

GMn Analysis Progress



Provakar Datta

(On behalf of the SBS collaboration)

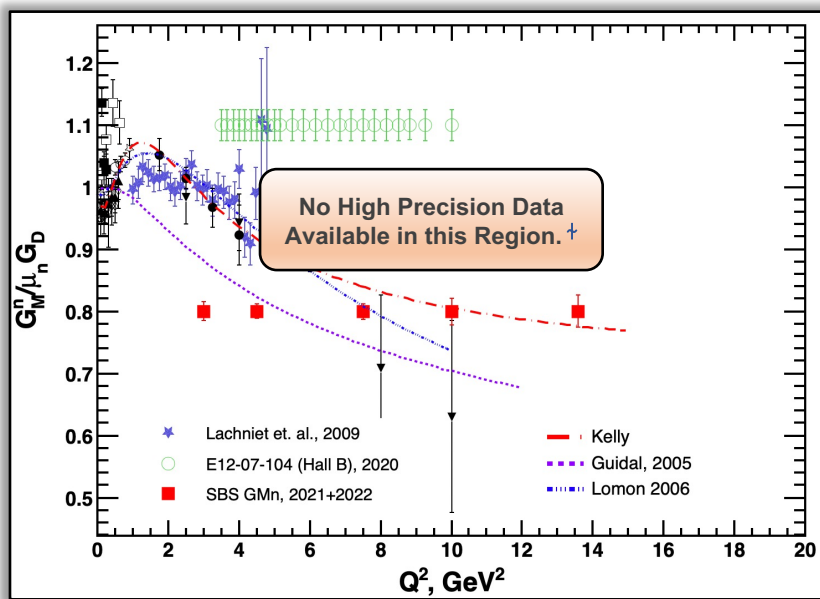
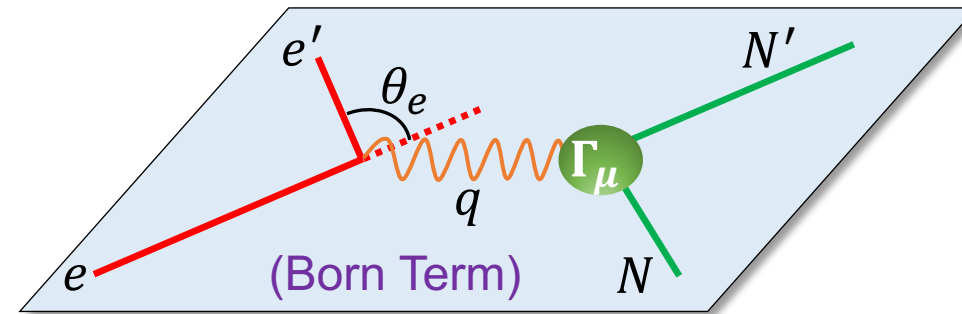
SBS-GMn – Theory and Motivation

- SBS-GMn ran in Jefferson Lab's Experimental Hall A from Fall 2021 to February 2022.

❖ **Goal:** High precision measurement of G_M^n at $Q^2 = 3, 4.5, 7.4, 9.9, \& 13.6 (GeV/c)^2$.

- Nucleon vertex (elastic e - N scattering):

$$\Gamma_\mu(q) = \gamma_\mu \underbrace{F_1(-q^2)}_{\text{Dirac FF}} + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N} \underbrace{F_2(-q^2)}_{\text{Pauli FF}}$$



- Defining Sachs Form Factors (FFs):
$$\begin{cases} G_E(Q^2) \equiv F_1(Q^2) - \tau F_2(Q^2) \\ G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2) \end{cases}$$

- G_E, G_M : Sachs Electric and Magnetic FFs, respectively.

- Differential Cross Section:

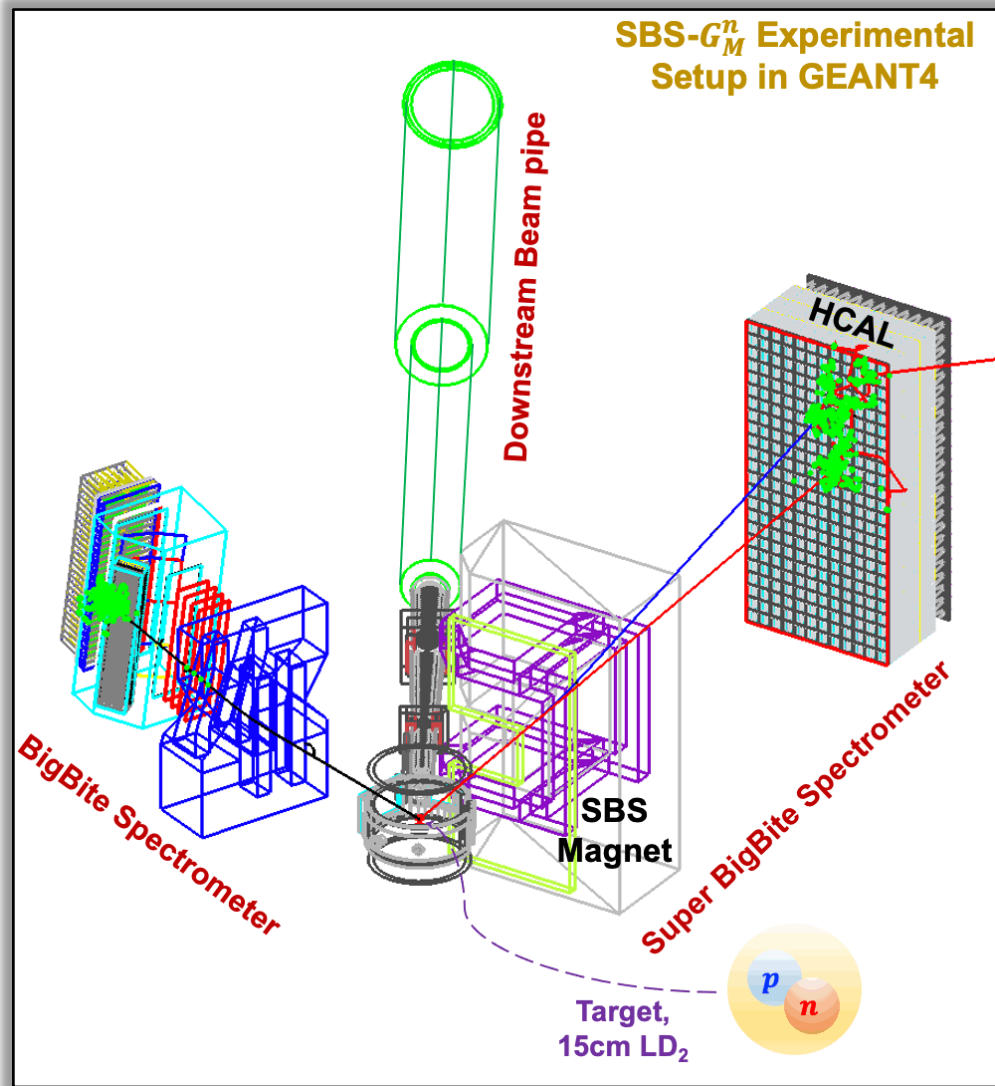
$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{Mott}}{1 + \tau} \left(G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right)$$

$$\begin{cases} \bullet Q^2 = -q^2 \\ \bullet \tau = Q^2/4M_N^2 \\ \bullet \epsilon = (1 + 2(1 + \tau)\tan^2(\theta_e/2))^{-1} \end{cases}$$

- ❖ Q^2 evolution of Sachs FFs reveal nucleon's internal structure.

† CLAS12 measured G_M^n up to $Q^2 = 10 GeV^2$, results are yet to be published.

SBS-GMn Measurement Technique (“Ratio method”)



- Simultaneous detection of elastically scattered electrons and nucleons lets us use “ratio method”^[1], which is way less sensitive to systematic errors than other measurement techniques.

- 3 major steps to get G_M^n :

- 1 Extracting QE cross section ratio, R'' , directly from the experiment:

$$R'' = \frac{\frac{d\sigma}{d\Omega} | d(e, e' n)}{\frac{d\sigma}{d\Omega} | d(e, e' p)}$$

- 2 Apply nuclear and radiative corrections to obtain:

$$R' = \frac{\frac{d\sigma}{d\Omega} | n(e, e')}{\frac{d\sigma}{d\Omega} | p(e, e')} \equiv \frac{\frac{\sigma_{Mott}}{1+\tau} \left(G_E^n^2 + \frac{\tau}{\epsilon} G_M^n^2 \right)}{\frac{d\sigma}{d\Omega} | p(e, e')}$$

- 3 Finally,

$$G_M^n = - \left[\frac{\epsilon(1+\tau)}{\tau \sigma_{Mott}} \frac{d\sigma}{d\Omega} \Big|_{p(e, e')} R' - \frac{\epsilon}{\tau} G_E^n^2 \right]^{\frac{1}{2}}$$

[1] L. Durand, Phys. Rev. 115 1020 (1959).

Kinematics of SBS-GMn

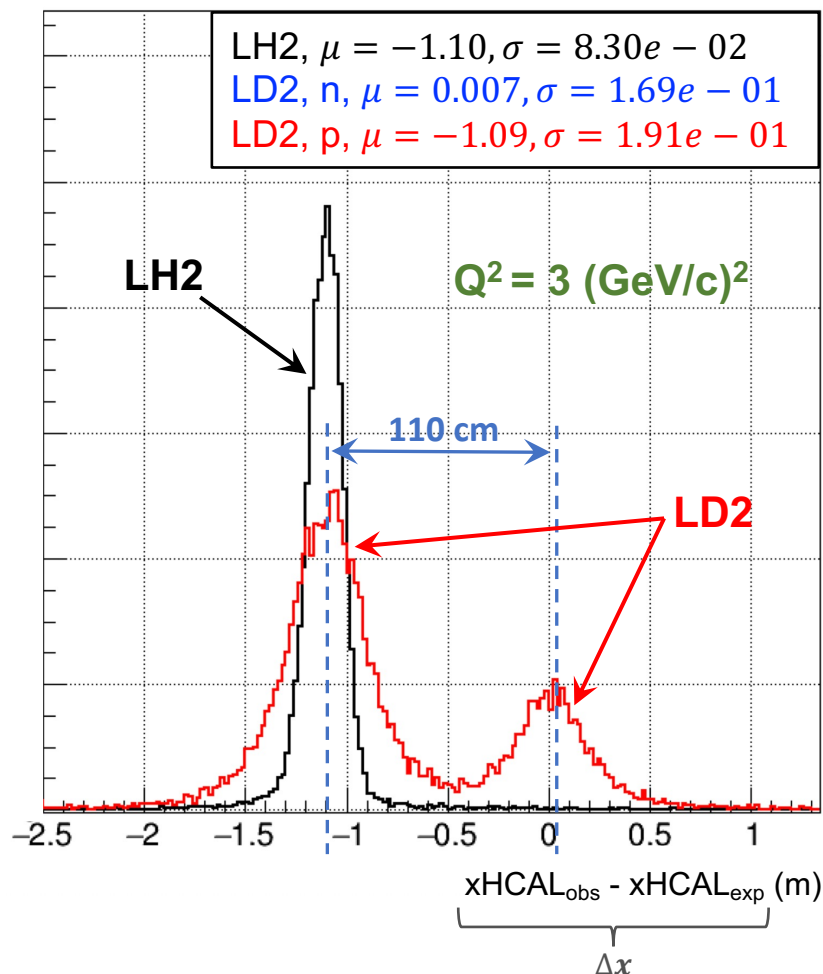
Table 1: Kinematics of SBS-GMn. Q^2 is the central Q^2 , E_{beam} is the beam energy, $\theta_{\text{BB}}(d_{\text{BB}})$ is the BigBite central angle (target-magnet distance), $\theta_{\text{SBS}}(d_{\text{SBS}})$ is the Super BigBite central angle (target-magnet distance), $\theta_{\text{HCAL}}(d_{\text{HCAL}})$ is the HCAL central angle (target-HCAL distance), ϵ is the longitudinal polarization of the virtual photon, $E_{e'}$ is the average scattered electron energy, and $E_{p'}$ is the average scattered proton energy.

SBS Config.	Q^2 (GeV/c) ²	E_{beam} (GeV)	θ_{BB} (deg)	d_{BB} (m)	θ_{SBS} (deg)	d_{SBS} (m)	θ_{HCAL} (deg)	d_{HCAL} (m)	ϵ	$E_{e'}$ (GeV)	$E_{p'}$ (GeV)
SBS-4	3.0	3.73	36.0	1.79	31.9	2.25	31.9	11.0	0.72	2.12	2.4
SBS-9	4.5	4.03	49.0	1.55	22.5	2.25	22.0	11.0	0.51	1.63	3.2
SBS-8	4.5	5.98	26.5	1.97	29.9	2.25	29.4	11.0	0.80	3.58	3.2
SBS-14	7.4	5.97	46.5	1.85	17.3	2.25	17.3	14.0	0.46	2.00	4.8
SBS-7	9.9	7.91	40.0	1.85	16.1	2.25	16.0	14.0	0.50	2.66	6.1
SBS-11	13.6	9.86	42.0	1.55	13.3	2.25	13.3	14.5	0.41	2.67	8.1

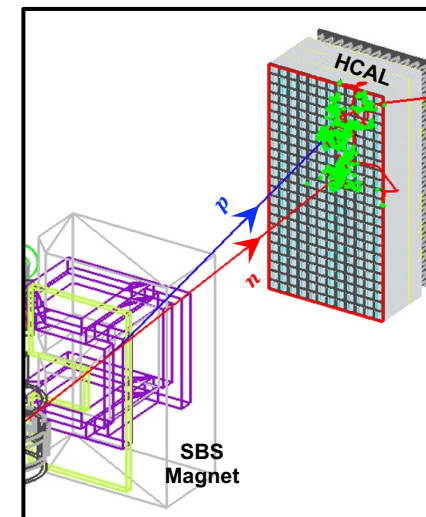
- We took data at five different spectrometer configurations for high- Q^2 G_M^n extraction.
- Data taken with **SBS-8** configuration in combination with SBS-9 dataset will be used for Rosenbluth separation to shed some light on the two-photon exchange (TPE) contribution in the elastic $e-n$ scattering. Stay tuned for Eric Fuchey's talk, "nTPE Analysis Progress"!

Physics Analysis Methods – Introducing HCAL Δx and Δy

❖ Introducing HCAL Δx plot:

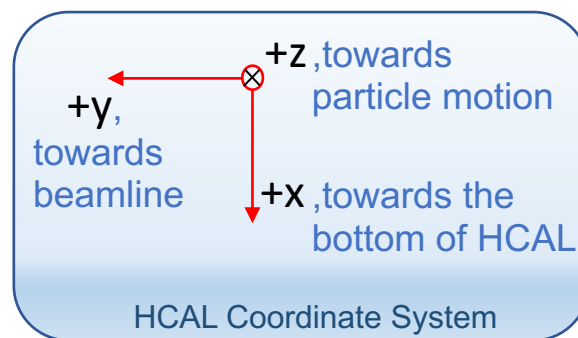


- **Definition of Δx :** The difference between the observed ($x\text{HCAL}_{\text{obs}}$) and expected ($x\text{HCAL}_{\text{exp}}$) nucleon position on HCAL in the vertical (dispersive) direction.
- **Definition of Δy :** The difference between the observed ($y\text{HCAL}_{\text{obs}}$) and expected ($y\text{HCAL}_{\text{exp}}$) nucleon position on HCAL in the horizontal (non-dispersive) direction.

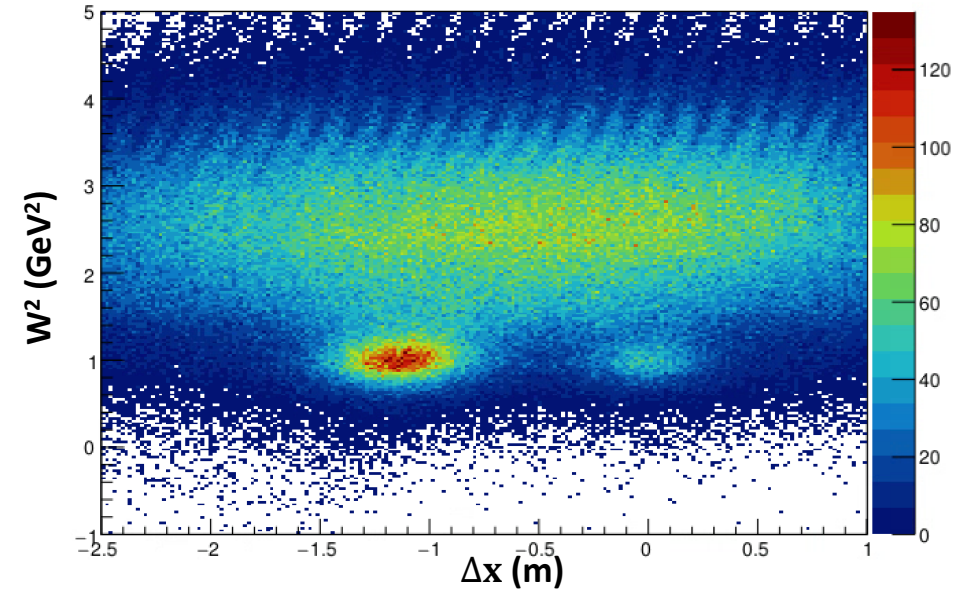
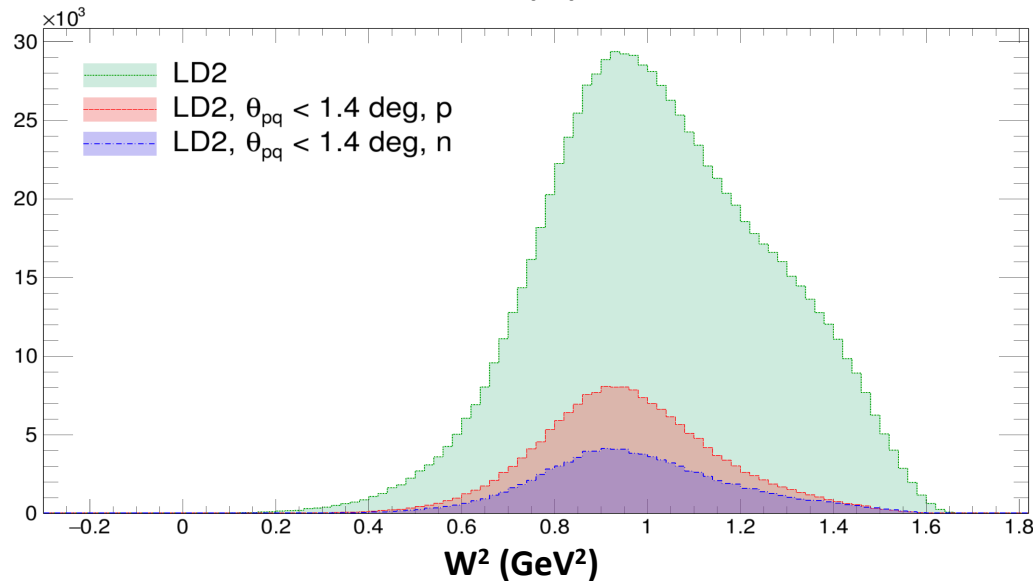
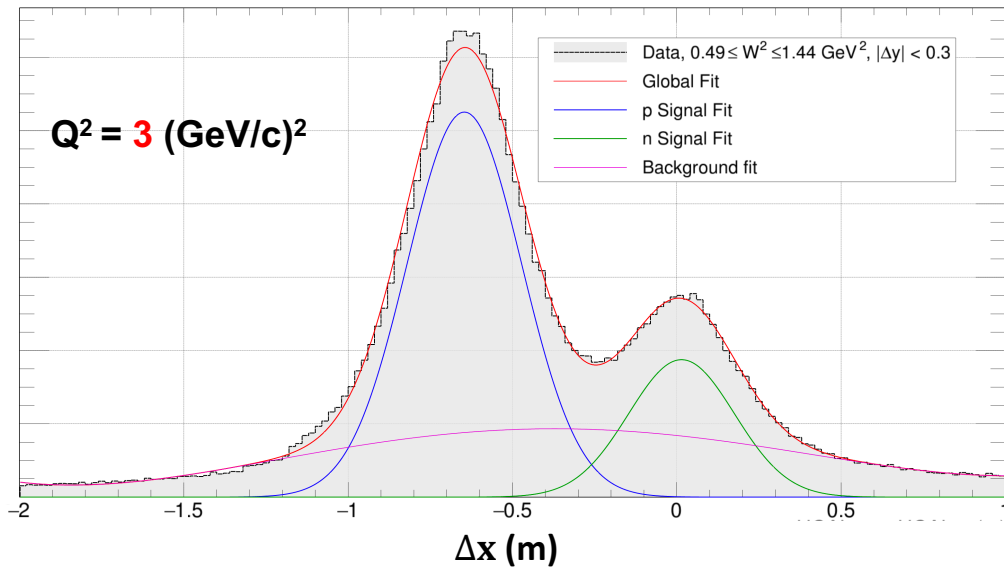


- ❖ Fitting Δx plot we can extract $d(ee'n)p$ & $d(ee'p)n$ yields and then form the ratio.

$$R'' = \frac{\frac{d\sigma}{d\Omega} | d(e, e'n)}{\frac{d\sigma}{d\Omega} | d(e, e'p)}$$



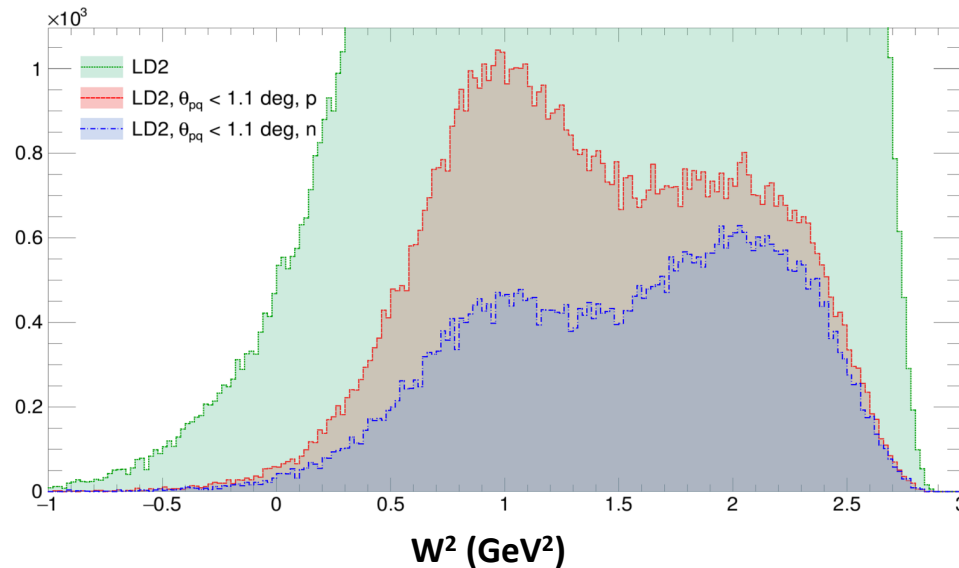
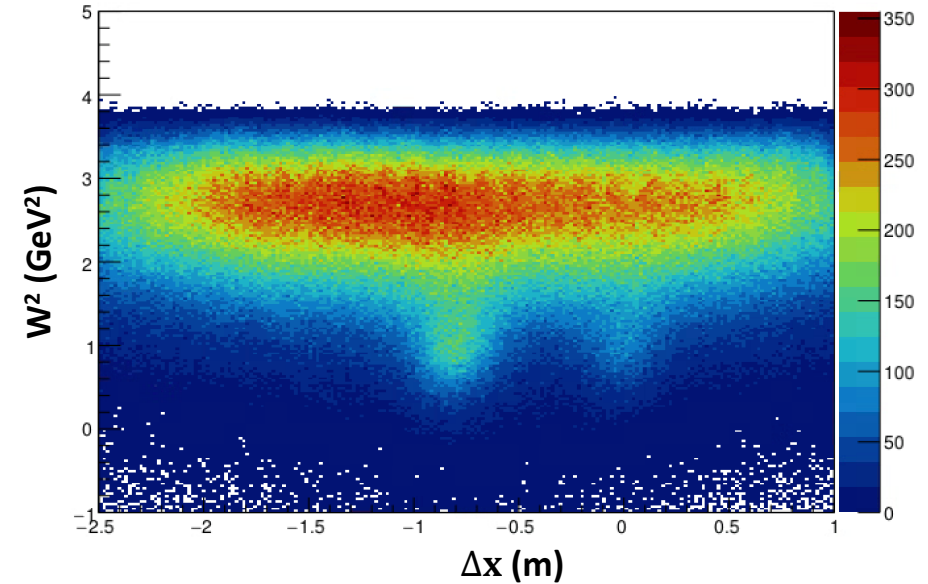
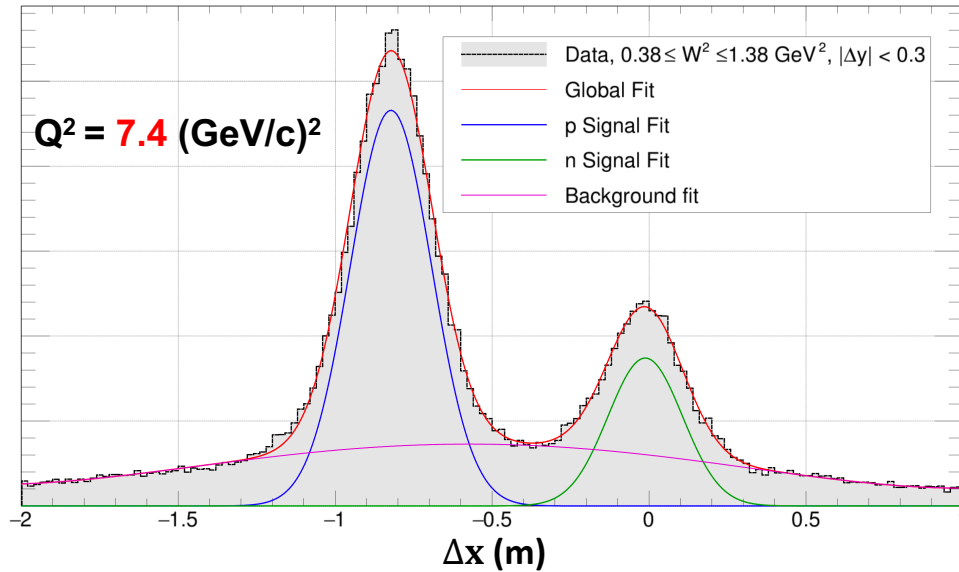
Quasi-Elastic (QE) Event Selection: $Q^2 = 3 \text{ (GeV/c)}^2$



Figures: HCAL Δx (Top Left), W^2 vs HCAL Δx (Top Right), W^2 (Bottom Left)

- List of cuts:
 - Primary cuts to choose good electron tracks.
 - Fiducial Cuts
 - $0.49 \leq W^2 \leq 1.44 \text{ GeV}^2$ (HCAL Δx plot)
 - $|\Delta y| < 0.3 \text{ m}$ (HCAL Δx plot)
 - $\theta_{pq} < 1.4^\circ$ with p hypothesis (W^2 plot)
 - $\theta_{pq} < 1.4^\circ$ with n hypothesis (W^2 plot)
- We fit the Δx distribution to sum of two Gaussian signals (p & n) along with a 4th degree polynomial background to extract raw $d(e, e'(p, n))$ yields.

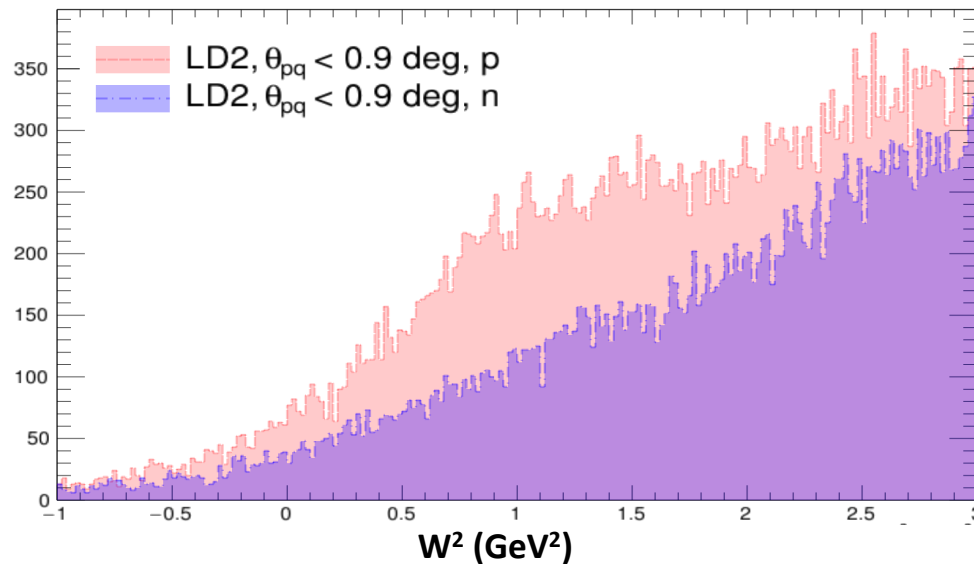
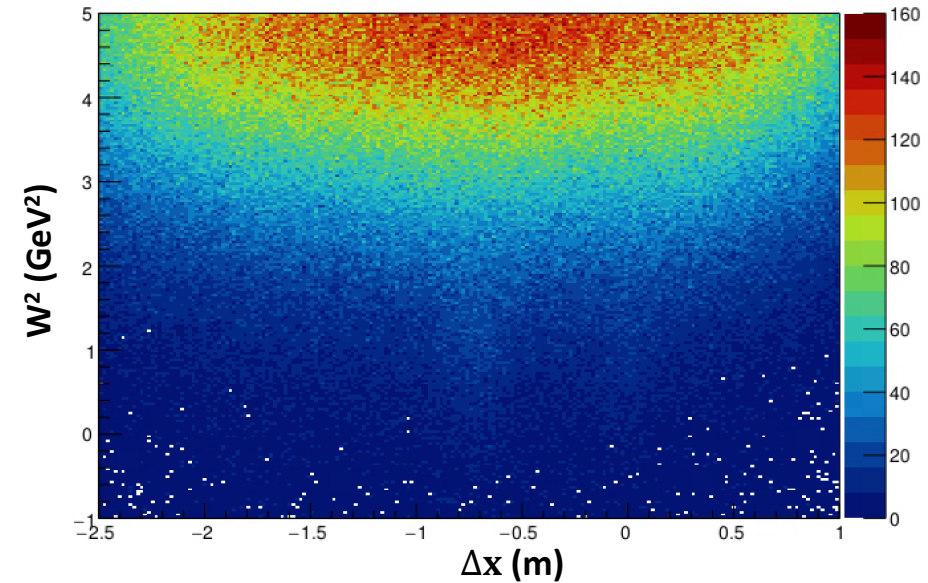
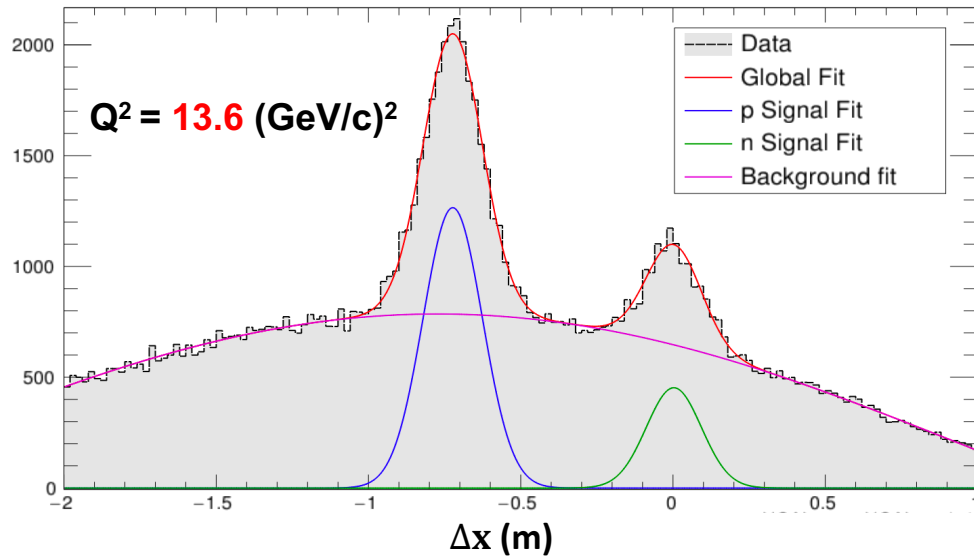
QE Event Selection: $Q^2 = 7.4 \text{ (GeV/c)}^2$



Figures: HCAL Δx (Top Left), W^2 vs HCAL Δx (Top Right), W^2 (Bottom Left)

- List of cuts:
 - Primary cuts to choose good electron tracks.
 - Fiducial Cuts
 - $0.38 \leq W^2 \leq 1.38 \text{ GeV}^2$ (HCAL Δx plot)
 - $|\Delta y| < 0.3 \text{ m}$ (HCAL Δx plot)
 - $\theta_{pq} < 1.1^\circ$ with p hypothesis (W^2 plot)
 - $\theta_{pq} < 1.1^\circ$ with n hypothesis (W^2 plot)
- We fit the Δx distribution to sum of two Gaussian signals (p & n) along with a 4th degree polynomial background to extract raw $d(e, e'(p, n))$ yields.

QE Event Selection: $Q^2 = 13.6 \text{ (GeV/c)}^2$



Figures: HCAL Δx (Top Left), W^2 vs HCAL Δx (Top Right), W^2 (Bottom Left)

- List of cuts:
 - Primary cuts to choose good electron tracks.
 - Fiducial Cuts
 - $-1 \leq W^2 \leq 2 \text{ GeV}^2$ (HCAL Δx plot)
 - $|\Delta y| < 0.3 \text{ m}$ (HCAL Δx plot)
 - $\theta_{pq} < 0.9^\circ$ with p hypothesis (W^2 plot)
 - $\theta_{pq} < 0.9^\circ$ with n hypothesis (W^2 plot)
- Same as other Q^2 points, we fit the Δx distribution to sum of two Gaussian signals (p & n) along with a 4th degree polynomial background to extract raw $d(e, e'(p, n))$ yields.

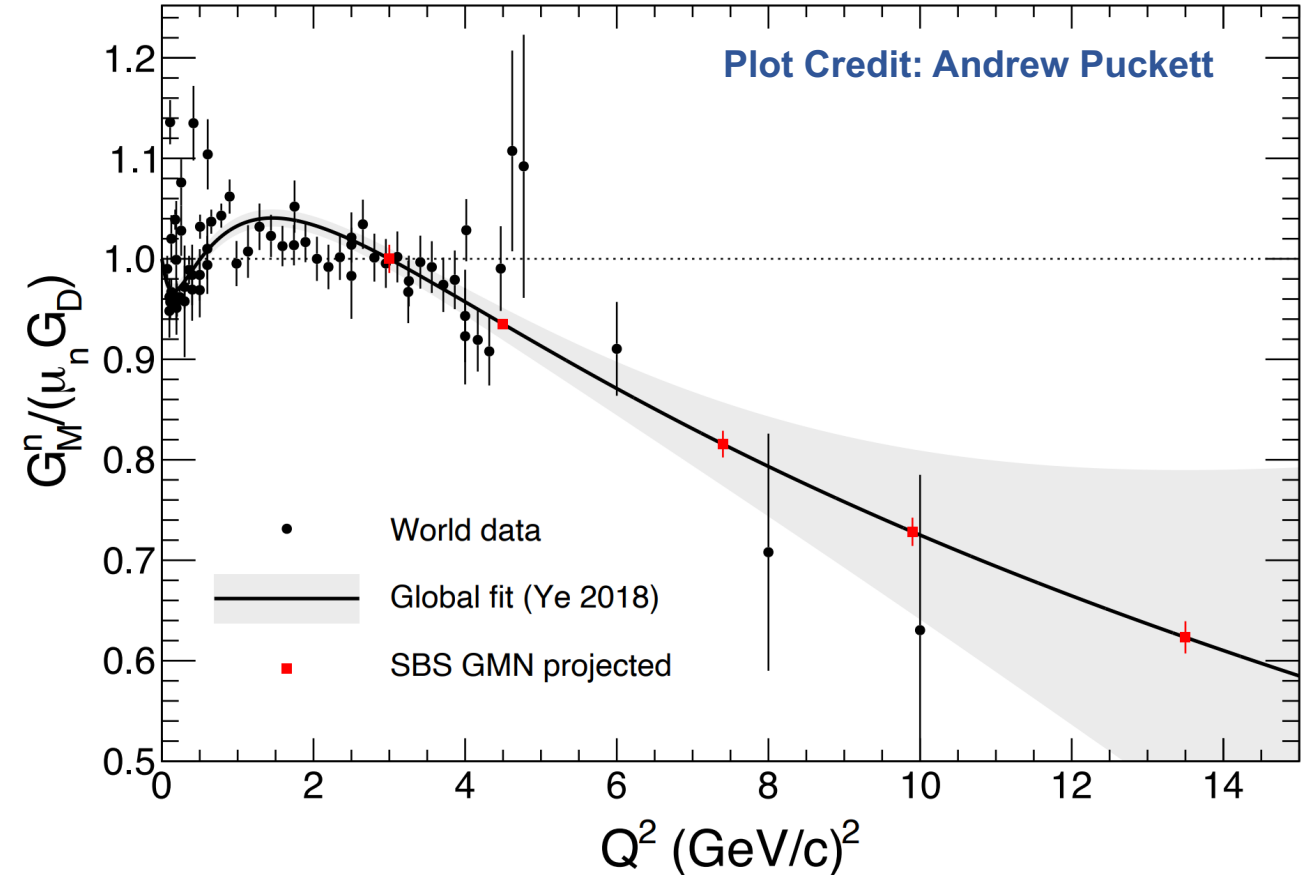
Raw Yields & Preliminary Uncertainty Projections

Table I: Estimated Raw QE Yields from SBS-GMn dataset

Q^2 (GeV/c) ²	E_{beam} (GeV)	Raw QE Yields	Projected $\Delta_{\text{stat}}(G_M^n/G_M^p)$	Projected $\Delta_{\text{syst}}(G_M^n/G_M^p)$
3.0	3.73	471,000	0.12%	1.4%
4.5	5.97	1,092,000	0.07%	0.6%
7.4	5.97	76,700	0.30%	1.6%
9.9	7.91	13,100	0.70%	1.8%
13.6	9.86	19,200	0.60%	2.5%

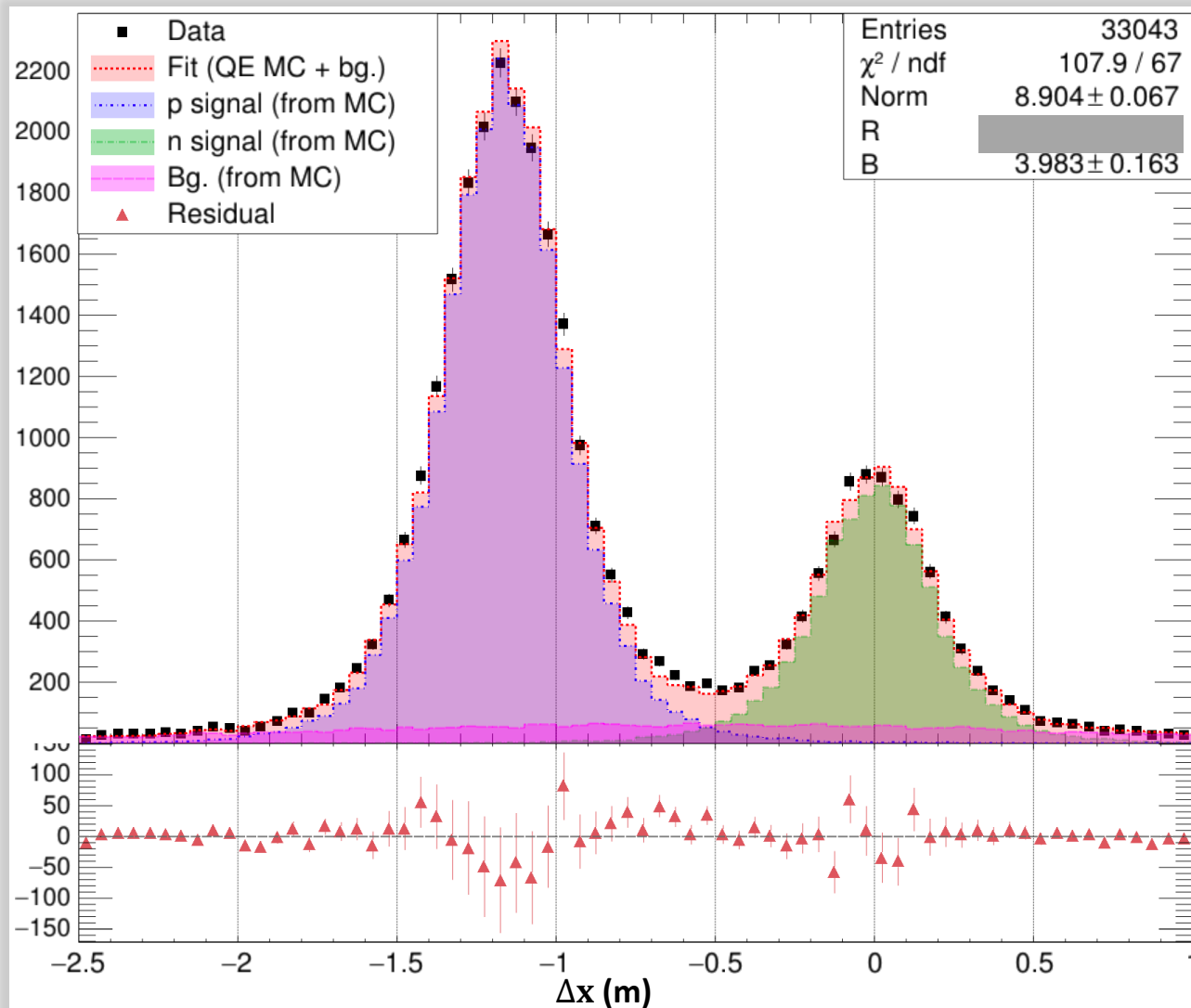
- Relative statistical uncertainties in G_M^n/G_M^p is estimated from the raw yields we got using the analysis shown in the previous slides.
- Projected systematic uncertainties have been taken from experiment proposal.
- ❖ Things we **haven't** considered:
 - HCAL p/n detection efficiency corrections
 - Radiative corrections
 - Nuclear corrections
 - Nucleon misidentification probabilities and many more

➤ **We now have the simulation machinery to estimate all the above-mentioned corrections.**



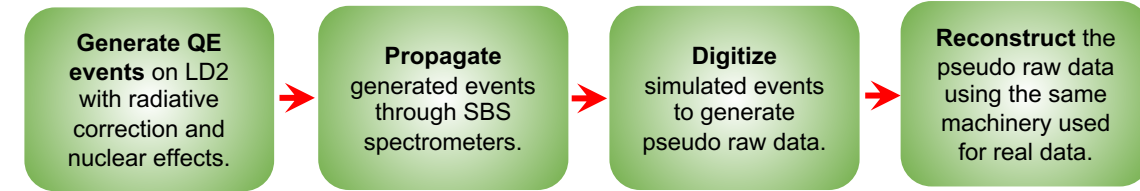
Data/MC Comparisons for Δx Dist.: $Q^2 = 3 \text{ (GeV/c)}^2$

$Q^2 = 3 \text{ GeV}^2$, $0.49 \leq W^2 \leq 1.44 \text{ GeV}^2$, Fiducial Cuts



❖ Steps to perform data/MC comparisons:

1. Generate realistic QE events via MC:



2. Plot Δx distribution using both data and reconstructed MC with same analysis cuts.
3. Fit data using QE MC signal and some estimated background.

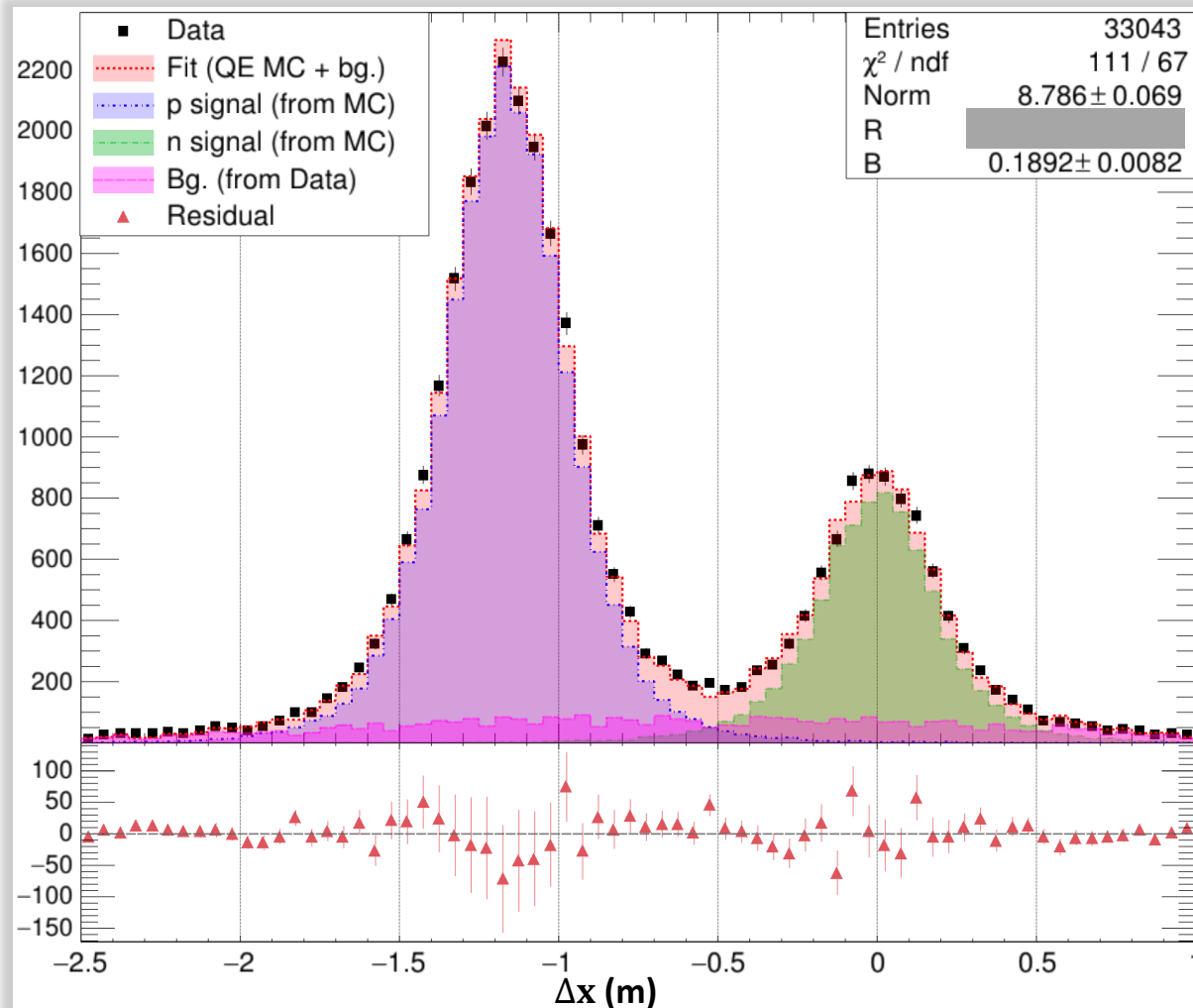
❖ Fit parameters:

1. **Norm:** Overall proton normalization.
2. **R:** Relative n/p normalization.
3. **B:** Overall background normalization.

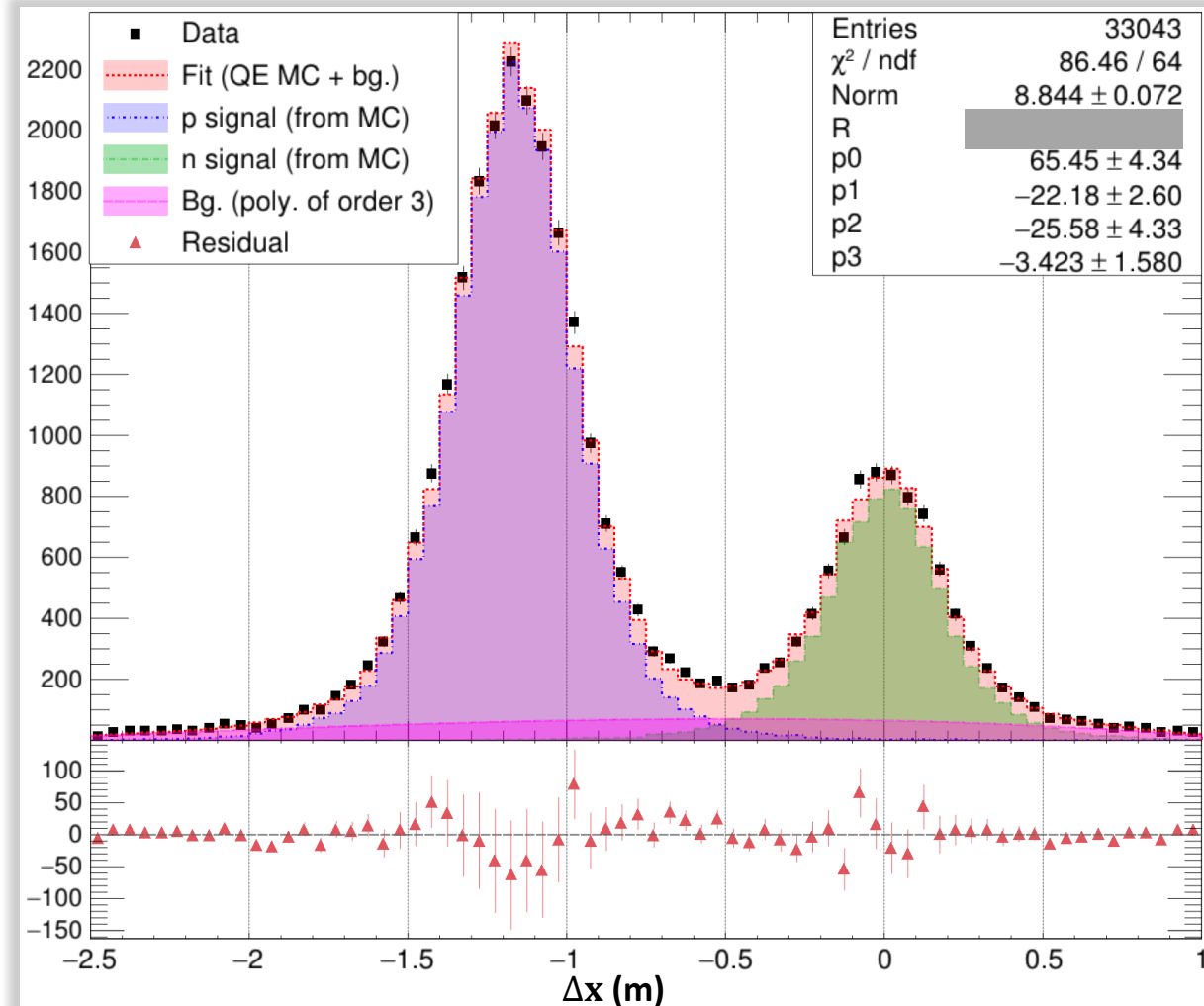
❖ Agreement of fit looks promising with preliminary analysis. Further optimization and systematic studies are ongoing.

Exploring Fit with Different Bg. Shapes: $Q^2 = 3 \text{ (GeV/c)}^2$

Using background shape from data

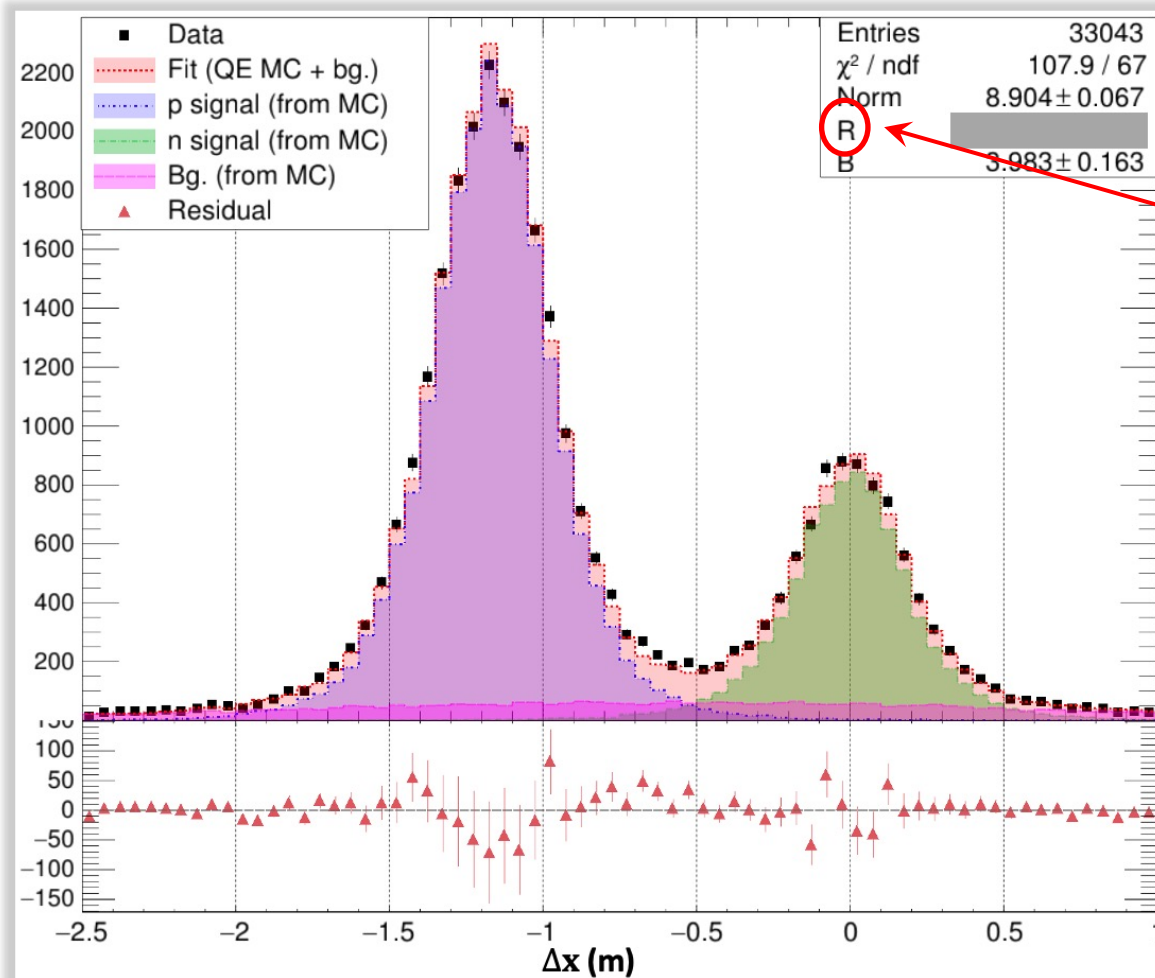


Using polynomial background



Method of GMn Extraction from Data/MC Fit

$Q^2 = 3 \text{ GeV}^2, 0.49 \leq W^2 \leq 1.44 \text{ GeV}^2, \text{ Fiducial Cuts}$



❖ Assumption:

- Simulation accurately represents nuclear, radiative, and detector effects that are present in data.

❖ Interpretation:

- The fit parameter R, i.e. the relative n/p normalization, is a measure of the discrepancy in the neutron to proton Born cross section ratio between simulation and data.

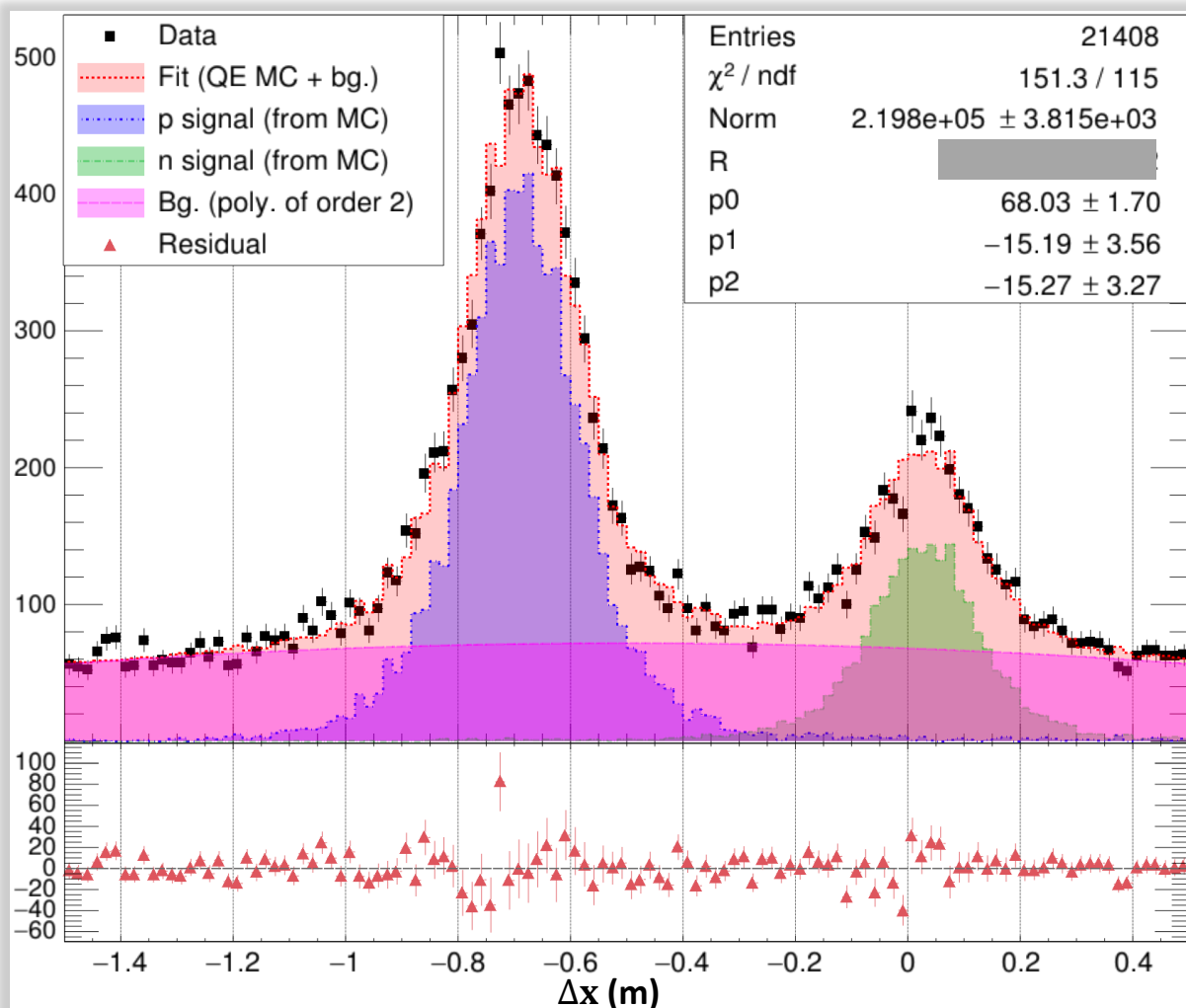
❖ GMn extraction:

$$R' = \frac{\frac{d\sigma}{d\Omega} | n(e,e')}{\frac{d\sigma}{d\Omega} | p(e,e')} = R * R'_{MC}$$

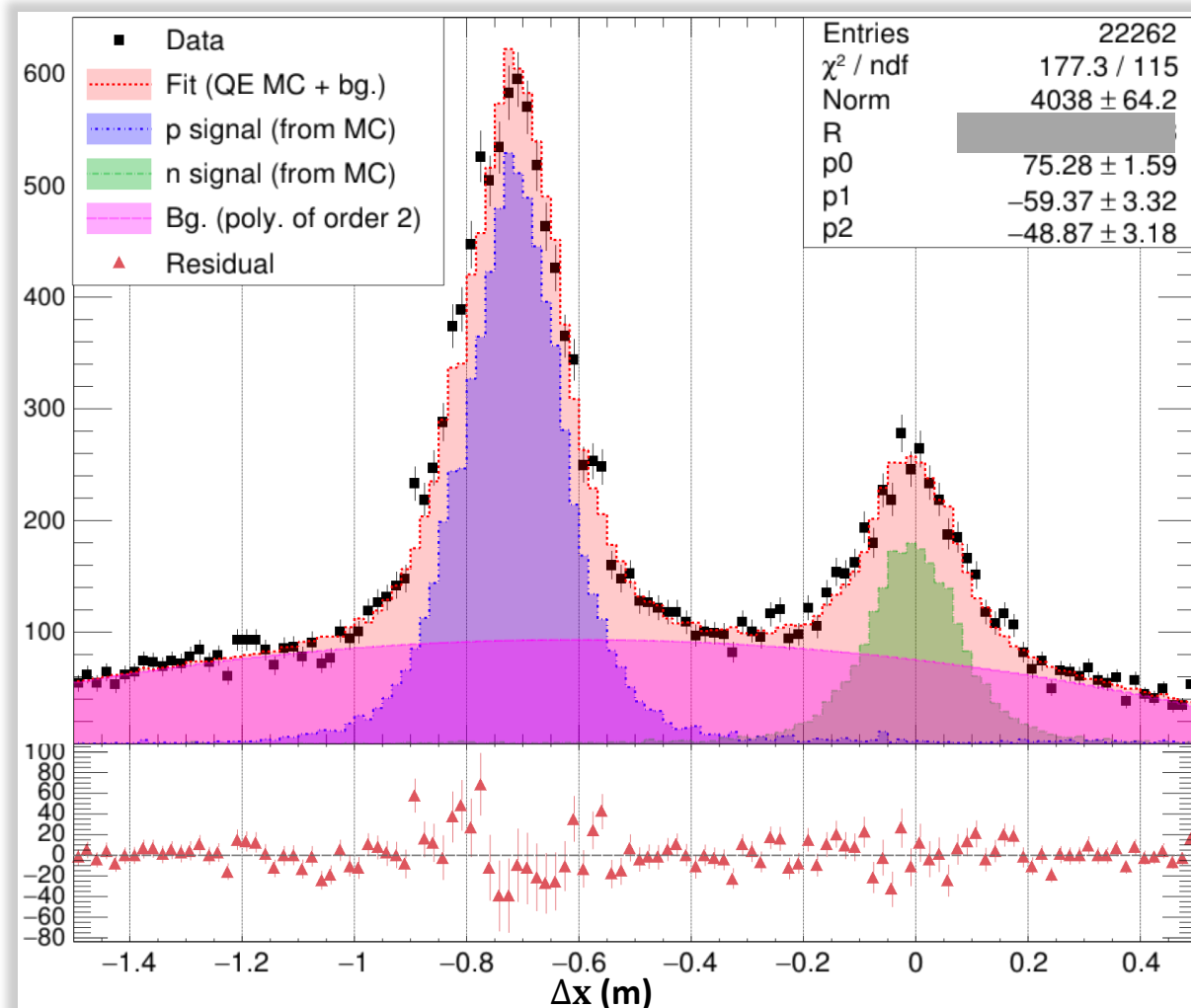
$$\Rightarrow G_M^m = - \left[\frac{\epsilon(1 + \tau)}{\tau \sigma_{Mott}} \frac{d\sigma}{d\Omega} |_{p(e,e')} R' - \frac{\epsilon}{\tau} G_E^m \right]^{\frac{1}{2}}$$

Data/MC Comparisons for High Q^2 Data

$Q^2 = 9.9 \text{ GeV}^2$, $0.49 \leq W^2 \leq 1.44 \text{ GeV}^2$, Fiducial Cuts



$Q^2 = 13.6 \text{ GeV}^2$, $0.49 \leq W^2 \leq 1.44 \text{ GeV}^2$, Fiducial Cuts



Summary, Outlook, & Acknowledgements

- SBS-GMn experiment finished data taking successfully for the high Q^2 measurement of the neutron's magnetic form factor in February 2022 at JLab's experimental hall A. A huge effort of data analysis is ongoing.
- Despite various challenges we finished the 1st pass reconstruction of the entire SBS-GMn dataset in January and now, we are all set to start the 2nd pass reconstruction!
- Preliminary projected uncertainties estimated from raw $d(e, e'(p, n))$ counts show promising results. **Precision of the highest Q^2 data point (13.6 GeV^2) is expected to stay unmatched for years to come.**
- **Realistic MC event generators including radiative and nuclear effects are in place and the preliminary data/MC comparisons look very encouraging.** So, we have all the tools necessary to get preliminary results out as soon as the 2nd pass reconstruction is complete.
- Currently, we are focused on performing various cut sensitivity studies to quantify systematic uncertainties. Inelastic contamination and HCAL's neutron detection efficiency (NDE) are going to be the two major sources of systematic errors.
- ❖ I would like to thank the entire Hall A collaboration and of course the SBS collaboration for letting me be a part of this program and write my thesis on the SBS-GMn experiment.
- ❖ I would also like to thank the US Department of Energy Office of Science, Office of Nuclear Physics, for supporting this work (Award ID DE-SC0021200).

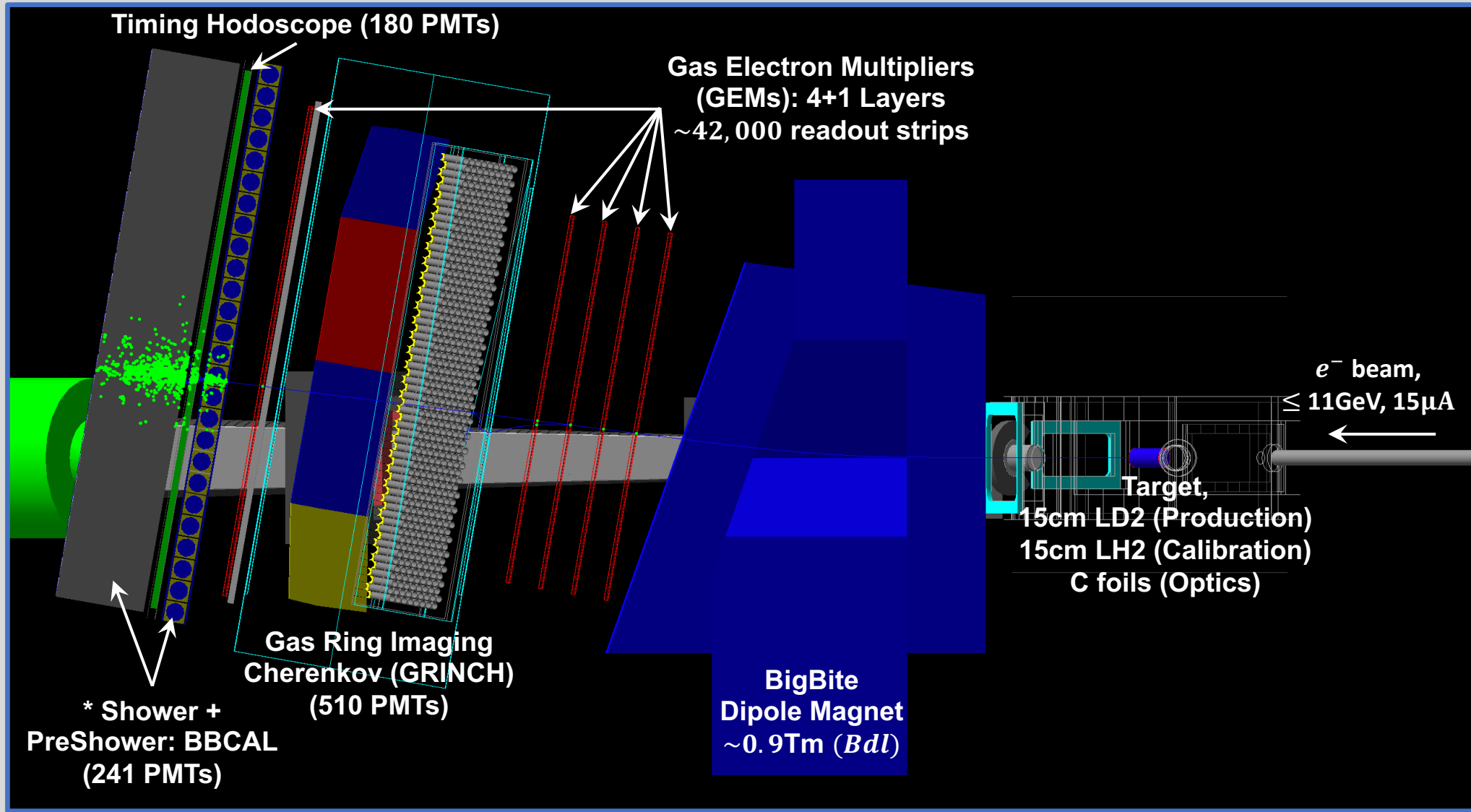


Thank You for Your Attention!
Questions? Comments?

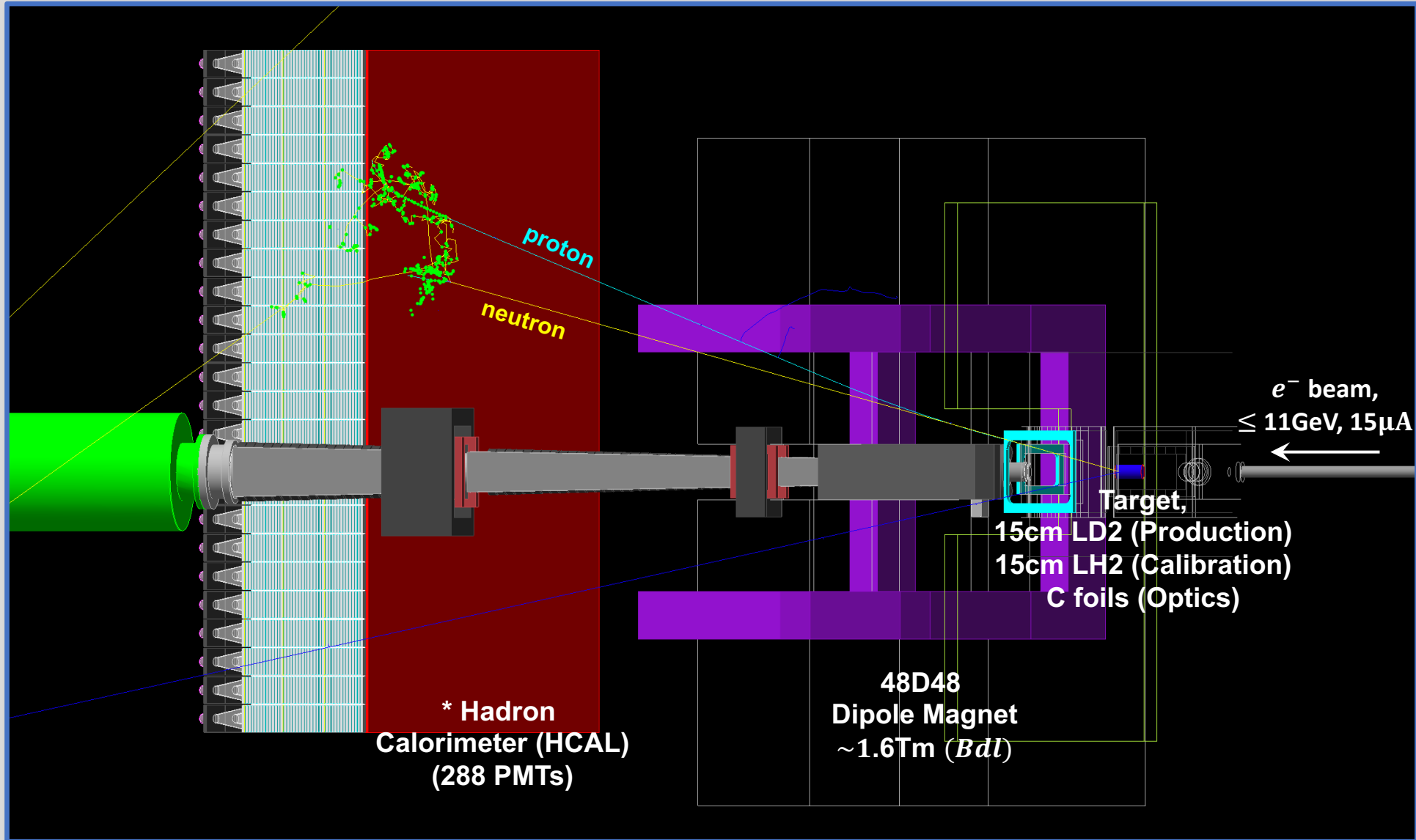
Team SBS in Hawaii!!

Backup Slides

Electron Arm: BigBite Spectrometer in Monte Carlo (Side View)

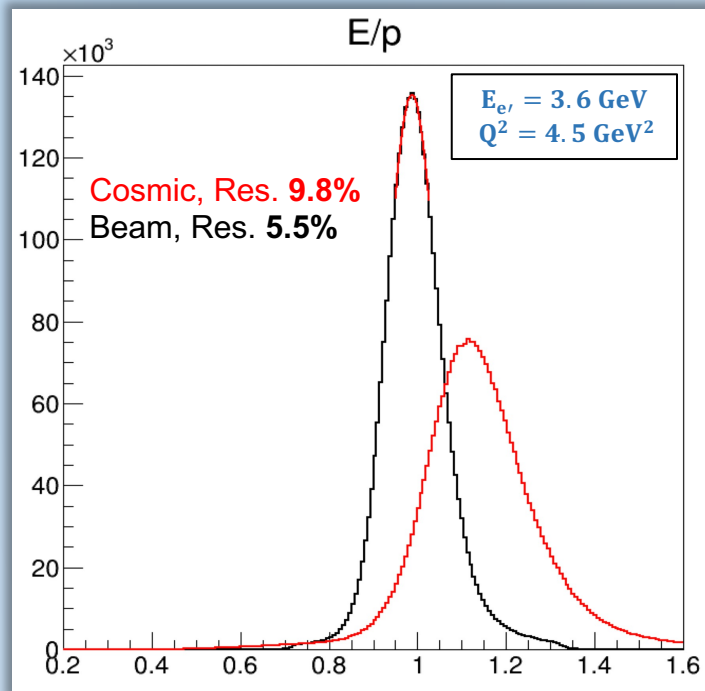


Hadron Arm: Super BigBite Spectrometer in Monte Carlo (Side View)



Data Analysis Status

- Despite the enormous raw data volume and reconstruction challenges, we finished 1st pass reconstruction of the entire SBS-GMn dataset in January this year. **Now, we are almost ready to start the 2nd pass.**
- **Realistic MC event generator including nuclear and radiative effects is now available for analysis.**
- Sophisticated physics analysis machineries for quasi-elastic event selection, yield extraction, data/MC comparison, nucleon detection efficiency estimation are in place. A huge effort is ongoing to optimize them.



- **Highlights of Detector Performance using pass 0/1 data:**

- BigBite Spectrometer:
 - Momentum resolution: $\frac{\sigma_p}{p} \approx 1 - 1.5\%$
 - Angular resolution (in-plane & out-of-plane): **1 – 2 mrad**
 - Vertex resolution: $\sigma_z \leq 1 \text{ cm}$
 - BigBite Calorimeter(BBCAL) energy resolution: **5.5% at 3.6 GeV** scattered e^- energy.
- Super BigBite Spectrometer:
 - Hadron Calorimeter (HCAL):
 - Time Resolution: $\sigma_t \approx 1.7 \text{ ns}$
 - Angular Resolution: **~2 mrad**

List of Analysis Cuts

■ Primary Cuts:

1. Presence of a track.
2. $|(\text{vertex})_z| < 0.08$ m, to ensure that the corresponding track was originated from the target.
3. Pre-Shower cluster energy > 0.2 GeV, to reject pions.
4. $|\text{BBCAL cluster energy} / \text{track momentum}| < 0.3$, to reduce fake tracks.

■ Quasi-Elastic (QE) Event Selection Cuts or Exclusivity Cuts: These cuts are used to strictly isolate elastic scattering events.

1. Cut on W^2 , the squared invariant mass of the virtual photon – nucleon system.
2. Cut on Δy , the difference between the observed and expected nucleon position on HCAL in the non-dispersive direction, as described on slide 18.
3. Cut on θ_{pq} , the angle between reconstructed nucleon momentum (\vec{p}) and the momentum transfer vector (\vec{q}).
4. Fiducial/Acceptance Cuts.

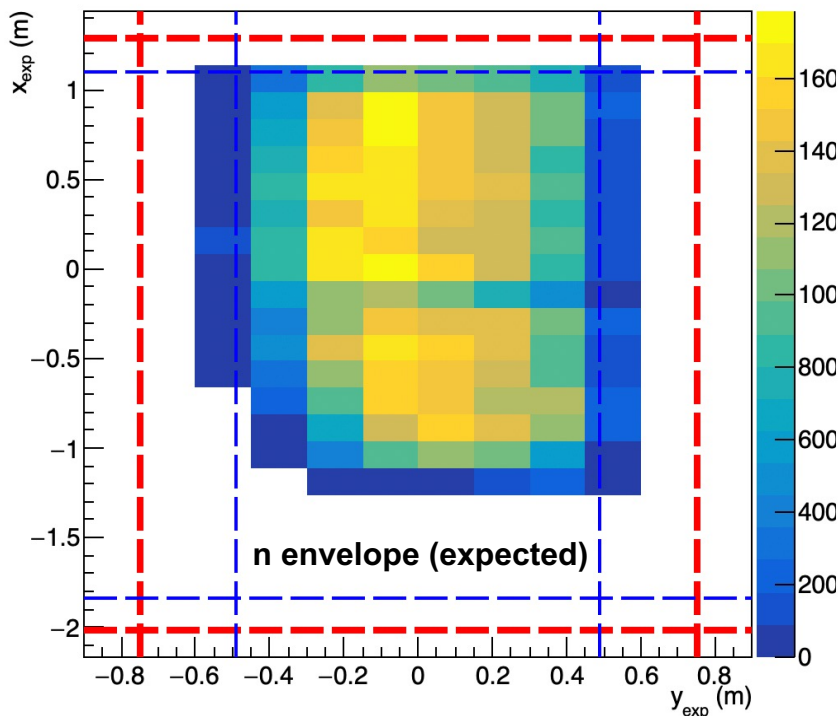
$$\begin{aligned} P_e^i + P_N^i &= P_e^f + P_N^f \\ \Rightarrow P_N^i + (P_e^i - P_e^f) &= P_N^f \\ \Rightarrow (P_N^i + q)^2 &= (P_N^f)^2 \\ \Rightarrow W^2 &= (P_N^f)^2 = M_N^2 \end{aligned}$$

Elastic e - N Scattering

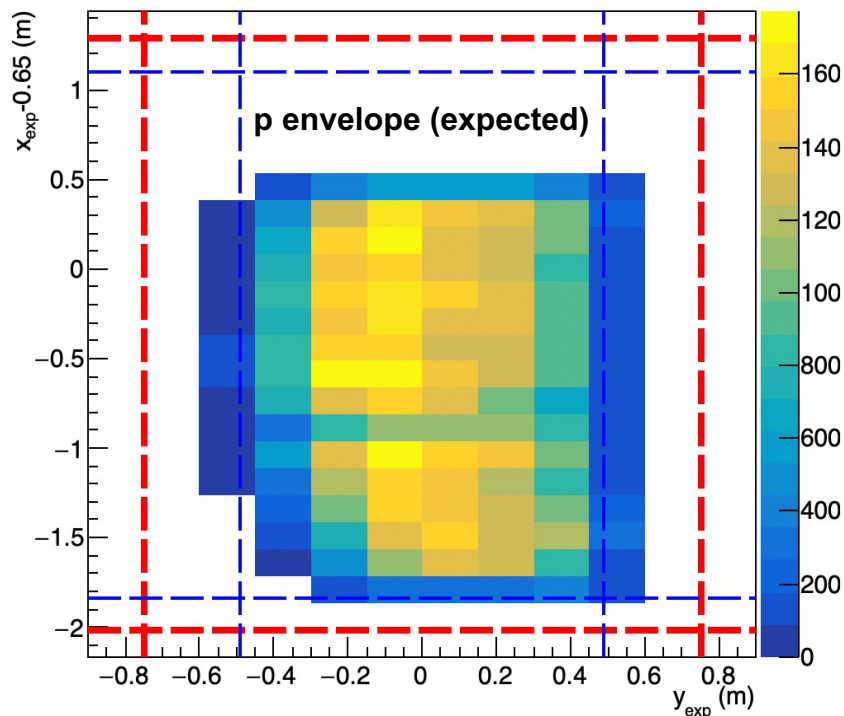
Implementation of Fiducial Cuts on \vec{q}

$$Q^2 = 3 \text{ (GeV/c)}^2$$

$|W^2 - 0.88| < 0.5$ & Fiducial Cuts



$|W^2 - 0.88| < 0.5$ & Fiducial Cuts



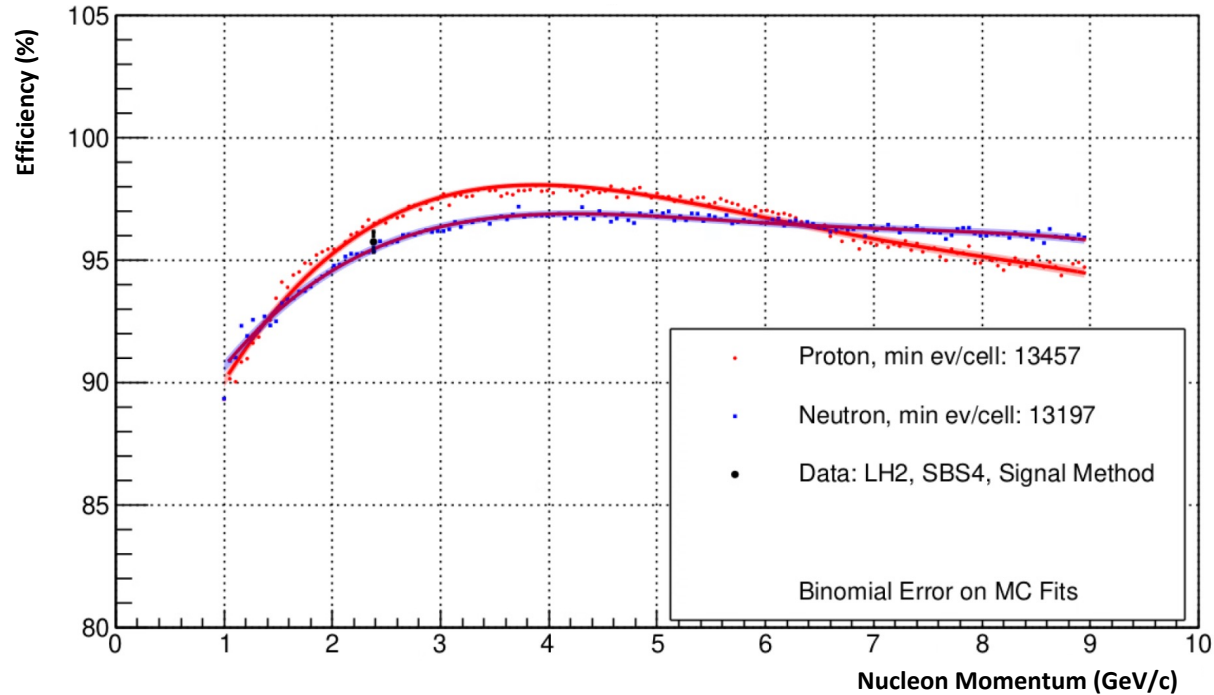
--- Top of HCAL ---

- The idea is to accept a n (p) event only if a p (n) event with equivalent kinematics would also be guaranteed to hit the active area of HCAL.
- The fiducial cut is only based on the scattered-electron angle and momentum measured by BigBite.
- As “active area” (red dashed lines) we consider entire HCAL excluding the outermost rows and columns.
- We also use an additional “safety margin” (blue dashed lines) based on the widths of the Δx & Δy distributions for p & n to encounter the effects of Fermi motion to some extent.

NOTE: W^2 represents the squared invariant mass of the virtual photon – nucleon system.

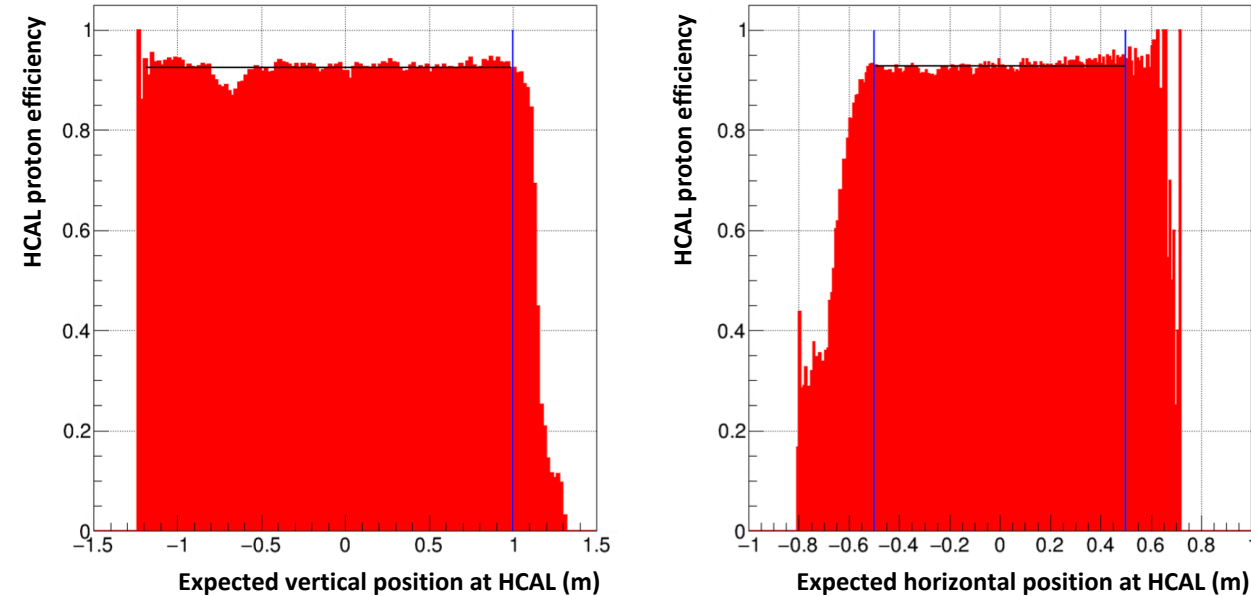
HCAL Nucleon Detection Efficiency

HCAL Nucleon Detection Efficiency from MC



Plot Credit: Sebastian Seeds

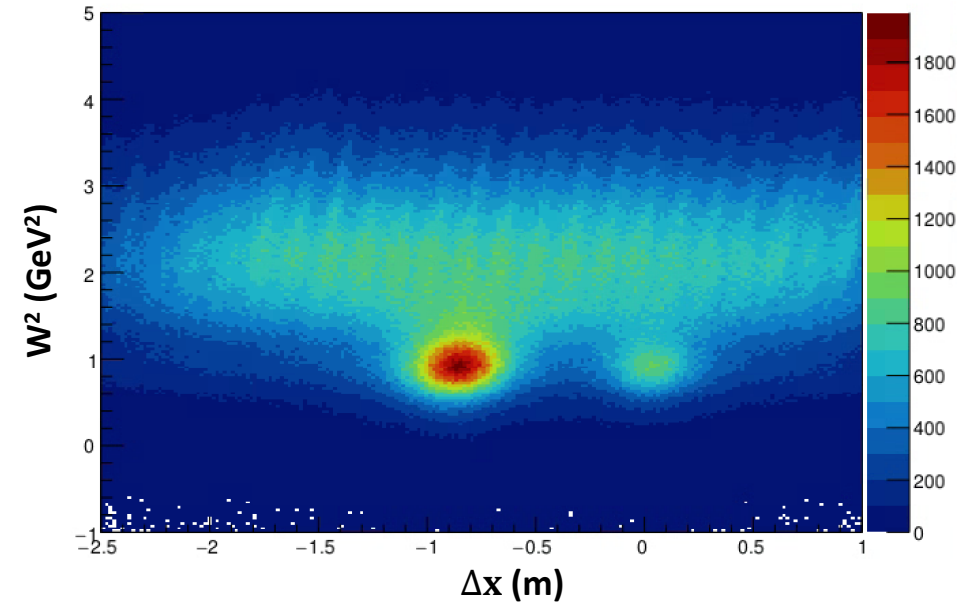
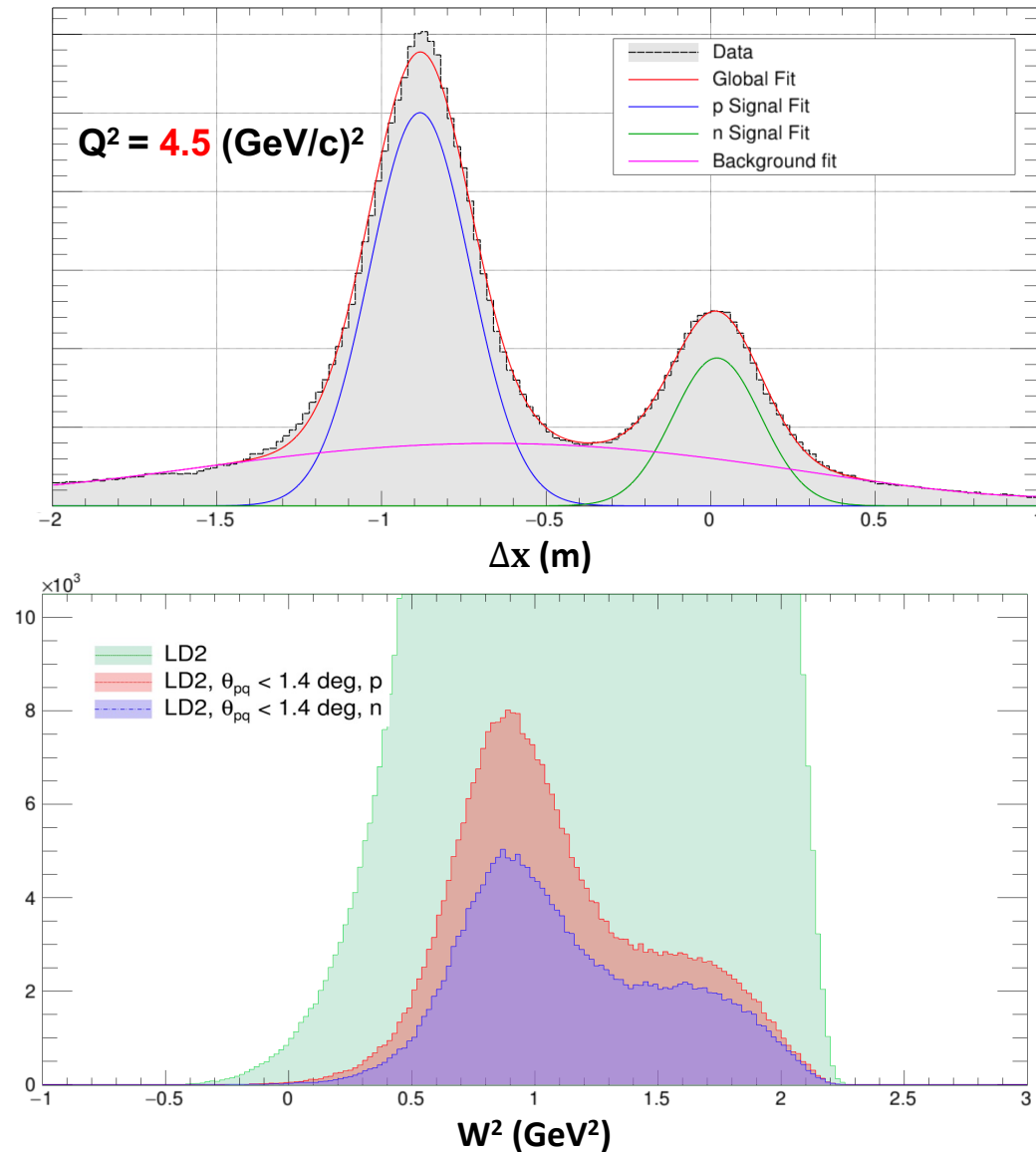
Uniformity of HCAL proton detection efficiency using $Q^2 = 3 \text{ GeV}^2$ data



Plot Credit: Andrew Puckett

- One of the biggest sources of systematic errors for SBS-GMn/nTPE analysis.
- MC shows very high (~95-98%) and comparable detection efficiencies for both protons and neutrons, as expected from the design of HCAL.
- Preliminary analysis of LH2 data from $Q^2 = 3 \text{ GeV}^2$ dataset shows reasonable uniformity of proton detection efficiency across HCAL, and the overall efficiency is comparable with MC predictions.

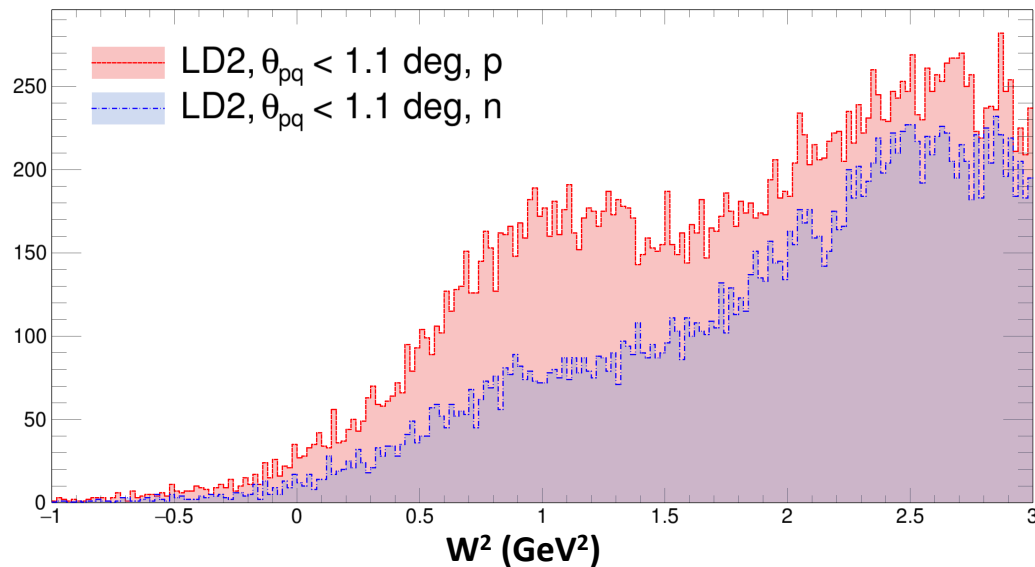
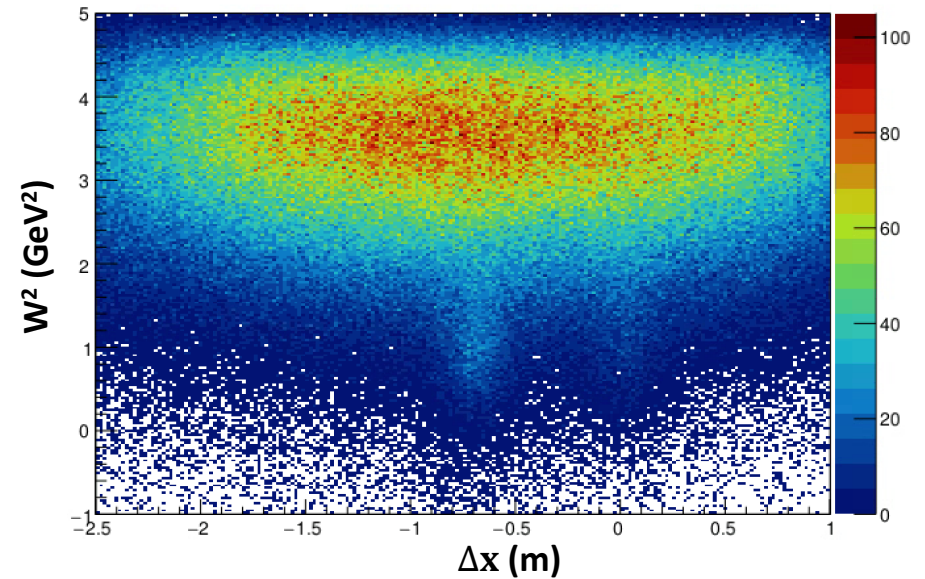
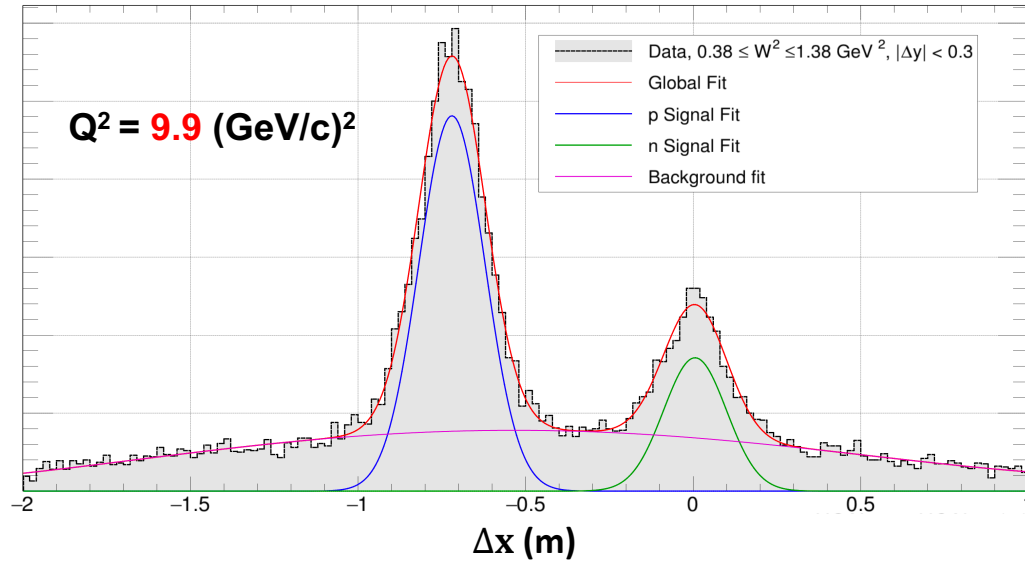
QE Event Selection: $Q^2 = 4.5 \text{ (GeV/c)}^2$



Figures: HCAL Δx (Top Left), W^2 vs HCAL Δx (Top Right), W^2 (Bottom Left)

- List of cuts:
 - All primary cuts listed on slide 7.
 - Fiducial Cuts
 - $0.49 \leq W^2 \leq 1.44 \text{ GeV}^2$ (HCAL Δx plot)
 - $|\Delta y| < 0.3 \text{ m}$ (HCAL Δx plot)
 - $\theta_{pq} < 1.4^\circ$ with **p** hypothesis (W^2 plot)
 - $\theta_{pq} < 1.4^\circ$ with **n** hypothesis (W^2 plot)
- We fit the Δx distribution to sum of two Gaussian signals (p & n) along with a 4th degree polynomial background to extract raw $d(e, e'(p, n))$ yields.

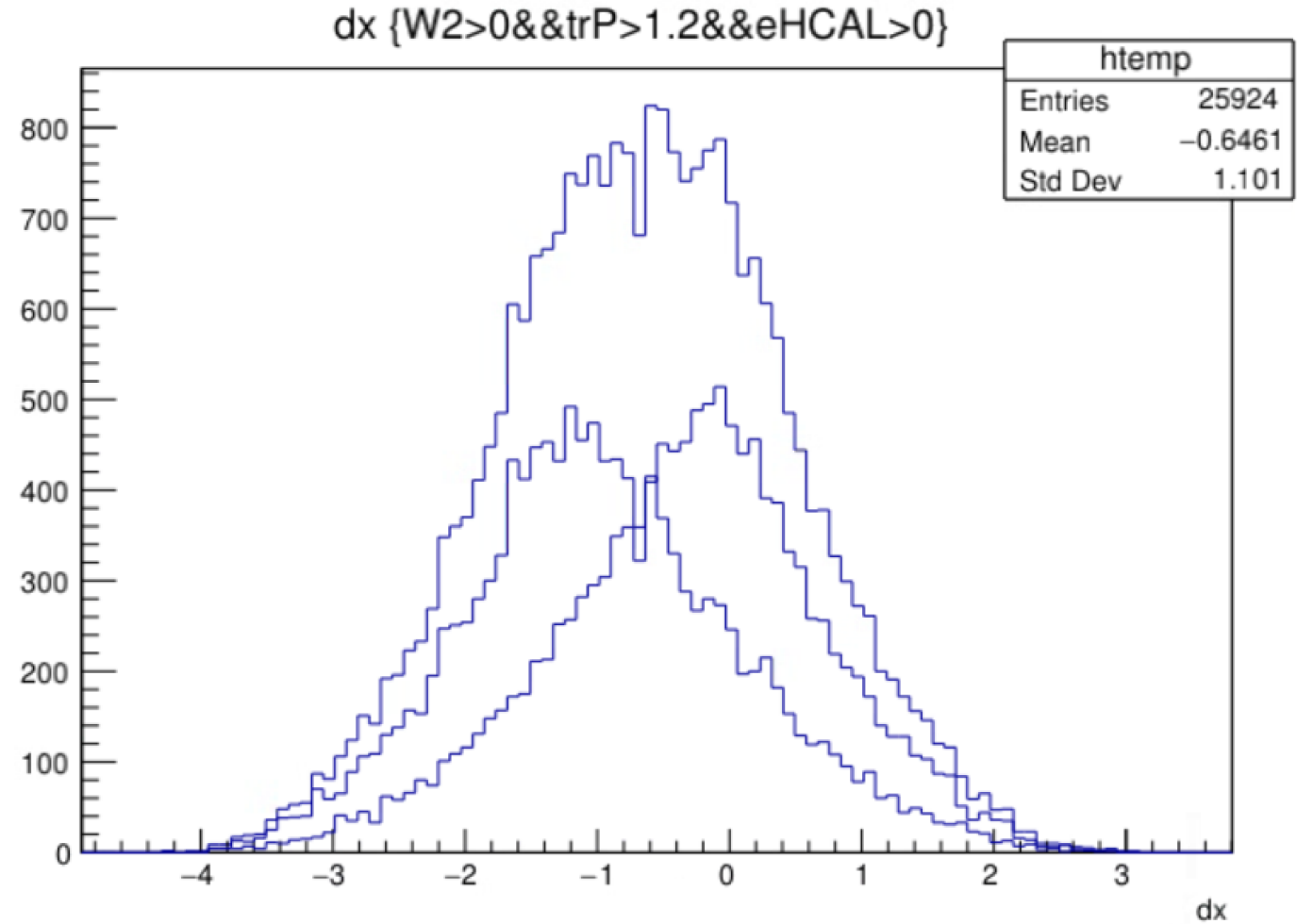
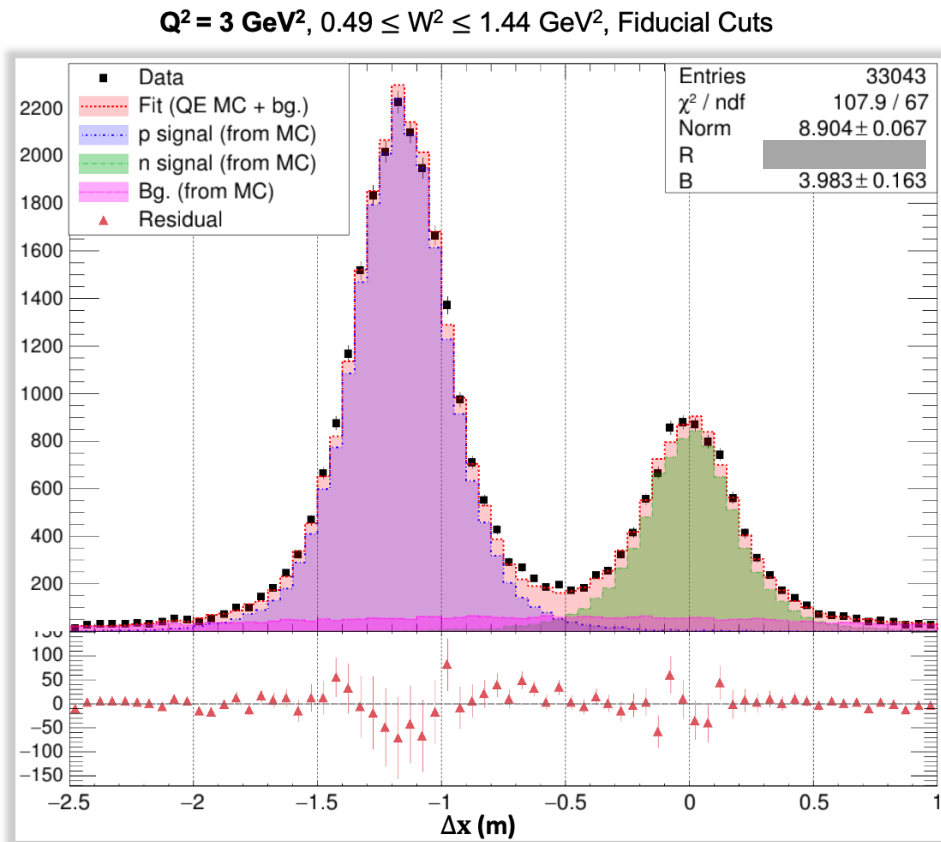
QE Event Selection: $Q^2 = 9.9 \text{ (GeV/c)}^2$



Figures: HCAL Δx (Top Left), W^2 vs HCAL Δx (Top Right), W^2 (Bottom Left)

- List of cuts:
 - All primary cuts listed on slide 7.
 - Fiducial Cuts
 - $0.38 \leq W^2 \leq 1.38 \text{ GeV}^2$ (HCAL Δx plot)
 - $|\Delta y| < 0.3 \text{ m}$ (HCAL Δx plot)
 - $\theta_{pq} < 1.1^\circ$ with p hypothesis (W^2 plot)
 - $\theta_{pq} < 1.1^\circ$ with n hypothesis (W^2 plot)
- We fit the Δx distribution to sum of two Gaussian signals (p & n) along with a 4th degree polynomial background to extract raw $d(e, e'(p, n))$ yields.

Understanding Inelastic Bg. Shape: $Q^2 = 3 \text{ (GeV/c)}^2$



Theoretical Interpretation of EMFFs & Nucleon Imaging

- At very low Q^2 when target recoil can be neglected, the Sachs FFs G_E and G_M can be interpreted as the 3D Fourier transforms of the spatial distributions of charge and current in the nucleon, respectively.

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} d^3x$$

- At higher Q^2 , $|p_f| \neq |p_i|$ **invalidates such a probability or density interpretation.**
 - In the Breit frame ($\vec{p}_i = -\vec{p}_f$), density interpretations of Sachs FFs can still be drawn but relating those to the nucleon rest frame require **model-dependent** relativistic corrections.
 - In the infinite momentum frame (IMF), however, a **model-independent** density interpretation can be drawn by relating the form factors to General Parton Distribution (GPD) moments. In IMF, the impact-parameter-space densities of charge and magnetization in the nucleon essentially become the 2D Fourier-Bessel transforms of the Dirac (F_1) and Pauli (F_2) form factors, respectively.

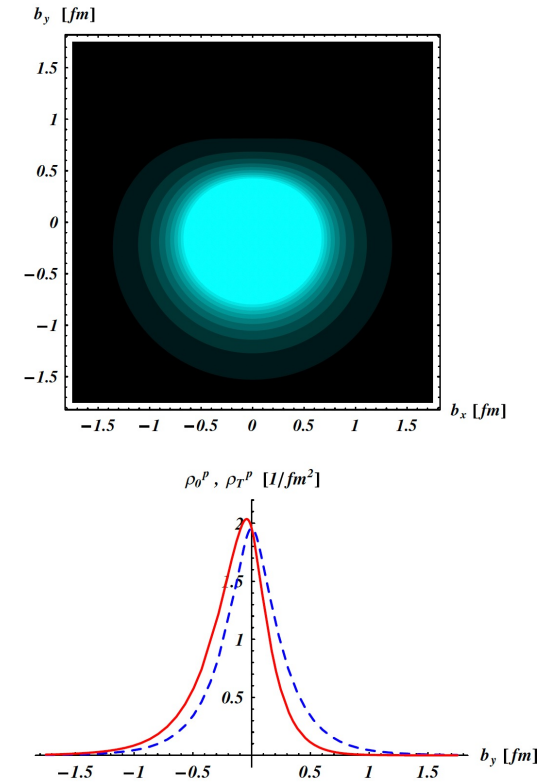


FIG. 1 (color online). Quark transverse charge densities in the *proton*. The upper panel shows the density in the transverse plane for a proton polarized along the x axis. The light (dark) regions correspond with largest (smallest) values of the density. The lower panel compares the density along the y axis for an unpolarized proton (dashed curve), and for a proton polarized along the x axis (solid curve). For the proton e.m. FFs, we use the empirical parameterization of Arrington *et al.* [15].

Ref: Carlson *et al.*: Phys. Rev. Lett. 100, 032004 (2008)