New developments on GPDs from lattice QCD

Martha Constantinou

Temple University

Winter Hall A Collaboration Meeting

January 16 - 17, 2024

Outline

★ Approaches to GPDs from lattice QCD

★ Recent results on Mellin moments for proton:

⇨ Axial form factors

⇨ E/M form factors

★ x-dependence of GPDs:

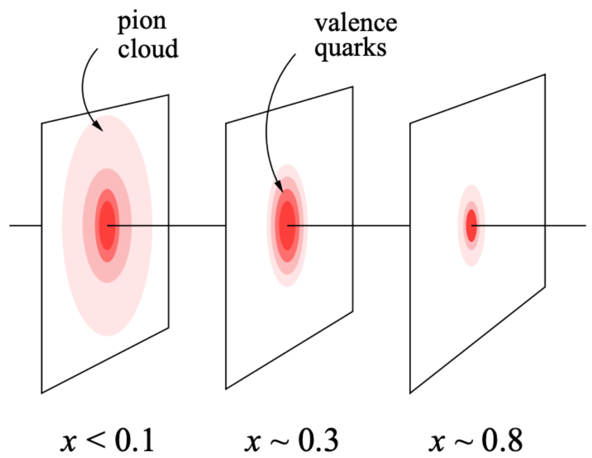
⇨ leading-twist results

⇨ subleasing-twist contributions

⇨ new promising method

★ Summary - Outlook

Motivation in a nutshell



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\text{mom}} + 2_{\text{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

- ★ Contain physical interpretation on mechanical properties
- ★ Mellin moments connected to e.g., E/M radii, axial mass, spin, mass, …
- ★ GPDs are not well-constrained experimentally:
  - x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

    (SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)
  - independent measurements to disentangle GPDs
  - GPDs phenomenology more complicated than PDFs (multi-dimensionality)
  - and more challenges …

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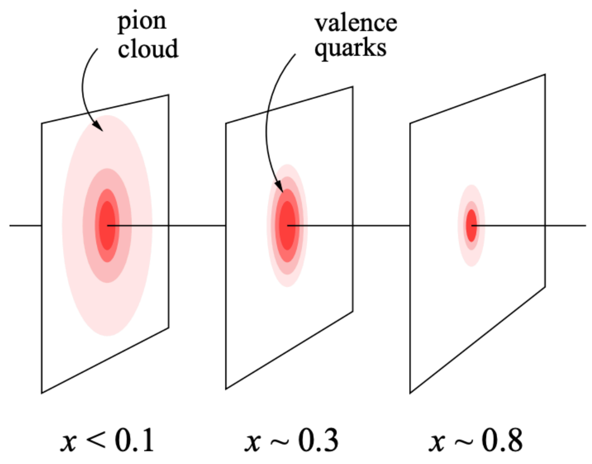

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Essential to complement the knowledge on GPD from lattice QCD

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t)-i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\}+\frac{\Delta^{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\underline{\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)}\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

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- ⇨ calculations at physical quark masses
- ⇨ precision calculations with controlled systematics (discretization, volume, excited states,…)

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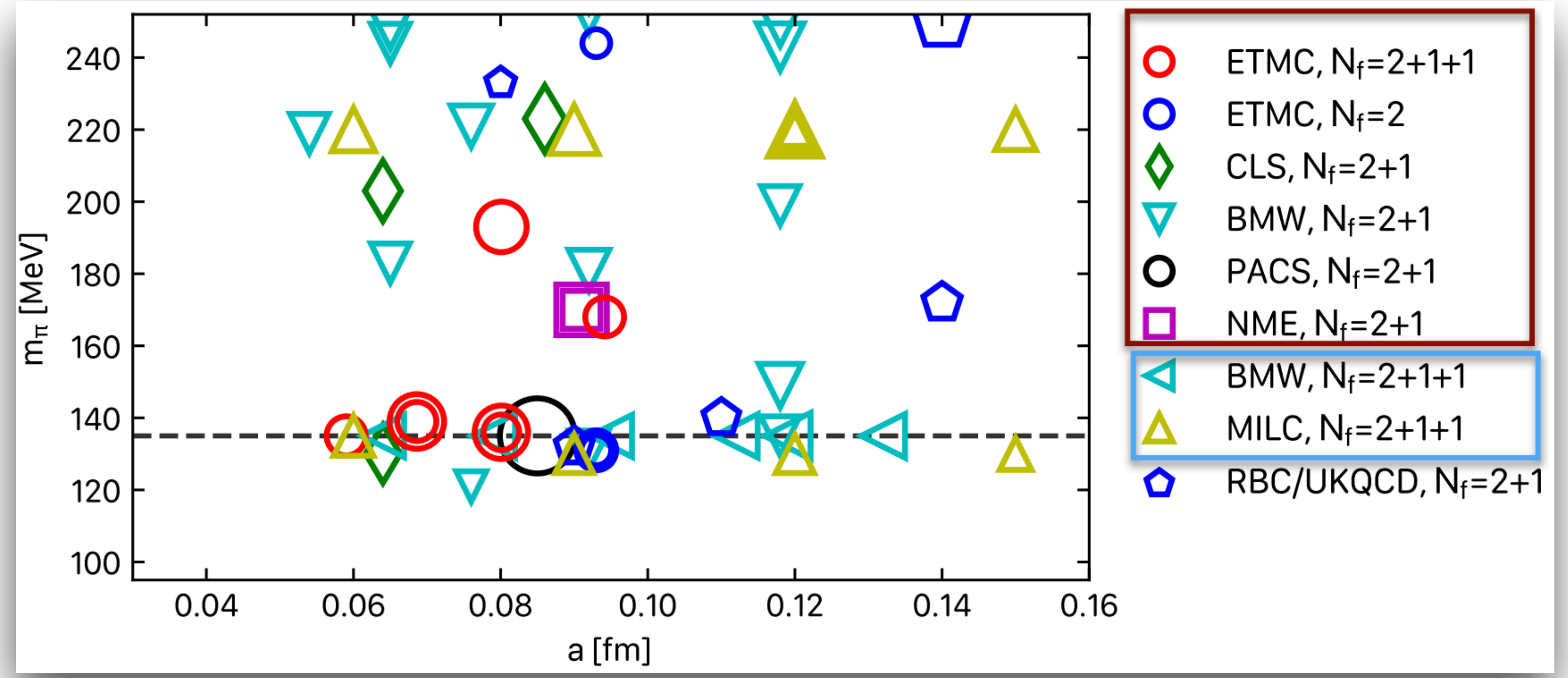

[Finkenrath, plenary talk, Lattice 2022]

Simulations for hadron structure and beyond

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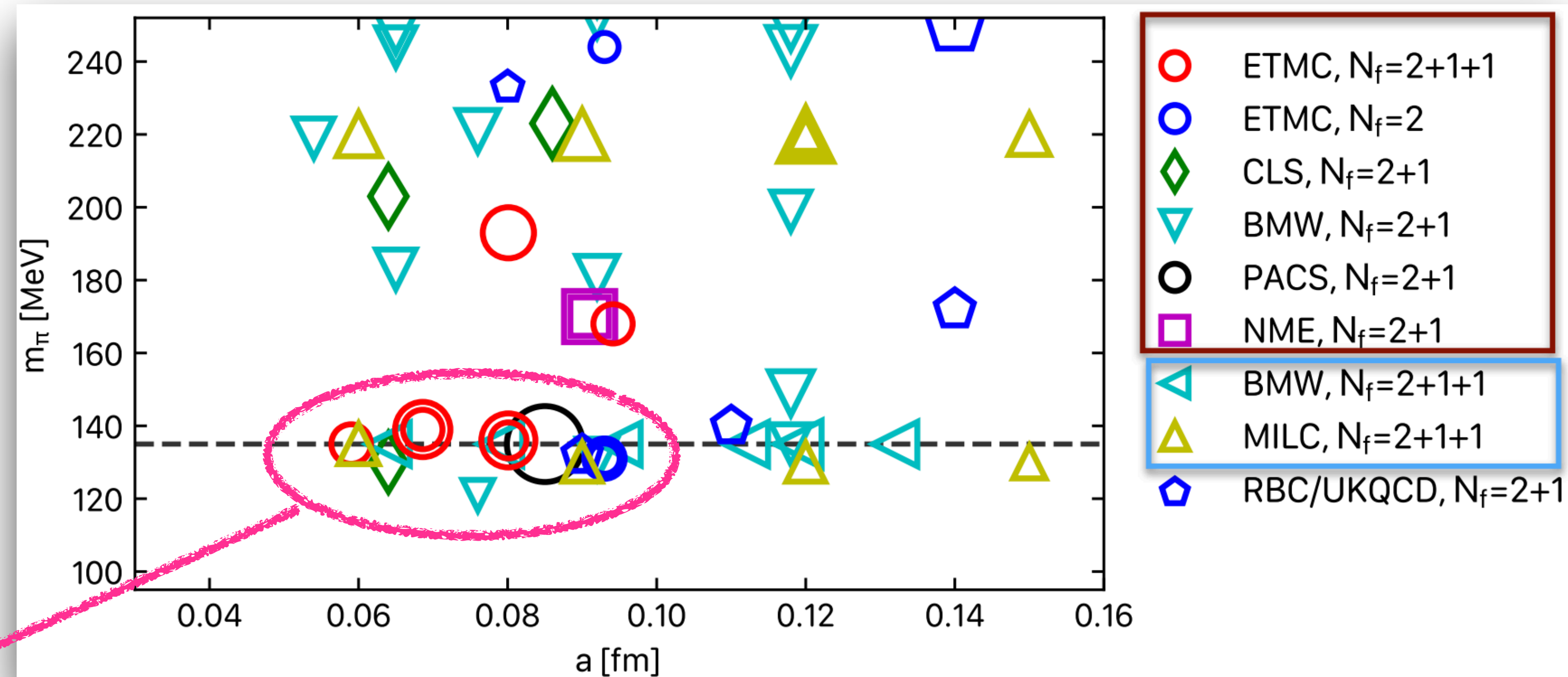
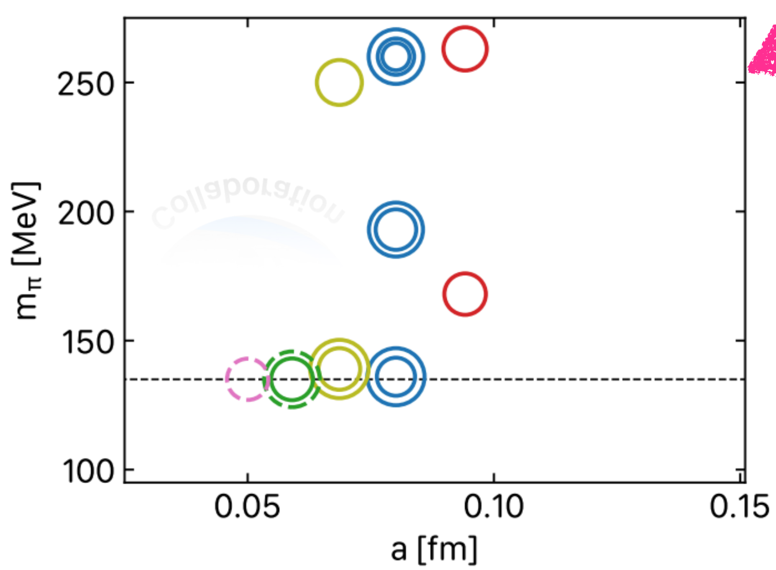


ETMC update



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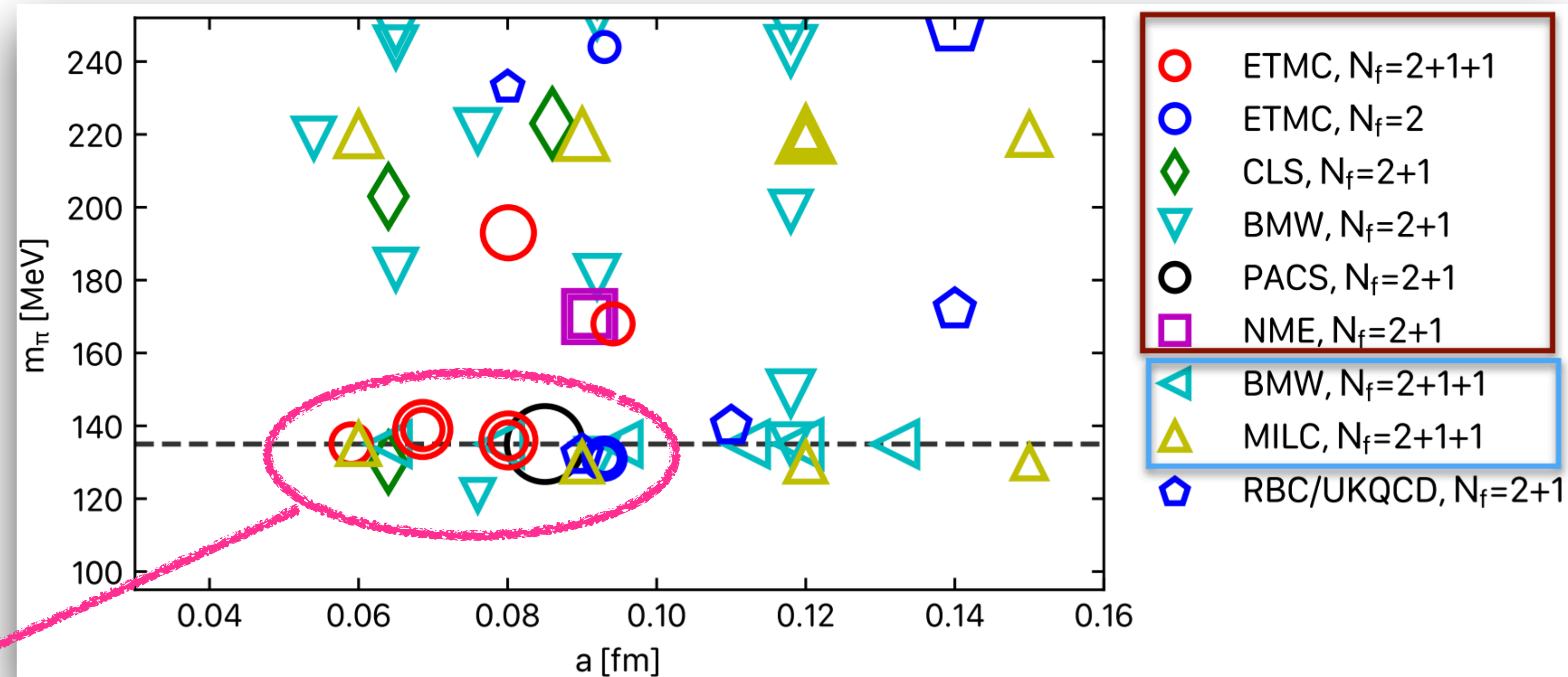

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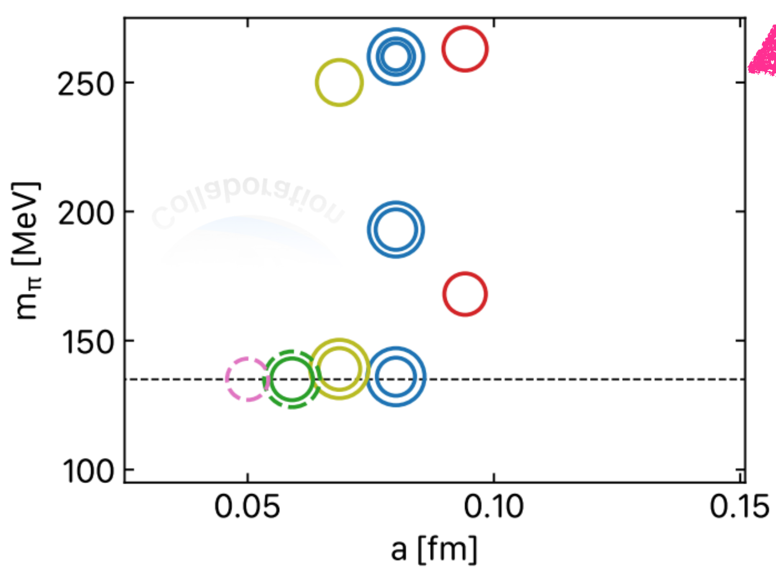

Simulations for hadron structure and beyond

Ensemble	V/a^4	β	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	1.836	0.06821(13)	136.7(2)	3.78
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Nucleon Form Factors

Axial form factors

[(ETMC) Alexandrou et al., (PRD) arXiv:2309.05774]

★ Matrix elements (including disconnected)

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

★ **Study of systematic uncertainties**

- excited states ($T_{\rm sink}$ up to ~1.6 fm)
- Q^2 parametrization (dipole, z-expansion)

$$G(Q^2) = \sum_{k=0}^{k_{\max}} a_k\, z^k(Q^2)$$

- continuum limit

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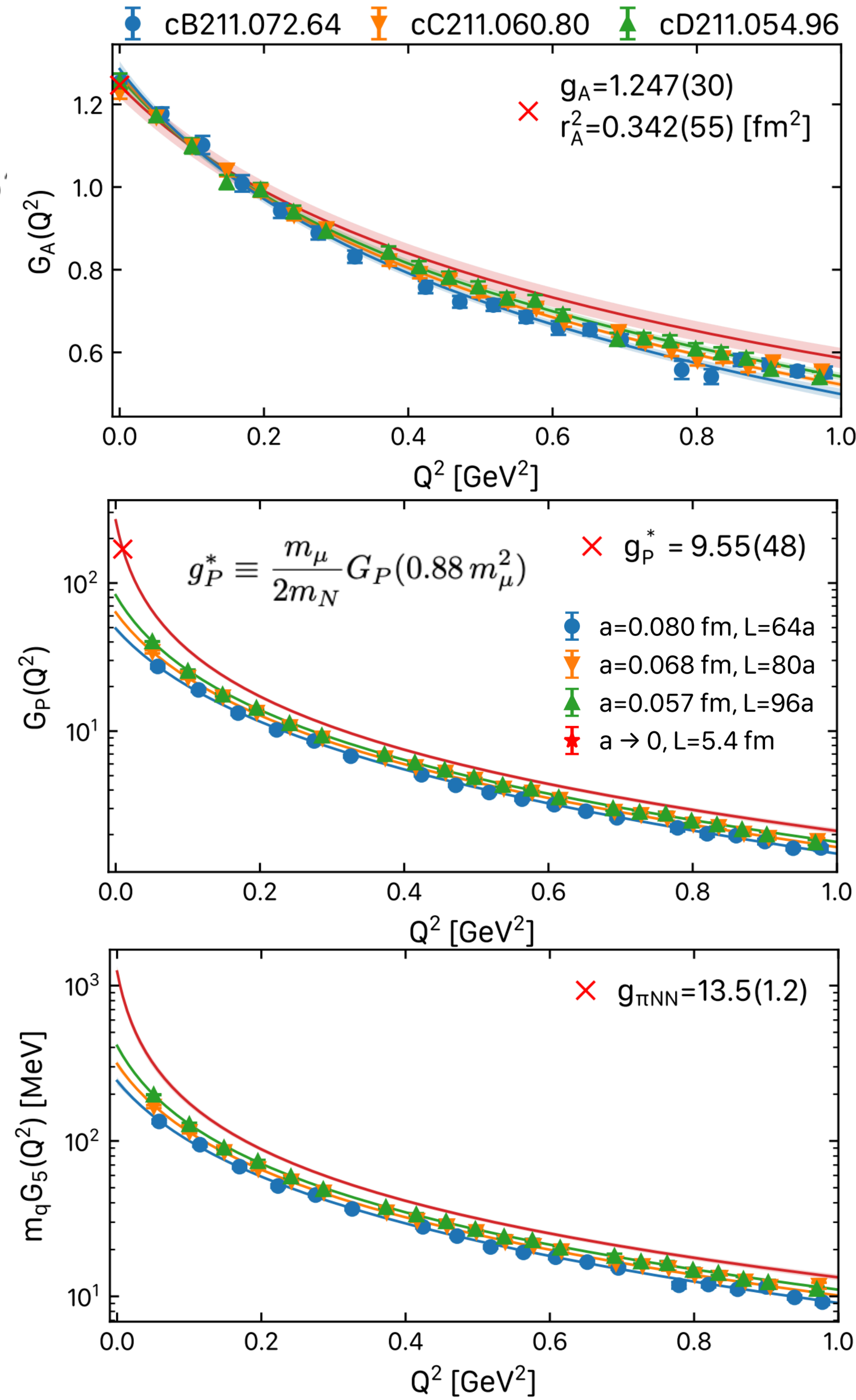

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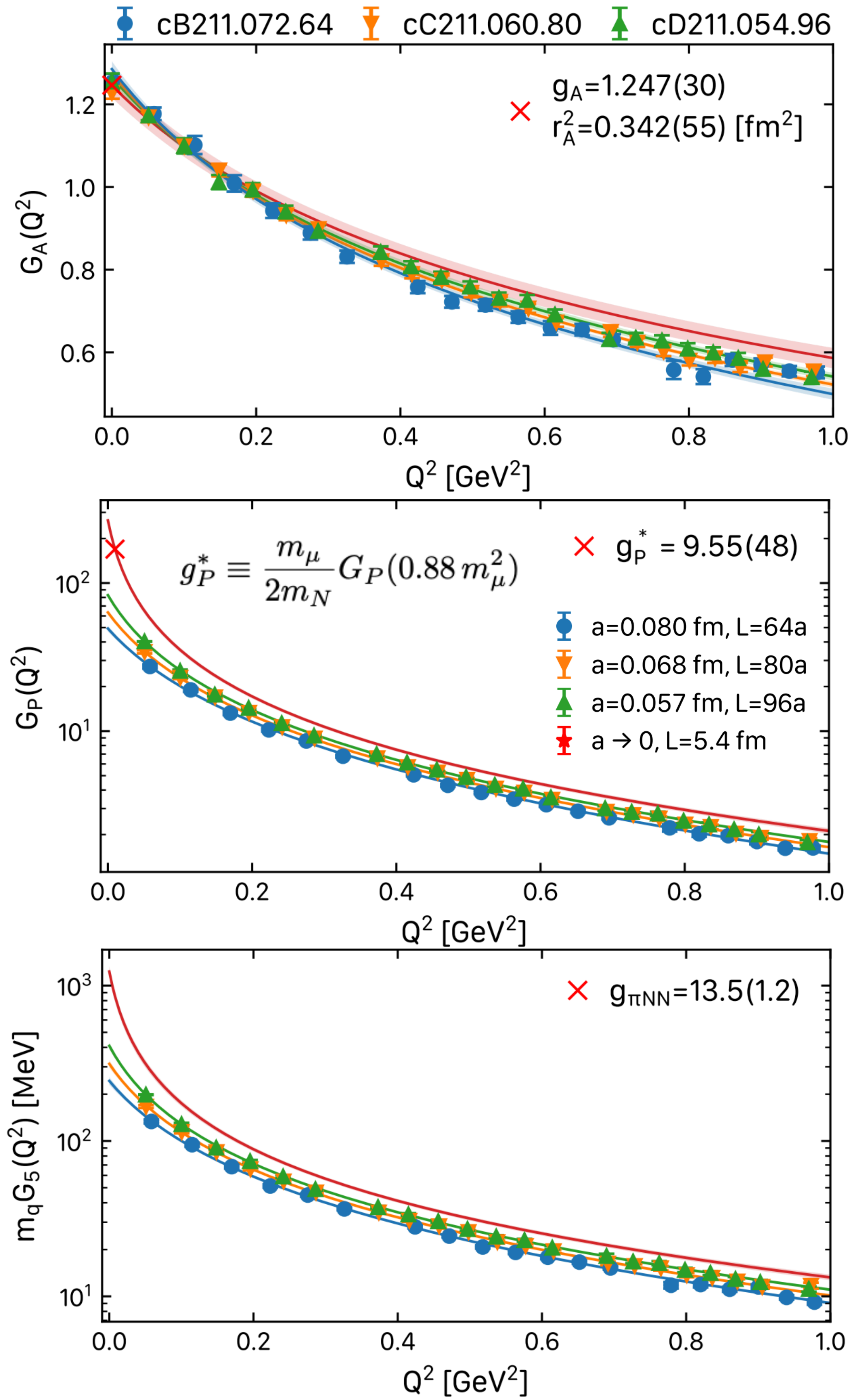
★ Pion pole dominance supported by lattice data of Gp

$$G_P(Q^2) = \frac{4m_N^2}{Q^2+m_\pi^2}G_A(Q^2)\Big|_{Q^2\to -m_\pi^2}$$



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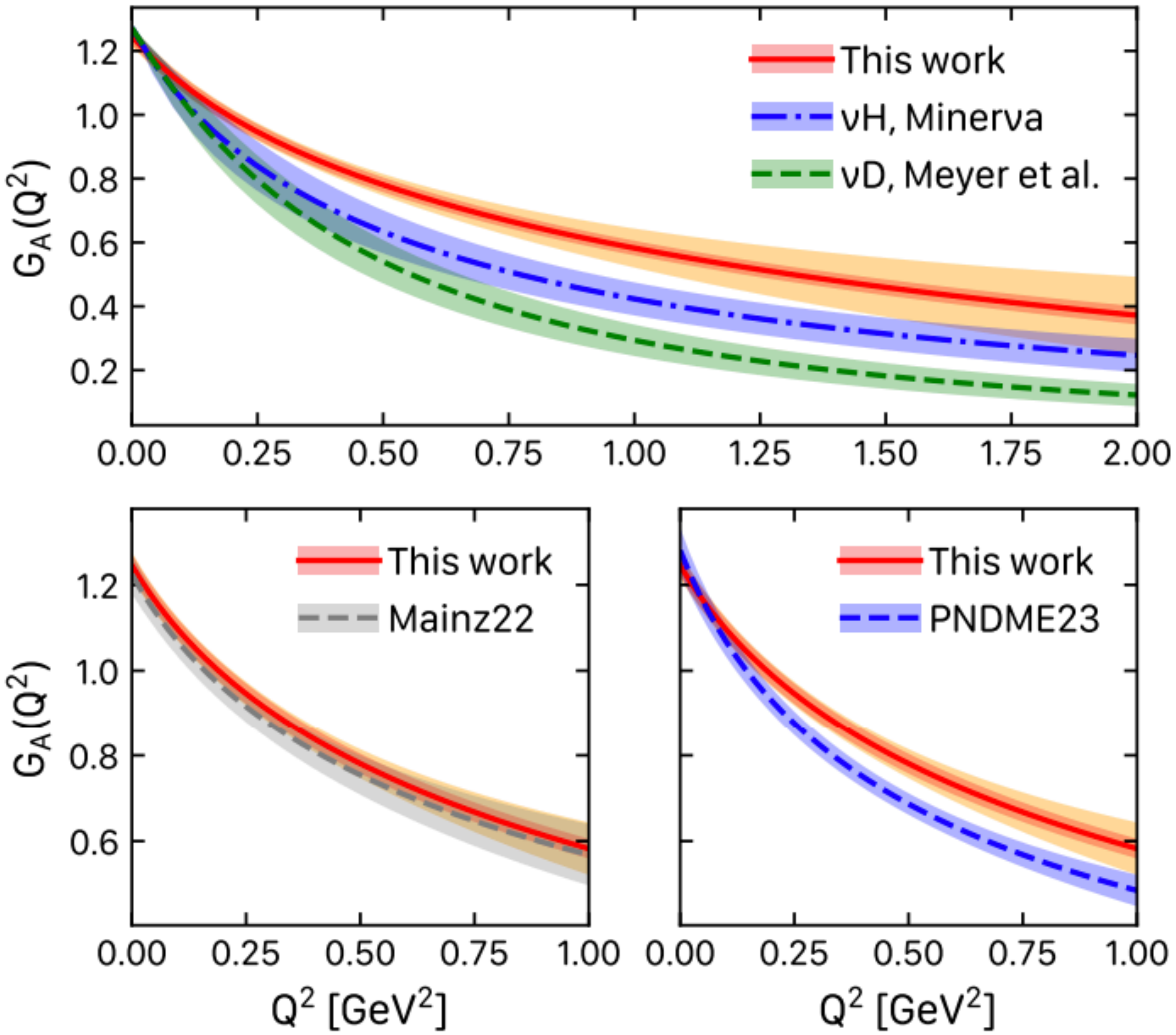
★ Comparison with other studies



★ Results closer to the new Minerva antineutrino-hydrogen data

[T. Cai et al., Nature 614, 48 (2023)]

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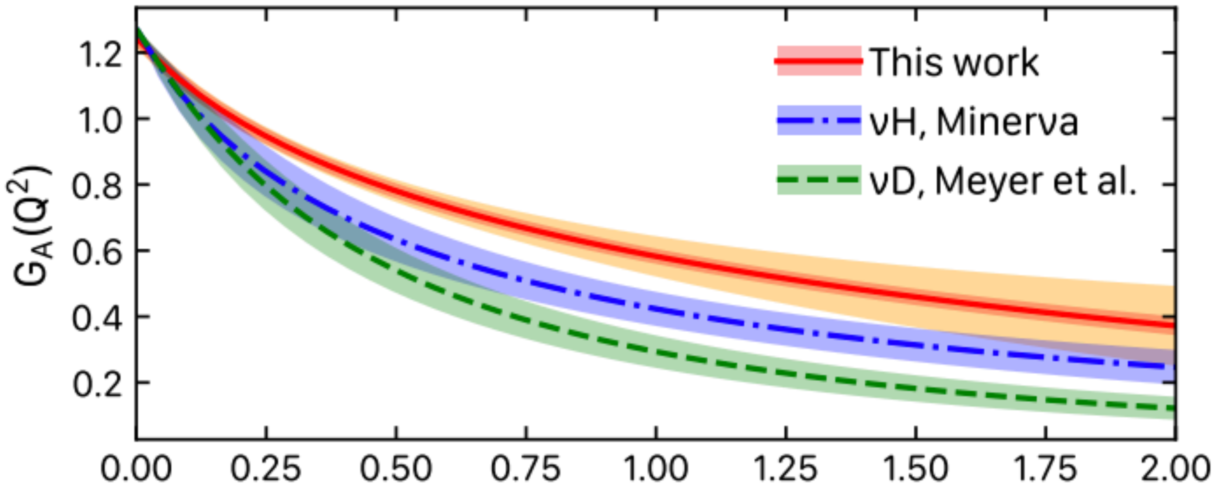

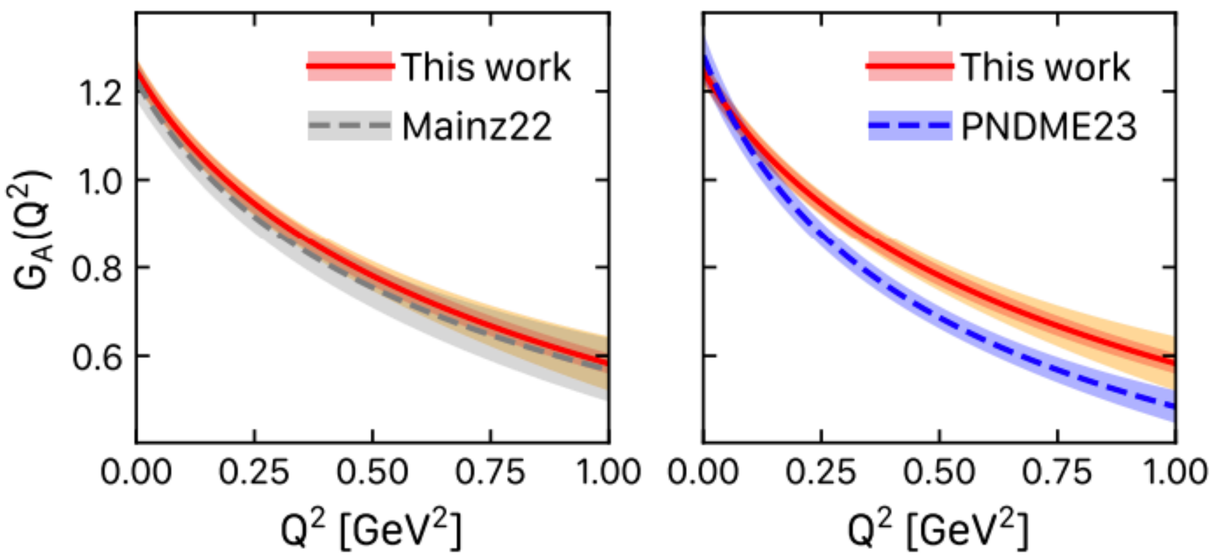

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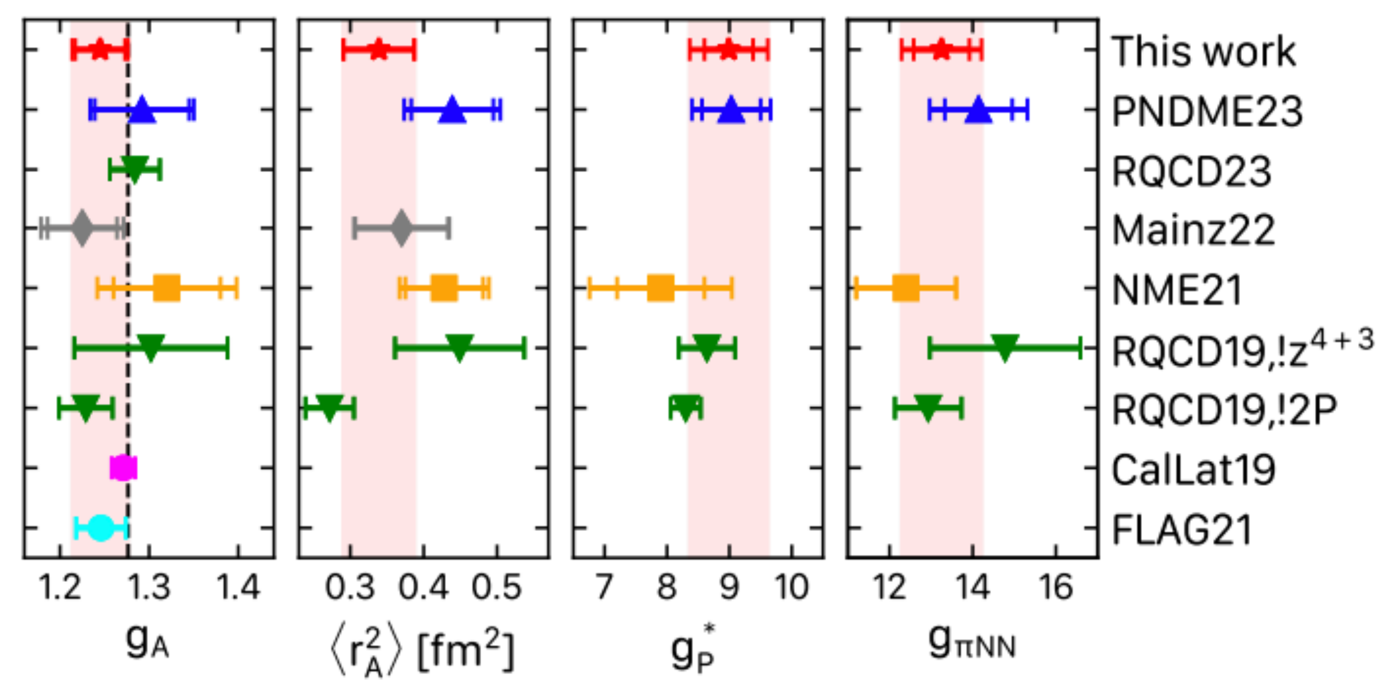
★ Axial radius

- dipole fit

$$G(Q^2) = \frac{g}{(1+\frac{Q^2}{m^2})^2} \qquad r^2 = \frac{12}{m^2}$$

- z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\rm max}} a_k \, z^k(Q^2) \qquad r^2 = -\frac{3a_1}{2a_0 t_{\rm cut}}$$



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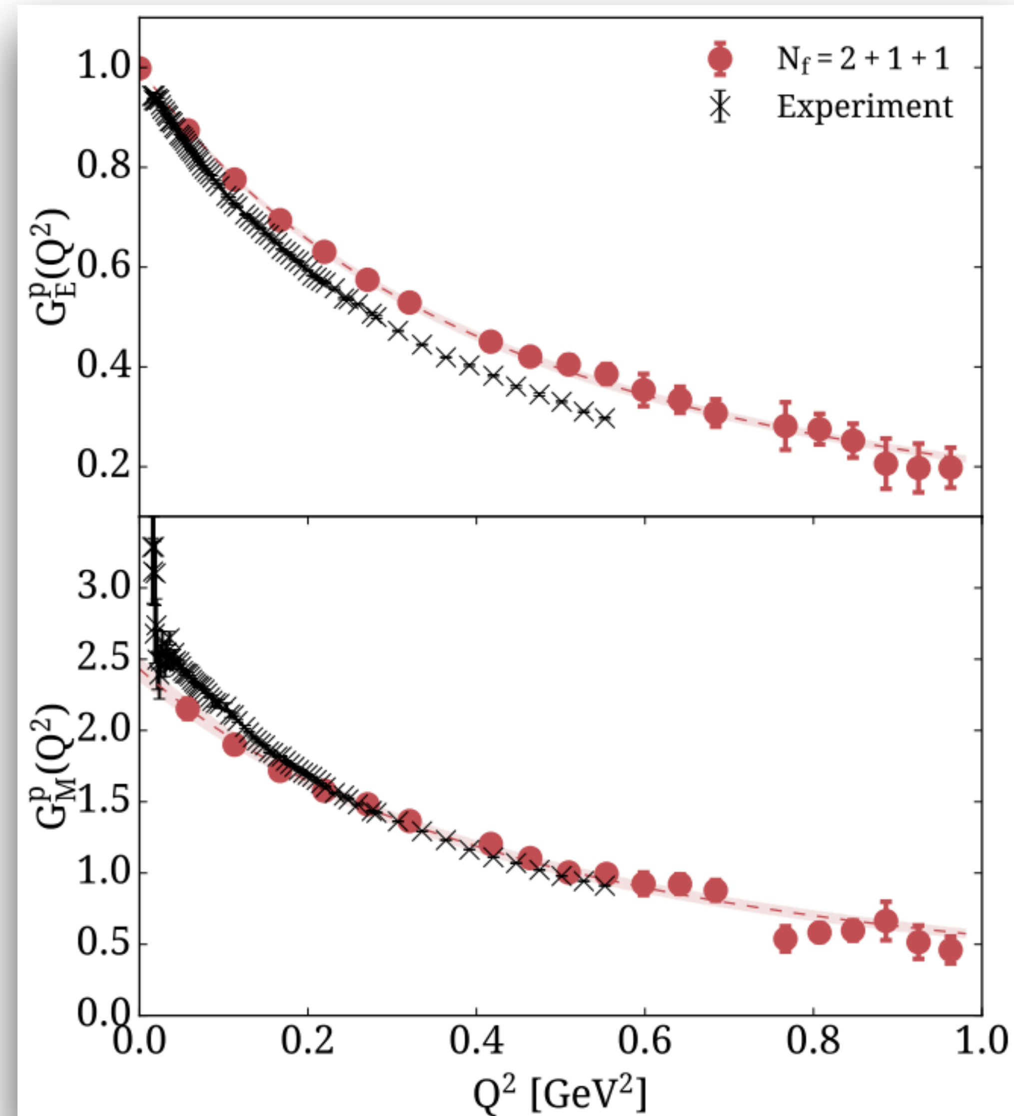

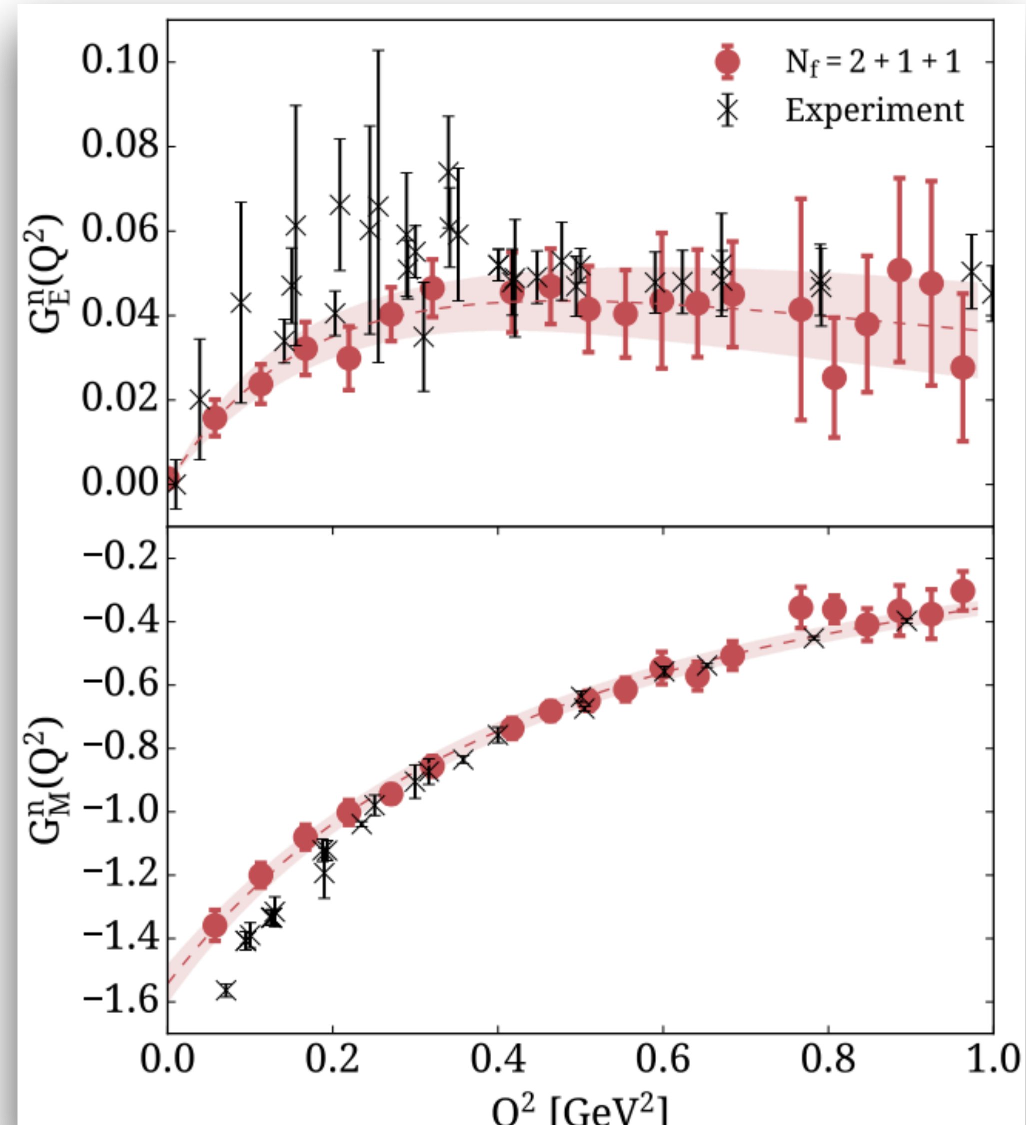

★ Results include disconnected contributions

★ High accuracy results may be valuable for experimental data

E/M form factors

★ Disconnected contributions non-negligible

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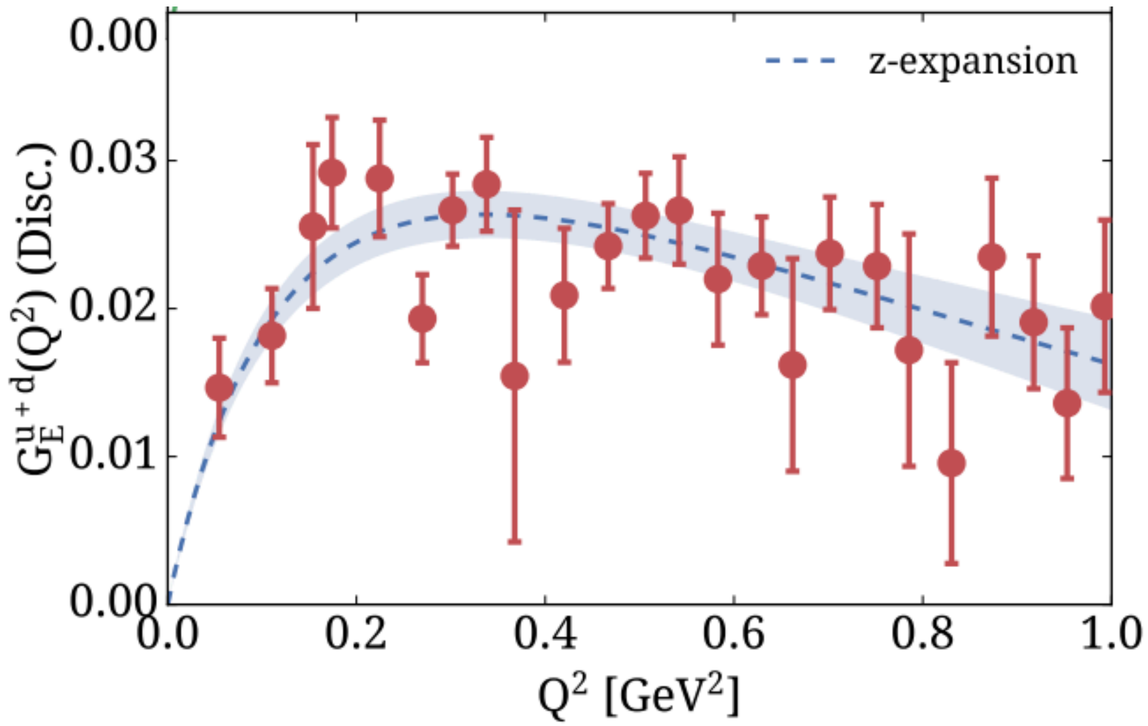

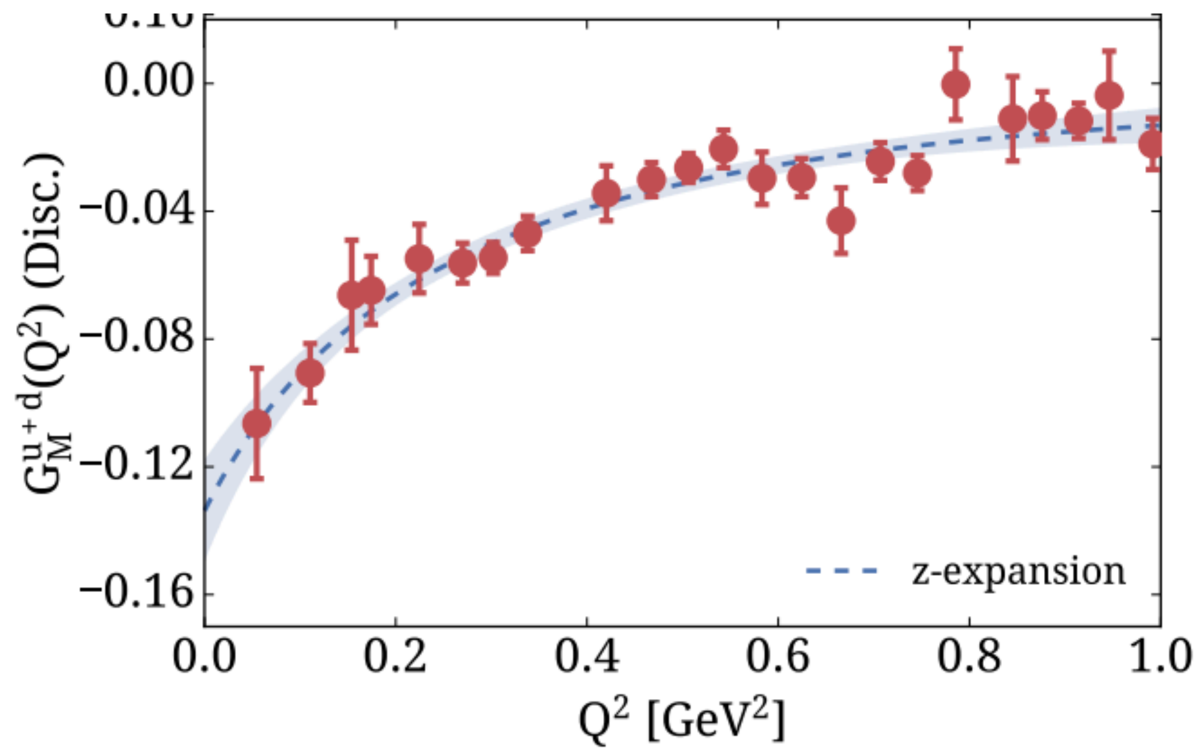

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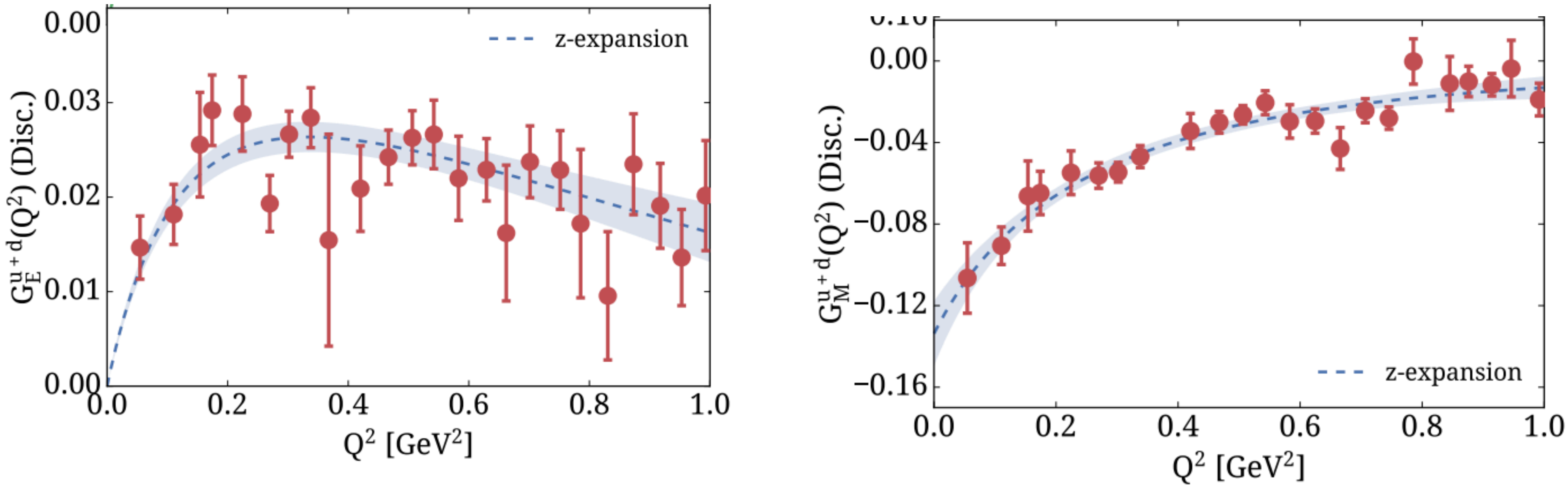
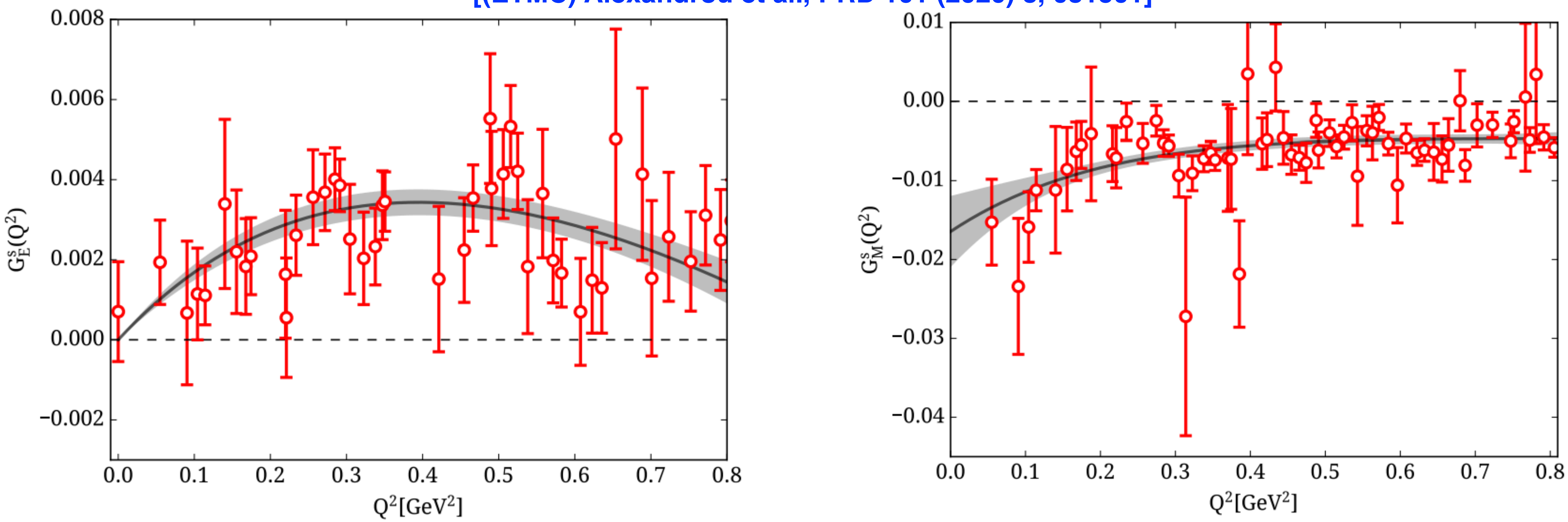
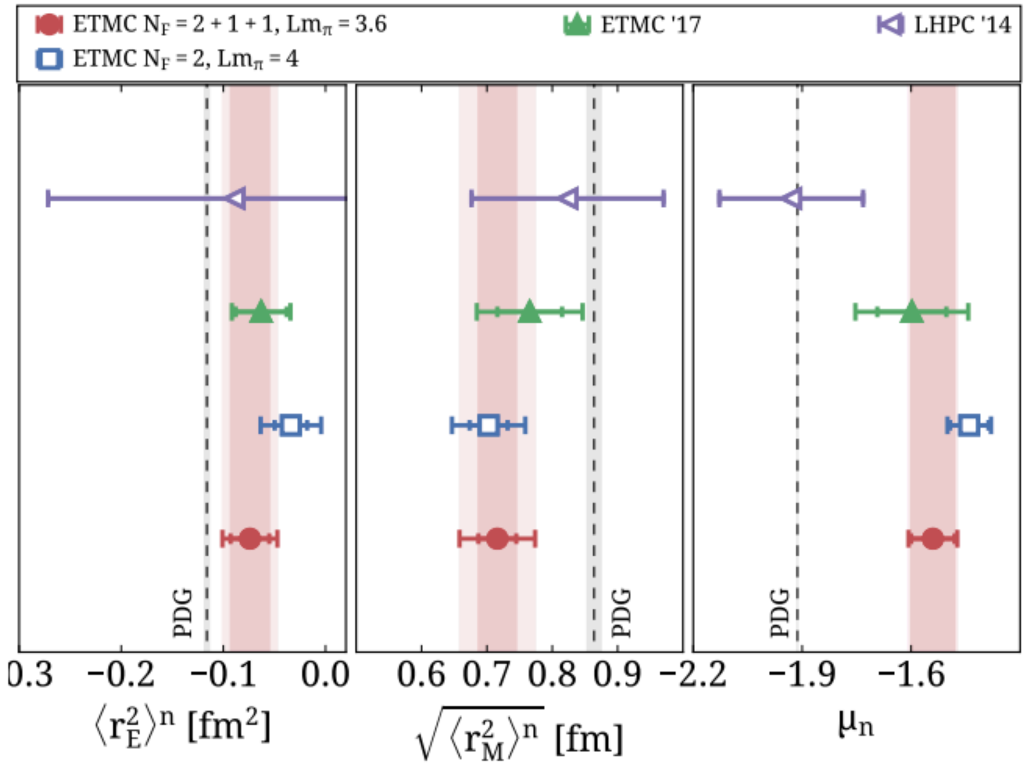
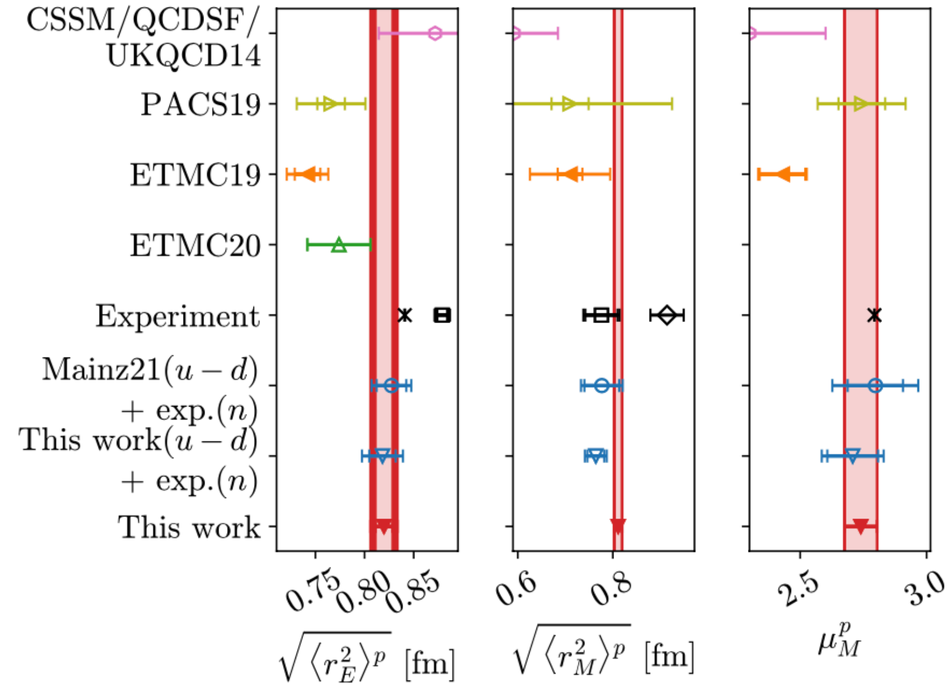


[(ETMC) Alexandrou et al., PRD 101 (2020) 3, 031501]



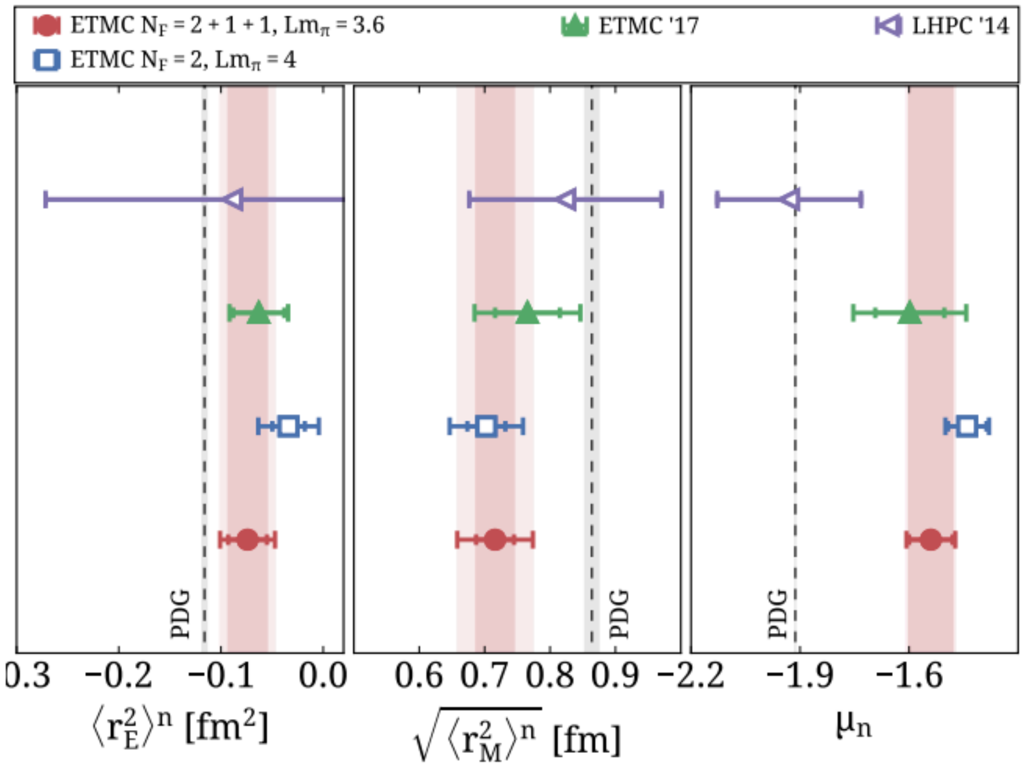
E/M form factors

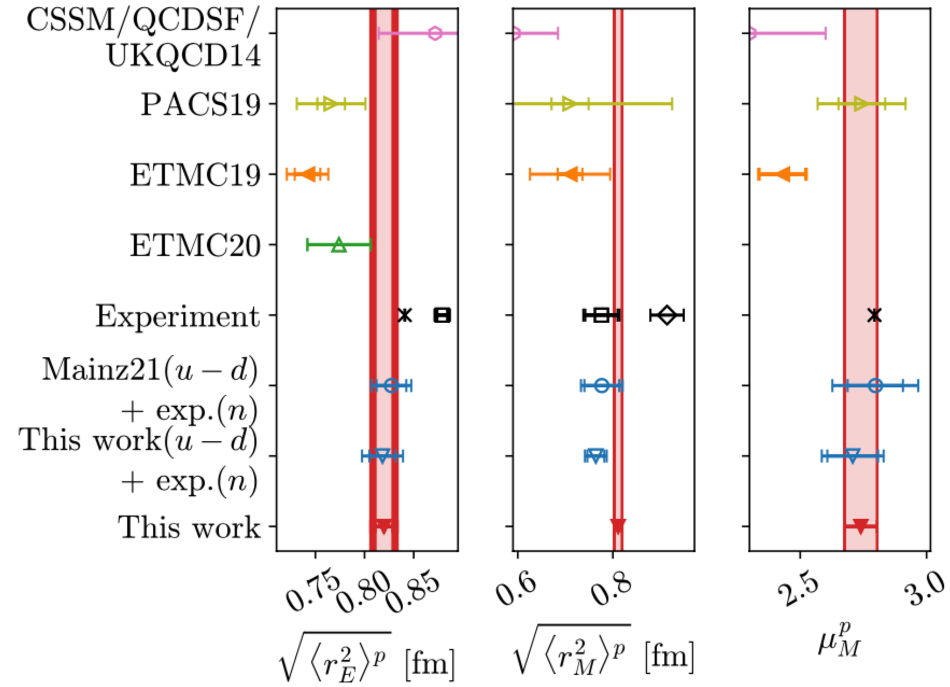




[(Mainz) Djukanovic et al., arXiv:2309.07491]

E/M form factors





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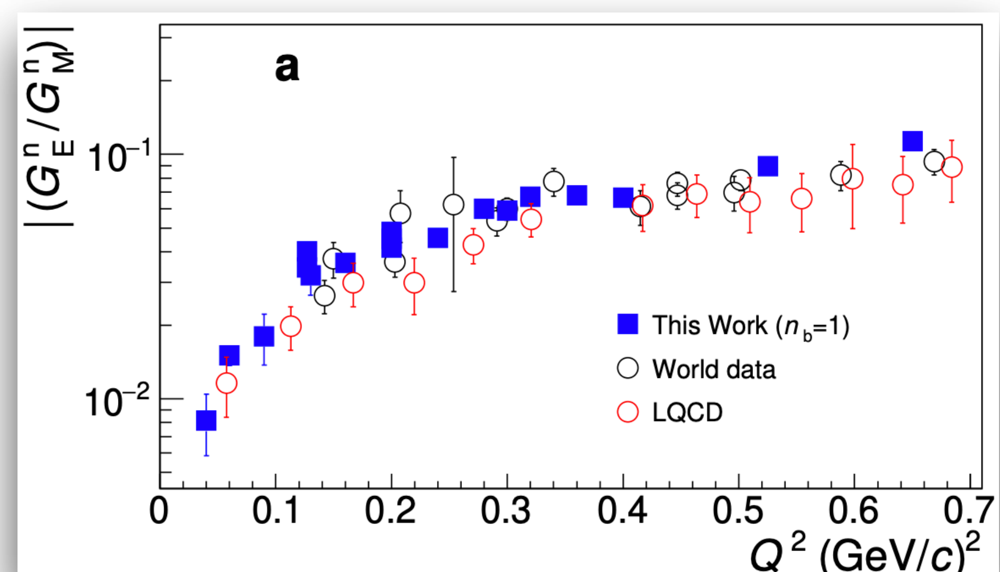
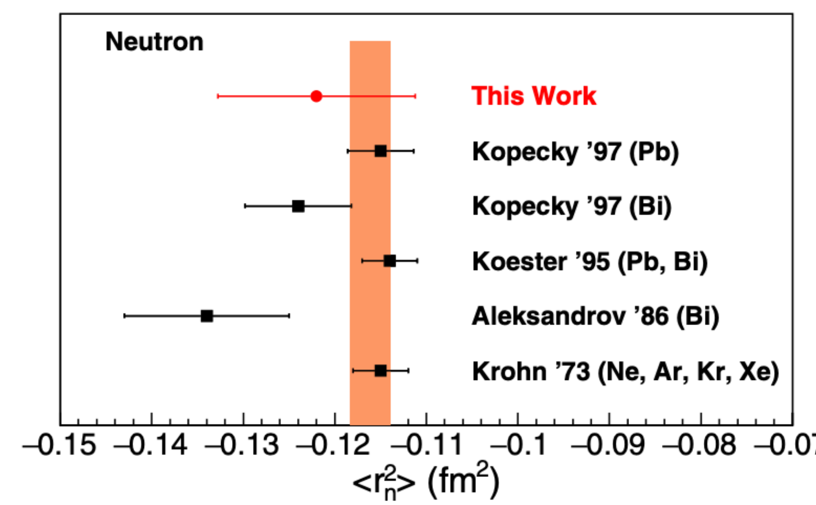
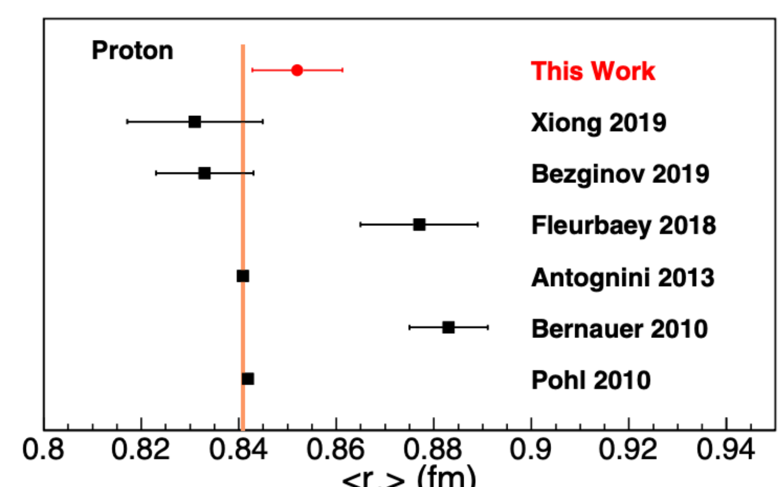
Synergy with experimental data



[Atac et al., Nature Comm. 12, 1759 (2021)]





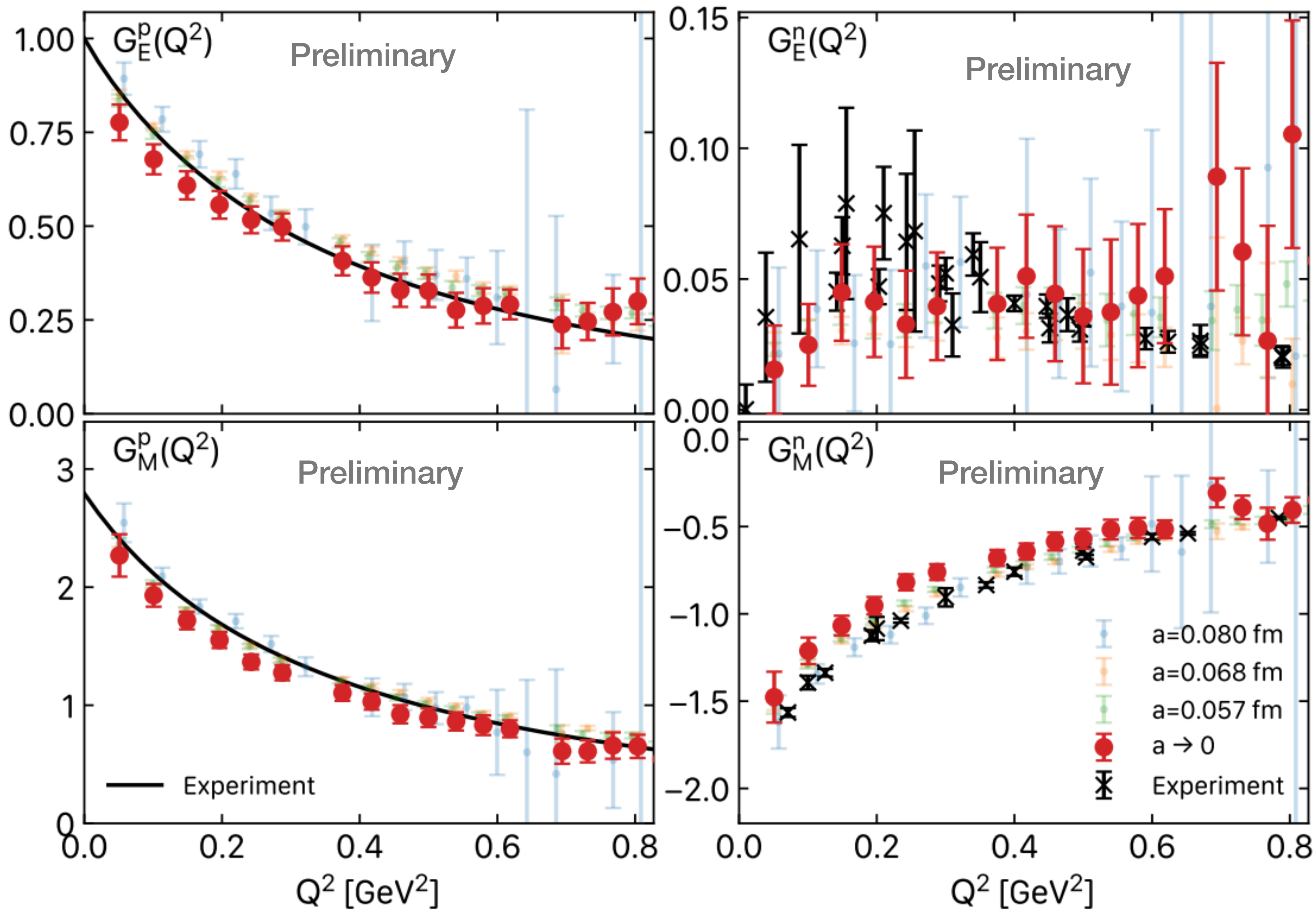
[H. Atac et al., Eur. Phys. J. A 57, 65 (2021)]

★ Coverage in regions where data is sparse

E/M form factors

★ Towards the continuum limit

[(ETMC) C. Alexandrou et al., PoS(LATTICE2022)114 (2023)]



★ Results are promising

★ Next step: extraction of radii (coming soon)

GPDs

Through non-local matrix elements

of fast-moving hadrons

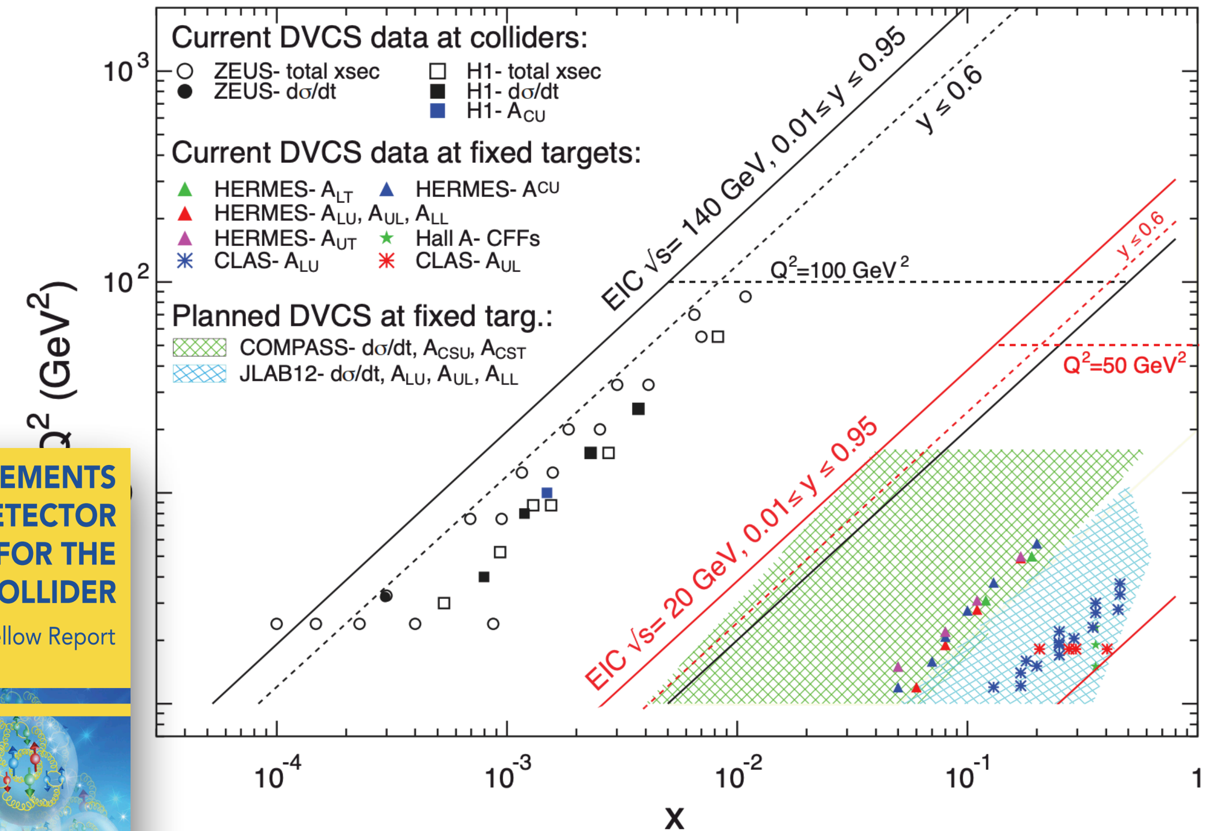
Hadron structure at core of nuclear physics

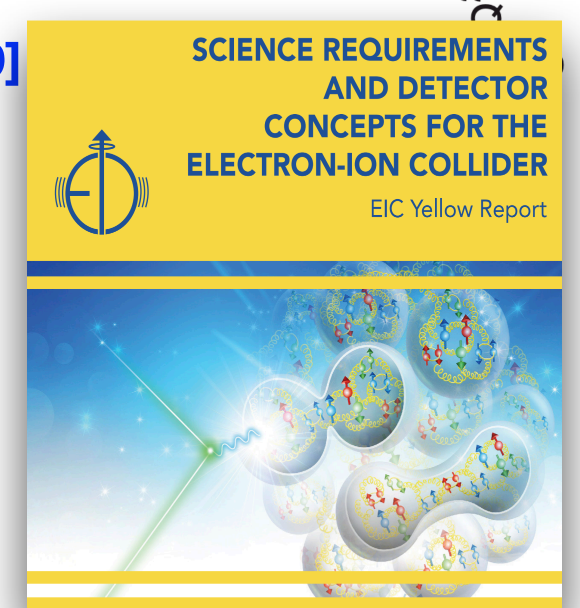
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

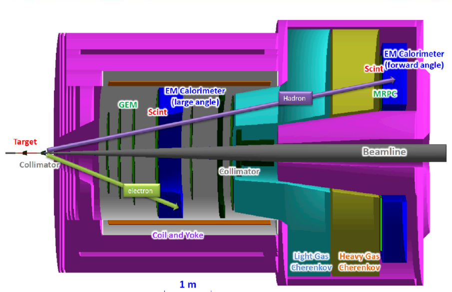
[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





SoLID
SOLENOIDAL LARGE INTENSITY DEVICE



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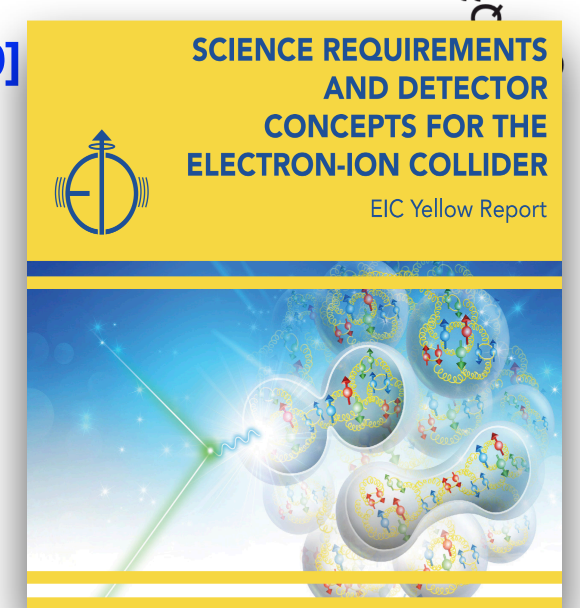

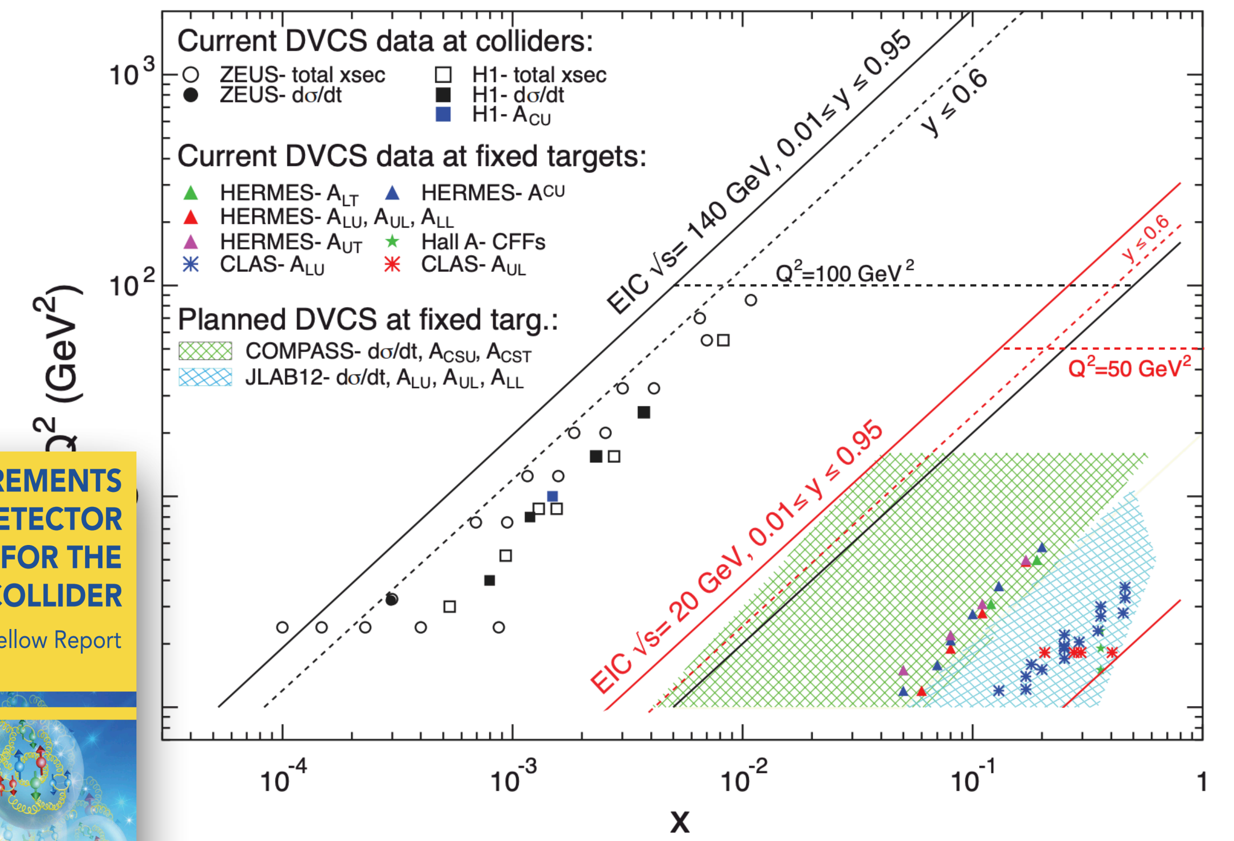

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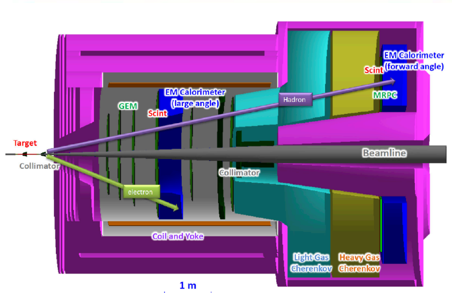
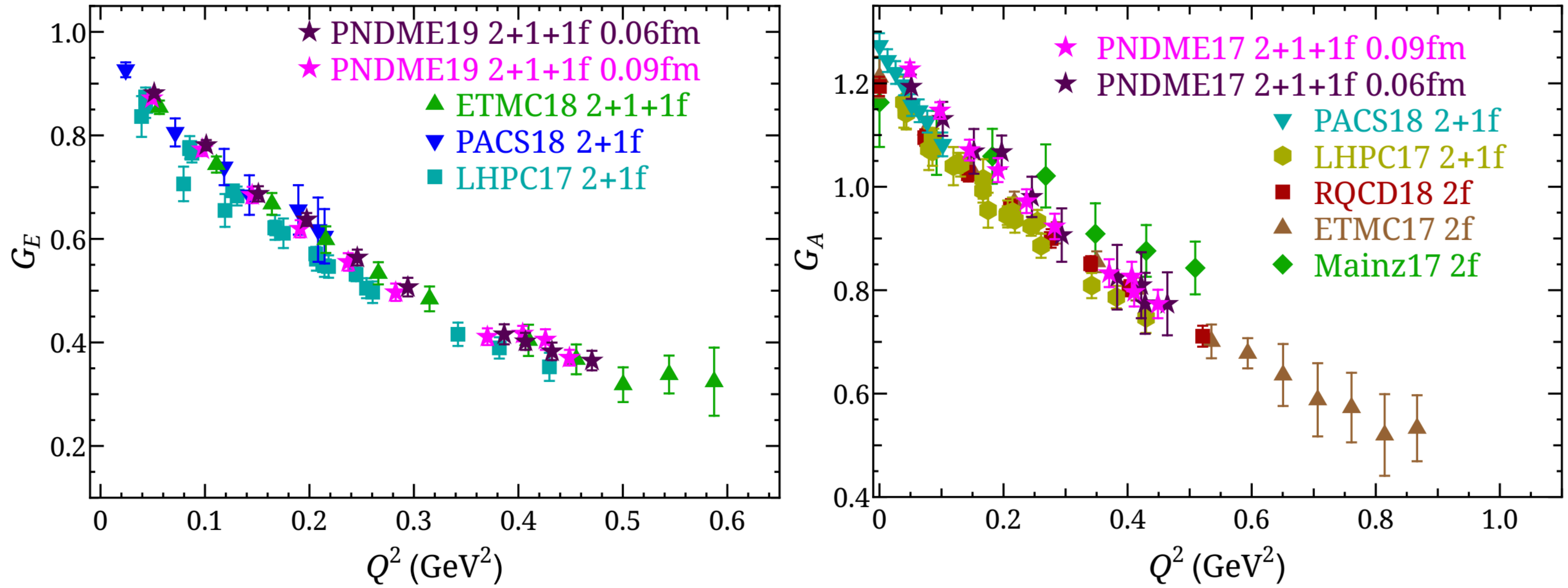


"The SoLID Collaboration should investigate the feasibility of carrying out a competitive GPD program. Such a program would seem particularly well suited to their open geometry and high luminosity."

Director's Review 2015

★ Form factors



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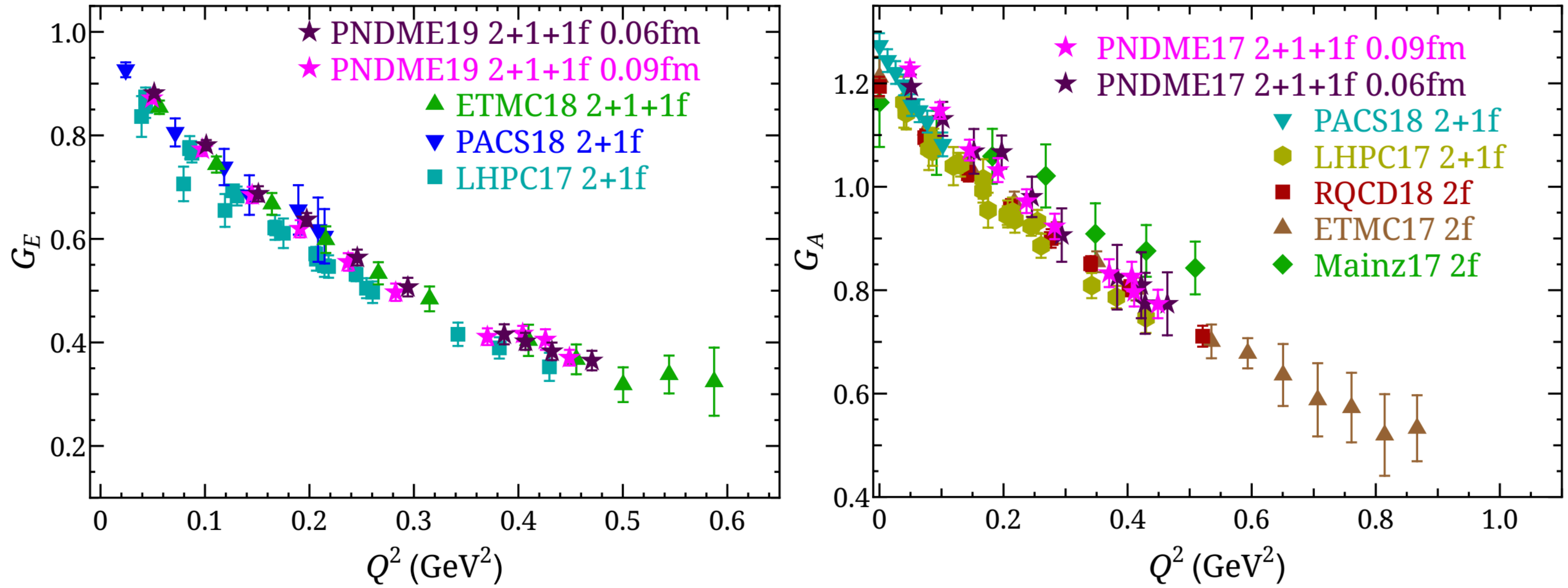

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- Precision data era (control of systematic uncertainties)

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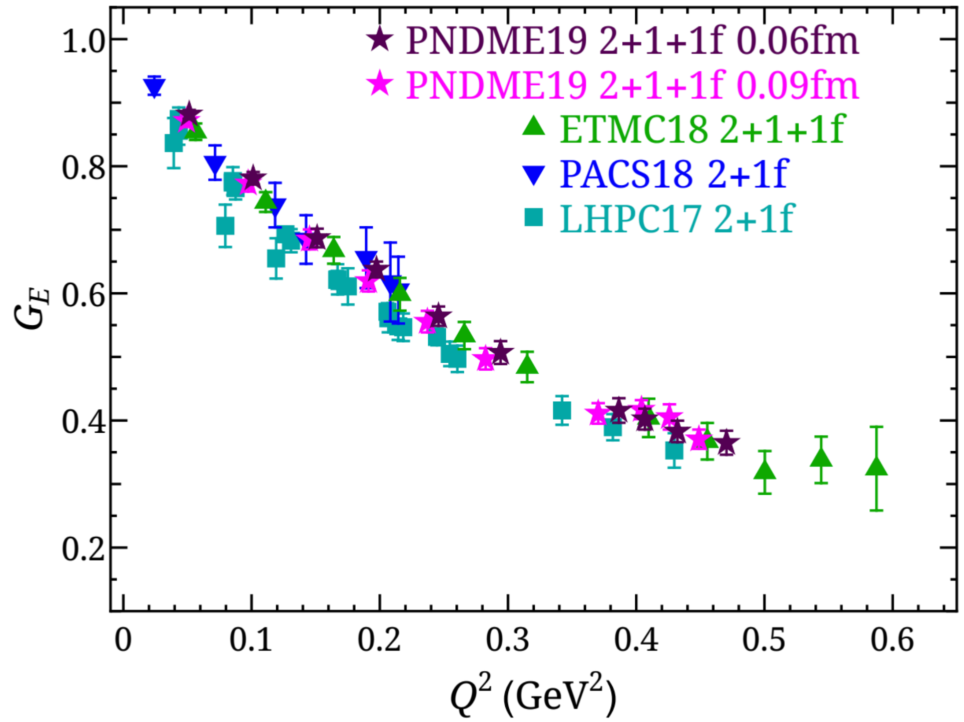

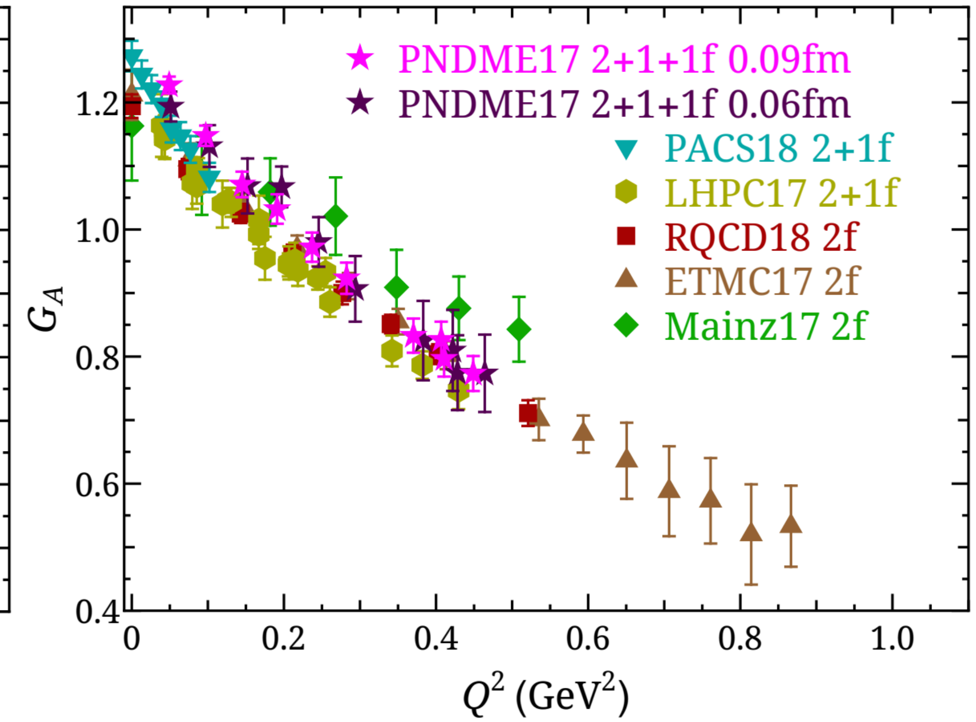

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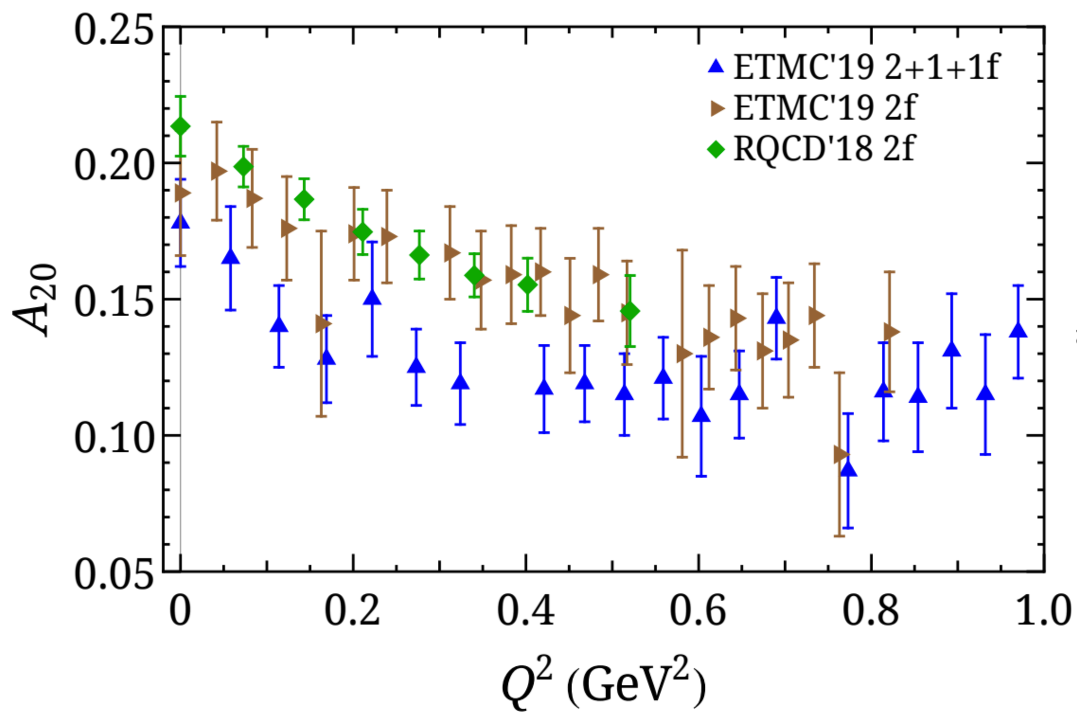
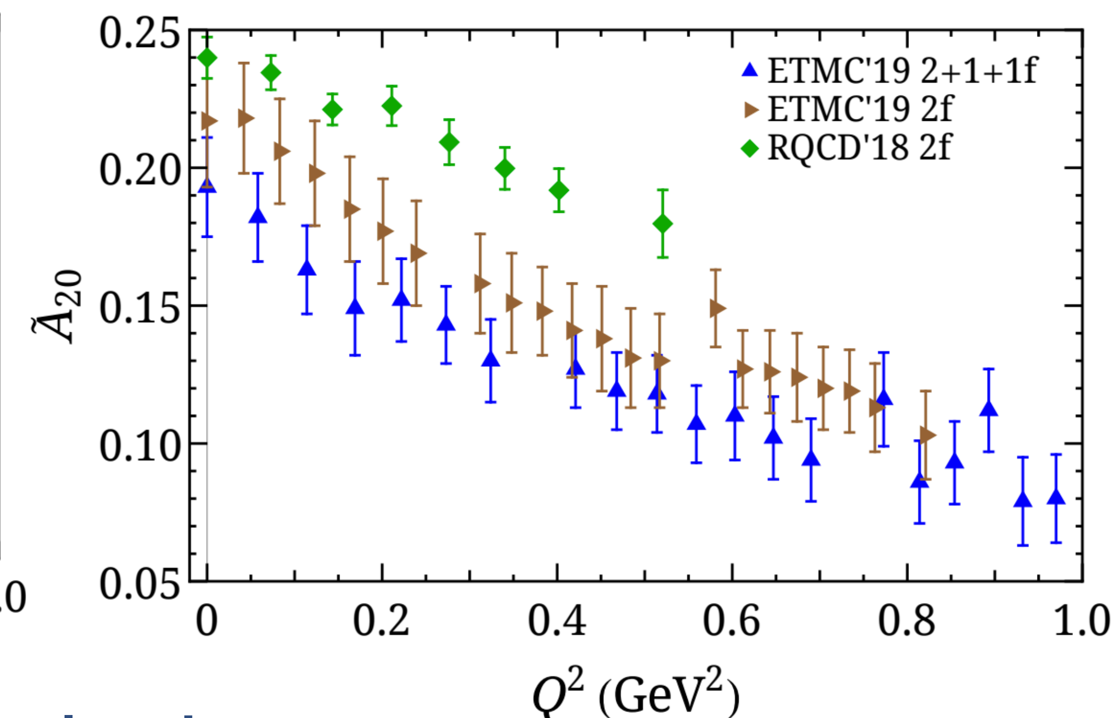
★ Generalized form factors





- Lesser studied compared to FFs at physical point
- Decay of signal-to-noise ratio

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

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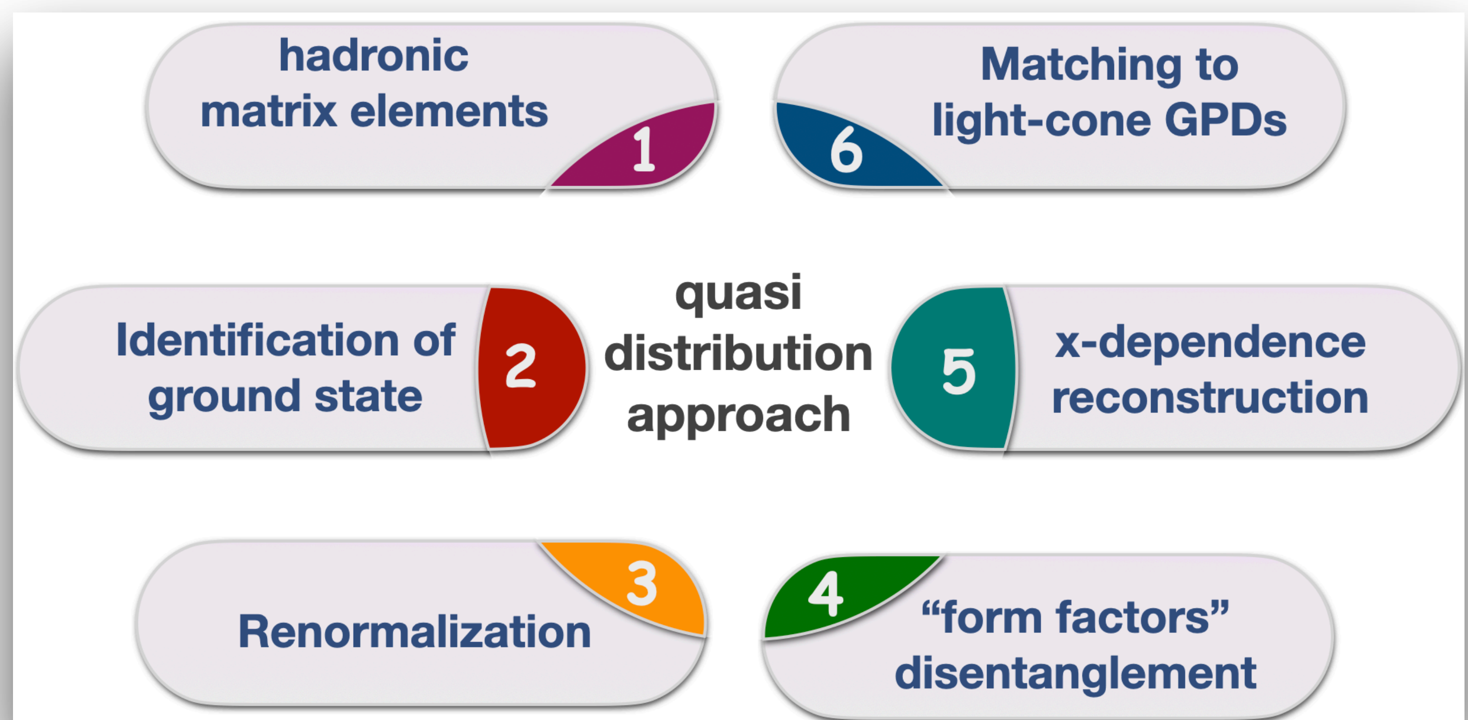
Accessing -t dependence: Computationally intensive



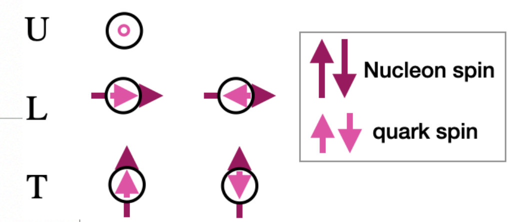
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) **Twist-3** $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
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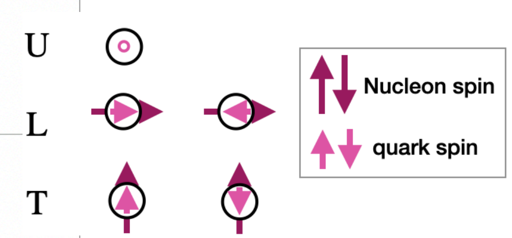

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- as sizable as twist-2
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- appear in QCD factorization theorems for various observables (e.g. g_2)
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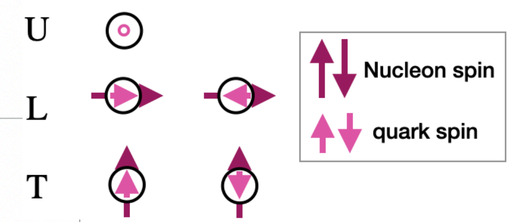

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Twist-2 GPDs

★ γ^+ inspired parametrization (symmetric frame)

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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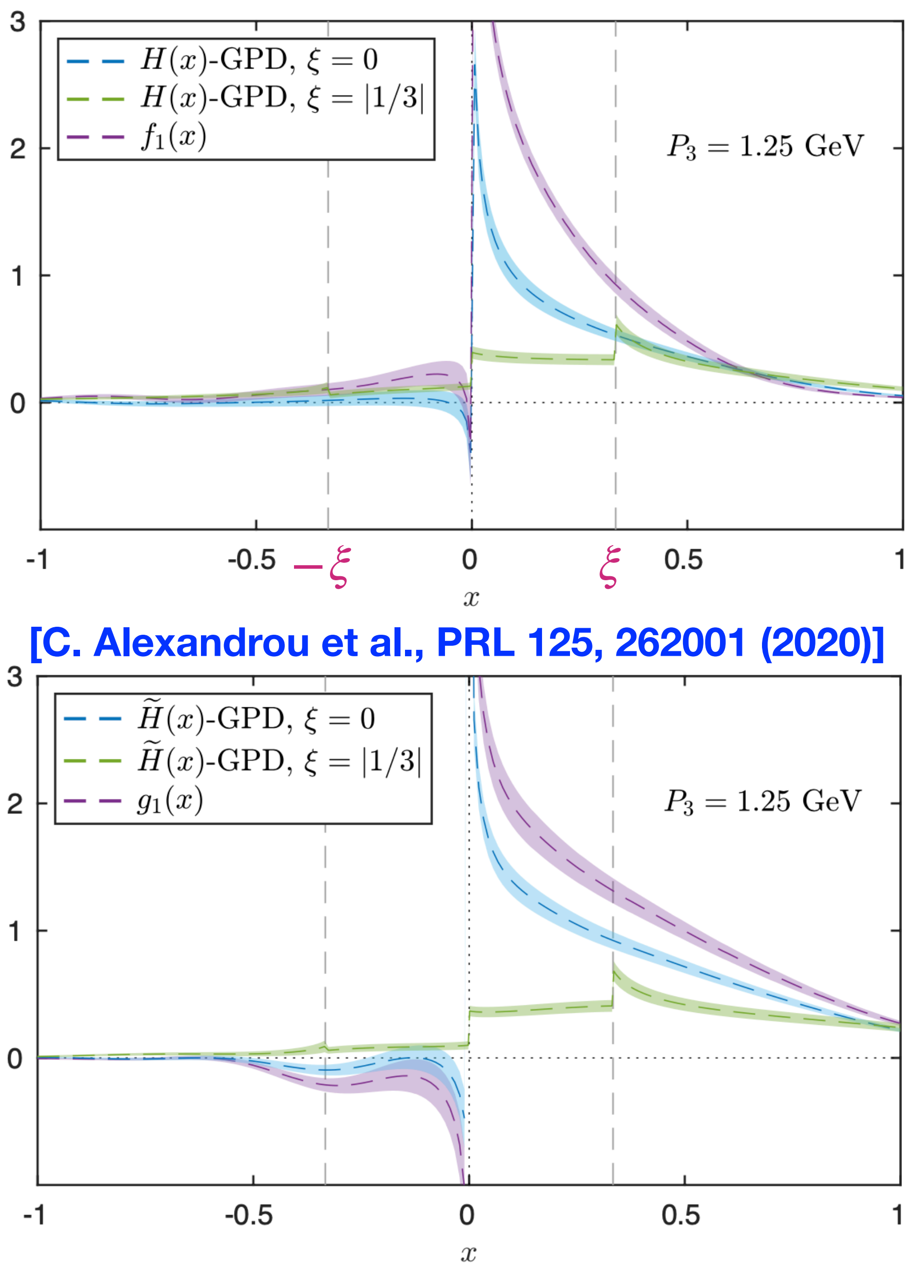

[C. Alexandrou et al., PRL 125, 262001 (2020)]

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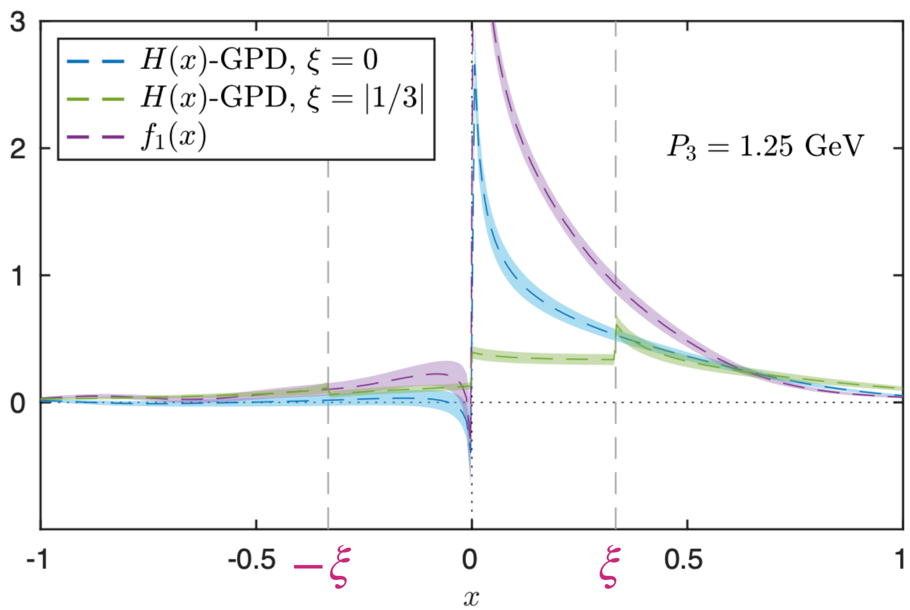

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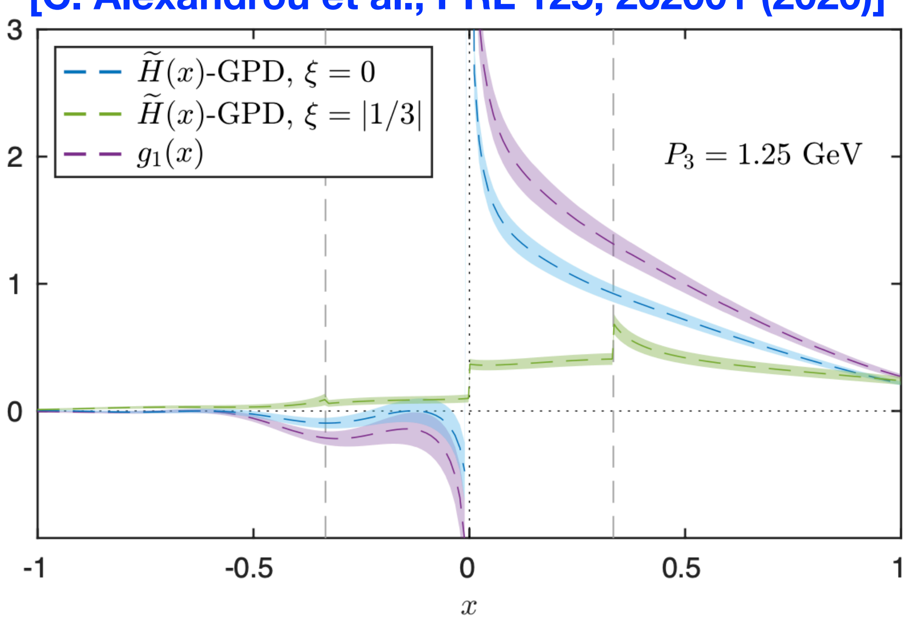

power counting analysis of GPDs ($x \to 1$)

[F. Yuan, Phys.Rev. D69 (2004) 051501, hep-ph/0311288]

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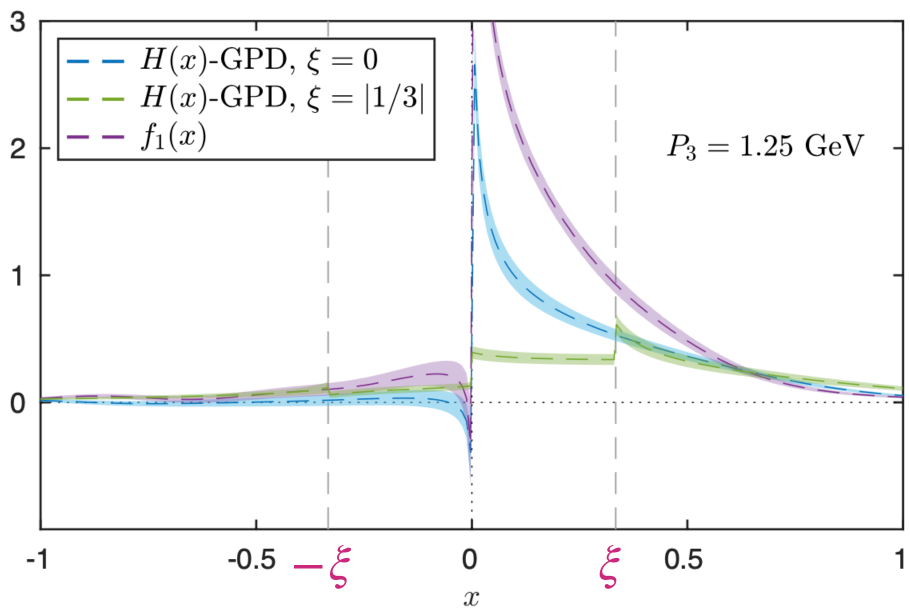

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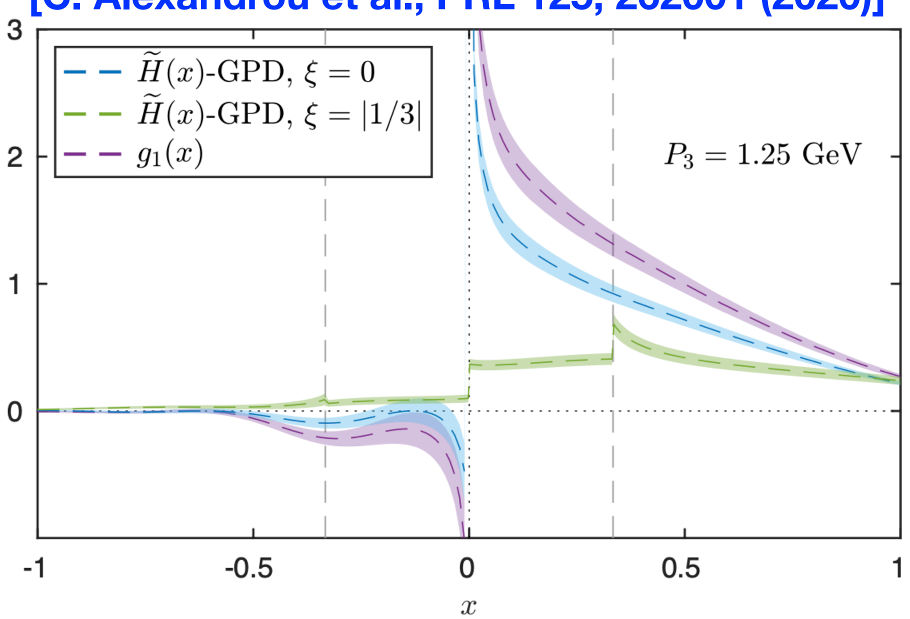

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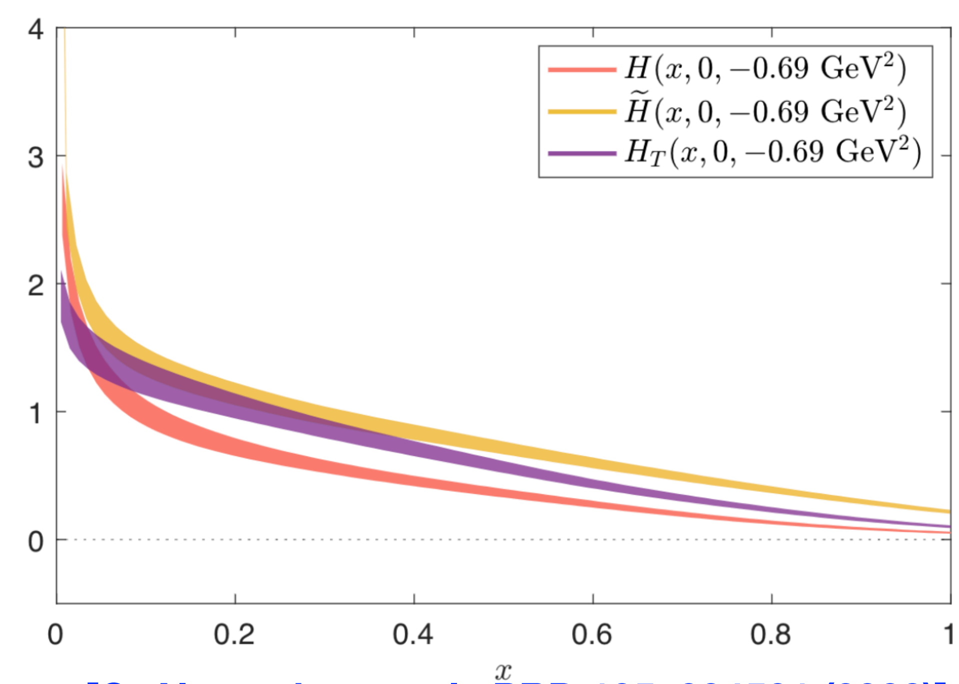

[C. Alexandrou et al., PRD 105, 034501 (2022)]

Twist-3 GPDs

PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1]
Aurora Scapellato,[1] and Fernanda Steffens[4]

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Bigg[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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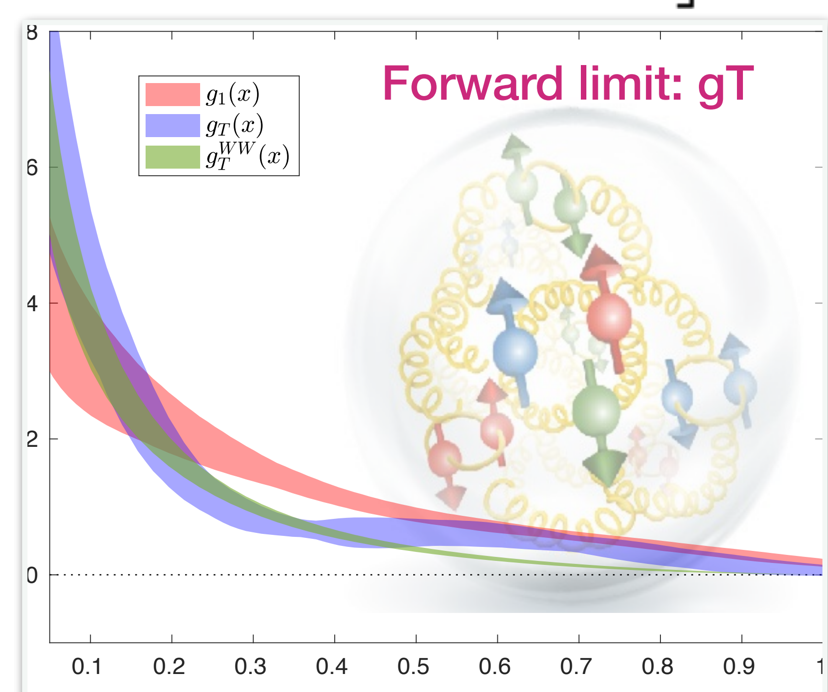

[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
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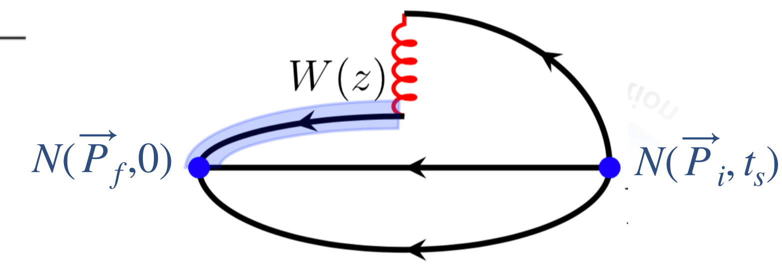

★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
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±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
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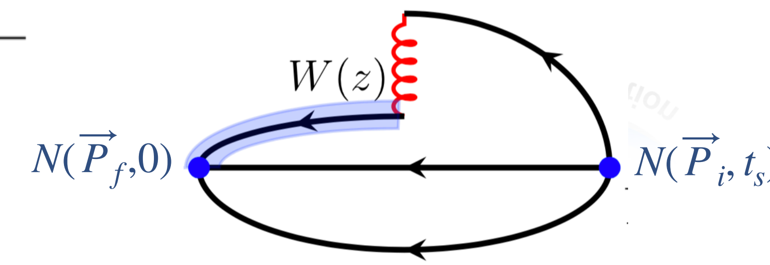

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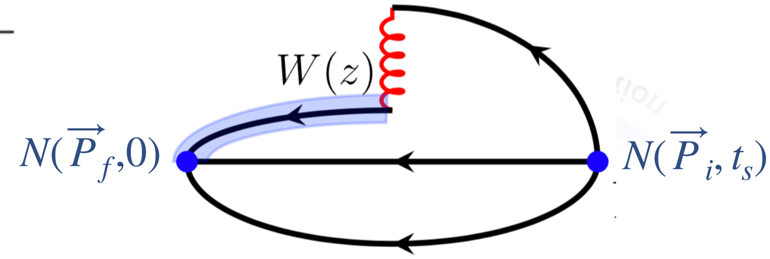

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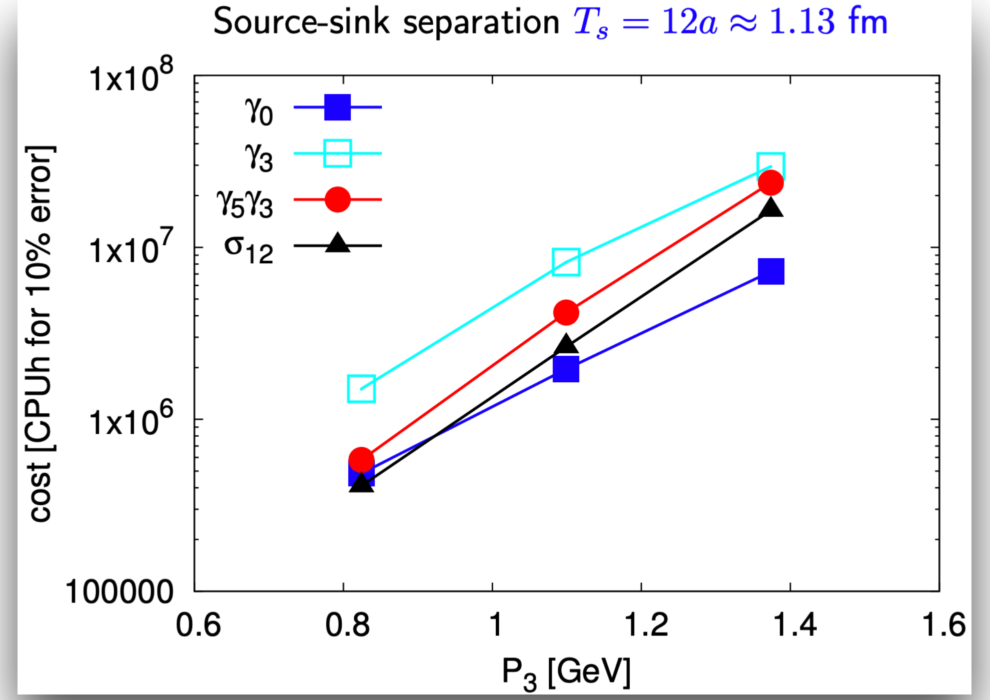

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! Suppressing gauge noise and reliably extracting the ground state comes at a significant computational cost

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

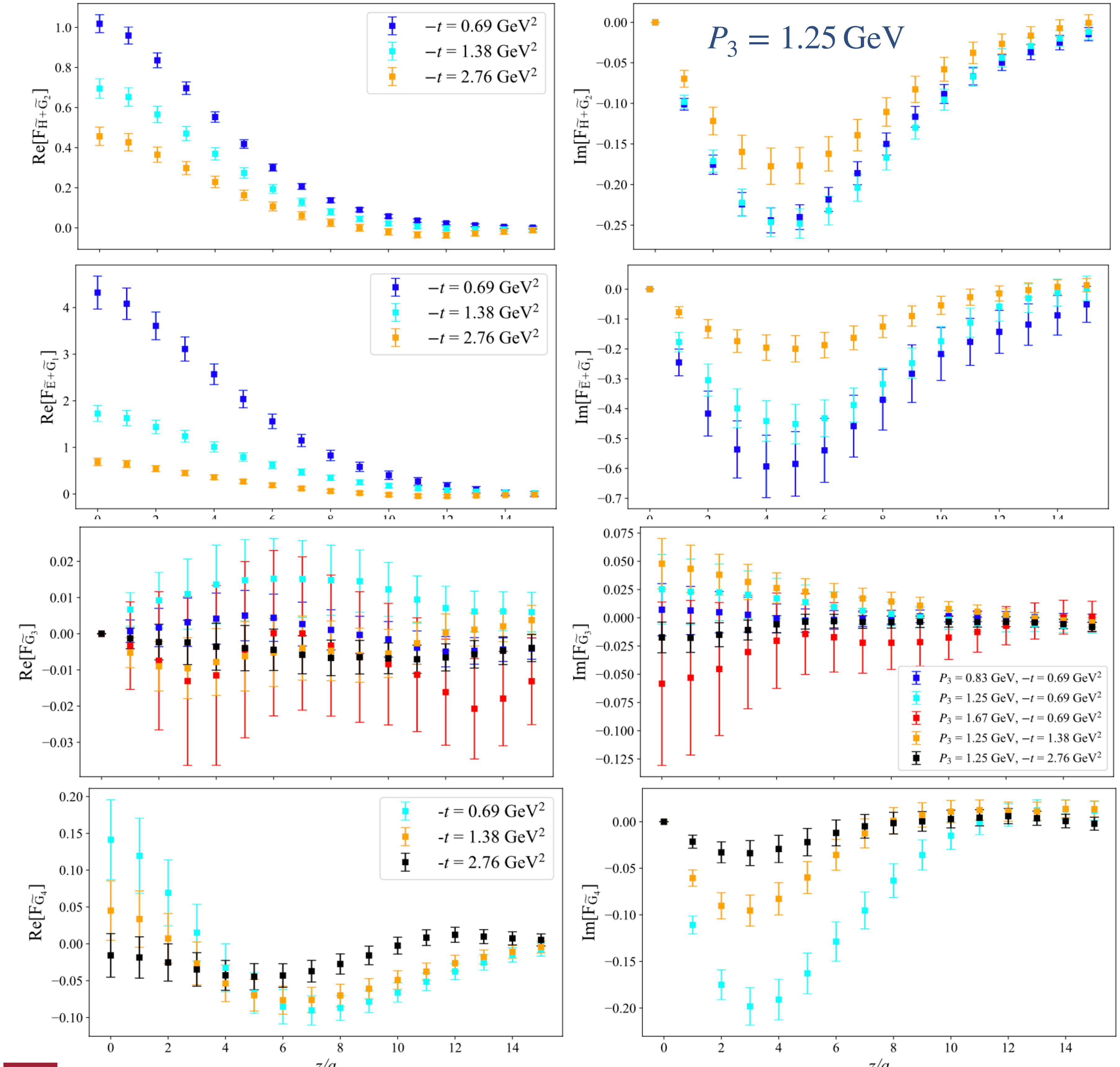
★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

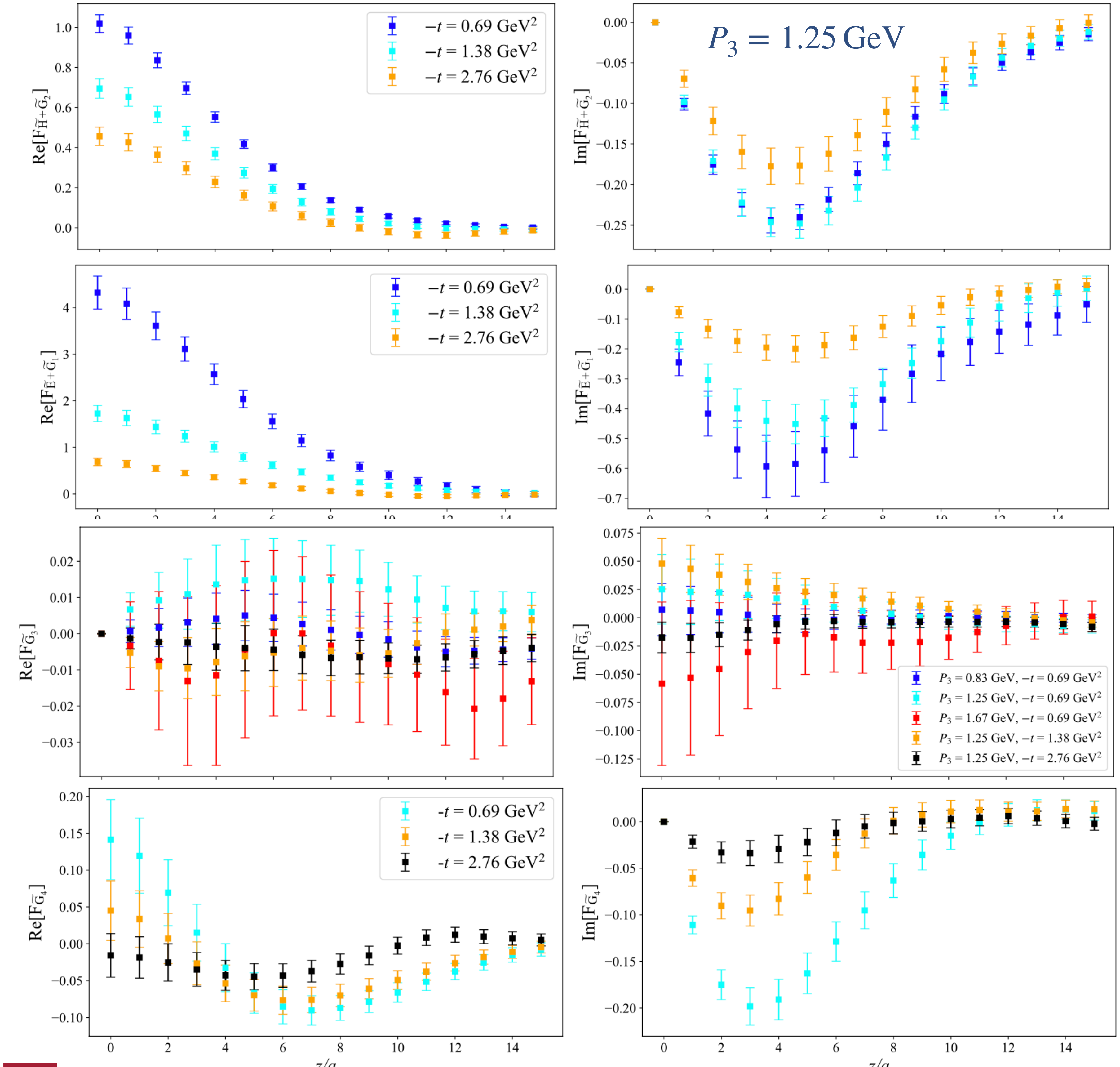
$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Lattice Results - quasi-GPDs



Lattice Results - quasi-GPDs



$$\int dx\, x\, \widetilde{G}_3 = \frac{\xi}{4} G_E(t)$$

Indeed, numerically found to be zero within uncertainties at ξ=0

Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\mathrm{M\overline{MS}}}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{\gamma_j\gamma_5}^{\mathrm{M\overline{MS}},\overline{\mathrm{MS}}}\left(\frac{x}{y},\frac{\mu}{yP_3}\right) G_X^{\overline{\mathrm{MS}}}(y,t,\mu) \;+\; \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda_{\mathrm{QCD}}^2}{x^2P_3^2}\right)$$

★ Operator dependent kernel

PHYSICAL REVIEW D **102,** 034005 (2020)

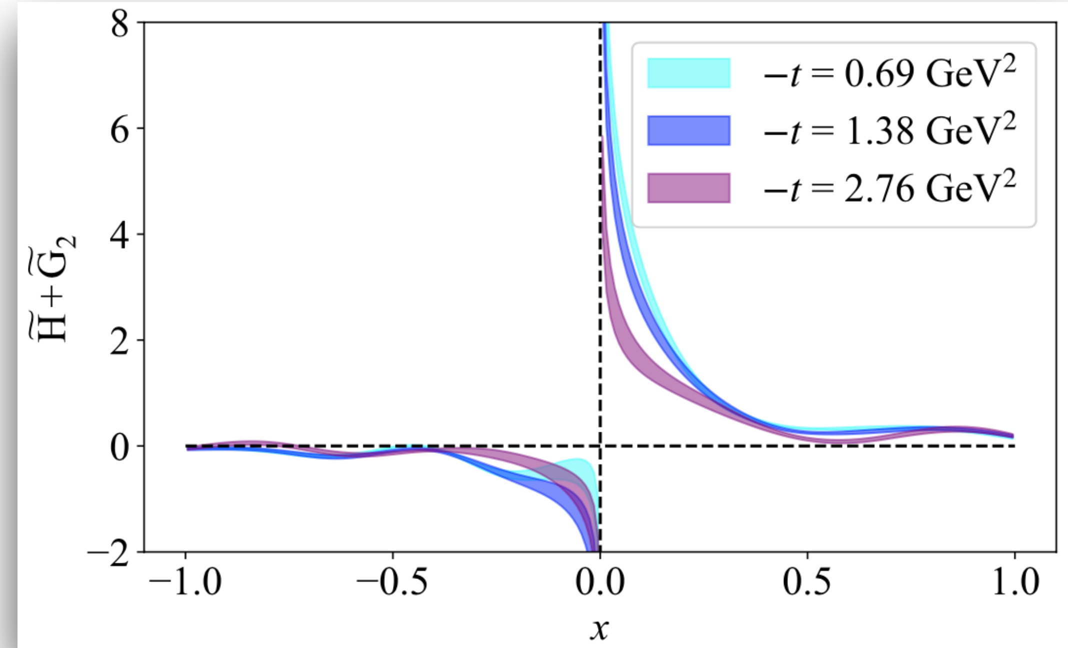
One-loop matching for the twist-3 parton distribution $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

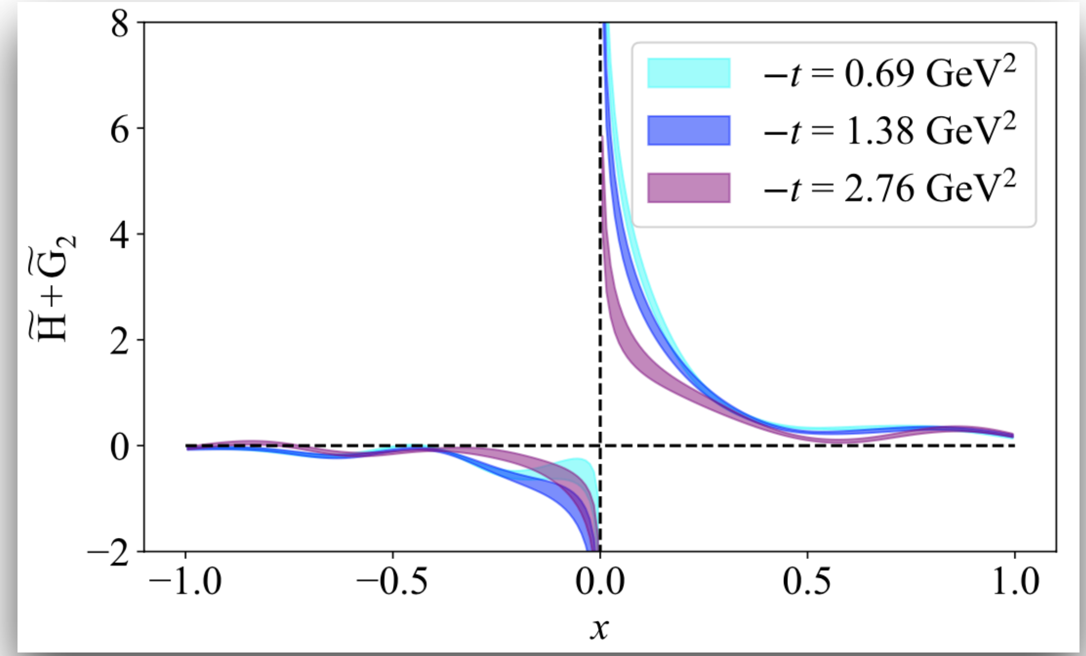
$$C^{(1)}_{\mathrm{M\overline{MS}}}\left(\xi,\frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} \;+\; \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ & \xi>1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ & \xi<0\,, \end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

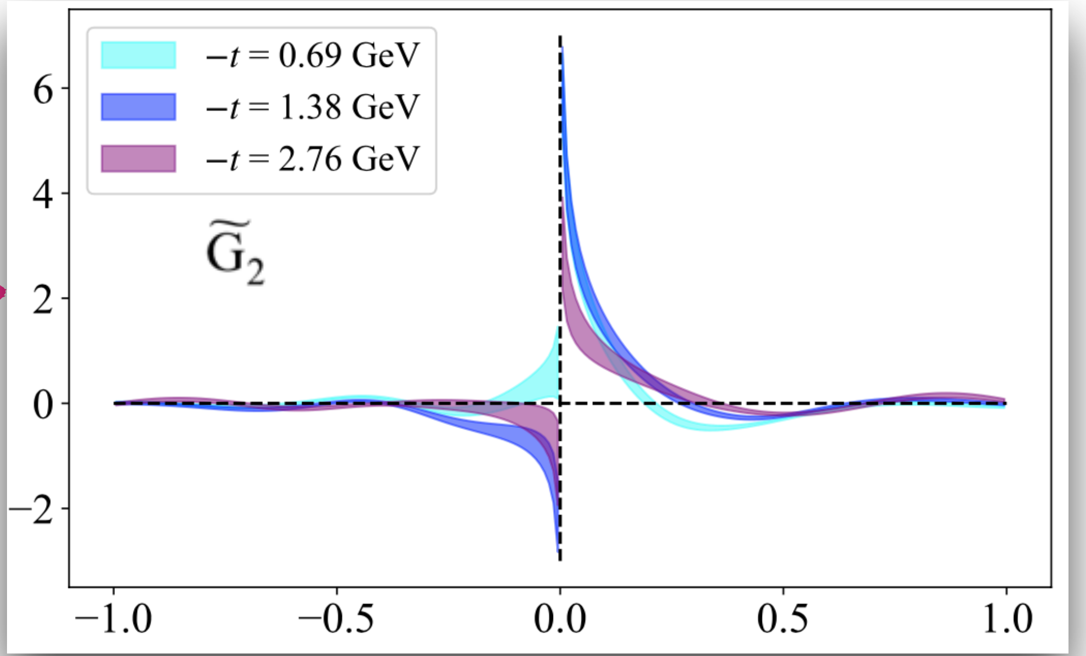
Lattice Results - light-cone GPDs



Lattice Results - light-cone GPDs



Isolating $\widetilde{G}_2$ using $\widetilde{H}$



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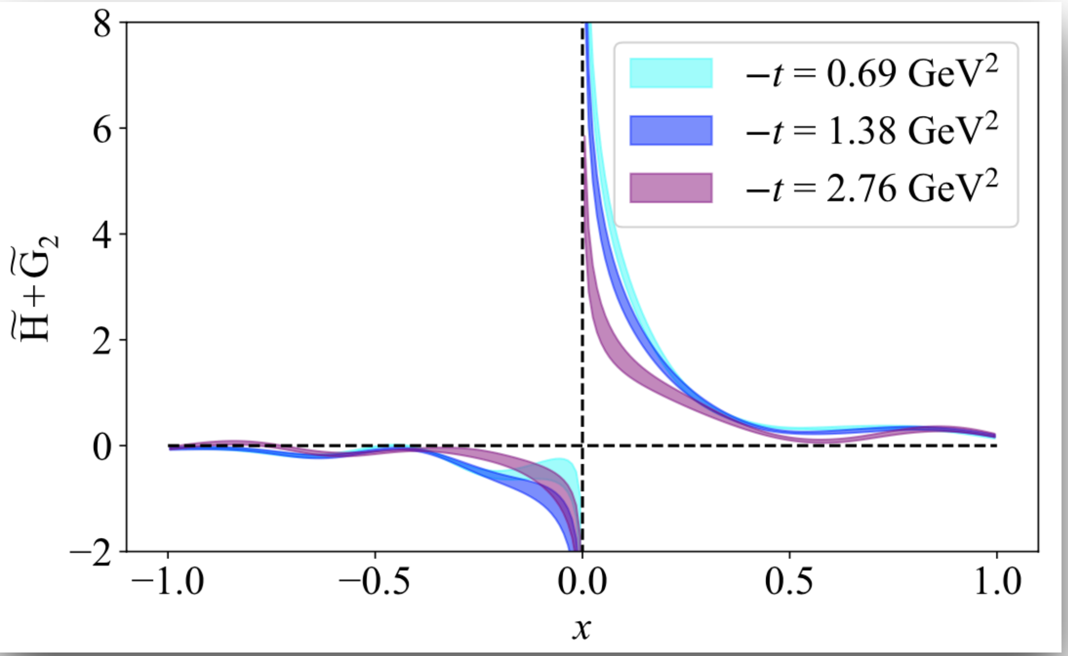

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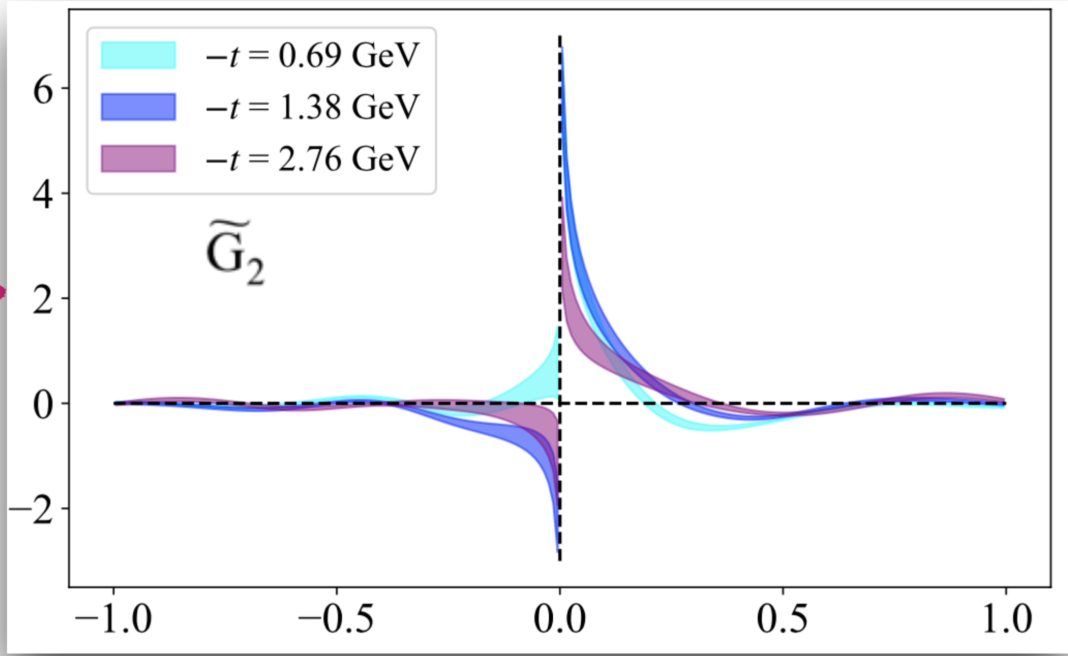

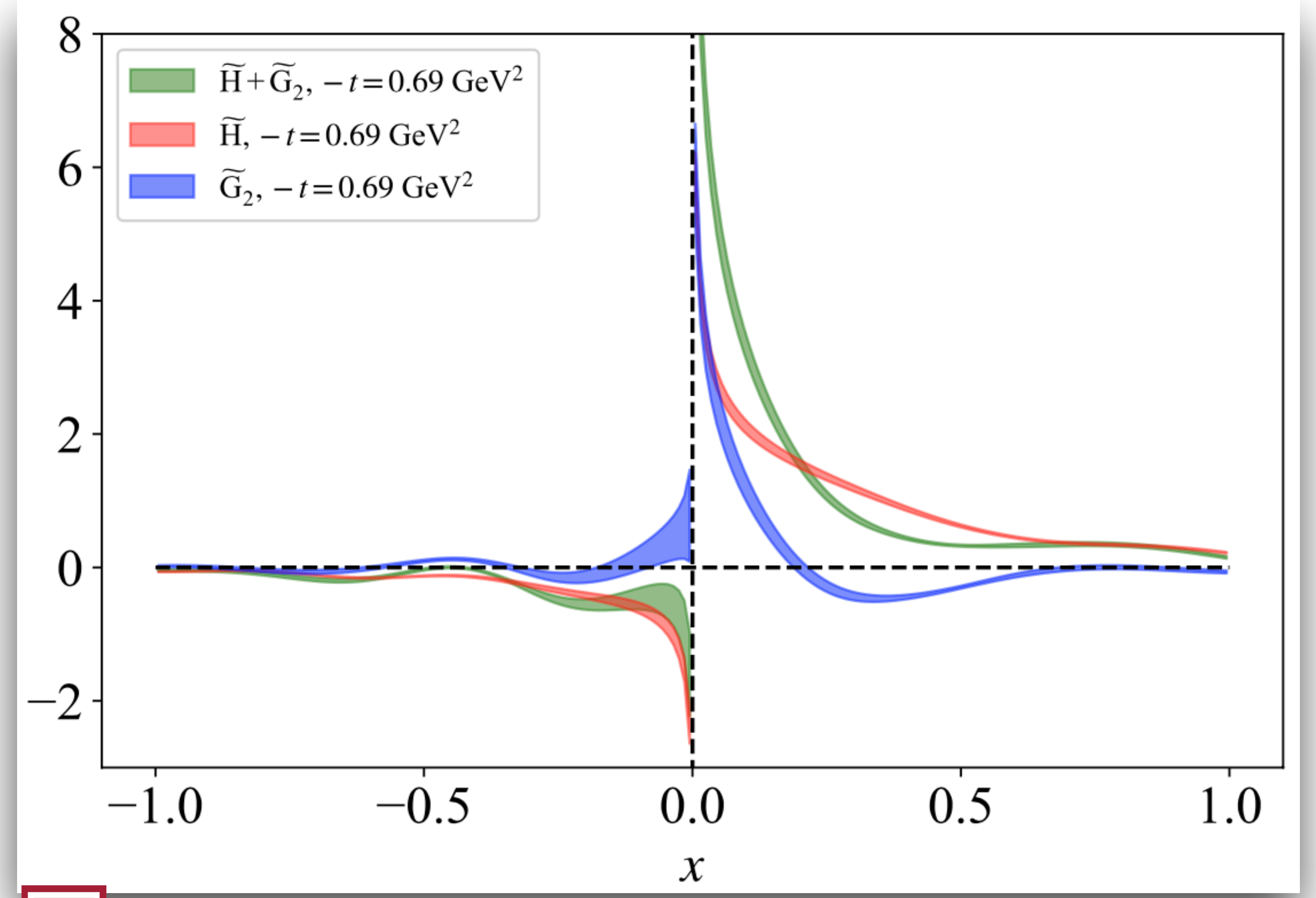

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

- ★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

- ★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

Lattice Results - light-cone GPDs

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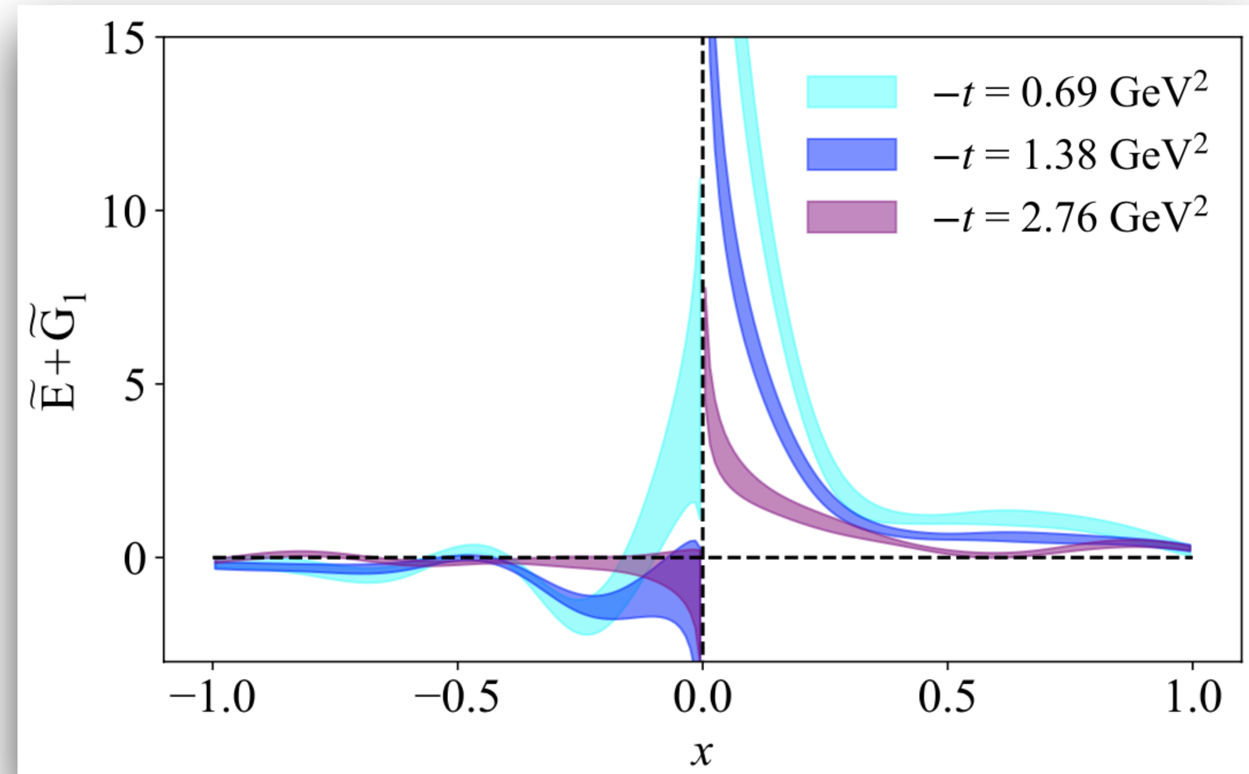

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

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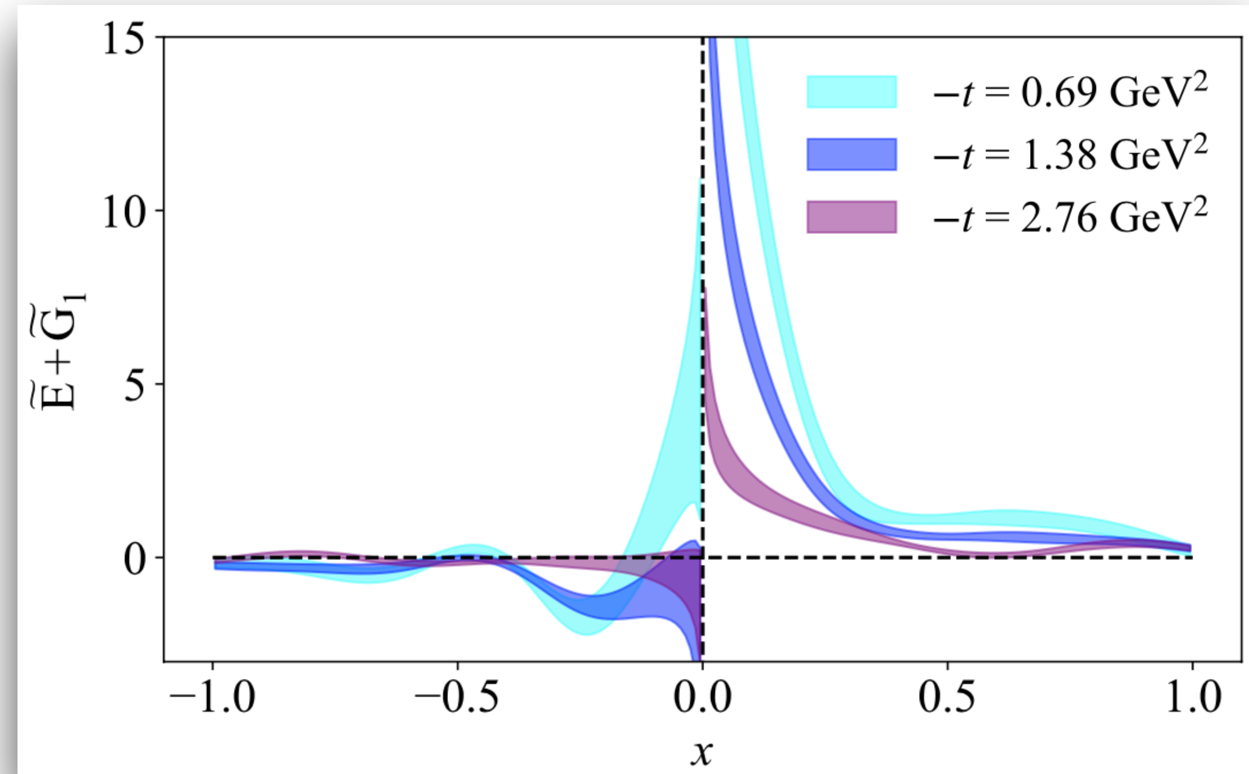

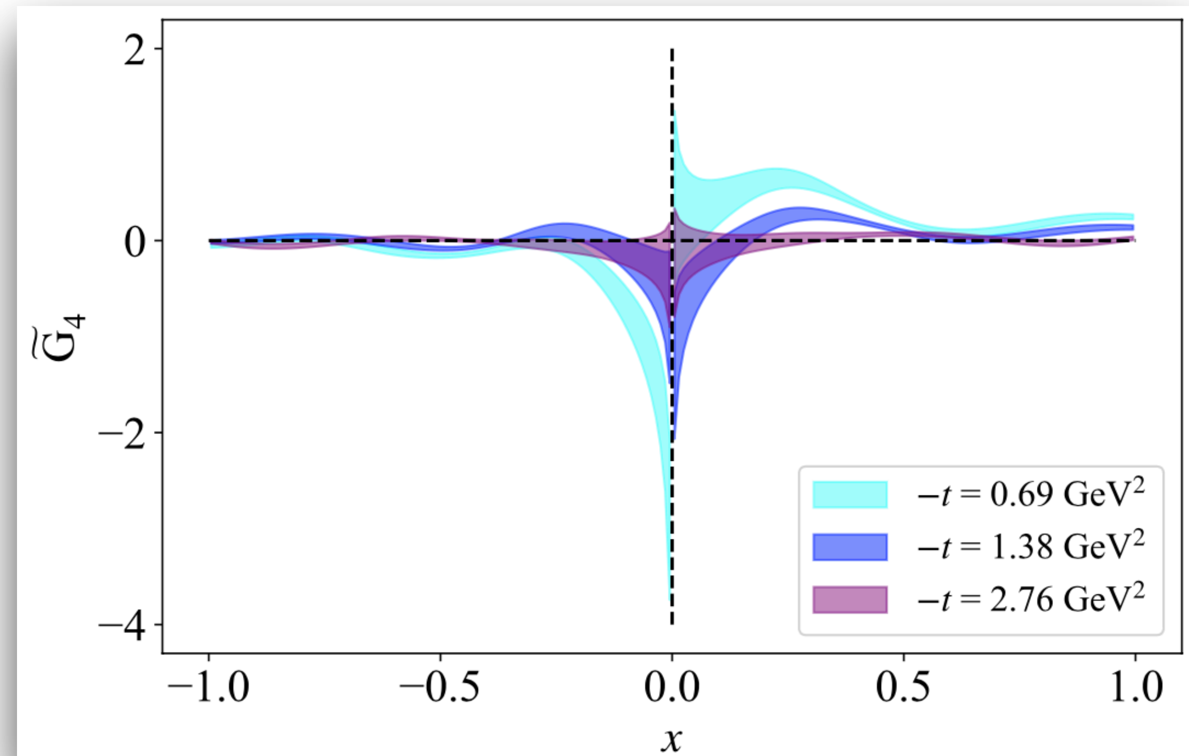

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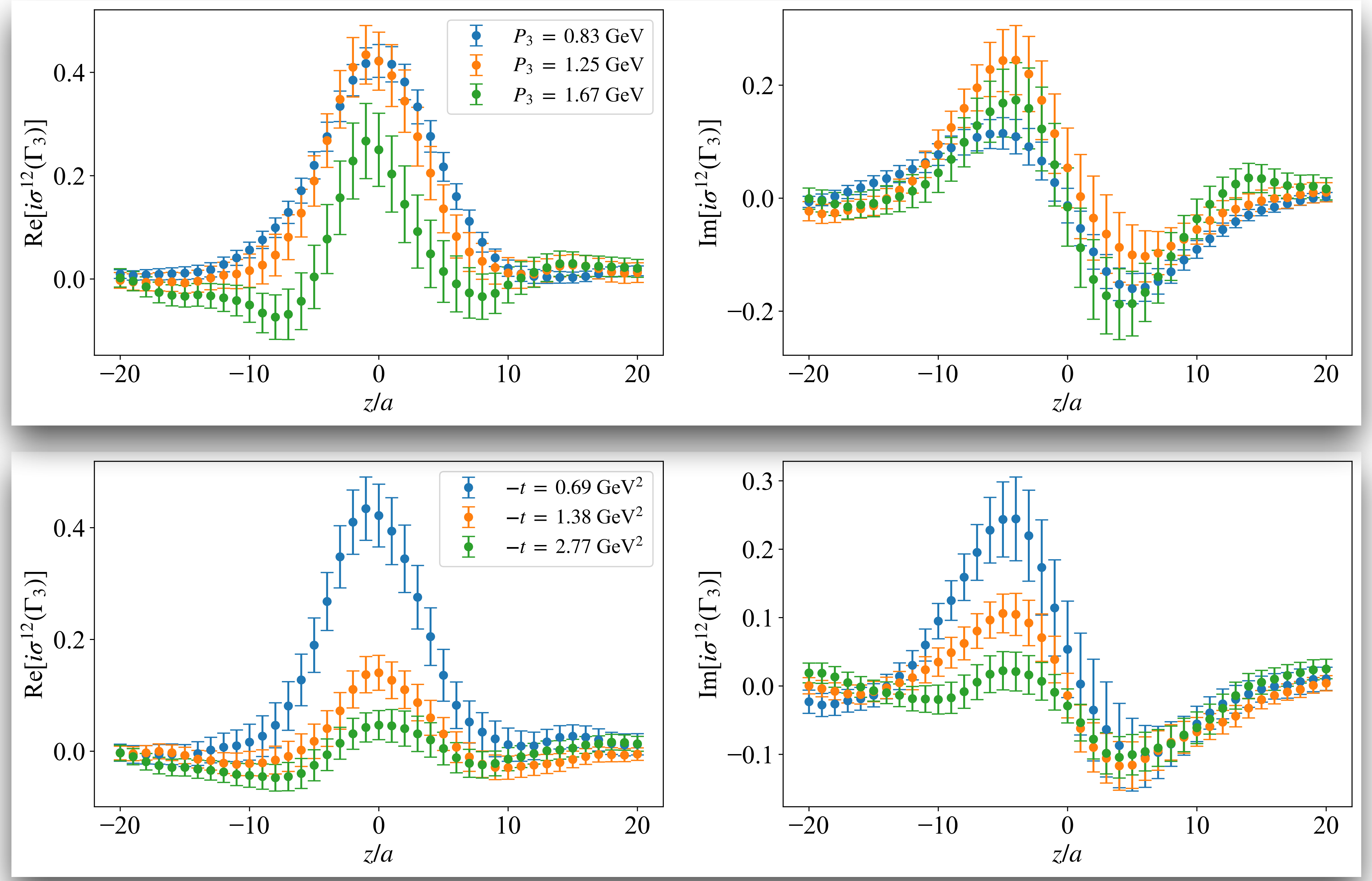
★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Extension to twist-3 tensor GPDs

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+ \gamma_5 \, \widetilde{H}'_2 + \frac{P^+ \gamma_5}{M} \, \widetilde{E}'_2 \right) u(p)$$



New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Theoretical setup

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(\cdots)} \cdots + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

Theoretical setup

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$$F^\mu_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

- ★ Extraction of standard GPDs using A_i obtained from any frame
- ★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

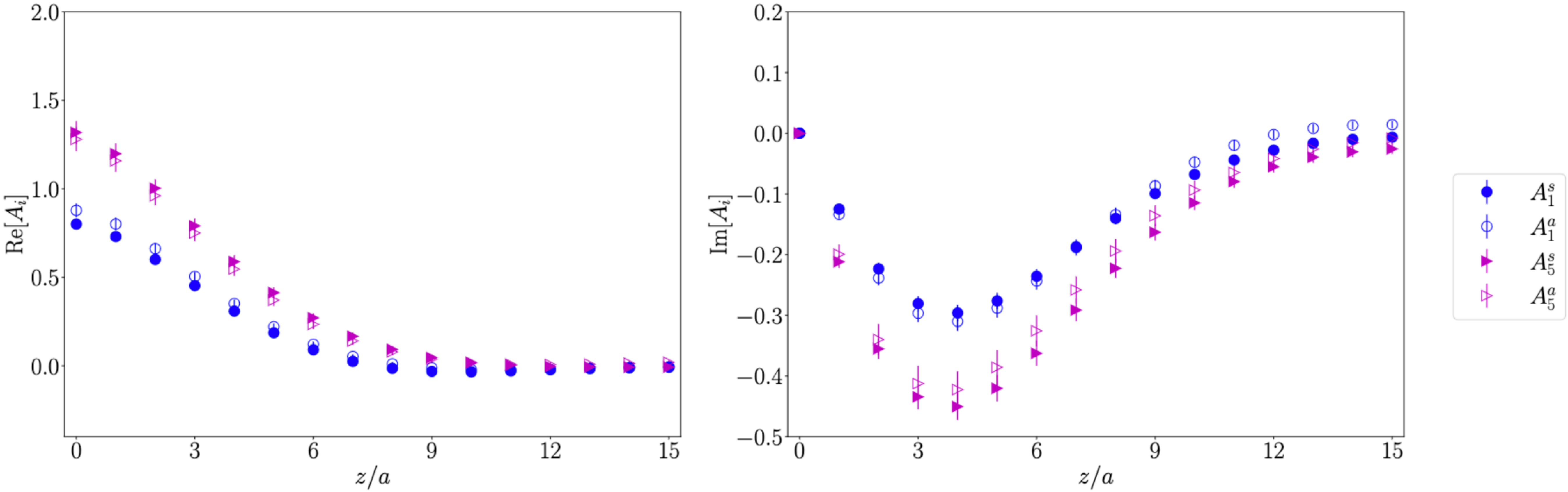
➡ **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \qquad -t^s = \vec{Q}^2 = 0.69\,GeV^2$

- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}, \qquad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

Comparison of A_i in two frames

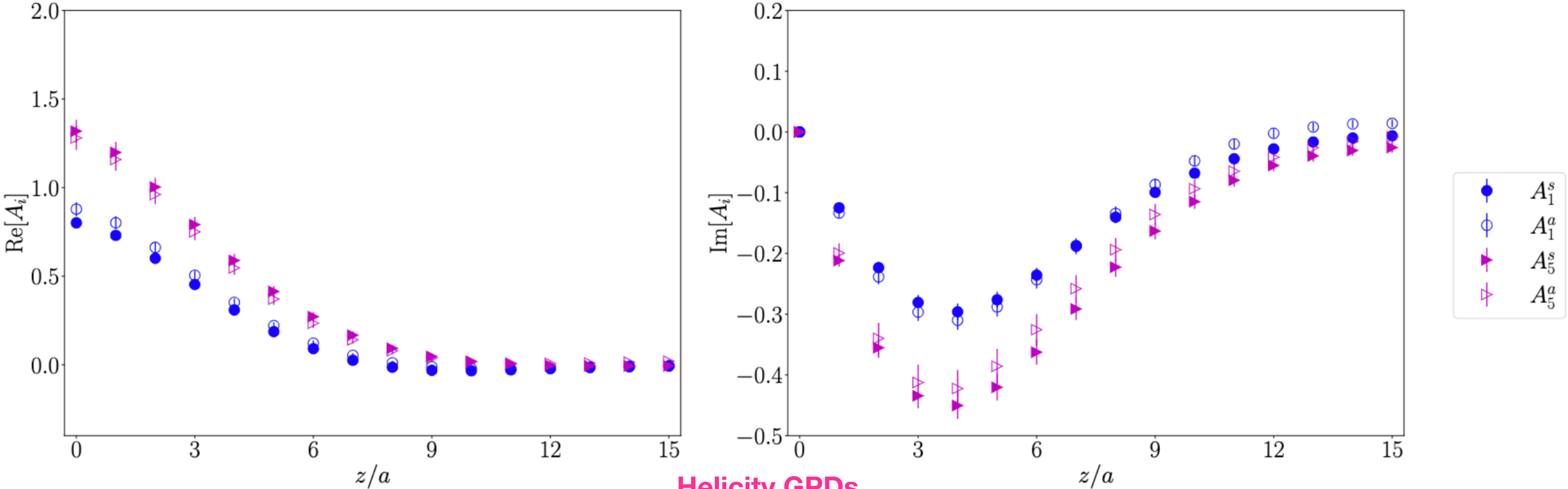
Unpolarized GPDs



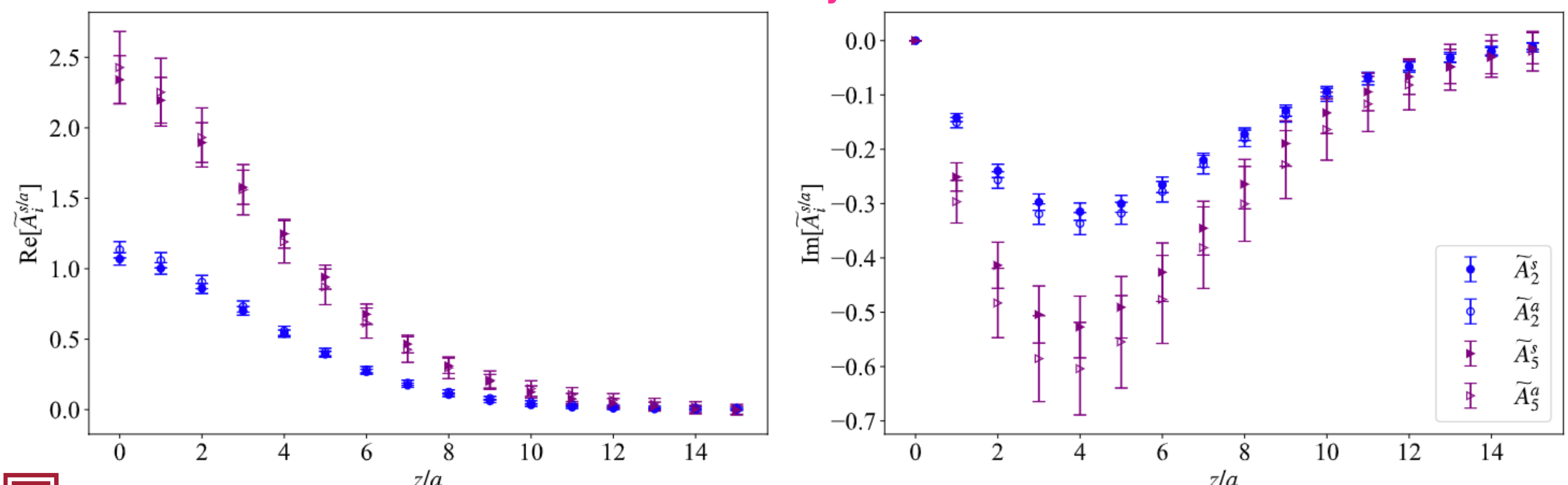
- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_3, A_4, A_8 zero at $\xi = 0$
- ★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

Comparison of A_i in two frames

Unpolarized GPDs



Helicity GPDs



Parameters of calculations

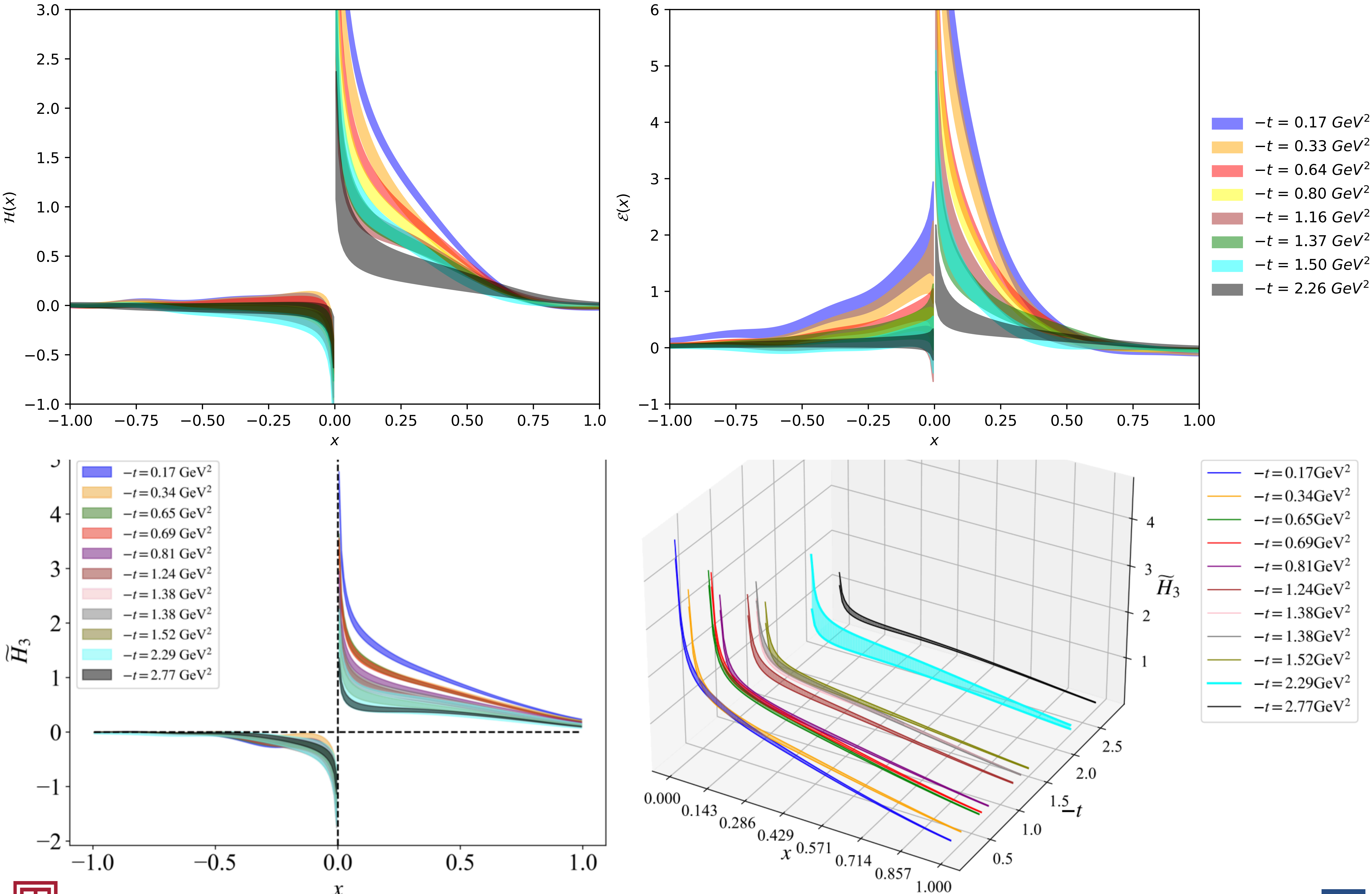
★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Light-cone GPDs



How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science
Award Number: DE-SC0023646

1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. Lattice QCD calculations of GPDs and related structures
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

QGT-related Publications

1. “Gluon helicity in the nucleon from lattice QCD and machine learning”, Khan, Liu, Sabbir Sufian, *Physical Review D*, Accepted, 2023.

2. "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO", Bhattacharya Cichy, Constantinou, Gao, Metz, Miller, Mukherjee, Petreczky, Steffens, Zhao, *Physical Review D*, DOI: 10.1103/PhysRevD.108.014507.

3. "Generalized parton distributions through universal moment parameterization: non-zero skewness case", Guo, Ji, Santiago, Shiells, Yang, *Journal of High Energy Physics*, DOI: 10.1007/JHEP05(2023)150.

4. "Hadronic structure on the light-front VI. Generalized parton distributions of unpolarized hadrons", Shuryak, Zahed, *Physical Review D*, DOI: 10.1103/PhysRevD.107.094005.

5. "Chiral-even axial twist-3 GPDs of the proton from lattice QCD", Bhattacharya, Cichy, Constantinou, Dodson, Metz, Scapellato, Fernanda Steffens, *Physical Review D*, DOI: 10.1103/PhysRevD.108.054501.

6. "Shedding light on shadow generalized parton distributions", Moffat, Freese, Cloët, Donohoe, Gamberg, Melnitchouk, Metz, Prokudin, Sato, *Physical Review D*, DOI: 10.1103/PhysRevD.108.036027.

7. "Colloquium: Gravitational Form Factors of the Proton", Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan, *Physical Review D*, Under Review.

8. "Synchronization effects on rest frame energy and momentum densities in the proton", Freese, Miller, eprint: 2307.11165.

9. "Proton's gluon GPDs at large skewness and gravitational form factors from near threshold heavy quarkonium photo-production", Guo, Ji, Yuan, e-Print: 2308.13006.

10. "Parton Distributions from Boosted Fields in the Coulomb Gauge", Gao, Liu, Zhao, e-Print: 2306.14960.

11. "Exactly solvable models of nonlinear extensions of the Schrödinger equation", Dodge, Schweitzer, e-Print: 2304.01183.

12. "Role of strange quarks in the D-term and cosmological constant term of the proton", Won, Kim, Kim, e-Print: 2307.00740.

13. "Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays", Ma, Feng, Gorchtein, Jin, Liu, Seng, Wang, Zhang, e-Print: 2308.16755.

Summary

- ★ Impressive progress in the calculation of Mellin moments of GPDs
- ★ Novel methods to access x dependence complementary to Mellin moments
- ★ New methods applicable beyond leading twist.
  Several improvements needed, e.g.,
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  - access to a broad range of t and ξ
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- ★ Synergy with phenomenology is an exciting prospect!
  QGT Collaboration will be instrumental in such effort

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Thank you

QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number:
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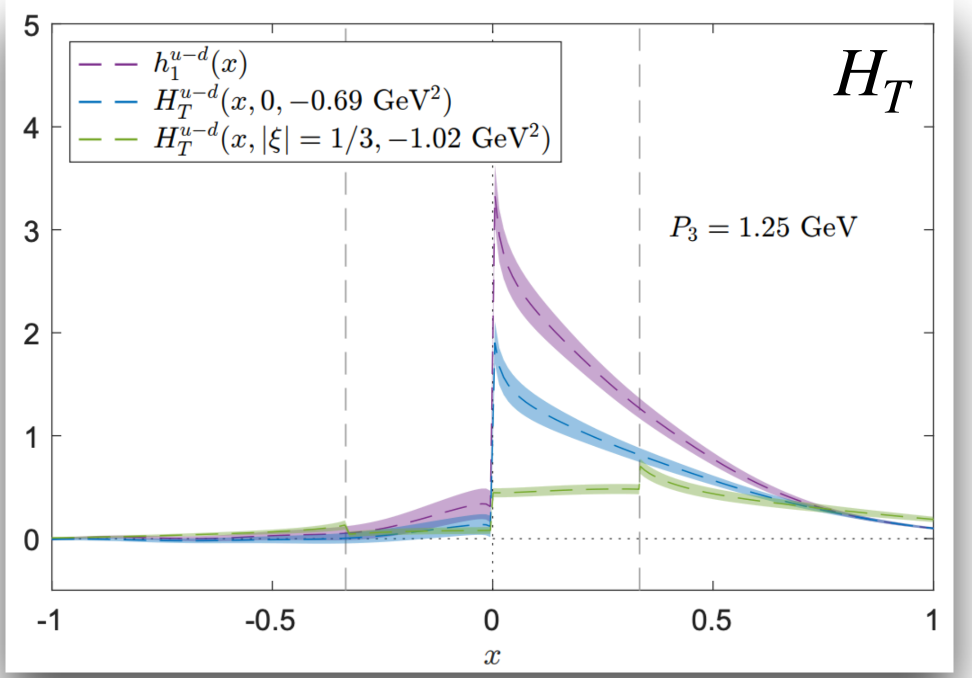
DOE Early Career Award (NP)
Grant No. DE-SC0020405

Miscellaneous

Transversity GPDs

Standard parametrization



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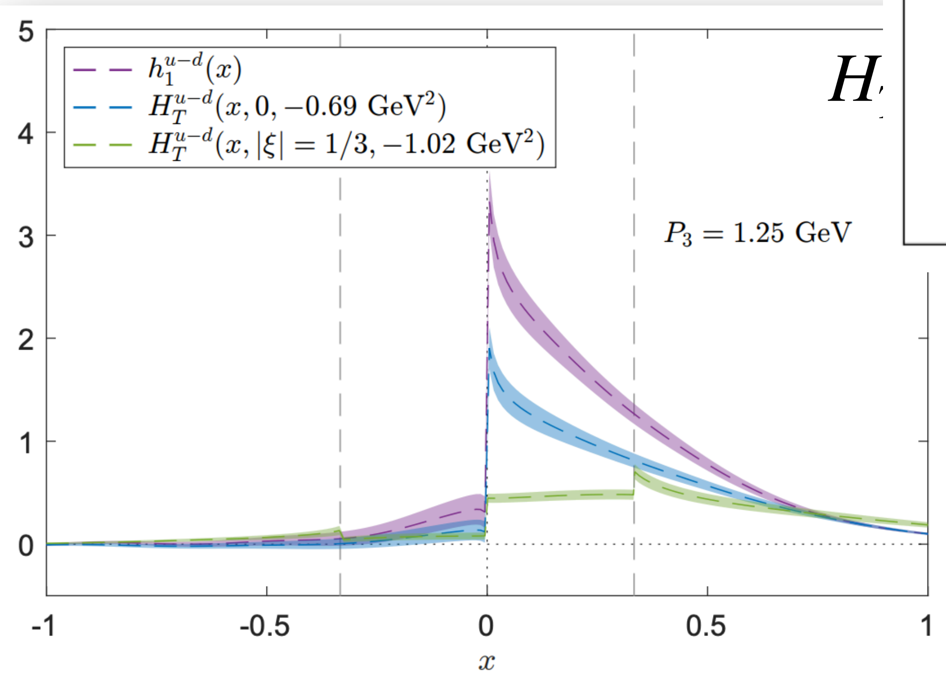

Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + M z^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M \not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_2}{16m^2} + A_{T12}\frac{\left(P_3^2 - E(E+m)\right)z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2 + 4P_3^2 + \Delta_1^2 - \Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 z E^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_1}{16m^2} + A_{T12}\frac{\left(E(E+m) - P_3^2\right)z\Delta_1}{8m^2}\right)$$

$$\Pi^s_{02}(\Gamma_1) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = i\,K\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m) - \Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_3) = i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 z E^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

Transversity GPDs

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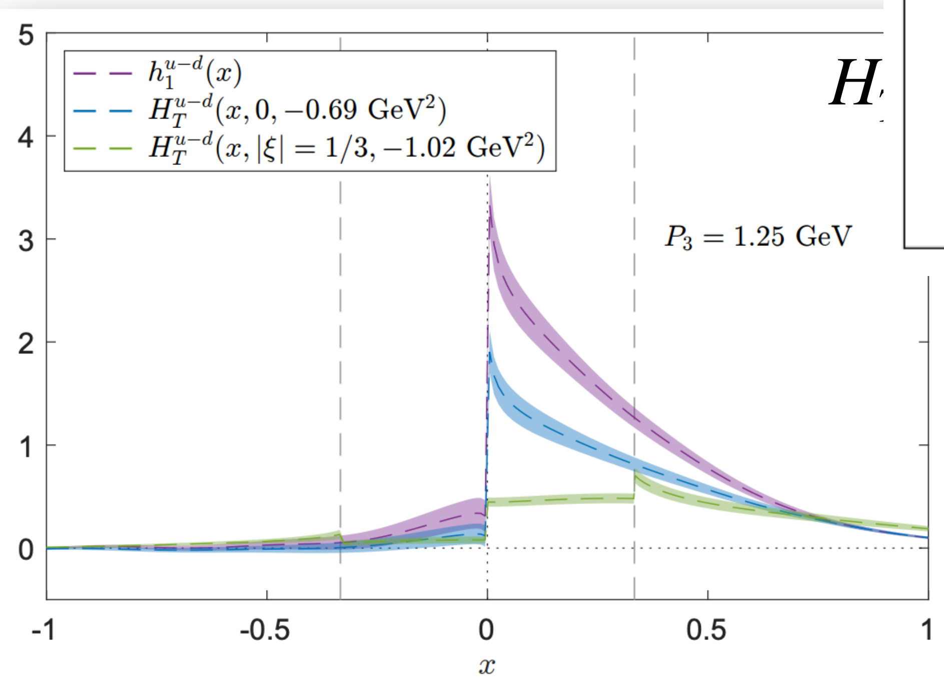

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$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_2}{16m^2} + A_{T12}\frac{(P_3^2-E(E+m))\,z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2+4P_3^2+\Delta_1^2-\Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_1}{16m^2} + A_{T12}\frac{(E(E+m)-P_3^2)\,z\Delta_1}{8m^2}\right)$$

$$\Pi^s_{02}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m)-\Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

On-going work

$$\Pi^s_{02}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

Decomposition

★ Requirement:
four independent matrix elements

$P_3\,[\text{GeV}]$	$\vec{q}\,[\frac{2\pi}{L}]$	$-t\,[\text{GeV}^2]$
± 0.83	$(0,0,0)$	0
± 1.25	$(0,0,0)$	0
± 1.67	$(0,0,0)$	0
± 0.83	$(\pm 2,0,0)$	0.69
± 1.25	$(\pm 2,0,0)$	0.69
± 1.67	$(\pm 2,0,0)$	0.69
± 1.25	$(\pm 2,\pm 2,0)$	1.38
± 1.25	$(\pm 4,0,0)$	2.76

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_y}{4m^2} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_y (E+m)}{2m^2} \right),$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_y^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^1(\Gamma_2) = i\,C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{\Delta_x \Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x \Delta_y (E+m)}{8m^3} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_x \Delta_y (E+m)}{4m^2 P_3} \right),$$

$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_x (E+m)}{2m^2 P_3} \right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_x}{4m^2} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_x (E+m)}{2m^2} \right),$$

$$\Pi^2(\Gamma_1) = i\,C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{\Delta_x \Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x \Delta_y (E+m)}{8m^3} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_x \Delta_y (E+m)}{4m^2 P_3} \right),$$

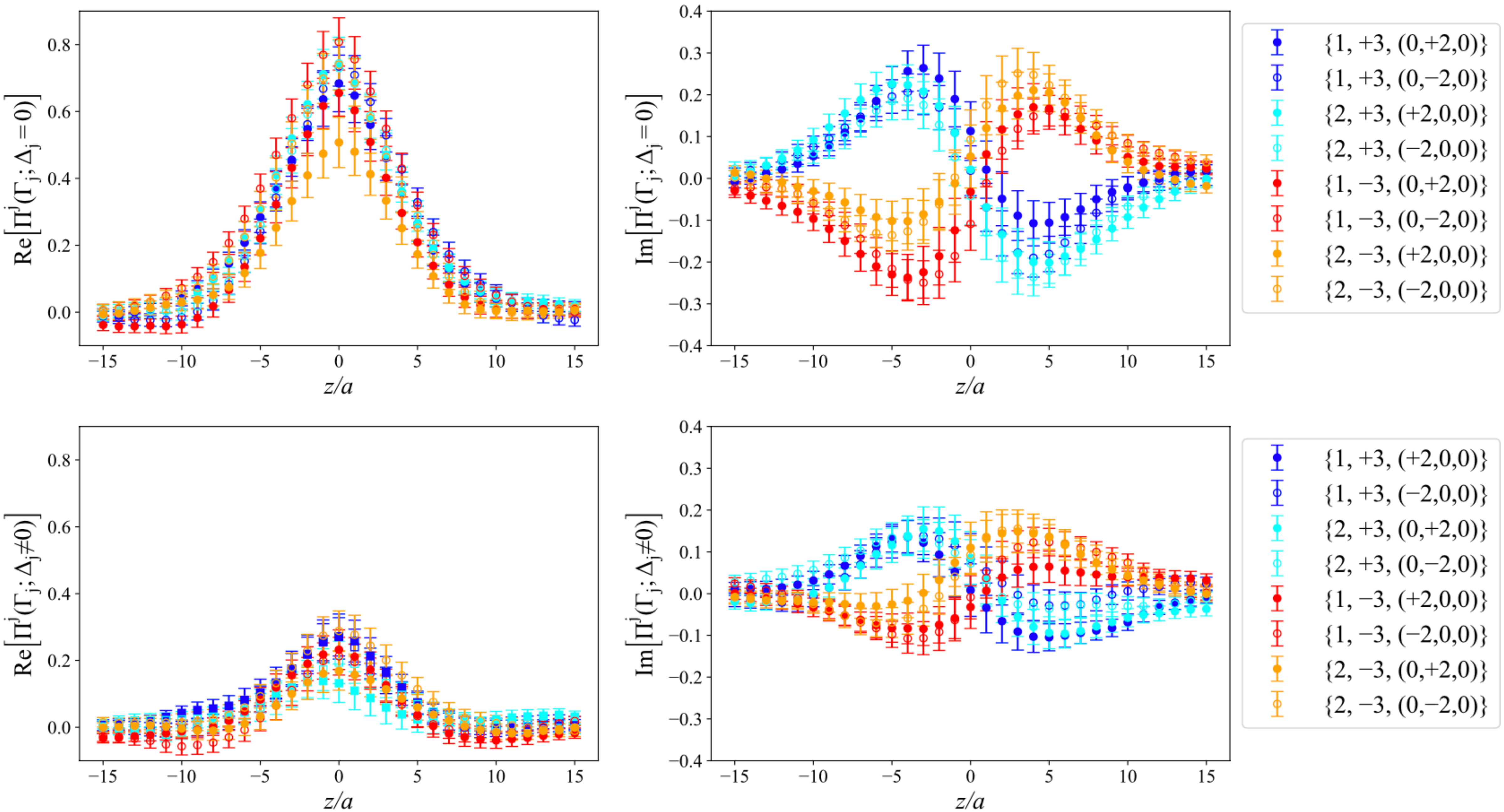
$$\Pi^2(\Gamma_2) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m)+\Delta_x^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_y^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\,\Delta_x^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^2(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_y (E+m)}{2m^2 P_3} \right),$$

Lattice Results - Matrix Elements

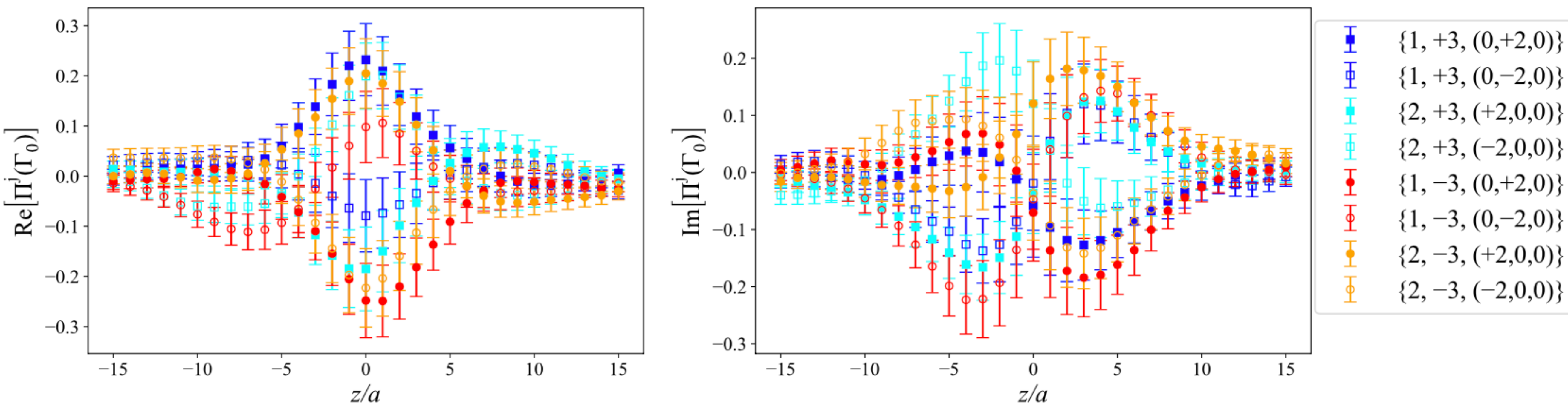
★ Bare matrix elements

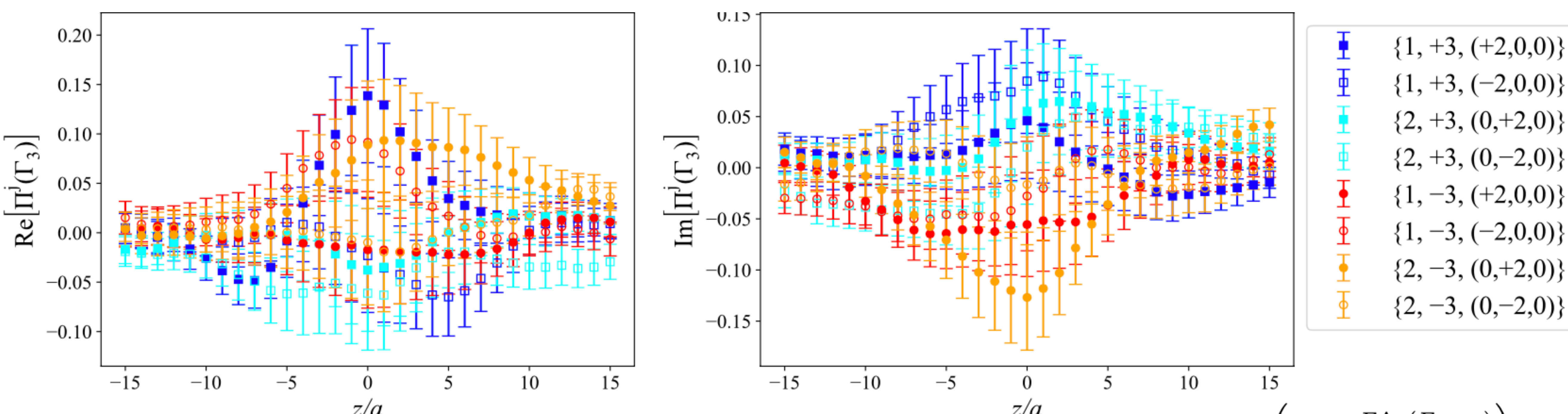
$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2 P_3}\right)$$



Lattice Results - Matrix Elements

★ Bare matrix elements





$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right)$$

★ Suppressed signal compared to $\gamma_+\gamma_5$ operators

Consistency checks

★ Norms satisfied   encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks

★ Norms satisfied encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

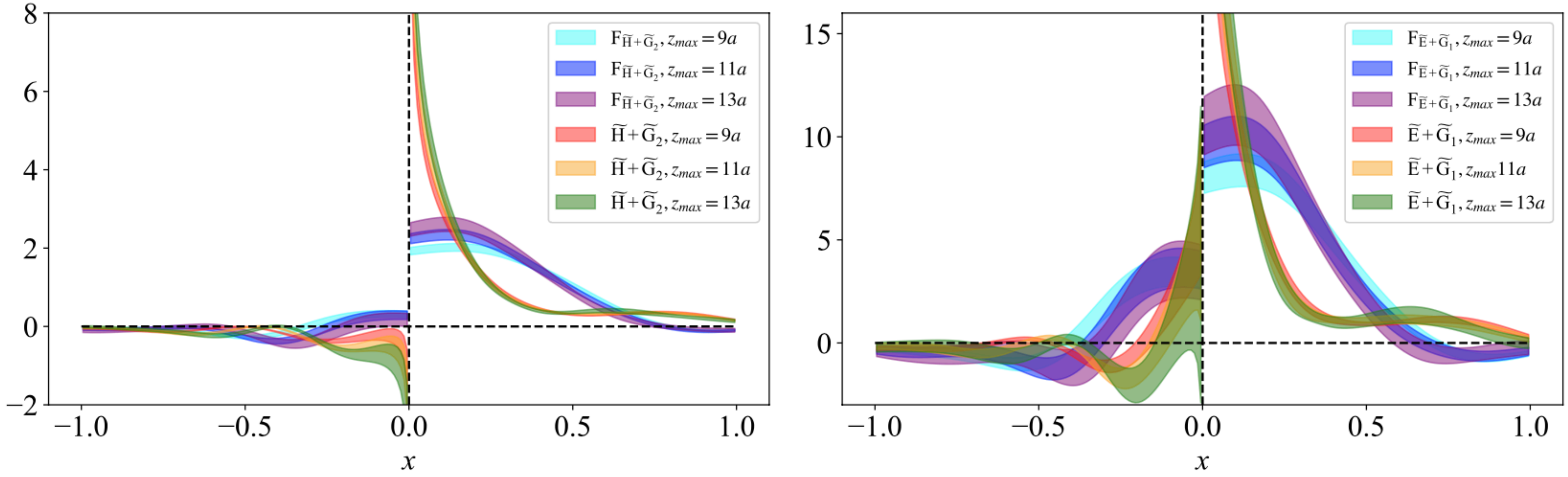


FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

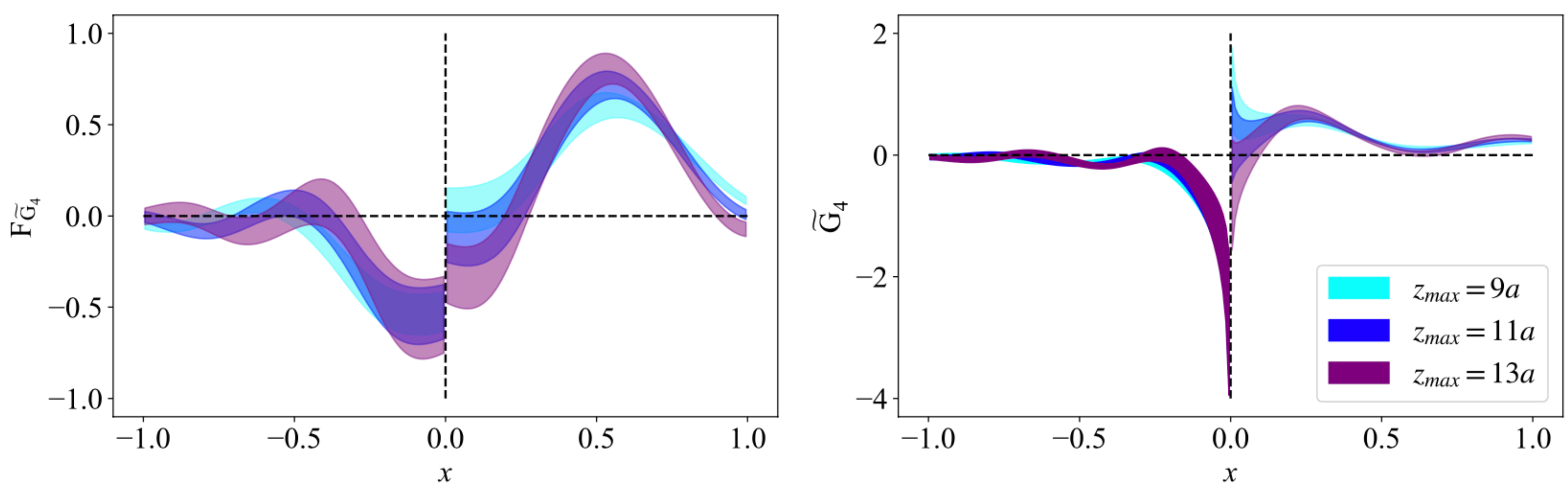


FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.