

Amplitude-level Searches for Dark Photons in Bhabha Scattering

D. Mack

Hall C Winter Meeting

January 18, 2024

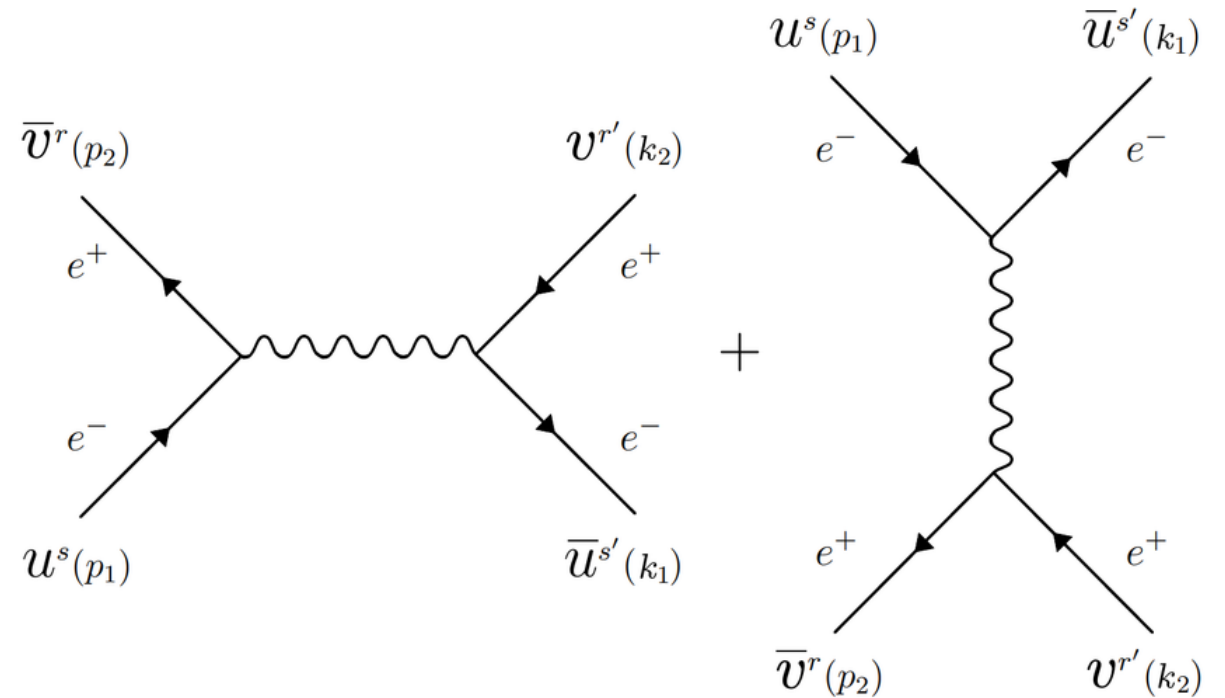
Bhabha Scattering: $e^+e^- \rightarrow e^+e^-$

Bhabha scattering is a purely leptonic reaction with very different behavior than Moller scattering.

The e^+ and e^- are of course not identical, and there is an s-channel annihilation diagram.

In the Standard Model (SM), the exchanged boson is a γ and a Z^0 .

Going Beyond the SM (BSM), considering neutral bosons only, there is also potentially an A' or Z' .



Research Gate uploaded by [Kort Beck](#)

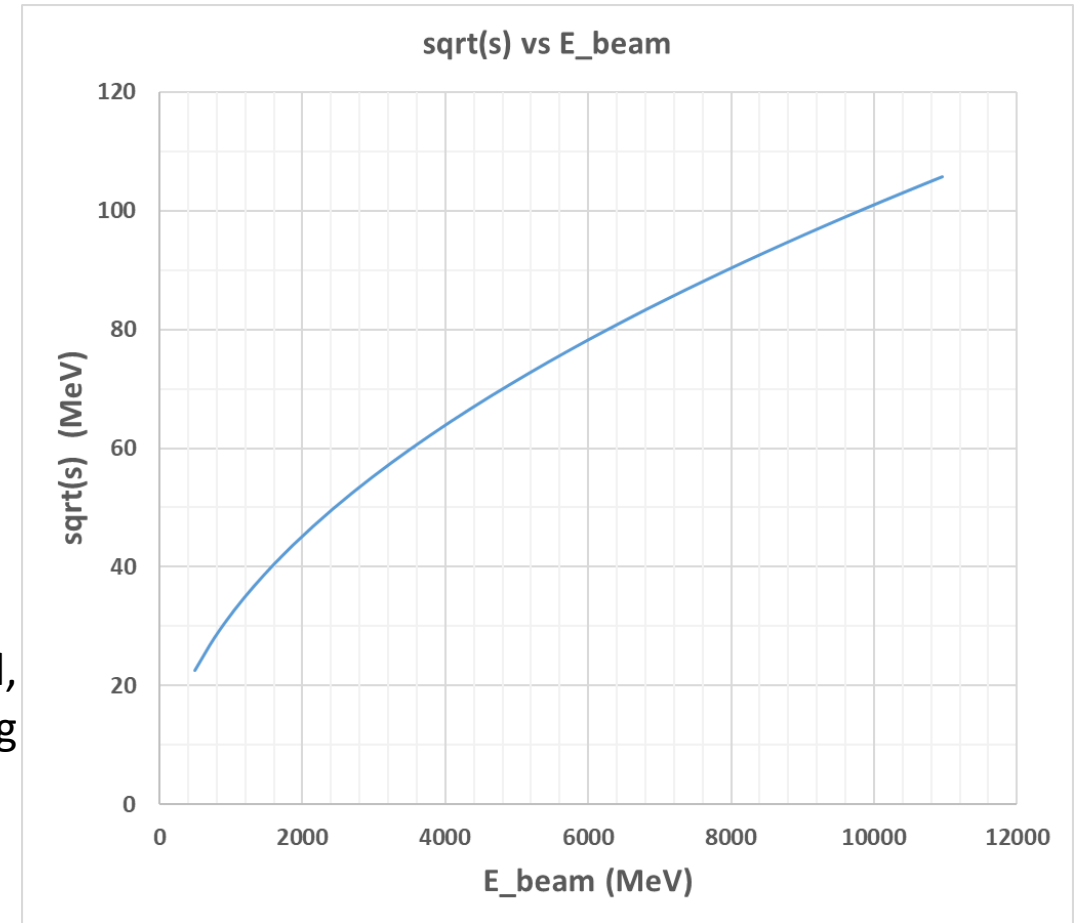
E_{cm} in Bhabha Scattering in Jlab Fixed Target Kinematics

At a 12 GeV CEBAF, the CM energy range will be ~ 20 -105 MeV/ c^2 .

$$E_{cm} = \sqrt{s} = \sqrt{2m_e^2 + 2E_{beam} * m_e} \\ \sim \sqrt{E_{beam}}$$

Notes:

- due to the sqrt factor above, it takes a roughly 100 MeV change in E_{beam} to produce a 1 MeV change in E_{cm} .
(Hold that thought for later!)
- since the differential xsect contains a factor of $1/s$, and s is small, the xsect is large by Jlab standards, $O(1)$ - $O(100)$ $\mu\text{B}/\text{sr}$ at 90deg CM.



So What's the Physics Interest?

Ignoring the cool factor of an anti-matter beam, $e^+e^- \rightarrow e^+e^-$ is in principle “just low-order QED”, and this is generally a well-studied reaction.

But due to the s-channel diagrams, one could expect dramatic changes in observables if the E_{cm} range included an unexpected resonance.

So I think the main physics interest is searches for physics beyond the SM, such as a dark photon or dark Z.

To my knowledge, the Jlab E_{cm} range has been poorly explored in Bhabha scattering, and certainly not with the weak interaction-scale sensitivity which will be needed a decade+ from now.

Along the journey, we may find that published calculations are inadequate for precision work in our fixed target kinematics due for example to ultra-relativistic approximations in older papers.

(Eg, see Epstein and Milner PRD 94, 033004 (2016)

<https://journals.aps.org/prd/abstract/10.1103/PhysRevD.94.033004>)

Excluded A' Phase Space (visible decays)

The mixing between the photon and dark photon is parameterized as ϵ .

The coupling of the dark photon to the electron is ϵ^*e .

There is a region of phase space, relevant to the Jlab positron program, which has proven resistant in visible searches (and to a much lesser extent for invisible decays).

The phase space in the red dashed box seems potentially excludable in a Jlab positron program, at least with amplitude-based searches.

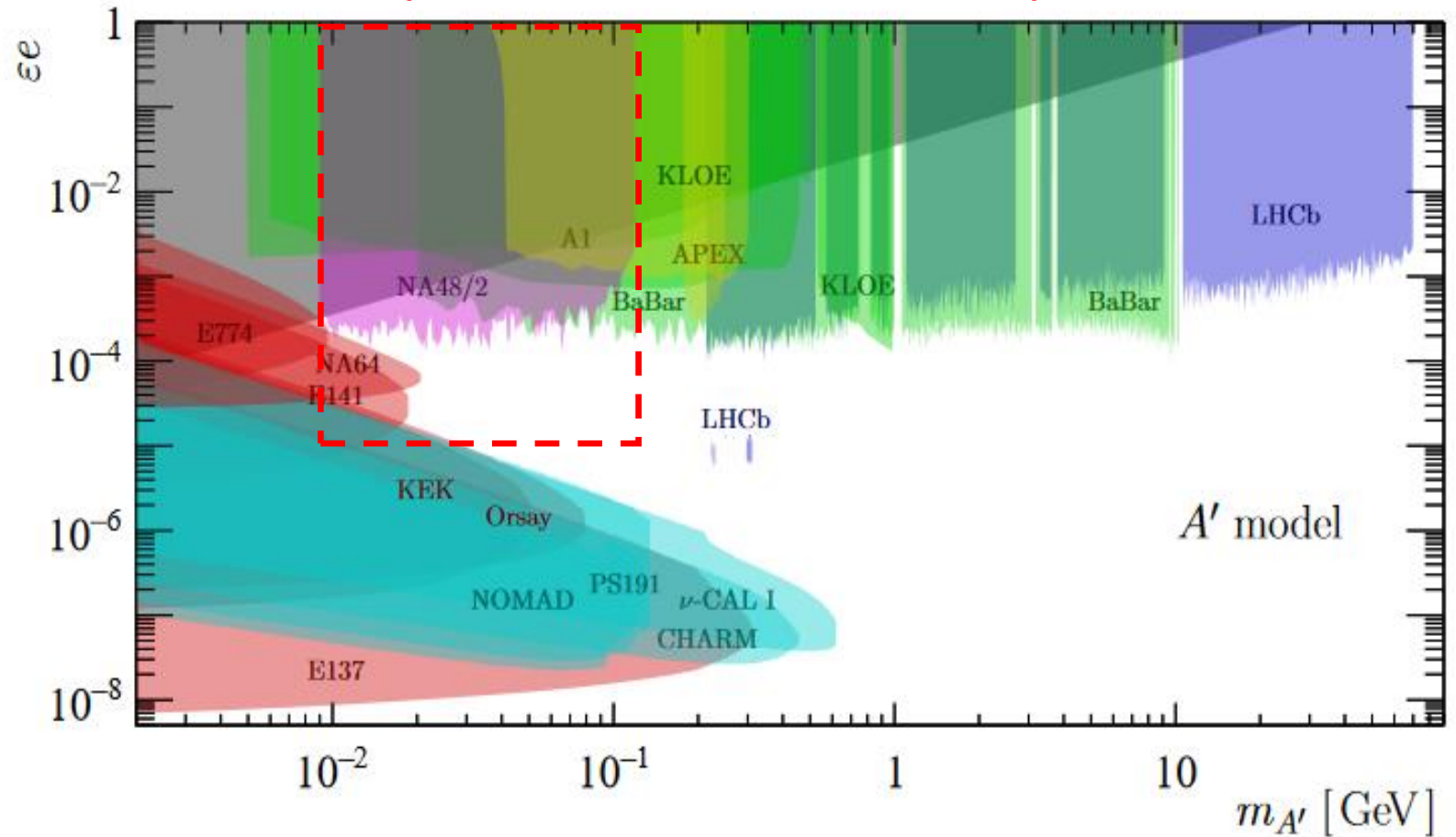


Figure 4. Constraints on visible A' decays considered in this study from (red) electron beam dumps, (cyan) proton beam dumps, (green) e^+e^- colliders, (blue) pp collisions, (magenta) meson decays, and (yellow) electron on fixed target experiments. The constraint derived from $(g-2)_e$ is shown in grey [90, 91].

Ambiguities in Hunting for an A'

- We don't know the mass.
- We don't know the coupling.
- We don't know the width (because we don't know the dominant decay mode):
 - i. Perhaps it's a very narrow state that reluctantly decays predominantly/"visibly" to $A' \rightarrow e^+e^-$.
 - ii. Or perhaps it is a broader state which decays quickly, effectively "invisibly", to a pair of dark particles, $\chi\bar{\chi}$.
 - iii. Or perhaps it decays semi-invisibly, and published visible and invisible constraints aren't as tight as we think

So to design an experiment which will still be a high priority 10+ years from now, we would like to:

- search a broad mass range,
- as sensitively as feasible in the coupling
- in a manner which is relatively insensitive to the A' decay mode

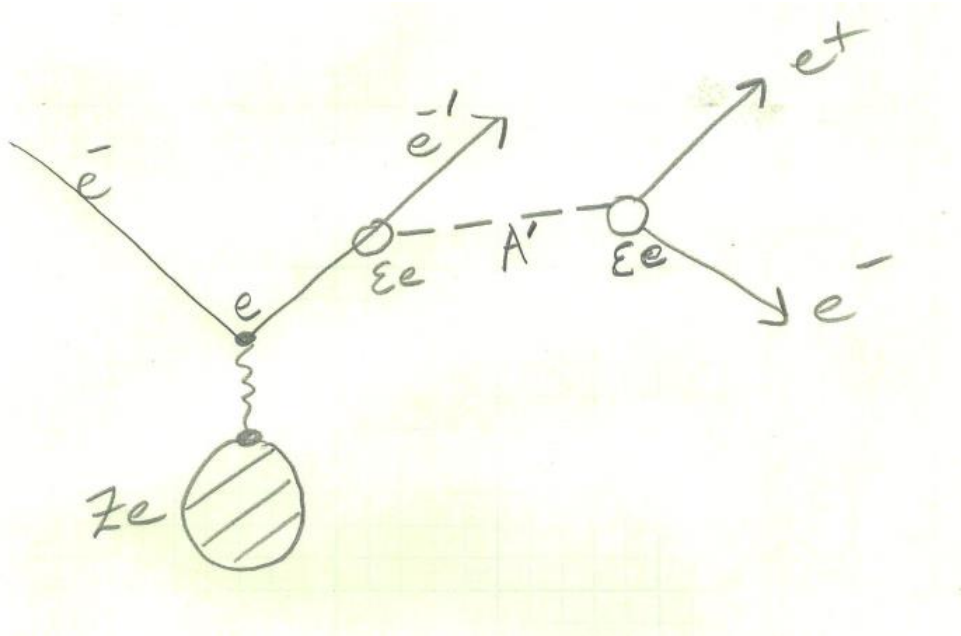
A' Signal Proportionality in terms of e and ϵ : example from dark Bremsstrahlung

For incoherent production and decay:

- Yield for A' production $\sim |Z F(q) e^3 \epsilon|^2$
- Yield for A' decay $\sim BR_{A' \rightarrow e^+e^-} |e\epsilon|^2$

Visible decay scenario: assume net signal yield for detecting $A' \rightarrow e^+e^-$

$$\sim Z^2 F^2(q) \alpha^4 \epsilon^4$$



A' Signal Proportionality in terms of e and ϵ : example from dark Bremsstrahlung

For incoherent production and decay:

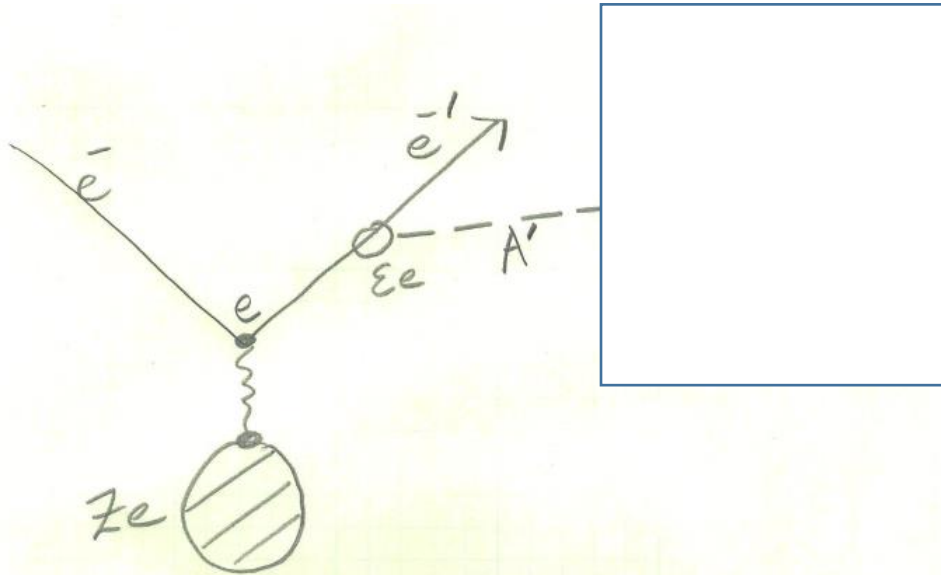
- Yield for A' production $\sim |Z F(q) e^3 \epsilon|^2$
- Yield for A' decay $\sim BR_{A' \rightarrow e+e-} |e\epsilon|^2$

Visible decay scenario: assume net signal yield for detecting $A' \rightarrow e+e-$

$$\sim Z^2 F^2(q) \alpha^4 \epsilon^4$$

All decays scenario: signal yield for detecting a narrow A' by MM_x^2 in $e + p \rightarrow e + p (X)$

$$\sim F^2(q) \alpha^3 \epsilon^2$$



A' Signal Proportionality in terms of e and ϵ : example from dark Bremsstrahlung

For incoherent production and decay:

- Yield for A' production $\sim |Z F(q) e^3 \epsilon|^2$
- Yield for A' decay $\sim BR_{A' \rightarrow e^+e^-} |e\epsilon|^2$

Visible decay scenario: assume net signal yield for detecting $A' \rightarrow e^+e^-$

$$\sim Z^2 F^2(q) \alpha^4 \epsilon^4$$

All decays scenario: signal yield for detecting a narrow A' by MM_x^2 in $e + p \rightarrow e + p (X)$

$$\sim F^2(q) \alpha^3 \epsilon^2$$

Because α and ϵ are small numbers, one would like to design an experiment with small exponents and low backgrounds.

Sensitivities of Yield-based Experiments

General Technique	Technique	Signal Proportional To:	Constrains	Comment
Dark Bremsstrahlung with e^- beams	$M(e^+e^-)$ peak in $e^- + A \rightarrow e' + e+e^- (X)$	$Z^2F(q)^2 \alpha^4 \epsilon^4$	Visible decays only	E.g., Hall A APEX and MAMI-A1 experiments. Especially sensitive when searching for a detached vertex (hence minimal bkg) as in Hall B HPS
“ “	MM_x^2 peak in $e^- + p \rightarrow e' + p' (X)$	$F(q)^2 \alpha^3 \epsilon^2$	All* decays	The DarkLight proposal planned to measure the fore-mentioned reaction as well as $e^- + p \rightarrow e' + p' + e+e^-$.
“ “	Missing energy in $e^- + \text{calorimeter} \rightarrow$ almost nothing	$Z^2F(q)^2 \alpha^3 \epsilon^2$	Invisible decays only	NA64 placed impressive constraints on <u>invisible</u> decays that would be extremely hard to beat.
Positron beams	$M(e^+e^-)$ peak in E_{cm} scan of $e^+e^- \rightarrow e+e^-$	$\alpha^2 \epsilon^4$	Visible decays only	Seems not particularly competitive.
	MM_x^2 peak in $e^+e^- \rightarrow \gamma (X)$	$\alpha^2 \epsilon^2$	All decays	Bogdan/Ashot proposal has attractive Figure of Merit!

*Any sort of peak search constraining invisible decays is potentially weakened if the A' width is broader than the resolution.

Alternate Strategy

Assume the total amplitude is the sum of a large SM and small BSM amplitude: $A_{\text{tot}} = A_{\text{EM}} + A_{\text{small}}$

The Yield is hand-wavingly proportional to

$$A_{\text{tot}}^2 = (A_{\text{EM}} + A_{\text{small}})^2$$
$$= A_{\text{EM}}^2 + 2A_{\text{EM}}A_{\text{small}} + A_{\text{small}}^2$$

Instead of looking for a dark photon in the far field as a bump proportional to A_{small}^2 , can we search sensitively for resonance signatures in an interference term proportional to $A_{\text{small}}/A_{\text{EM}}$?

The answer appears to be, “Yes”.

Sensitivities of Yield- and Amplitude-based Experiments

General Technique	Technique	Signal Proportional To:	Constrains	Comment
Dark Bremsstrahlung with e^- beams	$M(e^+e^-)$ peak in $e^- + A \rightarrow e' + e^+e^- (X)$	$Z^2F(q)^2 \alpha^4 \epsilon^4$	Visible decays only	E.g., Hall A APEX and MAMI-A1 experiments. Especially sensitive when searching for a detached vertex (hence minimal bkg) as in Hall B HPS
“ “	MM_x^2 peak in $e^- + p \rightarrow e' + p' (X)$	$F(q)^2 \alpha^3 \epsilon^2$	All decays	The DarkLight proposal planned to measure the fore-mentioned reaction as well as $e^- + p \rightarrow e' + p' + e^+e^-$.
“ “	Missing energy in $e^- + \text{calorimeter} \rightarrow$ almost nothing	$Z^2F(q)^2 \alpha^3 \epsilon^2$	Invisible decays only	NA64 placed impressive constraints on <u>invisible</u> decays that would be extremely hard to beat.
Positron beams	$M(e^+e^-)$ peak in Ecm scan of $e^+e^- \rightarrow e^+e^-$	$\alpha^2 \epsilon^4$	Visible decays only	Seems not particularly competitive.
	MM_x^2 peak in $e^+e^- \rightarrow \gamma (X)$	$\alpha^2 \epsilon^2$	All decays	Bogdan/Ashot proposal has attractive Figure of Merit!
	Asymmetry in $e^+e^- \rightarrow e^+e^-$	$\alpha \epsilon^2$	All decays	This talk.
	Asymmetry in $e^+e^- \rightarrow \gamma e^+e^-$	$\alpha^{1.5} \epsilon^2$	All decays	TBD

Sensitivities of Yield- and Amplitude-based Experiments

General Technique	Technique	Signal Proportional To	Constrains	Comment
Dark Bremsstrahlung with e^- beams	<p>The devil is in the details. To really compare experiments,</p> <ul style="list-style-type: none"> i. form factors have to be integrated over the acceptance, ii. backgrounds and resolutions vary, iii. the mass reach varies 			AMI-A1 sensitive atched vertex Hall B HPS
“ “	<p>But I think it's true that an amplitude search in Bhabha scattering:</p> <ul style="list-style-type: none"> i. has potentially competitive sensitivity, ii. is not affected by the “visible” vs “invisible” vs “partially visible” ambiguity (similar to the missing energy or missing mass class of experiments) iii. and one does not have to detect single photons close to the beamline. 			lanned to ed reaction o' +e+e- .
“ “				onstraints would be eat.
Bhabha scattering	M(e^+e^-) peak in Ecm scan of $e^+e^- \rightarrow e^+e^-$	$\alpha^2 \epsilon^4$	Visible decays only	Seems not particularly competitive.
	MM _x ² peak in $e^+e^- \rightarrow \gamma (X)$	$\alpha^2 \epsilon^2$	All decays	Bogdan/Ashot proposal has attractive Figure of Merit!
	Asymmetry in $e^+e^- \rightarrow e^+e^-$	$\alpha \epsilon^2$	All decays	This work.
	Asymmetry in $e^+e^- \rightarrow \gamma e^+e^-$	$\alpha^{1.5} \epsilon^2$	All decays	TBD

Formalism

SM Formalism with $\gamma + Z^0$ (with option for a Z')

I used H.A. Olsen and P. Osland, “Polarized Bhabha and Moller scattering in left-right-asymmetric theories”. This paper was clear, provided the xsect and two PC and two PV asymmetries, showed me how to include a Z' (which I could extend to an A' with purely vector couplings) and insightful comparisons to Moller scattering. But it does not include radiation which will be important for designing realistic experiments.

PHYSICAL REVIEW D

VOLUME 25, NUMBER 11

1 JUNE 1982

Polarized Bhabha and Møller scattering in left-right-asymmetric theories

Haakon A. Olsen

Institute of Physics, University of Trondheim, Norges Laererhøgskole, N-7000 Trondheim, Norway

Per Osland

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 30 November 1981)

We identify and calculate the independent quantities that determine arbitrarily polarized Bhabha and Møller scattering, for left-right-asymmetric theories. Longitudinal polarization of either beam appears most useful, in either Bhabha or Møller scattering, in discriminating between the $SU(2) \times U(1)$ theory and certain classes of extended theories. Transverse beam polarization would in Bhabha scattering at high energies, $\sqrt{s} \simeq M_Z$, provide a very clear distinction between theories in which the $e^+e^-Z^0$ coupling is dominantly axial vector and theories where it is dominantly vector.

I. INTRODUCTION

Present electron-positron accelerators have reached energies where weak-interaction effects are on the verge of being observed. The nonobservation of these effects has indeed served to constrain¹

a position to give a quantitative discussion of the dependence on beam polarization. It will be shown that, in contrast to the QED limit, the Bhabha cross section develops a strong dependence on transverse beam polarization, as the energy increases toward the Z^0 pole. Beyond the Z^0 pole

Suite of Observables in Bhabha Scattering

Eqn (1) of Olsen and Osland gives the different xsect and asymmetries for all combination e+ or e- longitudinal or transverse polarization.

Simplifying and dumbing down the notation a bit:

$$\sigma(\theta, \phi) = \sigma_0 \{ 1 + \mathbf{A}_{LL} P_-^{\text{para}} P_+^{\text{para}} + \mathbf{A}_{LU} (P_-^{\text{para}} - P_+^{\text{para}}) + P_-^{\text{perp}} P_+^{\text{perp}} [\mathbf{A}_{TT} \cos(2\phi) + \mathbf{A}_{TT}' \sin(2\phi)] \}$$

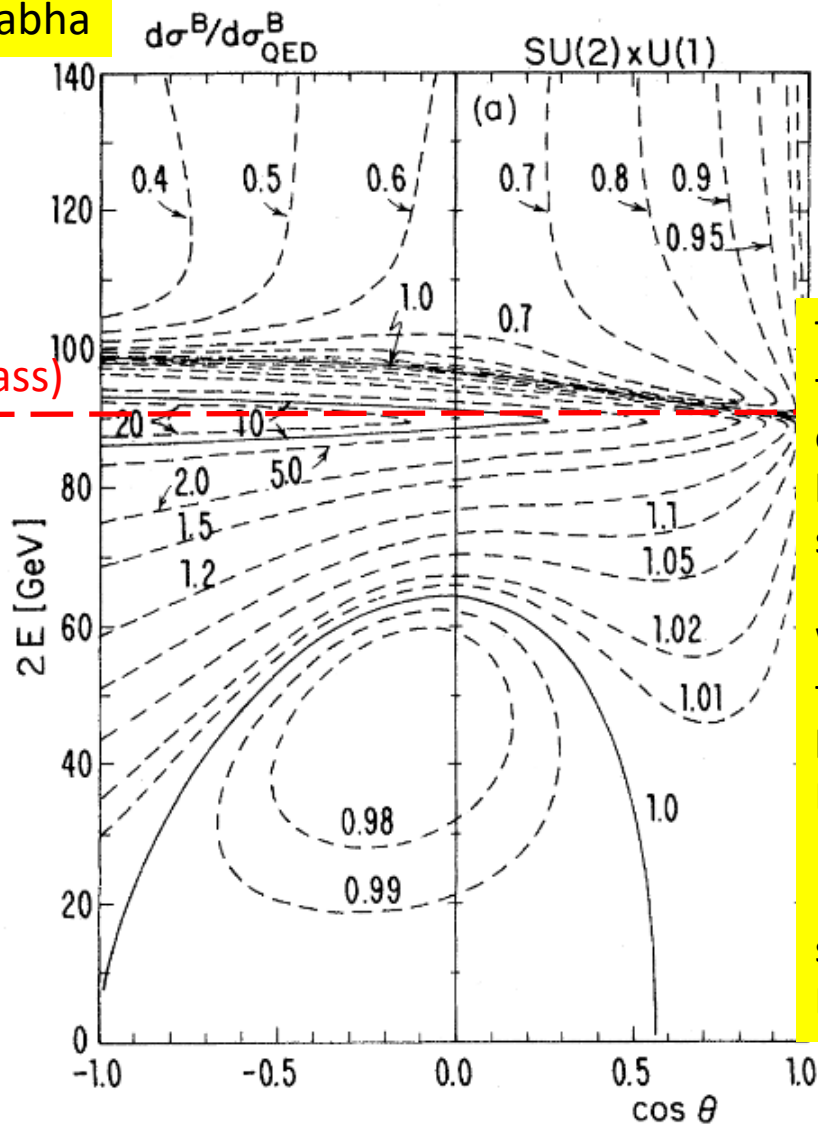
If I drop the predominantly PV terms, it looks just like the Moller polarimetry equations:

$$\sigma(\theta, \phi) = \sigma_0 \{ 1 + \mathbf{A}_{LL} P_-^{\text{para}} P_+^{\text{para}} + P_-^{\text{perp}} P_+^{\text{perp}} \mathbf{A}_{TT} \cos(2\phi) \}$$

Let's look at the σ_0 term first.

Bhabha vs Moller Comparison: $X_{\text{ssect}}/X_{\text{ssect}}_{\text{QED}}$ Up to $E_{\text{cm}} = 140 \text{ GeV}/c^2$

Bhabha



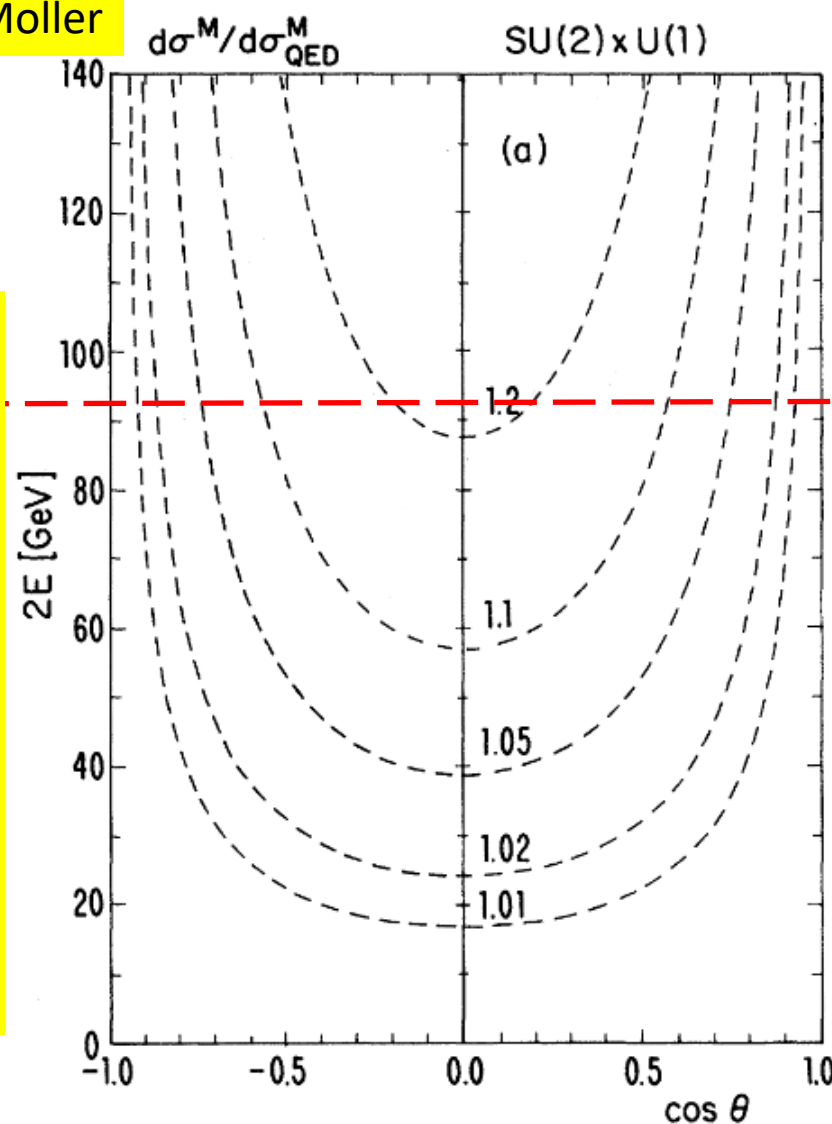
Z^0 Mass)

This is just one example from the Olsen and Osland paper of the dramatic difference between Bhabha and Moller scattering.

When a resonance is present, the effects on the xsect can be 10-100x larger than in Moller.

This motivates the idea of searching for an A' using Bhabha scattering.

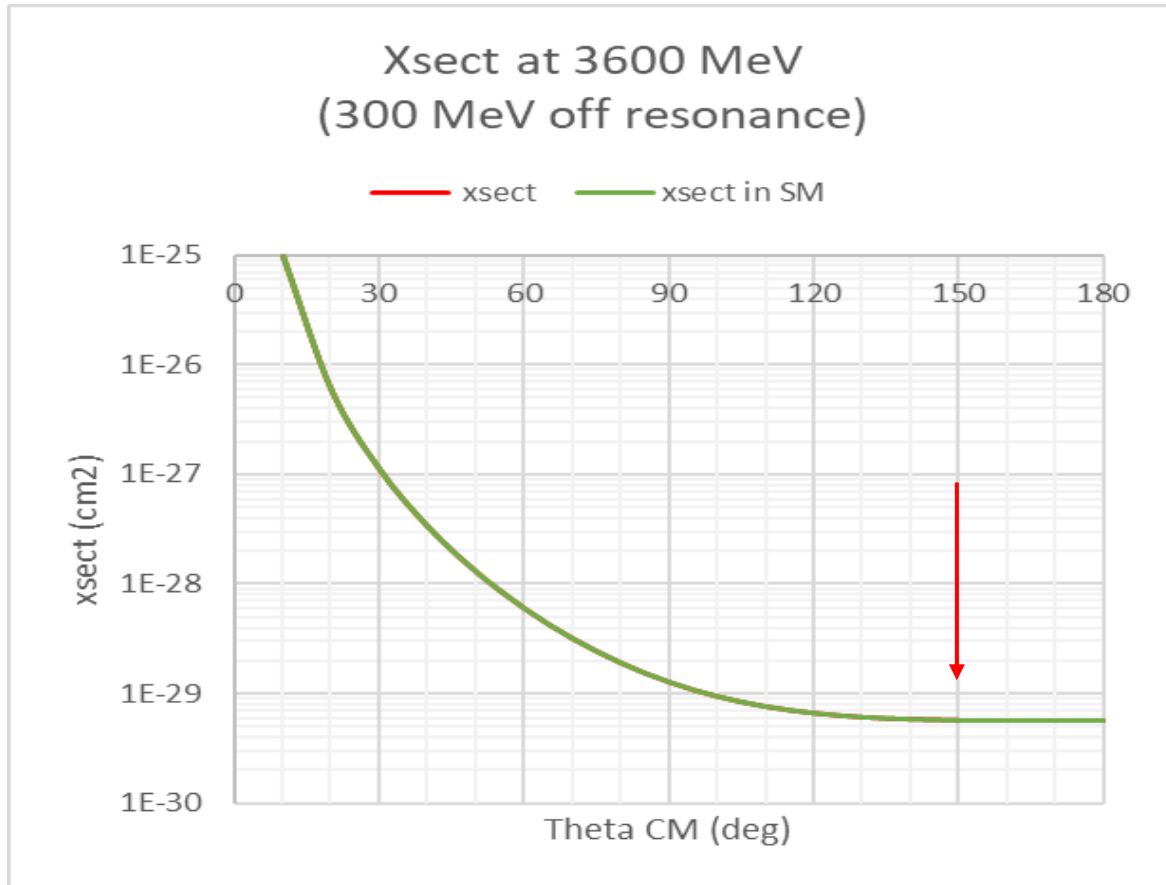
Moller



Z^0 Mass)

A' signals in the Yield

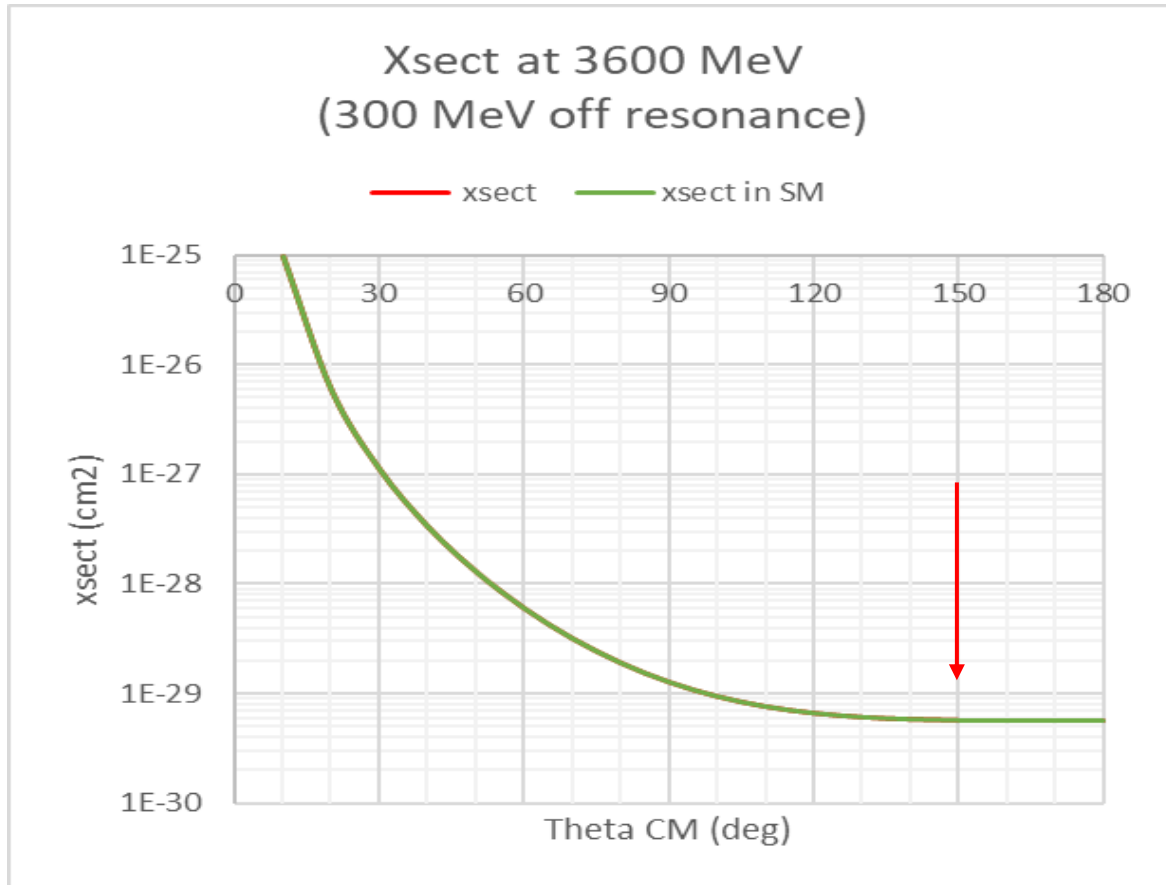
Yields: Purely Vector Coupling, $\epsilon = 1\text{E-}4$, $M_{A'} = 57.5 \text{ MeV}/c^2$



On this plotting scale, the A' effects are invisibly small.

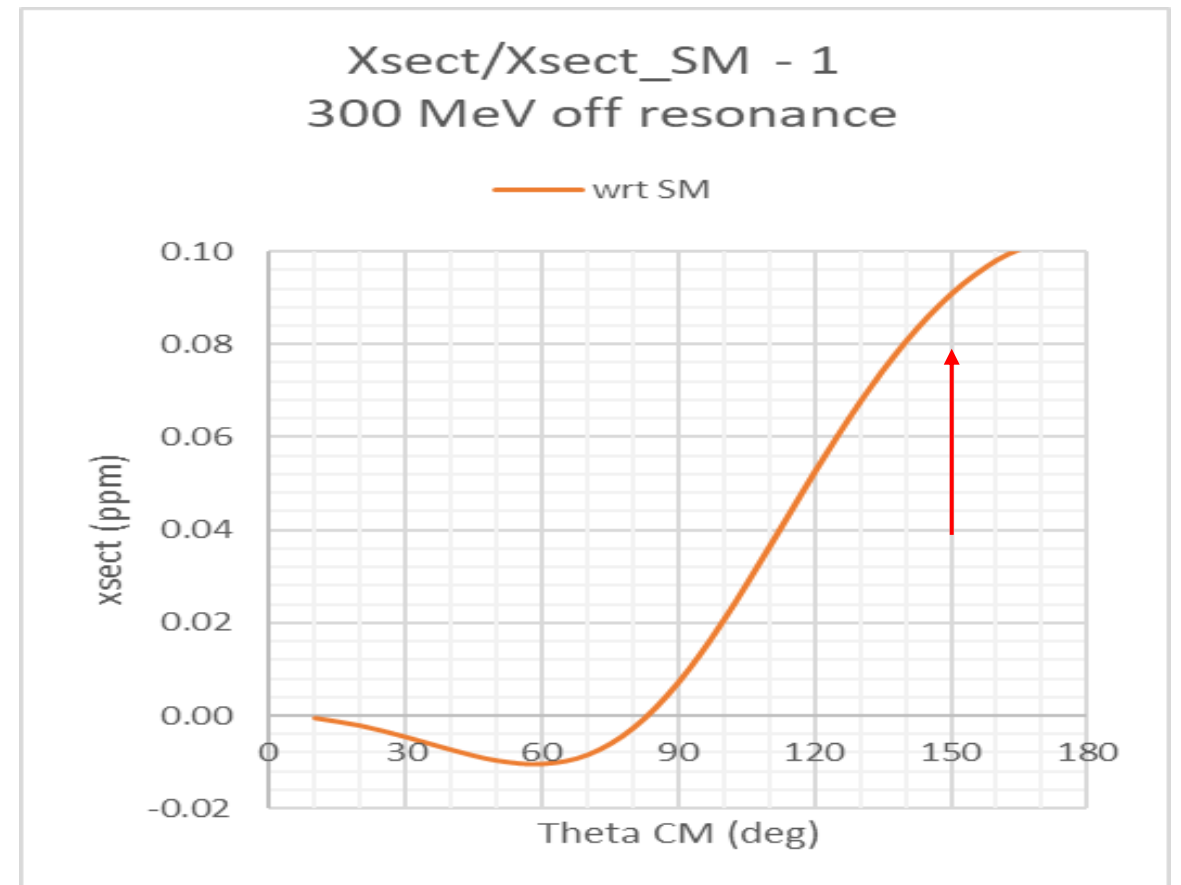
Note the flattening of the Bhabha xsect at backward angles, where t-channel photon exchange no longer dominates.

Yields: Purely Vector Coupling, $\epsilon = 1\text{E-}4$, $M_{A'} = 57.5 \text{ MeV}/c^2$



On this plotting scale, the A' effects are invisibly small.

Note the flattening of the Bhabha xsect at backward angles, where t-channel photon exchange no longer dominates.

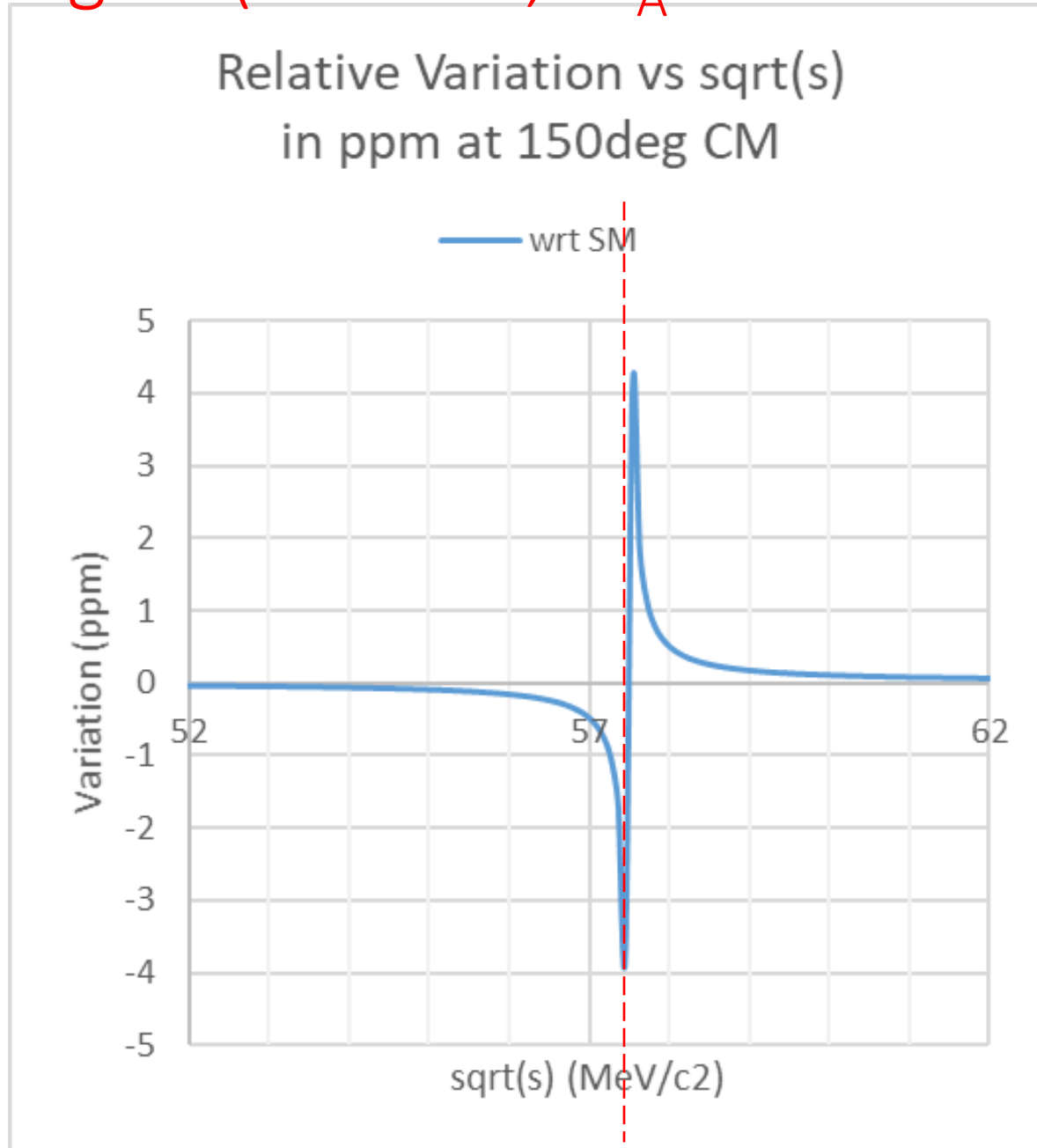


Taking the difference wrt the SM, off-resonance effects are tiny, comparable to Z^0 exchange.

As naively expected, A' effects are largest at backward angles.

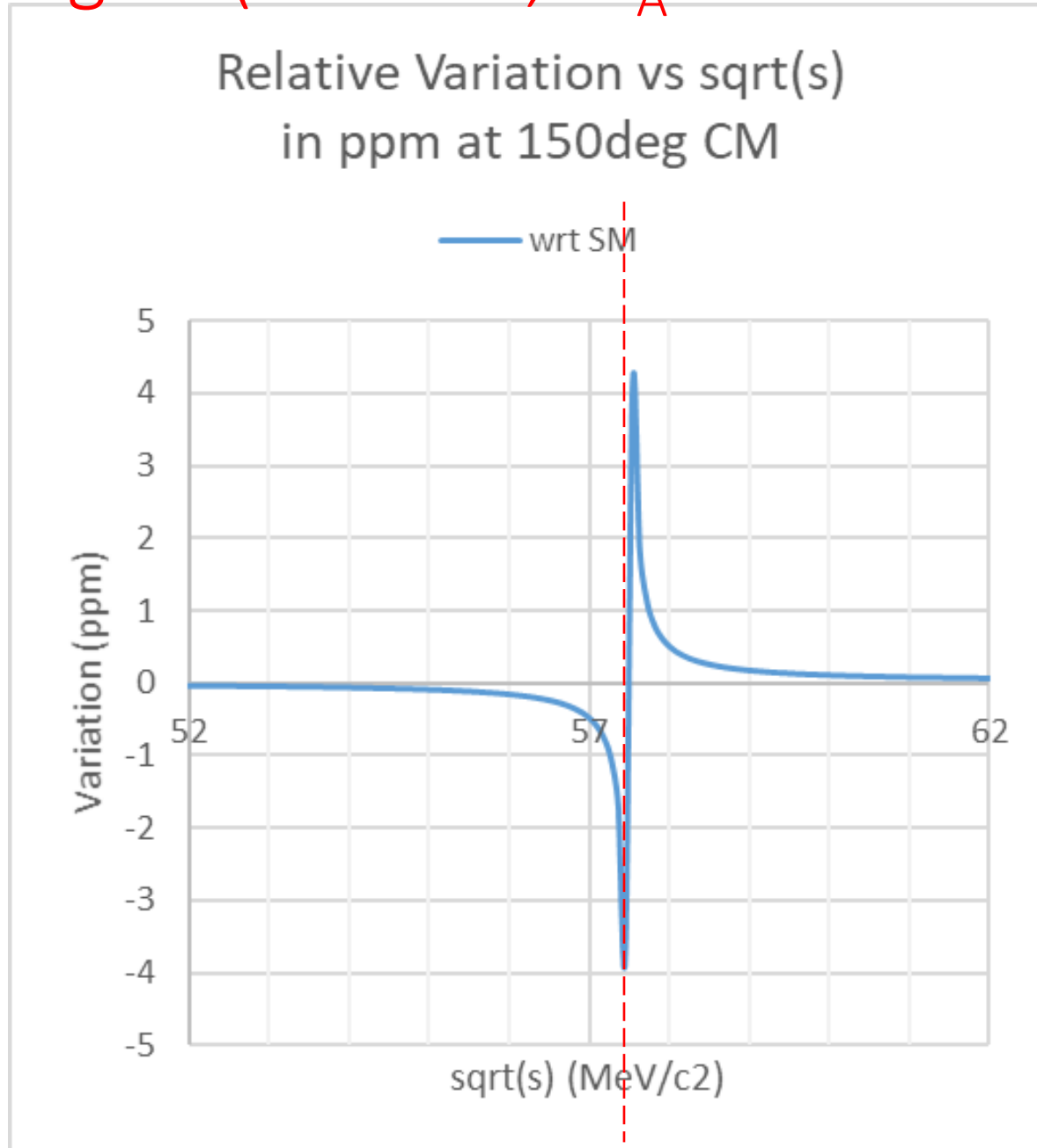
Yield Signal ($\epsilon = 1E-4$, $M_{A'} = 57.5 \text{ MeV}/c^2$)

Having established that A' effects are less diluted at large θ_{CM} , now scanning \sqrt{s} at 150deg CM:



Yield Signal ($\epsilon = 1E-4$, $M_{A'} = 57.5 \text{ MeV}/c^2$)

Having established that A' effects are less diluted at large θ_{CM} , now scanning \sqrt{s} at 150deg CM:



This is potentially measurable (about 0.5 ppm in the wings).

Naively, the beam energy would have to be within $\sim 50 \text{ MeV}$ of resonance to see a deviation from the SM.

In practice, if the beam energy is above the resonance, Initial State Radiation (ISR) will allow probing a broader range of lower E_{cm} .

The Glitch in the Matrix: Why is This Not a Bump Hunt?

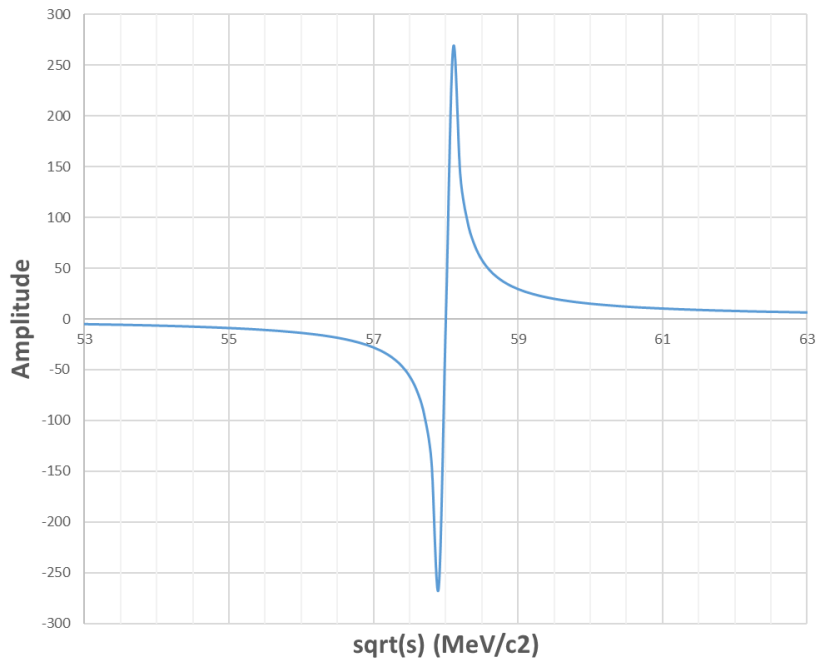
$M_{A'} = 57.5 \text{ MeV}/c^2$, Width = 57.5 keV

$$\frac{d\sigma_0^B}{d\Omega} = \frac{\alpha^2}{4s} \left\{ \cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1+f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1+f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] + 2 \sin^4(\theta/2) \left| 1 + f(s)g_R g_L \right|^2 + [2/\sin^4(\theta/2)][1+f(t)g_R g_L]^2 \right\}, \quad (14)$$

Real Part

f(s) for $M_{A'} = 57.5 \text{ MeV}/c^2$

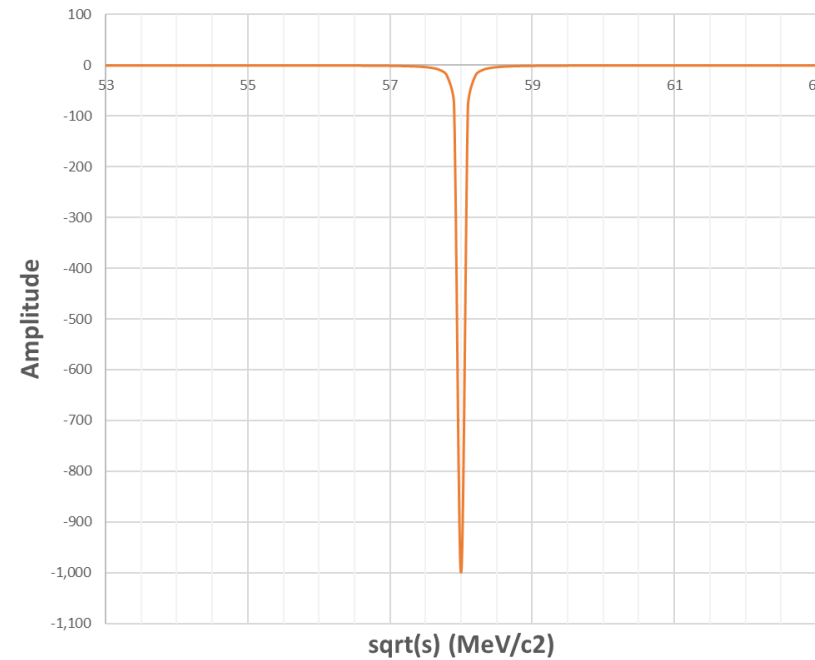
— Real part of f(s)



Imag. Part

f(s) for $M_{A'} = 57.5 \text{ MeV}/c^2$

— Imaginary part of f(s)



The Glitch in the Matrix: Why is This Not a Bump Hunt?

$M_{A'} = 57.5 \text{ MeV}/c^2$, Width = 57.5 keV

$$\frac{d\sigma_0^B}{d\Omega} = \frac{\alpha^2}{4s} \left\{ \cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1+f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1+f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] + 2 \sin^4(\theta/2) \left| 1 + f(s)g_R g_L \right|^2 + [2/\sin^4(\theta/2)][1+f(t)g_R g_L]^2 \right\},$$

$$X_{\text{ssect}} \sim |1 + f(s) g_R g_L|^2$$

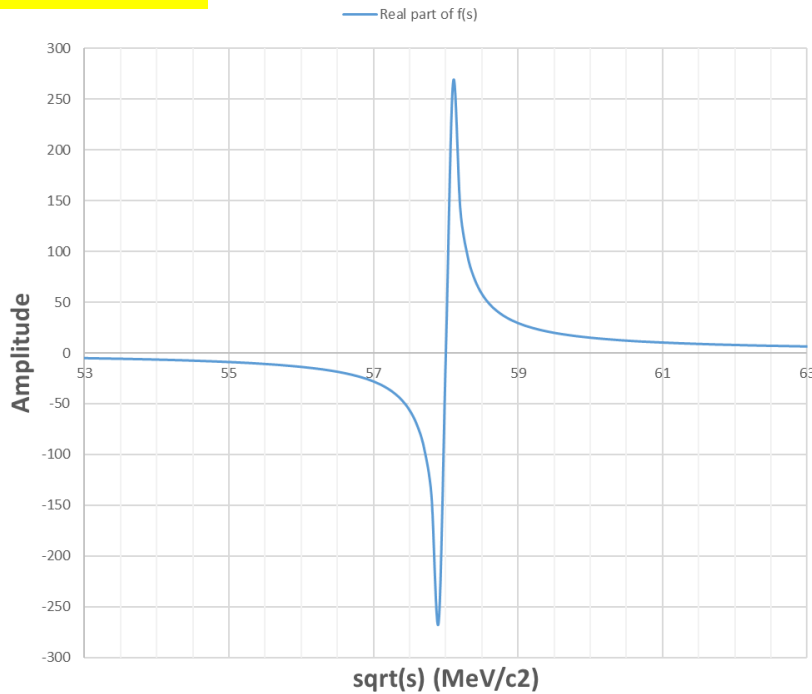
$$\sim |1 + \text{Re}f(s) g_R g_L + i \text{Im}f(s) g_R g_L|^2$$

$$\sim 1 + 2\text{Re}f(s) g_R g_L + [\text{Re}f(s)^2 + \text{Im}f(s)^2] (g_R g_L)^2$$

$$\sim 1 + 2\text{Re}f(s) g_R g_L + \text{H.O.T.}$$

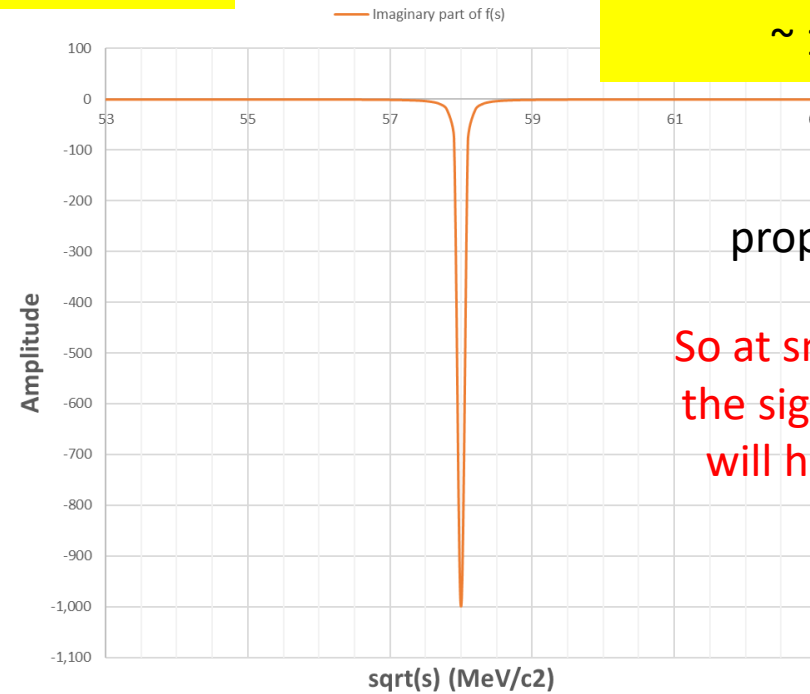
Real Part

f(s) for $M_{A'} = 57.5 \text{ MeV}/c^2$



Imag. Part

f(s) for $M_{A'} = 57.5 \text{ MeV}/c^2$



proportional to ϵ^2

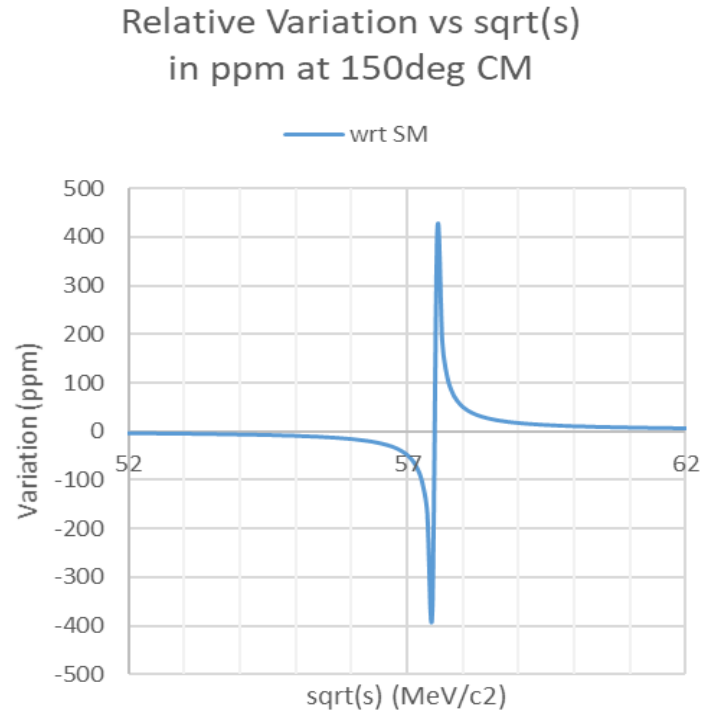
proportional to ϵ^4

So at small, relevant values of ϵ , the signal in Bhabha scattering will have the shape of $\text{Re}f(s)$.

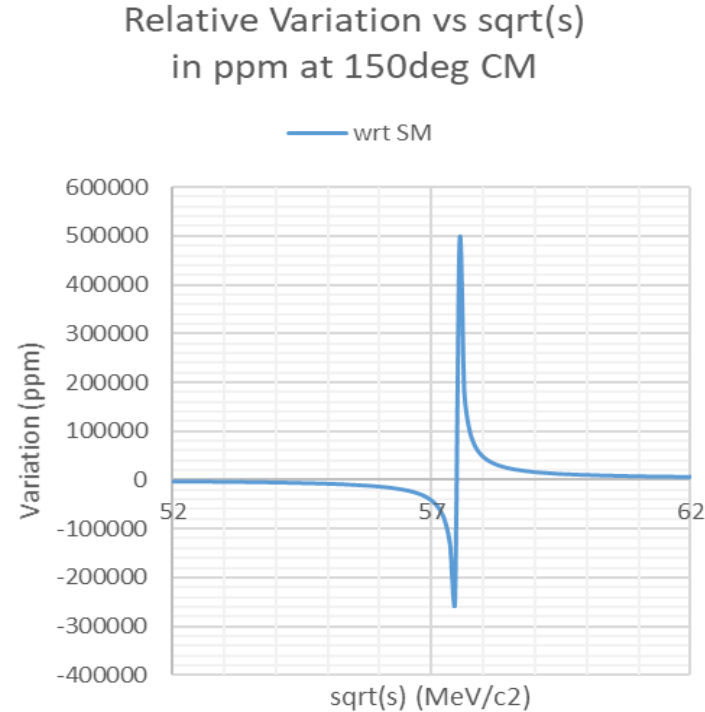
Evolution into a Bump with Increasing ϵ : idiot check

(Purely Vector Coupling, $M_{A'} = 57.5 \text{ MeV}/c^2$, Width = 57.5 keV)

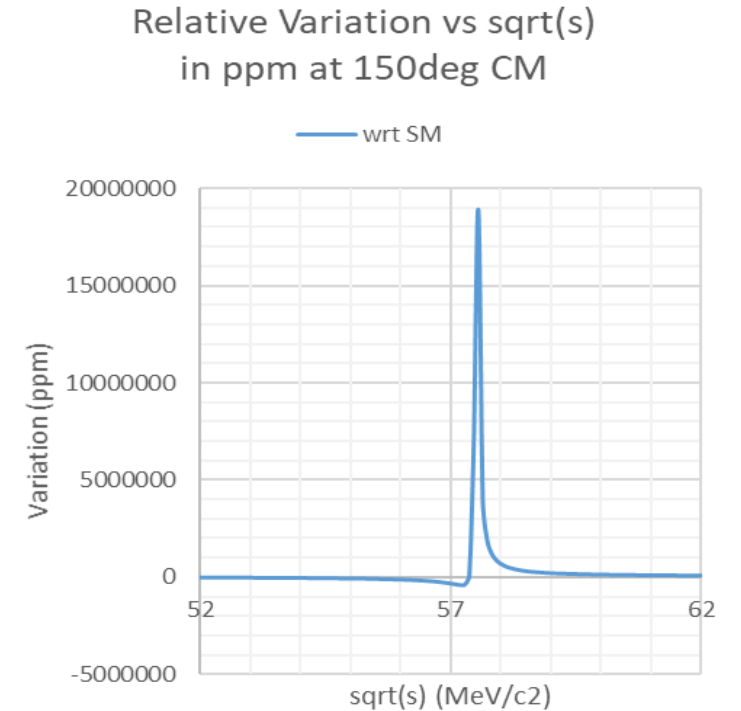
$\epsilon = 1e-3$



$\epsilon = 0.03$



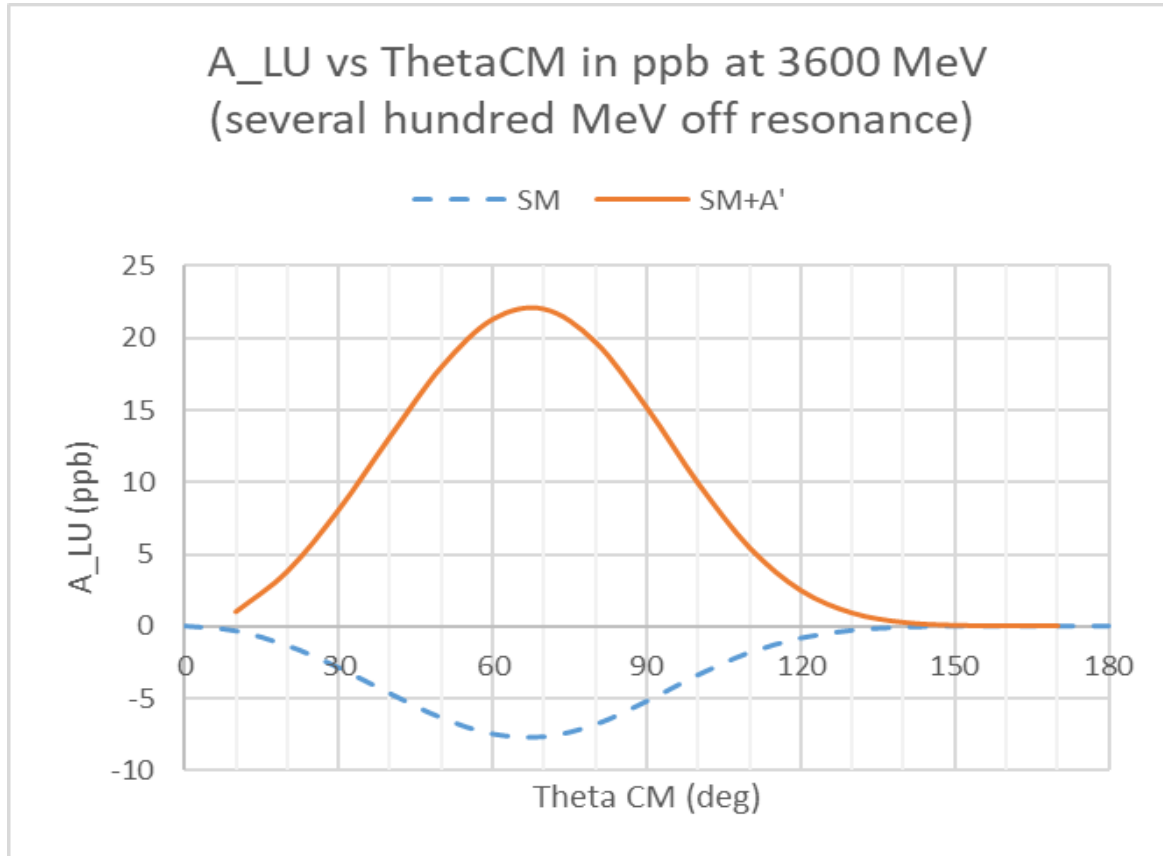
$\epsilon = 0.1$



The interference pattern ($A_{EM} * A_{small}$) shape evolves into a bump with increasing ϵ .
I think of the bump as representing real A' production (proportional to A_{small}^2).

Dark Z' signals in the PV Asymmetry

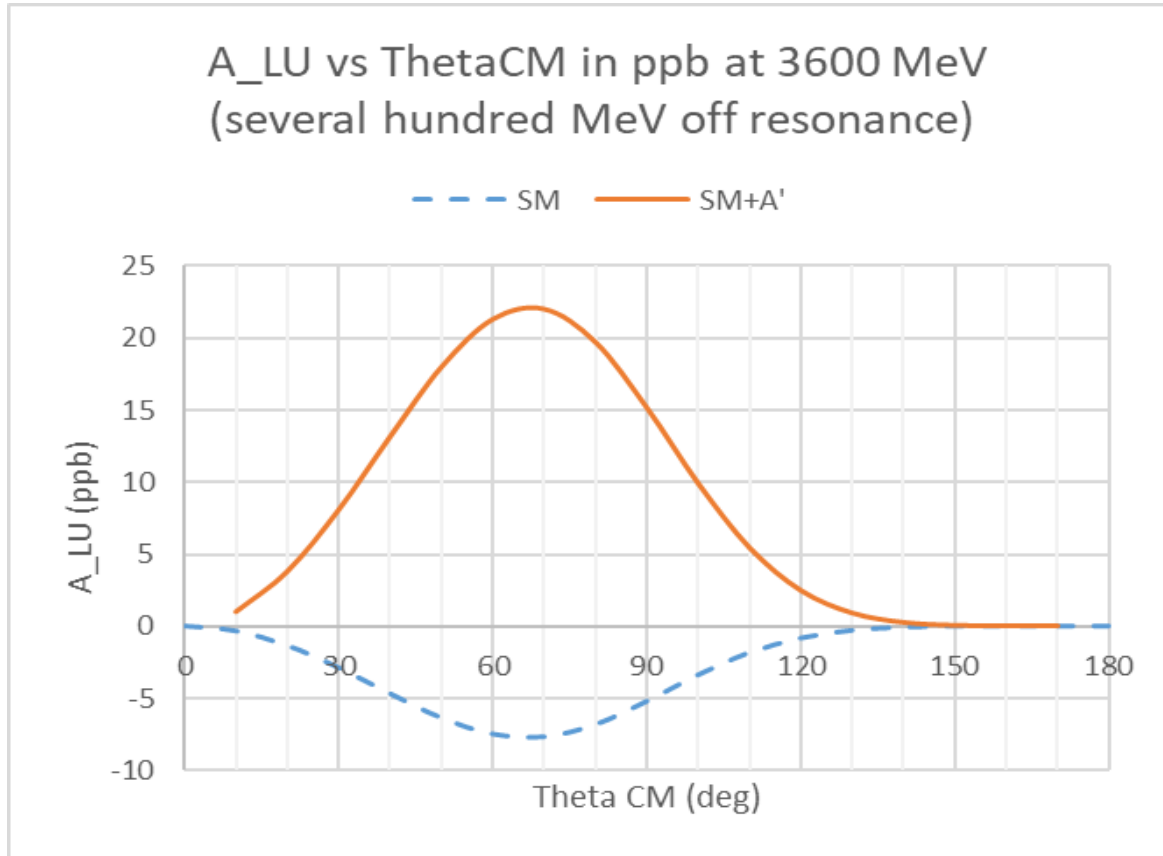
$$A_{pV}: g_A = g_V = 1, \varepsilon = 1E-4, M_{A'} = 57.5 \text{ MeV}/c^2$$



The tree-level SM A_{pV} in Bhabha is very small.
Even several hundred MeV in beam energy
off resonance, the effect on A_{pV} is dramatic.

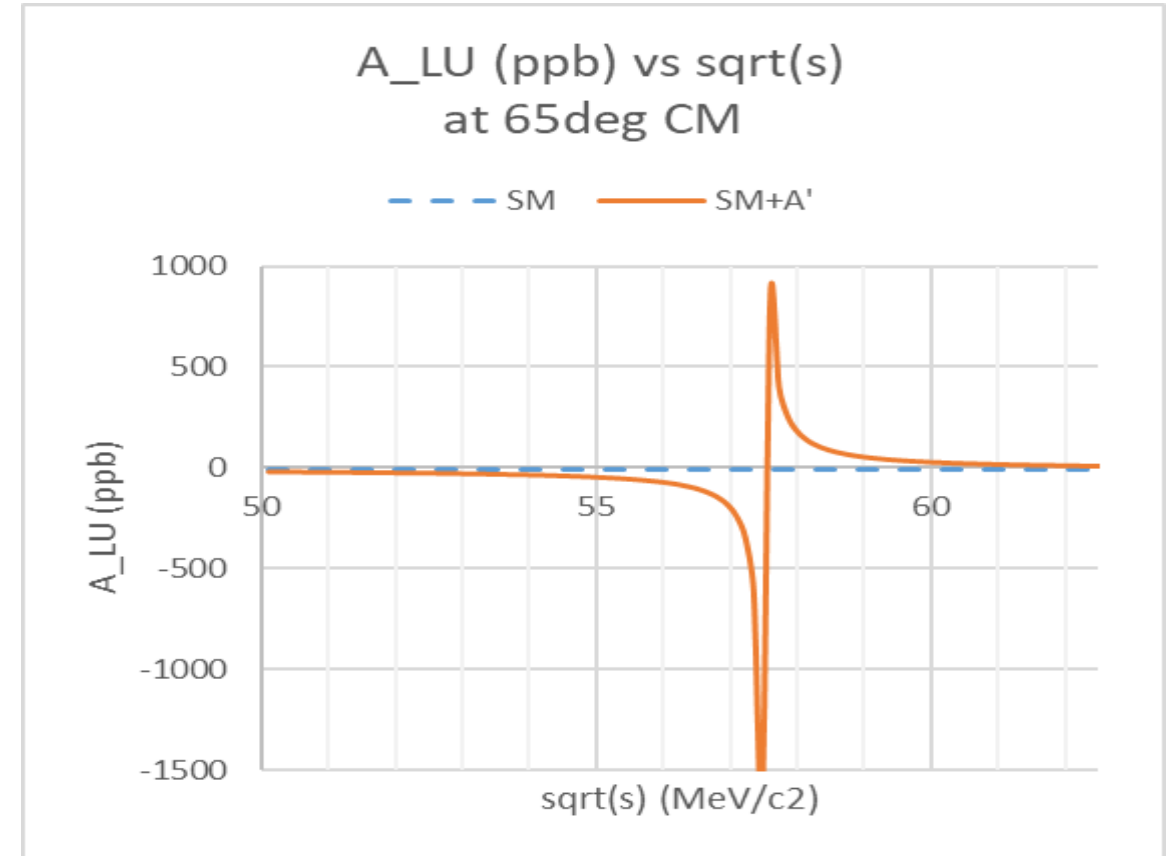
Of course, one would need 10 ppb sensitivity.

$$A_{pV}: g_A = g_V = 1, \varepsilon = 1E-4, M_{A'} = 57.5 \text{ MeV}/c^2$$



The tree-level SM A_{pV} in Bhabha is very small.
Even several hundred MeV in beam energy off resonance, the effect on A_{pV} is dramatic.

Of course, one would need 10 ppb sensitivity.

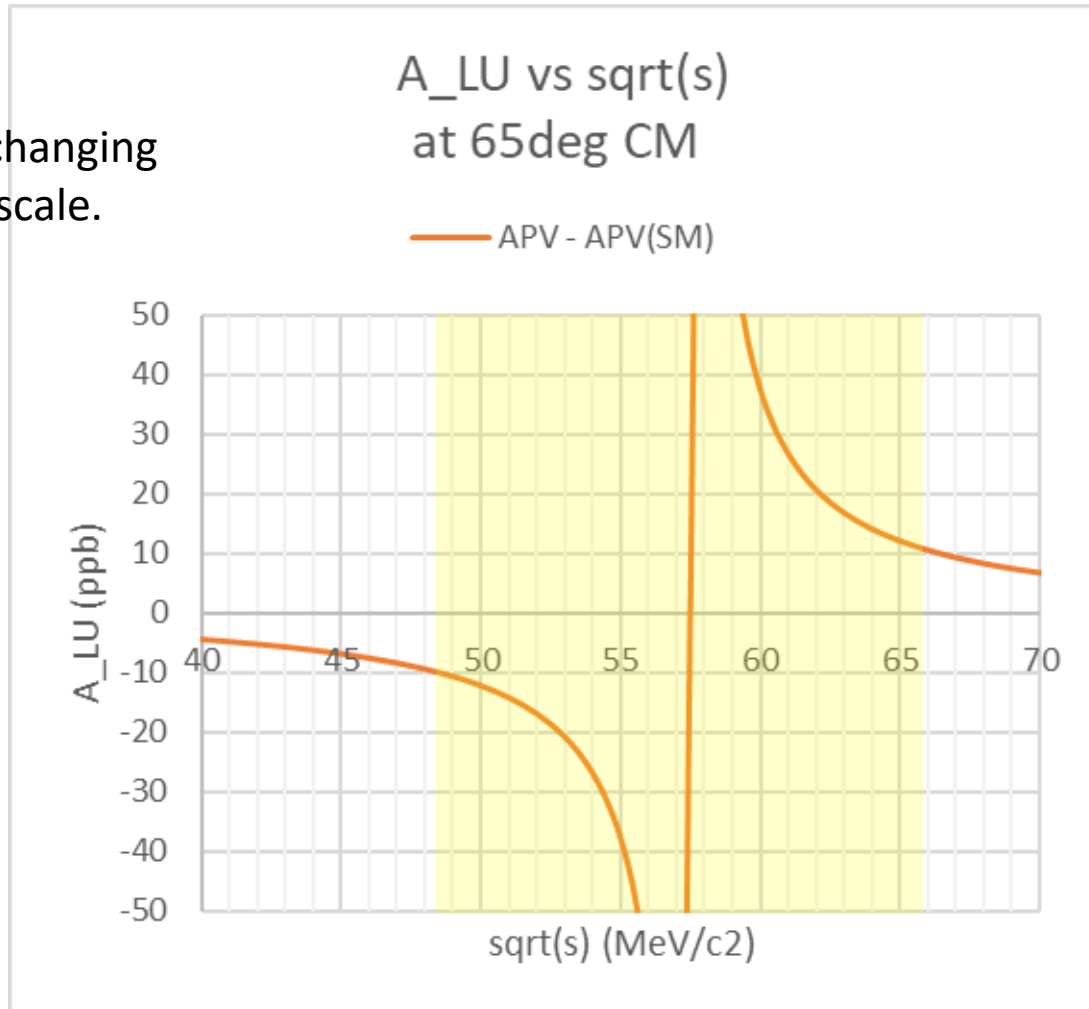


Near resonance, the asymmetry approaches the O(1) ppm level, which is 100x the SM value.

But we don't want to count on being near a resonance: we want to search a broad range.

$$A_{PV}: g_A = g_V = 1, \varepsilon = 1E-4, M_{A'} = 57.5 \text{ MeV}/c^2$$

Same plot, changing
the vertical scale.



The dark Z' effects exceed 10 ppb over a fairly large 20 MeV/c² mass range, corresponding to over 2 GeV in beam energy even without ISR.

Beam Normal Single Spin Asymmetry (BNSSA) (new!)

BNSSA in Bhabha Scattering

This observable is not available in the Olsen and Osland paper.

(It was not on the radar of the high energy e+e- collider community because it would be very small.)

Rather, I use **Fronsdal and Jaksic, Phys. Rev. 121, 916-919 (1961)**.

To 4th order in the EM coupling constant, e , their calculation should be valid from $\sqrt{s} = 2*m_e$ up to $\mu+\mu^-$ threshold.

I am not competent to add an A' to their formalism, but I make some dimensional arguments near the end.

BNSSA in Bhabha Scattering

This observable is not available in the Olsen and Osland paper.

(It was not on the radar of the high energy e+e- collider community because it would be very small.)

Rather, I use **Fronsdal and Jaksic, Phys. Rev. 121, 916-919 (1961)**.

To 4th order in the EM coupling constant, e , their calculation should be valid from $\sqrt{s} = 2m_e$ up to $\mu+\mu^-$ threshold.

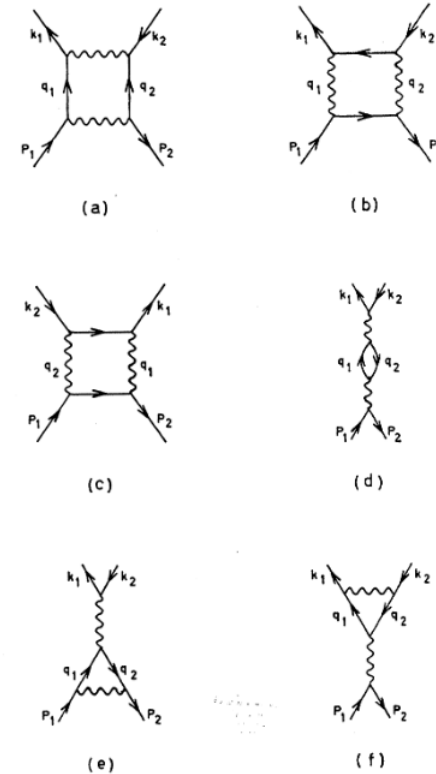
I am not competent to add an A' to their formalism, but I make some dimensional arguments near the end.

It's well known to the Jlab community that the BNSSA is the interference between 1-photon and the imaginary part of 2-photon exchange amplitudes.

$$P = 2i \operatorname{tr}\{M_2^* \sigma \bullet s \operatorname{Im} M_4\} / \operatorname{tr}\{M_2^* M_2\}$$

But most of the 2-photon exchange diagrams in Bhabha scattering at right are not Jlab business-as-usual ...

(on the next slide, I'm going to enlarge the diagrams, and tilt them clockwise so time goes from left to right)



Taking 2-Photon Exchange to the Next Level

To have an imaginary part, a 2-photon diagram must be **cuttable** in a way such that the two resulting lower order diagrams each represent an on-shell, physical process.

New feature in Bhabha: most of the 2-photon diagrams at right involve annihilation such as

$$e+e- \rightarrow \gamma^* \rightarrow e+e- \text{ (+self-energy correction)}$$

or

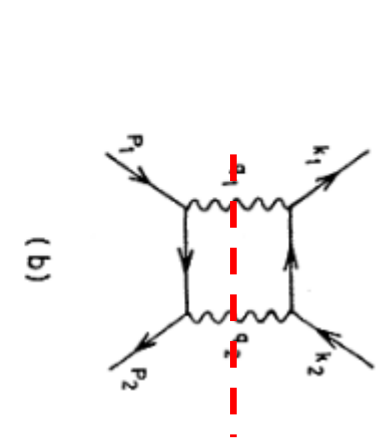
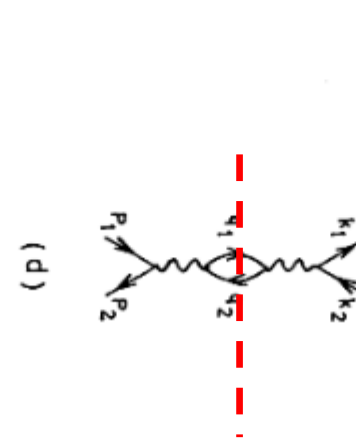
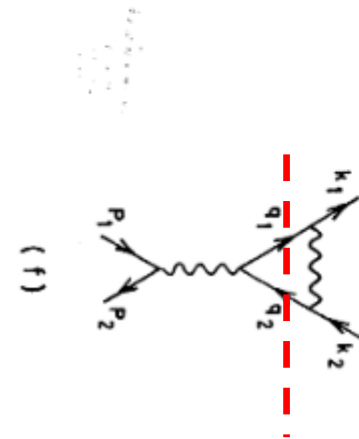
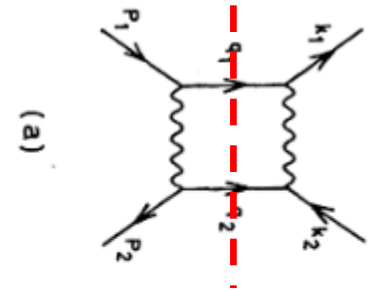
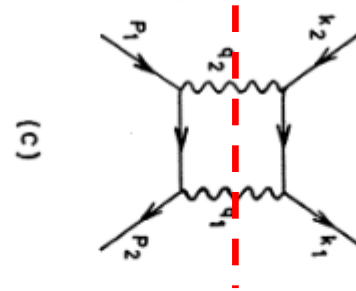
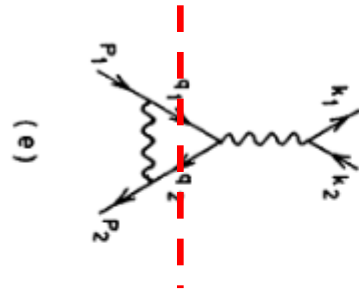
$$e+e- \rightarrow 2\gamma \rightarrow e+e-$$

One diagram even probes a simple vacuum:

$$e+e- \rightarrow \gamma^* \rightarrow (e+e-)_{\text{loop}} \rightarrow \gamma^* \rightarrow e+e-$$

Fronsdal and Jaksic suspected that the BNSSA in Bhabha might be very different than Moller.

They were both right and wrong.



2-photon box diagram as would appear in Moller scattering

e+e-loop diagram

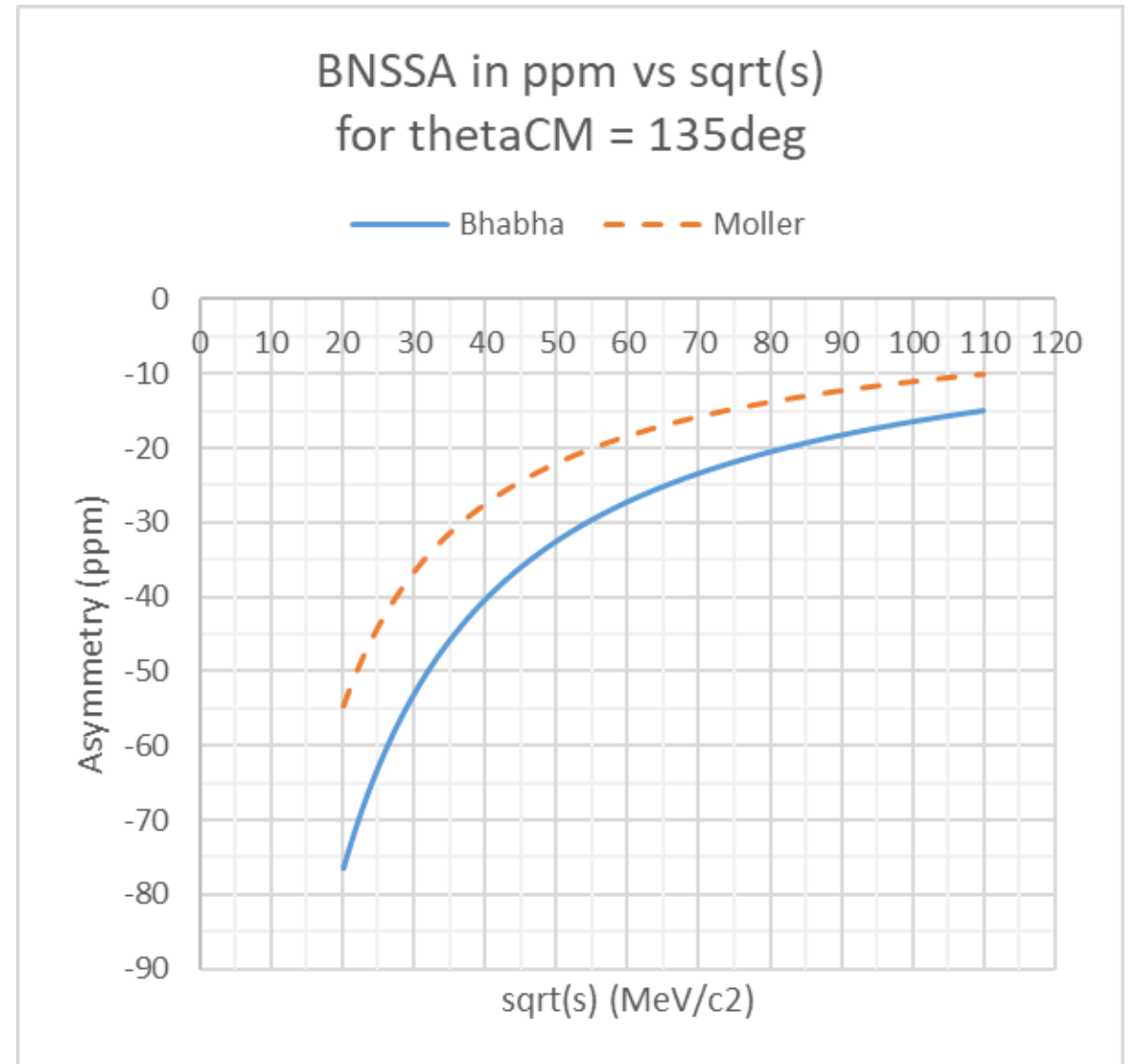
Bhabha Compared to Moeller - Magnitude

Over a wide range of beam energies, the magnitudes of the Bhabha and Moeller distributions (at fixed θ_{CM}) can be comparable.

Boring, right?

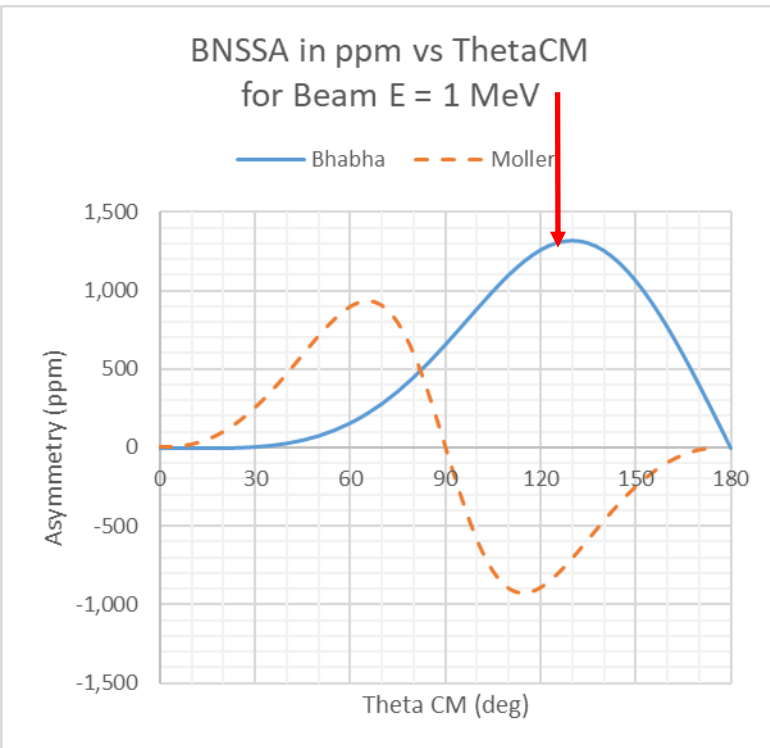
The disappointment in the conclusion of the Fronsdal and Jaksic paper was almost palpable.

But I find that the angular distribution in Bhabha has an interesting energy dependence. (next slide)



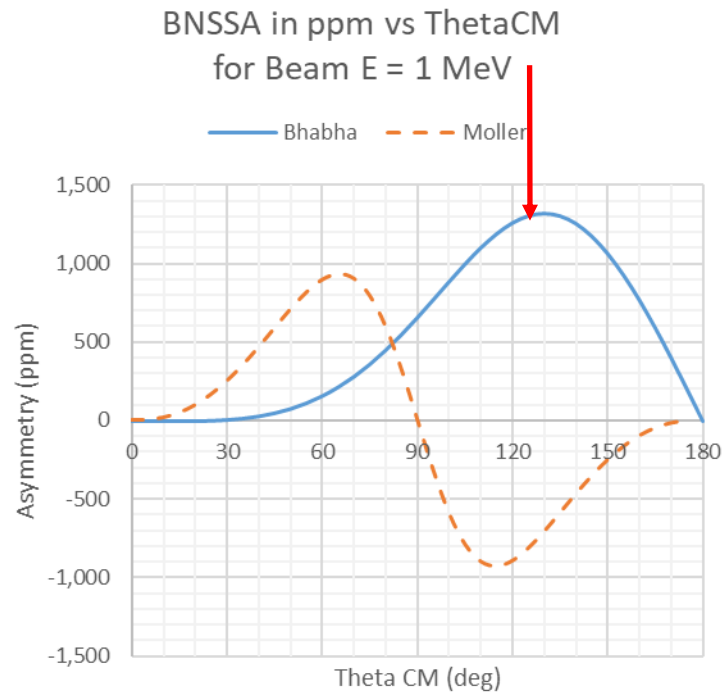
Bhabha Compared to Moeller – Angular Distributions

Non-relativistic

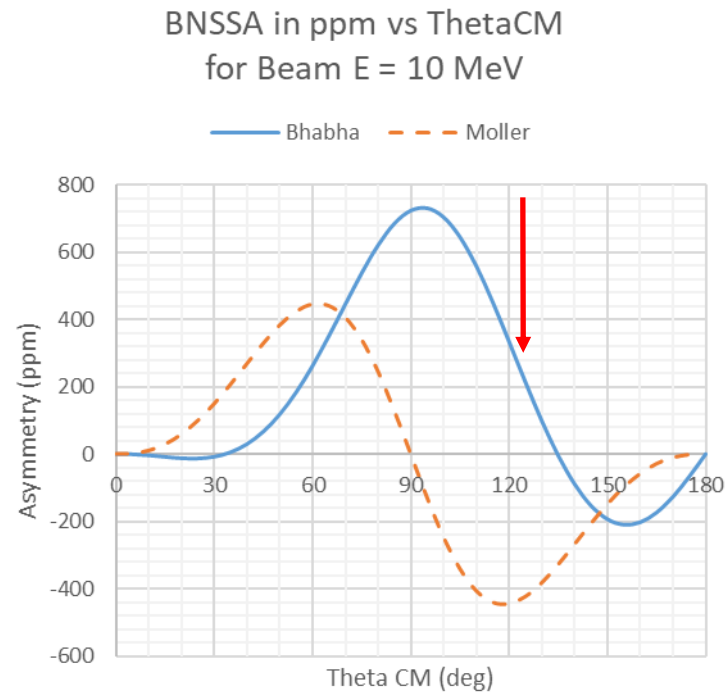


Bhabha Compared to Moeller – Angular Distributions

Non-relativistic

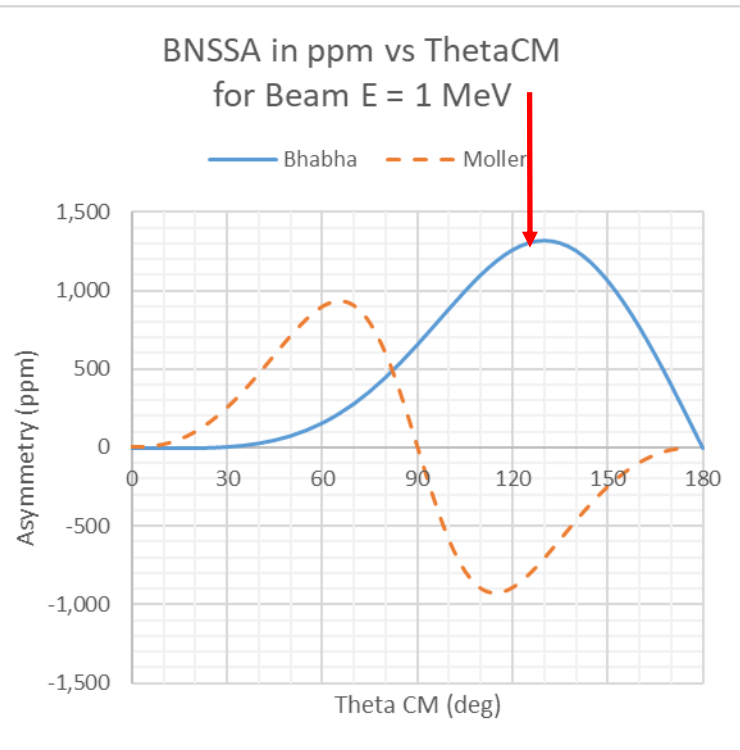


Relativistic

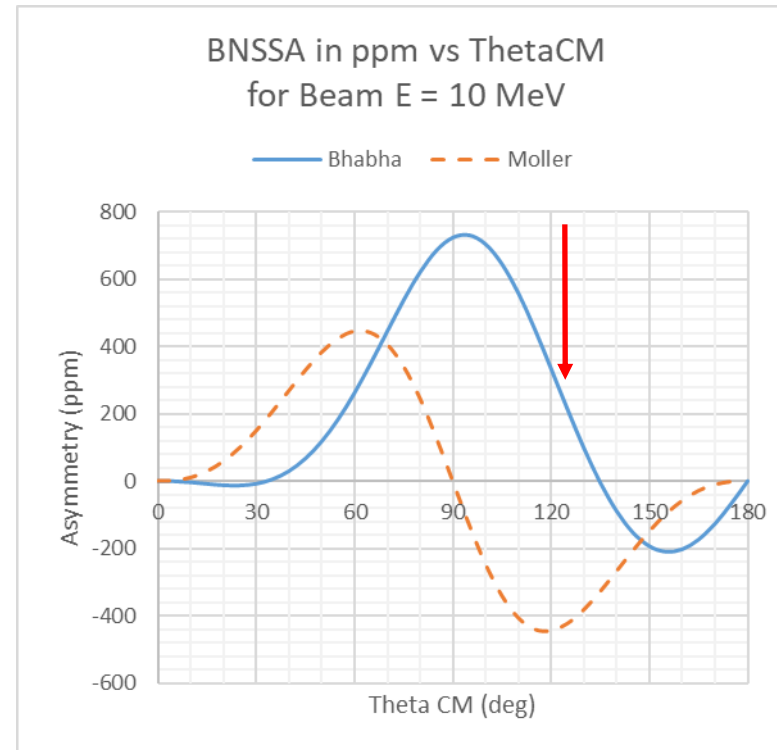


Bhabha Compared to Moeller – Angular Distributions

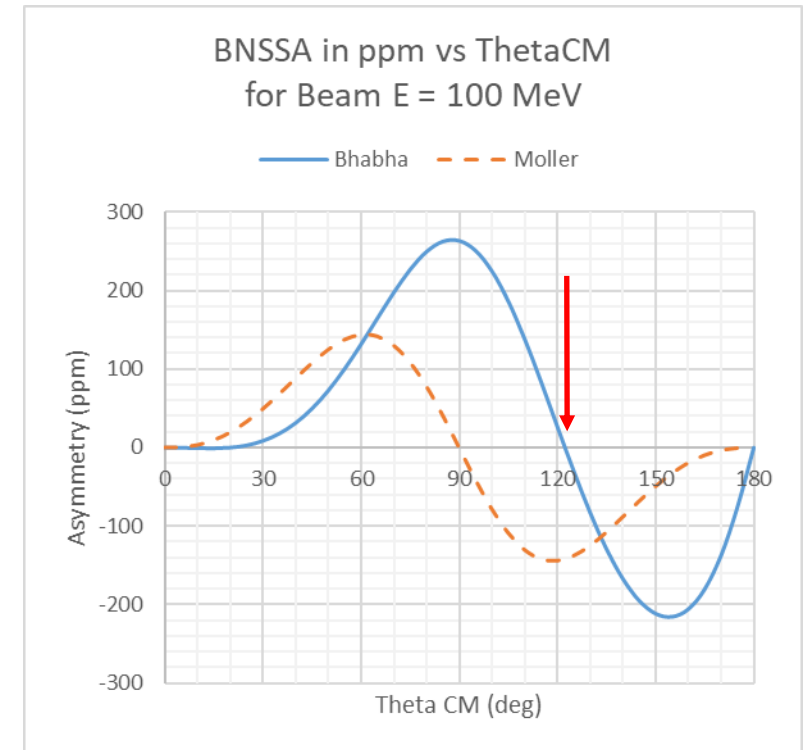
Non-relativistic



Relativistic



Highly relativistic



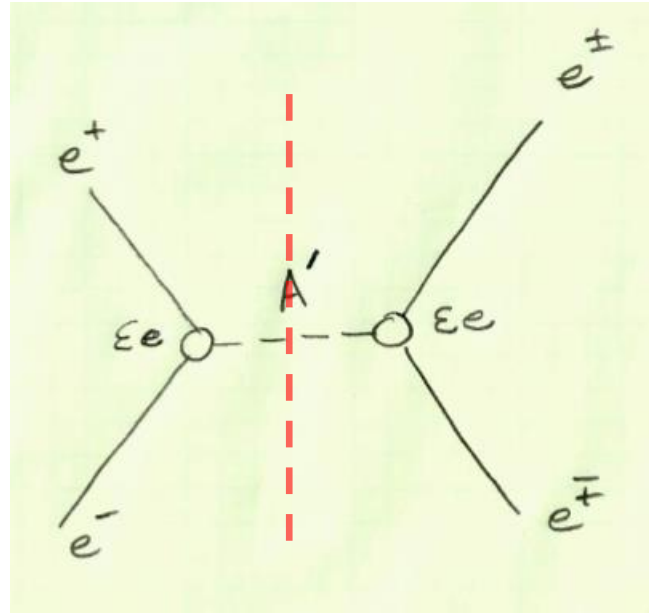
Between the non-relativistic and relativistic regimes, there's dramatic evolution of the shape of the Bhabha angular distribution. The magnitude near $\theta_{CM} \sim 122$ degrees in Bhabha goes from nearly a maximum to a zero crossing. The shape stabilizes when the gamma factor in the CM is > 10 .

By contrast, evolution of the shape of the Moeller angular distribution is subtle.

What Might a Dark Matter Contribution to the BNSSA in Bhabha Look Like?

Although the diagram below doesn't look 2 photon-ish at all, I believe it would satisfy the cut rules and contribute an Imaginary amplitude ... *but only when the measurement is above the threshold for producing an on-shell A' .*

(There is no equivalent $e^+e^- \rightarrow \gamma^* \rightarrow e^+e^-$ contribution because the γ^* is off shell.)

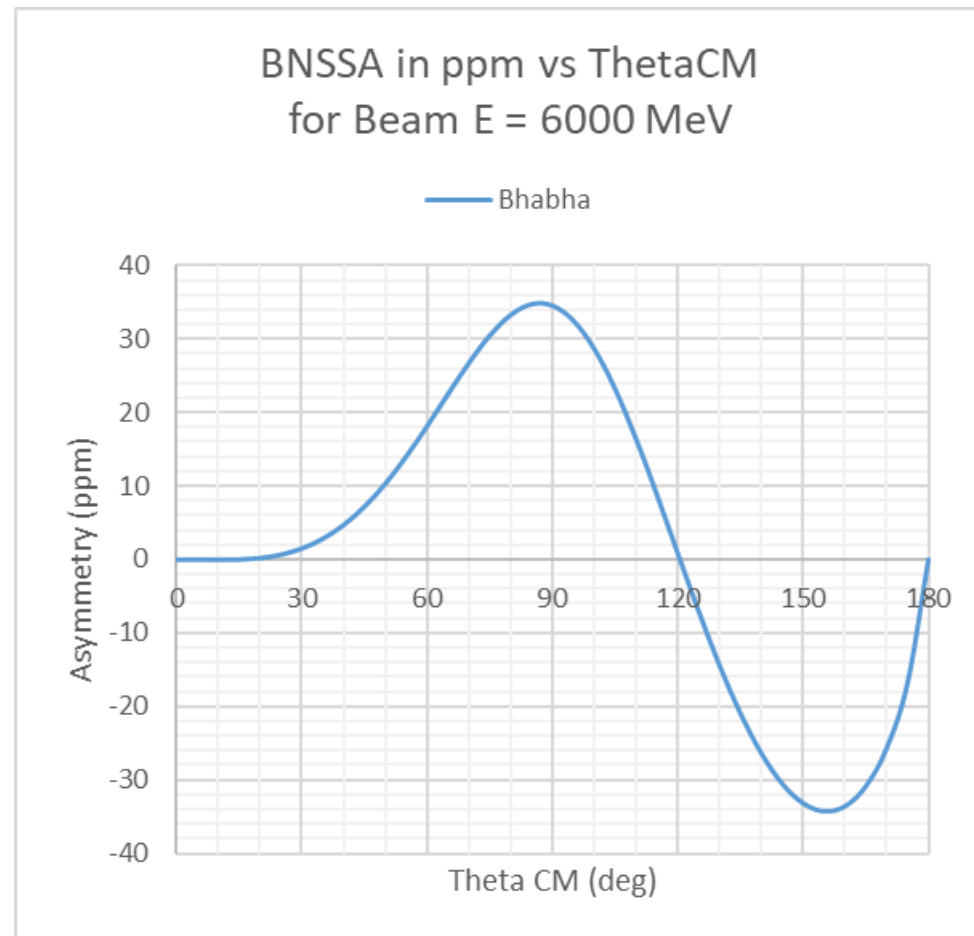


(Higher order diagrams, such as a box diagram or loop with both A' and photon exchange would involve an additional suppression factor of e^2 . These would be the major contributors in the BNSSA in Moeller scattering.)

This seems worth a closer look, because a measurement at \sqrt{s} could seemingly constrain the existence of an A' at all A' masses $< \sqrt{s}$. An experiment with broad mass sensitivity would definitely be worth doing.

Can We Make Use of This Cool QED Observable?

The energy dependence on a previous slide might prove useful. But for now let's assume the highly relativistic scenario where the **zero crossing** is extremely insensitive to beam energy uncertainties. Plotted at 6 GeV beam energy:

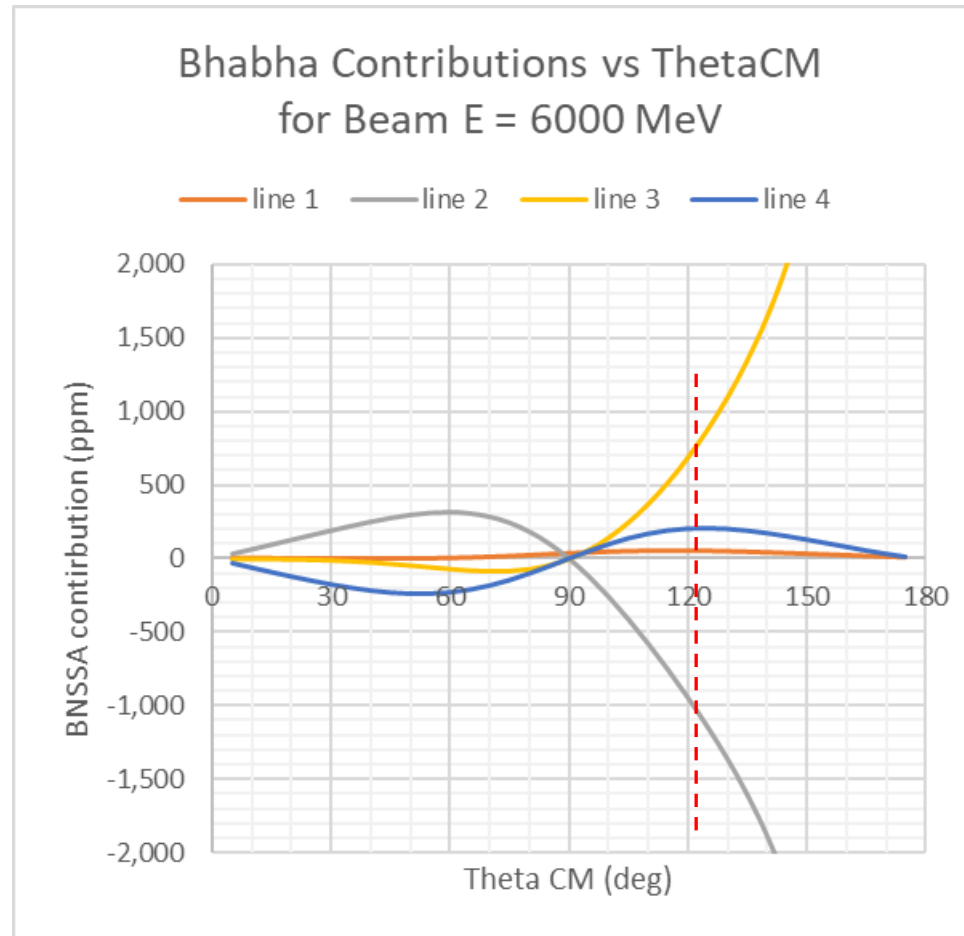


The asymmetry is not small (ie, by the standards of the Jlab parity program).

The location of the zero crossing could in principle be sensitive to the presence of an additional, small BSM amplitude.

Can We Make Use of This Cool QED Observable?

Here, I just split the BNSSA into various contributions (there is not a one-to-one correspondence with diagrams here). You can see how the zero crossing near 120deg arises from interference of two large terms.



The zero crossing is somewhat sensitive: changing one of the larger amplitudes by 1% would shift the zero crossing by 5 deg. (ie, 5deg/%)

My impression is that one could do exclusions at the level of only $\epsilon = 1E-2$.

But because the BNSSA is sensitive to the Imaginary amplitude, this would in principle exclude the existence of all lighter A's with a coupling greater than this straw-man $\epsilon = 1E-2$.

It would be nice if theorists could add the lowest order A' contribution to a BNSSA calculation for Bhabha scattering so rigorous sensitivity studies could be done. Maybe a nice Master's thesis project?

Overall Summary

The exchange of a virtual A' could indeed induce measurable, amplitude-level effects in Bhabha scattering. The examined observables were:

Observable	Comment	But	Theory community help needed on ...
Yield	Potentially very sensitive. Line shape is bi-polar.	What are the details of the normalization, and how to design an experiment that won't require 100 beam energies lol?	So far, the formalism seems to exist.
A_{LU}^{PV}	Potentially very sensitive. Line shape is bi-polar.	How thick can one make the target in a current-mode $e+e- \rightarrow e- (e+)$ current mode measurement without washing out the signal?	An overview paper on how/whether Bhabha could improve existing PV constraints is probably needed.
A_{TU}^{PC} (ie, the BNSSA)	Probably not terribly sensitive, but constraints would apply to a wide range of masses. Signal is a step change in the BNSSA at A' threshold.	How accurately can one reconstruct θ_{CM} from the scattered $e+$ and $e-$?	Design of an experiment will require a BNSSA containing at least the lowest order A' diagram.
Below the fold:			
A_{LL}^{PC}, A_{TT}^{PC}	Relatively insensitive due to dilution in the magnetized Fe foil. Line shape is bi-polar.	No problem with normalization though!	N/A
A_{TT}^{PV}	The line shape would be a robust bump (or a divot) proportional to $\text{Im } f(s)$.	I haven't figured out how to separate the PC $\cos(2\phi)$ and PV $\sin(2\phi)$ contributions.	Suggestions welcome. This is my white whale.

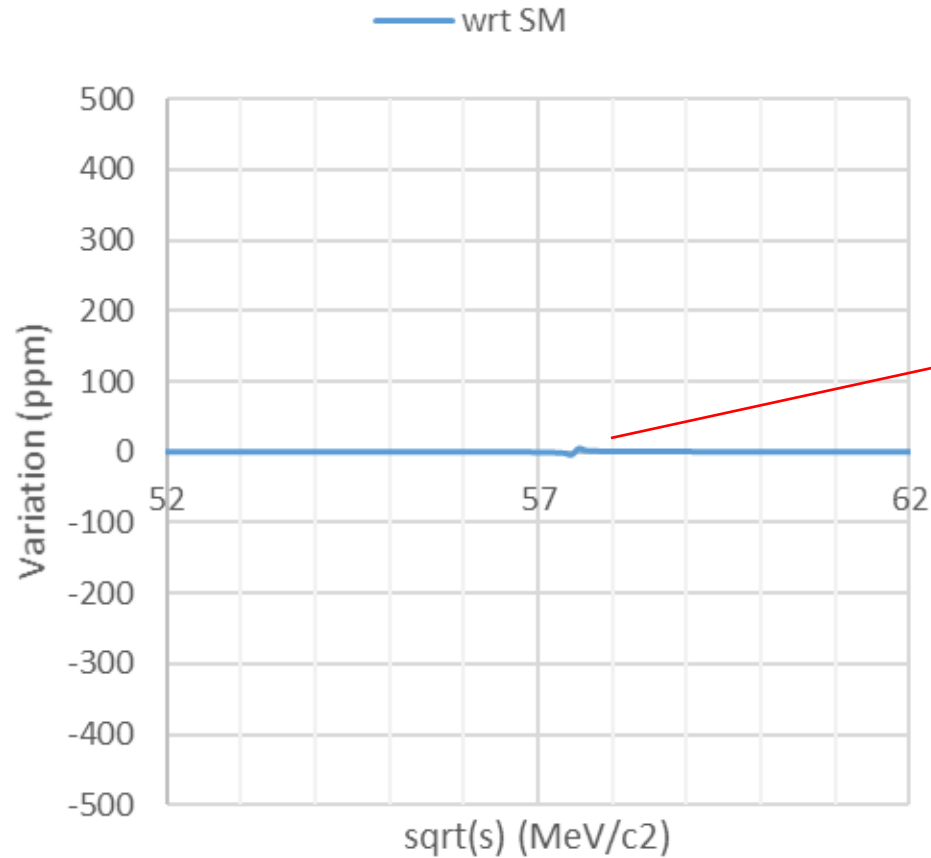
extras

Yield Signals Compared

($\epsilon = 1E-4$ and $1E-3$ on same vertical scale)

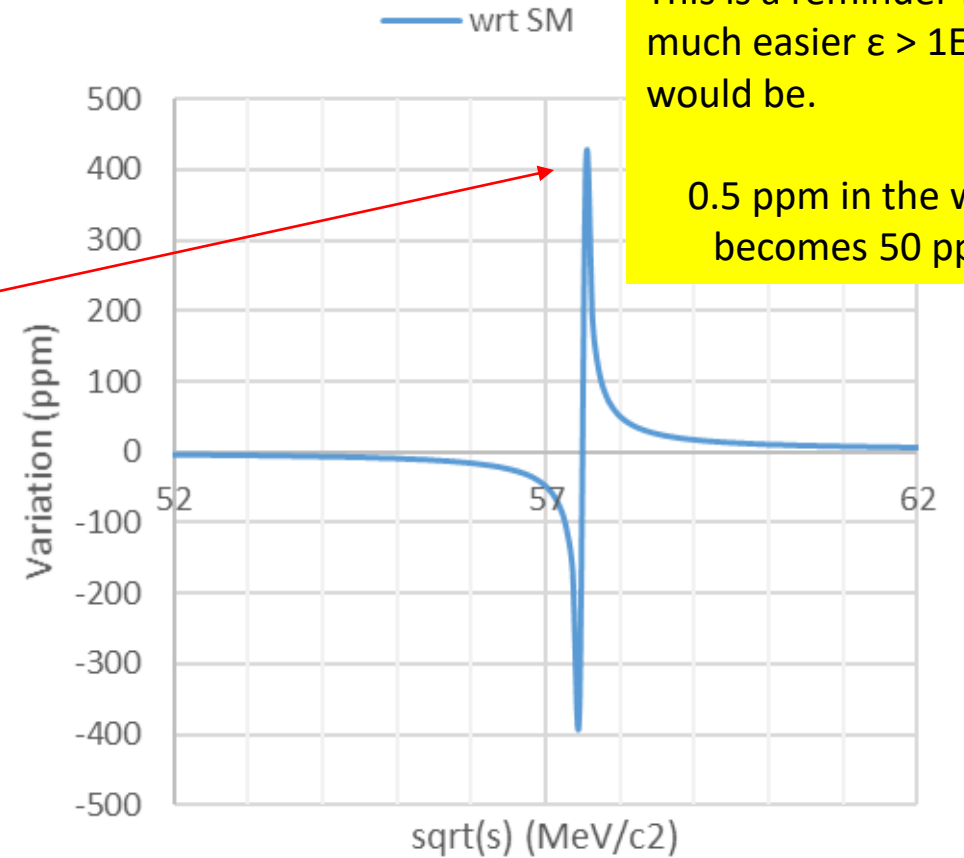
$\epsilon = 1E-4$

Relative Variation vs sqrt(s)
in ppm at 150deg CM



$\epsilon = 1E-3$

Relative Variation vs sqrt(s)
in ppm at 150deg CM



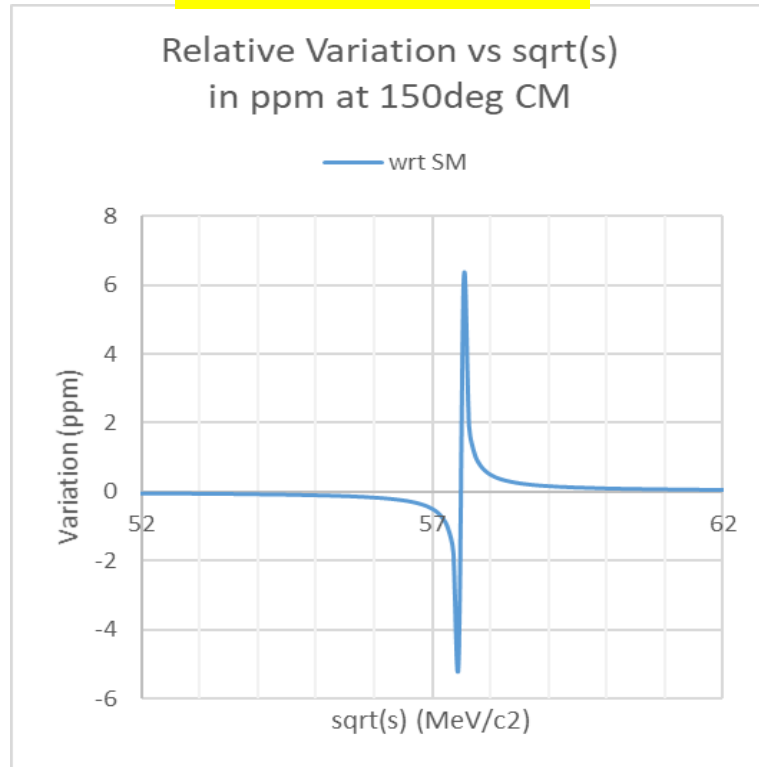
This is a reminder how much easier $\epsilon > 1E-3$ would be.

0.5 ppm in the wings becomes 50 ppm!

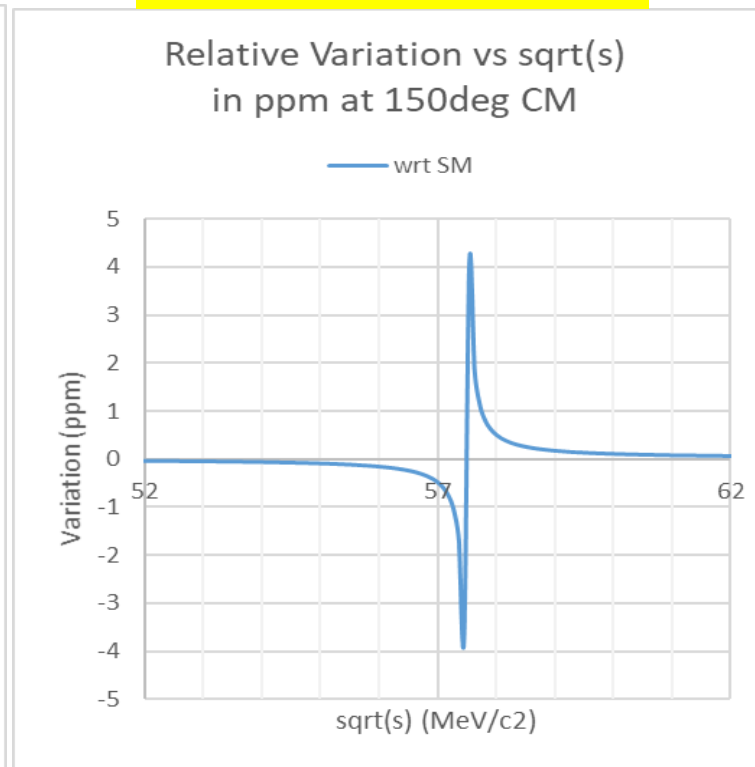
Yield Signal Dependence on the Decay Width

(Purely Vector Coupling, $\epsilon = 1E-4$, $M_{A'} = 57.5 \text{ MeV}/c^2$)

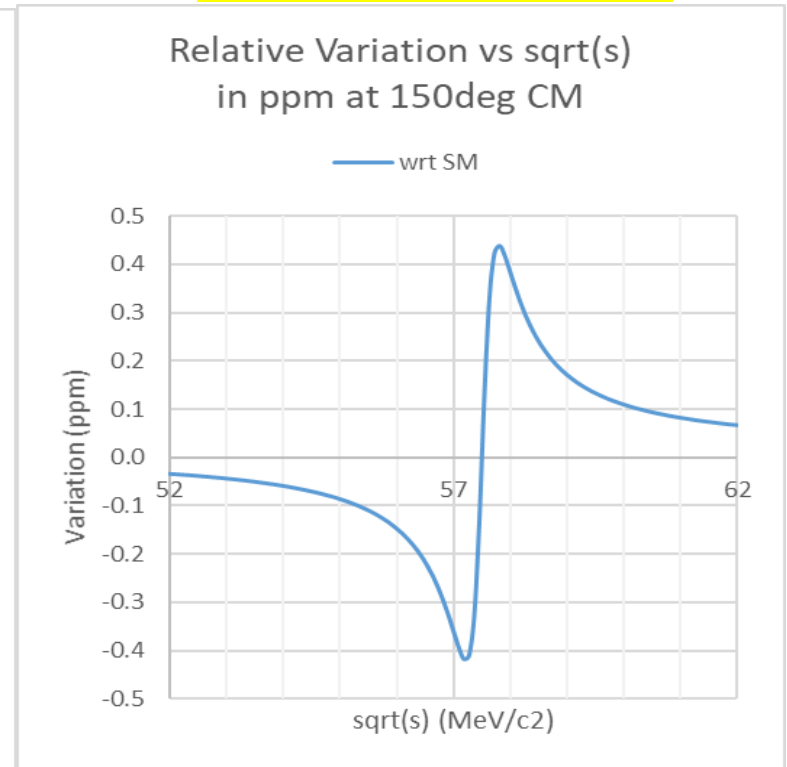
Width = 0% (0 keV)



Width = 0.1% (57.5 keV)



Width = 1% (57.5 keV)



Signal amplitudes get smaller with increasing decay width.
(But they also get broader in a way that seems to roughly preserve the area of the deviation.)

Thinking About the Problem its Figure of Merit

The stretch goal is to design experiments in the JLab energy range which exclude $\epsilon > 1\text{E-}4$.

This is 10x lower than existing limits of $\epsilon > 1\text{E-}3$. Let me remind you of how hard that is*:

- a hypothetical amplitude-level signal will be 100x smaller,
- an amplitude-based experiment will require $100^2 = 10,000$ times more Statistical Figure of Merit (FOM)

All other things being equal, an amplitude-based experiment which could exclude $\epsilon > 1\text{E-}4$ in 1 year of running would be able to reproduce existing $1\text{E-}3$ exclusions in less than 1 hour.

*For an amplitude signal which scales like " ϵ^2 ", the exclusion improves as $1/\text{FOM}^{1/4}$.

For a signal which scales like " ϵ^4 ", as in a dark Bremsstrahlung $M(e+e-)$ search, the exclusion improves as only as $1/\text{FOM}^{1/8}$.

Thinking About the Problem in a Figure of Merit

The stretch goal is to design experiments in the JLab energy range which exclude $\epsilon > 1E-4$.

This is 10x lower than existing limits of $\epsilon > 1E-3$. Let me remind you of how hard that is*:

- a hypothetical amplitude-level signal will be 100x smaller,
- an amplitude-based experiment will require $100^2 = 10,000$ times more Statistical Figure of Merit (FOM)

All other things being equal, an amplitude-based experiment which could exclude $\epsilon > 1E-4$ in 1 year of running would be able to reproduce existing $1E-3$ exclusions in less than 1 hour.

1 extra order of magnitude in ϵ would be a generational improvement. That is the reason why this gap is tough to fill, and why much of it may be still there in a decade.

*For an amplitude signal which scales like “ ϵ^2 ”, the exclusion improves as $1/\text{FOM}^{1/4}$.

For a signal which scales like “ ϵ^4 ”, as in a dark Bremsstrahlung $M(e+e^-)$ search, the exclusion improves as only as $1/\text{FOM}^{1/8}$.

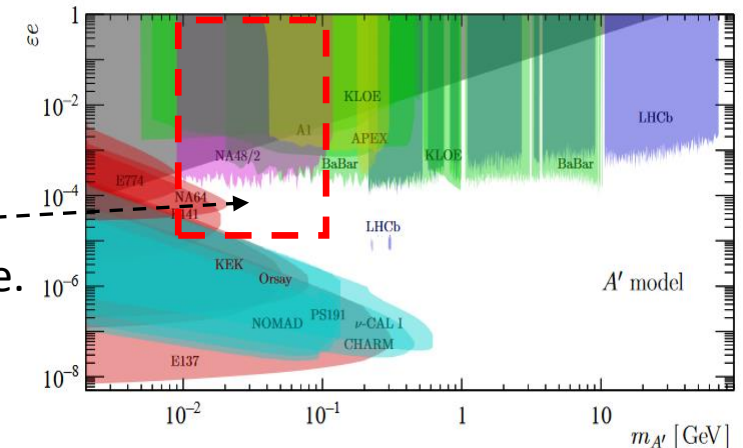


Figure 4. Constraints on visible A' decays considered in this study from (red) electron beam dumps, (cyan) proton beam dumps, (green) e^+e^- colliders, (blue) pp collisions, (magenta) meson decays, and (yellow) electron on fixed target experiments. The constraint derived from $(g-2)_e$ is shown in grey [90, 91].

Yield vs \sqrt{s} Kinematics Comments

Yield vs \sqrt{s} in $e^+e^- \rightarrow e^+e^-$:

Flexible as far as event-mode vs current-mode, or singles vs coincidence techniques.

Reaching sensitivity as low as $\epsilon \sim 1e-4$ level would probably require current-mode.

Big downside: to cover a large mass range, this would require a fine-toothed energy scan of the accelerator

Yield vs \sqrt{s} in $e^+e^- \rightarrow e^+e^- + \text{gamma}$ (ie, with Initial State Radiation):

Requires e^+e^- coincidence to determine \sqrt{s} , so would have to be event-mode.

Covers a larger range of masses, so needs fewer accelerator energies.

Downside: would have to settle for weaker exclusions ($\epsilon \sim 3e-4$) due to constraints on event-mode daq rate and limits on human patience.

A_{pV} Experiment Summary

A_{pV} in $e^+e^- \rightarrow e^+e^- + \gamma$ (ie, $e^+e^- \rightarrow e^+e^-$ with Initial State Radiation)

One would hope to do several measurements, covering the Jlab sqrt(s) range, each with 10 ppb statistical sensitivity.

→ Must be integrating mode measurement

→ no coincidence is possible, so need to choose which single particle to detect

→ e^- detection will likely have the lowest background (since the beam is e^+)

So there'd be no Mott scattering background (i.e., no $e^+ + A \rightarrow e^+ + A$)

Note that because the tree-level A_{pV} is naturally suppressed by the factor $1-4\sin^2\theta_W \sim 0.075$, then EW corrections are likely to make an O(100)% correction to A_{pV} , just as they do in Moller scattering.

I'm not sure yet how much theoretical support is needed here:

- i. I'm believe I can find those EW corrections for Bhabha from the old high energy collider literature, and
- ii. combine it with more recent work on the running of $\sin^2\theta_W$ to low energies scales, and
- iii. it appears that the ultra-relativistic approximation that $m_e^2 \ll$ the magnitude of s, or u, or t isn't too bad.
- iv. Bhabha is certainly sensitive, but it's not clear to me to what extent a realistic experiment would improve on constraints from E158 and Moller. A detailed study might be worth a theory paper.

Contributions to the Bhabha Xsect: s channel

The unpolarized xsect is proportional to α^2 :

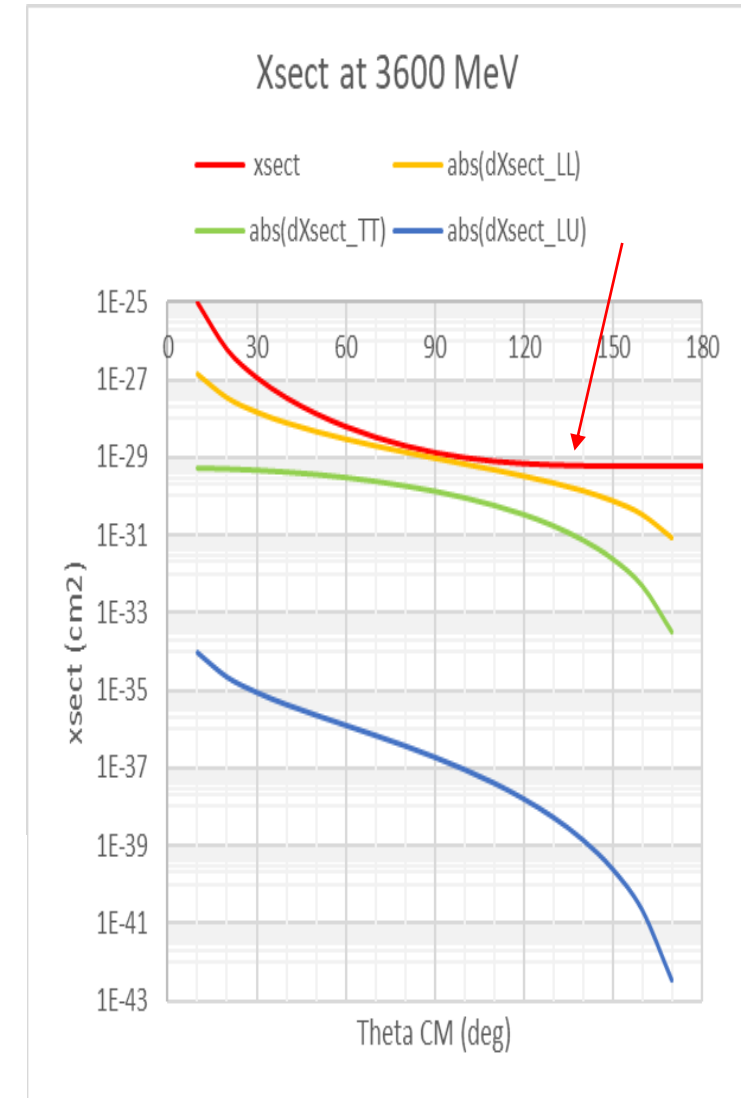
$$\frac{d\sigma_0^B}{d\Omega} = \frac{\alpha^2}{4s} \left\{ \cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1+f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1+f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] + 2 \sin^4(\theta/2) \left| 1 + f(s)g_R g_L \right|^2 + [2/\sin^4(\theta/2)] [1+f(t)g_R g_L]^2 \right\}, \quad (14)$$

(Polarized xsect differences can be defined from the asymmetries in Eqns 15-18.)

f(t) is for spacelike Z and is purely real. These terms tend to diverge as $\theta \rightarrow 0$ deg, which will dilute any interesting A' effects that we add to the s-channel.

f(s) is for time-like Z, has a Real part and an Imaginary part. Generally, effects from a resonant A' will be largest at backward angles (see red arrow at right, pointing to a "shelf" in the xsect).

$$f(q^2) = \begin{cases} \frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2 + iM_Z \Gamma_Z^{\text{tot}}}, & q^2 > 0 \text{ (q timelike), i.e., f(s)} \\ \frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2}, & q^2 \leq 0 \text{ (q spacelike). i.e., f(t)} \end{cases} \quad (12)$$



Contributions to the Bhabha Xsect: t channel

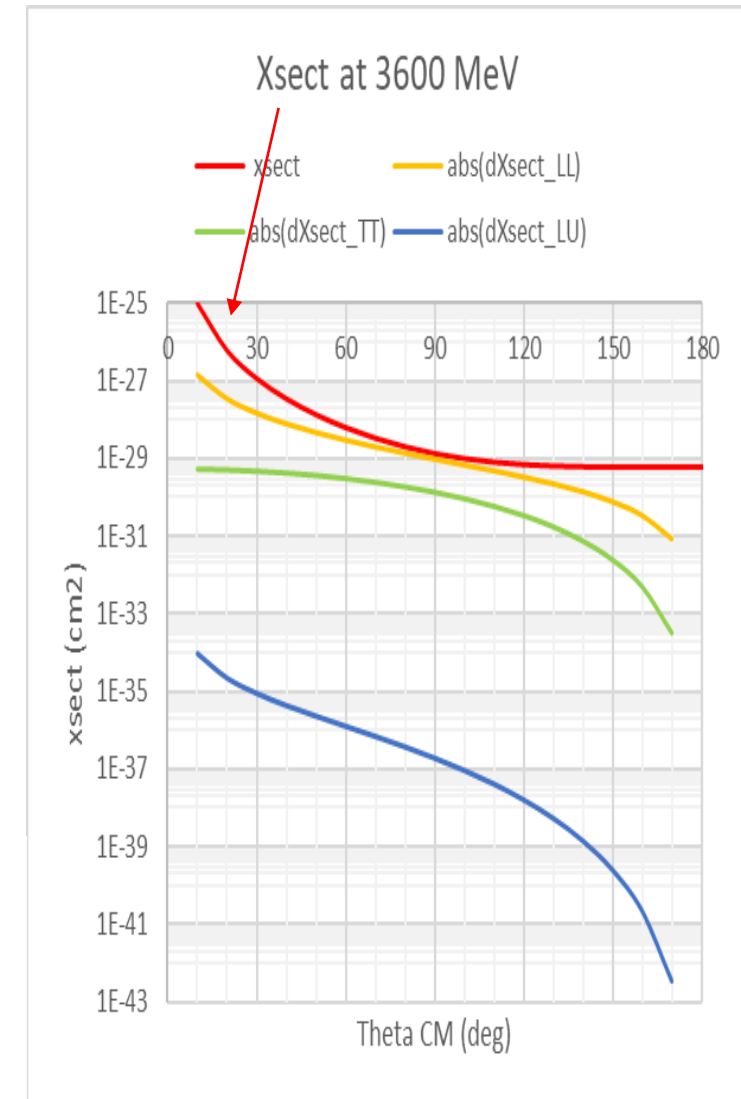
The unpolarized xsect is proportional to α^2 :

$$\frac{d\sigma_0^B}{d\Omega} = \frac{\alpha^2}{4s} \left\{ \cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1 + f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1 + f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] + 2 \sin^4(\theta/2) |1 + f(s)g_R g_L|^2 + [2/\sin^4(\theta/2)] [1 + f(t)g_R g_L]^2 \right\}, \quad (14)$$

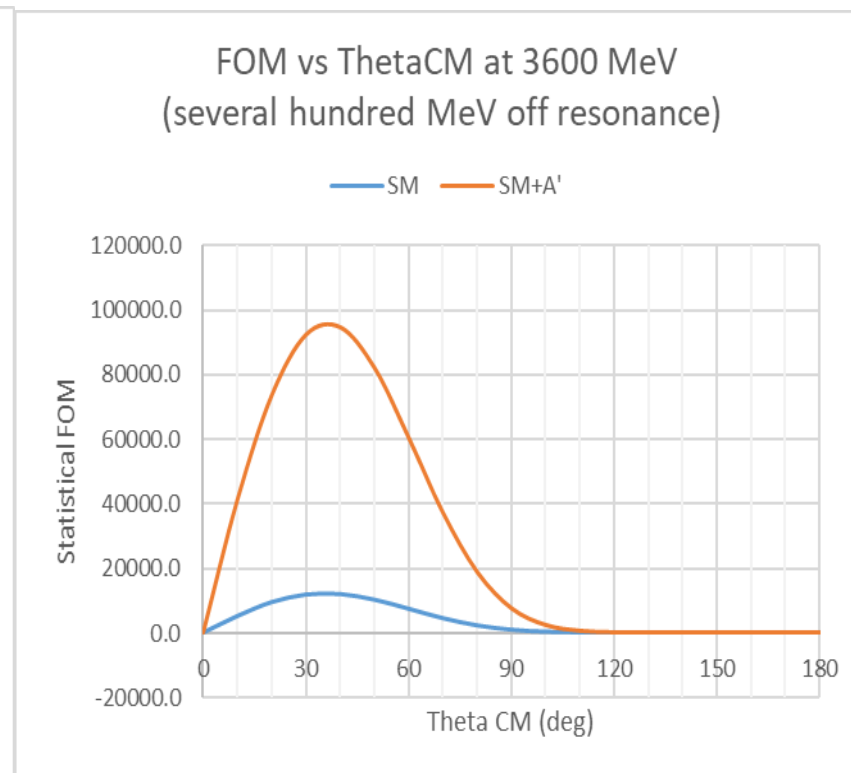
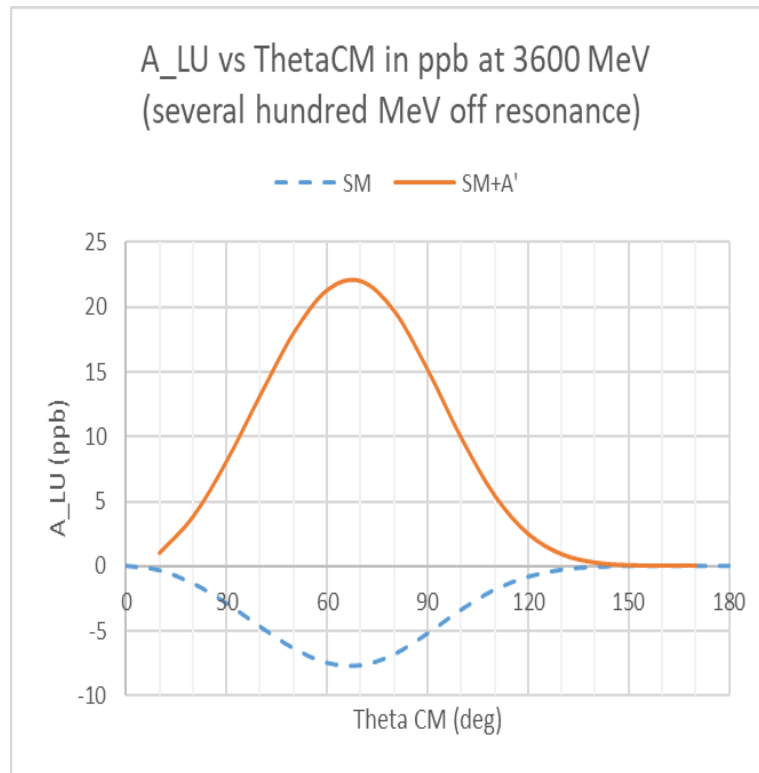
(Polarized xsect differences can be defined from the asymmetries in Eqns 15-18.)

$f(t)$ is for spacelike Z and is purely real. These terms tend to diverge as $\theta \rightarrow 0$ deg, which will dilute any interesting A' effects in the s-channel.

$$f(q^2) = \begin{cases} \frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2 + iM_Z \Gamma_Z^{\text{tot}}}, & q^2 > 0 \text{ (q timelike), i.e., } f(s) \\ \frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2}, & q^2 \leq 0 \text{ (q spacelike). i.e., } f(t) \end{cases} \quad (12)$$



$$A_{PV}: g_A = g_V = 1, \epsilon = 1E-4, M_{A'} = 57.5 \text{ MeV}/c^2$$



Same plot from a few slides ago.
With the assumed couplings, A_{PV} peaks around 65deg in CM.

With the assumed couplings, the statistical
 $FOM = Rate * A_{PV}^2$ peaks near 35 degCM
where rates are 1 GHz.

Issues:

- Can a spectrometer detecting only the e- in current mode provide crude binning in sqrt(s)?
- How much will ISR extend the sqrt(s) range covered by a single beam energy?
- Looking at the rates without ISR, it seems one would be able to complete at least one $O(10)$ ppb measurement in a calendar year. (100 nA on a 10% RL LH2 target)
- Will the accelerator have trouble delivering positron beams as low as 1 GeV in energy?

Purely Vector vs Purely Axial-Vector Couplings

There is some literature on BSM particles which have significant axial-vector couplings. It's easy to explore that in this formalism.

PREPARED FOR SUBMISSION TO JHEP
FERMILAB-PUB-16-385-PPD, UCI-HEP-TR-2016-15, MITP/16-098, PUPT 2507

Light Weakly Coupled Axial Forces: Models, Constraints, and Projections

Yonatan Kahn,^a Gordan Krnjaic,^b Siddharth Mishra-Sharma,^a and Tim M.P. Tait^c

^aPrinceton University,
Princeton, NJ USA

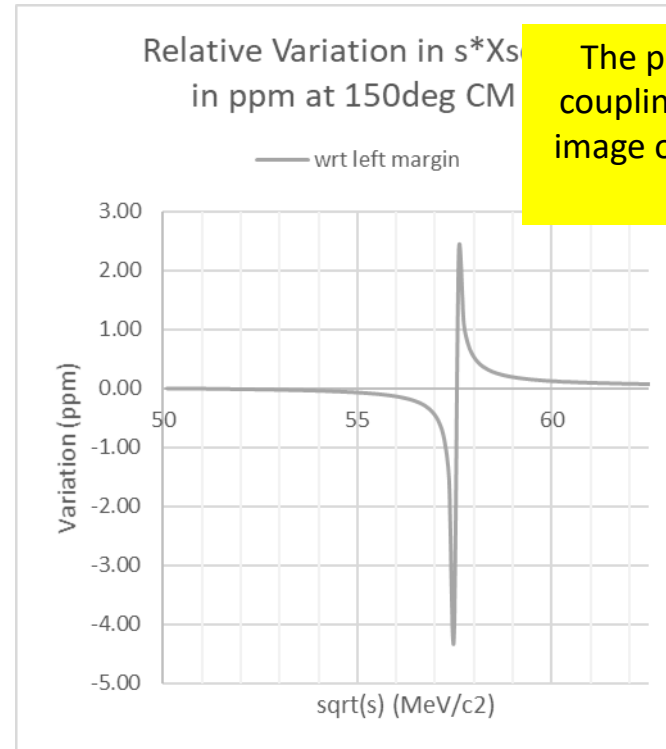
^bFermi National Accelerator Laboratory,
Batavia, IL USA

^cUniversity of California, Irvine,
Irvine, CA USA

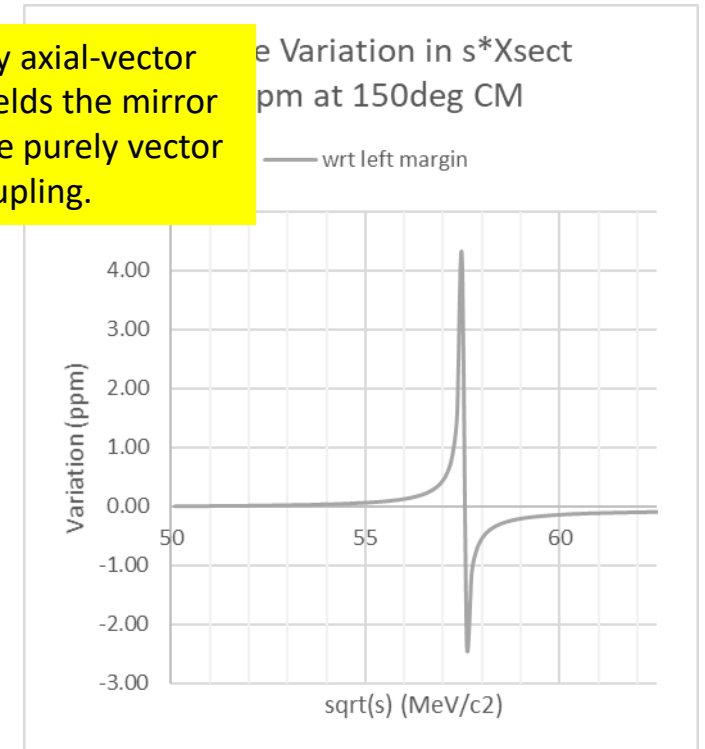
E-mail: ykahn@princeton.edu, krnjaicg@fnal.gov, smsharma@princeton.edu, ttait@uci.edu

ABSTRACT: We investigate the landscape of constraints on MeV-GeV scale, hidden $U(1)$ forces with nonzero axial-vector couplings to Standard Model fermions. While the purely vector-coupled dark photon, which may arise from kinetic mixing, is a well-motivated scenario, several MeV-scale anomalies motivate a theory with axial couplings which can be UV-completed consistent with Standard Model gauge invariance. Moreover, existing constraints on dark photons depend on products of various combinations of axial and vector couplings, making it difficult to isolate the effects of axial couplings for particular flavors of SM fermions. We present a representative renormalizable, UV-complete model of a dark photon with adjustable axial and vector couplings, discuss its general features, and show how some UV constraints may be relaxed in a model with nonrenormalizable Yukawa couplings at the expense of fine-tuning. We survey the existing parameter space and the projected reach of planned experiments, briefly commenting on the relevance of the allowed parameter space to low-energy anomalies in π^0 and $^8\text{Be}^*$ decay.

Purely vector



Purely axial-vector



The purely axial-vector coupling yields the mirror image of the purely vector coupling.

In the purely vector or purely axial-vector scenario, terms proportional to $g_v \cdot g_a$ vanish, leaving a $g_v^2 - g_a^2$ term which switches sign.

Modifying the Formalism to Turn a Z' into an A'

Z

Mass = 98,187.6 MeV/c²
Width = 2495 MeV/c²

$g_a = -1$
 $g_v = -0.0748$

$g_l = g_v + g_a$
 $g_r = g_v - g_a$

$4\sin^2(2\theta_W) = 2.845$
(a normalization factor in the propagator)

A'

Mass = 57.5 MeV/c²
Width = 0.575 MeV/c² (for example)

$g_a = 0$
 $g_v = 1$

$g_l = (g_v + g_a) \cdot \epsilon$
 $g_r = (g_v - g_a) \cdot \epsilon$

I just lamely set this to 1 for now.

$$\frac{d\sigma_0^B}{d\Omega} = \frac{\alpha^2}{4s} \left\{ \cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1+f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1+f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] + 2 \sin^4(\theta/2) |1 + f(s)g_R g_L|^2 + [2/\sin^4(\theta/2)][1 + f(t)g_R g_L]^2 \right\}, \quad (14)$$

A_LL (PC) →

$$A_1^B = \left\{ -\cos^4(\theta/2) \left[\left| 1 + f(s)g_L^2 - \frac{1+f(t)g_L^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s)g_R^2 - \frac{1+f(t)g_R^2}{\sin^2(\theta/2)} \right|^2 \right] - 2 \sin^4(\theta/2) |1 + f(s)g_R g_L|^2 + [2/\sin^4(\theta/2)][1 + f(t)g_R g_L]^2 \right\} \left/ \left[\frac{4s}{\alpha^2} \frac{d\sigma_0^B}{d\Omega} \right] \right., \quad (15)$$

A_LU (PV) →

$$A_2^B = \cos^4(\theta/2)(g_R^2 - g_L^2) \times \text{Re} \left\{ \left[f(s) - \frac{f(t)}{\sin^2(\theta/2)} \right] \left[2 + f^*(s)(g_R^2 + g_L^2) - \frac{2 + f(t)(g_R^2 + g_L^2)}{\sin^2(\theta/2)} \right] \right\} \left/ \left[\frac{4s}{\alpha^2} \frac{d\sigma_0^B}{d\Omega} \right] \right., \quad (16)$$

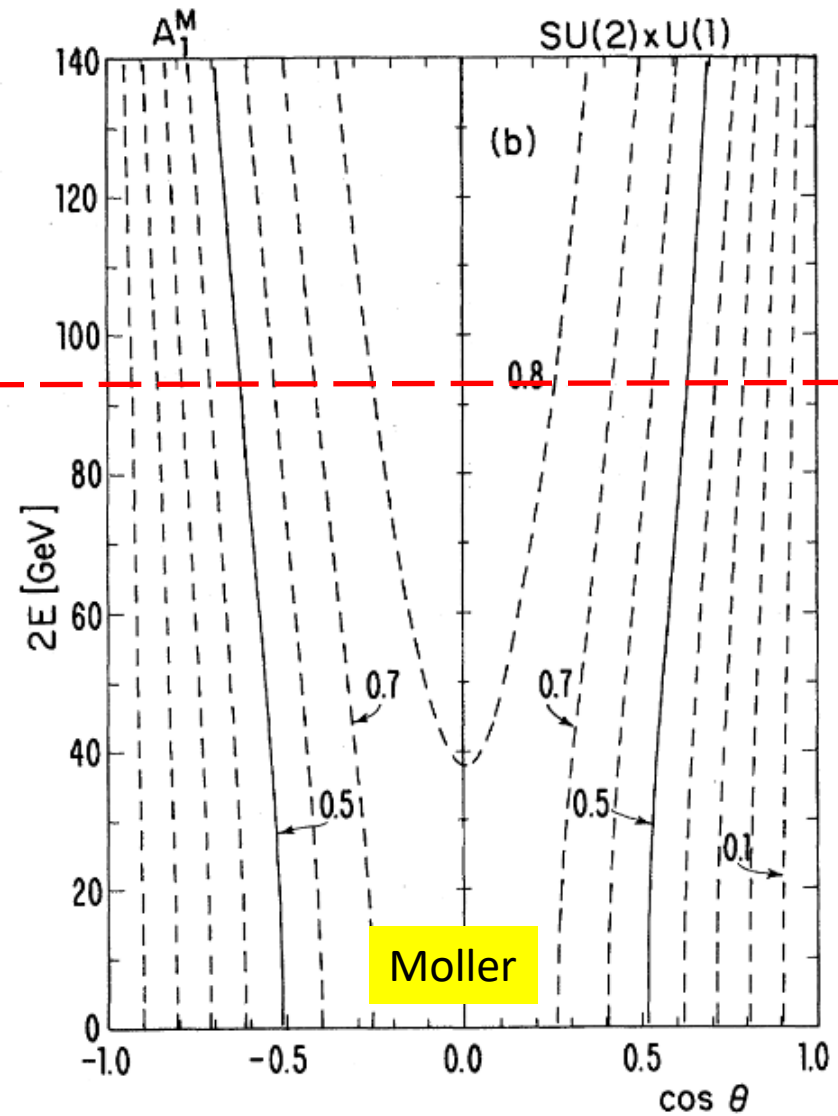
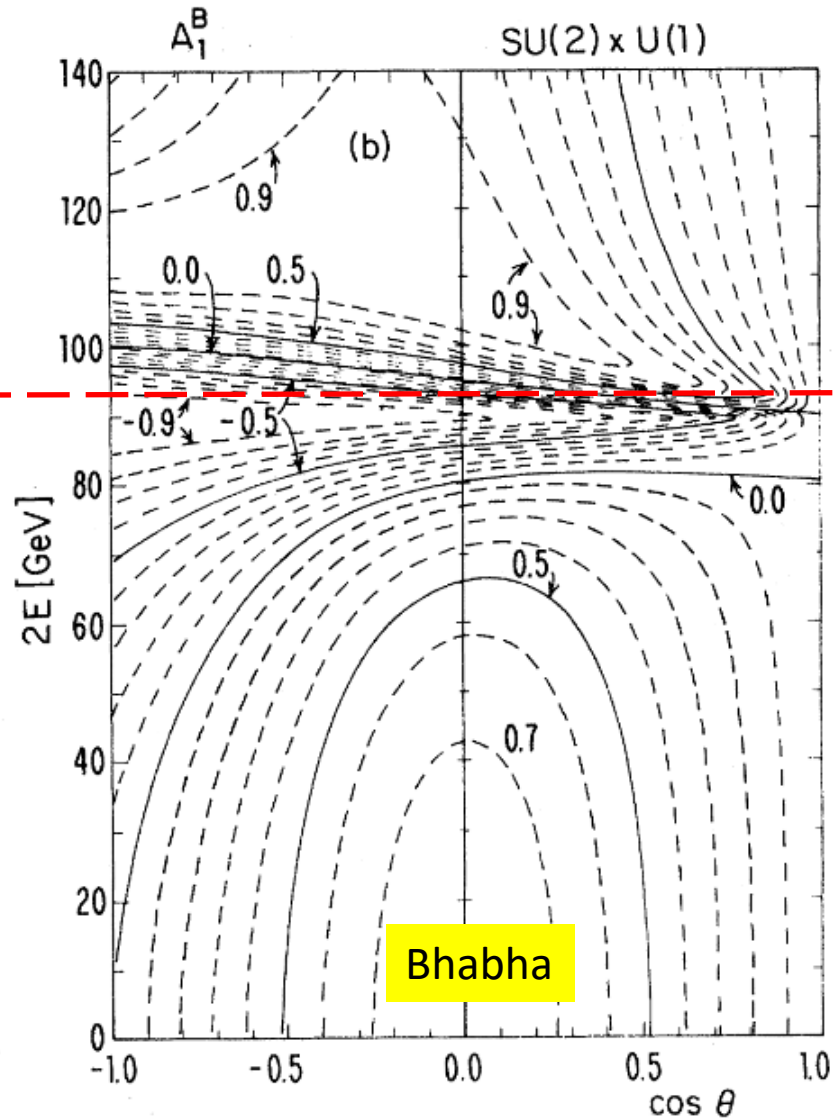
A_TT (PC) →

$$A_3^B = 2 \sin^2(\theta/2) \cos^2(\theta/2) \times \text{Re} \left\{ [1 + f(s)g_R g_L] \left[2 + f^*(s)(g_R^2 + g_L^2) - \frac{2 + f(t)(g_R^2 + g_L^2)}{\sin^2(\theta/2)} \right] \right\} \left/ \left[\frac{4s}{\alpha^2} \frac{d\sigma_0^B}{d\Omega} \right] \right., \quad (17)$$

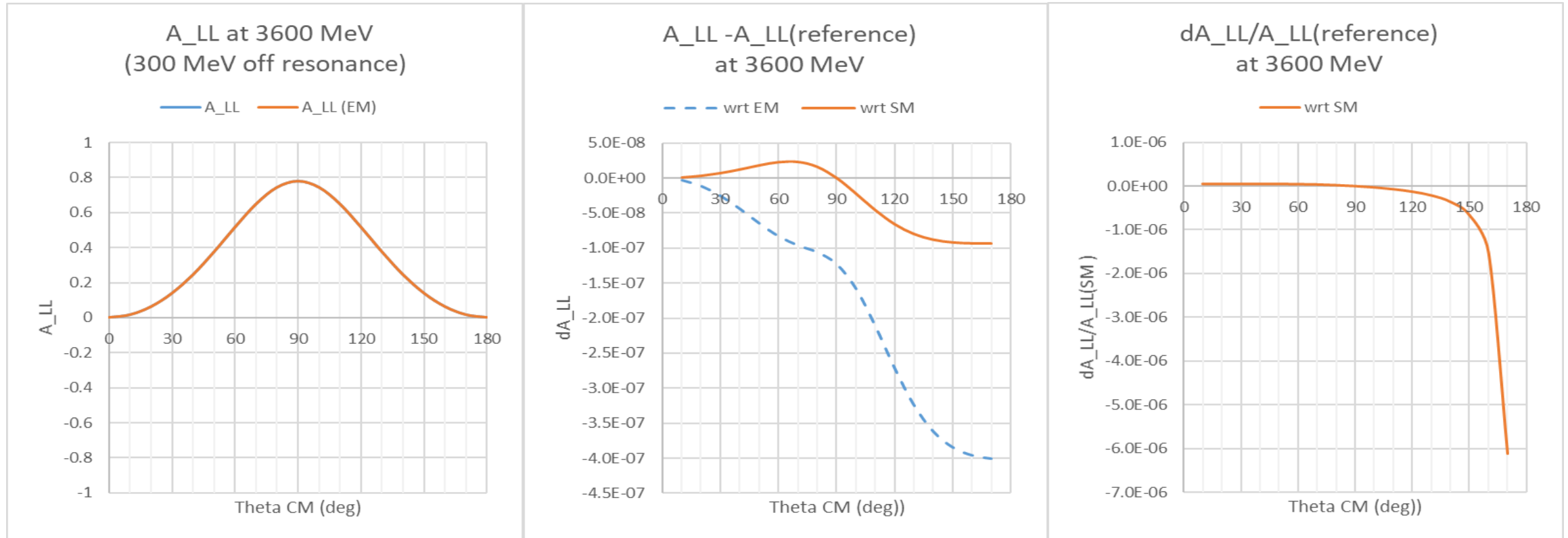
A'_TT (PV) →

$$A_4^B = -2 \sin^2(\theta/2) \cos^2(\theta/2) (g_R^2 - g_L^2) \times \text{Im} f(s) \left[1 + \frac{f(t)g_R g_L}{\sin^2(\theta/2)} \right] \left/ \left[\frac{4s}{\alpha^2} \frac{d\sigma_0^B}{d\Omega} \right] \right.. \quad (18)$$

A_{LL} Up to $E_{cm} = 140 \text{ GeV}/c^2$



A_{LL} : Purely Vector Coupling, $\epsilon = 1E-4$, $M_{A'} = 57.5 \text{ MeV}/c^2$



Again, on this plotting scale, the A' effects are invisibly small.

Near 180deg, ALL is not zero due to a surprisingly large PC contribution from Z exchange. Even this far off resonance, the A' can shift that result by O(100)%.

But the relative change in the asymmetry is $\ll 1\%$ so impossible to measure.

Moving on!

Dilution in A_LL and A_TT experiments

Dave Gaskell says the electron's in the Fe foil target are ~8% polarized.