gamma)| = 0.14 \rightarrow \boxed{1-(P_f/P_i)_{residual} = 0.02\%}$$

Amplitude error

- Dominated by optical error: coupling spin resonance ~ $\sqrt{\beta_x\beta_y}$
- Vertical beta beat ~25% (measured) >> horizontal
- Even if there was a *total* resonance amplitude error of 25%

$$\frac{|\Delta\varepsilon|}{|\varepsilon_{snk,max}|} = 0.25 \rightarrow \boxed{1-(P_f/P_i)_{residual} = 0.04\%}$$

Resonance *compensation* is robust with respect to likely errors. Even worst case over 90% of the correction effect persists.

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