Production of the $b_1(1235)$ Meson at the GlueX Experiment

PWA13/ATHOS8 - College of William & Mary
May 31, 2024

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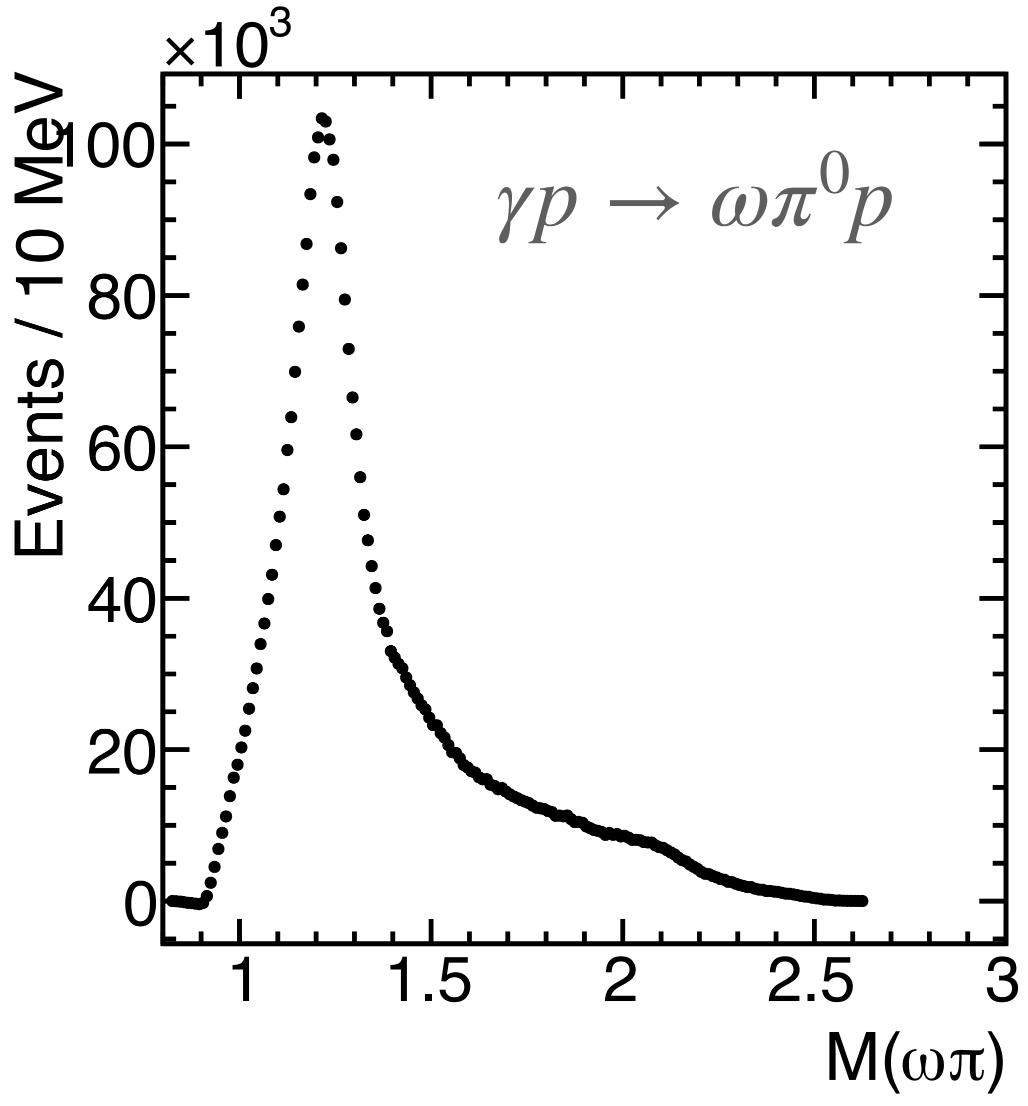
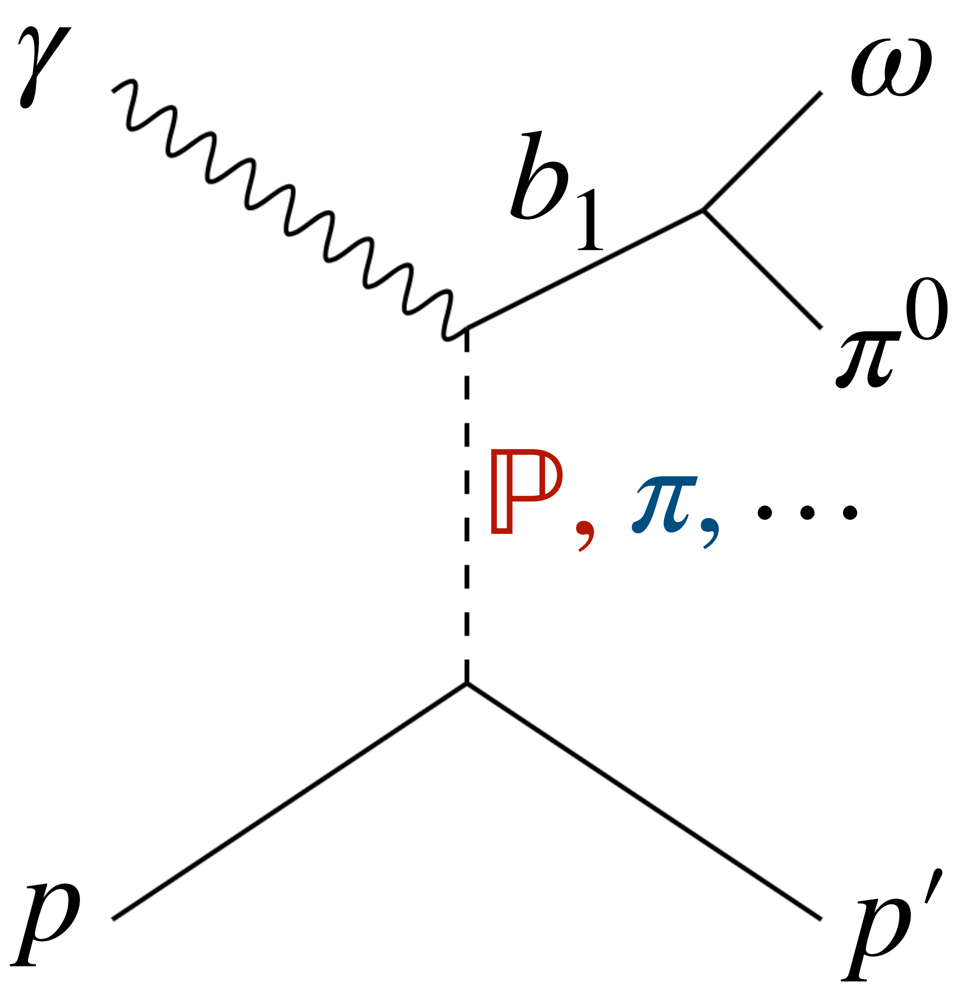
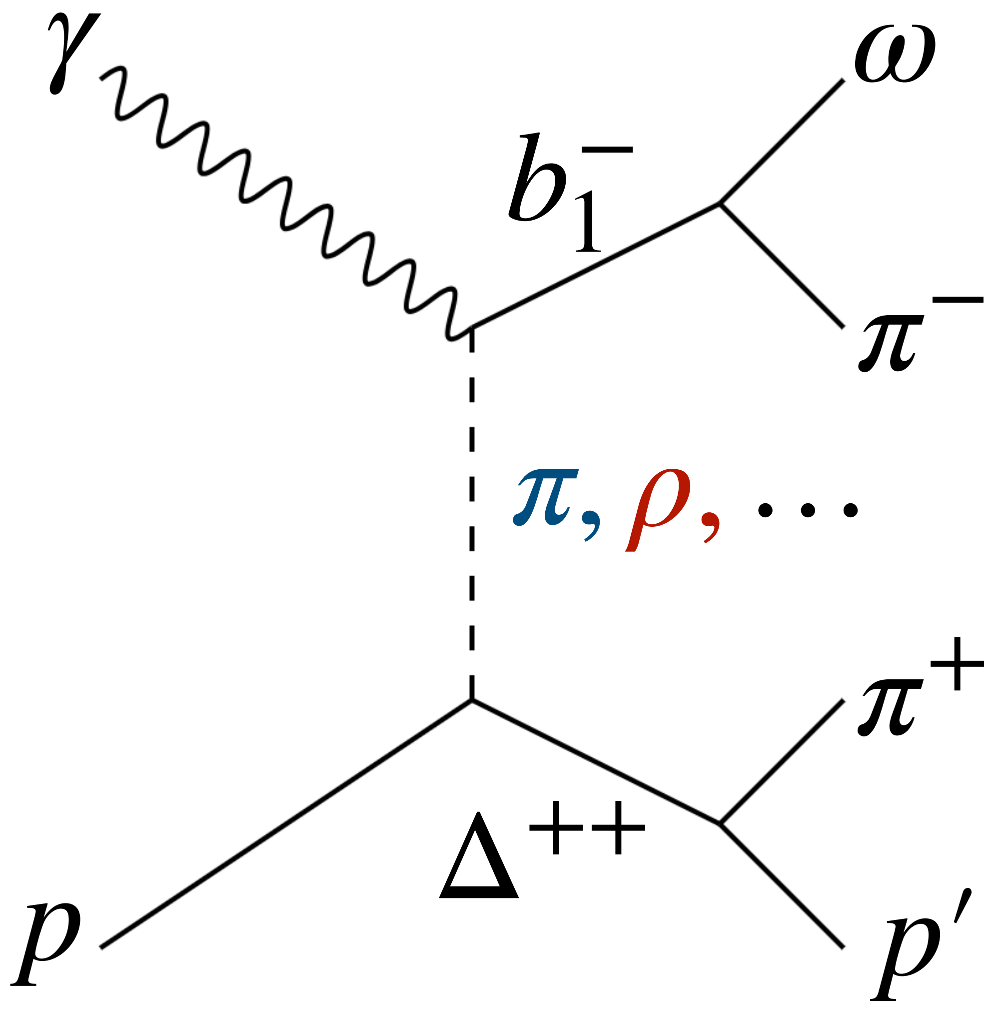
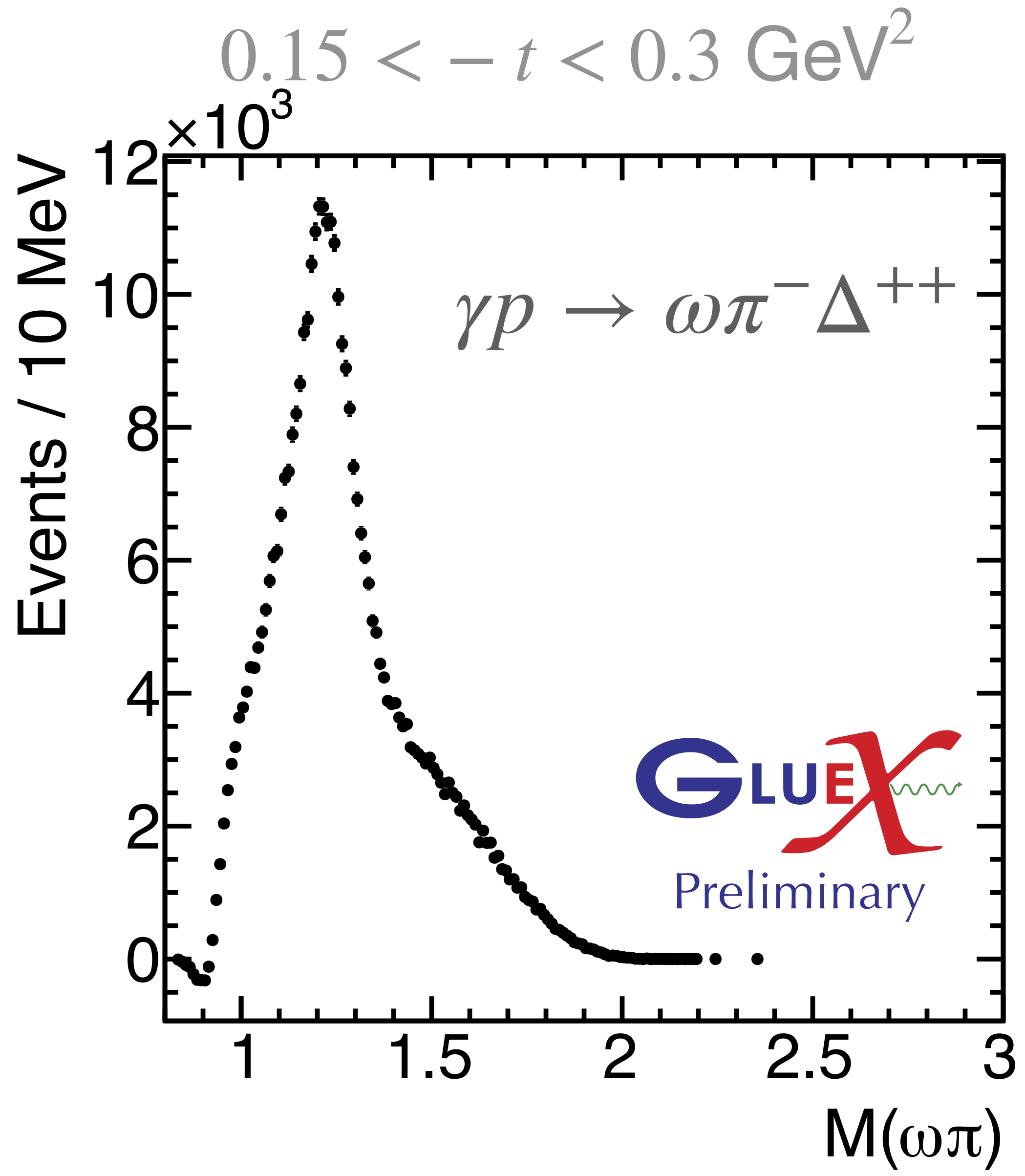
Outline

- Neutral b_1 photoproduction
  - Cross sections from PWA
    - Reflectivity and naturality
  - Cross section results
- Charged channels
  - Simplest case: $\gamma p \to \pi^- \Delta^{++}$ SDMEs
  - Strategy for $\omega\pi^-\Delta^{++}$ PWA





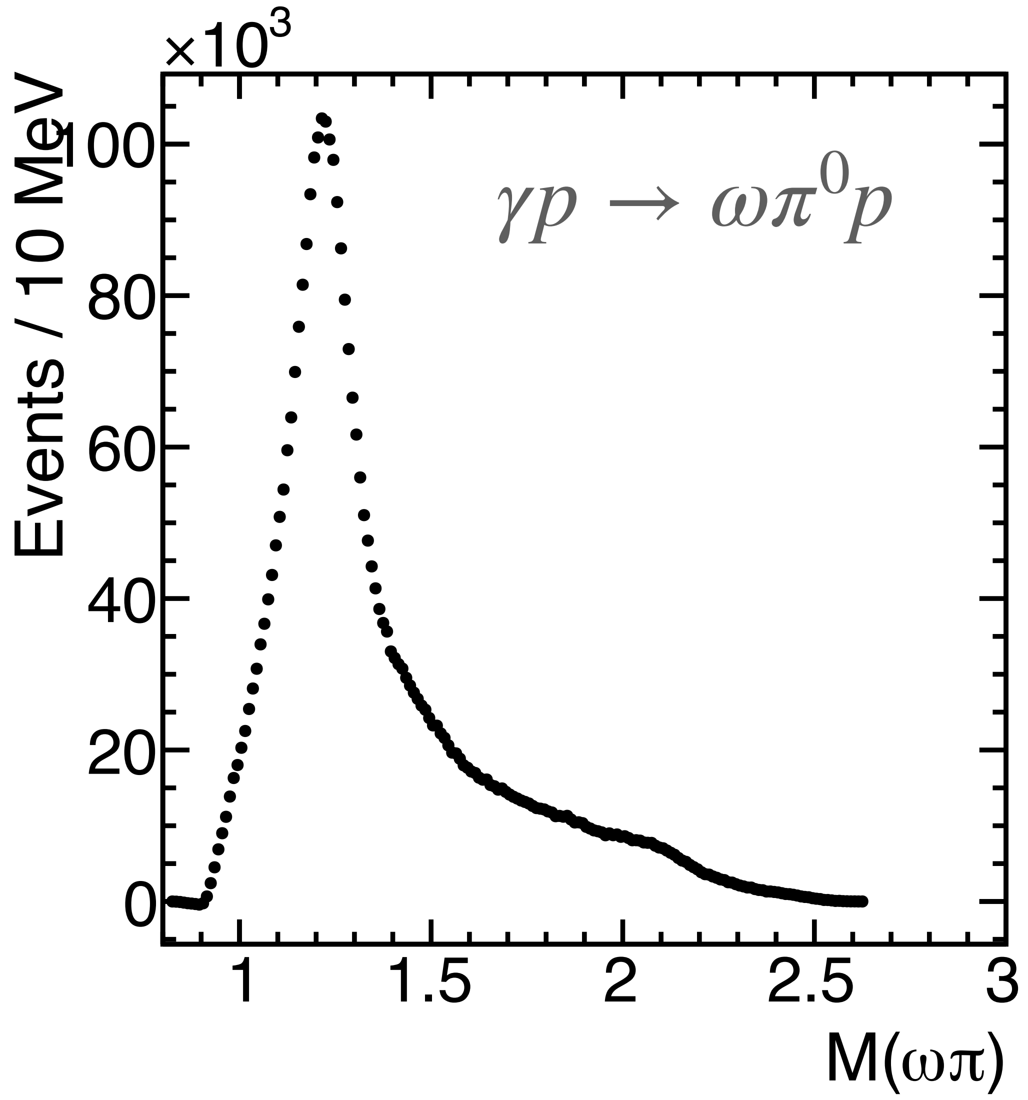


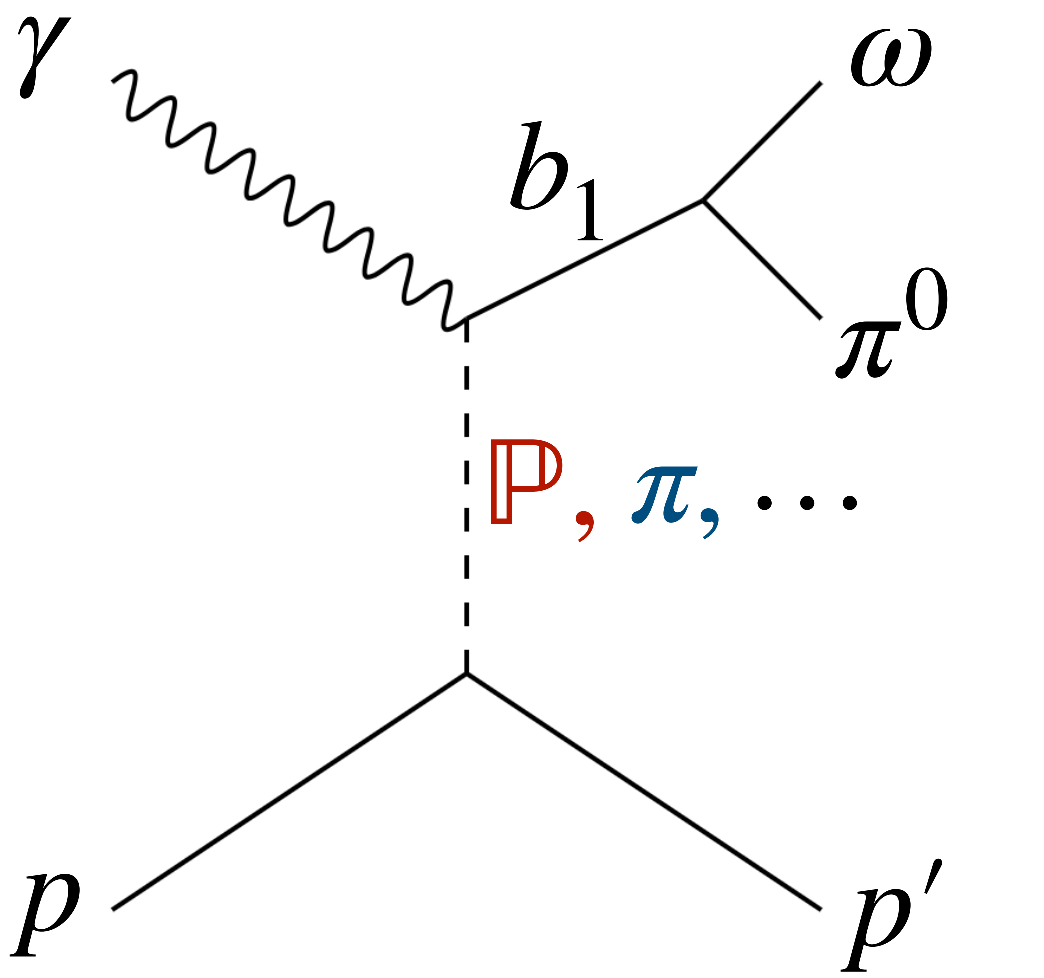


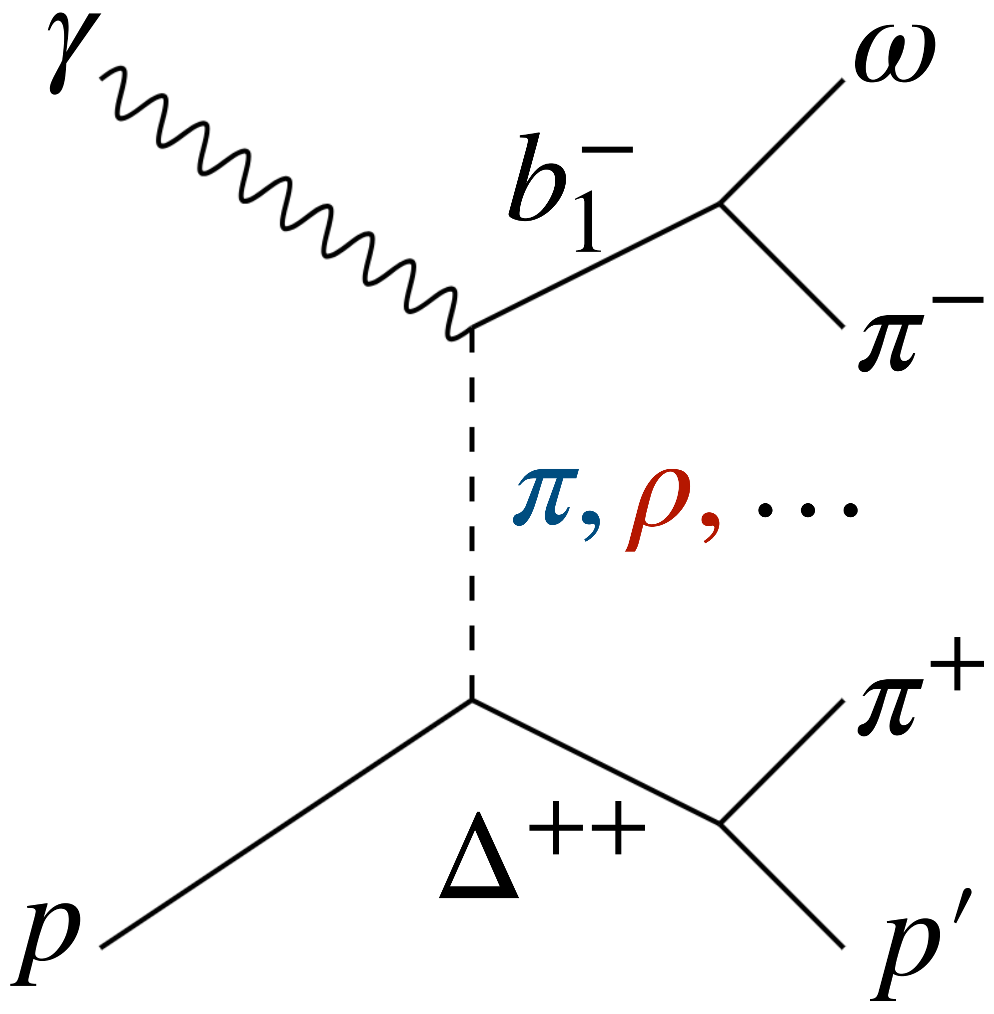
Photoproduction of the b_1

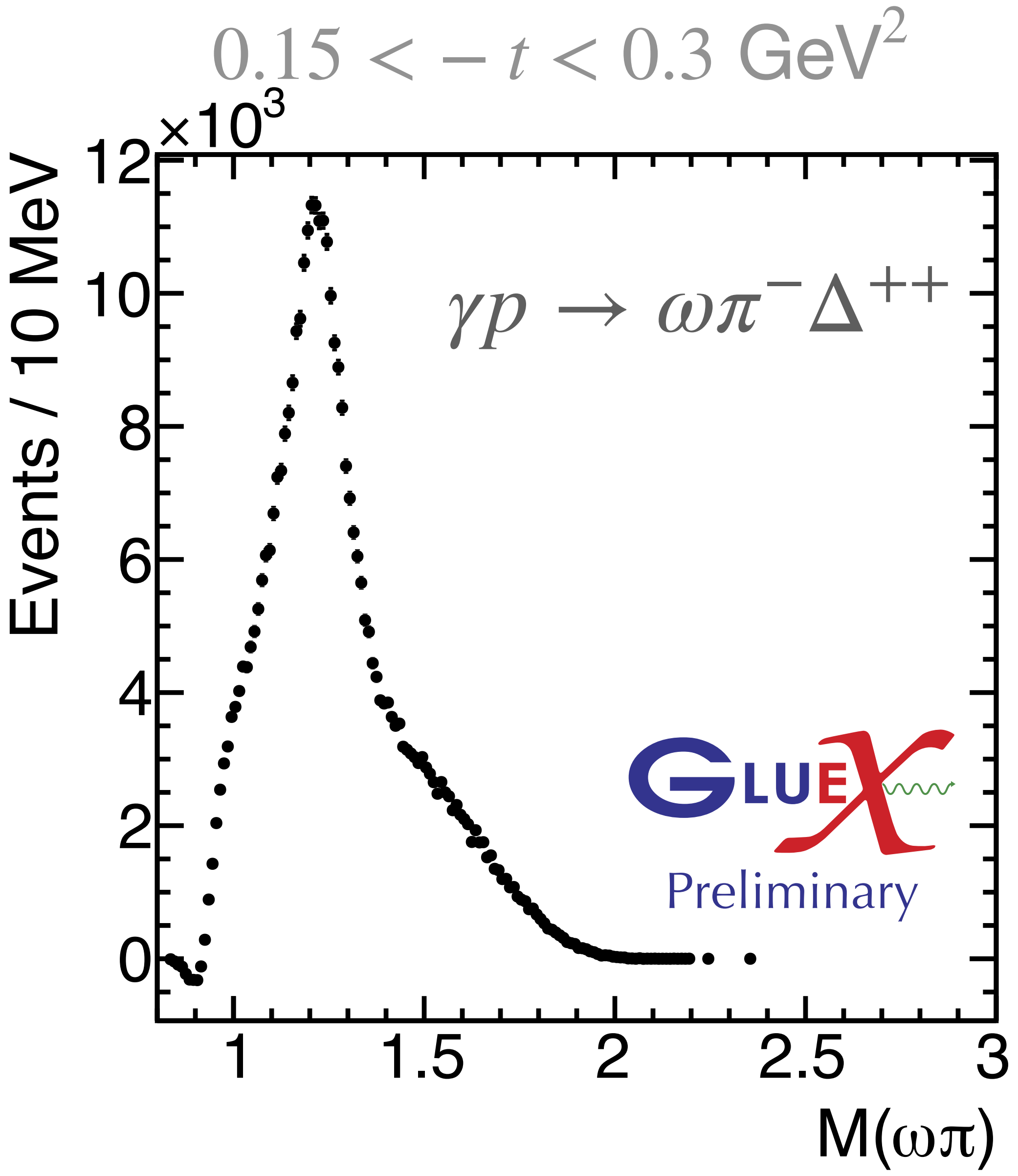
Amplitude analysis of $\omega\pi$

- $\omega\pi$ is the only well-studied decay mode of the b_1 meson
  - Clean samples of both charged and neutral b_1 mesons at GlueX
- Clean and clear enough to use b_1 as a “standard candle” for other resonances that decay to $\omega\pi$, some of which are predicted to include gluonic excitation in their wavefunctions
- Amplitude analysis framework for $\omega\pi$ can be used for other vector-pseudoscalar channels, comes with challenges (see earlier talk by E. Barriga)
  - Amplitudes for charged $\omega\pi^-$ can be used for other unstable recoil processes, including $\gamma p \to \eta'\pi^-\Delta^{++}$, with high discovery potential (see earlier talk by M. Albrecht)





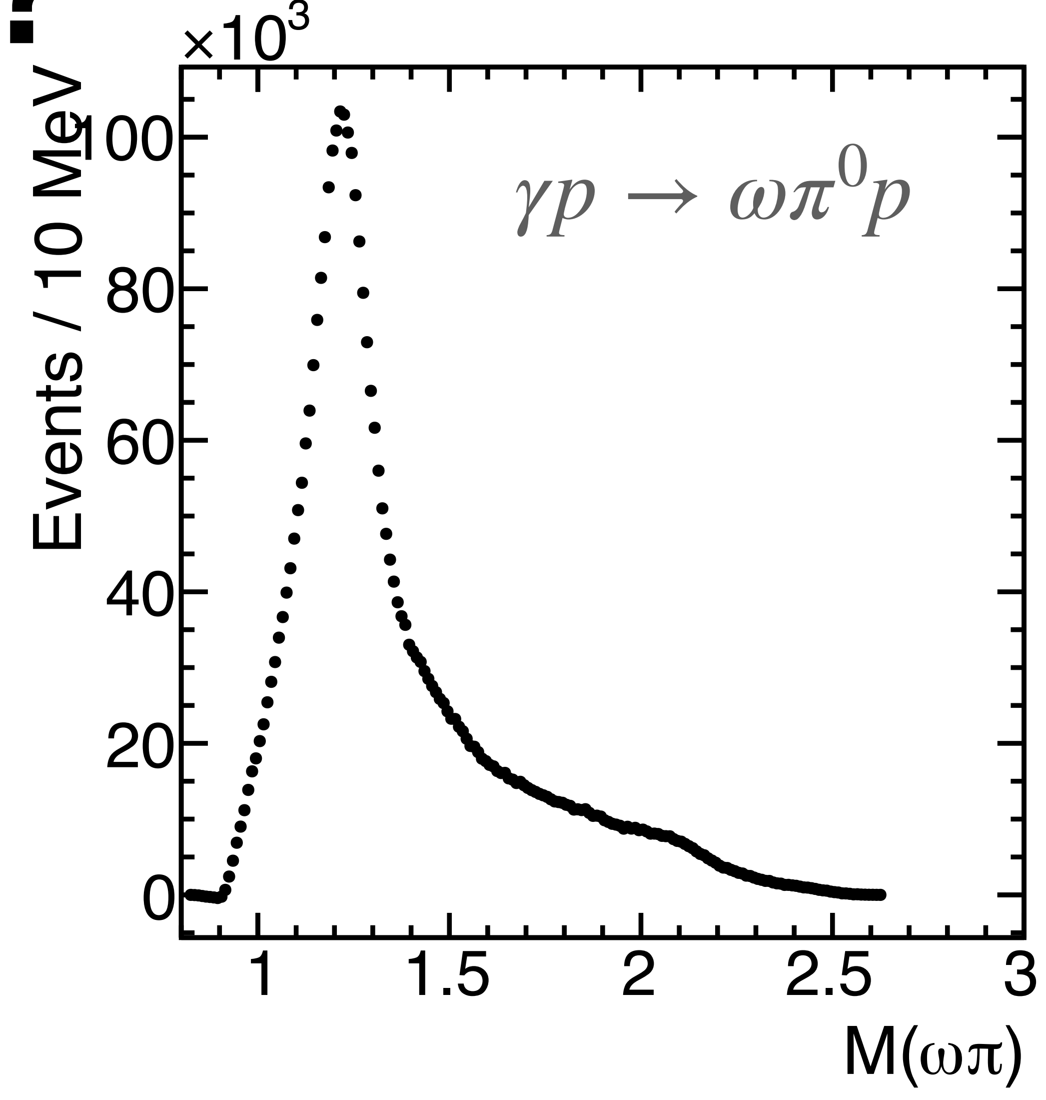


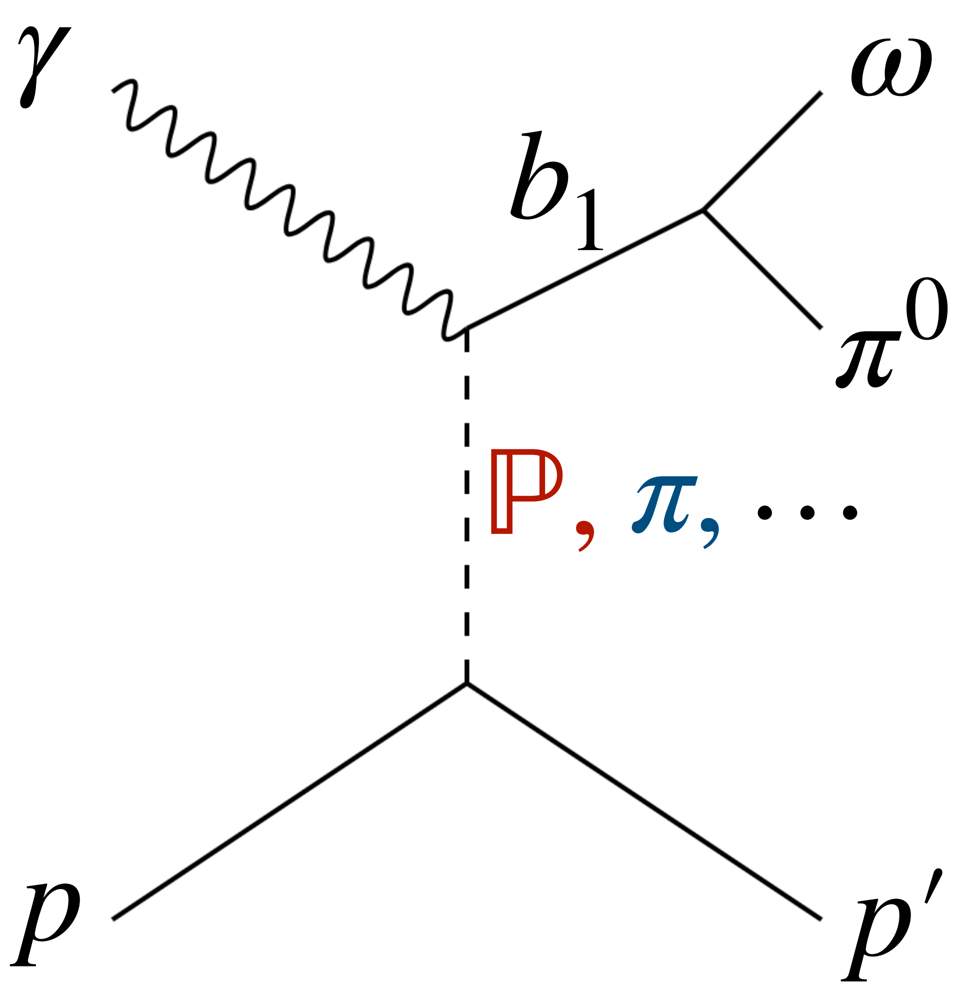


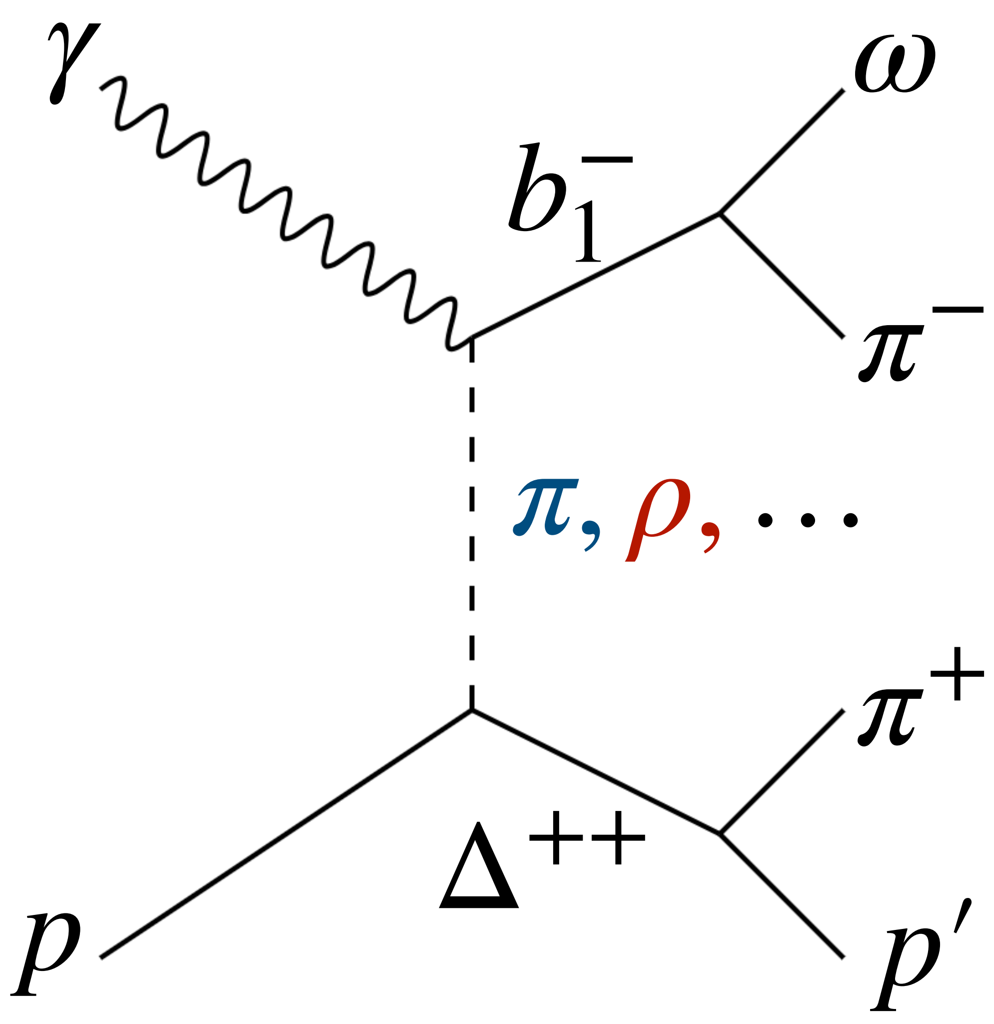
Is the b_1 interesting on its own?

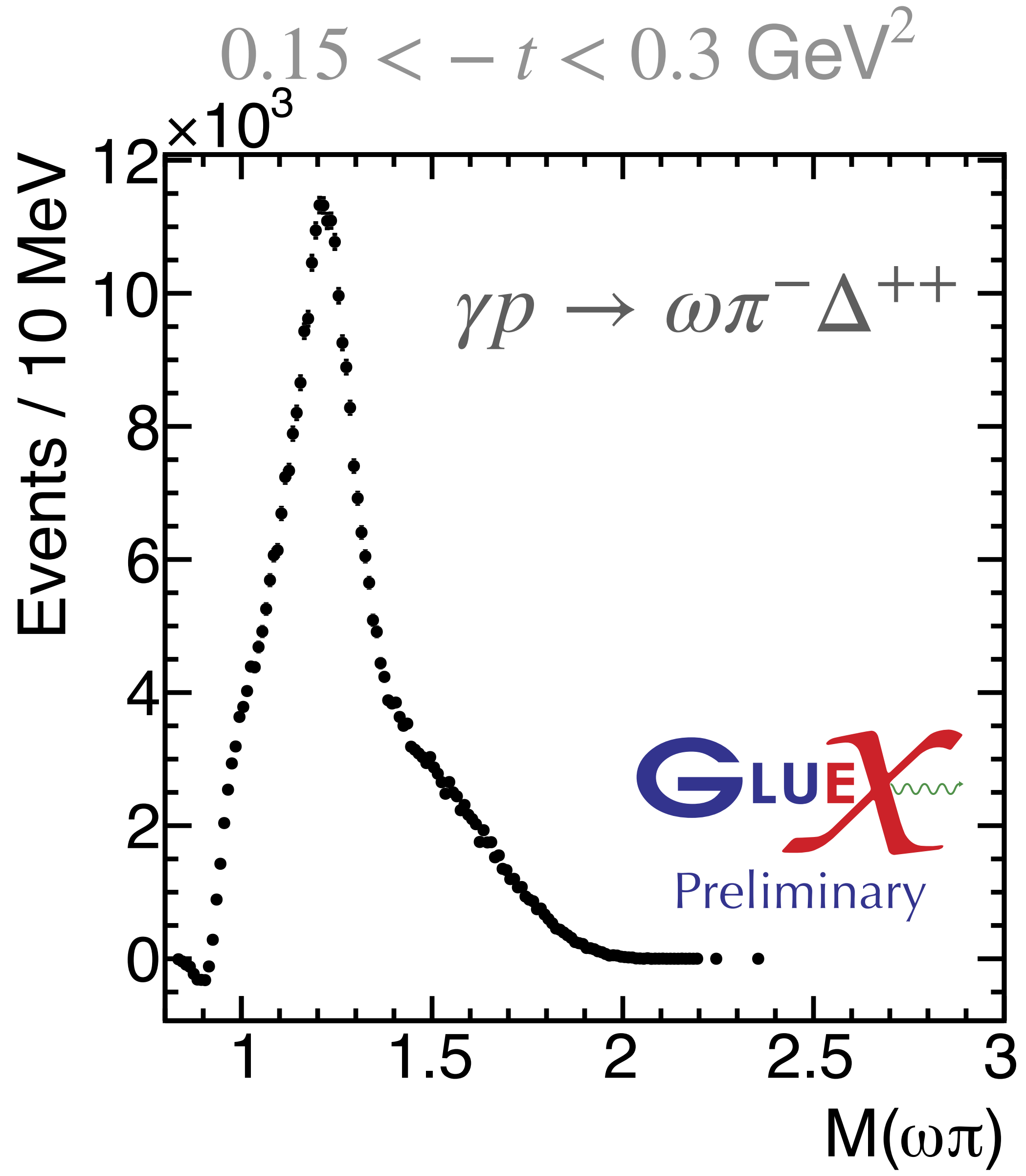
Unexpected production mechanism?

- Naive intuition tells us that the b_1 should be photoproduced mostly through π (unnatural) exchange
  - Beam photon has $J^{PC} = 1^{--}$ like a vector meson, which, when paired with a virtual exchange π, should couple to b_1 like $\omega\pi$
- GlueX measurements of the neutral b_1 cross section show natural exchange is dominant - natural cross section is an order of magnitude greater than unnatural
- What can we learn about this seemingly anomalous unnatural exchange by comparing the charged and neutral channels?





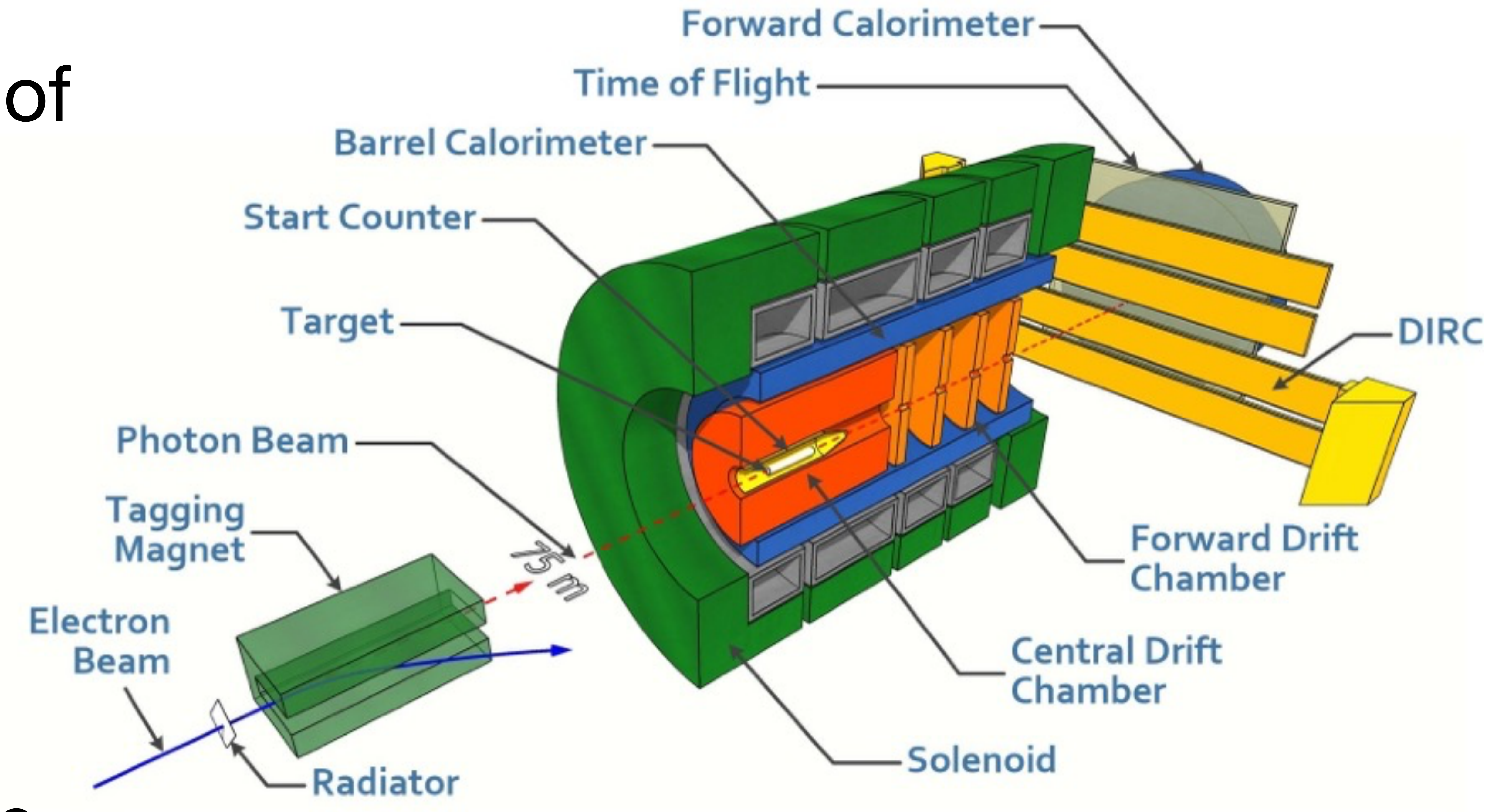




GlueX at Jefferson Lab

Photoproduction in Hall D

- Photoproduction experiment with goal of measuring and understanding the light quark meson spectrum
- Linear polarization of the photon beam gives insight into production mechanisms
- Near-hermetic acceptance allows reconstruction and analysis of exclusive final states

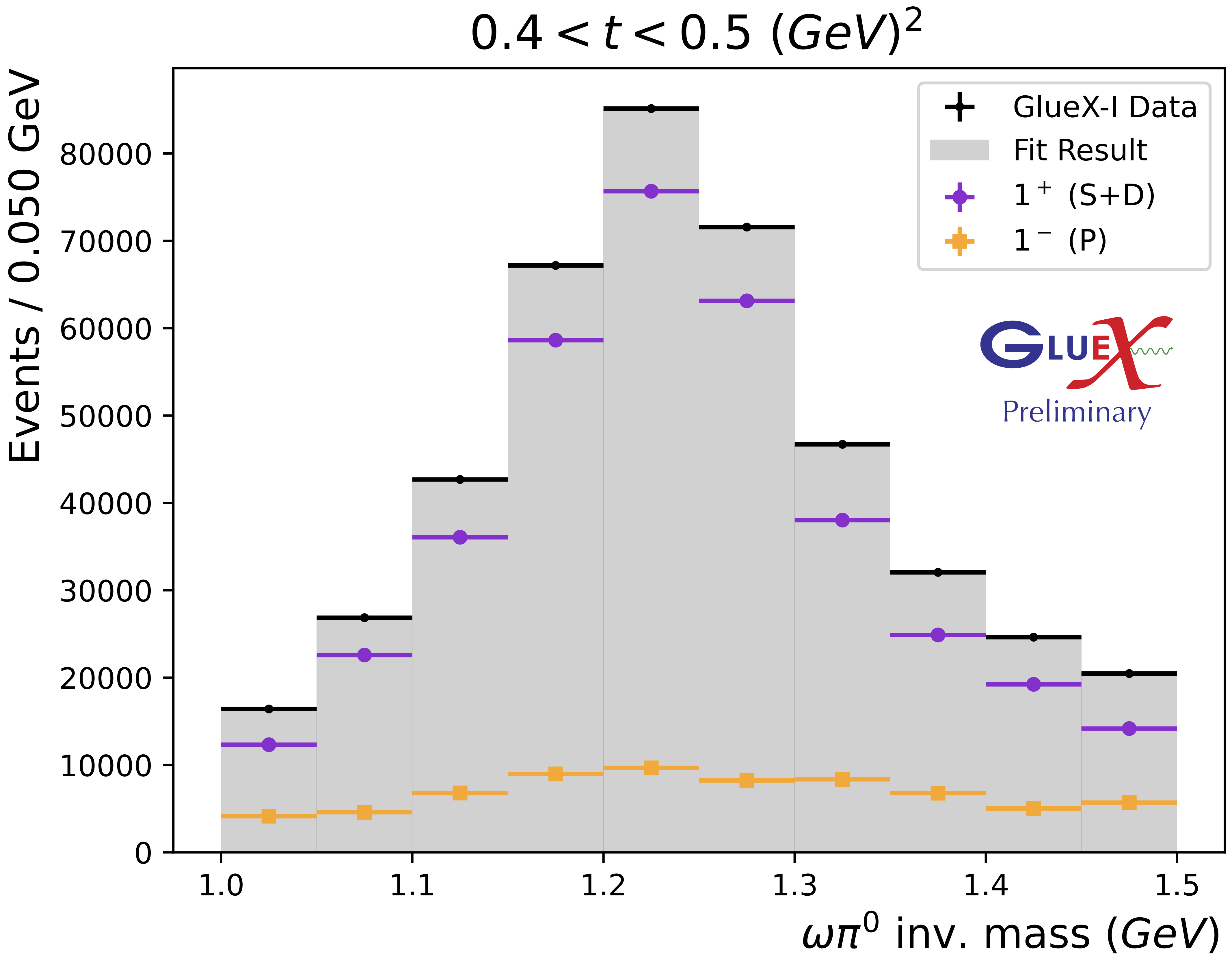


Neutral b_1 Production in $\gamma p \to \omega\pi^0 p$

- PWA fits show clean b_1 sample
- Amplitudes extracted from PWA fits can be used to calculate the b_1 cross section

$$\frac{d\sigma}{dt} = \frac{N_{obs}}{\epsilon} \frac{1}{\mathscr{L}\mathscr{B}(\omega \to 3\pi) f_{b_1} \Delta t}$$

- Cross section can be split into natural and unnatural contributions



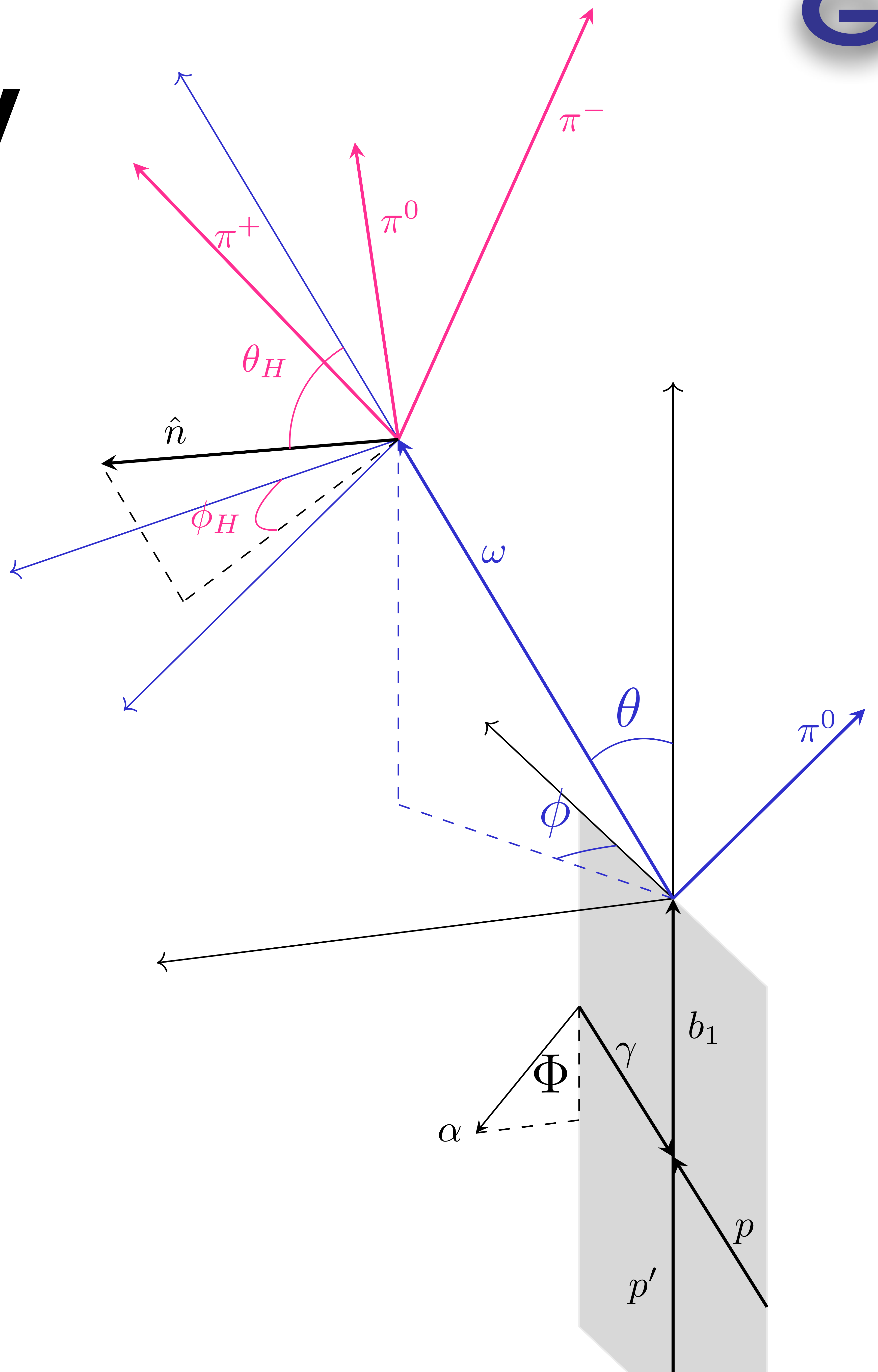
Plot courtesy K. Scheuer. Statistical errors only

Reflectivity and Naturality

- The reflectivity operator $\Pi_y = PR_y(\pi)$ reflects the reaction through the production plane
- Choose a basis where the amplitudes have definite reflectivity, $\varepsilon = \pm 1$

$$\left[J^P\right]_m^{(\varepsilon)} = \frac{1}{2}\left[T^j_{+1,m} - \tau_j \varepsilon (-1)^m T^j_{-1,-m}\right]$$

- At the energies used by GlueX, the reflectivity of a t-channel reaction directly corresponds to the naturality, $\tau = P(-1)^J$, of the exchange particle



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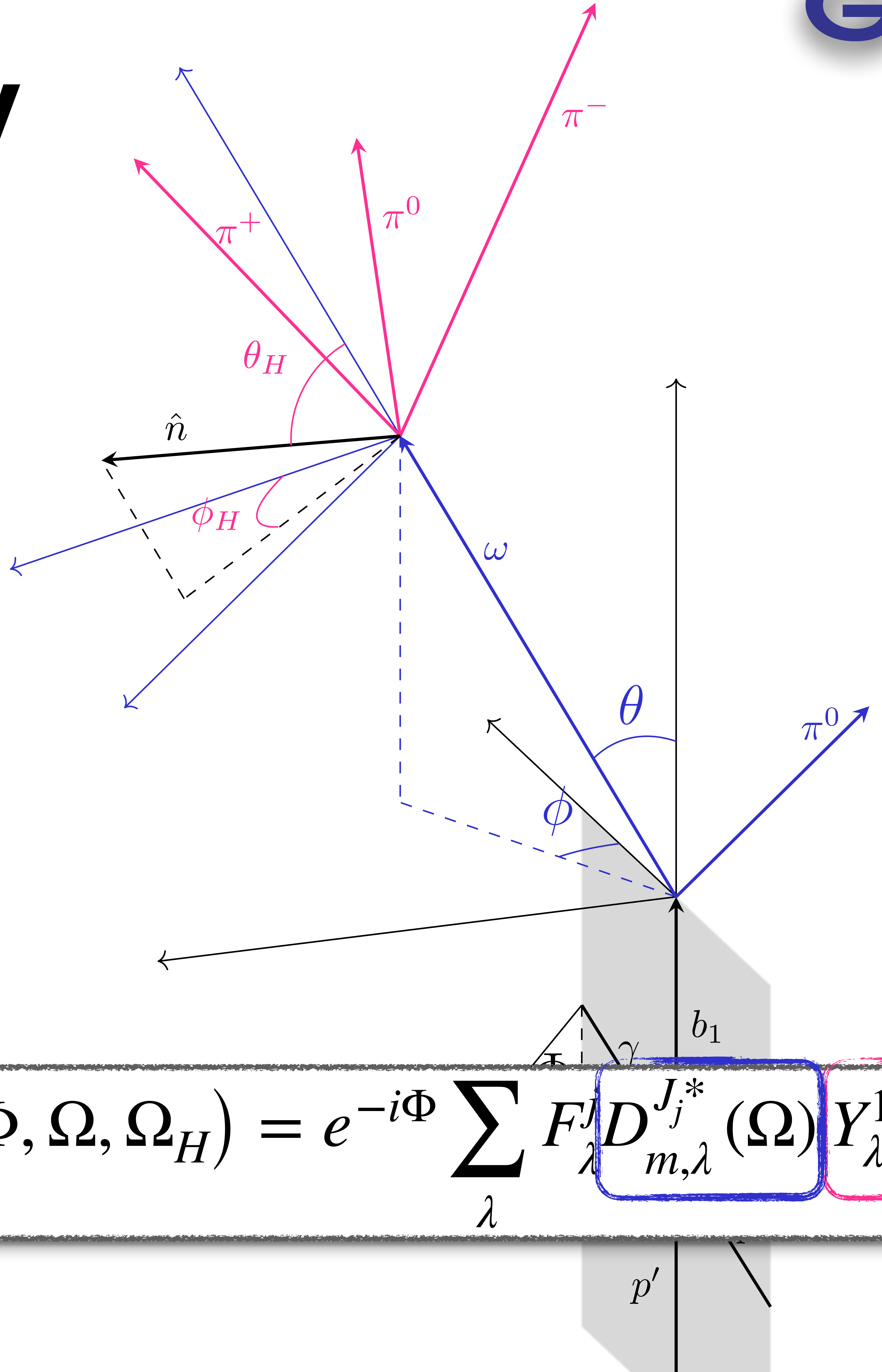


$$Z^j_m\left(\Phi, \Omega, \Omega_H\right) = e^{-i\Phi} \sum_\lambda F^j_\lambda D^{J_j *}_{m,\lambda}(\Omega)\, Y^1_\lambda\left(\Omega_H\right) G$$

Vector-Pseudoscalar Amplitudes

No reflectivity interference

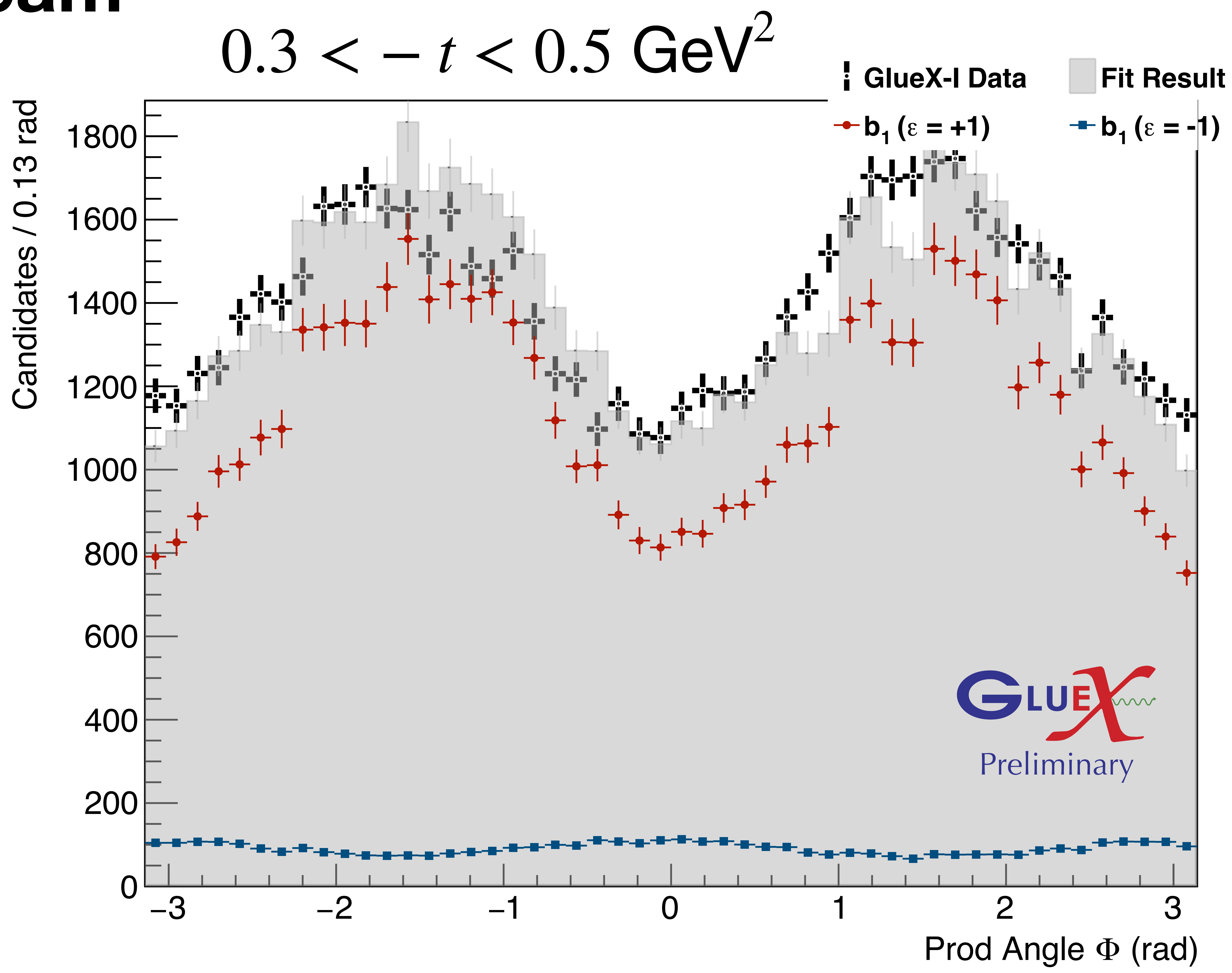
$$I \propto \left(1 - P_\gamma\right) \left[\left| \sum_{j,m} \left[J^P\right]_m^{(-)} ImZ_m^j \left(\Phi, \Omega, \Omega_H\right) \right|^2 + \left| \sum_{j,m} \left[J^P\right]_m^{(+)} ReZ_m^j \left(\Phi, \Omega, \Omega_H\right) \right|^2 \right]$$

$$+ \left(1 + P_\gamma\right) \left[\left| \sum_{j,m} \left[J^P\right]_m^{(+)} ImZ_m^j \left(\Phi, \Omega, \Omega_H\right) \right|^2 + \left| \sum_{j,m} \left[J^P\right]_m^{(-)} ReZ_m^j \left(\Phi, \Omega, \Omega_H\right) \right|^2 \right]$$

Production Angle Distinguishes Reflectivities

Thanks to a linearly polarized beam

- When $m = 0$:
  - Positive reflectivity -> $\sin^2 \Phi$
  - Negative reflectivity -> $\cos^2 \Phi$
- NB: This is a simplified illustration, valid for $m = 0$. In reality the fit uses information from all production and decay angles



Plot courtesy K. Scheuer. Statistical errors only

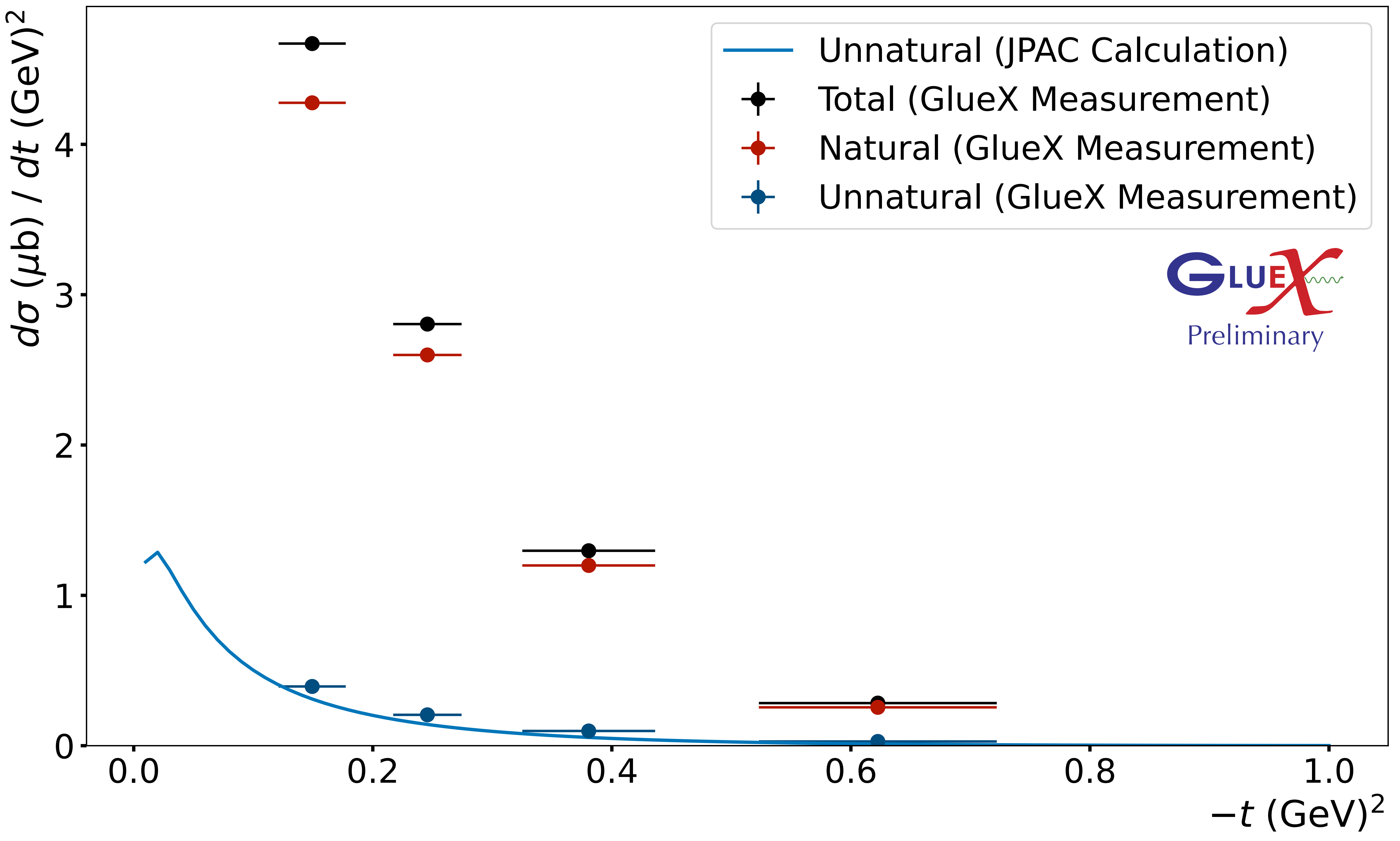
Cross Section of the Neutral b_1

Contrary to naive expectations…

- Preliminary cross section measurements of the b_1 show dominant natural parity exchange by roughly an order of magnitude

- Unnatural contribution to the b_1 cross section agrees well with JPAC calculation of b_1 photoproduced through pion exchange in the *t*-channel
    - Based off the decay width $\Gamma\left(b_1^{\pm} \to \pi^{\pm}\gamma\right) = 230 \pm 60$ keV

$\gamma p \to b_1 p$



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JPAC Calculation courtesy V. Mathieu: private communication

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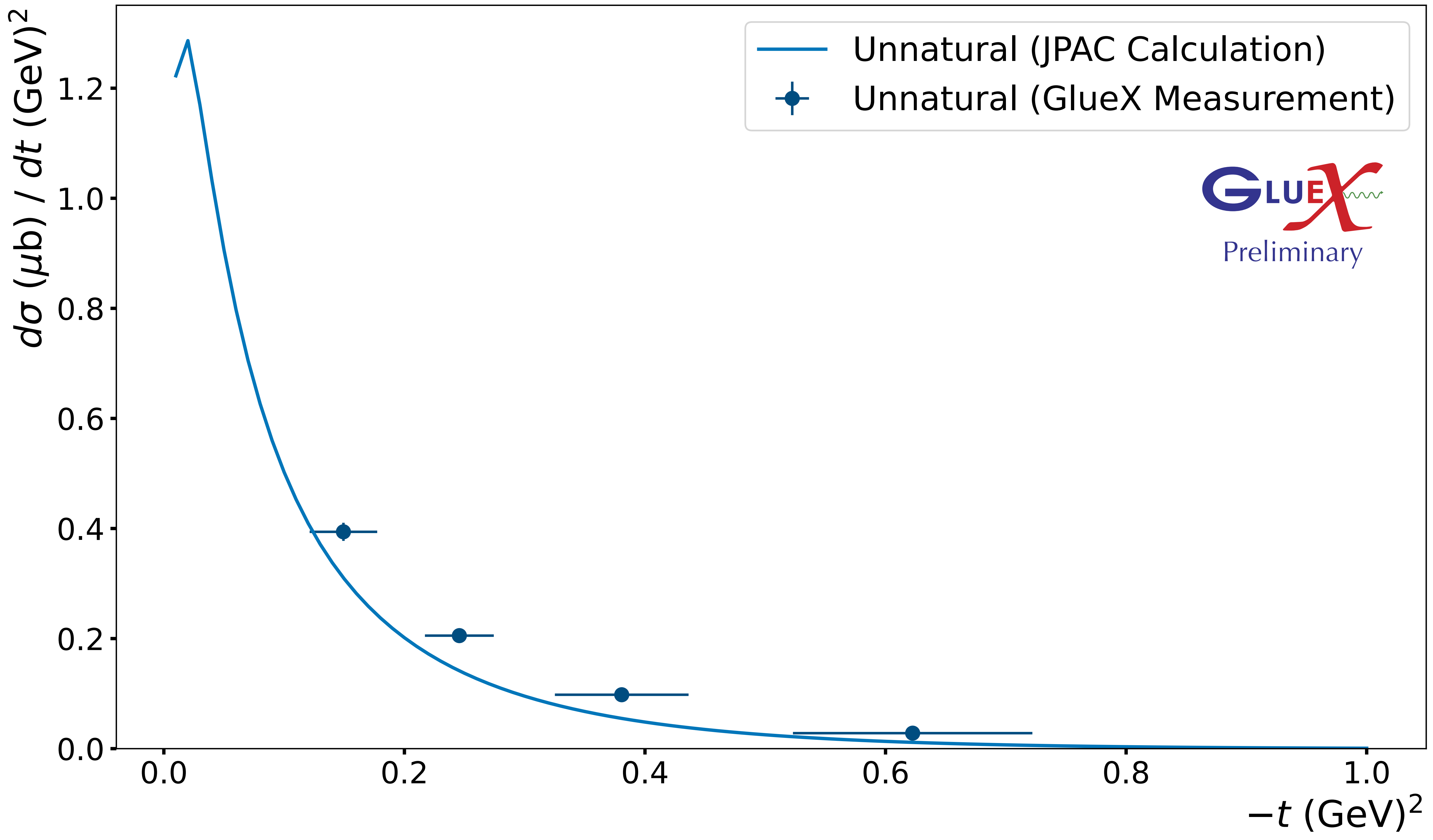

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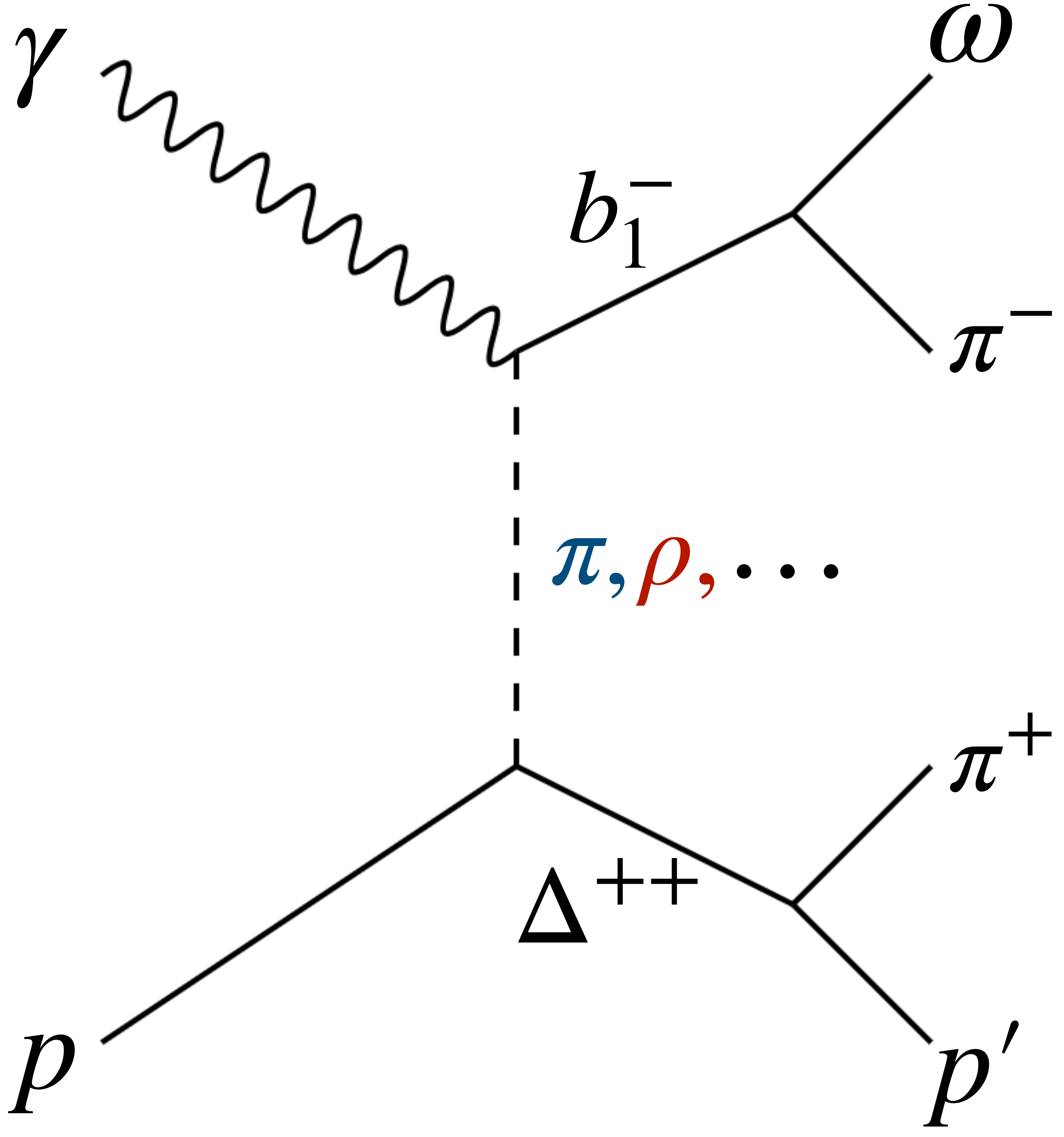
Neutral to Charged Channels

- Measurement of the charged b_1^- cross section adds another aspect to production mechanism analysis
  - Electrically neutral exchange particles such as pomerons or isoscalars are not allowed
  - Don't have to conserve *C*-parity
- Involves an unstable Δ^{++} baryon at the lower vertex
  - A complication not present in the neutral channel
- Can learn from simpler reactions involving charged exchange with a Δ^{++} at the lower vertex, such as $\gamma p \to \pi^- \Delta^{++}$



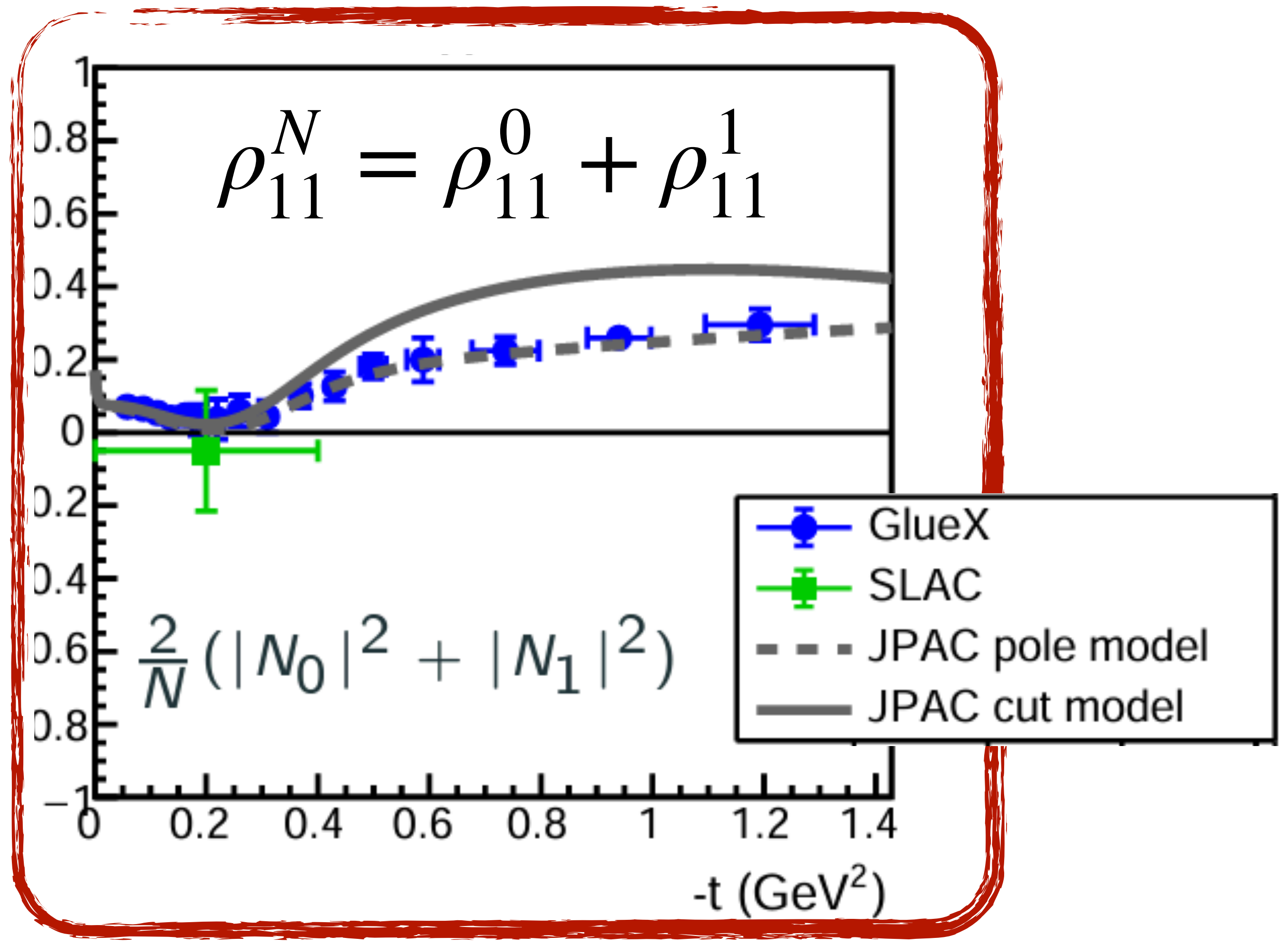
SDMEs in $\gamma p \to \pi^- \Delta^{++}$

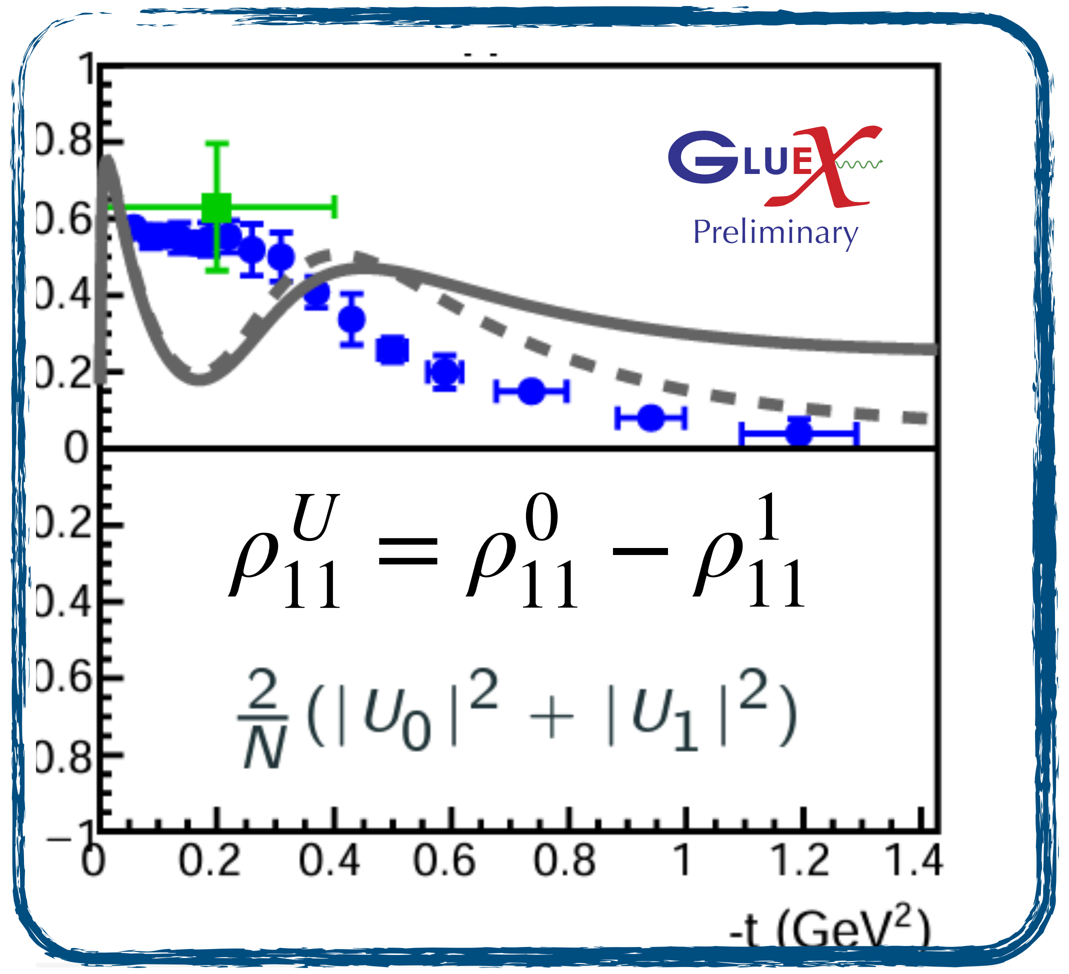
- SDMEs compared with theoretical models demonstrate how well natural and unnatural exchanges are understood

- Published theoretical calculations (PLB 779, 77 (2018)) model natural exchange well, but don't match preliminary GlueX measurements of unnatural exchange

- Indicates that unnatural exchange mechanism ~~is~~ *was* not well understood

- See talk by V. Shastry tomorrow



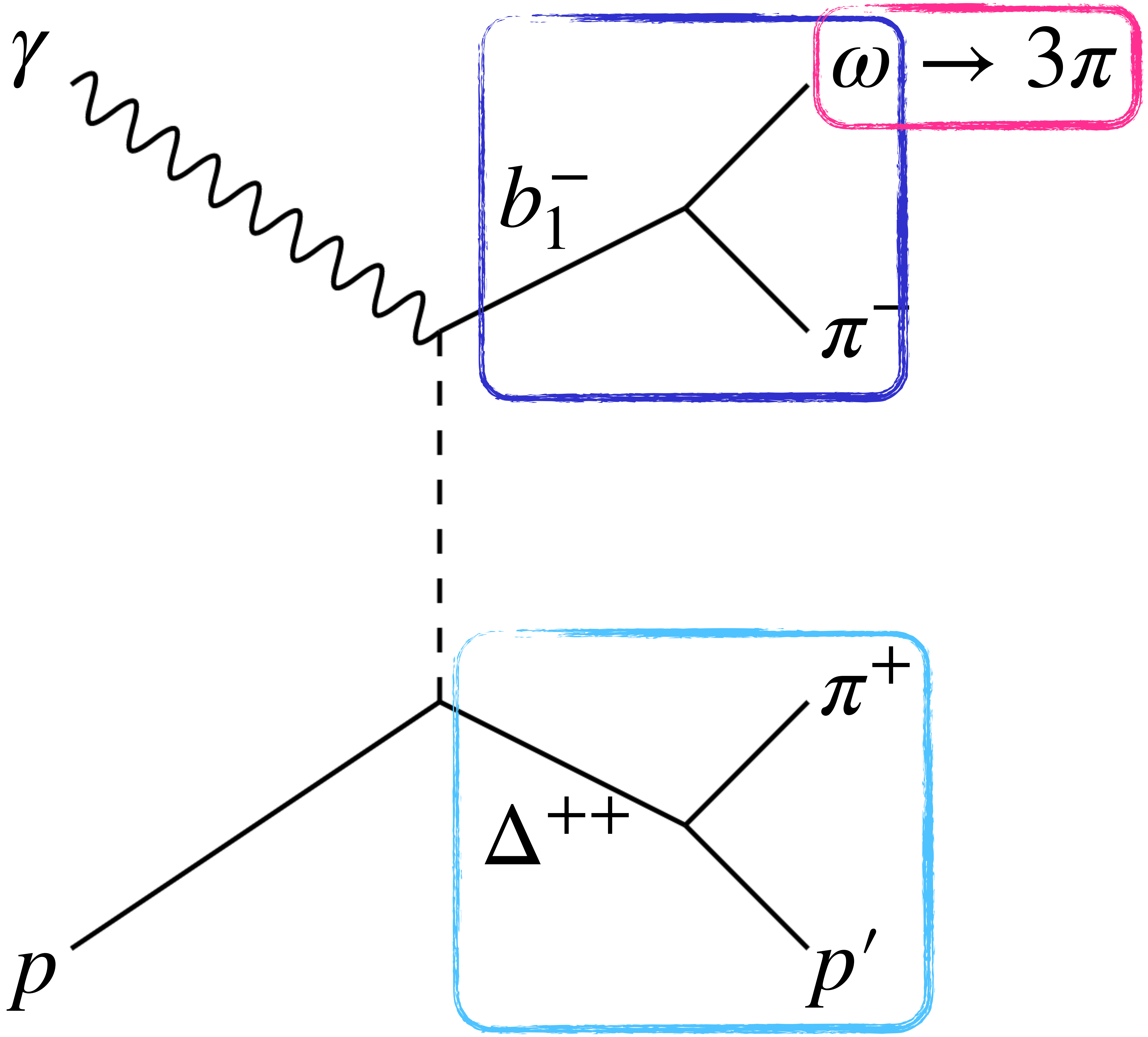


Plots courtesy F. Afzal. Publication in preparation
SLAC: Ballam et al, PRD 7 (1973), 3150
JPAC: Nys et al, PLB 779, 77 (2018)

Putting it all together: $\gamma p \to \omega\pi^-\Delta^{++}$

Production and Decay Angles

- The production and decay of $\omega\pi^-$ against a Δ^{++} requires 7 angles to properly describe
- The decay of $b_1^- \to \omega\pi^-$ is described by $\Omega = (\theta, \phi)$
- The decay of $\omega \to 3\pi$ is described by $\Omega_H = (\theta_H, \phi_H)$
- The decay of $\Delta^{++} \to p\pi^+$ is described by $\Omega_p = (\theta_p, \phi_p)$
- And Φ is the angle between the production and beam polarization planes



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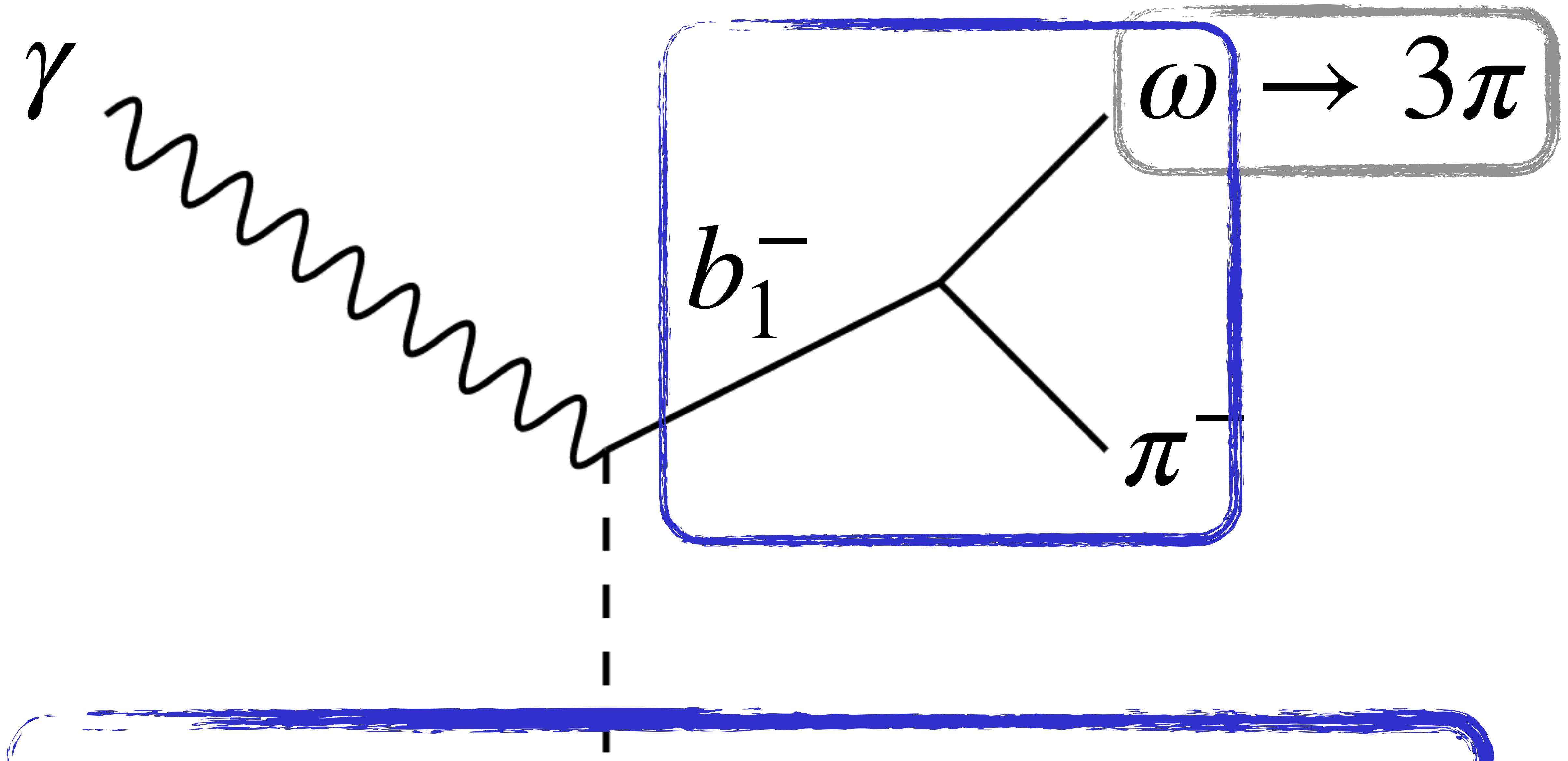

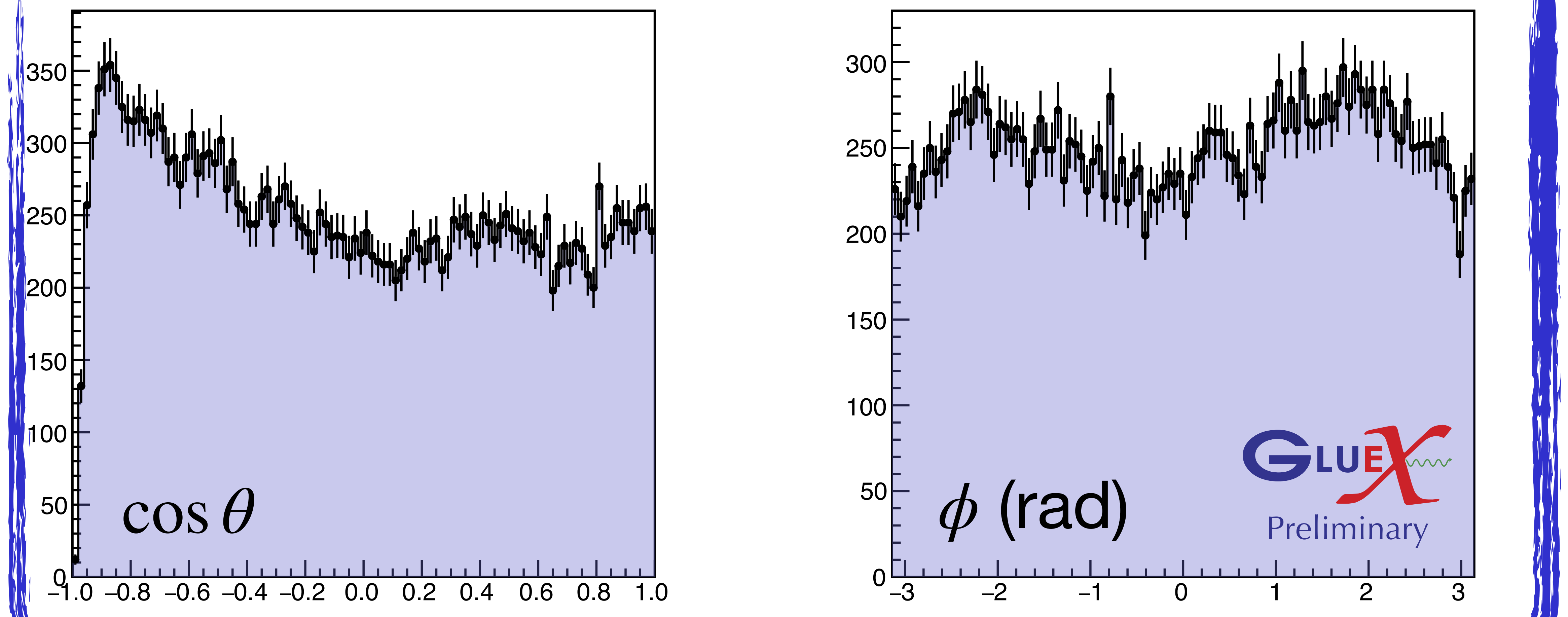

Not corrected for acceptance

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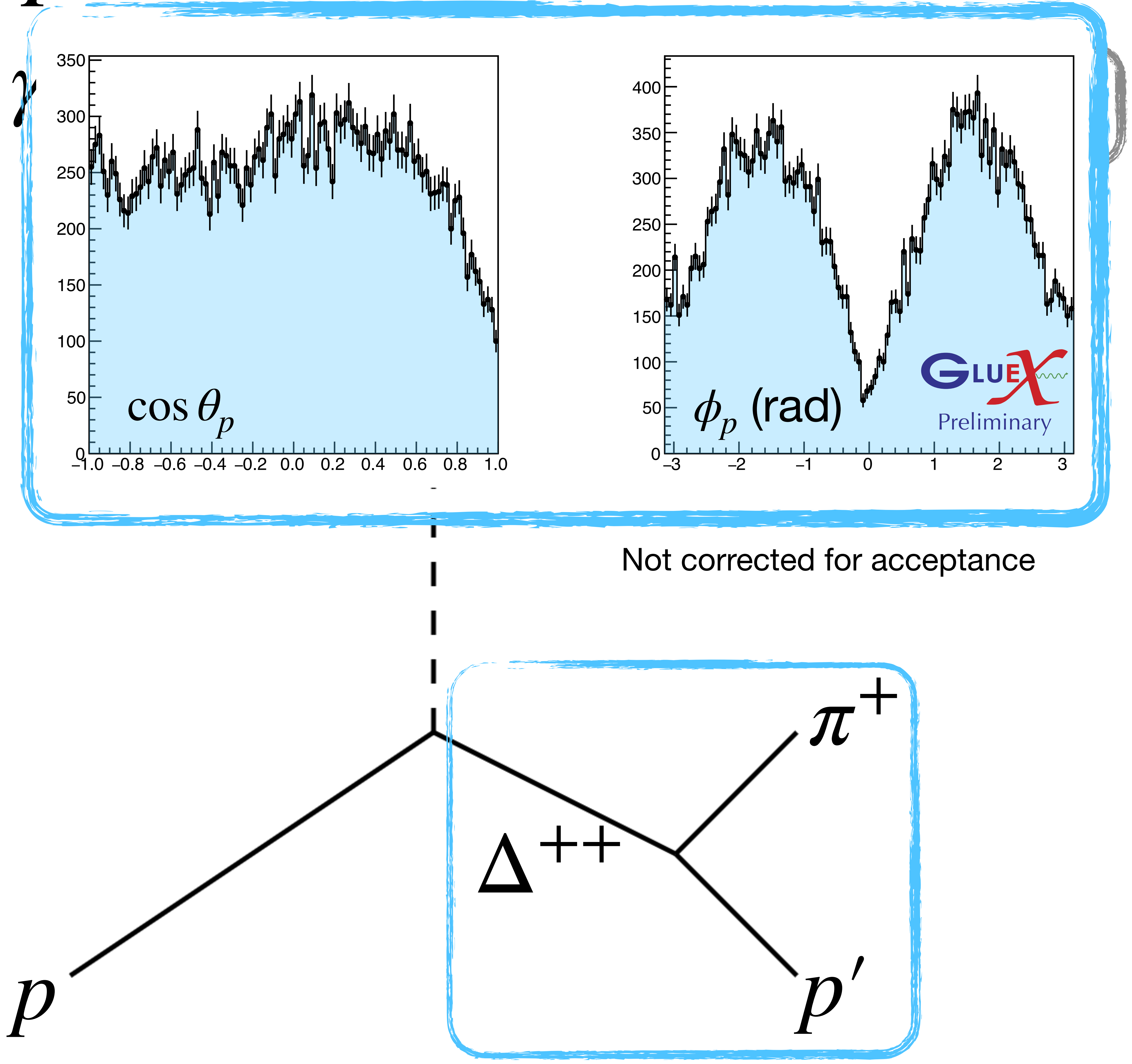

Not corrected for acceptance

Vector-Pseudoscalar Amplitudes

Unstable recoil leads to possible reflectivity interference

$$
\begin{aligned}
I \propto \sum_{\lambda_2} \left(1 - P_\gamma\right) & \left| \sum_{j,m,\lambda_\Delta} \tilde{F}D^{3/2*}_{\lambda_\Delta,\lambda_2}\left(\Omega_p\right) \left(\left[J^P\right]^{(+)}_{m,\lambda_\Delta} ReZ^j_m\left(\Phi,\Omega,\Omega_H\right) + i \left[J^P\right]^{(-)}_{m,\lambda_\Delta} ImZ^j_m\left(\Phi,\Omega,\Omega_H\right) \right) \right|^2 \\
+ \left(1 - P_\gamma\right) & \left| \sum_{j,m,\lambda_\Delta} \tilde{F}D^{3/2}_{\lambda_\Delta,\lambda_2}\left(\Omega_p\right) \left(\left[J^P\right]^{(+)}_{m,\lambda_\Delta} ReZ^j_m\left(\Phi,\Omega,\Omega_H\right) - i \left[J^P\right]^{(-)}_{m,\lambda_\Delta} ImZ^j_m\left(\Phi,\Omega,\Omega_H\right) \right) \right|^2 \\
+ \left(1 + P_\gamma\right) & \left| \sum_{j,m,\lambda_\Delta} \tilde{F}D^{3/2*}_{\lambda_\Delta,\lambda_2}\left(\Omega_p\right) \left(\left[J^P\right]^{(+)}_{m,\lambda_\Delta} ImZ^j_m\left(\Phi,\Omega,\Omega_H\right) - i \left[J^P\right]^{(-)}_{m,\lambda_\Delta} ReZ^j_m\left(\Phi,\Omega,\Omega_H\right) \right) \right|^2 \\
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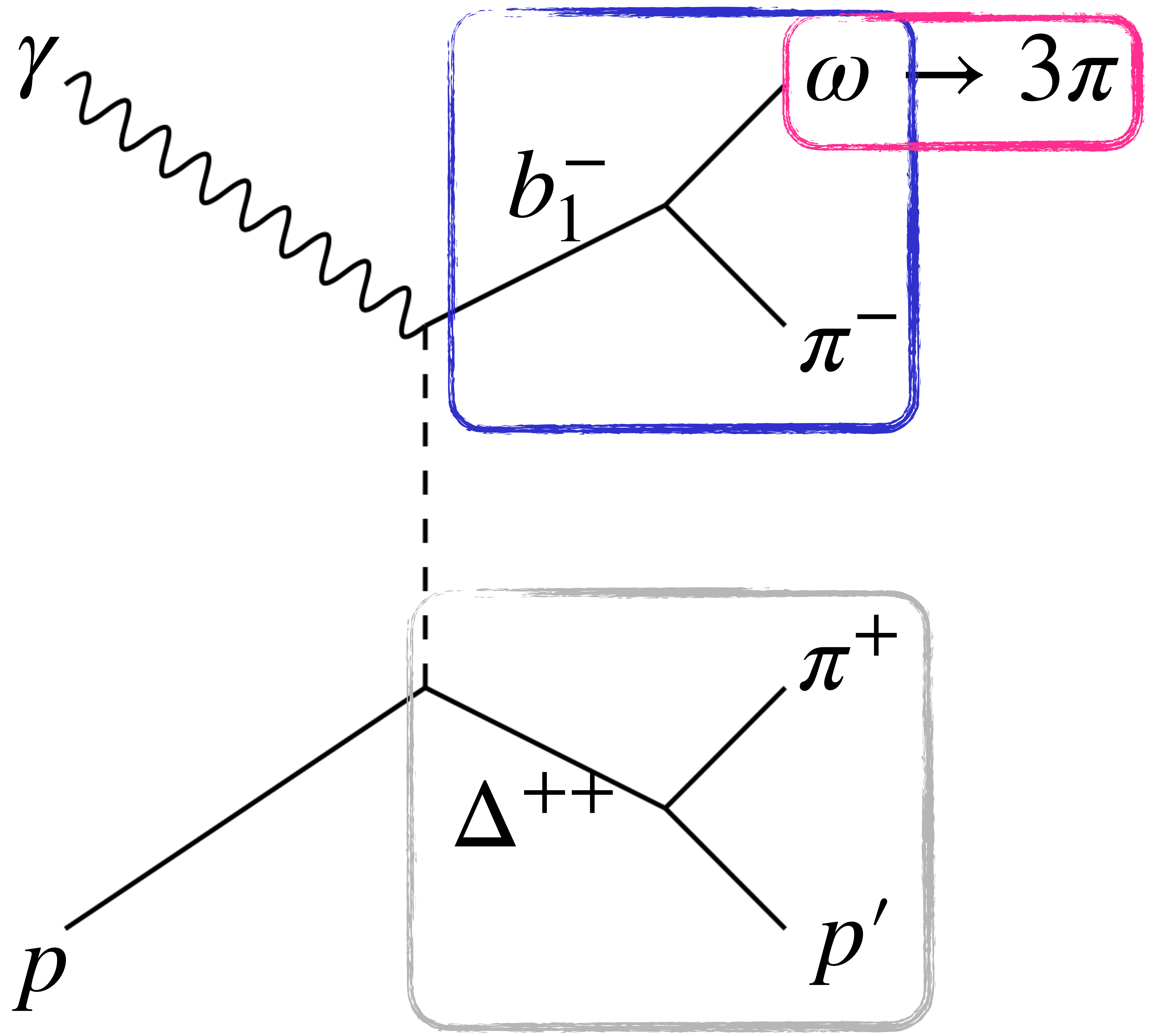
\end{aligned}
$$

Charged b_1^-

Can we separate the natural and unnatural cross sections?

- Unstable recoil -> possible reflectivity interference
- In general, separate “cross sections” don’t make sense for amplitudes that can interfere with each other
- If no interference between reflectivities, can integrate over the Δ^{++} angles and fit only the $b_1^- \to \omega\pi^-$ and $\omega \to 3\pi$ angles
    - Use the same intensity function and amplitudes as for the neutral channel
- Does this simpler fit method get the right answer?
    - Testable with Monte Carlo



Monte Carlo I/O Study

Step one

- Generate Monte Carlo with the full description of the intensity - taking both vertices into account
- Generated waveset consists of an axial vector resonance decaying to $\omega\pi^-$ in an *S* wave
  - Partial wave contributions $[J^P]^{(\varepsilon)}_{m,\lambda_\Delta}$ of varying strengths

Wave	Gen Value
$[1^+]^{(+)}_{+1,+3/2}$	0.030
$[1^+]^{(+)}_{+1,+1/2}$	0.004
$[1^+]^{(+)}_{+1,-1/2}$	0.004
$[1^+]^{(+)}_{0,+3/2}$	0.280
$[1^+]^{(+)}_{0,+1/2}$	0.031
$[1^+]^{(+)}_{0,-1/2}$	0.031
$[1^+]^{(-)}_{+1,+3/2}$	0.030
$[1^+]^{(-)}_{+1,+1/2}$	0.262
$[1^+]^{(-)}_{+1,-1/2}$	0.267
$[1^+]^{(-)}_{0,+3/2}$	0.004
$[1^+]^{(-)}_{0,+1/2}$	0.030
$[1^+]^{(-)}_{0,-1/2}$	0.030

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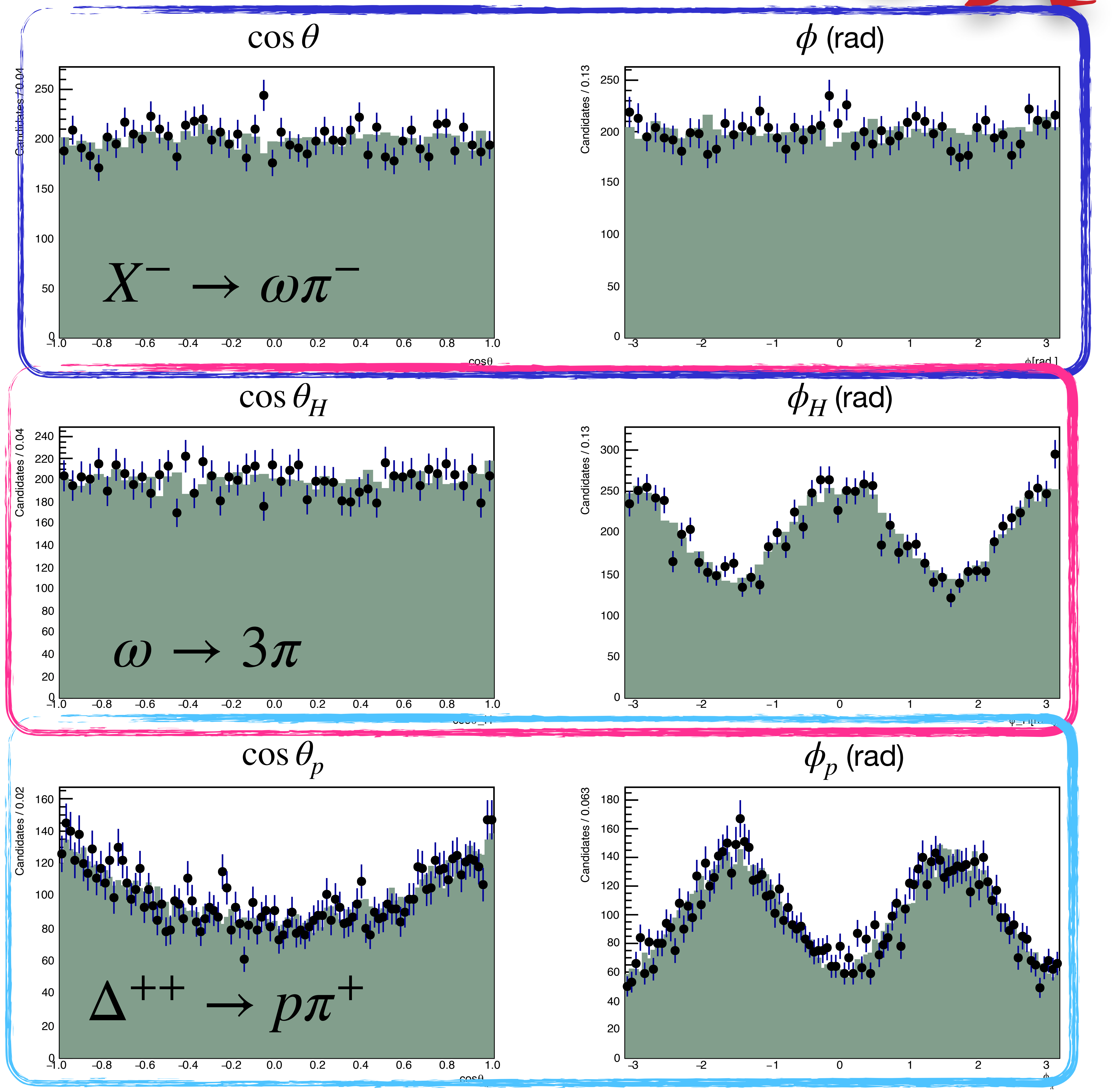
Wave	Gen Value
$[1^+]^{(+)}_{+1}$	0.04
$[1^+]^{(+)}_{0}$	0.33
$[1^+]^{(+)}_{-1}$	0
$[1^+]^{(-)}_{+1}$	0.56
$[1^+]^{(-)}_{0}$	0.06
$[1^+]^{(-)}_{-1}$	0

Monte Carlo I/O Study

Step two

- Fit generator-level Monte Carlo with the full description of the intensity - taking both vertices into account
- Fit results are the partial waves contributing to the intermediate resonance X^-, for each possible value of λ_Δ: $\left[J^P\right]^{(\varepsilon)}_{m,\lambda_\Delta}$
- Fitting with a $J^P = 1^+$ resonance decaying in an S wave requires 24 different partial waves
  - A waveset consisting of a proper b_1^- and a vector meson would require 72 partial waves



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Wave	Fit Fraction	Gen Value
$[1^+]^{(+)}_{+1}$	0.08 ± 0.03	0.04
$[1^+]^{(+)}_{0}$	0.33 ± 0.03	0.33
$[1^+]^{(+)}_{-1}$	0.00 ± 0.01	0
$[1^+]^{(-)}_{+1}$	0.54 ± 0.04	0.56
$[1^+]^{(-)}_{0}$	0.04 ± 0.02	0.06
$[1^+]^{(-)}_{-1}$	0.00 ± 0.01	0

λ_Δ	Fit Fraction	Gen Value
$\pm 1/2$	0.67 ± 0.01	0.66
$\pm 3/2$	0.33 ± 0.01	0.34

Monte Carlo I/O Study

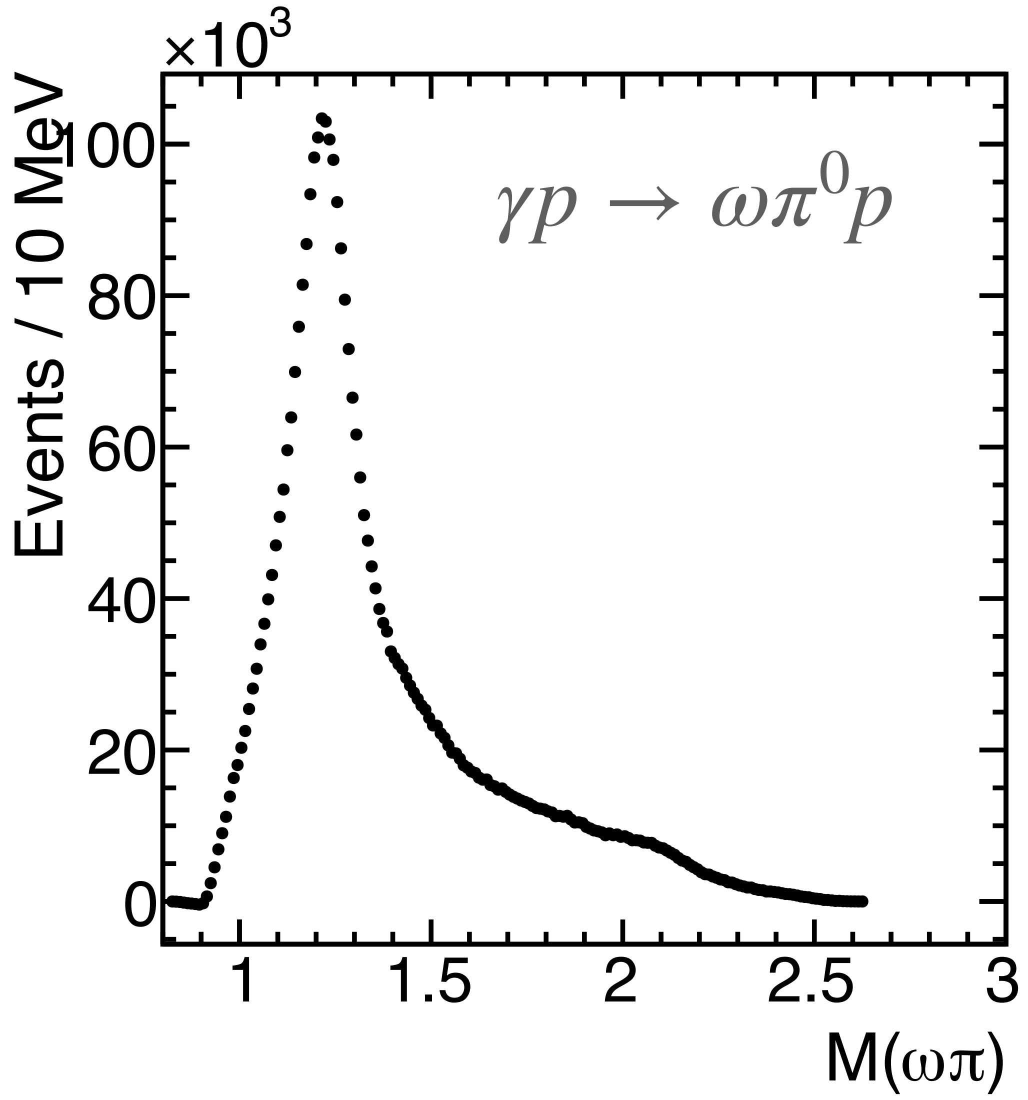
Step three

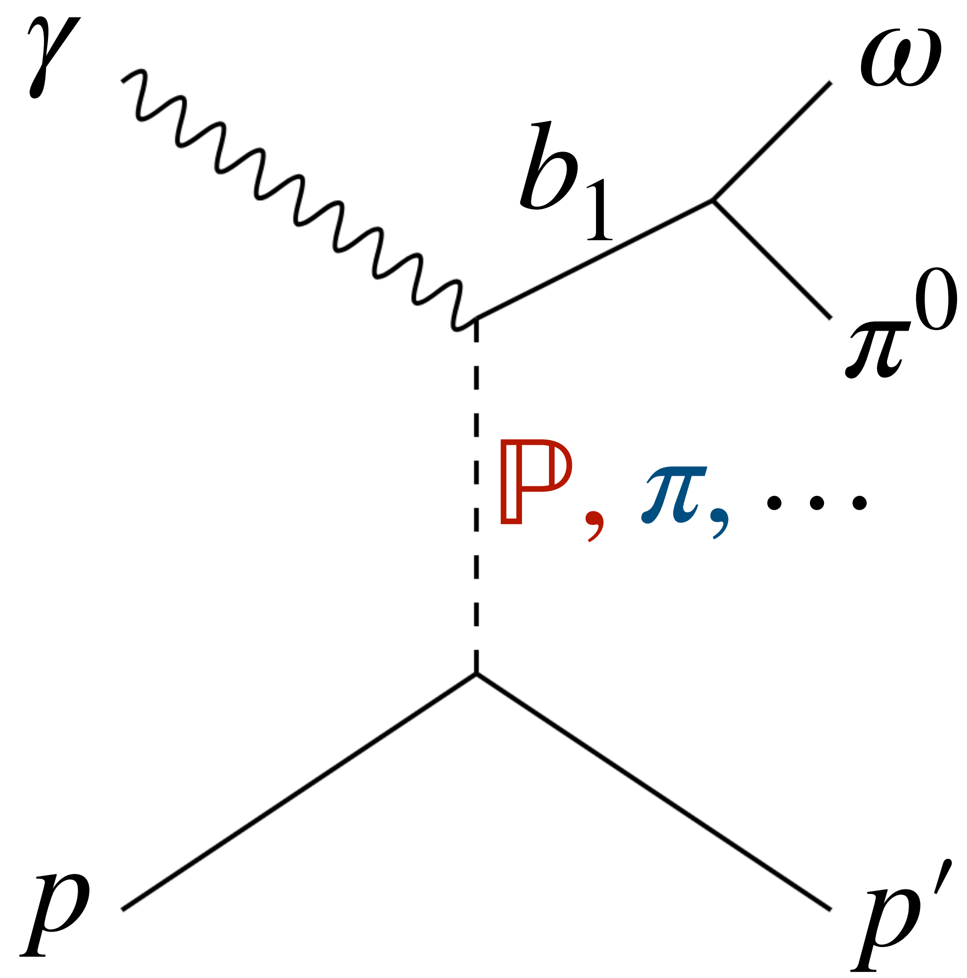
- Integrate over the lower vertex decay angles - this fit requires 1/4 as many partial waves as the full model fit
- Fit results are the partial waves contributing to the intermediate resonance X^-: $\left[J^P\right]_m^{(\varepsilon)}$ with no information about Δ^{++} polarization
- Results are consistent with generated values
- Caveat: Tested without reconstruction and acceptance effects, with a limited waveset

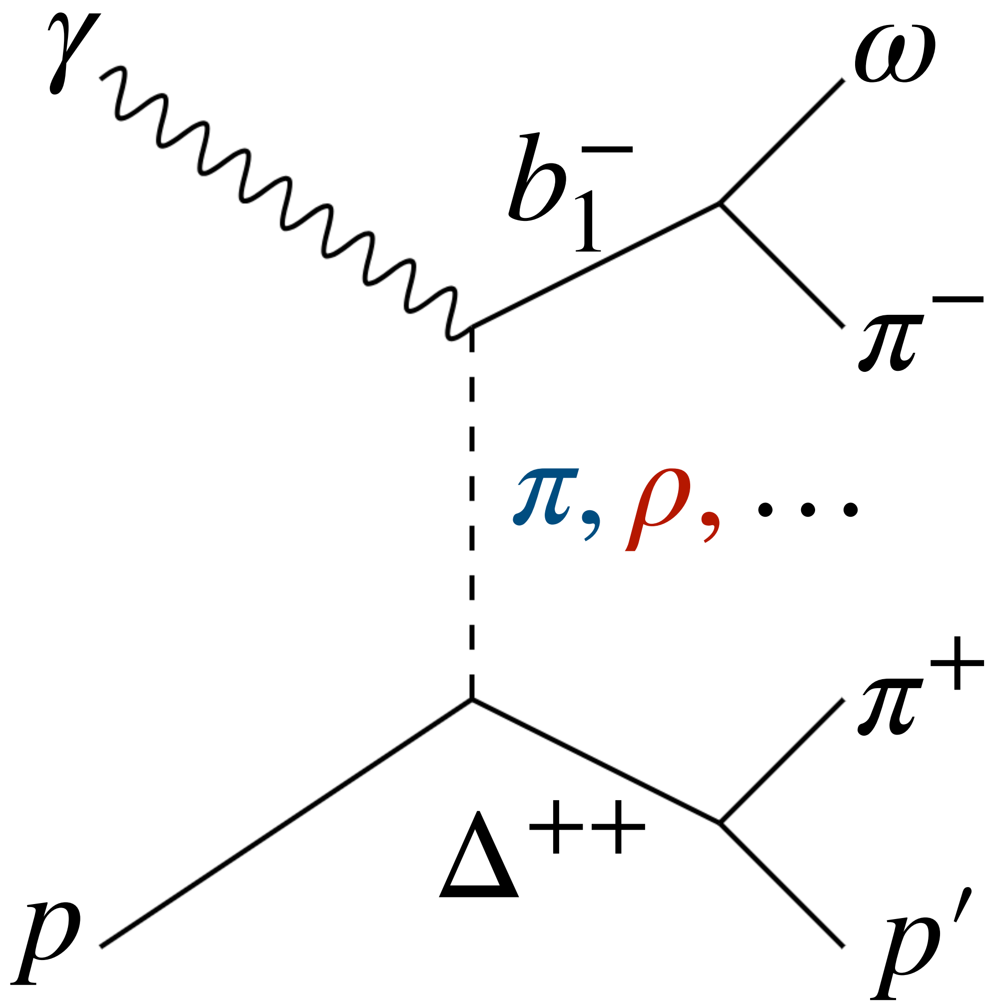
Wave	Fit Fraction	Gen Value
$[1^+]_{+1}^{(+)}$	0.00 ± 0.00	0.04
$[1^+]_{0}^{(+)}$	0.29 ± 0.04	0.33
$[1^+]_{-1}^{(+)}$	0.08 ± 0.08	0
$[1^+]_{+1}^{(-)}$	0.52 ± 0.11	0.56
$[1^+]_{0}^{(-)}$	0.07 ± 0.03	0.06
$[1^+]_{-1}^{(-)}$	0.04 ± 0.08	0

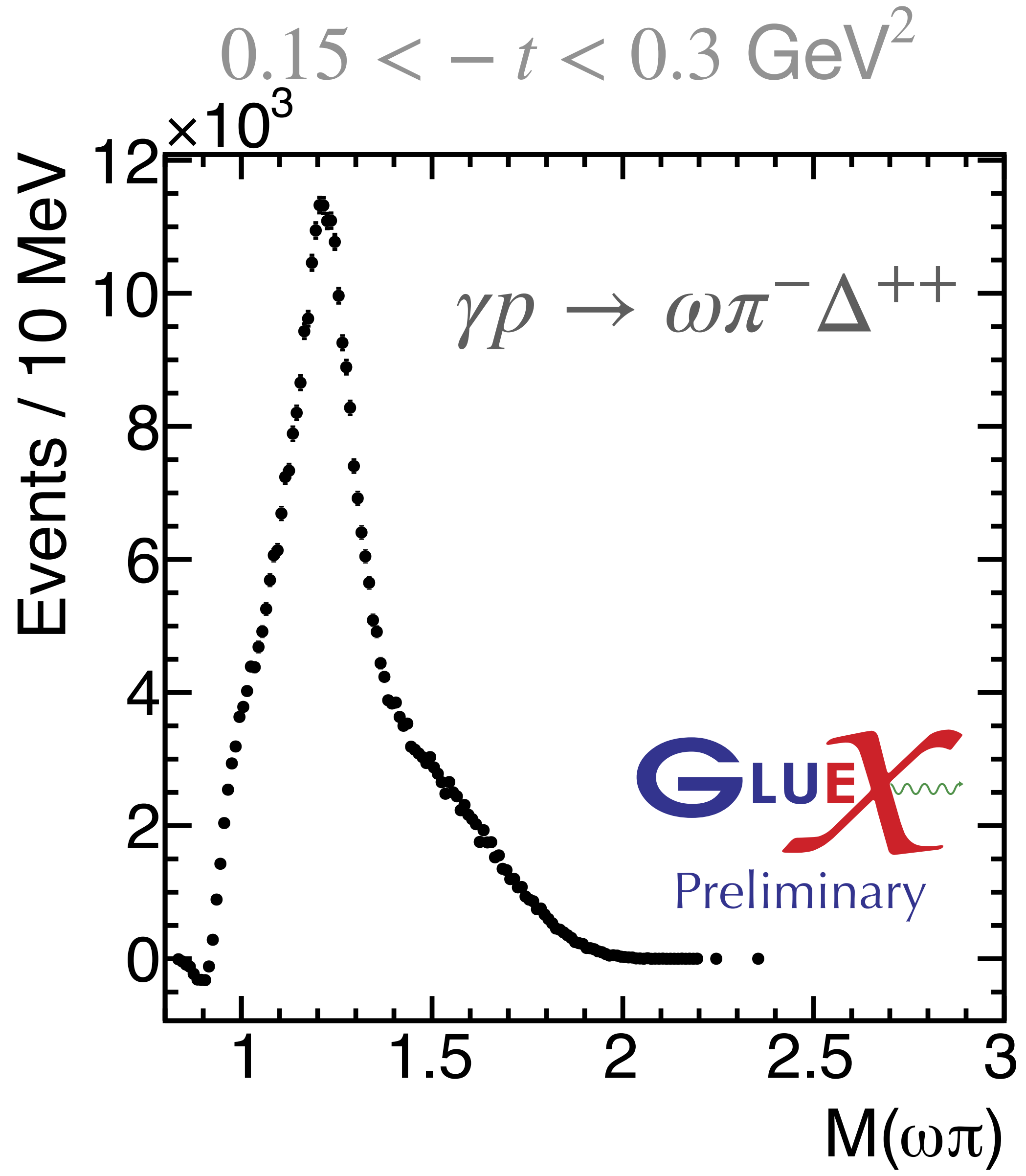
Summary

- Our $\omega\pi$ "standard candle" is the b_1 meson, we can get a large clean sample in both charged and neutral exchange
- Natural parity exchange appears to dominate neutral b_1 production
- Simplest charged exchange process, $\gamma p \rightarrow \pi^- \Delta^{++}$, proceeds through a combination of natural and unnatural exchange - not just pion exchange!
- Charged b_1^- analysis in progress
- Future plans: use this analysis framework to look for excited states in the $\omega\pi$ and other vector-pseudoscalar channels









GlueX gratefully acknowledges the support of several funding agencies and computing facilities: gluex.org/thanks

Cross Sections from PWA

$$\frac{d\sigma}{dt} = \frac{N_{obs}}{\epsilon} \frac{1}{\mathscr{L}\mathscr{B}(\omega \to 3\pi) f_{b_1} f_{\Delta} \Delta t}$$

Extracted from fit

Luminosity factor

Branching fraction (89.2%)

b_1 and Δ^{++} lineshape factors

-*t* bin width

SDME Description

A description of the lower vertex

$$W(\Omega_p, \Phi) \propto \rho^0_{33} \sin^2\theta_p + \rho^0_{11}\left(\frac{1}{3} + \cos^2\theta_p\right) - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^0_{31} \sin 2\theta_p \cos\phi_p - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^0_{3-1} \sin^2\theta_p \cos 2\phi_p$$

$$-P_\gamma \cos 2\Phi \left[\rho^1_{33} \sin^2\theta_p + \rho^1_{11}\left(\frac{1}{3} + \cos^2\theta_p\right) - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^1_{31} \sin 2\theta_p \cos\phi_p - \frac{2}{\sqrt{3}}\mathrm{Re}\rho^1_{3-1} \sin^2\theta_p \cos 2\phi_p\right]$$

$$-P_\gamma \sin 2\Phi \left[\frac{2}{\sqrt{3}}\mathrm{Im}\rho^2_{31} \sin 2\theta_p \sin\phi_p + \frac{2}{\sqrt{3}}\mathrm{Im}\rho^2_{3-1} \sin^2\theta_p \sin 2\phi_p\right]$$

NB: $\rho^0_{11} + \rho^0_{33} = \frac{1}{2}$ and $\Sigma = 2\left(\rho^1_{11} + \rho^1_{33}\right)$

Lower Vertex SDME Fit

- Integrate over the upper vertex decay angles
- Fit results are the Δ^{++} SDMEs, which can be used to extract Δ^{++} helicities, and in special cases like $\pi^-\Delta^{++}$, extract exchange naturalities

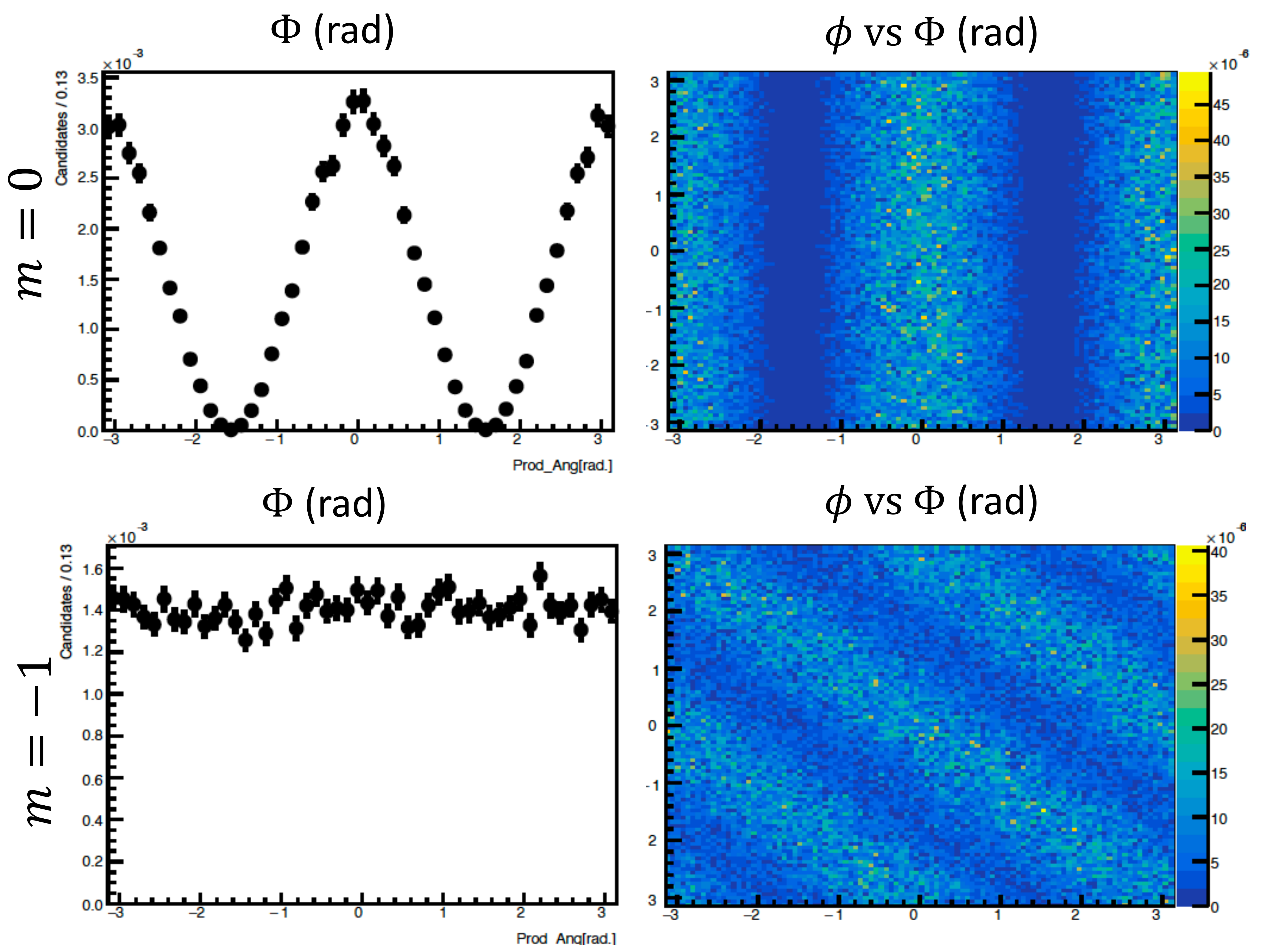
SDME	Fit Value
ρ^0_{11}	0.33 ± 0.01
ρ^0_{33}	0.17 ± 0.01
$\mathrm{Re}\rho^0_{31}$	0.08 ± 0.01
$\mathrm{Re}\rho^0_{3-1}$	0.20 ± 0.01
ρ^1_{11}	0.15 ± 0.06
ρ^1_{33}	0.22 ± 0.06
$\mathrm{Re}\rho^1_{31}$	-0.40 ± 0.05
$\mathrm{Re}\rho^1_{3-1}$	-0.51 ± 0.05
$\mathrm{Im}\rho^2_{31}$	0 (fixed)
$\mathrm{Im}\rho^2_{3-1}$	0 (fixed)

$$FF_{1/2} = 2\rho^0_{11}$$
$$FF_{3/2} = 2\rho^0_{33}$$

λ_Δ	Fit (LV)	Fit (Full)	Gen Value
$\pm 1/2$	0.67 ± 0.02	0.67 ± 0.01	0.66
$\pm 3/2$	0.33 ± 0.02	0.33 ± 0.01	0.34

What we can and can’t learn

- The full description of the reaction allows us to generate realistic Monte Carlo sets. When we fit them with subsets of the full description, we can learn if and how these subset fits are biased
- For example: the production angle Φ has a clear distribution when the spin projection at the upper vertex is zero, but is flat otherwise, only showing structure when plotted against ϕ from the upper vertex
  - The lower vertex fits by themselves don’t have access to ϕ - can’t learn anything meaningful about reflectivity
- Right: Angular distributions of generator-level MC, produced with 100% polarization



SDME to Amplitudes $(\pi^-\Delta^{++})$

$$\rho^0_{\frac12\frac12} + \rho^1_{\frac12\frac12} = \frac{2}{N}\left(|N_0|^2 + |N_1|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac32\frac12} + \rho^1_{\frac32\frac12}\right) = \frac{2}{N}\mathrm{Re}\left(N_{-1}N_0^* - N_1N_2^*\right) \quad (15a)$$

$$\rho^0_{\frac12\frac12} - \rho^1_{\frac12\frac12} = \frac{2}{N}\left(|U_0|^2 + |U_1|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac32\frac12} - \rho^1_{\frac32\frac12}\right) = \frac{2}{N}\mathrm{Re}\left(U_{-1}U_0^* - U_1U_2^*\right) \quad (15b)$$

$$\rho^0_{\frac32\frac32} + \rho^1_{\frac32\frac32} = \frac{2}{N}\left(|N_{-1}|^2 + |N_2|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac32-\frac12} + \rho^1_{\frac32-\frac12}\right) = \frac{2}{N}\mathrm{Re}\left(N_0N_2^* + N_1N_{-1}^*\right) \quad (15c)$$

$$\rho^0_{\frac32\frac32} - \rho^1_{\frac32\frac32} = \frac{2}{N}\left(|U_{-1}|^2 + |U_2|^2\right) \qquad \mathrm{Re}\left(\rho^0_{\frac32-\frac12} - \rho^1_{\frac32-\frac12}\right) = \frac{2}{N}\mathrm{Re}\left(U_0U_2^* + U_1U_{-1}^*\right) \quad (15d)$$

$$\mathrm{Im}\,\rho^2_{\frac32\frac12} = \frac{1}{N}\mathrm{Re}\left(N_1U_2^* + N_0U_{-1}^* - N_{-1}U_0^* - N_2U_1^*\right) \quad (15e)$$

$$\mathrm{Im}\,\rho^2_{\frac32-\frac12} = \frac{1}{N}\mathrm{Re}\left(N_1U_{-1}^* - N_0U_2^* - N_{-1}U_1^* + N_2U_0^*\right) \quad (15f)$$

$$\mathrm{Im}\,\rho^2_{\frac32\frac12} = \frac{1}{N}\mathrm{Im}\left(U_2N_1^* + U_{-1}N_0^* - N_{-1}U_0^* - N_2U_1^*\right) \quad (15g)$$

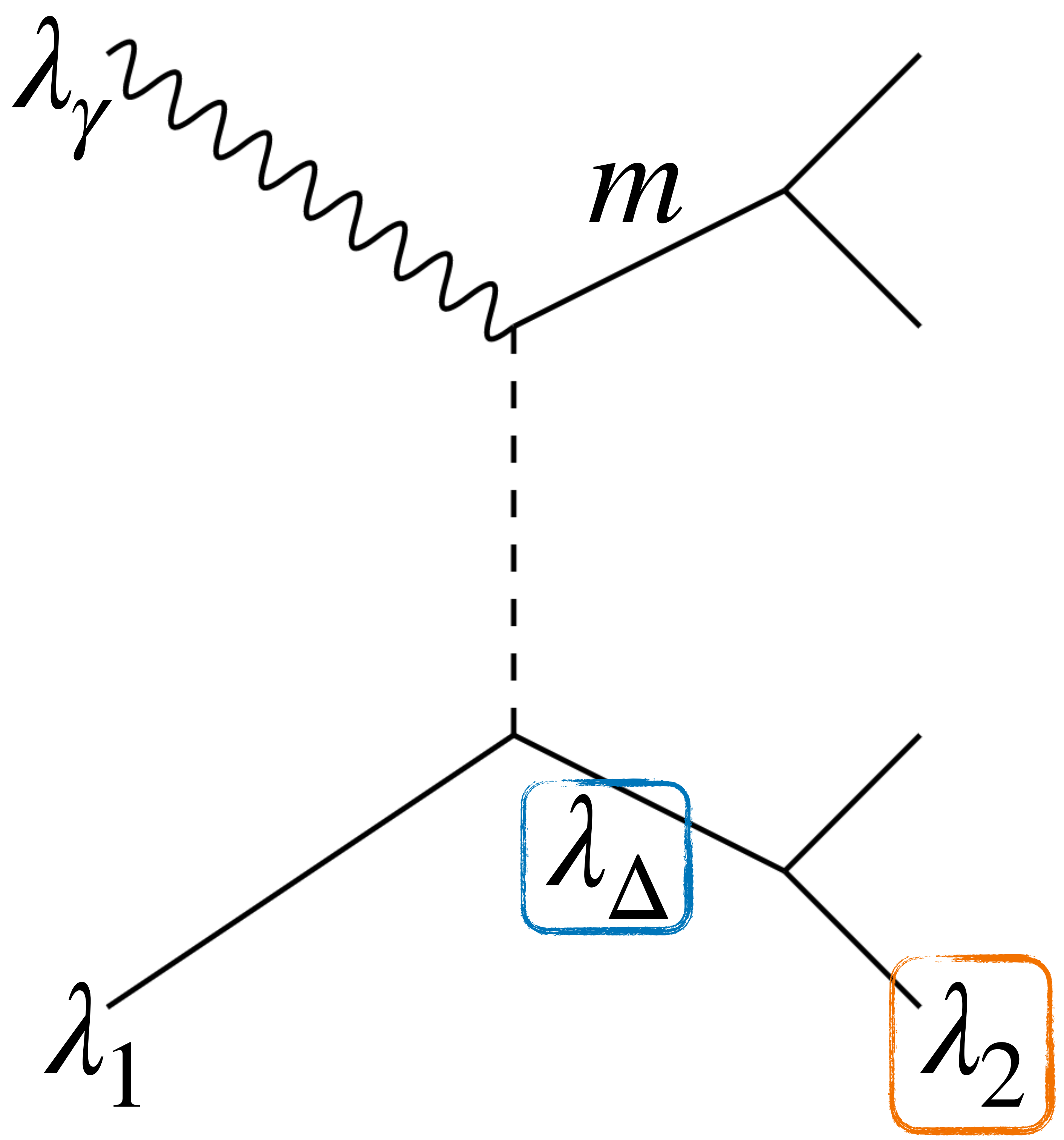
$$\mathrm{Im}\,\rho^2_{\frac32-\frac12} = \frac{1}{N}\mathrm{Im}\left(U_{-1}N_1^* - U_2N_0^* - N_{-1}U_1^* + N_2U_0^*\right) \quad (15h)$$

$$N = 2\left(|N_{-1}|^2 + |N_0|^2 + |N_1|^2 + |N_2|^2 + |U_{-1}|^2 + |U_0|^2 + |U_1|^2 + |U_2|^2\right) \quad (15i)$$

Comparison Between Proton and Δ^{++} Recoil

$$\tilde{A}^{\lambda_\gamma}_{\lambda_1,\lambda_2} = \sum_{j,m} T^j_{\lambda_\gamma,m;\lambda_1,\lambda_2} Z^j_m\left(\Phi,\Omega,\Omega_H\right)$$

$$\tilde{A}^{\lambda_\gamma}_{\lambda_1,\lambda_2} = \sum_{j,m,\lambda_\Delta} T^j_{\lambda_\gamma,m;\lambda_1,\lambda_\Delta} Z^j_m\left(\Phi,\Omega,\Omega_H\right) \tilde{F}_{\lambda_2} D^{3/2*}_{\lambda_\Delta,\lambda_2}\left(\Omega_p\right)$$



Vector-Pseudoscalar Amplitudes

No reflectivity interference

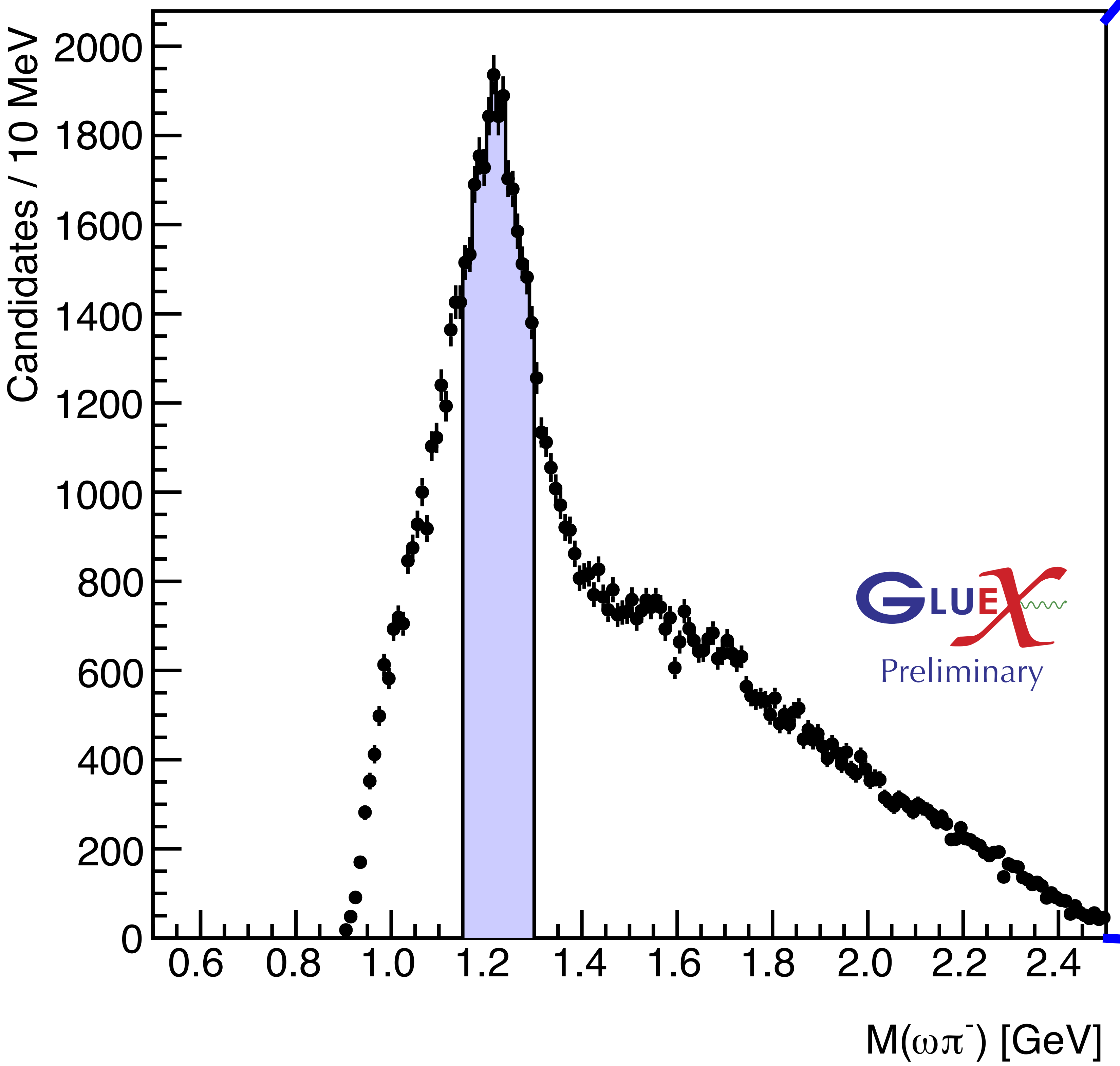
$$I \propto \sum_{k} \left(1 - P_\gamma\right) \left[\left| \sum_{j,m} \left[J^P\right]^{(-)}_{m,k} ImZ^j_m \left(\Phi, \Omega, \Omega_H\right) \right|^2 + \left| \sum_{j,m} \left[J^P\right]^{(+)}_{m,k} ReZ^j_m \left(\Phi, \Omega, \Omega_H\right) \right|^2 \right]$$

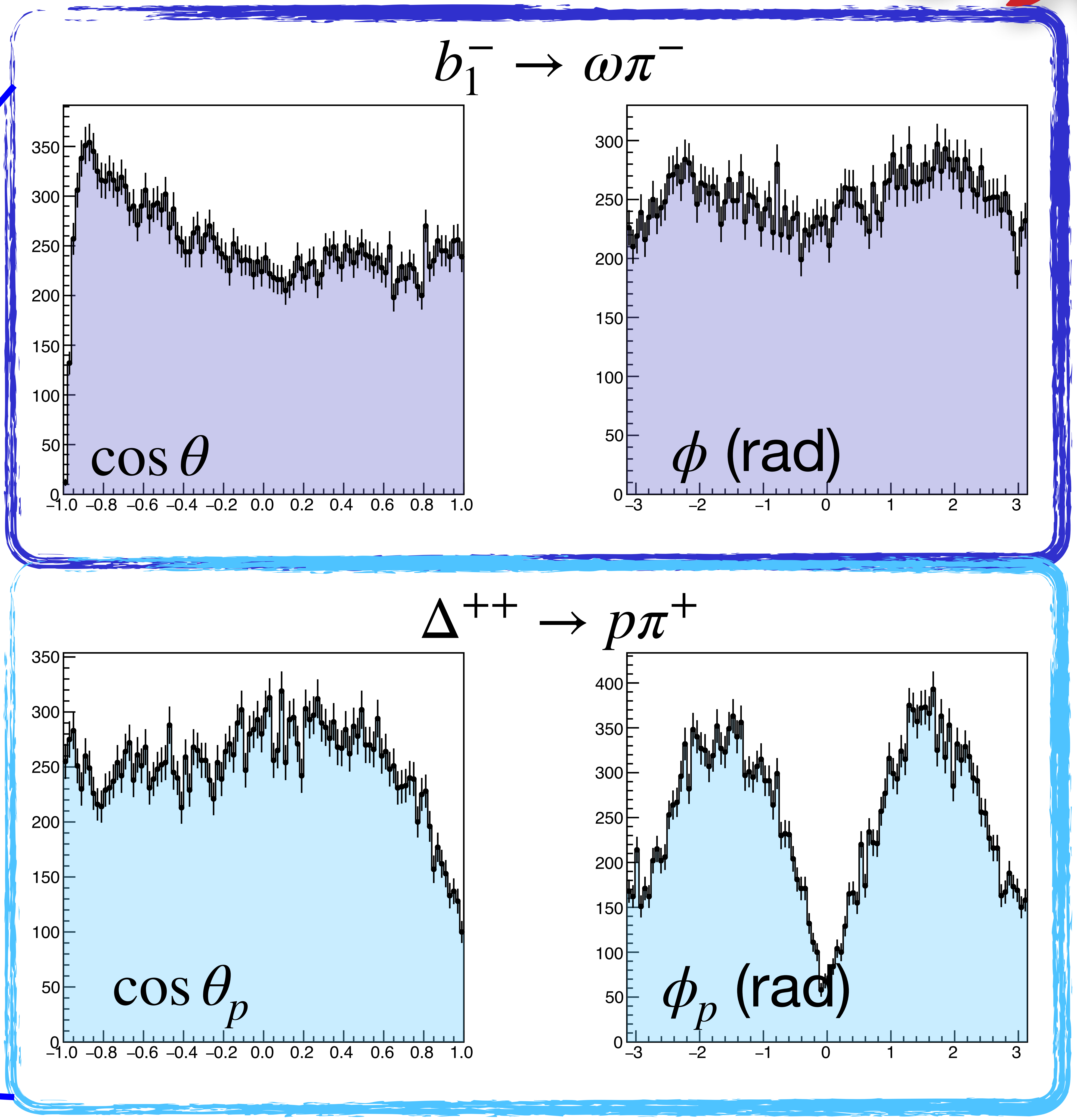
$$+ \left(1 + P_\gamma\right) \left[\left| \sum_{j,m} \left[J^P\right]^{(+)}_{m,k} ImZ^j_m \left(\Phi, \Omega, \Omega_H\right) \right|^2 + \left| \sum_{j,m} \left[J^P\right]^{(-)}_{m,k} ReZ^j_m \left(\Phi, \Omega, \Omega_H\right) \right|^2 \right]$$

k indexes spin flip or non-flip, but can't be measured at GlueX. It is assumed that one value is dominant, and this sum is neglected in practice

Charged b_1^-

- Mass and angular distributions from GlueX-I





Not corrected for acceptance

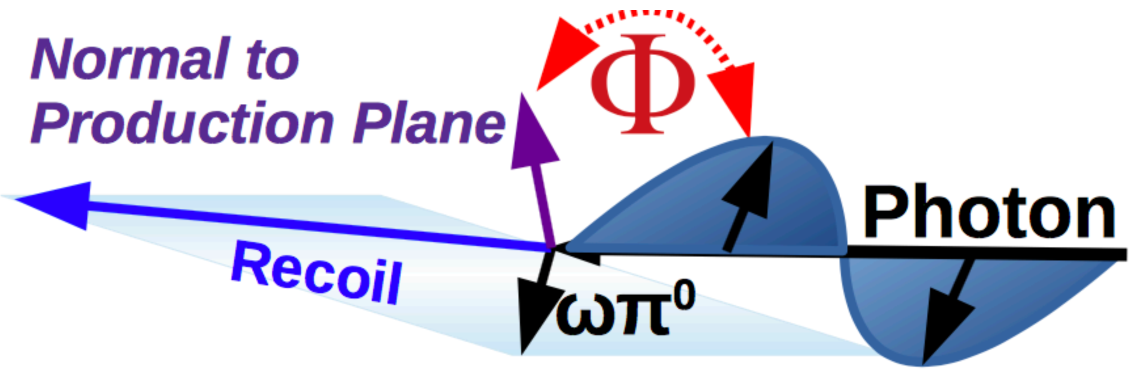
Reflectivity and Naturality

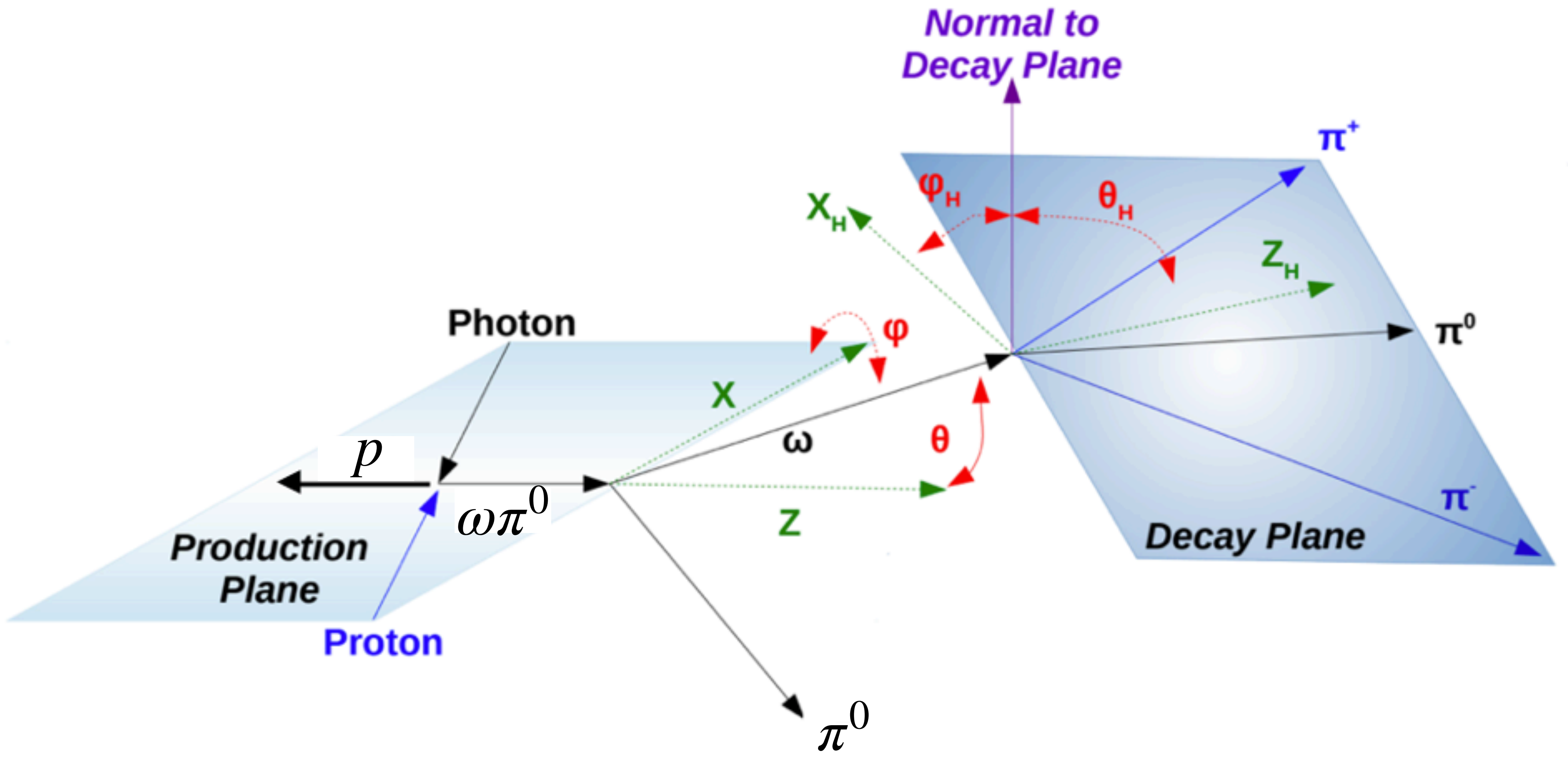
- The reflectivity operator $\Pi_y = PR_y(\pi)$ reflects the reaction through the production plane

- Choose a basis where the amplitudes have definite reflectivity, $\varepsilon = \pm 1$

$$[J^P]^{(\varepsilon)}_m = \frac{1}{2}\left[T^j_{+1,m} - \tau_j \varepsilon (-1)^m T^j_{-1,-m}\right]$$

- At the energies used by GlueX, the reflectivity of a *t*-channel reaction directly corresponds to the naturality, $\tau = P(-1)^J$, of the exchange particle





$$Z^j_m\left(\Phi, \Omega, \Omega_H\right) = e^{-i\Phi} \sum_{\lambda} F^j_\lambda D^{J_j *}_{m,\lambda}(\Omega)\, Y^1_\lambda\left(\Omega_H\right) G$$