Impact of a positron beam at JLab on the extraction of GPDs

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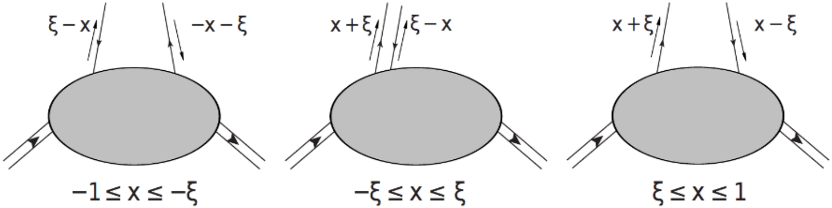
Outline

1. The deconvolution problem: example with gravitational form factors
2. Impact of a positron beam

Spin-1/2 hadron, unpolarized quark GPDs H^q and E^q in the lightcone gauge **[Müller et al, 1994]**, **[Radyushkin, 1996]**, **[Ji, 1997]**

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| p_1 \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$
$$= \frac{1}{2P^+}\left(H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}u(p_1)\right) \tag{1}$$

$$p_2 - p_1 = \Delta,\ t = \Delta^2,\ P = \frac{1}{2}(p_1+p_2),\ \xi = -\frac{\Delta^+}{2P^+}\,. \tag{2}$$



- More functions than in the inference of PDFs from DIS, with more kinematic dependences, non-analyticities, and a more difficult experimental access.
- When $x \gg \xi$, negligible asymmetry between incoming $(x - \xi)$ and outgoing $(x + \xi)$ parton longitudinal momentum fraction $\rightarrow$ **smooth limit of GPDs**

$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \text{ for } x \gg \xi \,. \tag{3}$$

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \tag{4}$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**

If one collects mostly information about $x \sim \xi$, important modelling issue in the ξ dependence to reach $\xi = 0$.

Gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997]

Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s\rangle = \bar{u}(p', s')\Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t) + B_a(t)\right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t)\Bigg\} u(p, s) \quad (5)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \quad (6)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \quad (7)$$

Factorization of DVCS [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \frac{H^a(x, \xi, t, \mu^2)}{|x|^{p_a}} \tag{8}$$

At leading-order in α_s, defining the singlet GPD $H^{(+)q} = H^q(x) - H^q(-x)$

$$\mathrm{Im}\,\mathcal{H}(\xi, t, Q^2) \propto \sum_q e_q^2 H^{(+)q}(\xi, \xi, t, Q^2) \tag{9}$$

$$\mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) \propto \sum_q e_q^2 \int_0^1 \mathrm{d}x \left[\frac{1}{\xi - x} - \frac{1}{\xi + x} \right] H^{(+)q}(x, \xi, t, Q^2) \tag{10}$$

Factorization-scale dependence of GPDs [Müller et al, 1994]

$$\frac{1}{|x|^{p_a}} \frac{\partial}{\partial \log(\mu^2)} H^a(x, \xi, t, \mu^2) = \sum_b \int_{-1}^{1} \frac{\mathrm{d}y}{\xi} K^{ab} \left(\frac{y}{\xi}, \frac{\xi}{x}, \alpha_s(\mu^2) \right) \frac{H^b(y, \xi, t, \mu^2)}{|y|^{p_b}} \tag{11}$$

where $p_q = 0$, $p_g = 1$

DVCS dispersion relation [Anikin, Teryaev, 2007], [Diehl, Ivanov, 2007]

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \tag{12}$$

$$\overset{LO}{=} 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z, t, Q^2)}{1 - z} \tag{13}$$

$$\textbf{Desired GFF: } GFF(t, \mu^2) = \int_{-1}^{1} \mathrm{d}z\, z D^q(z, t, \mu^2) \tag{14}$$

Only model-independent strategy to directly hop from one to the other using the LO scale dependence of the D-term (ERBL equation). How effective is evolution to constrain it?

Shadow distributions

Find a distribution with reasonable shape such that it gives no experimental contribution at one scale, and check how big its contribution becomes as you move from the initial scale → measures worst case uncertainty propagation from experiment to fit

Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2)\, C_n^{3/2}(z) \tag{15}$$

Then

GFF C_a extraction

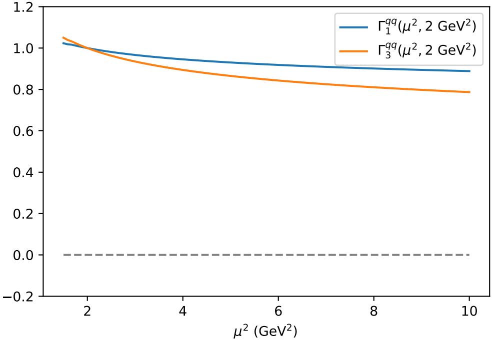
$$\int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z\, z D^q(z,t,\mu^2) = \frac{4}{5}\, d_1(t,\mu^2) \tag{16}$$

- There is a shadow D-term for

$$d_1(\mu_0^2) = -d_3(\mu_0^2)\,! \tag{17}$$

[HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, 2021]: allowing two free parameters d_1 and d_3 results in an **inflation of uncertainty by a factor 20 with full correlation between fitted parameters** compared to just d_1 over a range of $Q^2 \in [1.5, 4]$ GeV^2

in preparation



Simplified evolution in the qq sector

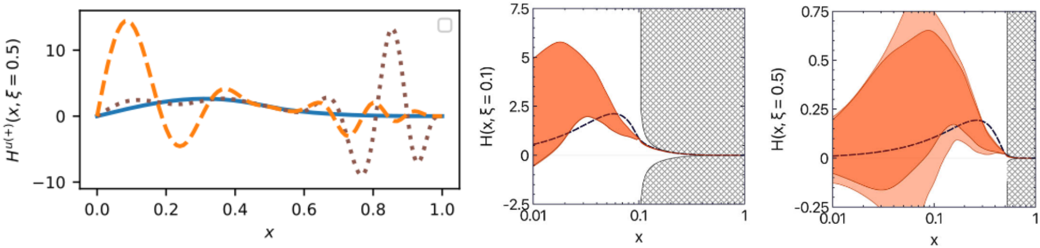
$$d_n^q(\mu^2) = \Gamma_n^{qq}(\mu^2, 2\ \mathrm{GeV}^2) d_n^q(2\ \mathrm{GeV}^2) \tag{18}$$

- current range of most DVCS data : [1.5, 4] GeV^2
- Over this range, Γ_1^{qq} and Γ_3^{qq} are numerically very close $\rightarrow$ little actual leverage in evolution to separate the two
- Estimate of the inflation on uncertainty when fitting jointly d_1 and d_3 compared to the sole d_1 :

$$\propto \left(1 - \frac{\Gamma_3^{qq}(Q^2_{\mathrm{max}}, Q^2_{\mathrm{min}})}{\Gamma_1^{qq}(Q^2_{\mathrm{max}}, Q^2_{\mathrm{min}})}\right)^{-1} \tag{19}$$

- **An increase thanks to EIC from [1.5, 4] GeV^2 to [1.5, 50] GeV^2 could yield a decrease by 3 times of the uncertainty on (d_1, d_3) due to the sole effect of increase in Q^2 range, without taking account a better experimental precision.**

- **Moral of the story**: Pure DVCS extraction of GPDs is not possible without large model dependence (at least in the regime of moderate x_B probed by JLab, situation somewhat different at very small x_B [HD, Bertone, Winn, 2023]).
- Shadow distributions [Bertone, HD, Mezrag, Moutarde, Sznajder, 2021] introduced to deal with the more complicated general problem of extracting a full GPD from a Compton form factor at higher-order in perturbation theory, including **polynomiality** and **positivity** constraints [HD, Grocholski, Moutarde, Sznajder, 2021]:



See Pierre Chatagnon's talk for details on more elaborated analyses

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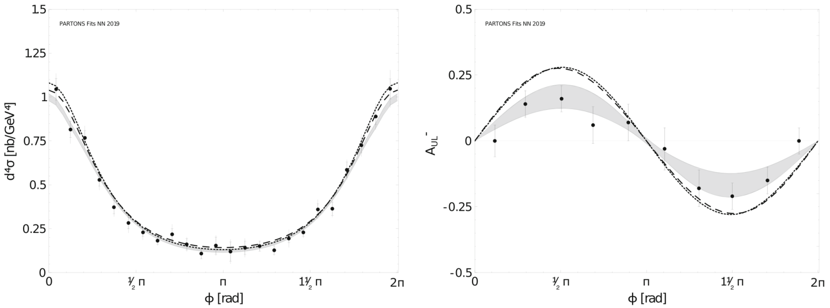
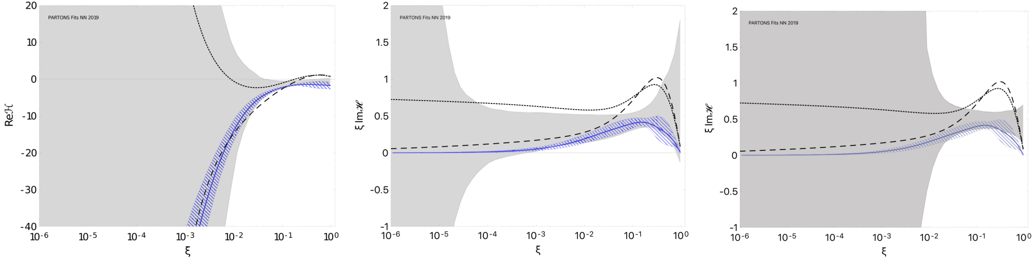
- GPDs from CFFs: very ill-defined, need either models, other experimental processes (DDVCS, meson pair production, ...) or lattice data.
- But CFFs from experimental data in principle can be data-driven: for instance **[Moutarde, Sznajder, Wagner, 2019]**, with 8 independent neural networks for the real and imaginary parts of the leading-twist CFFs



CLAS data (2015) **[Pisano et al, 2015], [Jo et al, 2015]**, $x_B \approx 0.25$, $t \approx -0.2$ GeV^2, $Q^2 \approx 2$ GeV^2, comparison with GK and VGG models

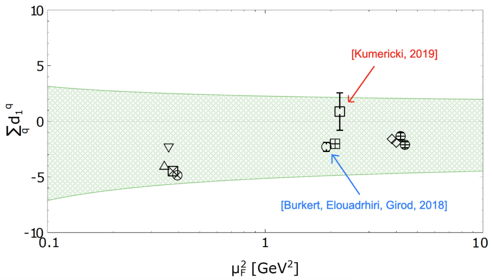
With such a flexible approach and using the world DVCS dataset in 2019, barely more than $\mathrm{Im}\,\mathcal{H}$ is significantly constrained



(left) $\mathrm{Re}\,\mathcal{H}(\xi, t = -0.3, Q^2 = 2\ \mathrm{GeV}^2)$ – (center) $\mathrm{Im}\,\mathcal{H}$ – (right) $\mathrm{Im}\,\mathcal{H}$ without the HERA and COMPASS data. (blue bands: parametric fit with physical parameters)

The uncertainty on $\mathrm{Re}\,\mathcal{H}$ drives the uncertainty on the subtraction constant, and therefore the ability to extract the D-term.

Assuming pure dominance of d_1 in the subtraction constant, a tripole t-dependence, enforcing LO evolution in Q^2,



But a positron beam is ideally designed to reduce the uncertainty on $\mathrm{Re}\,\mathcal{H}$!

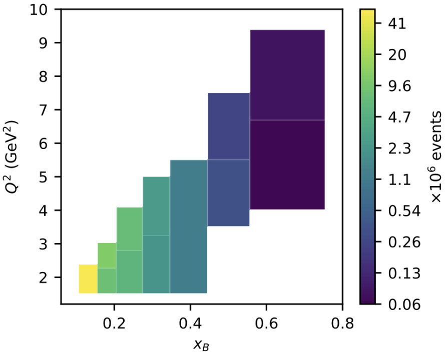
- Beam Charge Asymmetry: $A_C(x_B, t, Q^2) = (\mathrm{d}^4\sigma^+ - \mathrm{d}^4\sigma^-)/(\mathrm{d}^4\sigma^+ + \mathrm{d}^4\sigma^-)$

$$A_C^{\cos\phi} \propto \mathrm{Re}\left[F_1\mathcal{H} + \xi(F_1 + F_2)\widetilde{\mathcal{H}} - \frac{t}{4m^2}F_2\mathcal{E}\right] \qquad (20)$$

[Kroll, Moutarde, Sabatie, 2013] - leading order, leading twist

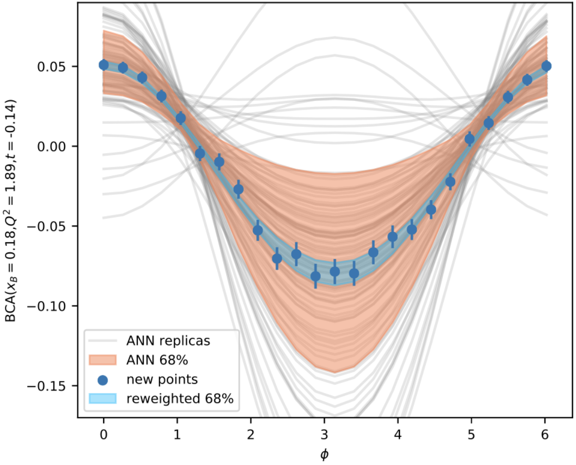
Settings of the impact study: **[HD, Bertone, Moutarde, Sznajder, 2021]**

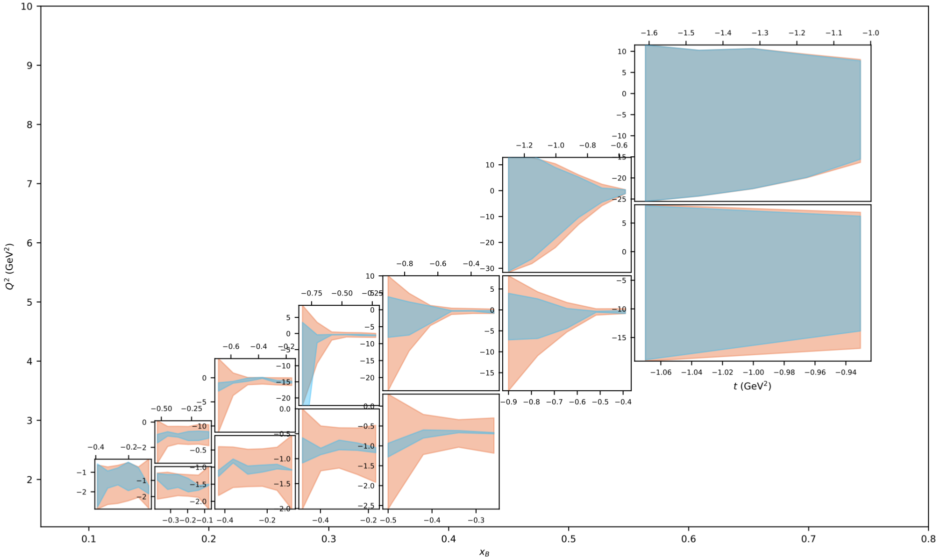
- CLAS12 10.6 GeV, cuts $Q^2 > 1.5$ GeV^2 and $-t < 0.2Q^2$
- 80 days of ($e^+ + e^-$ beam), luminosity 0.6×10^{35} cm^{-2}.s^{-1}, perfect acceptance and efficiency, 3% systematic uncertainty
- we use a parametric model of CFFs to estimate the BH/DVCS cross-section (the neural network is too unconstrained in some regions of the phase-space to provide a good prediction) $\rightarrow$ number of events per bin



Bayesian reweighting of the BCA in one bin (x_B, t, Q^2):

$$\omega_k = \frac{1}{Z}\chi_k^{n-1}\exp(-\chi_k^2/2) \tag{21}$$





Q^2 (GeV^2)
x_B
t (GeV^2)

Conclusion

- A positron beam is a great opportunity to constrain some poorly known CFFs
- Although this is not in itself enough to extract GPDs in a data-driven fashion, it is still the most prominent experimental sensitivity to GPDs as of now
- We will soon enter an era of calculations of GPDs on the lattice with realistic uncertainties and a domain in $x \approx 0.15 - 0.85$. Lattice picks directly the value of (ξ, t, μ^2) at which it operates by selecting the external momenta, so no "deconvolution" problem. Instead there an x-reconstruction issue, but arguably more manageable:

$$\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP\cdot z} \left\langle p_2 \middle| \bar{\psi}^q \left(-\frac{z}{2}\right) \gamma^\mu \psi^q \left(\frac{z}{2}\right) \middle| p_1 \right\rangle \tag{22}$$

Lattice will give first-principles modelling tools of GPDs in the moderate x_B range, while experiment will validate the lattice calculation in a crucial kinematic domain.

Thank you for your attention!