Spin Transport in Particle Accelerators

Fanglei Lin (ORNL)

20th International Workshop on Polarized Sources, Targets and Polarimetry (PSTP24)

Sept. 22-27, Jefferson Lab, Newport News, VA

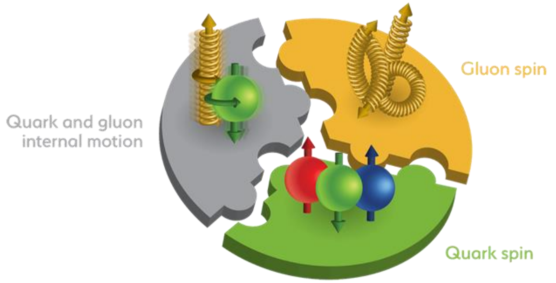
Several accelerator facilities in US

Presentations on Spin Transport in PSTP24

- **Monday**, September 23, **15:50 – 17:10**
  - Paolo Lenisa, "Search for Electron Dipole Moments and Axions/ALPS with Polarized Beams in Storage Rings"
  - Riad Suleiman, "Small Polarized Electron Storage Ring for Fundamental Physics Experiments"
  - Vincent Schoefer, "Correction of Partial Snake Resonances with Betatron Coupling at the Brookhaven AGS"
  - Vera Shmakova, "Impact of the Crab Cavities on Polarization in EIC"
- **Tuesday**, September 24, **11:00– 12:20**
  - Haixin Huang, "*Hadron Polarization at EIC*"
  - Eiad Hamwi, "*Polarization Transmission to High Energy at the EIC's HSR*"
  - Vahid Ranjbar, "*Electron Polarization in the RCS at EIC*"
  - Vadim Ptitsyn, "*Electron polarization at EIC*"

Why Polarization

- Study **distribution of angular momentum** inside protons and neutrons: HERA, RHIC, LHeC, EIC
  - SPIN is one of the fundamental properties of matter
  - All elementary particles, but the Higgs carry spin
  - Spin cannot be explained by a static picture of the proton
  - It is more than the number ½ ! It is the interplay between the intrinsic properties and interactions of quarks and gluons



$$\frac{1}{2}\hbar = \left\langle P,\frac{1}{2}\middle| J^z_{QCD} \middle| P,\frac{1}{2}\right\rangle = \overbrace{\frac{1}{2}\int_0^1 dx\Delta\Sigma(x,Q^2)}^{\text{total quark spin}} + \overbrace{\int_0^1 dx\Delta G(x,Q^2)}^{\text{gluon spin}} + \overbrace{\int_0^1 dx\left(\sum_q L_q^z + L_g^z\right)}^{\text{angular momentum}}$$

~ 30% ~ 40% ~?

Y. Furletova

- Deep study of the **electroweak interactions** with electron-positron linac: ILC, CLIC
- Measure **gyromagnetic anomalies** of particles
- Measure **electron dipole moment** of particles (puts a strong constraint on CP violation)
- Measure **beam energies** very precisely and thereby particle masses

History of (some) Polarized Electrons and Protons

	Machine Name	Energy (GeV)	Polarization (%)
e^-	VEPP	0.65	80
	SPEAR	3.7	90
	LEP	47	57
	PETRA	16.5	70
	TRISTAN	29	75
	HERA	27.5	70
p	ZGS	12	70
	KEK PS	12	>25
	AGS	24	70
	SATURNE	3	>75
	HERA	920	70
	RHIC	250	60

Figure of merit in a collider ∞ $\sqrt{L \cdot P_1^2 \cdot P_2^2}$

Definition of Polarization

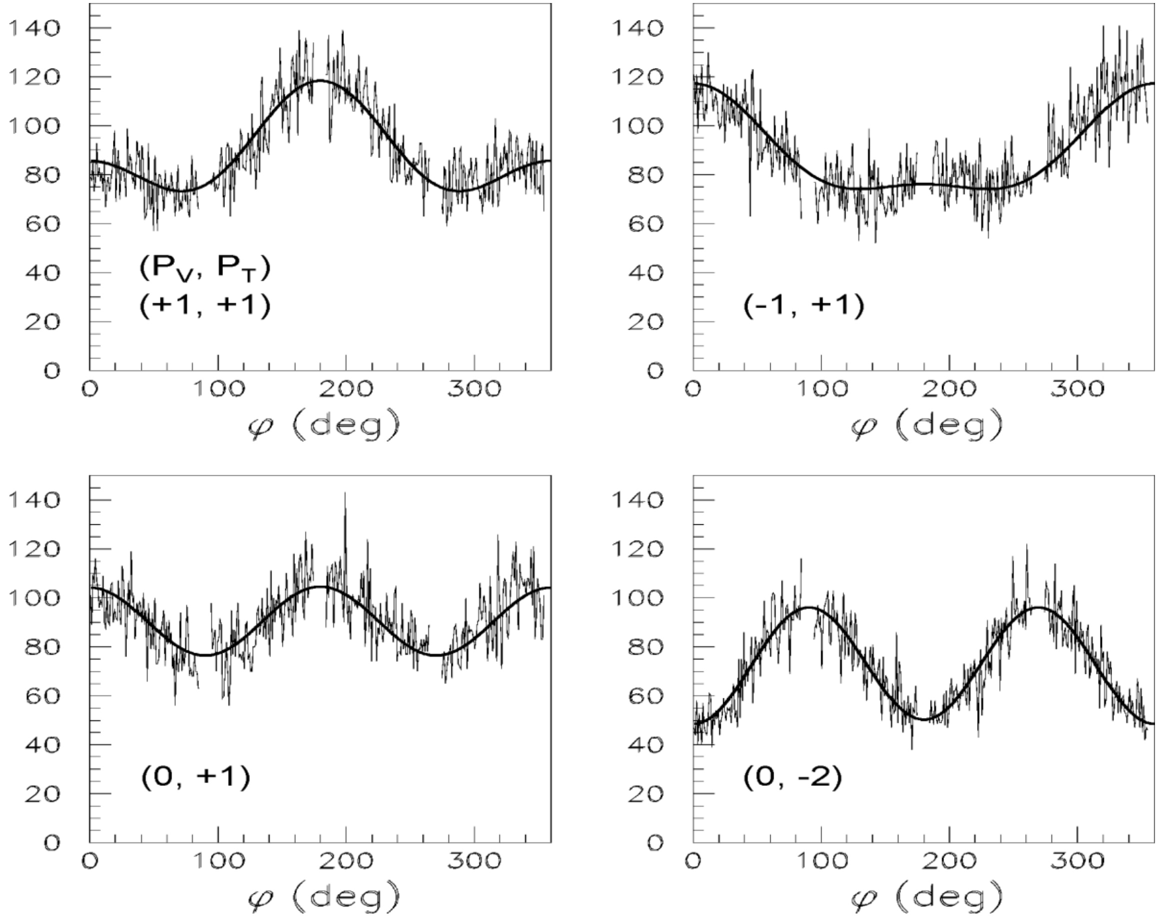
- Polarization: $\vec{P} = \frac{1}{|\langle\vec{S}_i\rangle|}\frac{\sum_{i=1}^{1\to N}\langle\vec{S}_i\rangle}{N}$ $\langle\vec{S}\rangle$ is a single-particle spin expectation value.
- Spin-1/2 fermions:
  - $\langle\vec{S}\rangle = (\Psi^*)^T\frac{\hbar}{2}\vec{\sigma}\Psi$ ($\vec{\sigma}$ is the vector of Pauli spin matrices and Ψ is the spinor)
  - $P = \frac{N_+ - N_-}{N_+ + N_-} \le 1$ ($N_\pm$ are the numbers of particles in two spin state $|\frac{1}{2},\pm\frac{1}{2}\rangle$ along a quantization axis)
  - The polarization for spin-1/2 system is called **vector polarization** $\vec{P}$, characterized by a direction and a magnitude P
- Spin-1 bosons:
  - Cartesian vector and tensor spin operators (z is the quantization axis)
  - $\langle\vec{S}\rangle = (\Psi^*)^T\hbar\vec{S}$ with $S_x = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$, $S_y = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}$, $S_z = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$, $S_{ij} = \frac{1}{2}(S_iS_j + S_jS_i) - \frac{2}{3}\delta_{ij}I$
  - **Vector polarization**: $P_V = \frac{N_+ - N_-}{N_+ + N_0 + N_-} = \frac{N_+ - N_-}{N}$
  - **Tensor polarization** : $P_T = 1 - 3\frac{N_0}{N}$

Measurement of Polarization

- Differential spin-dependent cross-section (using lab-frame polarizations)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}\left[1 + \frac{3}{2}\underline{A_y P_V \cos\varphi} - \frac{1}{4}A_{zz}P_T - \frac{1}{4}\underline{\left(A_{xx} - A_{yy}\right)P_T \cos(2\varphi)}\right]$$

- pp, pC for protons; dp, dC for deuterons
- Polarizations extracted using azimuthal count rate dependence and effective analyzing powers A_y, $(A_{xx} - A_{yy})$.
- For fast measurement, one can calibrate P_V in terms of "left-right" scattering asymmetry and P_T in terms of "left-right-top-bottom" asymmetries



V. Morozov, experiments @ IUCF

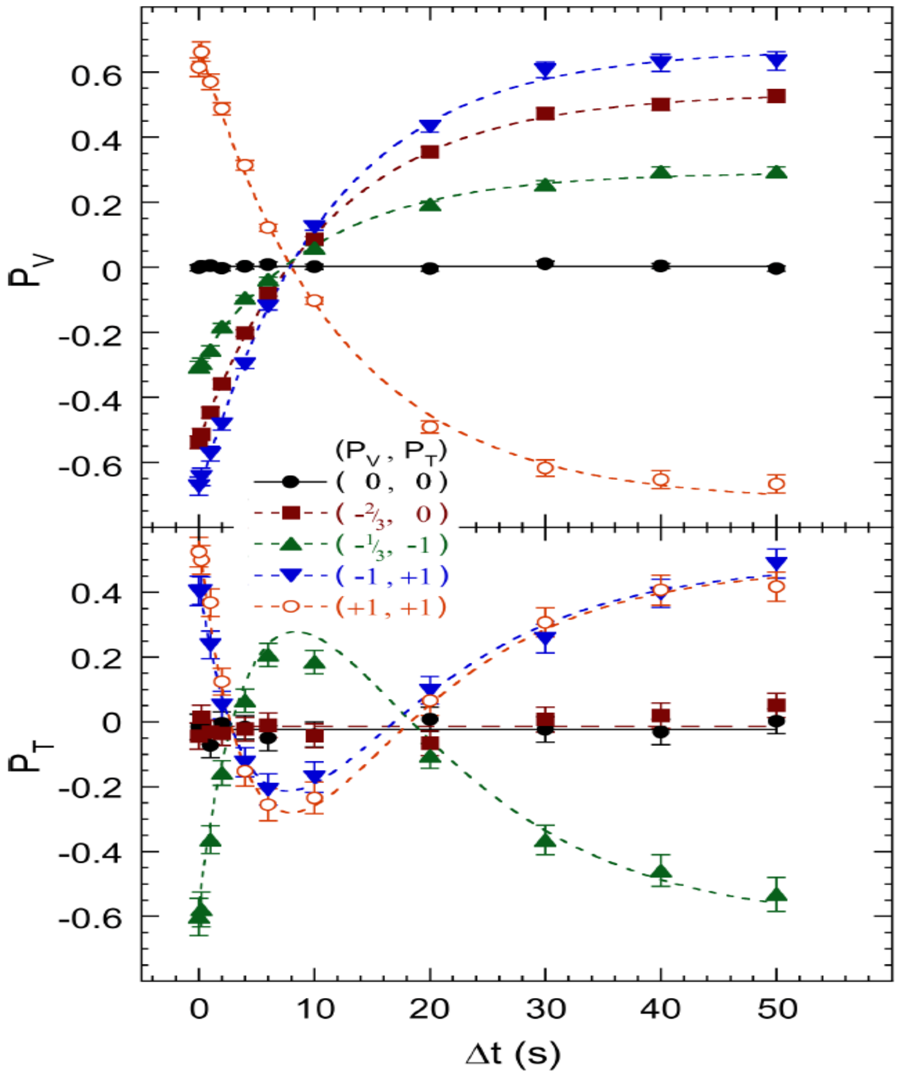
Vector and Tensor Polarization

- Experimental Demonstration
  - Sweeping an RF magnet's frequency through a spin resonance can rotate the polarization
  - Spin rotation and Froissart-Stora formula

$$\frac{P_V}{P_V^i} = 2\exp\left(-\frac{(\pi w f_c)^2}{\Delta f/\Delta t}\right) - 1$$

$$\frac{P_T}{P_T^i} = \frac{3}{2}\left(\frac{P_V}{P_V^i}\right)^2 - \frac{1}{2} = \frac{3}{2}\left[2\exp\left(-\frac{(\pi w f_c)^2}{\Delta f/\Delta t}\right) - 1\right]^2 - \frac{1}{2}$$

- Preservation and control of spin-1/2 vector polarization = preservation and control of spin-1 tensor polarization



V. Morozov, experiments @ COSY

Spin Dynamics in a Ring Accelerator

- Thomas-BMT (Bargmann-Michel-Telegdi) equation in the laboratory frame

$$\frac{d\vec{s}}{dt} = -\frac{e}{\gamma m}\left[(1+G\gamma)\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel} + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

- Lorentz Force equation

$$\frac{d\vec{v}}{dt} = -\frac{e}{\gamma m}\left[\vec{B}_{\perp}\right]\times\vec{v}$$

- Spin equation of motion in the rotating frame

$$\frac{d\vec{s}}{dt} = -\frac{e}{\gamma m}\left[G\gamma\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel} + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

- Usually, electric field contribution to the spin precession is small, therefore
  - For pure vertical field, spin rotation is $G\gamma$ times faster than motion
  - For spin manipulation
    - At low energy, use longitudinal fields
    - At high energy, use transverse field

Particle	$\frac{g}{2}$	$G\ (or\ a) = \frac{g}{2} - 1$
e⁻	1.001159652	0.001159652
p	2.7928474	1.7928474
d	0.85699	-0.14301

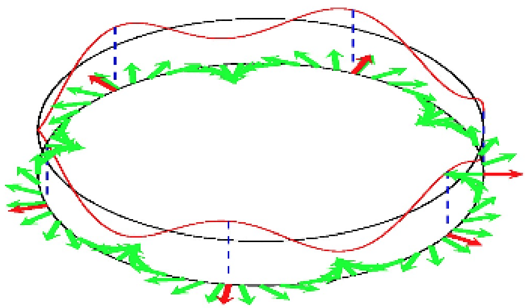
Spin Tune and Depolarizing Resonances

- Spin tune is the number of spin precession per turn in a conventional ring

$$\nu_{sp} = G\gamma$$

- Depolarizing resonance condition: the spin precession is synchronized with the frequency of spin perturbing fields
- Resonance types
  - Imperfection resonances due to field errors and misalignments

$$\nu_{sp} = n$$
$$E_e = 0.441n \text{ GeV},\ \ E_p = 0.523n \text{ GeV},\ E_d = 13.12n \text{ GeV}$$



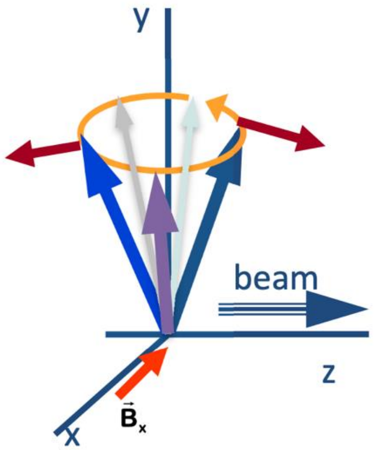
W.W. MacKay

  - Intrinsic resonances due to betatron oscillations

$$\nu_{sp} = n \pm \nu_y$$

  - Coupling and higher-order resonances

$$\nu_{sp} = n + l\nu_x + m\nu_y + k\nu_{syn}$$



Spin Resonance Crossing

- Spin resonance strength ε can be obtained from Fourier amplitude of the spin perturbing field divided by 2π

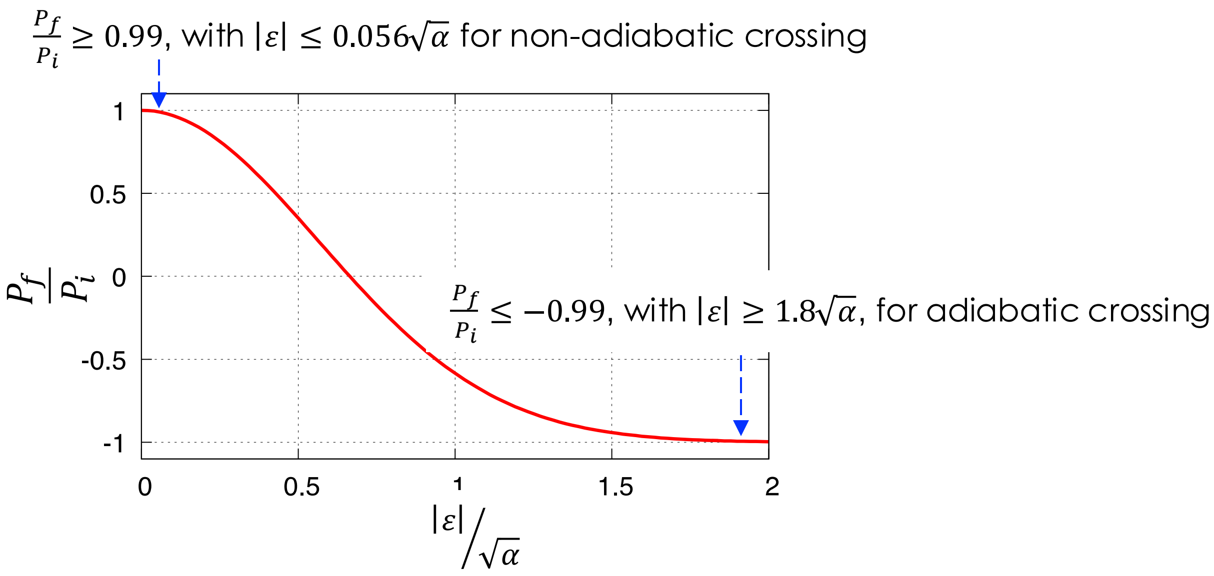
$$\varepsilon_K = \frac{1}{2\pi}\oint\left[(1+G\gamma)\frac{\Delta B_x}{B\rho} + (1+G)\frac{\Delta B_{//}}{B\rho}\right]e^{iK\theta}ds \quad \begin{cases} \varepsilon_k^{imp} \propto G\gamma\sqrt{\langle y_{co}^2\rangle} \\ \varepsilon_k^{int} \propto G\gamma\sqrt{\epsilon_y^N/\beta\gamma} \end{cases}$$

  - Weak resonances => some depolarization, strong resonances => partial or complete spin flip
  - If $\nu_{sp} = G\gamma$ changes during the acceleration, many resonances crossing cause depolarization.
  - At a fixed energy, a finite spread of $\Delta\nu_{sp}$ may overlap with higher-order resonances limiting polarization lifetime

- Froissart-Stora formula:

$$\frac{P_f}{P_i} = 2e^{-\frac{\pi|\varepsilon|^2}{2\alpha}} - 1$$

  - ε: resonance strength
  - α: resonance crossing speed

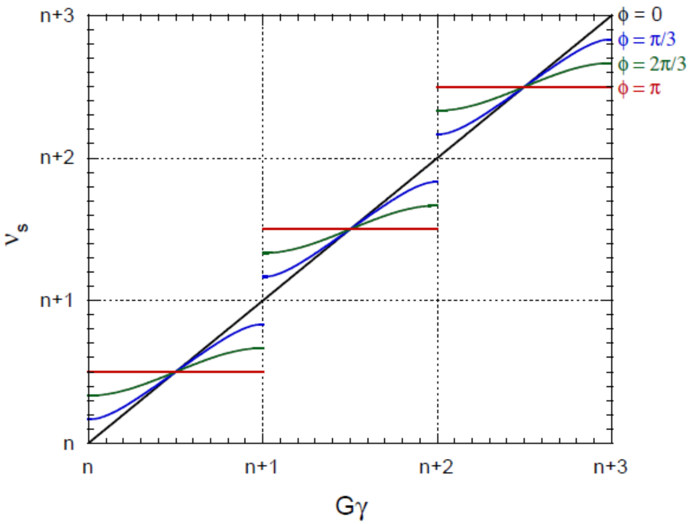


Siberian Snake to Overcome Spin Resonances

- Device rotating the spin by some angle about an axis in horizontal plane (by Derbenev and Kondratenko, 1976)
  - A "full" Siberian snake rotates the spin by 180°
  - Overcomes all imperfection and most intrinsic resonances
- Spin tune with one snake

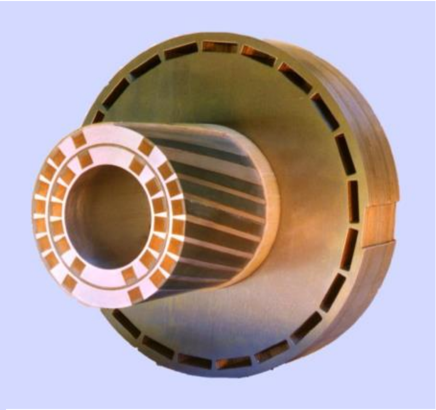
$$\nu_{sp} = \frac{1}{\pi}\cos^{-1}\left[\cos(G\gamma\pi)\cos\frac{\phi}{2}\right]$$

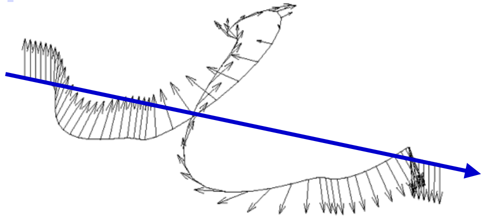
$$\phi = \pi \Rightarrow \nu_{sp} = 1/2$$



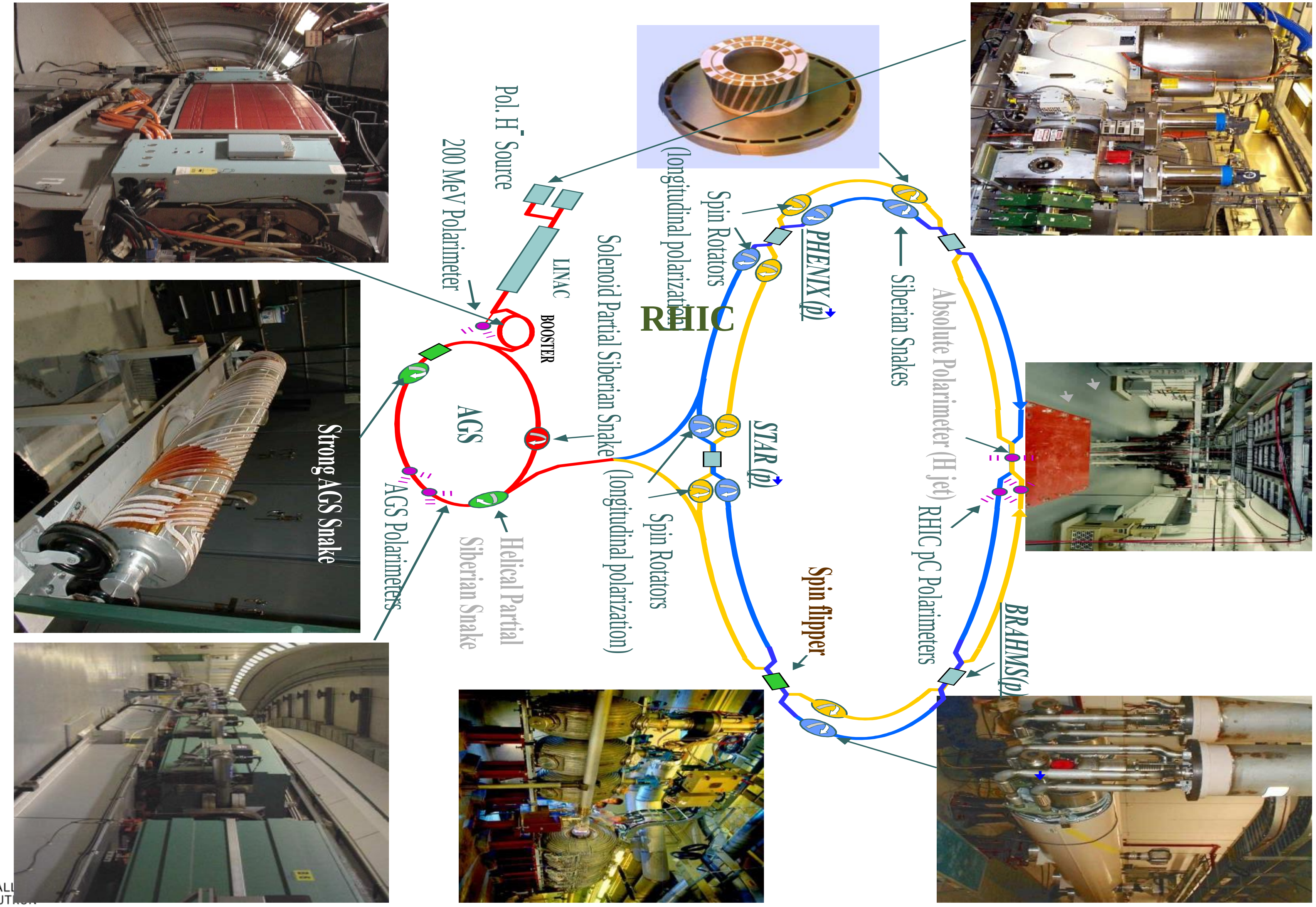
First Siberian Snake (solenoid) Test at IUCF

AGS and RHIC helical dipole snakes





Polarized Protons: RHIC @ BNL (USA)



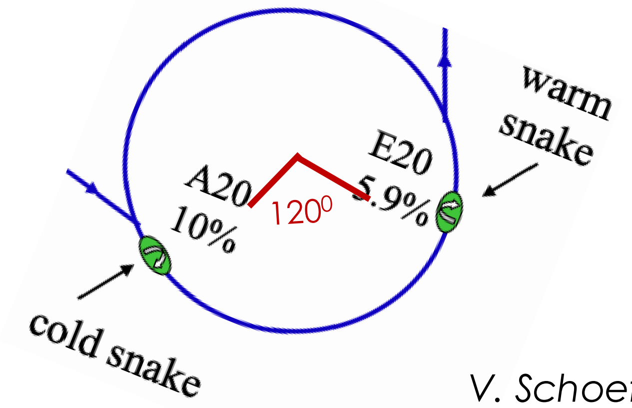
Polarized Proton in the AGS

- Polarized proton beam in AGS ramps up from 2.4 to 23.8 GeV => $G\gamma$ varies from 4 to 45
- Two partial Siberian snakes generate a spin tune gap

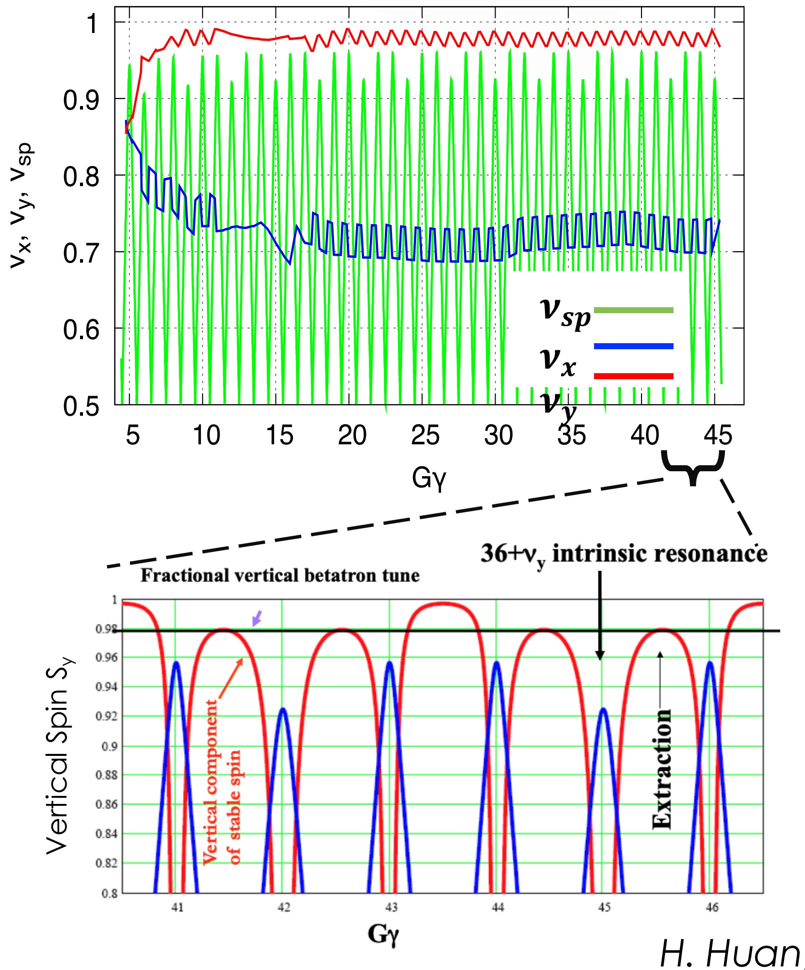
$$\nu_{sp} = \frac{1}{\pi}\cos^{-1}\left(\cos(\frac{\chi_c}{2})\cos(\frac{\chi_w}{2})\cos(G\gamma\pi) - \sin(\frac{\chi_c}{2})\sin(\frac{\chi_w}{2})\cos(G\gamma\frac{\pi}{3})\right)$$

  - Vertical tunes are put in the spin tune gap, avoiding imperfection and vertical intrinsic resonance conditions with $\nu_{sp} \neq n, or\ n \pm \nu_y$
  - Horizontal intrinsic resonance $\nu_{sp} = n \pm \nu_y$ are weak and mostly overcome by the tune jump
  - The stronger the partial snake, the larger the spin tune gap. However, a strong snake will deviate the stable spin direction from the vertical direction

- **> 65% polarized protons for RHIC**



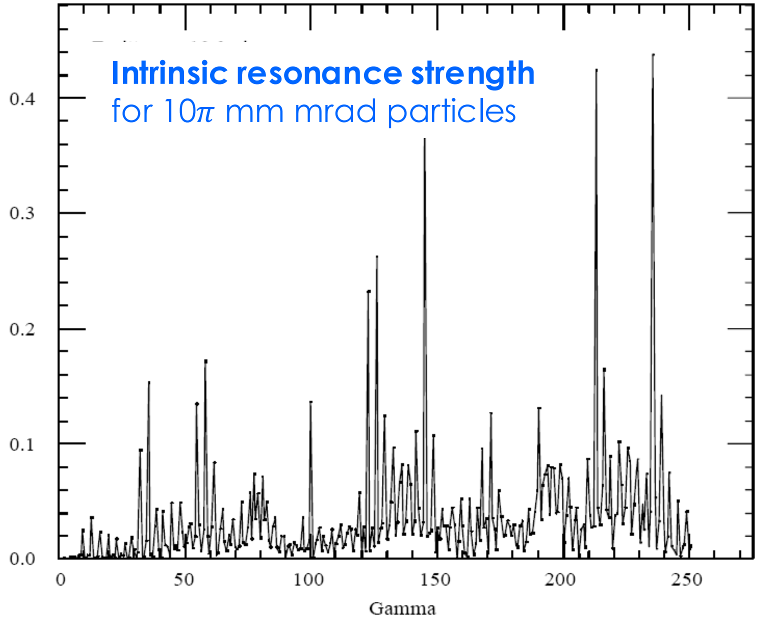
V. Schoefer

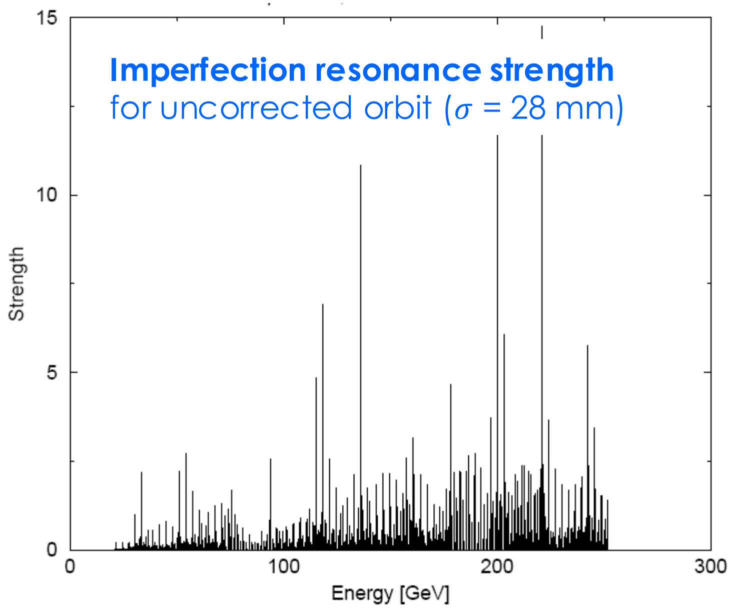


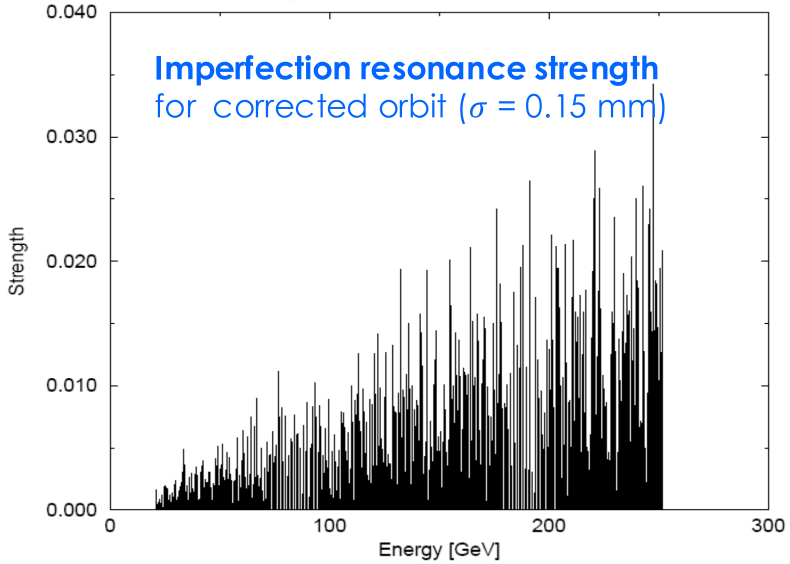
H. Huang

Polarized Protons in the RHIC

- Polarized proton beams are accelerated from 23.8 GeV to up to 250 GeV at RHIC
- Imperfection and intrinsic resonance strengths are calculated without snakes

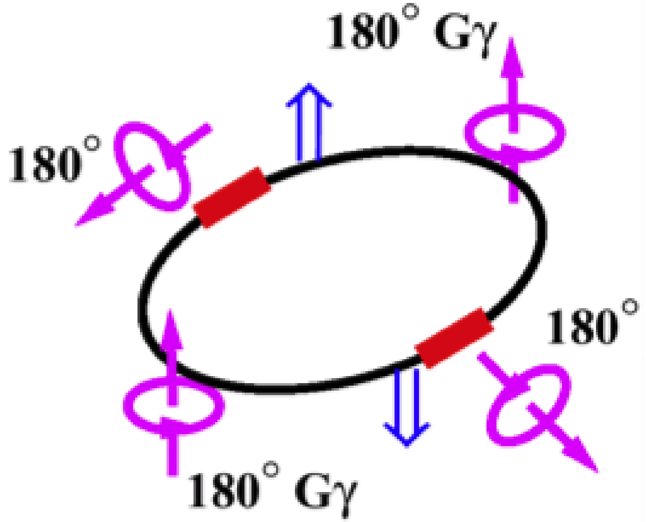




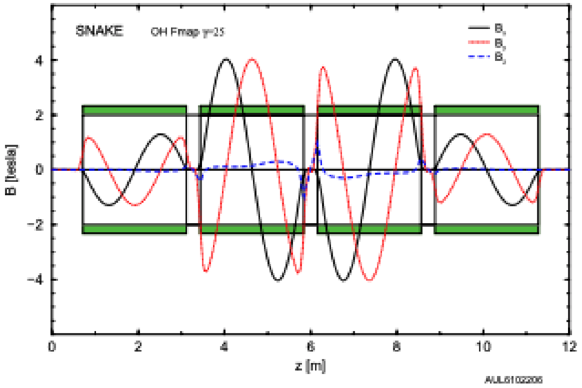


Siberian Snakes in the RHIC

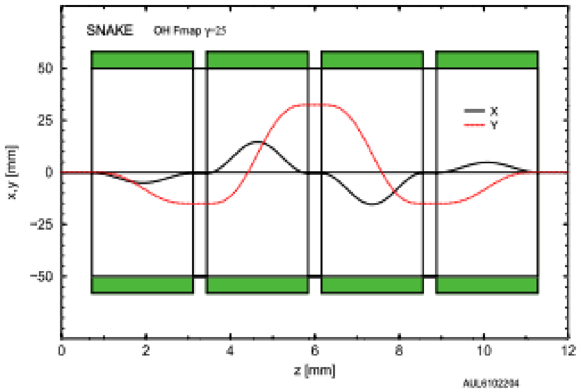
- RHIC has two full Siberian snakes: 180º apart in orbital bending angle in the ring and 90º between two snake axes.
- This orthogonal snake axes lead to $\nu_{sp} = \frac{1}{2}$, independent of beam energy and avoiding imperfection and intrinsic spin resonances.
- Each Siberian snake has four 2.4 m long superconducting helical dipoles, with a field up to 4 T.
- **60% polarization** has been achieved at 255 GeV



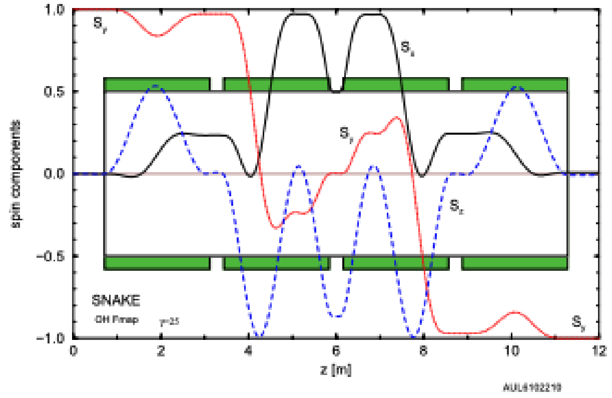
Magnet fields in one Snake



Orbits in one Snake

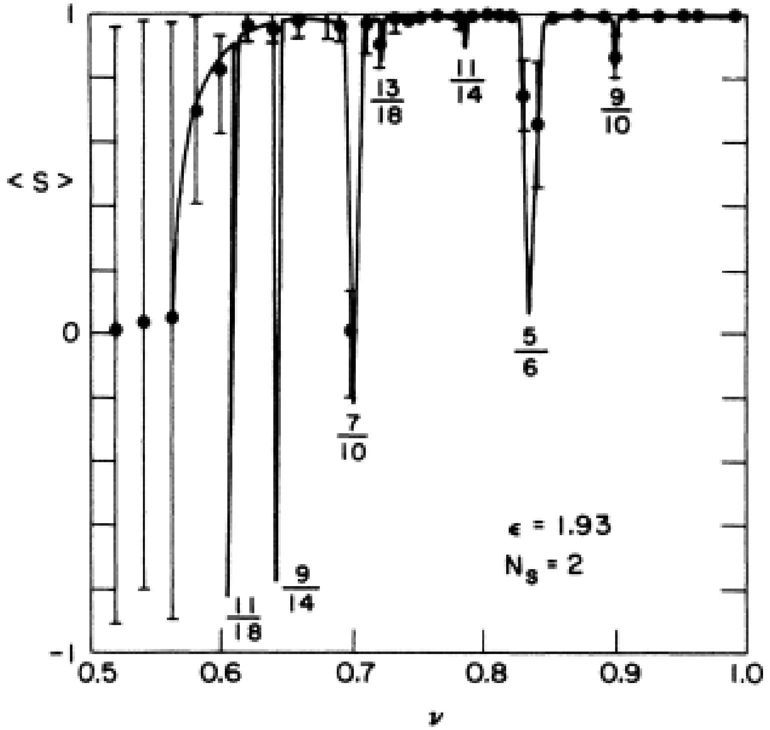


Spin trajectory in one snake



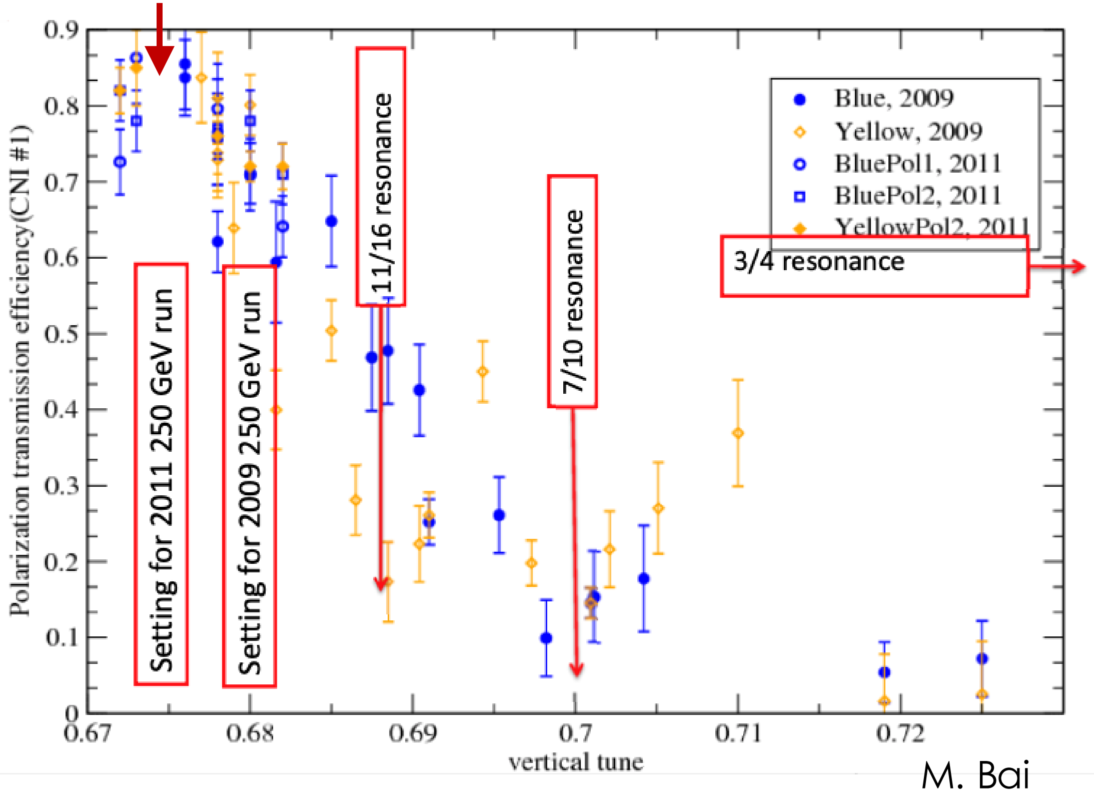
Snake Resonances

- Snake depolarizing resonances occur at $\nu_{sp} = \frac{1}{2} = m \pm n\nu_y$, $m, n =$ integer
- Even order snake resonances, $n = 2k$, are driven by the imperfection resonances
- Odd order snake resonances, $n = 2k + 1$, are driven by the intrinsic resonances



S. Y. Lee and S. Tepikian

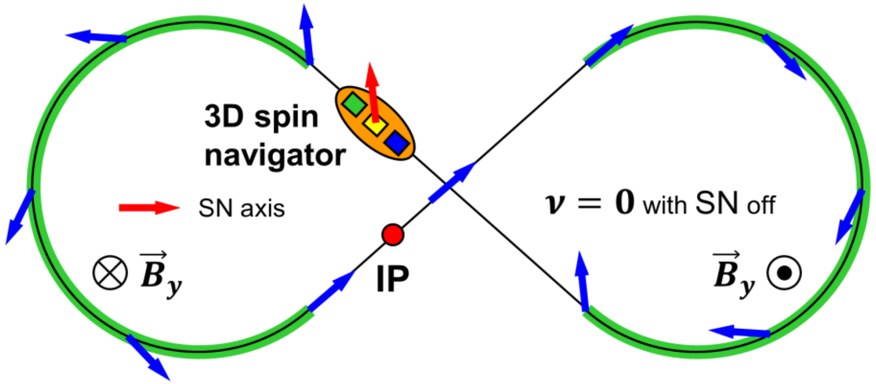
Best polarization transmission

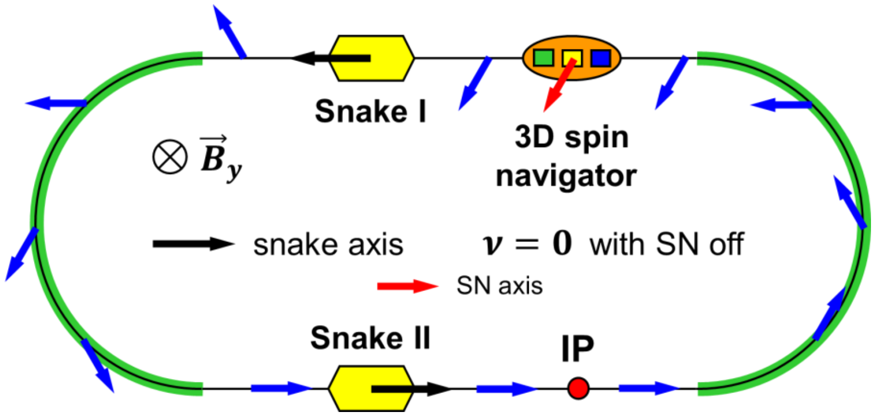


M. Bai

Spin Transparency Concept and its Features

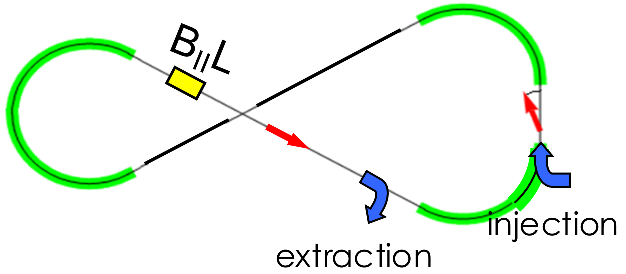
- **Figure-8 ring** or **racetrack with two identical Siberian snakes** separated by 180° bends makes energy-independent $\boldsymbol{\nu_{sp} = 0}$ (one arc cancels the spin rotation of the other)
- The spin is **highly sensitive to small perturbing fields** due to closed orbit excursion and beam emittances
- The high sensitivity makes it easy to control the spin **by low magnetic field integrals** as long as induced spin rotation >> spin rotation due to field errors
- A 3D spin rotator (navigator) conducting small rotations about different axes provides any polarization orientation at any point in the collider ring, has no effect on the orbit and can perform frequence adiabatic spin flips



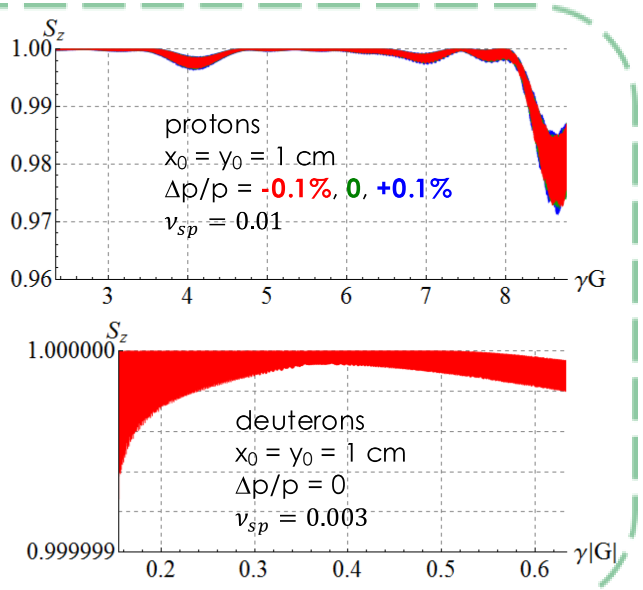


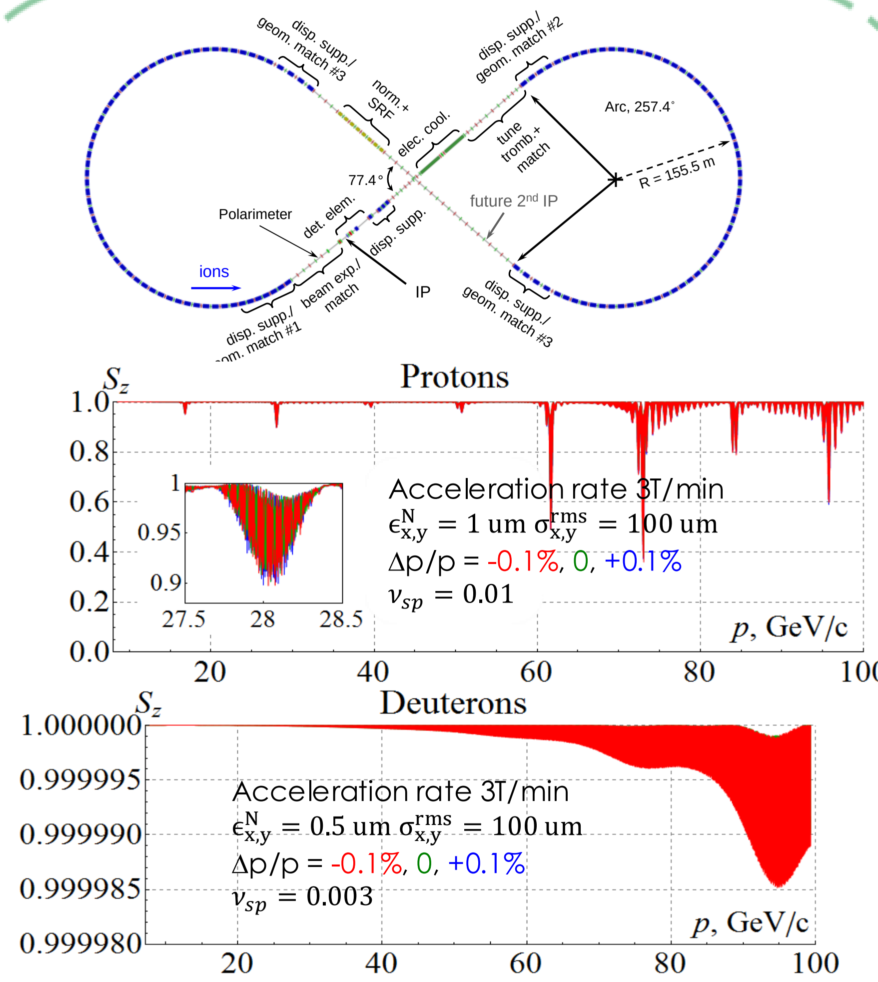
3D spin navigator: *Yu.N. Filatov et al., PRAB, 24, 061001 (2021)*

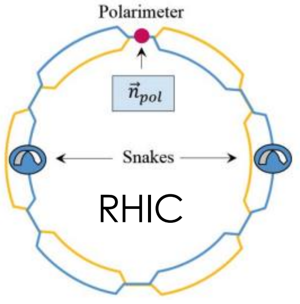
Simulation and Test of Spin Transparency



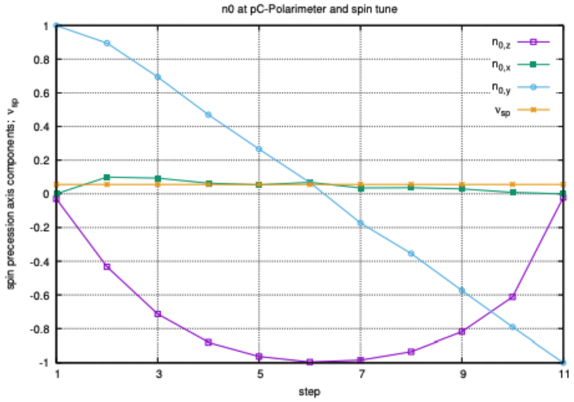
Polarization is stabilized by a single weak solenoid at 8 GeV/c with **0.6 T·m** field vs. fields in conventional accelerators **~30 Tm** for protons and **~100 Tm** for deuterons







Step	Polarization	Angle (°)	Polarization ratio
initial	46.6±1.7	2.0±3.2	
mid 1	37.9±3.2	-22.0±6.2	0.81±0.07
mid 2	40.3±3.8	-67.6 ±3.8	1.06±0.13
final	39.6±2.3	-94.0±2.3	0.98±0.11

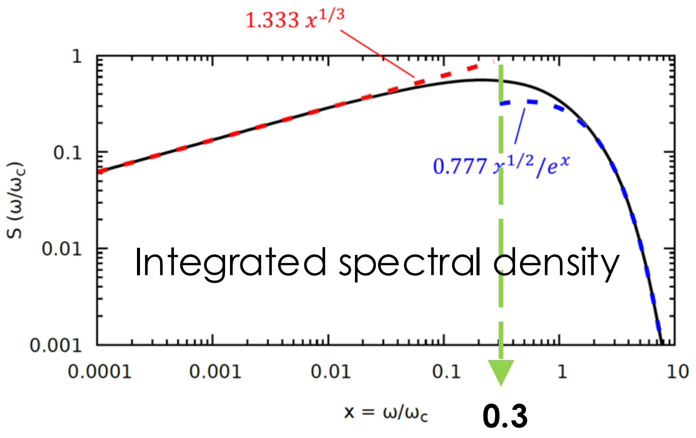


Observed some depolarization

H. Huang: *IPAC'23 TUOGB3*

Electron Radiative Polarization – Sokolov-Ternov Effect

- Synchrotron radiation in dipoles: majority of photon emission is not associated with a spin flip: $\frac{w_{\uparrow\downarrow}+w_{\downarrow\uparrow}}{w_{\uparrow\uparrow}+w_{\downarrow\downarrow}} \sim \xi^2 \sim 10^{-12}$. However, a small amount of radiation cause spin flip with slightly different spin flip rates: $\frac{w_{\uparrow\downarrow}-w_{\downarrow\uparrow}}{w_{\uparrow\uparrow}+w_{\downarrow\downarrow}} = \frac{8}{5\sqrt{3}} \approx 0.924$



- **Sokolov-Ternov** (ST) effect in a uniform magnetic field: self-polarization with polarization antiparallel to the guiding dipole field.

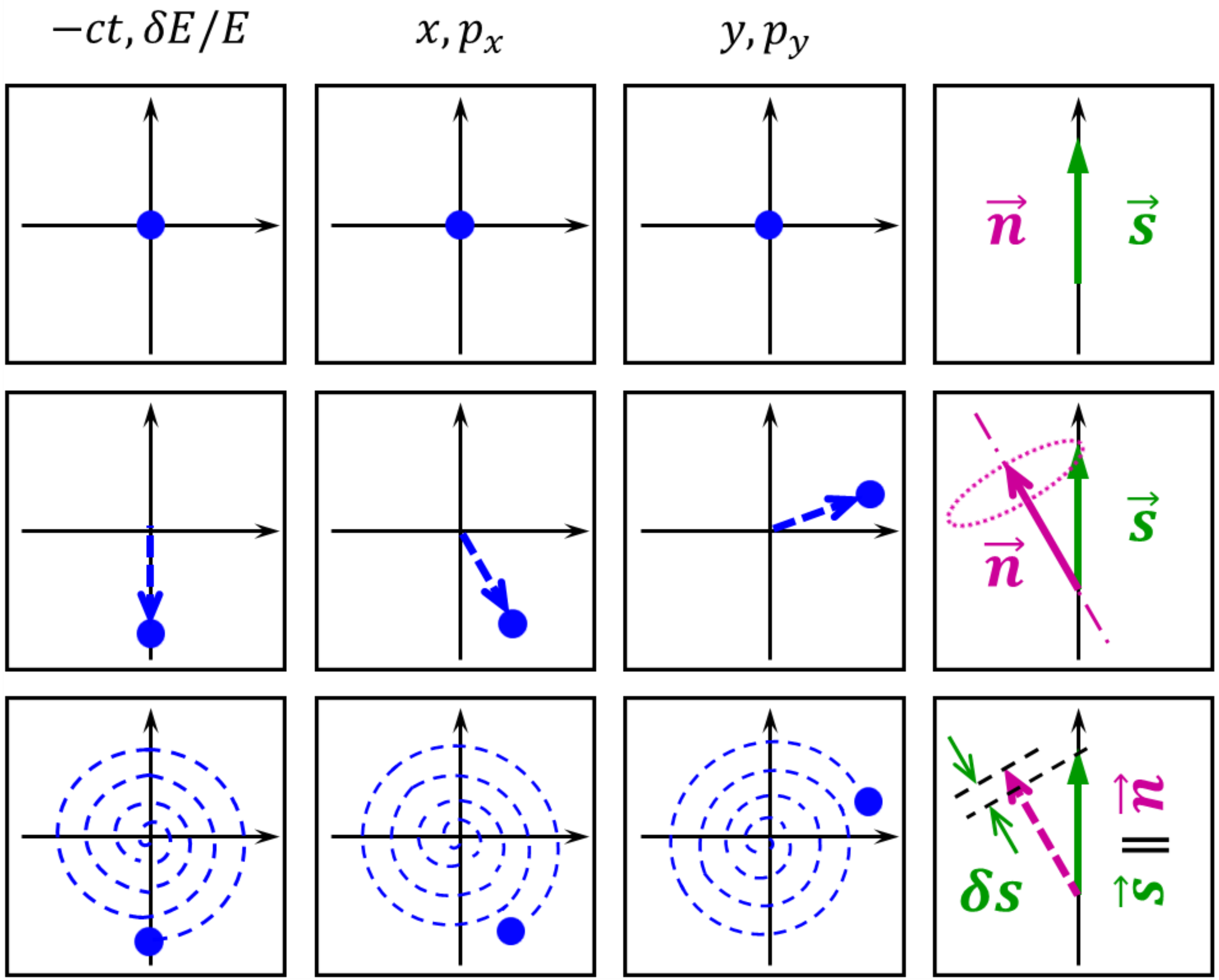
  - $P_{st} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = \frac{8}{5\sqrt{3}} = \mathbf{92.4}\%$ with a build-up rate of $\tau_{st}^{-1} = \frac{5\sqrt{3}}{8}\frac{\hbar r_e \gamma^5}{m_e|\rho|^3}$

- **Baier-Katkov-Strakhovenko** (BKS) in piecewise homogeneous fields:

  - $P_{bks} = \frac{8}{5\sqrt{3}} \frac{\oint ds \frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds \frac{1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2}{|\rho(s)|^3}}$ with a built-up rate: $\tau_{st}^{-1} = \frac{5\sqrt{3}}{8}\frac{\hbar r_e \gamma^5}{m_e C}\oint ds \frac{1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2}{|\rho(s)|^3}$

- in the EIC ESR, $P_{bks} = 82.8\%$, $\tau_{bks} = 36$ minutes at 18 GeV

Electron Radiative Polarization – Spin Diffusion



Derbenev-Kondratenko formula considers both self-polarization and spin diffusion.

$$P_{dk} = -\frac{8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho(s)|^3} \hat{b} \cdot \left(\hat{n} - \frac{\partial \hat{n}}{\partial \delta} \right) \right\rangle_s}{\oint ds \left\langle \frac{1}{|\rho(s)|^3} \left(1 - \frac{2}{9} (\hat{n} \cdot \hat{s})^2 + \frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2 \right) \right\rangle_s}$$

$$\tau_{dk}^{-1} = \frac{5\sqrt{3}}{8} \frac{\hbar r_e \gamma^5}{m_e C} \oint ds \left\langle \frac{1 - \frac{2}{9} (\hat{n} \cdot \hat{s})^2 + \frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2}{|\rho(s)|^3} \right\rangle_s$$

spin-orbit coupling function: $\left(\frac{\partial \hat{n}}{\partial \delta} \right)^2$

Electron Polarization – Equation and Calculation

- Thomas-BMT Equation: $\frac{d\vec{s}}{dt} = -\frac{e}{\gamma m}\left[(1+G\gamma)\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel} + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right] \times \vec{S}$

$$\Rightarrow \frac{d\vec{s}}{dt} = \vec{\Omega}(x, p_x, y, p_y z, \delta, s) \times \vec{S} = \Omega(x, p_x, y, p_y z, \delta)\hat{n}(s) \times \vec{S}$$

$\hat{n}_0(s)$: stable spin direction for particles on the closed orbit (periodic solution of Thomas-BMT equation)

$\hat{n}(s)$: invariant spin field (ISF) or stable spin direction, when particles return to the same phase space.

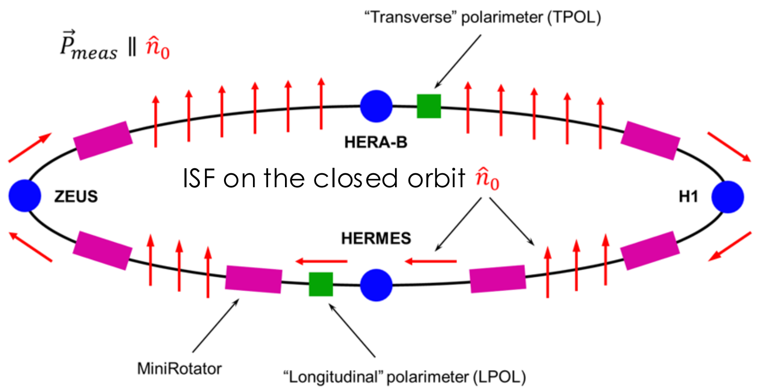
- SLIM Algorithm (by Chao and Barber) calculates $(\frac{\partial\hat{n}}{\partial\delta})^2$ using orbit and spin eigenvectors, in a linear approximation of orbit and spin motions $\hat{n}(\vec{u};s) = \hat{n}_0(s) + \alpha(\vec{u};s)\hat{m}(s) + \beta(\vec{u};s)\hat{l}(s)$:

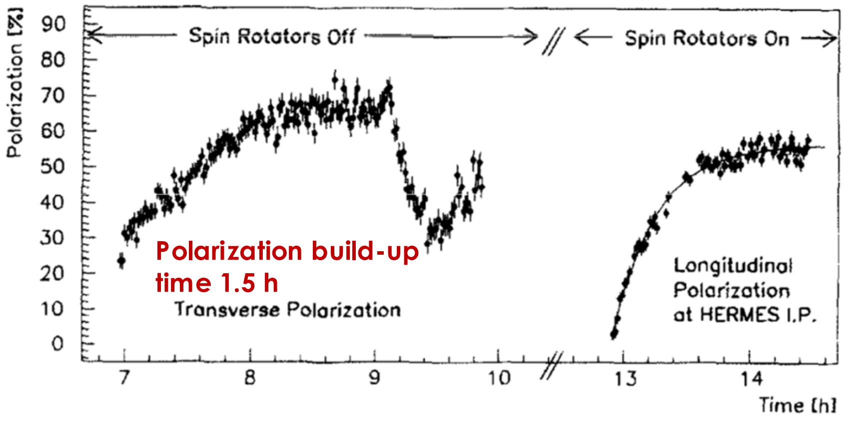
$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ \sigma \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_1) = \begin{pmatrix} M_{6\times6} & 0_{6\times2} \\ G_{2\times6} & D_{2\times2} \end{pmatrix}(s_1, s_0) \begin{pmatrix} x \\ x' \\ y \\ y' \\ \sigma \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_0) \Rightarrow (\frac{\partial\hat{n}}{\partial\delta})^2 = 4\sum_{\mu=1}^{2}(\mathrm{Im}\sum_{k=I,II,III} v_{k5}^{\ *} w_{k\mu})^2$$

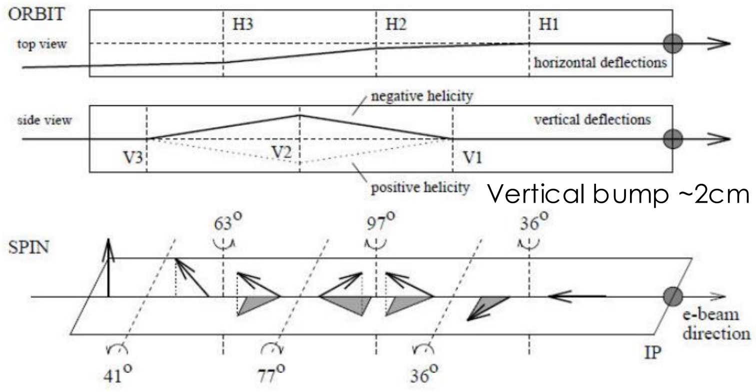
- Spin matching by **minimizing** $(\frac{\partial\hat{n}}{\partial\delta})^2$ to make the spin transparent to the orbital motion can be applied to extend the equilibrium polarization and depolarization time.

Polarized Electrons: HERA @ DESY

- HERA (1992-2007)
  - 820-920 GeV protons
  - 27.5 GeV electrons/positrons







HERA Mini-Rotator: Buon + Steffen

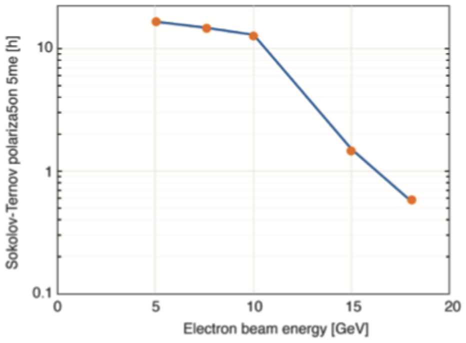
56 m ("short") → no quads.

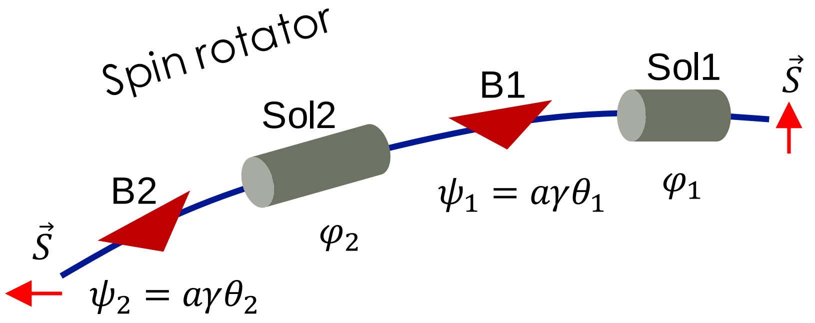
27 – 39 GeV, both helicities at the IP, variable geometry

D.P. Barber

Polarized Electrons: EIC @ BNL

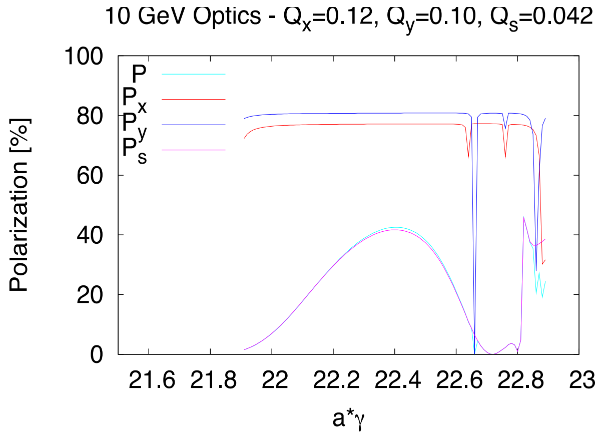
- Unique Electron Polarization Design in the EIC ESR:
  - Electrons: 5-18 GeV => ST time varies
  - Electron bunches with two opposite polarization are injected into and stored in the ESR
  - Polarization is vertical in arcs to avoid spin diffusion and longitudinal at IP for experiments
  - Interleaved solenoid and dipole fields spin rotators rotate spin between vertical and longitudinal directions
  - Spin matching is implemented to preserve high equilibrium(asymptotic) polarization and extend the depolarization (polarization relaxation) time
  - Regular replacement of electron bunches down to a few minutes at 18 GeV is needed to obtain a high average polarization

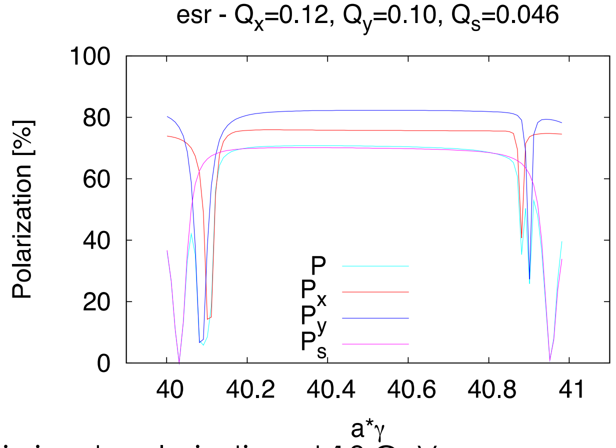




$$\tan\varphi_1 = \pm\frac{\cos\psi_2}{\sqrt{-\cos(\psi_1+\psi_2)\cos(\psi_1-\psi_2)}}$$

$$\cos\varphi_2 = \cot\psi_1\,\cot\psi_2$$





- Spin matching is performed to minimize depolarization at 18 GeV area
- Longitudinal spin matching can not be done perfectly at 10 GeV. However, the depolarization at 10 GeV area is ~16 times slower. Thus, averaged polarization >70% can still be achieved under the imperfect spin matching

Summary

- Collision of polarized beams offer a unique probe to study and understand the fundamental structure of matter and original mass.
- Decades of continuous effort all around the world makes highly polarized beams generated, accelerated, maintained, manipulated and measured in an accelerator.
- What is not covered in this talk
  - Spin rotators
  - Spin flipping
  - Spin matching
  - Machine and beam effects on the polarization
  - High order depolarizing resonances
  - Spin tracking
  - And more

Literature / Acknowledgement

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Thank you for your attention !