Finite- and infinite-volume analysis of the tetraquark $T_{cc}^{+}(3875)$

Sebastian M. Dawid

with the honorable
F. Romero-López & S. Sharpe

SUMMARY

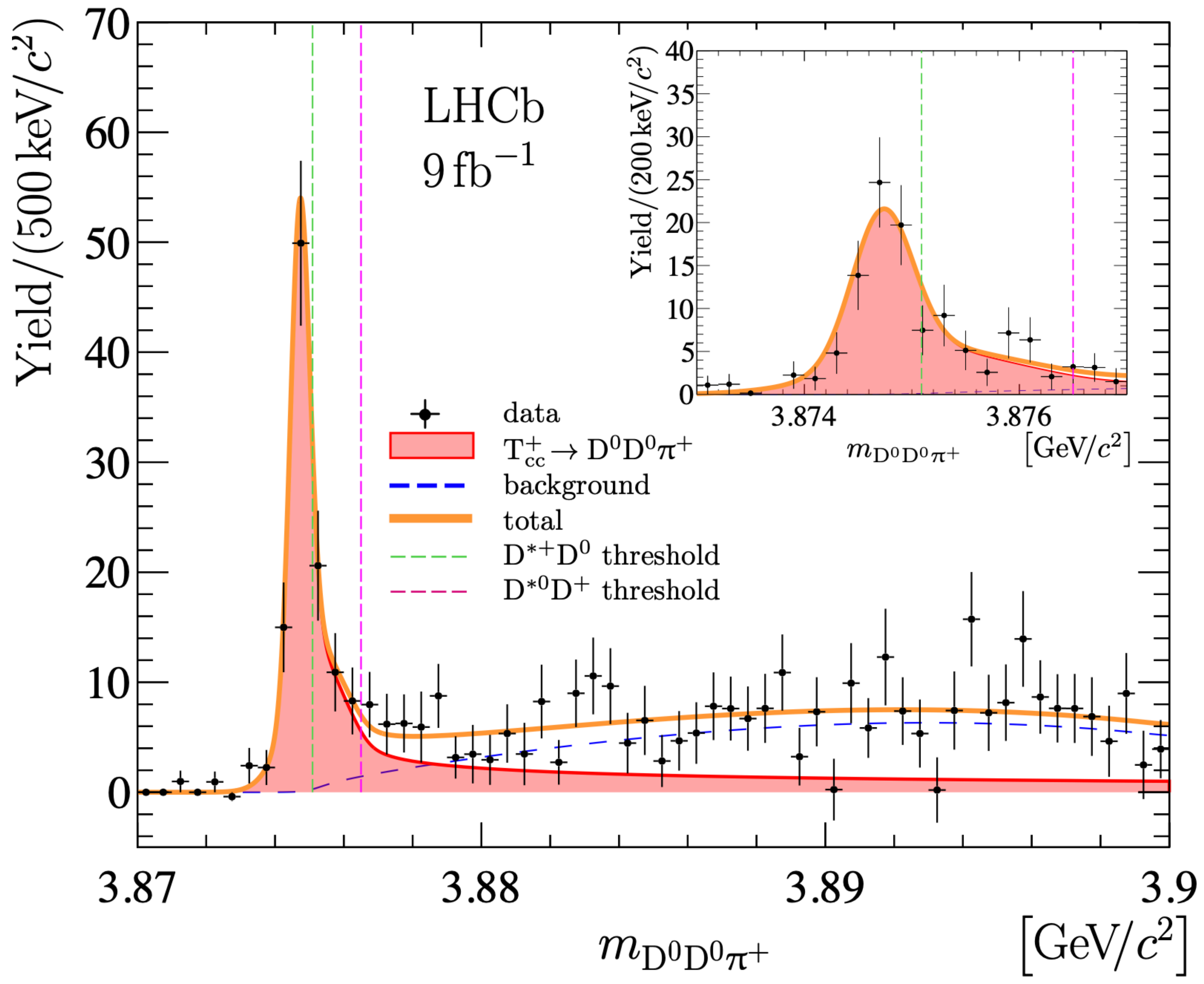
1) We lay out a strategy for a rigorous determination of T_{cc} and related systems from Lattice QCD
2) We propose resolution of the so-called "left-hand cut problem" both in the finite volume and in the continuum
3) We generalize solutions of the relativistic three-body equations to non-degenerate particles, non-zero partial waves and non-negligible short-range couplings

UNIVERSITY of WASHINGTON

PWA13/ATHOS8, William & Mary, 30.05.2024

Doubly-charmed tetraquark

Observation of an exotic narrow doubly charmed tetraquark
R. Aaij et al. [LHCb], Nature Physics 18 (2022) 751



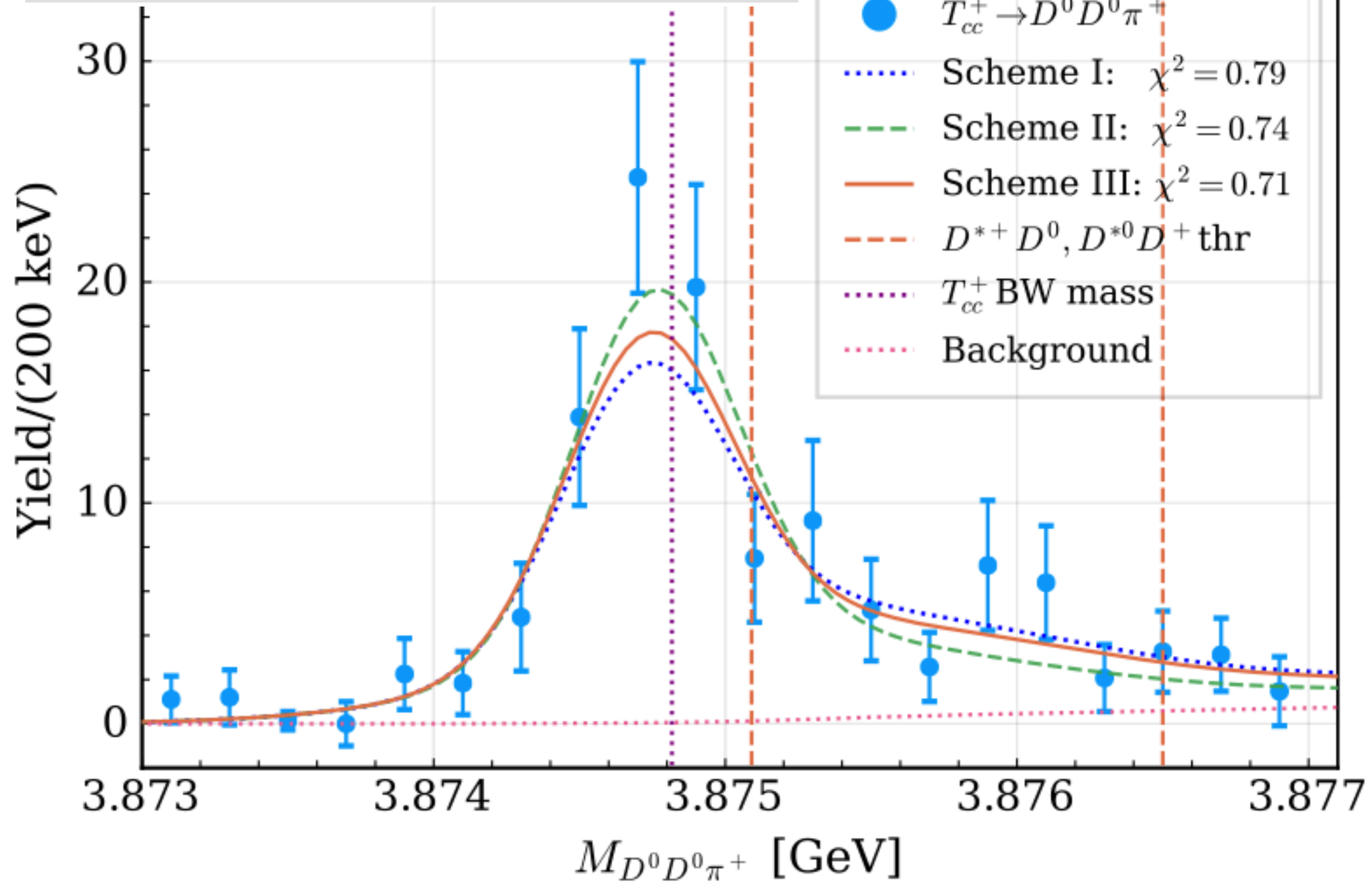
$(I)\, J^P = (0)\, 1^+,\ cc\bar{u}\bar{d}$

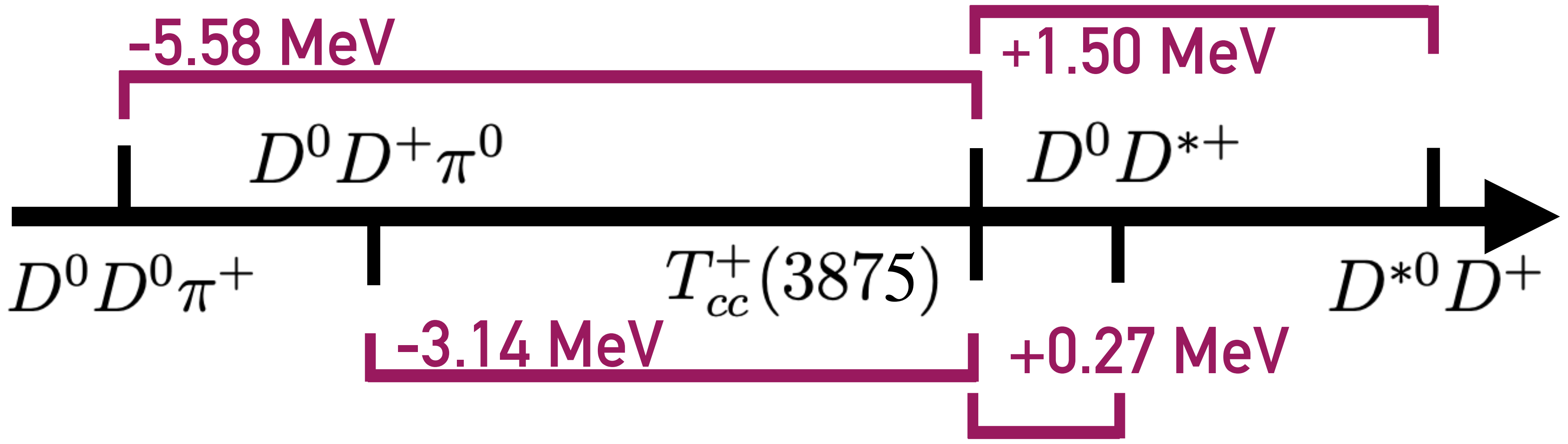
$\delta m_{\text{pole}} \approx -360$ keV, $\Gamma_{\text{pole}} \approx 48$ keV

$a \approx (-7.16 + 1.85\, i)$ fm

The three-body effects are expected to have a strong impact on the properties of the tetraquark due to the proximity of the three-body thresholds.

Meng-Ling Du et al., PRD 105, 014024 (2022)
Xu Zhang, PRD 109, 094010 (2024)





Available lattice results

Signature of a doubly charm tetraquark pole in DD scattering on the lattice*
Padmanath, Prelovsek, PRL 129, 032002 (2022)

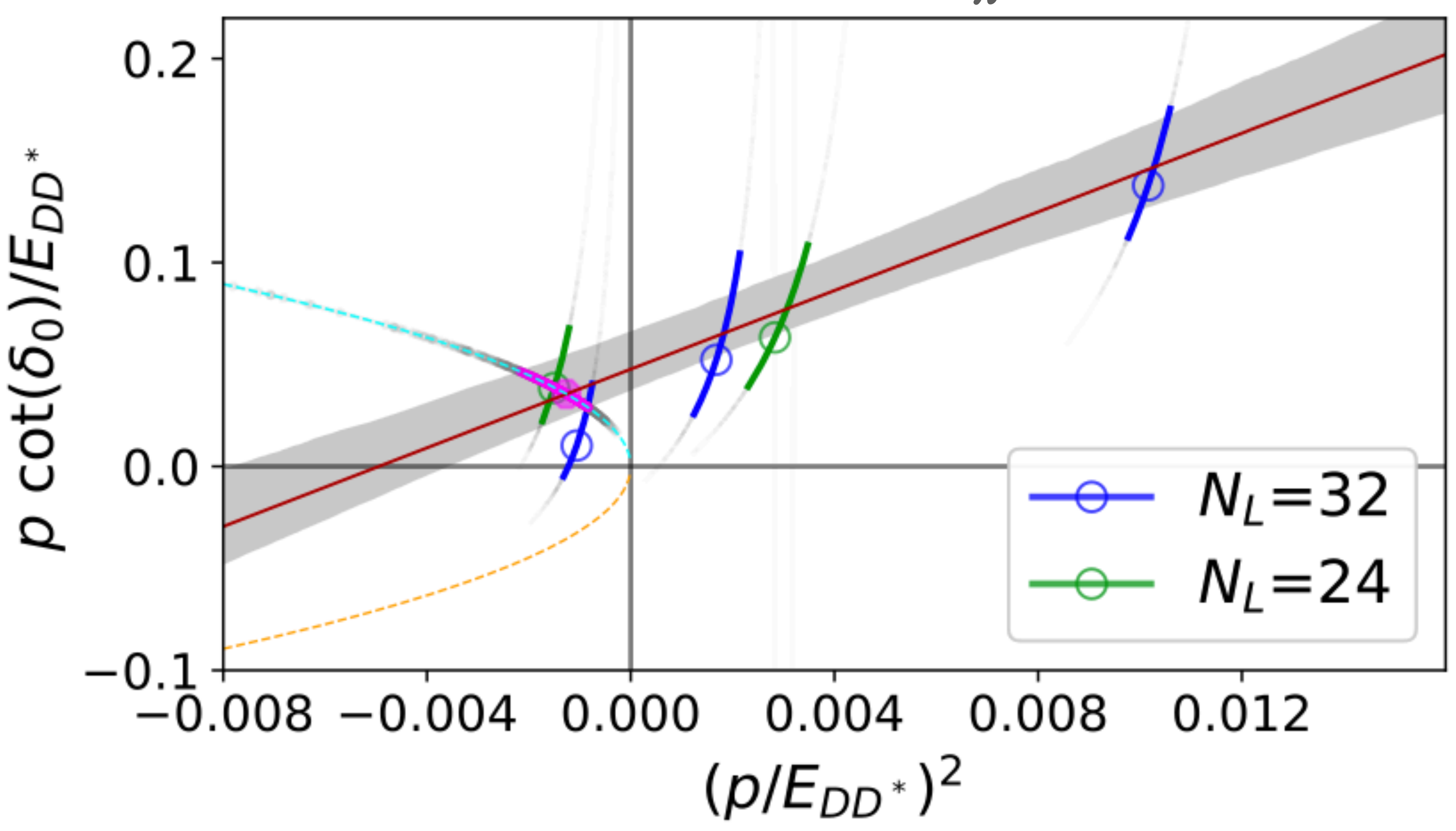
Towards the quark mass dependence of from lattice QCD
Collins, Nefediev, Padmanath, Prelovsek, PRD 109 (2024) 9, 094509

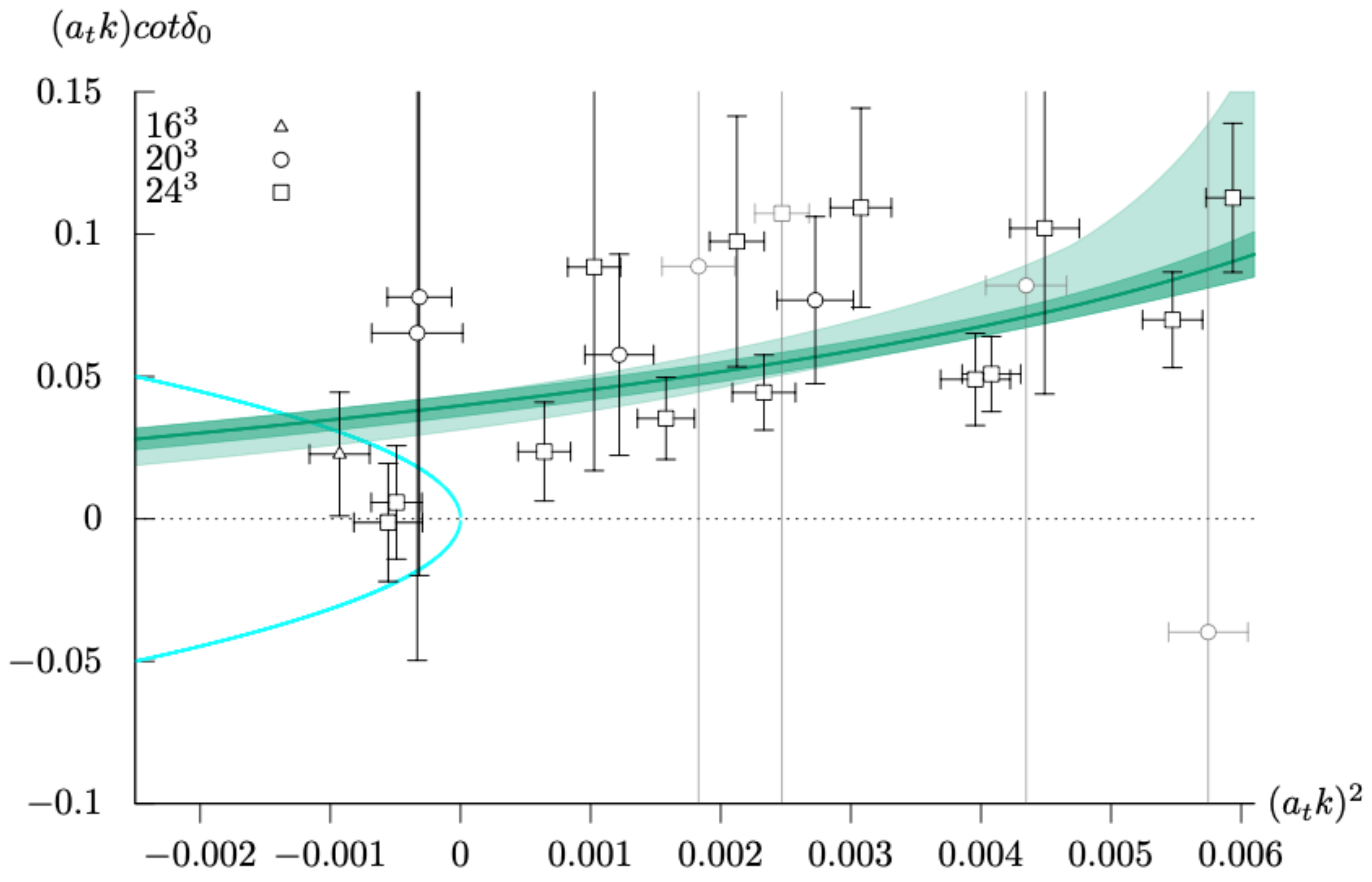
Lyu et al., PRL 131, 161901 (2023) $m_\pi \approx 146$ MeV

Chen et al. PLB 833, 137391 (2022) $m_\pi \approx 350$ MeV

Whyte, Wilson, Thomas, arXiv:2405.15741 $m_\pi \approx 391$ MeV

$m_\pi \approx 280$ MeV



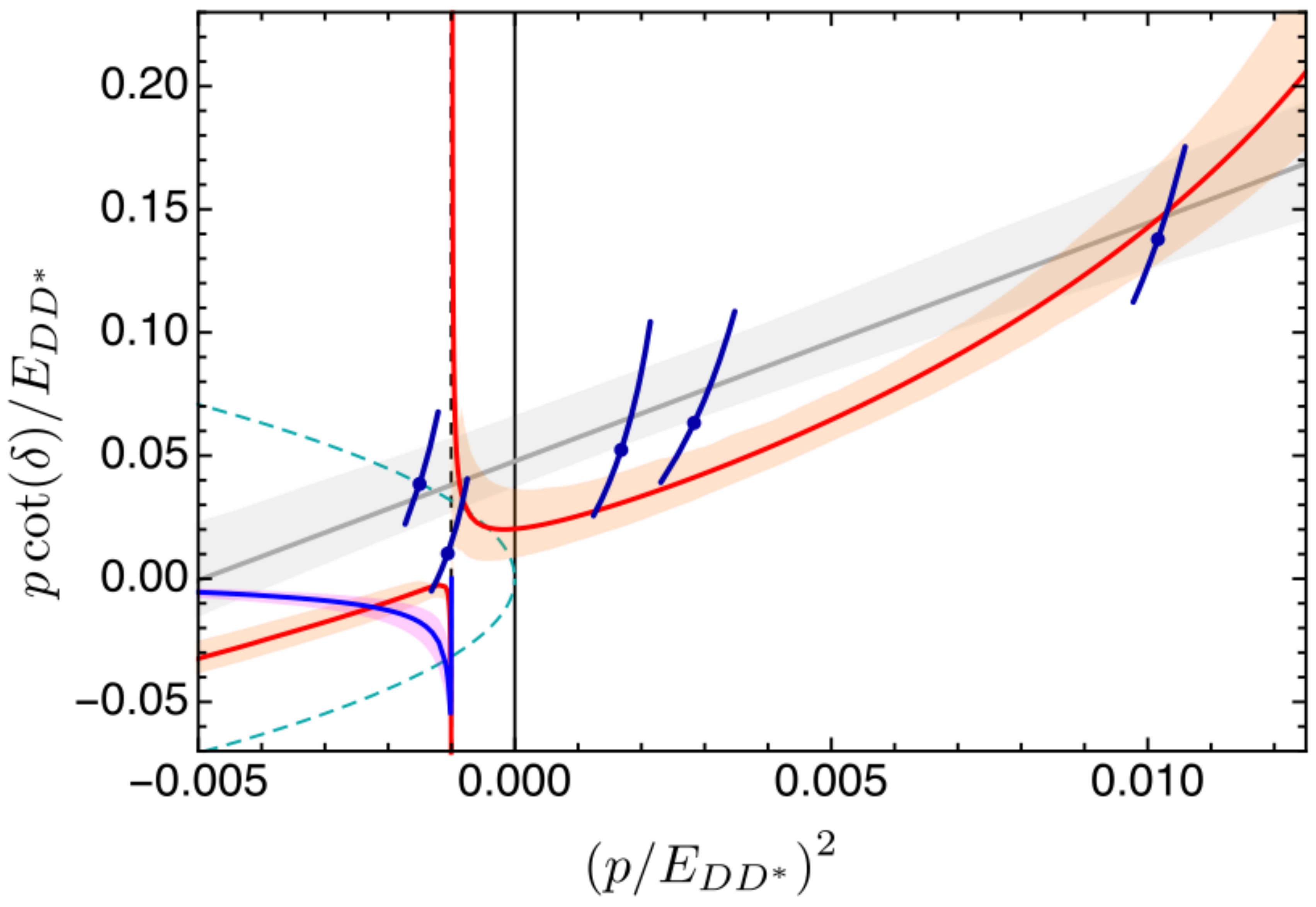


Thresholds are inverted but the three-body effects still play an important role in the analysis

$m_\pi \approx 280$ MeV

The left-hand cut problem

Role of the left-hand cut contributions on pole extractions from lattice data...
Meng-Lin Du et al., PRL 131, 131903 (2023)



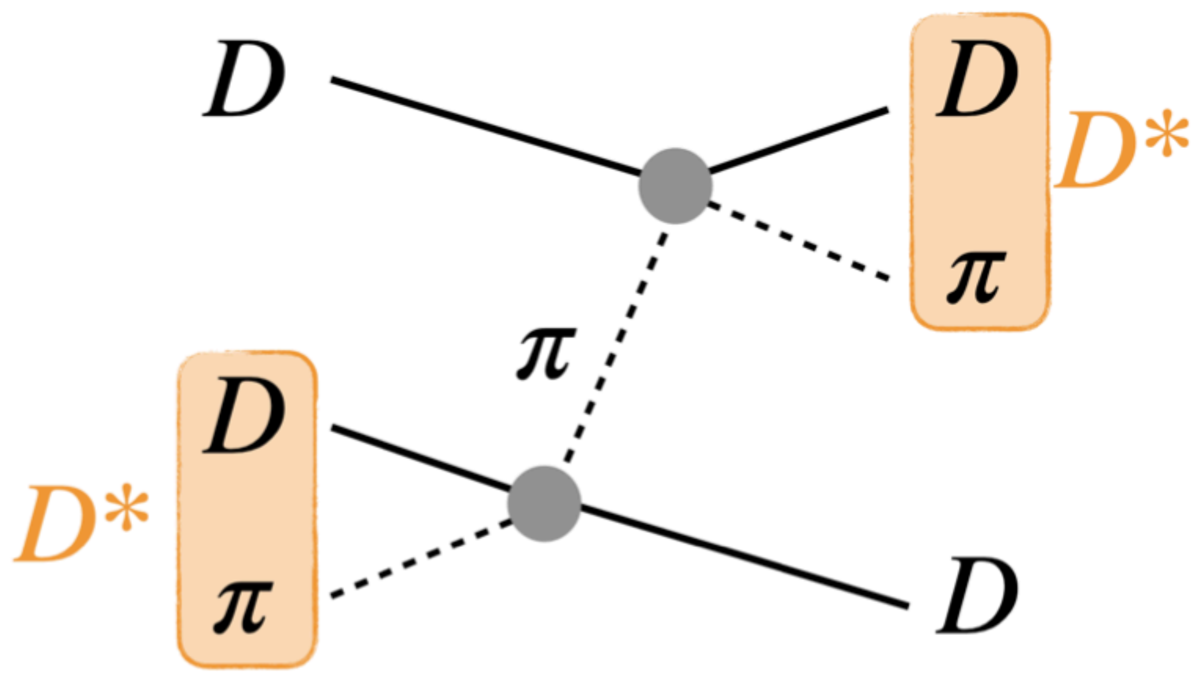
$$s_{\text{lhc}} = s_{\text{thr}} - m_\pi^2 + (m_{D^*} - m_D)^2$$

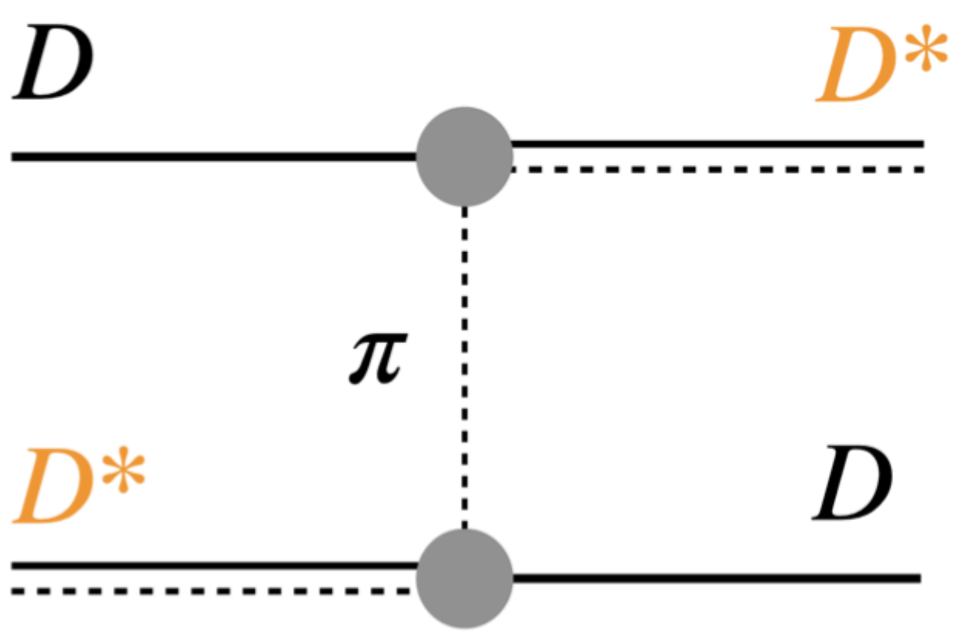
$$\sqrt{s_{\text{lhc}}} \approx 3966\ \text{MeV} \qquad \sqrt{s_{\text{thr}}} \approx 3975\ \text{MeV}$$

Presence of the left-hand cut:
a) invalidates the Lüscher formalism
b) invalidates the effective-range expansion

Incorporating DDπ effects and left-hand cuts in lattice QCD studies of T_{cc}^+
Hansen, Romero-López, Sharpe, arXiv:2401.06609

Raposo, Hansen, arXiv:2311.18793
Lu Meng et al., Phys.Rev.D 109, L071506 (2024)
Bubna et al. JHEP 05 (2024)





$$s_{\text{lhc},2} = s_{\text{thr}} - 4m_\pi^2 + (m_{D^*} - m_D)^2$$

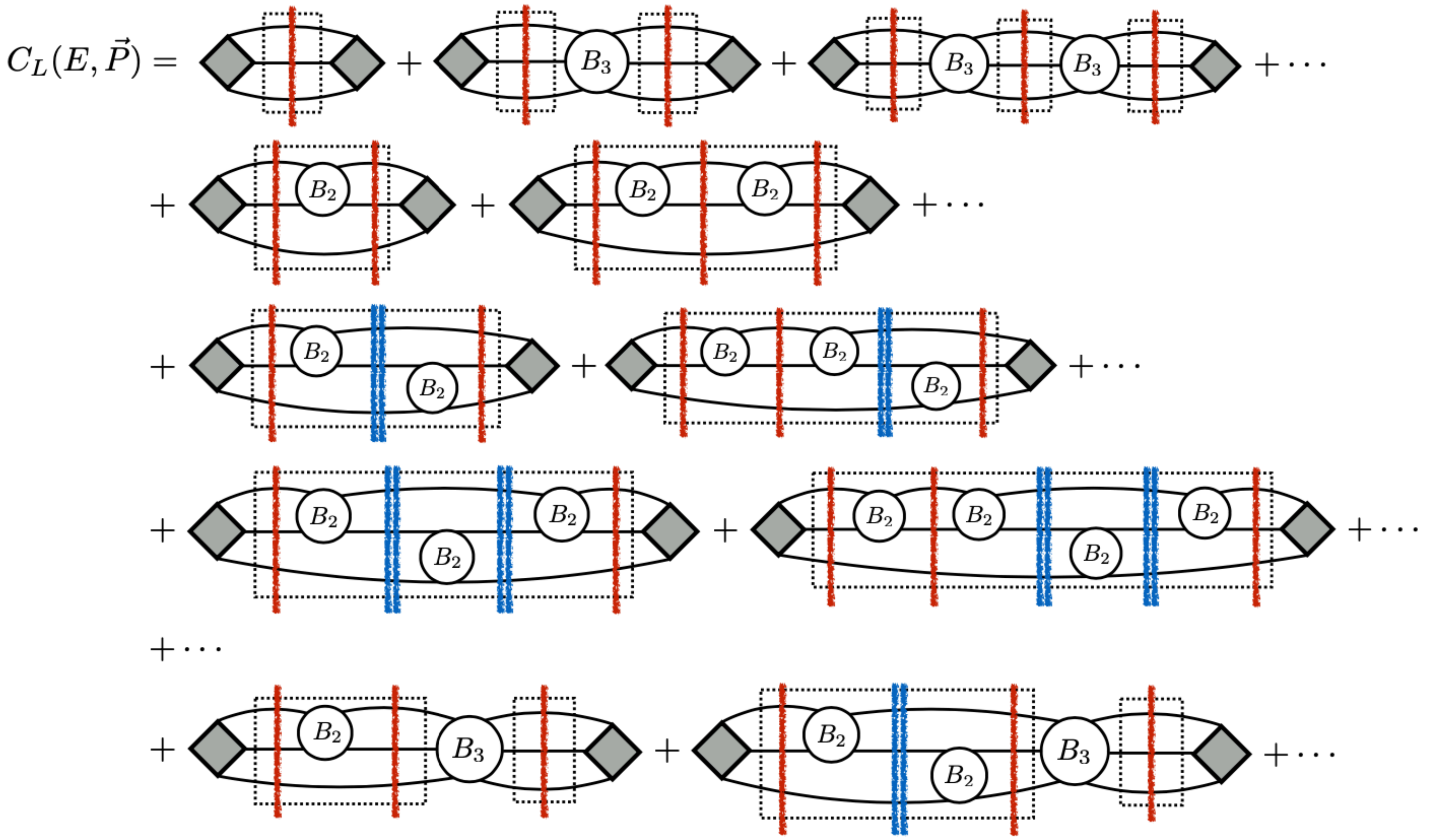
$$\sqrt{s_{\text{lhc},2}} \approx 3937\ \text{MeV} \qquad \sqrt{s_{\text{thr}}} \approx 3975\ \text{MeV}$$

REFT finite-volume quantization

Relativistic three-particle quantization condition for non-degenerate scalars
Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ and related systems
Blanton, Sharpe, PRD 103 (2021) 5, 054503 and PRD 104 (2021) 3, 034509

Lattice QCD and three-particle decays of resonances
Hansen, Sharpe, Ann. Rev. Nucl. Part. Sci. 69 (2019) 65-107

Incorporating DDπ effects and left-hand cuts in lattice QCD studies of T_{cc}^+
Hansen, Romero-López, Sharpe, arXiv:2401.06609



$$\begin{array}{cc} (D\pi)D & (DD)\pi \end{array}$$

$$\begin{pmatrix} \mathcal{K}_3^{(11)} & \mathcal{K}_3^{(12)} \\ \mathcal{K}_3^{(21)} & \mathcal{K}_3^{(22)} \end{pmatrix} \begin{array}{c} (D\pi)D \\ (DD)\pi \end{array}$$

Generalization to the relevant isospin

$$\mathbf{\frac{1}{2} \otimes \frac{1}{2} \otimes 1 = 0 \oplus 1 \oplus 1 \oplus 2}$$

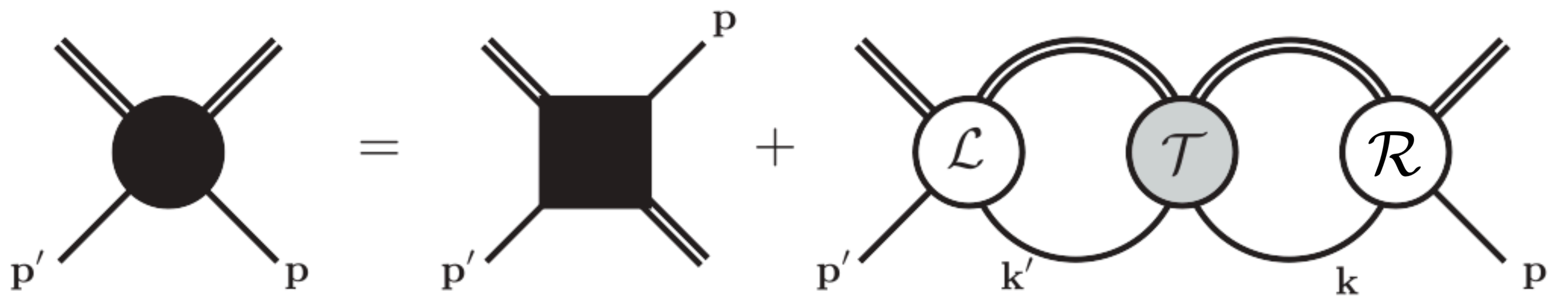
$$\det_{k,\ell,m} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3(E^\star)\boldsymbol{F}_3(E,\boldsymbol{P},L)\right] = 0$$

$$\prod_{I\in\{0,1,2\}} \det_{k,\ell,m,f} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3^I(E^\star)\boldsymbol{F}_3^I(E,\boldsymbol{P},L)\right] = 0$$

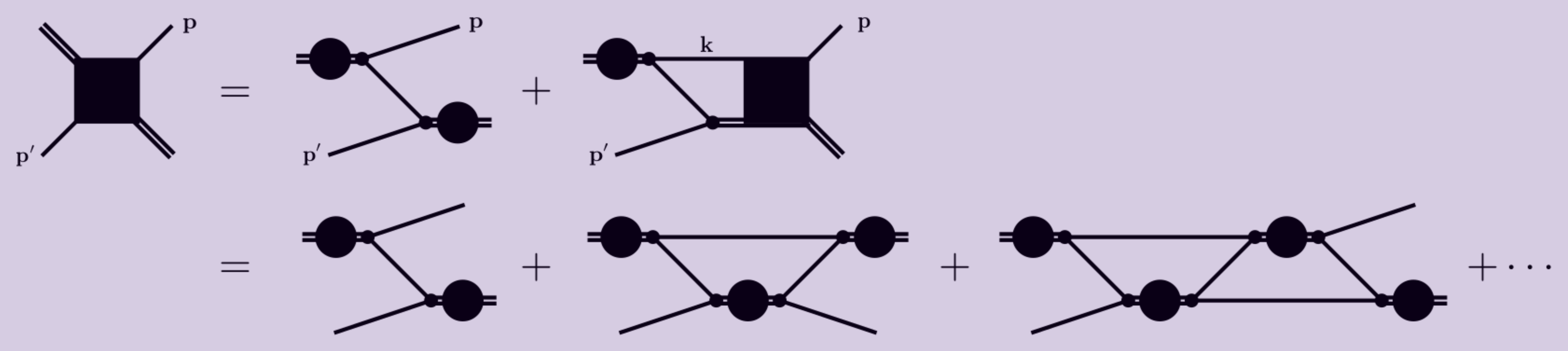
REFT three-body integral equations

diagrams by Andrew Jackura

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{3,\mathrm{df}}$$



One−particle exchanges

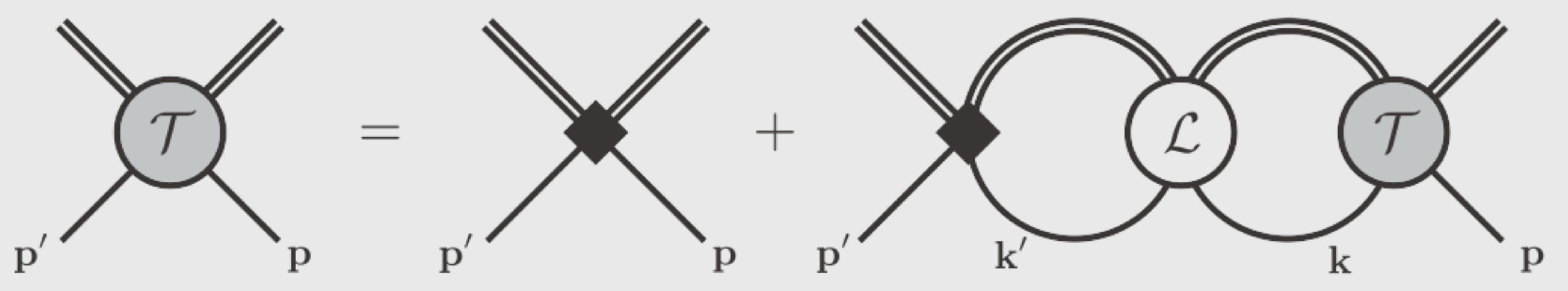


Three-body scattering: Ladders and Resonances
Mikhasenko, Wunderlich, Jackura, et al., *JHEP* 08 (2019) 080

Equivalence of three-particle scattering formalisms
Jackura, Dawid, Fernandez-Ramirez, et al., PRD 100 (2019) 3, 034508

Equivalence of relativistic three-particle quantization conditions
Blanton, Sharpe, PRD 102 (2020) 5, 054515

Short−range amplitude



External−state rescatterings

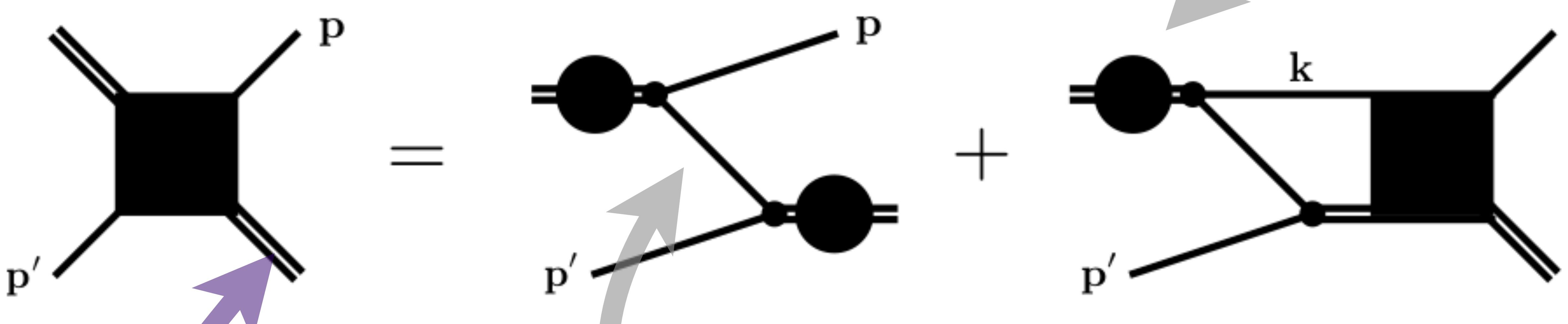
Simple example at J=0

diagrams by Andrew Jackura

Solving relativistic three-body integral equations in the presence of bound states
Jackura, Briceño, Dawid, Islam, McCarty, Phys.Rev.D 104 (2021) 1, 014507

Two-body amplitude

$$\mathcal{M}_2^{-1} \sim -\frac{1}{a} - i\rho_2$$



Bound state of mass

$m_b = 1.732\, m \quad (ma = 2)$

$m_b = 1.996\, m \quad (ma = 16)$

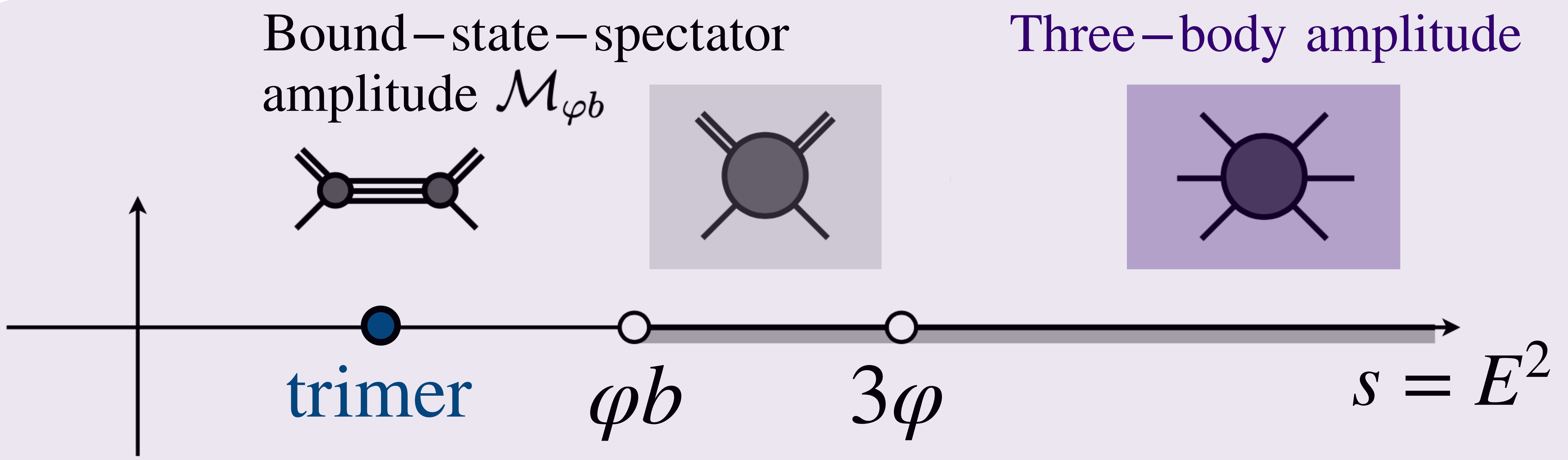
$$\mathcal{M}_2 \approx \frac{-g^2}{\sigma - m_b^2}$$

One-particle exchange

$$G(p', s, p) \propto \log\left(\frac{1 + z(p', s, p)}{1 - z(p', s, p)}\right)$$

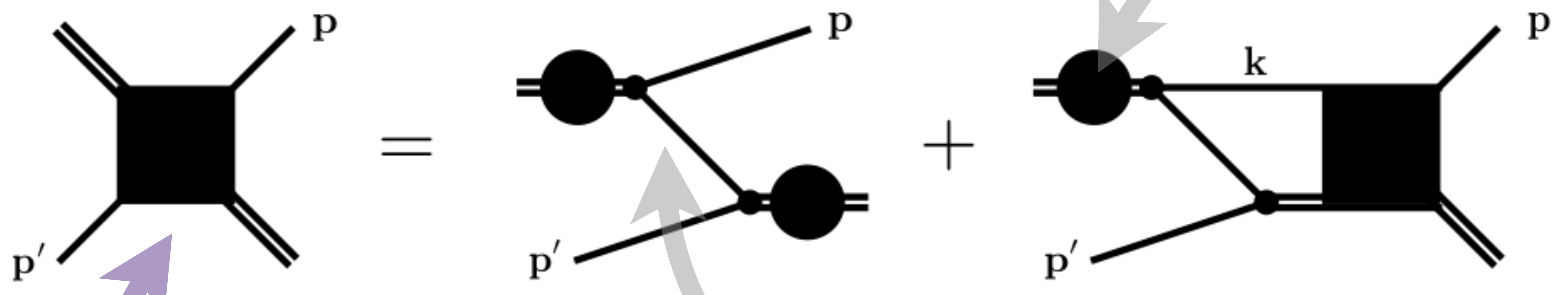
LSZ reduction

$$\mathcal{M}_{\varphi b} = \lim_{\sigma', \sigma \to m_b^2} (\sigma' - m_b^2)\, \mathcal{D}\, (\sigma - m_b^2)$$



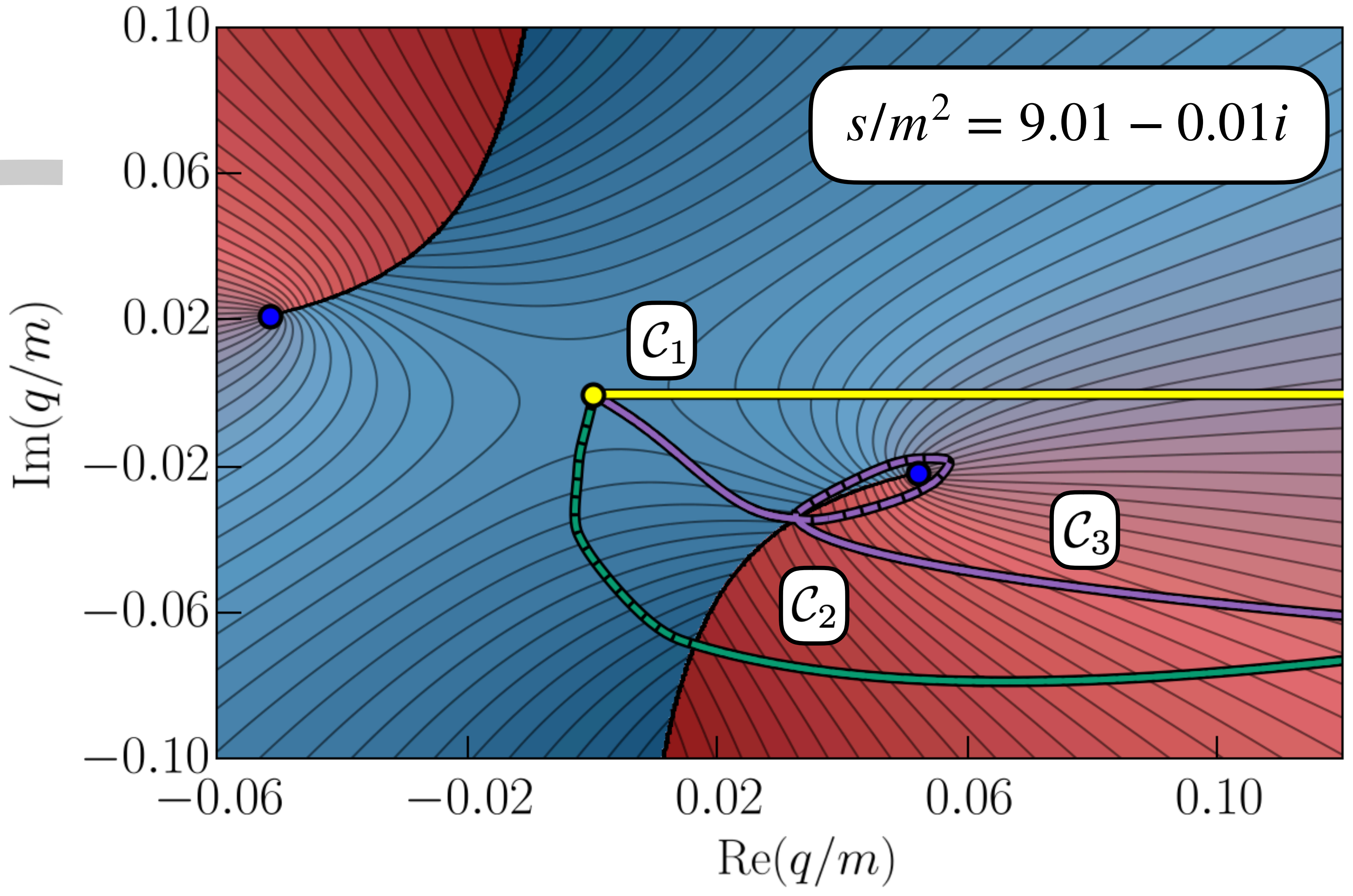
Analytic continuation*

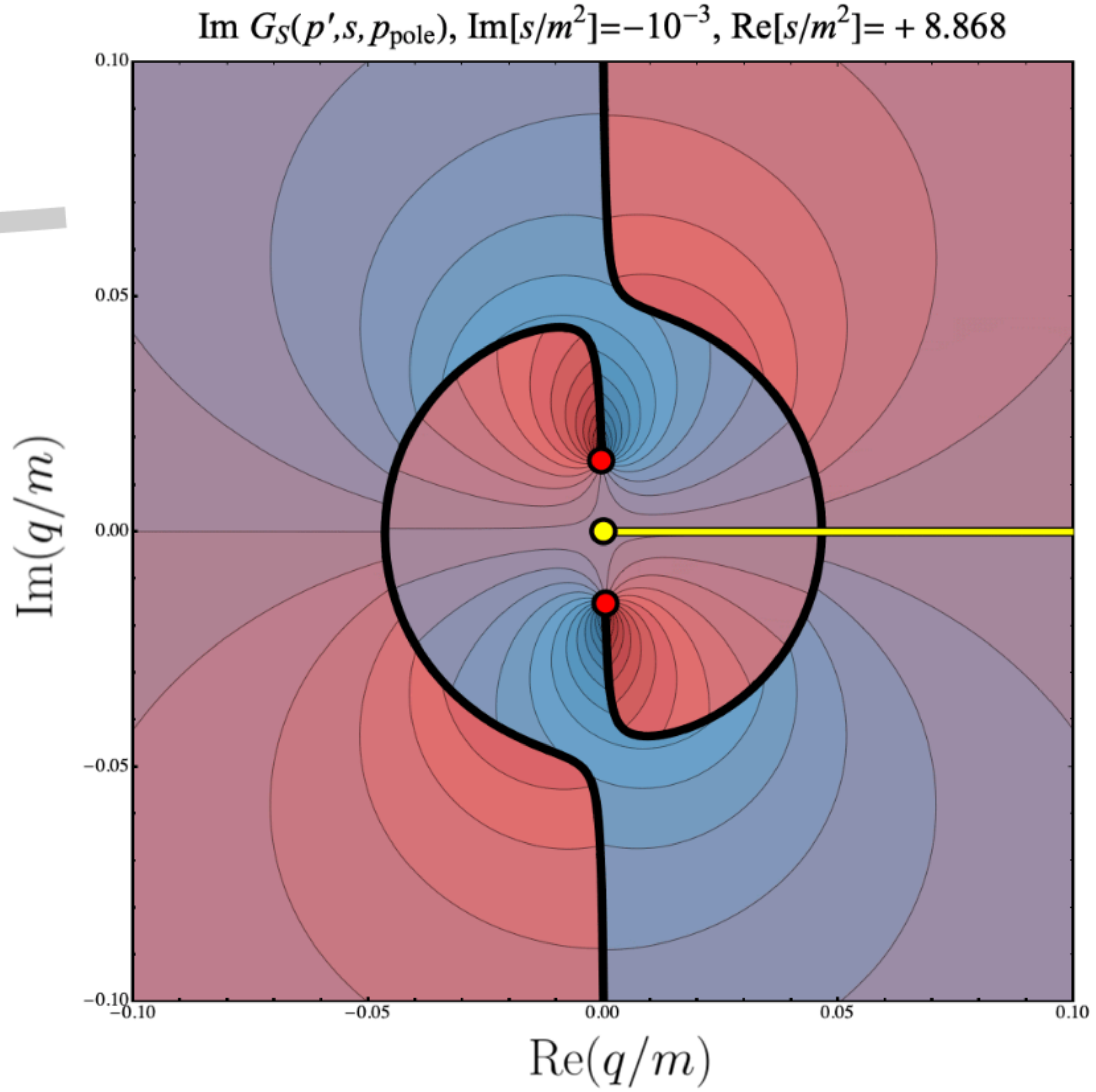
Two-body amplitude



Singularities

One-particle exchange



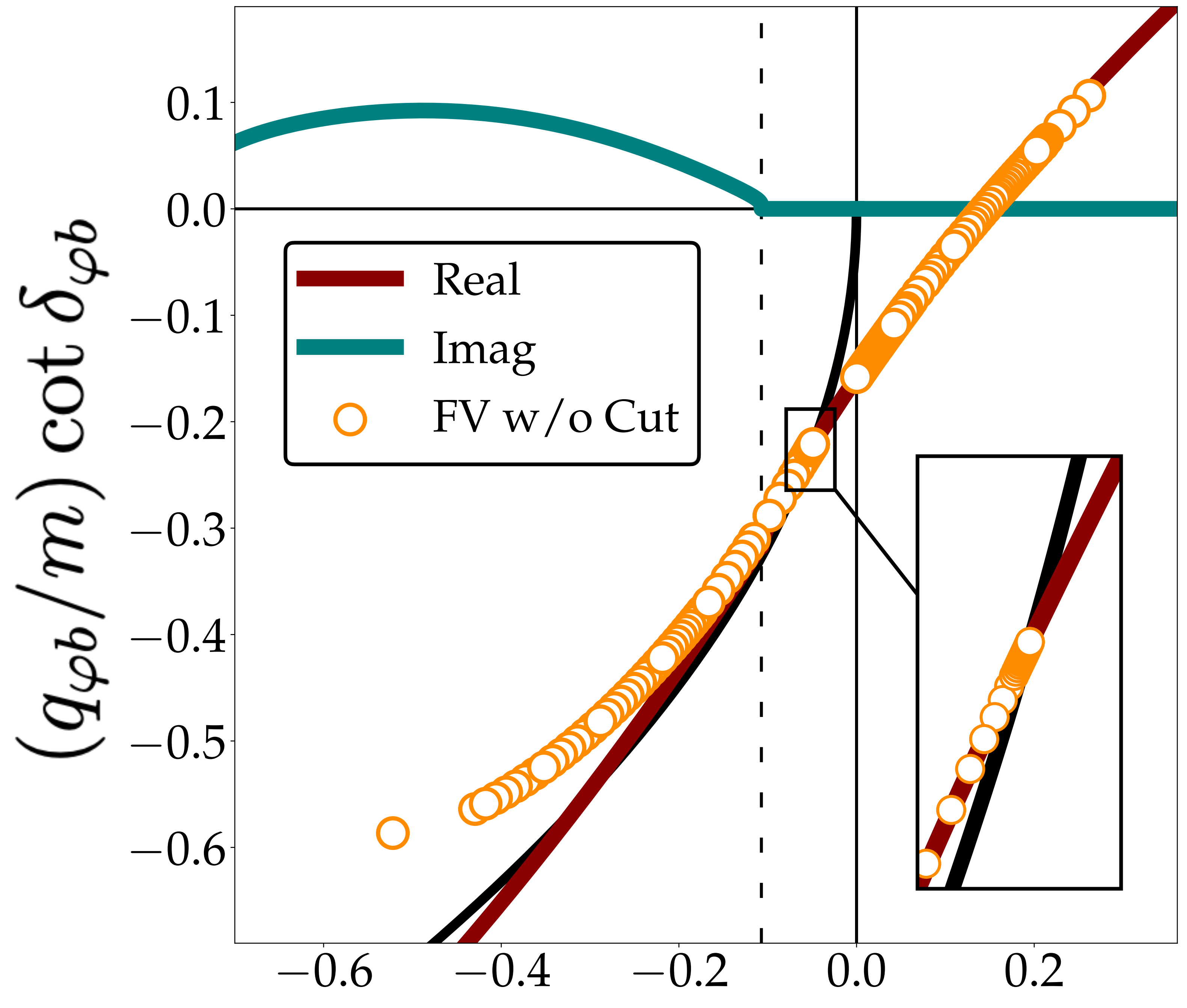


In a nutshell

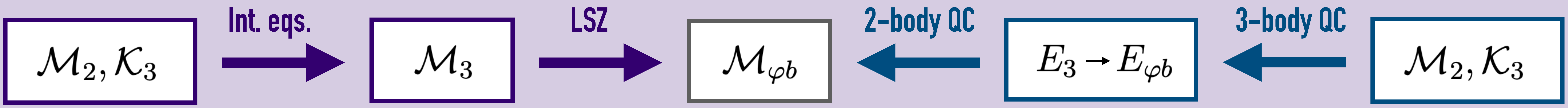
- need to avoid crossing the singularities in the integration
- achieved by contour deformations, addition of discontinuities
- Multi-valuedness of the amplitude originates from collisions of the contour with: poles (two-body threshold) and branch points (three-body threshold),
- Riemann sheets defined by a monodromy

Breakdown of the Lüscher formalism

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, PRD 108 (2023) 3, 034016
Numerical exploration of three relativistic particles...
Romero-Lopez et al. JHEP 10 (2019) 007



$ma = 2$



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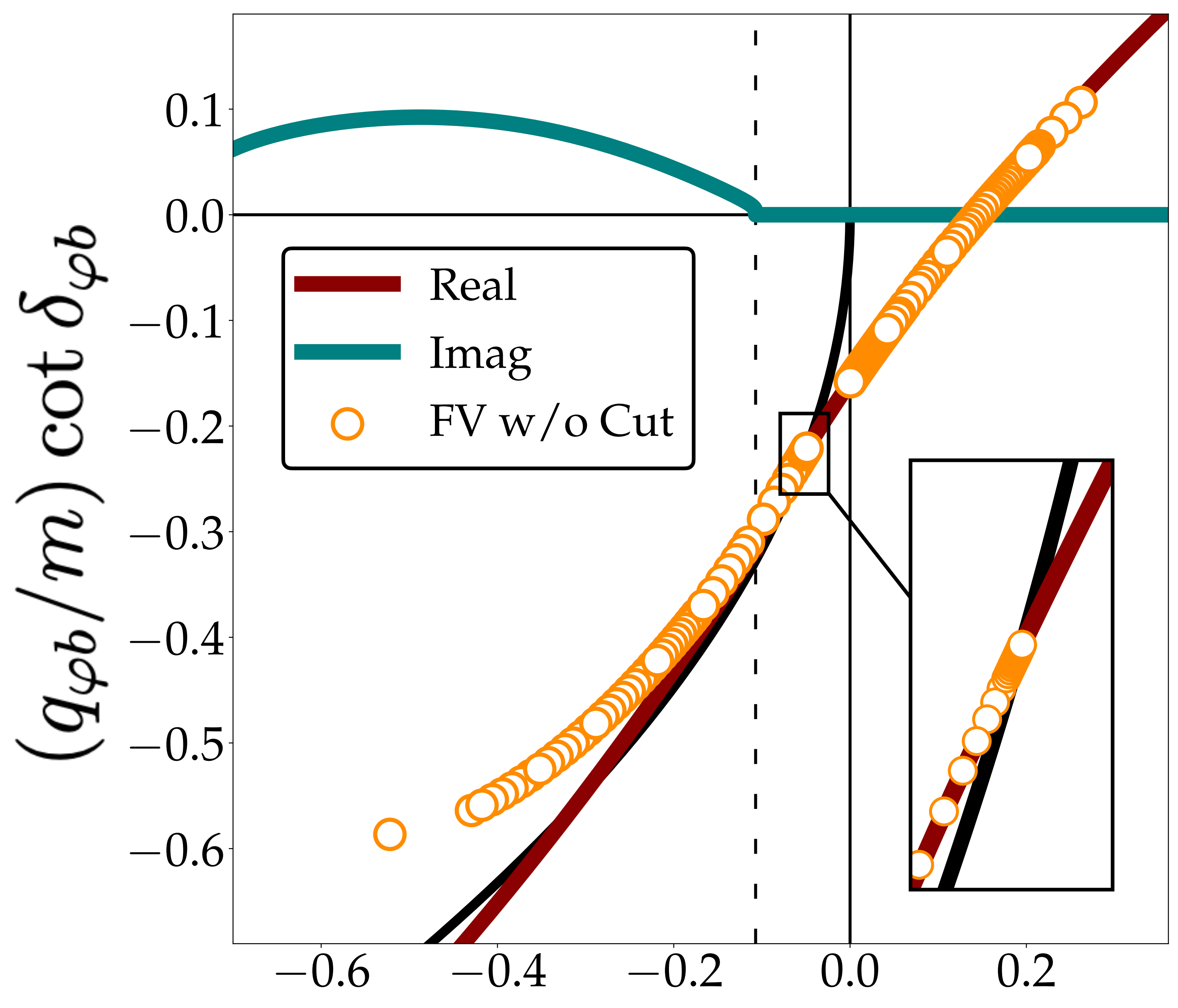
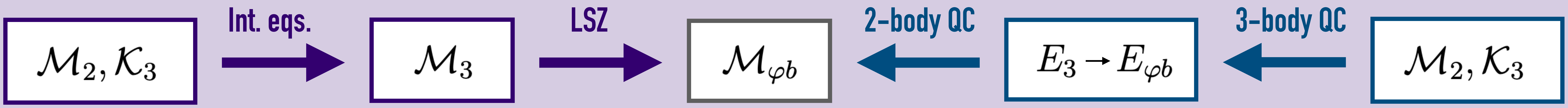


$ma = 2$

DDπ STRATEGY

1. Apply the three-body quantization condition to states with DD* quantum numbers (regardless of the pion mass)
2. Extract the DDπ-relevant two-and three-body K matrices
3. Solve the integral equations relating these objects to the continuum DDπ scattering amplitude
4. Employ the LSZ reduction formula to obtain the DD* amplitude that accounts for the pion exchanges



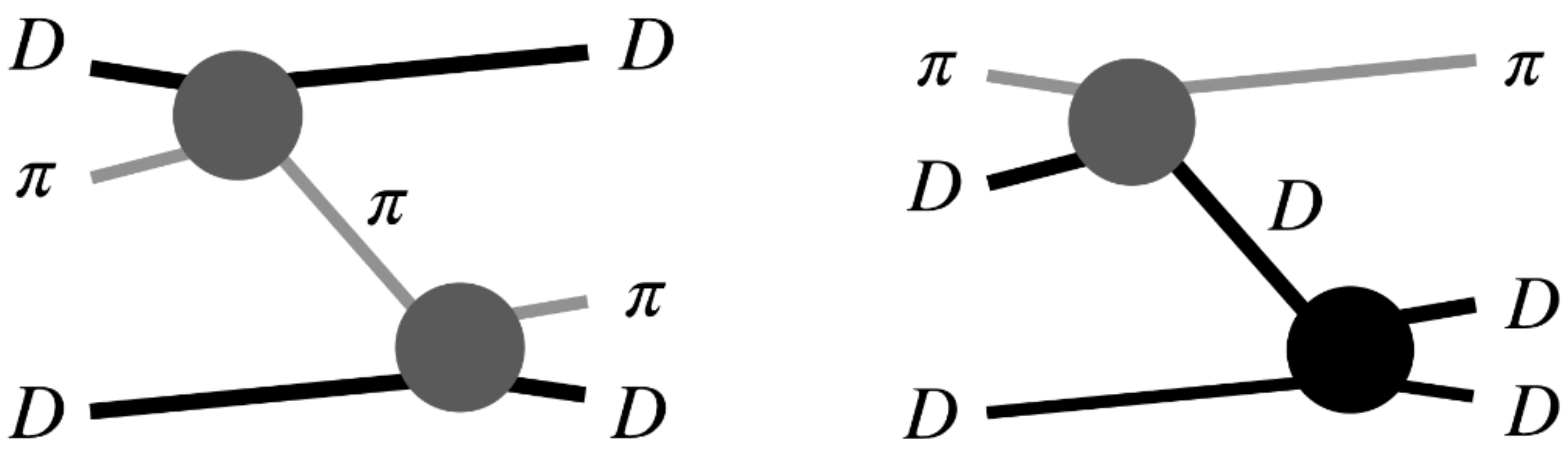
Generalizing to DDπ

$J^P = 1^+$

Dawid, Romero-López, Sharpe, in preparation

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 G \mathcal{D}$$

The amplitude becomes a matrix describing coupled-channel scattering between pairs and spectators of different angular momenta (PW mixing allowed)



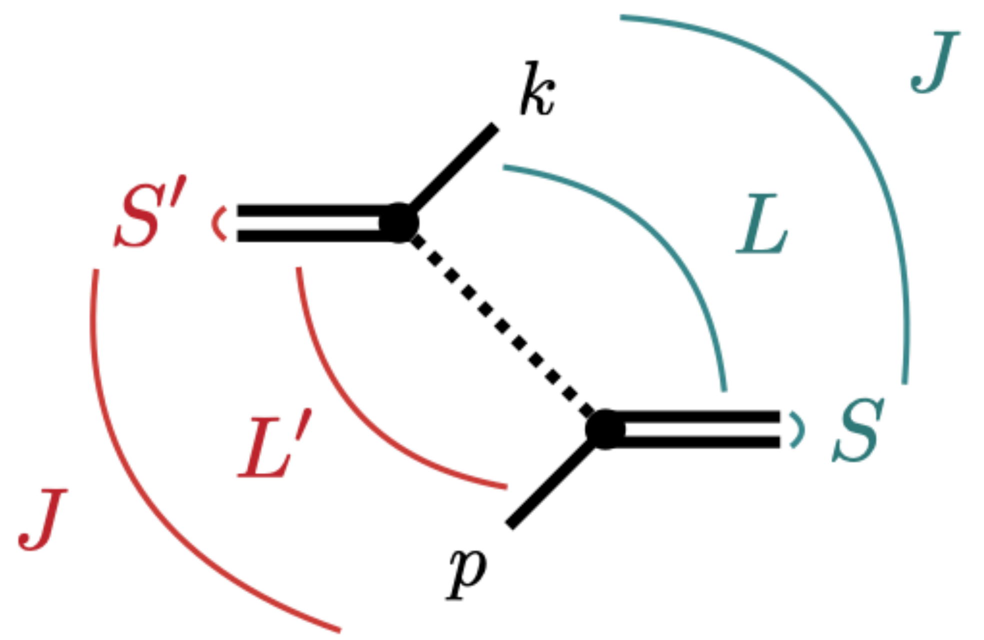
$$\begin{array}{cccc} (D\pi)D & (D\pi)D & (D\pi)D & (DD)\pi \end{array}$$

$$\begin{bmatrix} \mathcal{M}_3(^1P_1|^1P_1) & \mathcal{M}_3(^1P_1|^3S_1) & \mathcal{M}_3(^3P_1|^3D_1) & \mathcal{M}_3(^1P_1|^1P_1) \\ \mathcal{M}_3(^3S_1|^1P_1) & \mathcal{M}_3(^3S_1|^3S_1) & \mathcal{M}_3(^3S_1|^3D_1) & \mathcal{M}_3(^3S_1|^1P_1) \\ \mathcal{M}_3(^3D_1|^1P_1) & \mathcal{M}_3(^3D_1|^3S_1) & \mathcal{M}_3(^3D_1|^3D_1) & \mathcal{M}_3(^3D_1|^1P_1) \\ \mathcal{M}_3(^1P_1|^1P_1) & \mathcal{M}_3(^1P_1|^3S_1) & \mathcal{M}_3(^1P_1|^3D_1) & \mathcal{M}_3(^1P_1|^1P_1) \end{bmatrix} \begin{array}{c} (D\pi)D \\ (D\pi)D \\ (D\pi)D \\ (DD)\pi \end{array}$$

$$\mathcal{M}_{DD^*}(E) = [\mathcal{M}_{DD^*}(^3S_1|^3S_1)]$$

Partial-wave projection of the one-particle exchange in three-body scattering amplitudes
Jackura, Briceño, PRD 109, 096030 (2024)

$$\mathcal{G}(^{2S'+1}L'_J|^{2S+1}L_J) =$$



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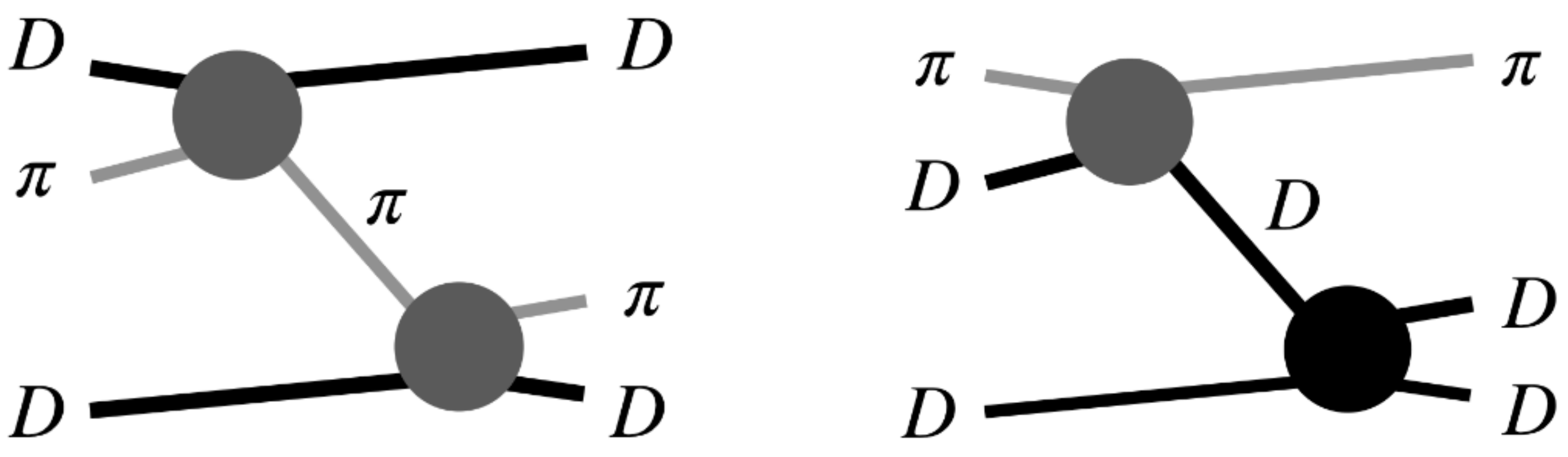
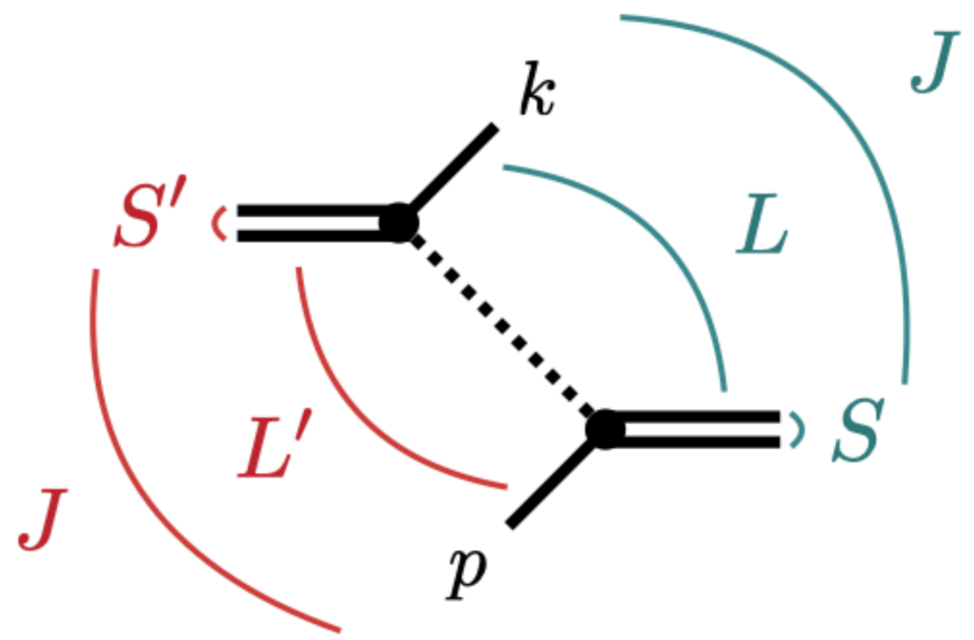


$$\begin{array}{cccc} (D\pi)D & (D\pi)D & (D\pi)D & (DD)\pi \end{array}$$
$$\begin{bmatrix} \mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^3P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1) \\ \mathcal{M}_3({}^3S_1|{}^1P_1) & \mathcal{M}_3({}^3S_1|{}^3S_1) & \mathcal{M}_3({}^3S_1|{}^3D_1) & \mathcal{M}_3({}^3S_1|{}^1P_1) \\ \mathcal{M}_3({}^3D_1|{}^1P_1) & \mathcal{M}_3({}^3D_1|{}^3S_1) & \mathcal{M}_3({}^3D_1|{}^3D_1) & \mathcal{M}_3({}^3D_1|{}^1P_1) \\ \mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^1P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1) \end{bmatrix} \begin{array}{c} (D\pi)D \\ (D\pi)D \\ (D\pi)D \\ (DD)\pi \end{array}$$

$$\mathcal{M}_{DD^*}(E) = \begin{bmatrix} \mathcal{M}_{DD^*}({}^3S_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3S_1|{}^3D_1) \\ \mathcal{M}_{DD^*}({}^3D_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3D_1|{}^3D_1) \end{bmatrix}$$

Partial-wave projection of the one-particle exchange in three-body scattering amplitudes
Jackura, Briceño, PRD 109, 096030 (2024)

$$\mathcal{G}({}^{2S'+1}L'_J|{}^{2S+1}L_J) \;=$$



Including three-body forces $J^P = 1^+$

Implementing the three-particle quantization condition for ππK and related systems
Blanton, Romero-López, Sharpe, JHEP 02 (2022) 098

$$\mathcal{T} = \mathcal{K}_3 - \mathcal{K}_3\,\rho\,\mathcal{L}\,\mathcal{T}$$

Solution of another integral equation is unnecessary for certain models of the three-body K matrix

$$\mathcal{K}_3^{(ij)}(p,k) = \sum_a \mathcal{K}_{L,a}^{(i)}(p)\,\mathcal{K}_{R,a}^{(j)}(k)$$

$$\mathcal{T} = \mathcal{K}_L^T\,[1+\mathcal{I}]^{-1}\,\mathcal{K}_R$$

Inversion of a 2 x 2 matrix composed of double integrals of the rescattering function.

Threshold expansion

$$\mathcal{K}_3 = \mathcal{K}_3^{\text{iso},0} + \mathcal{K}_3^{\text{iso},1}\Delta + \mathcal{K}_3^B\Delta_2^S + \mathcal{K}_3^E t'_{22}$$

$$\Delta = \frac{s-(2m_D+m_\pi)^2}{(2m_D+m_\pi)^2} \qquad \tilde{t}_{22} = \frac{(p_2-p'_2)^2}{(2m_D+m_\pi)^2}$$

The last term contributes, for instance

$$\mathcal{K}_3({}^3S_1|{}^3S_1) = \frac{2}{27}\mathcal{K}_3^E\,q_p^\star\,q_k^\star\,(\gamma_p+2)(\gamma_k+2)$$

Relative two−body momentum in a pair

Boost to pair's rest frame

Details of the two-body interactions

Mohler et al. PRD 87, 034501 (2012)
Moir et al. (HadSpec) JHEP 10 (2016) 011 (2016)
Gayer et al. (HadSpec), JHEP 07 (2021) 123
Yan et al. arXiv: 2404.13479 (2024)

Six parameters

$$\{a^S_{D\pi}, r^S_{D\pi}, a^P_{D\pi}, r^P_{D\pi}, a^S_{DD}, \mathcal{K}^E_3\}$$

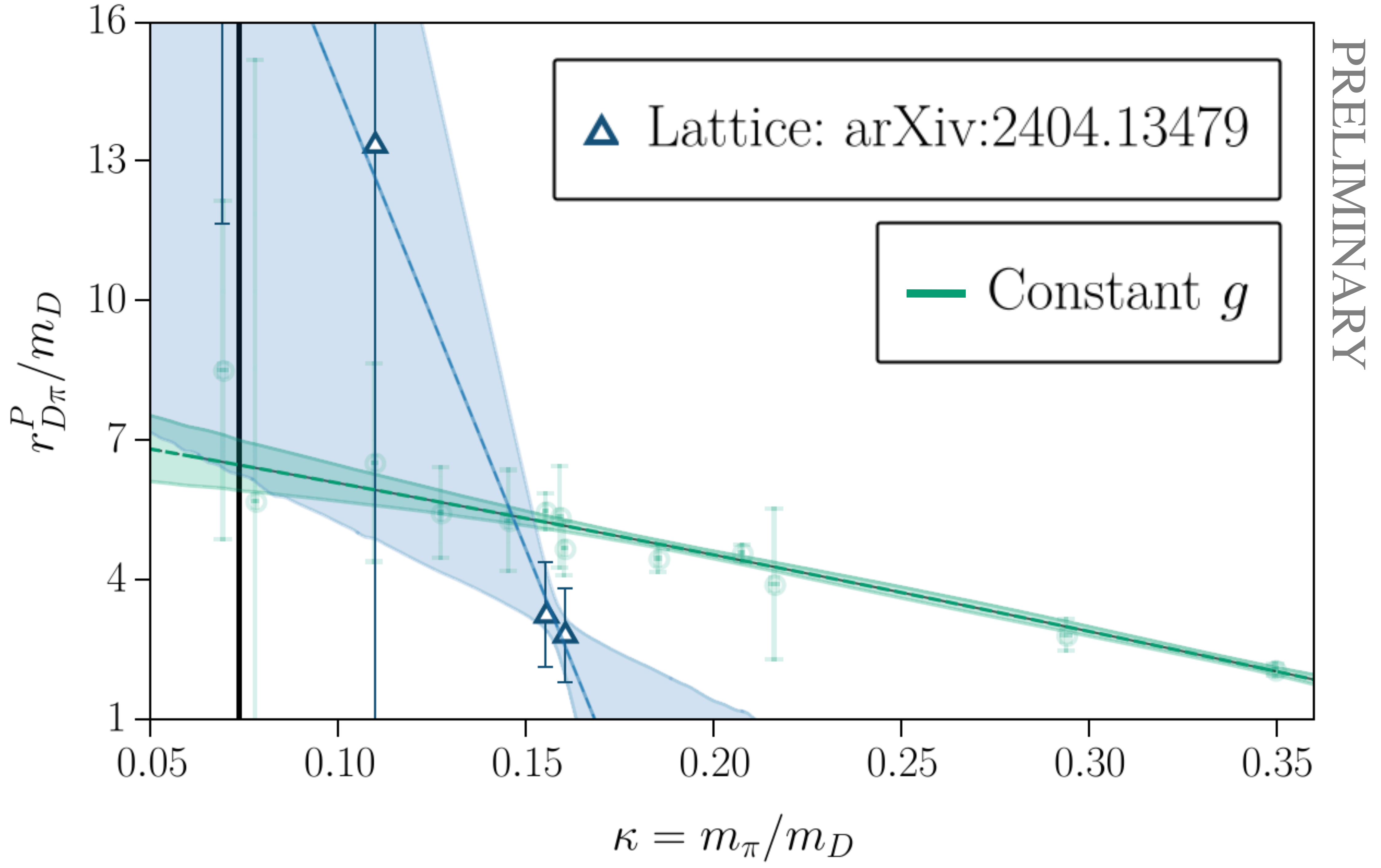
$$k^{2s+1}\cot\delta_n^{(s)} = -\frac{1}{a_n^{(s)}} + \frac{1}{2} r_n^{(s)} k^2$$

$m_\pi \approx 280$ MeV
$m_D \approx 1927$ MeV
$m_{D^*} \approx 2049$ MeV

$\kappa = m_\pi / m_D \approx 0.145$

$\kappa' = m_{D^*}/m_D \approx 1.063$

physical: $\kappa \approx 0.072$, $\kappa' \approx 1.075$



Heavy−light meson ChPT

$$\mathcal{L} = \frac{f_\pi^2}{8}\partial^\mu\Sigma_{ab}\,\partial_\mu\Sigma^\dagger_{ab} + \lambda_0[\hat{m}\Sigma + \hat{m}\Sigma^\dagger]_{aa}$$
$$- \mathrm{Tr}[\bar{H}_a i v_\mu D^\mu_{ba} H_b] + g\,\mathrm{Tr}[\bar{H}_a H_b \not{A}_{ba}\gamma_5] + \ldots$$

Details of the two-body interactions

Mohler et al. PRD 87, 034501 (2012)
Moir et al. (HadSpec) JHEP 10 (2016) 011 (2016)
Gayer et al. (HadSpec), JHEP 07 (2021) 123
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$$k^{2s+1}\cot\delta^{(s)}_n = -\frac{1}{a^{(s)}_n} + \frac{1}{2}\, r^{(s)}_n\, k^2$$

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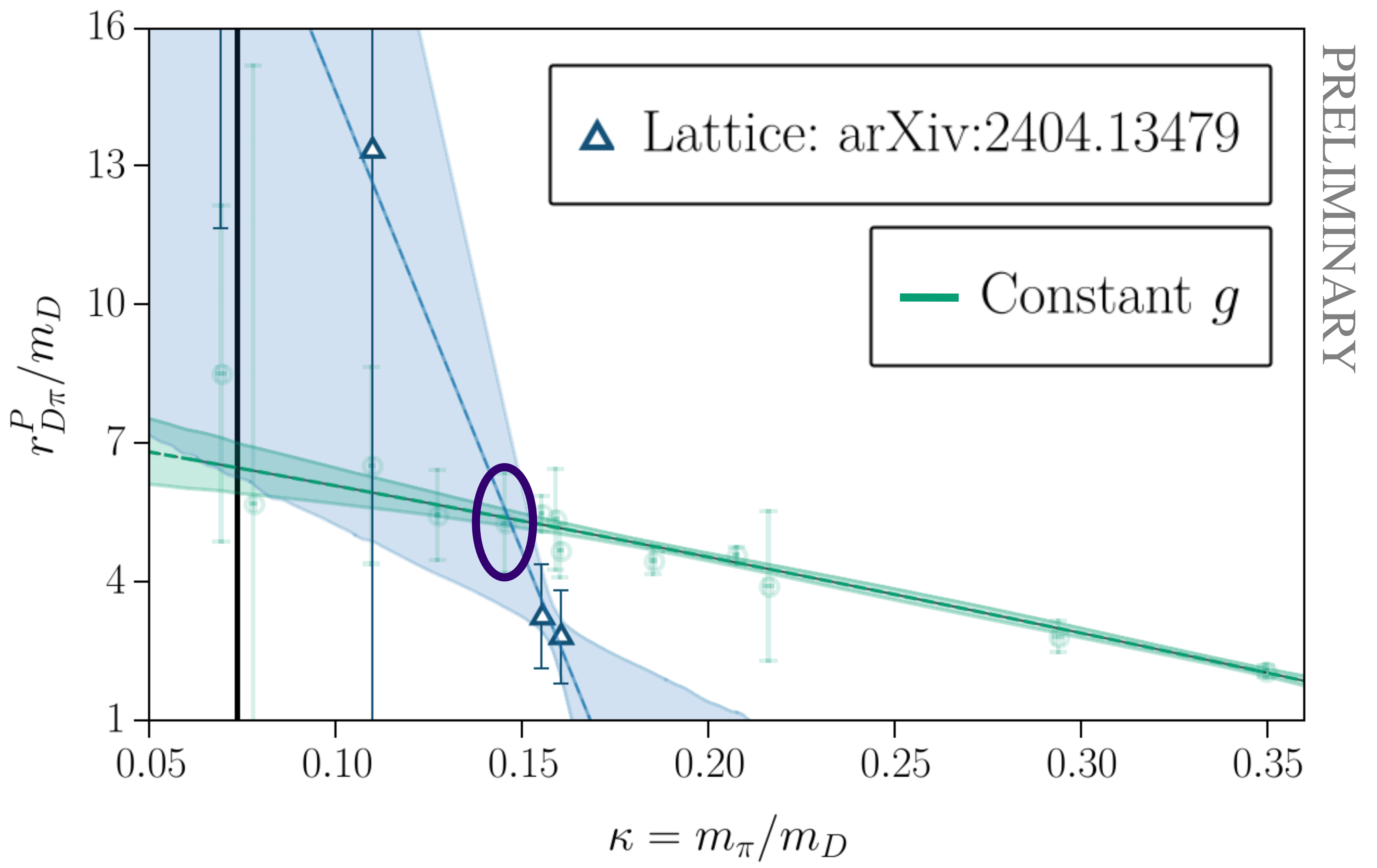

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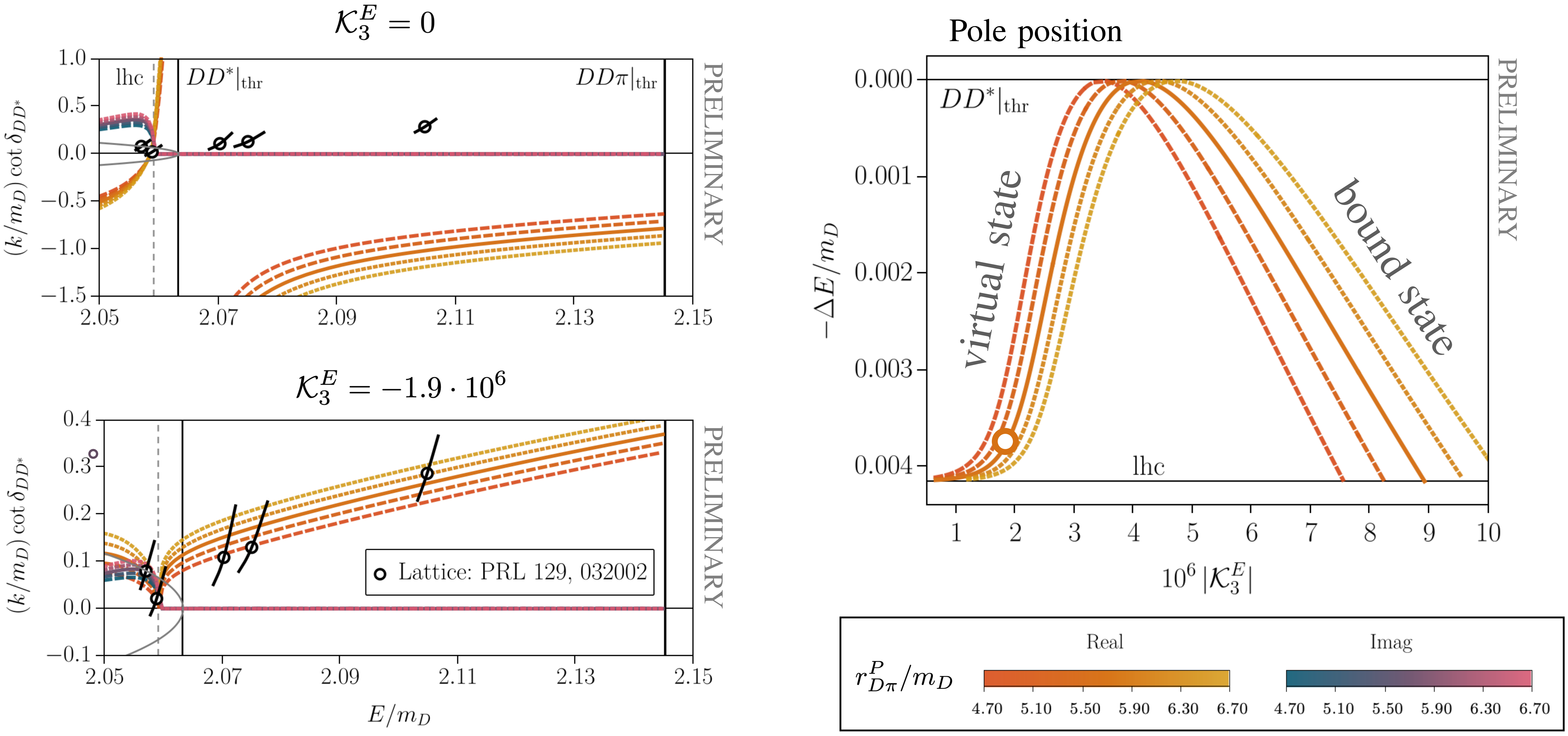
$$- \mathrm{Tr}[\bar{H}_a i v_\mu D^\mu_{ba} H_b] + g\,\mathrm{Tr}[\bar{H}_a H_b \not{A}_{ba}\gamma_5] + \ldots$$

Single-channel approximation

$[\mathcal{M}_3({}^3S_1|{}^3S_1)]$ $(\kappa = m_\pi/m_D = 0.145)$

$J^P = 1^+$



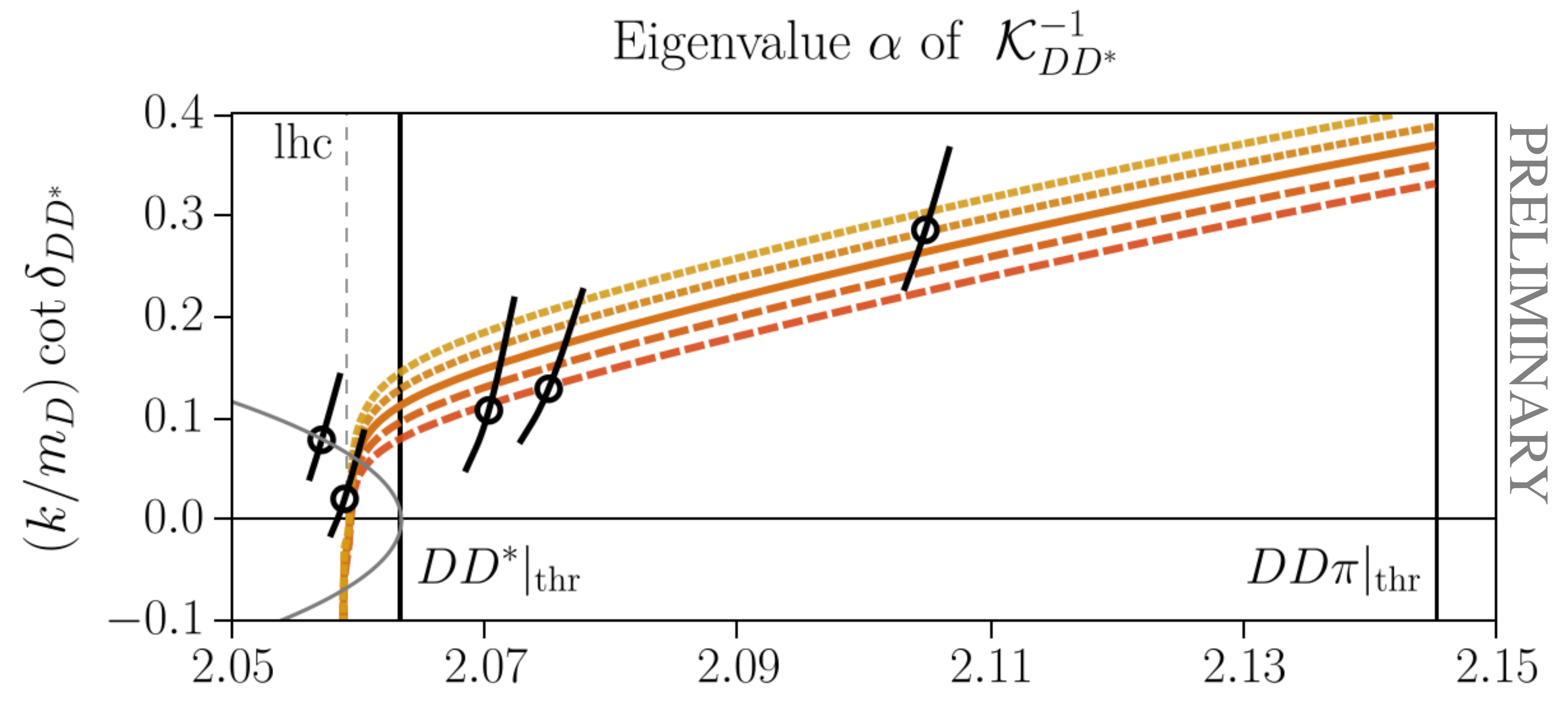
Simple lessons - part 1

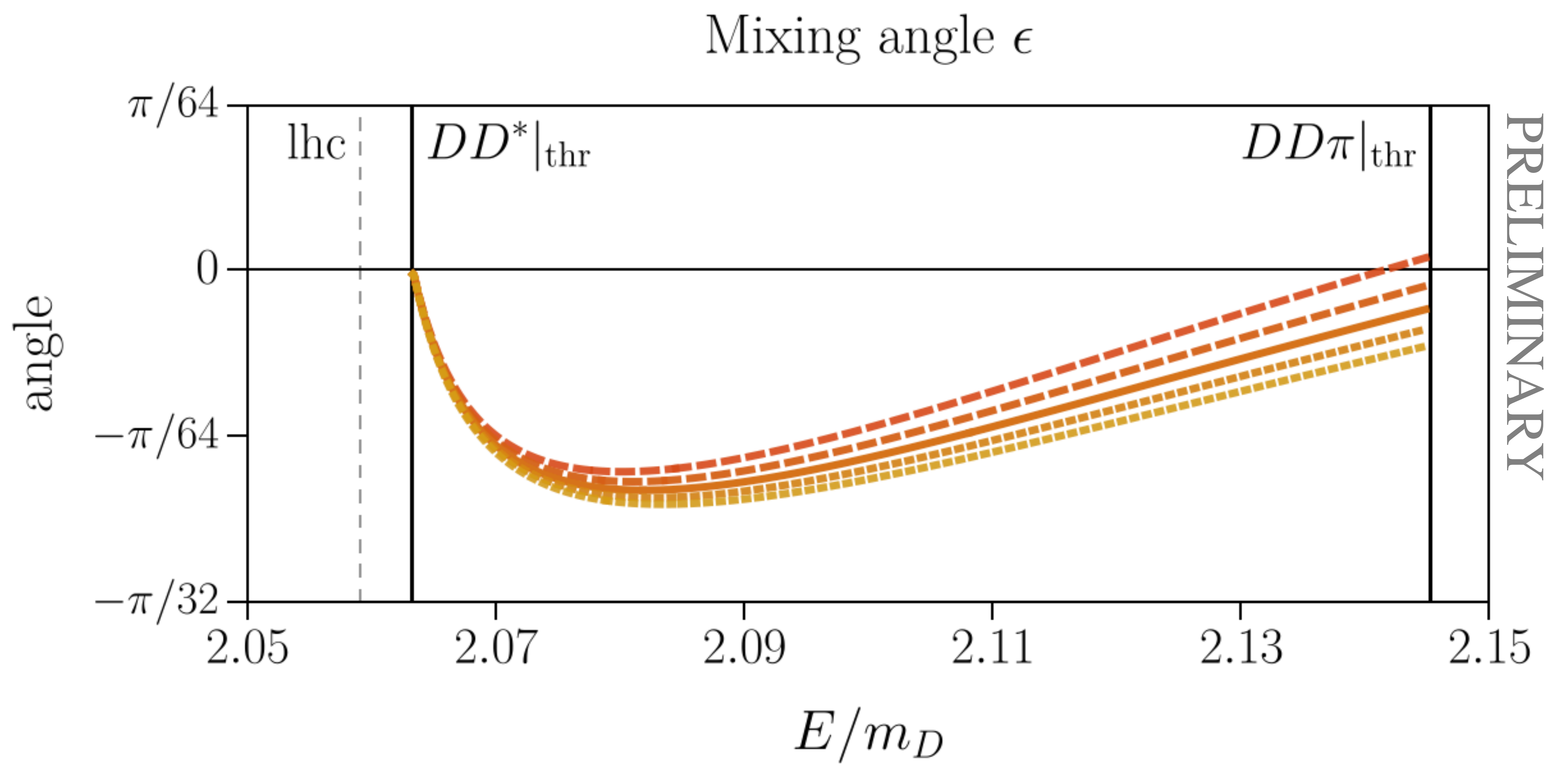
- presence of a virtual state
- status of the Effective Range Expansion
- one-particle exchange does not describe the data
- inclusion of three-body forces leads to an agreement

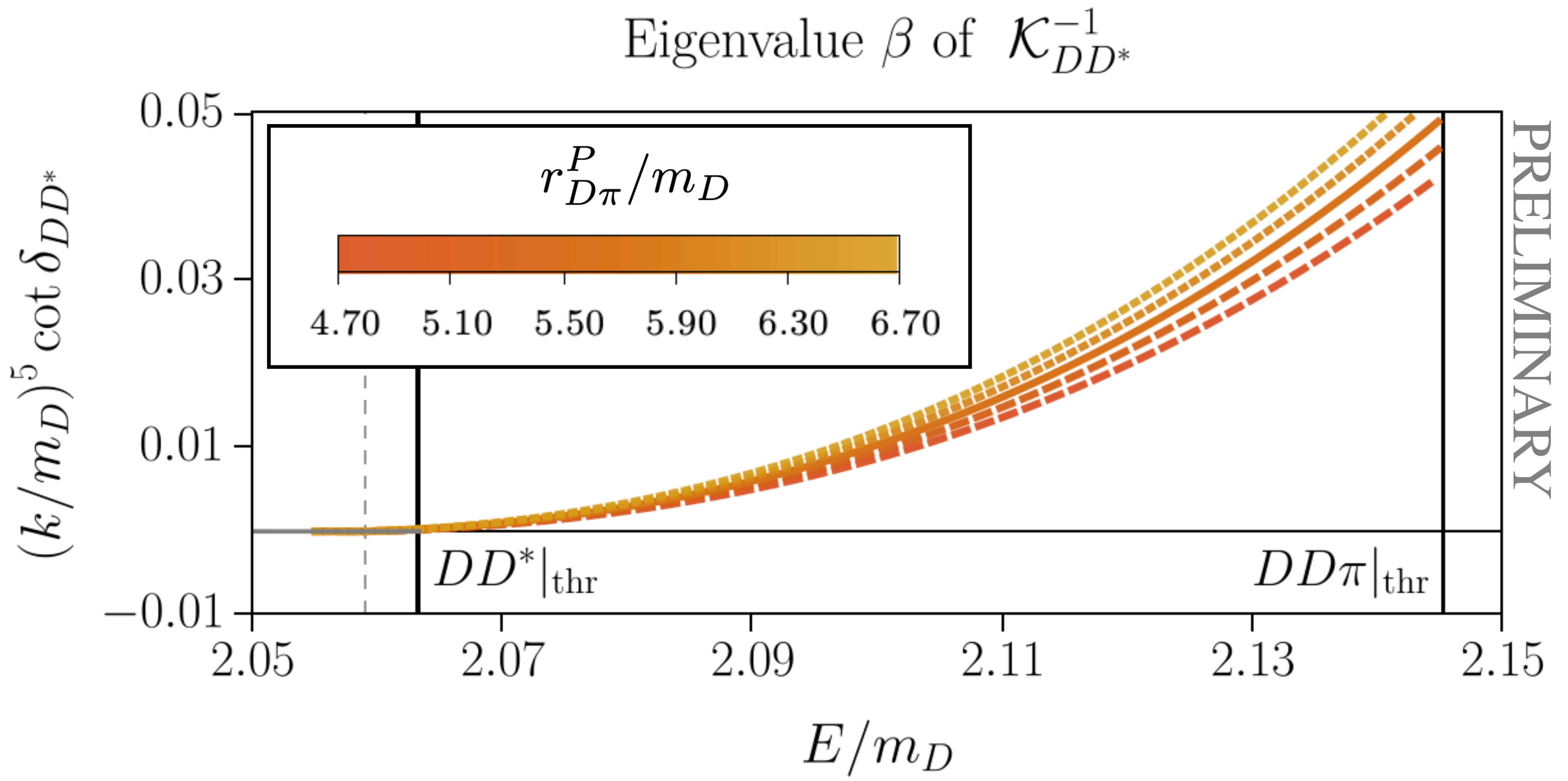
Partial-wave mixing included

$$\begin{bmatrix} \mathcal{M}_{DD^*}({}^3S_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3S_1|{}^3D_1) \\ \mathcal{M}_{DD^*}({}^3D_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3D_1|{}^3D_1) \end{bmatrix}$$

$J^P = 1^+$





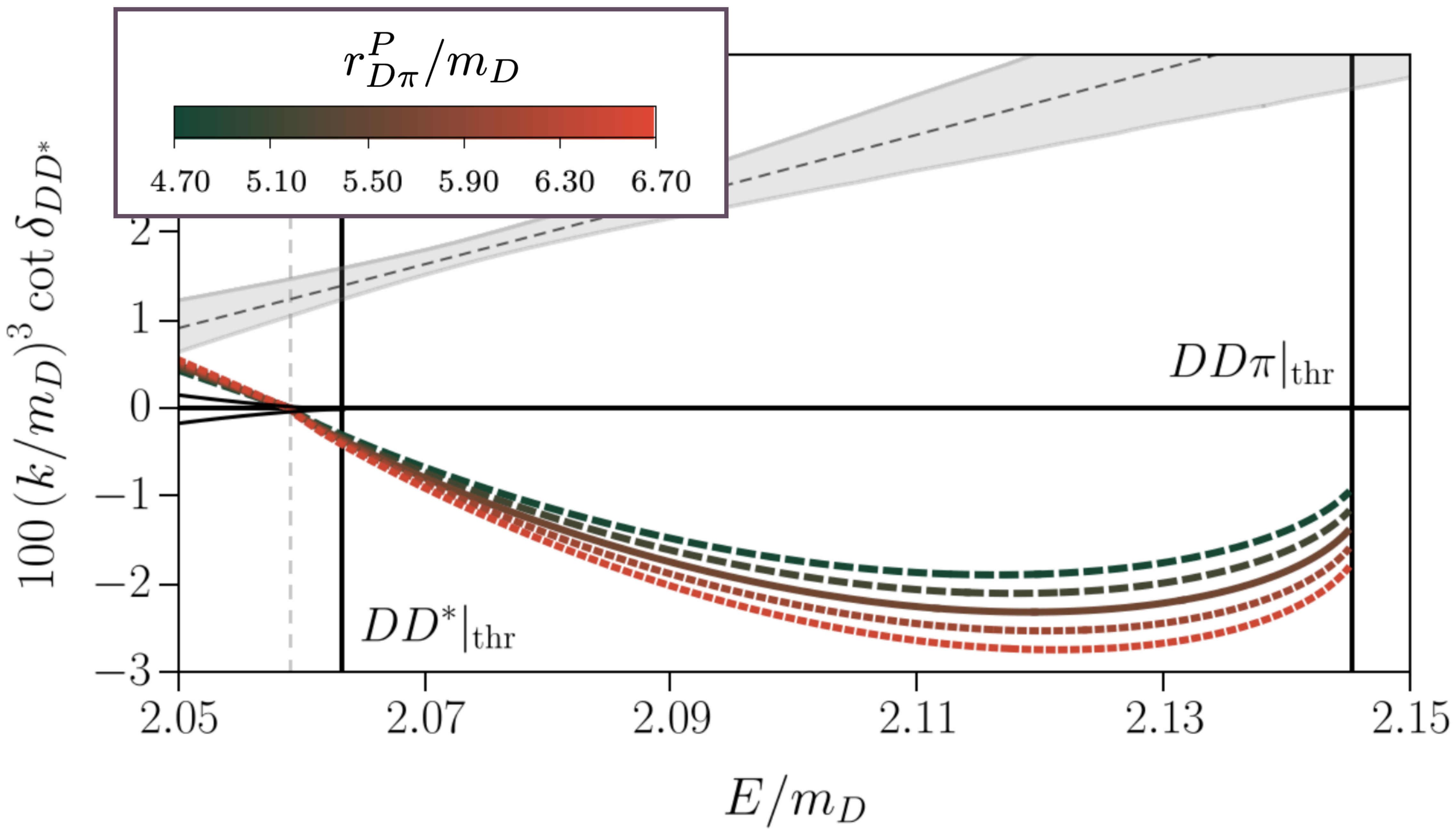


Blatt−Biederharn parametrization

$$\mathcal{K}^{-1}_{DD^*} = \begin{bmatrix} \cos(\epsilon) & -\sin(\epsilon) \\ \sin(\epsilon) & \cos(\epsilon) \end{bmatrix} \begin{bmatrix} k\cot\delta_0 & 0 \\ 0 & k\cot\delta_2 \end{bmatrix} \begin{bmatrix} \cos(\epsilon) & \sin(\epsilon) \\ -\sin(\epsilon) & \cos(\epsilon) \end{bmatrix}$$

Simple lessons - part 2

- partial-wave mixing is small
- $D\pi$ S-wave scattering is (almost) negligible



$$\begin{bmatrix} \mathcal{M}_3({}^1S_0|{}^1S_0) & \mathcal{M}_3({}^1S_0|{}^3P_0) \\ \mathcal{M}_3({}^3P_0|{}^1S_0) & \mathcal{M}_3({}^3P_0|{}^3P_0) \end{bmatrix}$$

$J^P = 0^-$

Two−body quantization condition

$$\det\left(\mathcal{K}_2^{-1} + U \cdot F \cdot U^\dagger\right) = 0$$

$$U_{j,m_j,\ell;m_\ell m_s} = \langle j m_j | \ell m_\ell s = 1 m_s \rangle$$

$$\begin{bmatrix} \mathcal{M}_3({}^1S_0|{}^1S_0) & \mathcal{M}_3({}^1S_0|{}^3P_0) \\ \mathcal{M}_3({}^3P_0|{}^1S_0) & \mathcal{M}_3({}^3P_0|{}^3P_0) \end{bmatrix}$$

$J^P = 0^-$

$m_\pi L = 3.9$

irrep	$E_{\text{free,cm}}$	$E_{\text{cm}}(\text{QC2})$	$E_{\text{cm}}(\text{QC2}^{\#})$	$E_{\text{cm}}(\text{QC3})$	$E_{\text{cm}}^{\text{Lattice}}$
$T_{1g}(0)$	2.06331	2.06008	same	2.060029	2.059(1)
$T_{1g}(0)$	2.11581	2.10467	same	2.104562	2.105(2)
$T_{1g}(0)$	2.11581	2.11523	same	2.114690	
$A_{1u}(0)$	2.11581		2.12227	2.123407	2.108(2)
$A_{1u}(0)$	2.145304*			2.145304	
$A_2(1)$	2.07562	2.07200	2.07203	2.072027	2.070(2)
$A_2(1)$	2.07720	2.07655	2.07807	2.078165	2.075(2)
$A_2(1)$	2.12785	2.11541	2.11542	2.115449	
$A_2(1)$	2.12785	2.12754	2.12788	2.127656	
$A_2(1)$	2.12932	2.12909	2.12876	2.131103	
$A_2(1)$	2.12932		2.13799	2.139797	
$A_2(1)$	2.15969*			2.159553	
$E(1)$	2.07562	2.07074	same	2.070441	
$E(1)$	2.07720	2.07651	same	2.076208	
$E(1)$	2.12785	2.11597	same	2.11583	
$E(1)$	2.12785	2.12731	same	2.125667	
$E(1)$	2.12785	2.12901	same	2.126917	
$E(1)$	2.12932			2.127326	
$E(1)$	2.12932			2.128819	
$E(1)$	2.12932			2.129434	

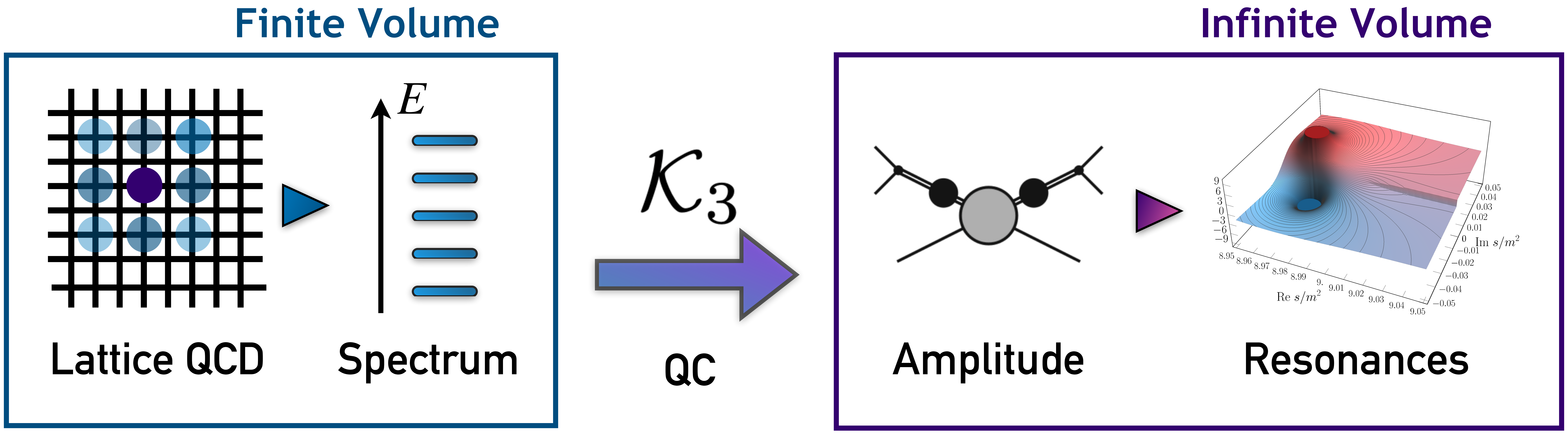
$m_\pi L = 2.93$

irrep	$E_{\text{free,cm}}$	$E_{\text{cm}}(\text{QC2}^{\#})$	$E_{\text{cm}}(\text{QC3})$	$E_{\text{cm}}^{\text{Lattice}}$
$T_{1g}(0)$	2.06331	no level	2.055713	2.057(1)
$T_{1g}(0)$	2.15576	2.13615	2.135876	
$A_{1u}(0)$	2.15576	2.06001	no level	2.141(2)
$A_2(1)$	2.08494	2.07779	2.077231	2.075(3)
$A_2(1)$	2.08769	2.09112	2.090232	2.081(2)
$E(1)$		2.05989*		
$E(1)$	2.08494	2.07505	2.073859	
$E(1)$	2.08769	2.08649	2.085316	

Simple lessons - part 3

- two-body quantization condition clearly breaks down near the lhc for small lattice volumes
- repulsive interaction in J=0 inconsistent with the lattice

Summary



Towards the tetraquark from lattice QCD

- ☑ resolution of the left-hand cut problem
- ☑ generalization of the three-body formalism
- ☑ check of the finite-volume effects
- ☑ models as an initial condition for Lattice QCD studies

Next steps

- ☐ Systematics of the K matrices
- ☐ Application of the formalism to the experimental data
- ☐ Formalism for the Roper resonance

THANK YOU

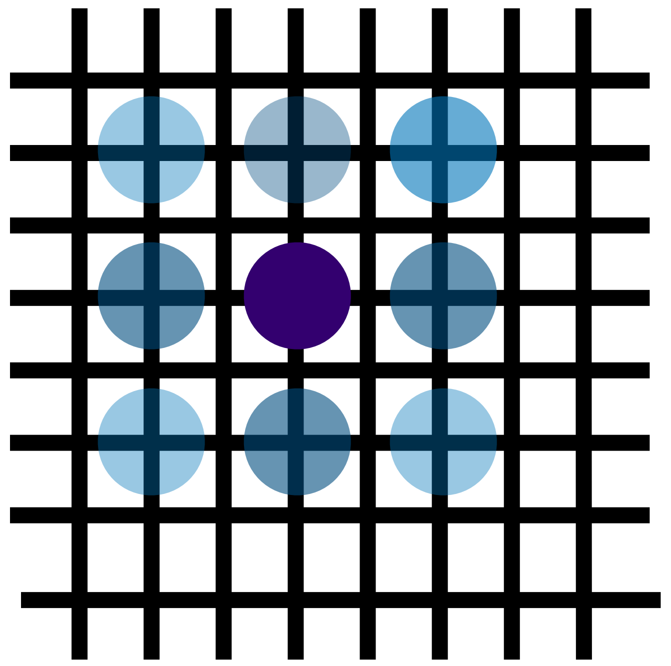
The three-body program

Relativistic, model-independent, three-particle quantization condition
Hansen, Sharpe, PRD 90 (2014) 11, 116003

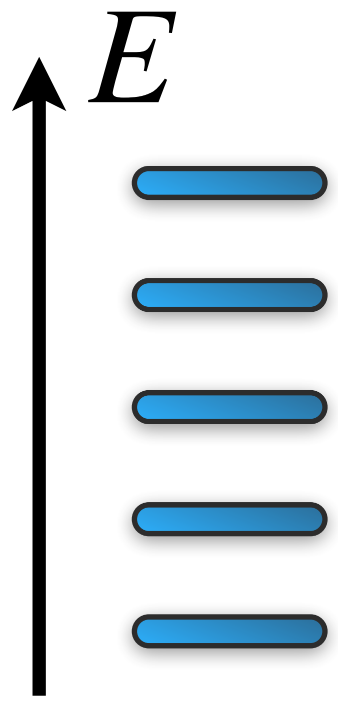
Three-body unitarity in finite volume
Mai, Döring, EPJ A 53 (2017) 12, 240

Relativistic-invariant formulation of the NREFT three-particle quantization condition
Müller, Pang, Rusetsky, Wu, JHEP 02 (2022), 158

Finite Volume



Lattice QCD

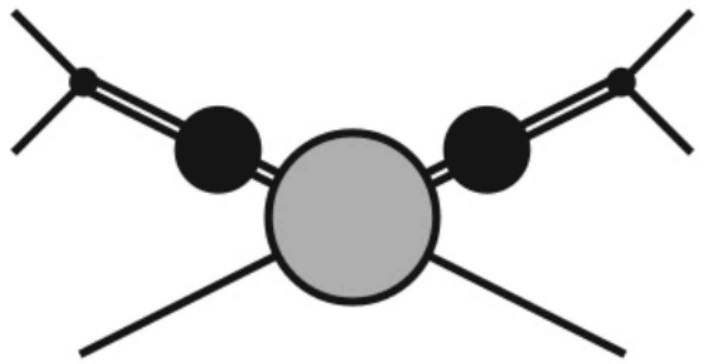


Spectrum

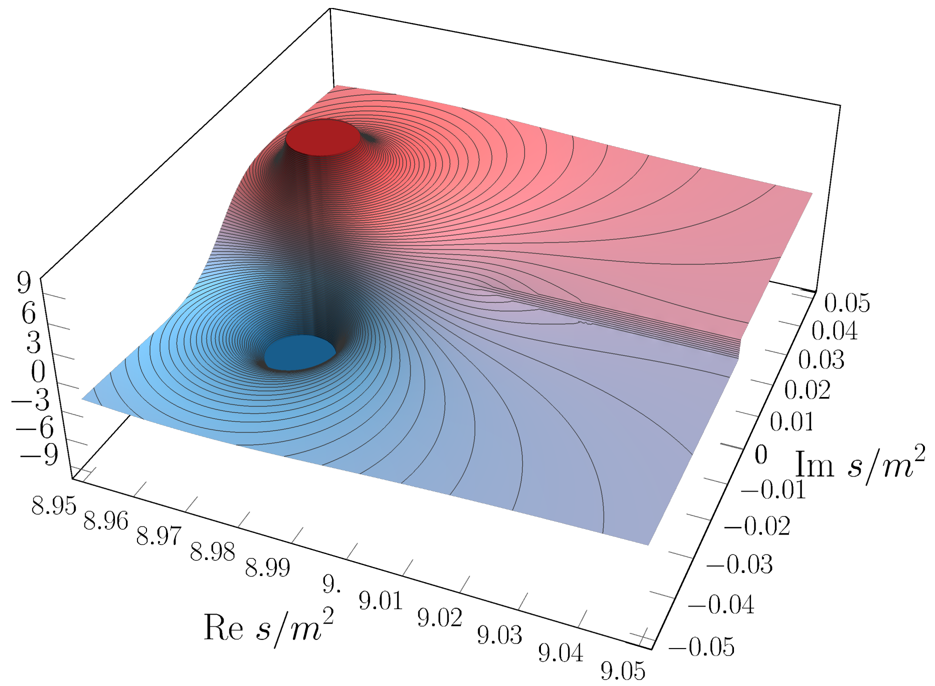
$\mathcal{K}_3$

Quantization Condition

Infinite Volume



Amplitude

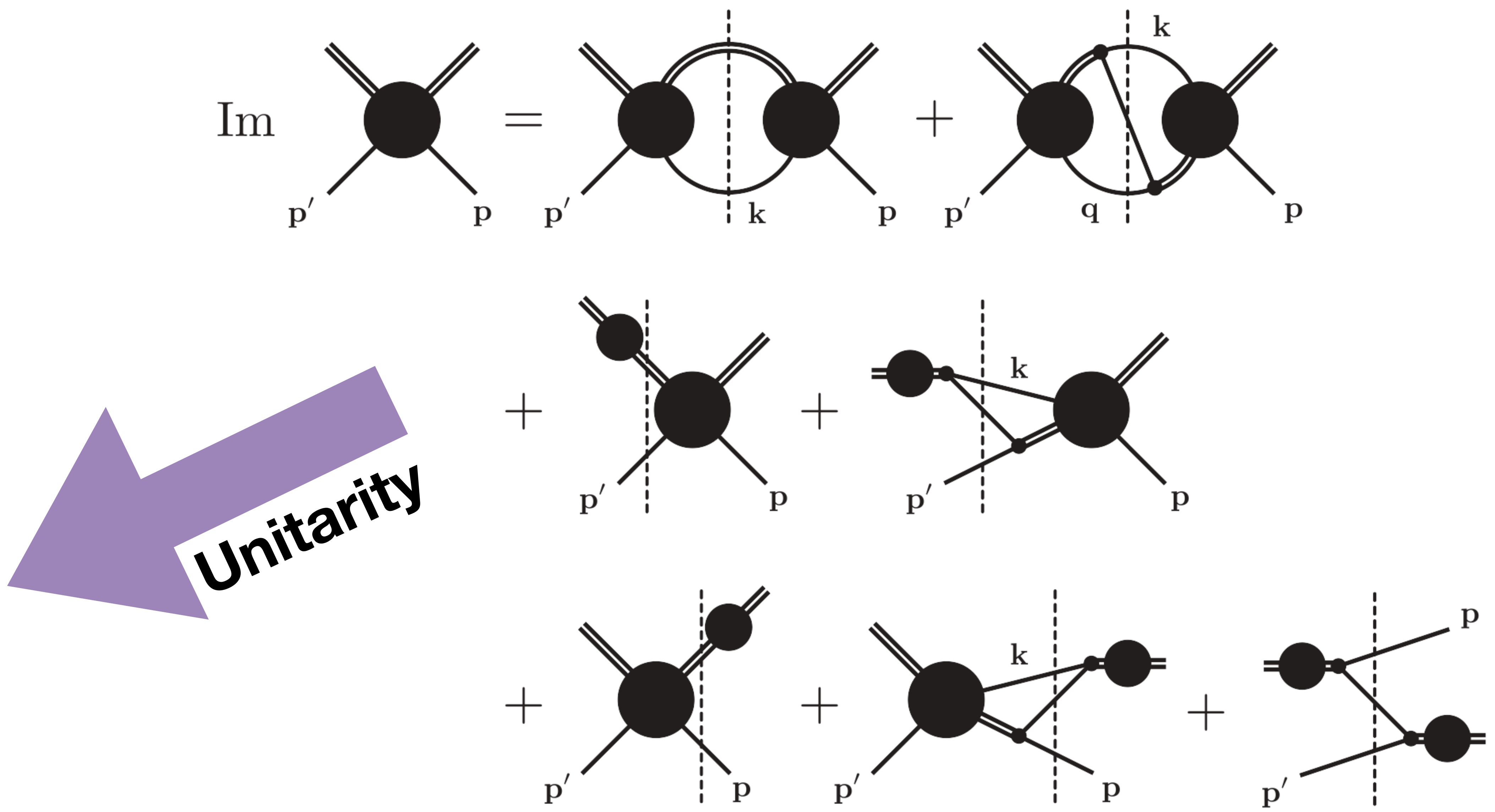


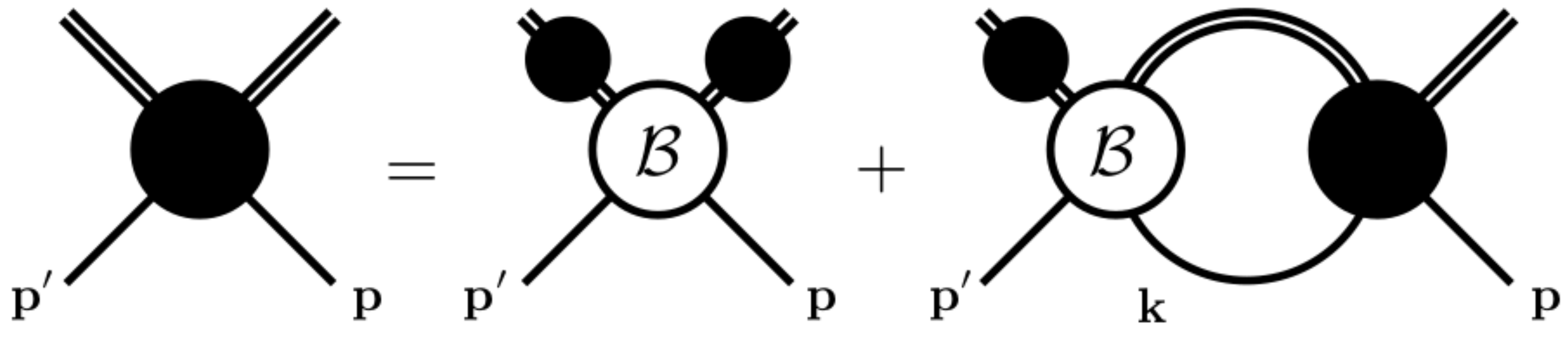
Resonances

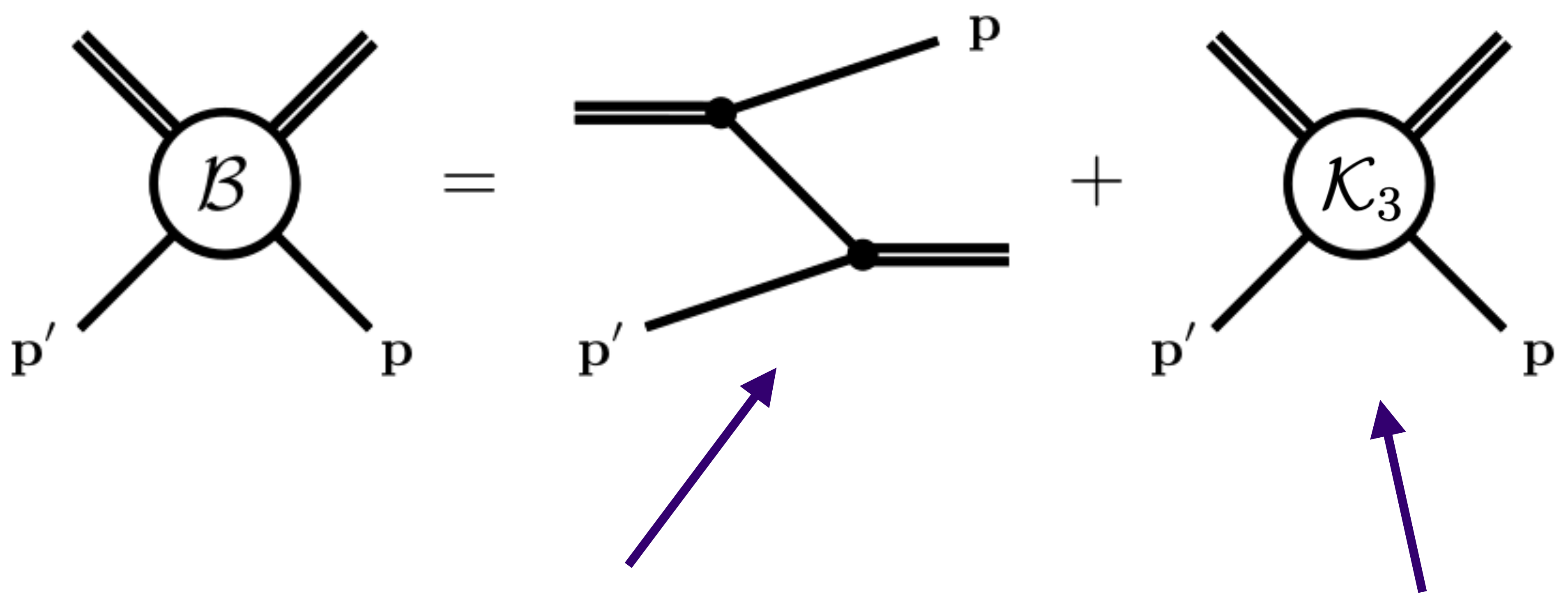
$$\det\left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3(E^\star)\boldsymbol{F}_3(E,\boldsymbol{P},L)\right] = 0$$

S-matrix parametrization

Diagrams by Andrew Jackura







One Particle Exchange

Short Range Interactions

Three-body amplitude

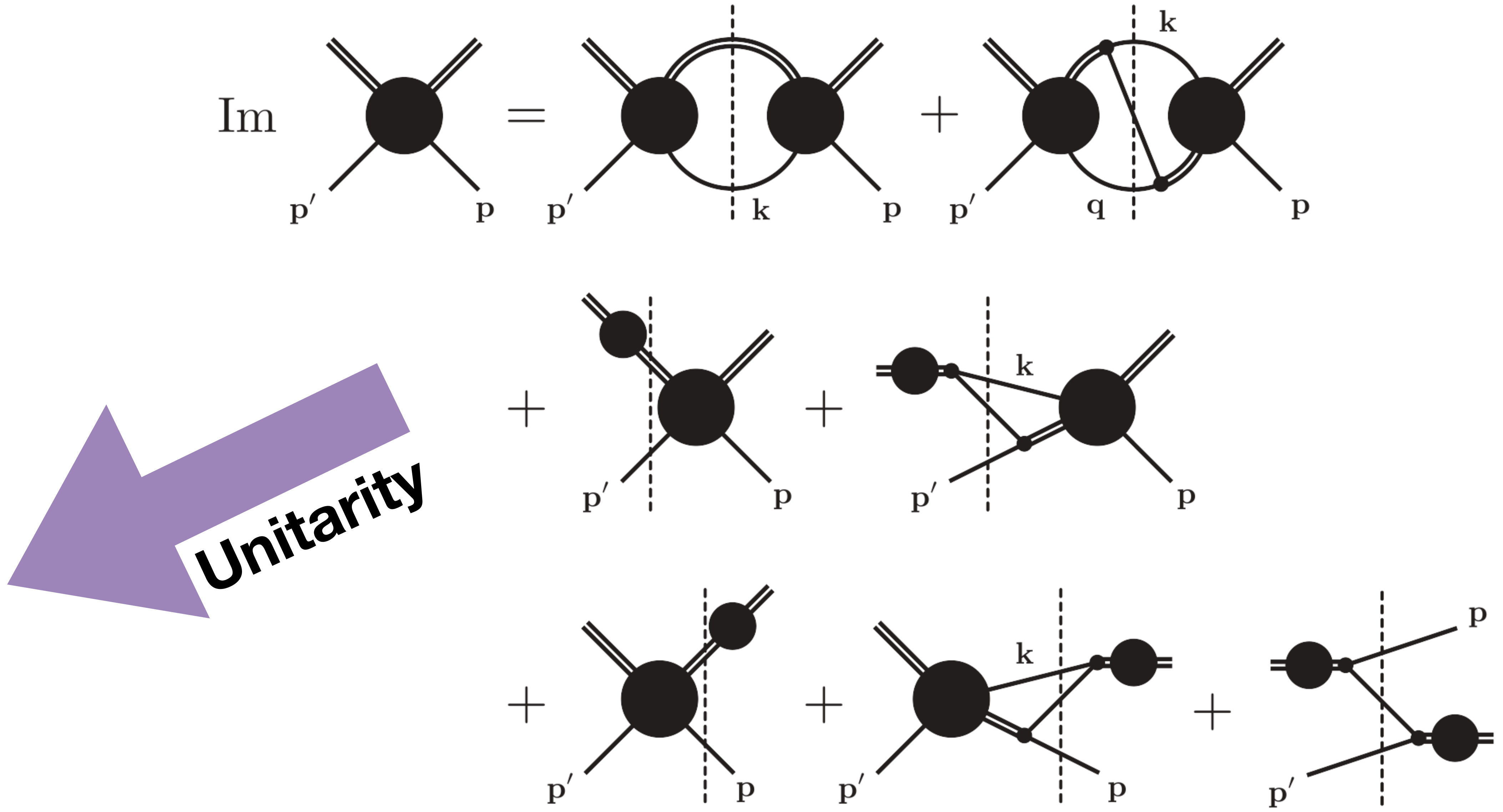
$$[\mathcal{M}_3]^J_{\ell' m'_\ell;\ell m_\ell}(p', s, p)$$

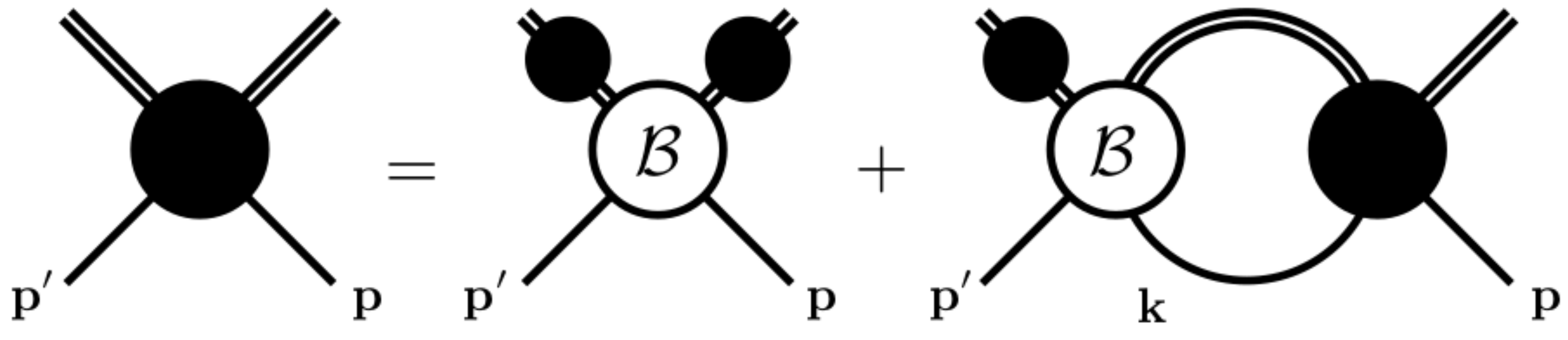
- pair-spectator
- partial waves
- symmetrization

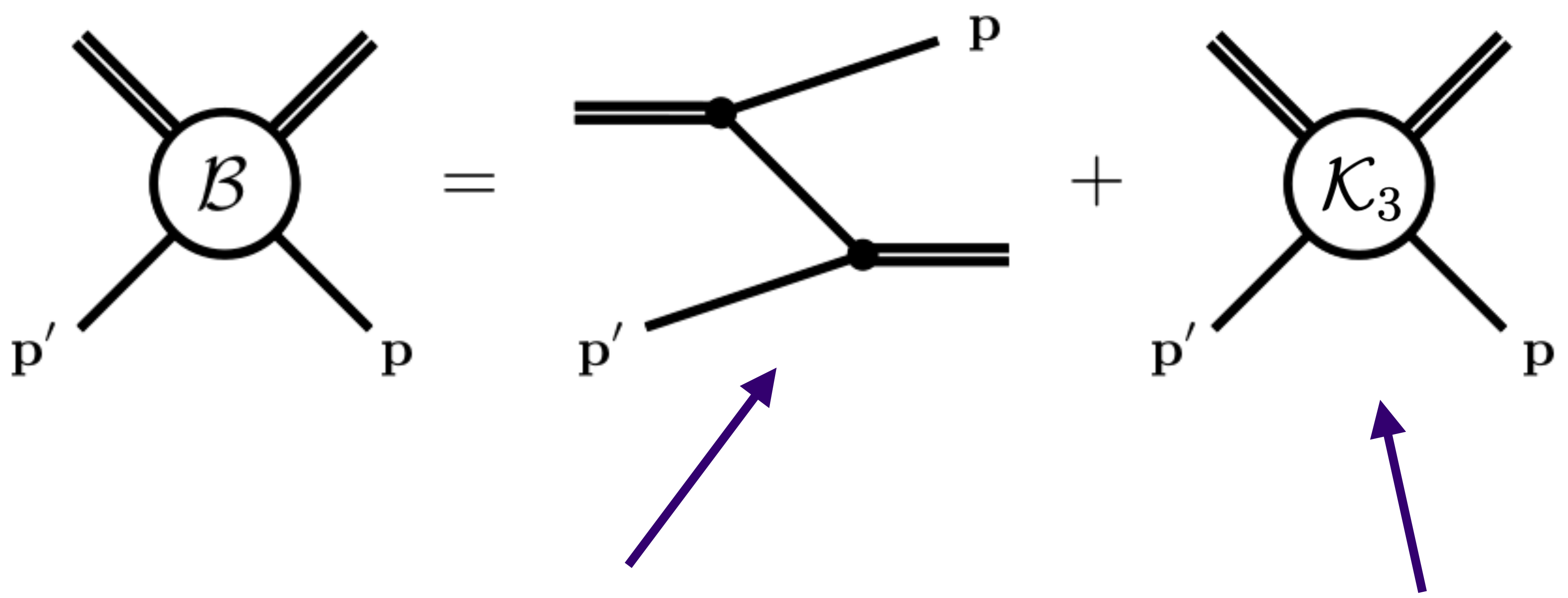
$$\mathcal{M}_3 = \mathcal{M}_2 \mathcal{B} \mathcal{M}_2 + \mathcal{M}_2 \int \mathcal{B}\, \rho_3\, \mathcal{M}_3$$

S-matrix parametrization

Diagrams by Andrew Jackura







One Particle Exchange

Short Range Interactions

Three-body amplitude

$$[\mathcal{M}_3]^J_{\ell' m'_\ell;\ell m_\ell}(p', s, p)$$

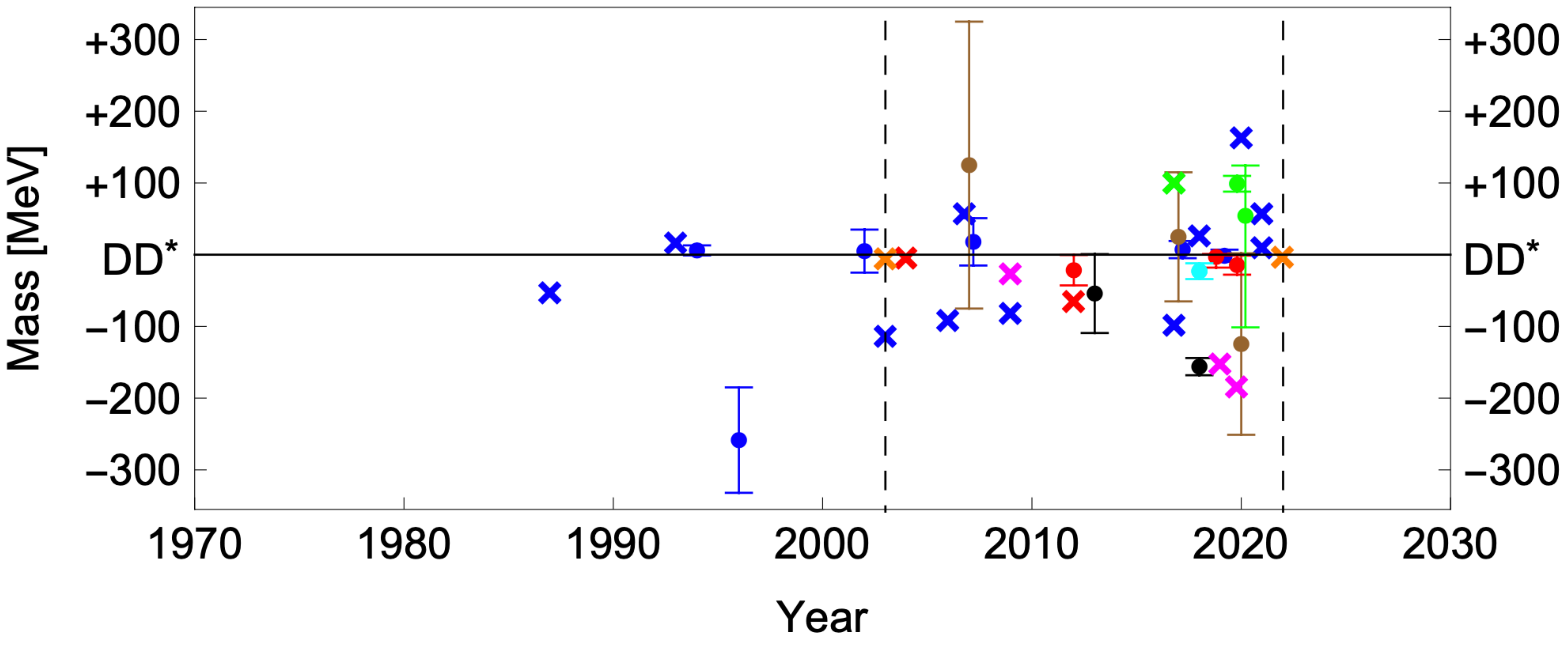
- pair-spectator
- partial waves
- symmetrization

$$\widetilde{\mathcal{M}}_3 = \mathcal{B} + \int \mathcal{B}\,\mathcal{M}_2\,\rho_3\,\widetilde{\mathcal{M}}_3$$

$\mathcal{B}\ \rho_3$

$$\mathcal{M}_2 = \mathcal{K}_2 + \mathcal{K}_2 i\rho_2\mathcal{M}_2$$

Problems with interpretations



QCD sum rules
various quark models
meson-meson & diquark-antidiquark
Heavy quark symmetry
Hadronic molecule
Lattice QCD
Other

Chen et al. arXiv:2204.02649v1

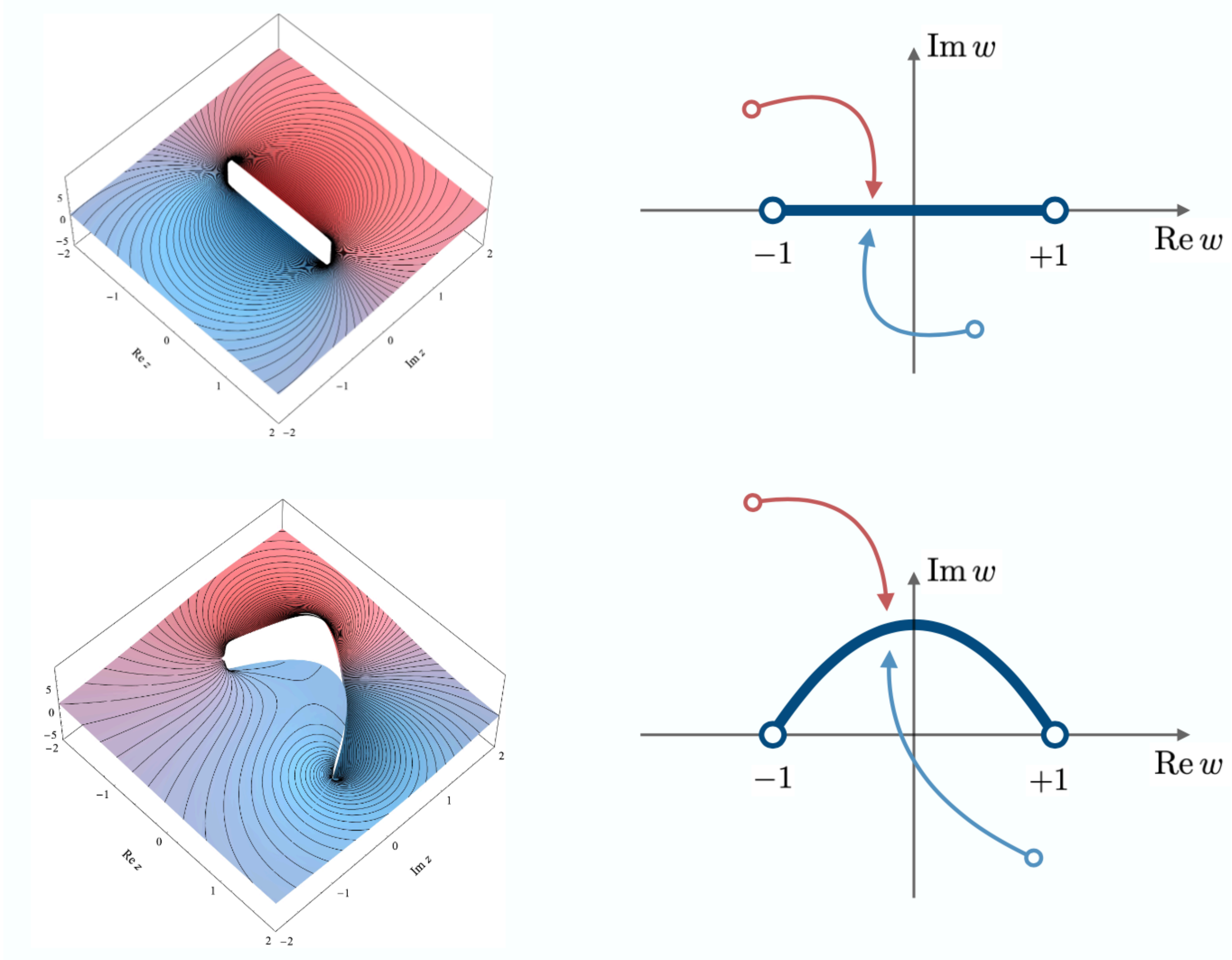
Brief introduction to analytic continuation

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, arXiv:2303.04394

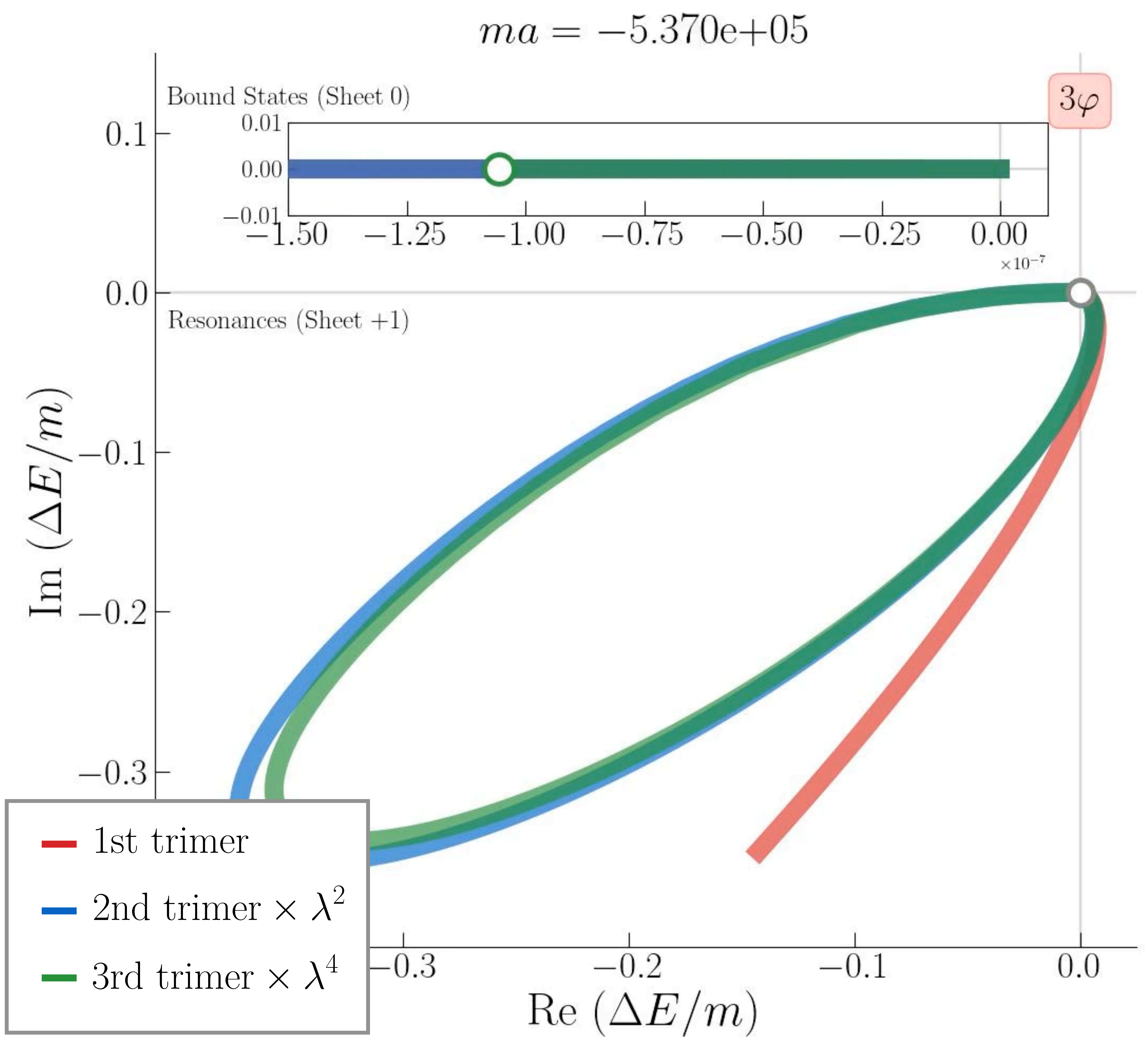
$$I(z) = \int_{\mathcal{C}(w_1,w_2)} f(w,z)\,dw\,,$$

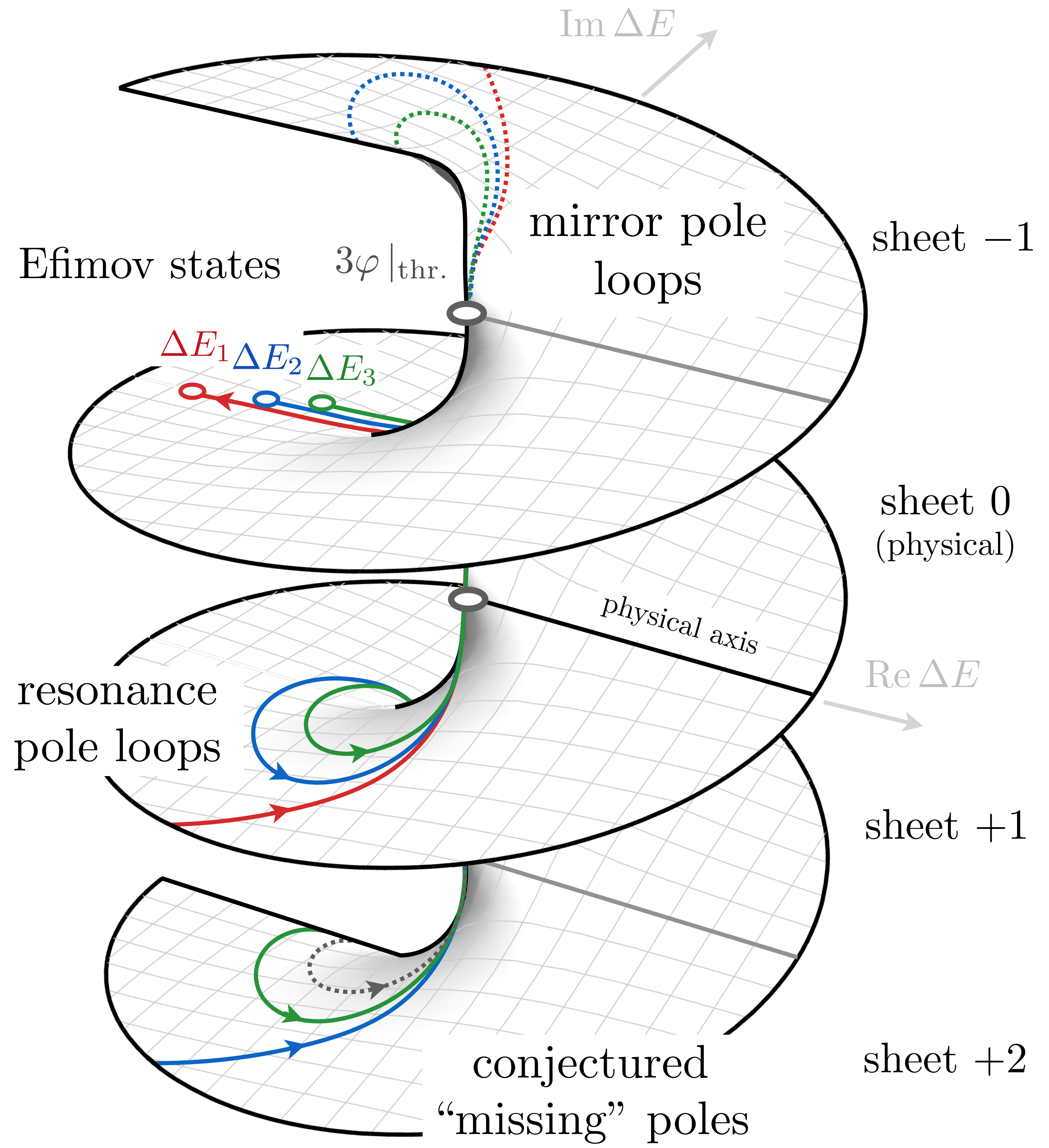
$$I(x) = \int_{-1}^{1} \frac{dw}{w-x} = \log\left(\frac{x-1}{x+1}\right)$$

$$x \in (-\infty,-1) \cup (1,\infty)$$



Three-body resonances*





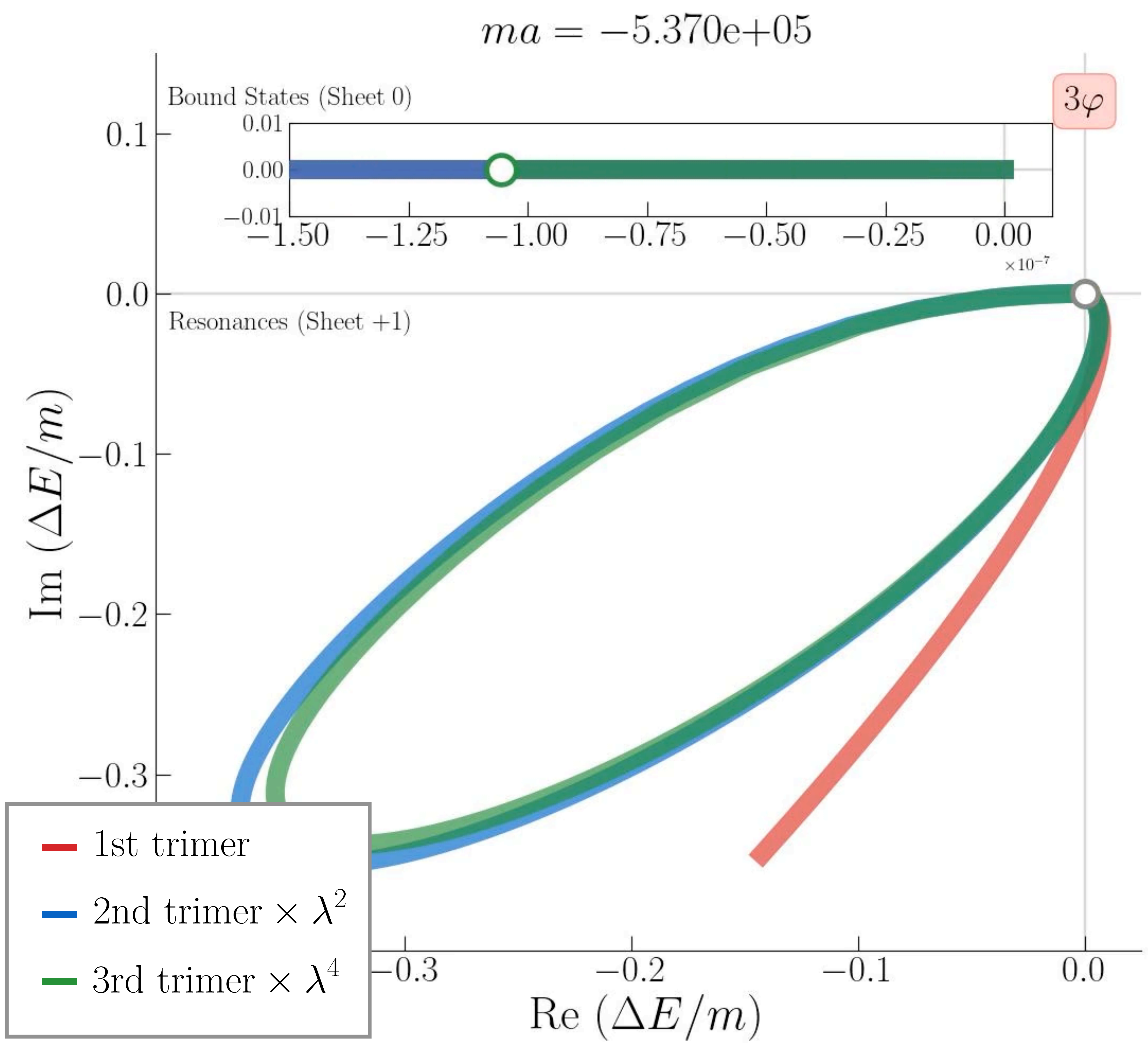
Trimers evolve on cyclic trajectories obeying the Efimov scaling.

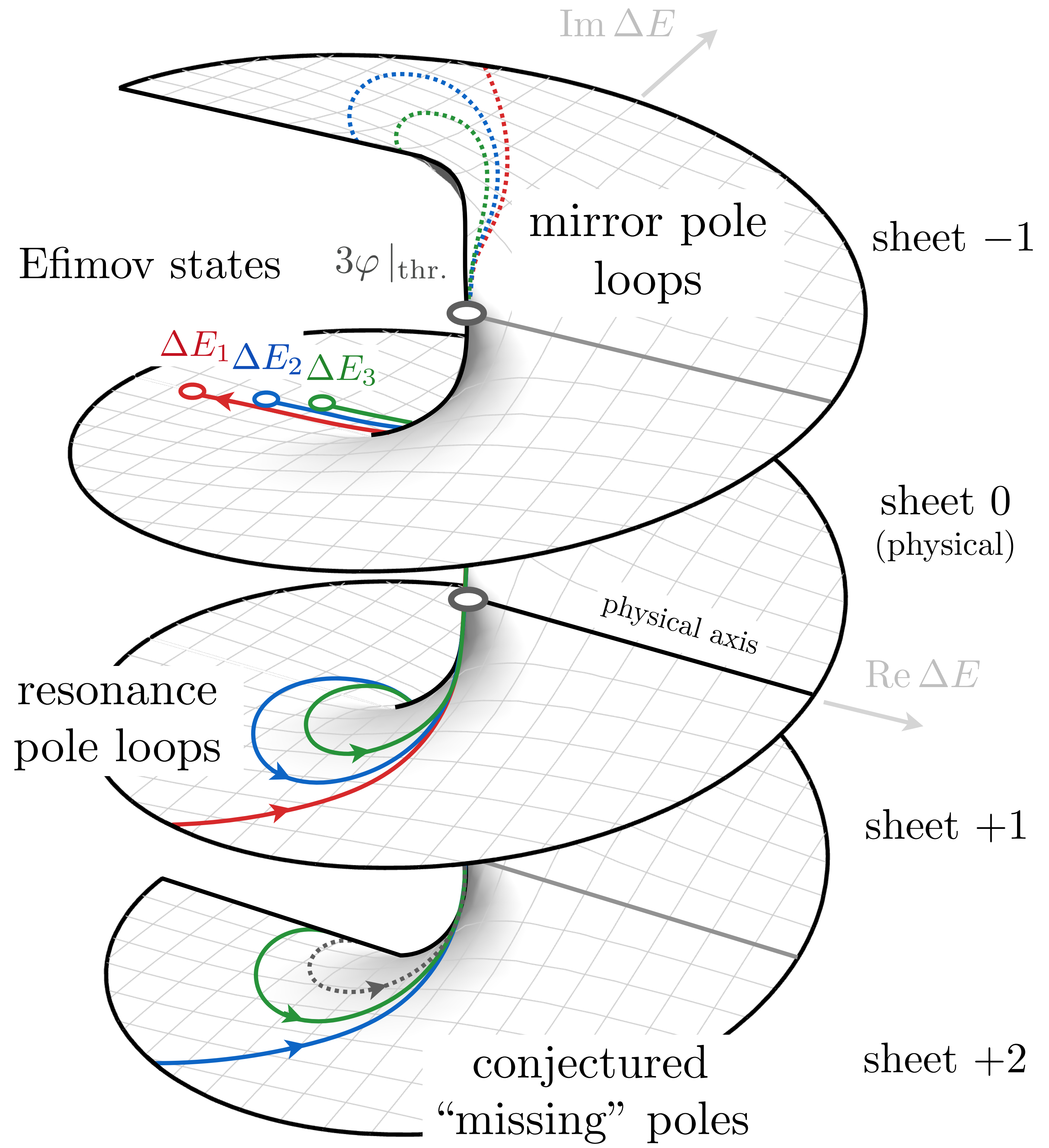
They travel through multiple Riemann sheets and…

…seem to disappear!

Evolution of Efimov states
Dawid, Islam, Briceño, Jackura, arXiv:2309.01732

Three-body resonances*





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Dawid, Islam, Briceño, Jackura, arXiv:2309.01732

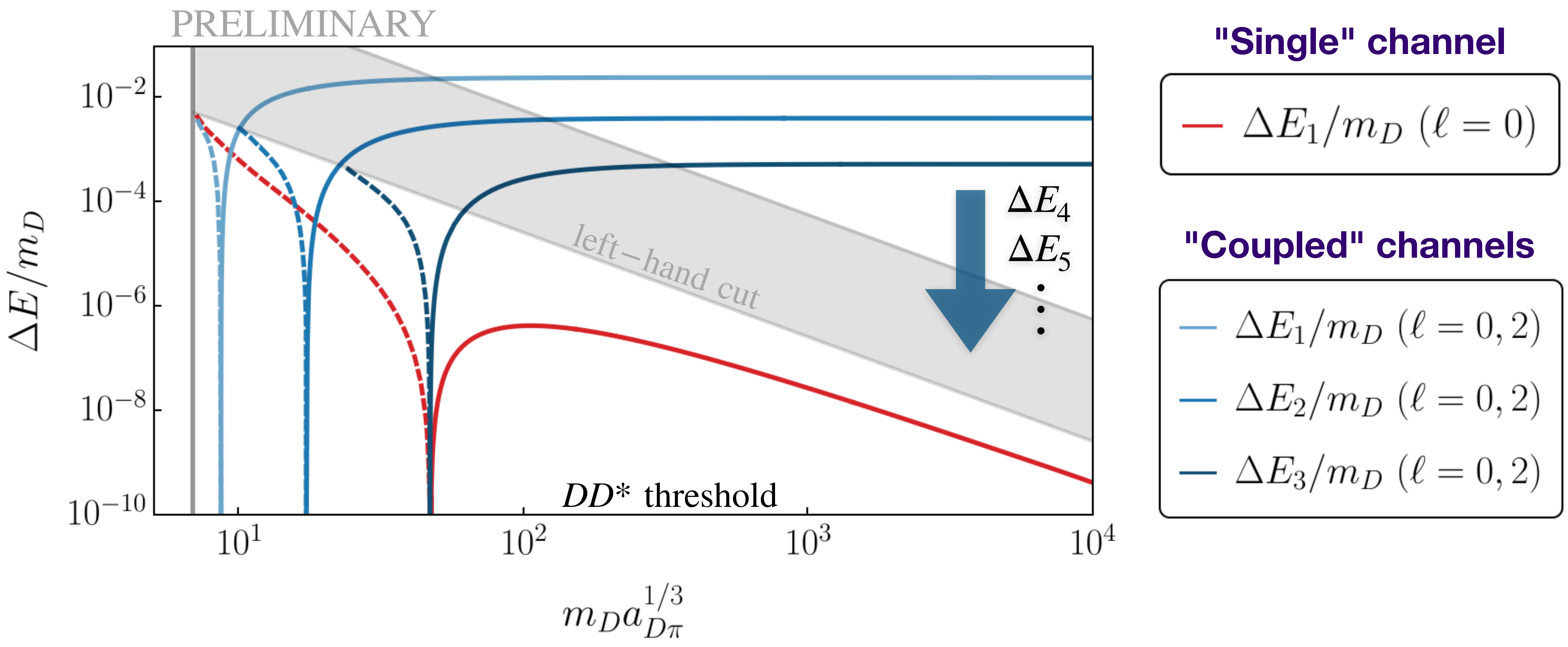
Spectrum of simplified models

$$m_D^3\, a^P_{D\pi} = \text{large}\,, \quad r^P_{D\pi}/m_D = 0.0$$

D^* no longer fixed

Additional lesson:

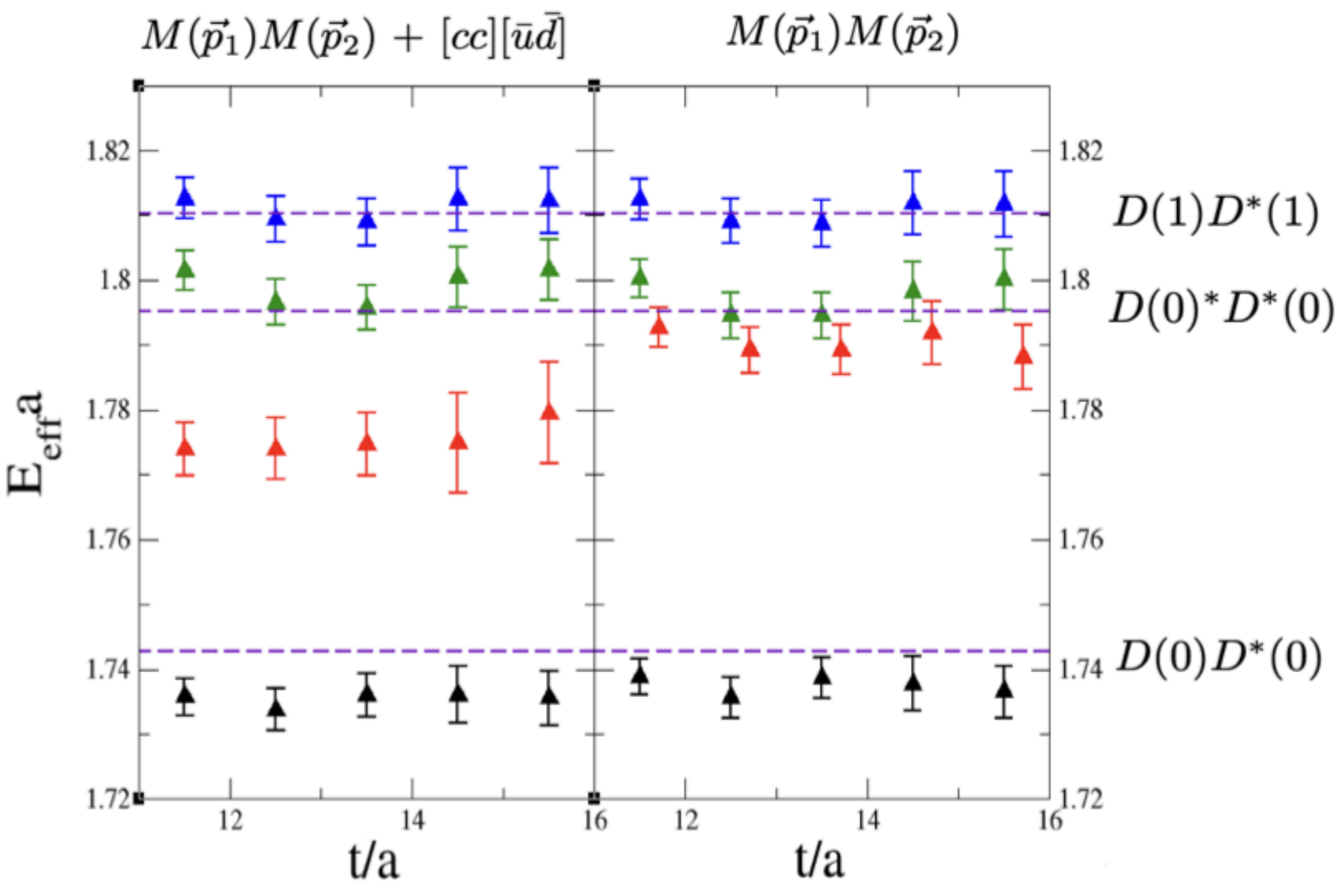
- "Small" refinement of a model can lead to a significantly different spectrum.
- Efimov physics emerges in the different corners of the parameter space in the three-body problem.



The Cloud of Unknowing

Finite-volume:

- excited-state energy affected by the inclusion of the diquark-antidiquark operators,
- see Ortiz-Pacheco et al. arXiv:2312.13441



Infinite-volume:

- LS equation with three-body effects gives an interesting evolution of singularities,
- two sub-threshold resonances turn into virtual states