Two-Photon Exchange for Semi-inclusive Meson Production

Positron Working Group Workshop

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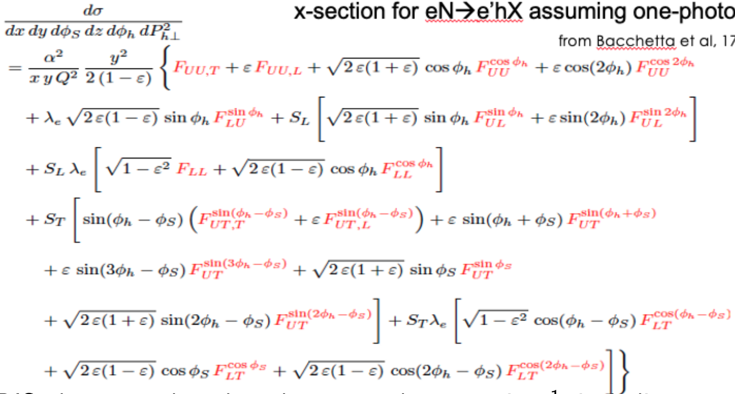
- Motivation & Introduction
- Background
- Assumptions & Calculation
- Results
- Conclusion

Semi-Inclusive production of hadrons in DIS



x-section for eN→e'hX assuming one-photon exchange

from Bacchetta et al, 1703.10157

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}\\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.\\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]\\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]\\
&\quad + S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right.\\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}\\
&\qquad \left.+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right.\\
&\qquad \left.\left.+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\right\}
\end{aligned}
$$

SIDIS phenomenology based on several assumptions[1], including:

- One-photon exchange dominates;
- Transverse photon cross section dominates, and F_{UU}^{L} can be ignored.

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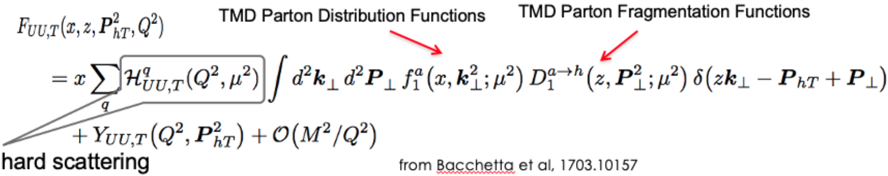
[1] Bacchetta et al. 1703.10157 [1]

TMDs in SIDIS

Assuming the one-photon exchange and dominance of the transverse photon. SIDIS phenomenology for last decades was extracting the underlying transverse momentum dependent (TMD) distribution and fragmentation functions from multiplicities and single spin asymmetries in SIDIS.

Analysis of multiplicities was done based on factorization of the x-section from transverse part.



TMD Parton Distribution Functions

TMD Parton Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = x\sum_q \boxed{\mathcal{H}^q_{UU,T}(Q^2,\mu^2)} \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

hard scattering

from Bacchetta et al, 1703.10157

Several JLab proposals focused on extraction of the longitudinal photon contributions.

No measurement so far available for evaluation of systematics from two-photon exchange.

SIDIS cross section: separating $F_{UU,L}$

Semi-Inclusive:

from Avakian et al., Eur. Phys. J. A (2016) **52**: 150

ratio of longitudinal and transverse photon flux

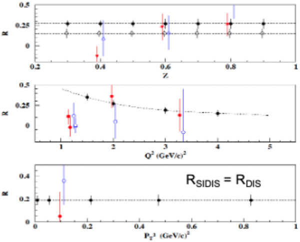
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ F_{UU,T} + \varepsilon F_{UU,L} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+ S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] + S_{\parallel}\lambda_e\sqrt{1-\varepsilon^2}\,F_{LL}\Bigg\}$$

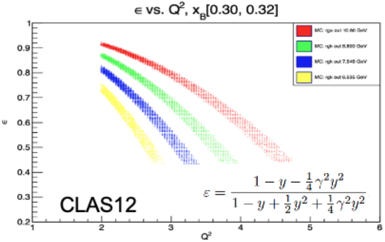
Hall-C E12-06-104
E12-23-014
Hall-B E12-16-010C

Separation of contributions from longitudinal and transverse photons critical for interpretation

Expected E12-06-104 assume $R=F_{UU,L}/F_{UU,T}$



Wide ε-coverage needed!!!

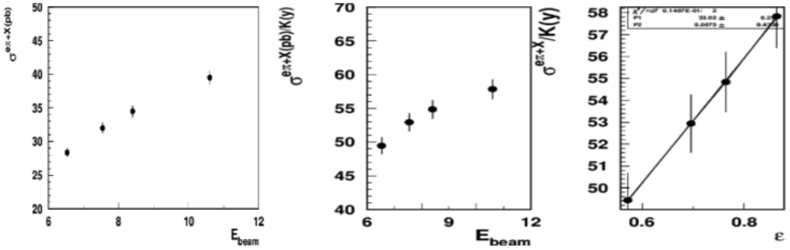


So far the P_T-dependence is neglected (DIS=SIDIS)!!

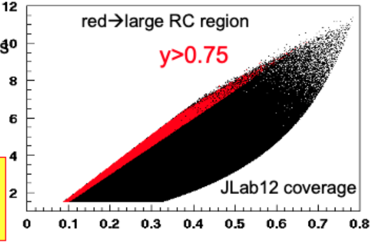
$F_{UU,L}$ studies at Hall-B

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_T} = GK(y)\left(F_{UU,T} + \epsilon F_{UU,L}\right)$$

Measurements in a given small bin in x(0.3-0.32) and Q^2(2.4-2.6 GeV^2)



For evaluation of the longitudinal part, measurements at different beam energies are needed to fit the x-sections as a function of epsilon and extract the $F_{UU,L}$(or the ratio R)



Understanding the systematics from two-photon exchange, in particular at large P_T is critical for evaluation of systematics of TMD extractions

Considering the correction δ^{TPE},

$$\frac{d\sigma_{tot}}{dxdzdQ^2d^2P_T} \equiv d\sigma_{tot} = d\sigma_{exp}/(1+\delta^{TPE})$$
$$\sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}}) + \sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$
$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\} \tag{1}$$

with x is Bjorken-x, transverse momentum of the detected meson P_T, Q^2 relates to the momentum transfer of the virtual photon.

Background

$$K(y) = 1 - y + y^2/2 + \gamma^2 y^2/4 \tag{2}$$

$$\epsilon = \frac{1 - y - \gamma^2 y^2}{K(y)} \tag{3}$$

$$\gamma = 2Mx/Q \tag{4}$$

$$\nu = E_{lab} - E' \tag{5}$$

$$x = \frac{Q^2}{2M\nu} \tag{6}$$

$$y = \nu/E \tag{7}$$

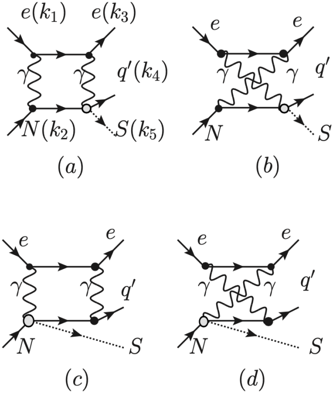
$$z = E_h/\nu \tag{8}$$

$$P_T = P_h \sin(\theta_{h,\gamma}) \tag{9}$$

where E_{lab} and E' are the energies of the incoming electron beam and the scattered electron, respectively.

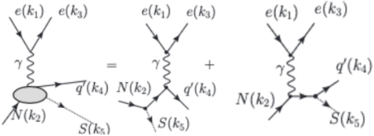
Assumptions & Calculations

$$e(k_1) + N(k_2) \to e(k_3) + q'(k_4) + S(k_5),$$



For quark-diquark model, q' represents quark and S represents diquark.

Assumptions & Calculations



Born-level one photon models, which equals to the sum of the "quark graph" and the "proton pole graph". q' and S stand for quark and diquark.[2]

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[2] Afanasev and Carlson Phys. Rev. D 74.114027[2].

Assumptions & Calculations

Using soft-photon approximation (SPT [3]) by neglecting the momentum for one of the photon while calculating the amplitude, such that

$$\begin{aligned} M^{2\gamma} =& M^{1\gamma} \cdot \sum_{l} [\frac{-e^2}{2\pi} \cdot \sum_{i,j} (2k_i \cdot k_j) \\ & \cdot C_0(\{k_i, m_i\}, \{\mp k_j, m_j\}) \qquad (10) \\ =& \sum_{l=N,q',s} \sum_{i=a,b,c} M^{1\gamma} M_{l,i,box}, \qquad (11) \end{aligned}$$

where the Passarino-Veltman three-point scalar integral

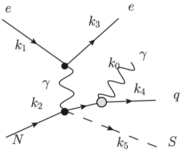
$$\begin{aligned} & C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \\ & \frac{1}{i\pi^2} \int d^4q \frac{1}{q^4} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}. \qquad (12) \end{aligned}$$

The correction

$$\delta_{box} = \frac{2Re[M^{2\gamma} M^{1\gamma\dagger}]}{|M^{1\gamma}|^2} = 2Re[\sum_{l;i} M_{l,i,box}]. \qquad (13)$$

Assumptions & Calculations

Regularization of the infrared divergent integrals.



One of the possibilities for the Bremsstrahlung process [4].

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[4] Afanasev et al. Phys. Rev. D 88, 053008 [3]

Assumptions & Calculations

The IR-divergence cancellation approximation of the photon in the numerator, the correction is

$$\delta_\gamma \sim \sum \frac{\alpha}{(2\pi)^2}(k_i \cdot k_j) I(k_i, k_j), \tag{14}$$

where $i, j = 1, 2, 3, 4$ correspond to the momenta from the Feynman diagram, and k_0 is the momentum of the virtual photon in Bremsstrahlung process.

$$I(k_i, k_j) = \int \frac{d^3 k_0}{\sqrt{\mathbf{k_0}^2 + \lambda^2}} \frac{1}{(k_i \cdot k_0)(k_j \cdot k_0)}. \tag{15}$$

Therefore,

$$\boxed{\delta^{TPE} = \delta^{TPE}_{box} + \delta_\gamma} \tag{16}$$

Assumptions & Calculations

$$d\sigma_{tot} \sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}})+\sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$

$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\}$$
(17)

The moments of $\cos(n\phi)$,

$$\langle\cos(n\phi)\rangle \sim \int d\sigma_{exp}d\phi(1-\delta^{TPE})\cos(n\phi) \qquad (18)$$

The $\cos(n\phi)$ moments with the corrected terms only,

$$\langle\delta\cos(n\phi)\rangle \sim \int d\sigma_{exp}d\phi\,\delta^{TPE}\cos(n\phi) \qquad (19)$$

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Results

- $E_{lab} = 10.6$ GeV;
- $Q^2 \approx 2.5$ GeV^2;
- $y < 0.75$ to avoid the region most susceptible to radiative effects and lepton-pair symmetric background;
- $x = 0.31$ (the invariant mass $W \approx 2.7$ GeV);
- $z = 0.5$;
- The polar angle of the detected meson is $\cos\theta = 0.8$ ($P_T \approx 0.35$) for P_T independent figures;
- The azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent;
- $F_{UU,L}/F_{UU,T} \approx 0.2$;
- $F_{UU}^{\cos\phi}/F_{UU,T} \approx -0.05$;
- $F_{UU}^{\cos(2\phi)}/F_{UU,T} \approx 0.1$.

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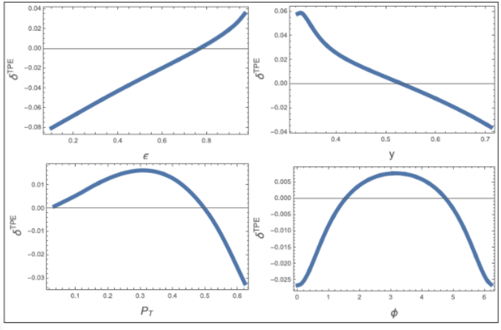
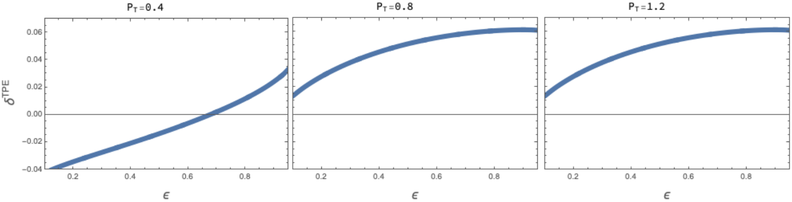
Results



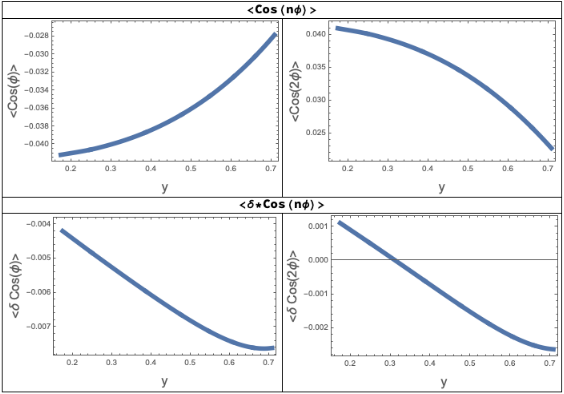
$Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$ for P_T independent figures, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of speculator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

Results



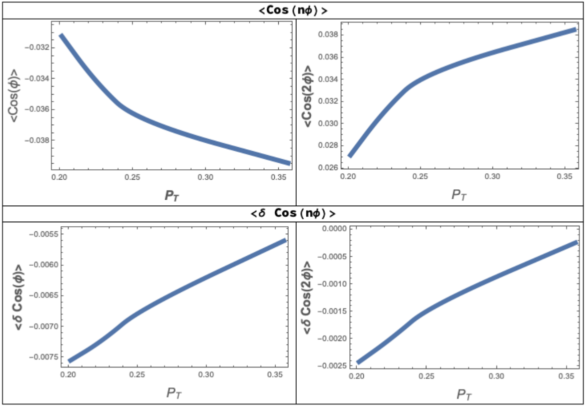
$Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of speculator $m_x = 2$ GeV.

Results



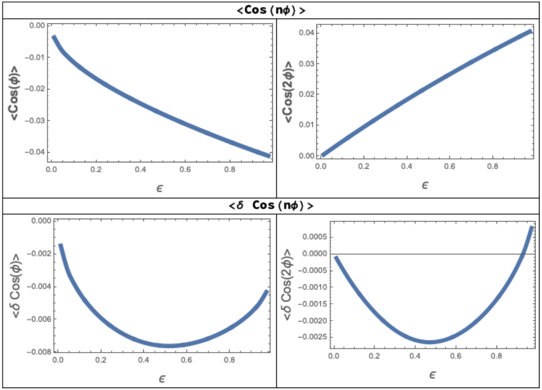
The cosine moments in terms of y. $Q^2 \approx 2.5\ \text{GeV}^2$, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark $(m_{quark} = 0.14\ \text{GeV})$, and the mass of speculator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

Results



The cosine moments in terms of transverse momentum of the detected meson. $Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of speculator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

Results



The cosine moments in terms of polarization factor ϵ. $Q^2 \approx 2.5$ GeV^2, $x = 0.31$, $z = 0.5$, $P_T \approx 0.35$, and the azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent. The masses for the incoming particles (m_e, M) are the mass of electron and neutron, respectively. The mass of quark ($m_{quark} = 0.14$ GeV), and the mass of speculator $m_x = 2$ GeV. Using data from JLab E12-06-104 [5].

Conclusion

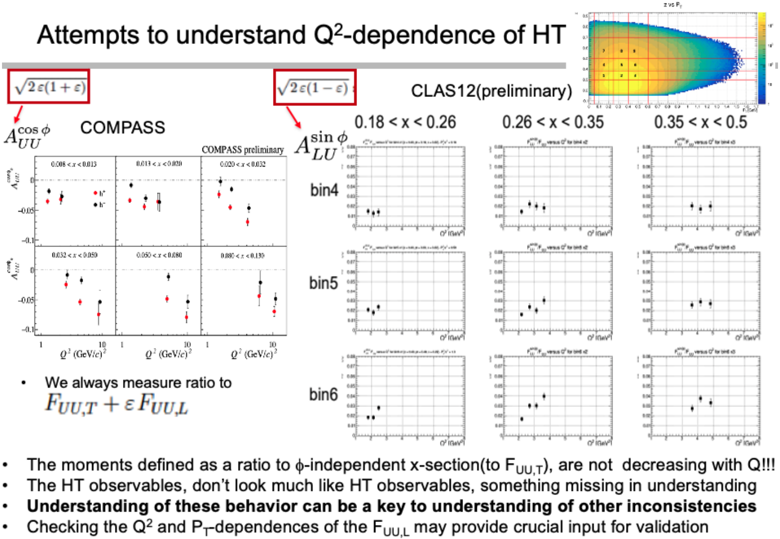
- Measurements of two-photon effects in SIDIS are important for validation of the phenomenology to extract 3D PDFs and FFs;
- The corrections of the two-photon exchange are in the interval of $-4(-8) \sim 5\%$ for y, ϵ & P_T dependents;
- The corrections can affect the moments of $\cos(\phi)$ by nearly $0.5 \sim 1\%$ and 0.3% for $\langle \cos(2\phi) \rangle$;
- The experiments of multiparticle final-state observables in a multidimensional space in x, Q^2, z, P_T with the electron beam energies of $6.5, 7.5, 8.5, 10.5$ GeV have been measured at JLab;
- A proposal is being prepared to measure two-photon effects with positron beam by using the CLAS12 detector.

The importance of calculating the cosine moments, $\langle \cos(n\phi) \rangle$:

- It is crucial for probing the transverse momentum distribution of partons;
- Constraining quark and gluon polarization;
- Testing quantum chromodynamics (QCD) factorization

Supporting Slides

Supporting

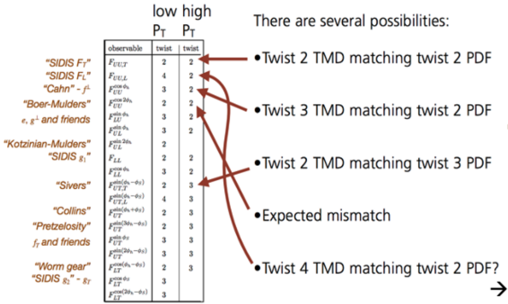


Attempts to understand Q^2-dependence of HT

- We always measure ratio to $F_{UU,T} + \varepsilon F_{UU,L}$

- The moments defined as a ratio to ϕ-independent x-section(to $F_{UU,T}$), are not decreasing with Q!!!
- The HT observables, don't look much like HT observables, something missing in understanding
- **Understanding of these behavior can be a key to understanding of other inconsistencies**
- Checking the Q^2 and P_T-dependences of the $F_{UU,L}$ may provide crucial input for validation

Longitudinal photon contributions in SIDIS

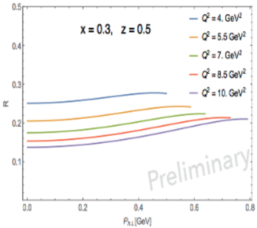


		low P_T	high P_T
	observable	twist	twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF
- Expected mismatch
- Twist 4 TMD matching twist 2 PDF?

A. Bacchetta



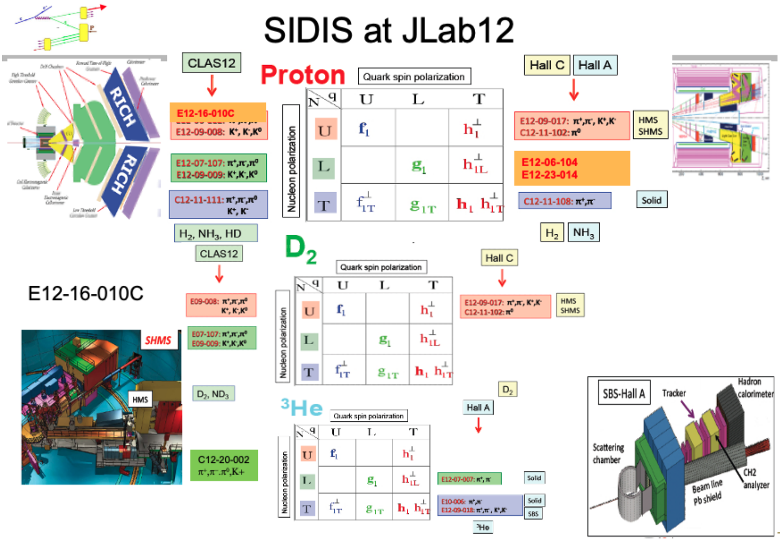
→ Contributes everywhere,
→ we know nothing!!!

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h\perp}^2} = \frac{2\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{2\pi F_{UU,T}(x,z,P_{h\perp}^2,Q^2) + \varepsilon\,2\pi F_{UU,L}(x,z,P_{h\perp}^2,Q^2)\right\}$$

$$\frac{d\sigma}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T}(x,z,Q^2) + \varepsilon\,F_{UU,L}(x,z,Q^2)\right\}$$

$$R = \frac{F_{UU,L}}{F_{UU,T}}$$

Supporting



SIDIS at JLab12

Proton

CLAS12

E12-16-010C
E12-09-008: K⁺, K⁻,K⁰

E12-07-107: π⁺,π⁻,π⁰
E12-09-009: K⁺,K⁻,K⁰

C12-11-111: π⁺,π⁻,π⁰ K⁺, K⁻

H_2, NH_3, HD

Quark spin polarization

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Nucleon polarization

Hall C | Hall A

E12-09-017: π⁺,π⁻, K⁺,K⁻
C12-11-102: π⁰

HMS
SHMS

E12-06-104
E12-23-014

C12-11-108: π⁺,π⁻

Solid

H_2 | NH_3

D_2

CLAS12

E09-008: π⁺,π⁻,π⁰ K⁺, K⁻,K⁰

E07-107: π⁺,π⁻,π⁰
E09-009: K⁺,K⁻,K⁰

D_2, ND_3

Quark spin polarization

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Nucleon polarization

Hall C

E12-09-017: π⁺,π⁻, K⁺,K⁻
C12-11-102: π⁰

HMS
SHMS

D_2

E12-16-010C

SHMS

HMS

C12-20-002
π⁺,π⁻,π⁰,K+

^{3}He

Quark spin polarization

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Nucleon polarization

Hall A

E12-07-007: π⁺, π⁻

Solid

E10-006: π⁺,π⁻
E12-09-018: π⁺,π⁻ , K⁺,K⁻

Solid
SBS

^{3}He

TMDs in Semi-Inclusive DIS

TMD Parton Distribution Functions

TMD Parton Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)$$

$$= x\sum_q \boxed{\mathcal{H}^q_{UU,T}(Q^2,\mu^2)} \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$+ Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Major advance in theory in last years

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} f_1^q(x,k_\perp^2;\mu_f,\zeta_f)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x,b_T^2;\mu_f,\zeta_f) = [C \otimes f_1](x,\mu_{b_*})\, e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu}\left(\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}}+g_K} f_{1\,NP}(x,b_T^2;\zeta_f,Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

$$g_K(b_T^2) = -g_2^2\frac{b_T^2}{4}$$

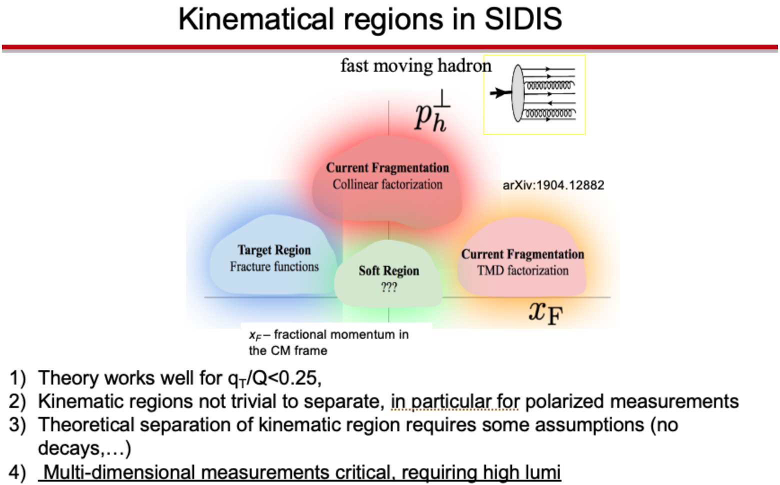
CS kernel discribes the interaction of out-going parton with the confining potential
Provides nonperturbative part of evolution for TMDs

CS-kernel→independent on any other variables

Possible sources of large P_T behavior

1. Perturbative contributions and P_T-dependence of unpolarized FFs (so far unlikely...);
2. Significantly wider in k_T distributions of u-quarks with spin opposite to proton spin (possible sign flips in asymmetries related to polarization of partons);
3. Significantly wider in k_T distributions of d-quarks (possible sign flips in asymmetries related to polarization of partons);
4. Significantly wider in k_T sea quark distributions (study contributions dominated by sea, K^-,..);
5. Increasing fraction of hadrons due to F_{UU}^L (needed for proper interpretation→separation of F_{UU}^L from total);
6. Significant contributions from VMs to low P_T pion multiplicities, with direct pions showing up at large P_T (needed for proper interpretation→much wider in k_T original parton distributions);
7. Radiative corrections (need the full x-section, typically applied to pions, while may be needed for underlying VMs,...).

Supporting

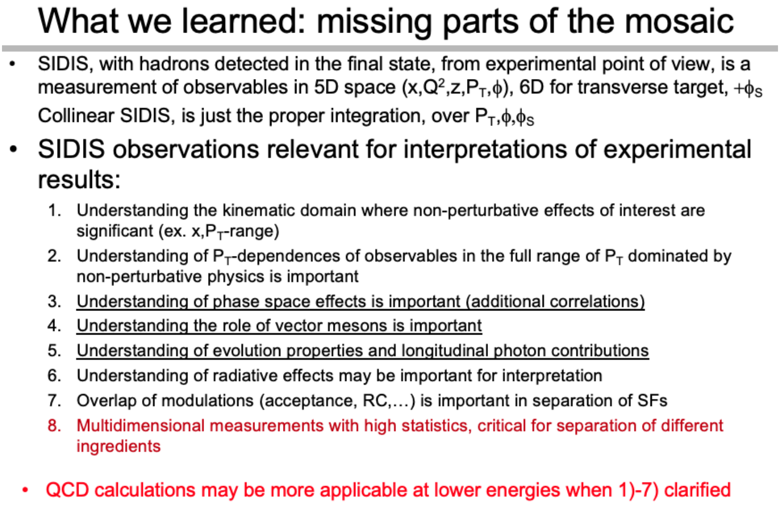


Kinematical regions in SIDIS

x_F – fractional momentum in the CM frame

1) Theory works well for $q_T/Q<0.25$,
2) Kinematic regions not trivial to separate, in particular for polarized measurements
3) Theoretical separation of kinematic region requires some assumptions (no decays,…)
4) Multi-dimensional measurements critical, requiring high lumi

Supporting



What we learned: missing parts of the mosaic

- SIDIS, with hadrons detected in the final state, from experimental point of view, is a measurement of observables in 5D space (x,Q^2,z,P_T,ϕ), 6D for transverse target, $+\phi_S$
  Collinear SIDIS, is just the proper integration, over P_T,ϕ,ϕ_S
- SIDIS observations relevant for interpretations of experimental results:
  1. Understanding the kinematic domain where non-perturbative effects of interest are significant (ex. x,P_T-range)
  2. Understanding of P_T-dependences of observables in the full range of P_T dominated by non-perturbative physics is important
  3. Understanding of phase space effects is important (additional correlations)
  4. Understanding the role of vector mesons is important
  5. Understanding of evolution properties and longitudinal photon contributions
  6. Understanding of radiative effects may be important for interpretation
  7. Overlap of modulations (acceptance, RC,…) is important in separation of SFs
  8. Multidimensional measurements with high statistics, critical for separation of different ingredients

- QCD calculations may be more applicable at lower energies when 1)-7) clarified

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The End