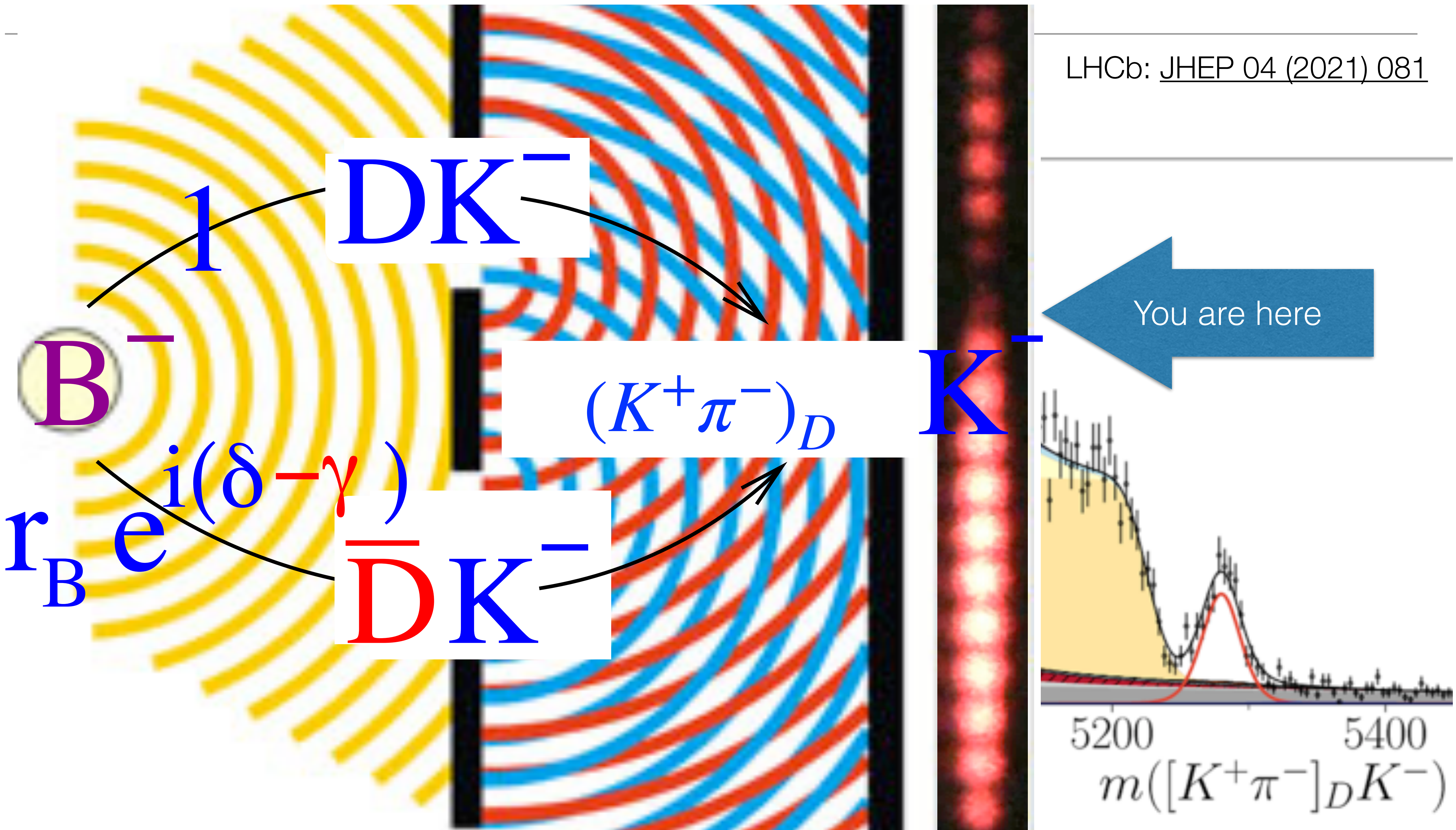
Model-independent phase correction to amplitude models

J Cottee-Meldrum, E Gersabeck, J Lane, C Normand, J Rademacker, S Zheng

JHEP 09 (2023) 007 plus new developments

CP violation is an interference effect



LHCb: JHEP 04 (2021) 081

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003, A. Bondar, "Proceedings of BINP special analysis meeting on Dalitz analysis, 24-26 Sep. 2002, unpublished."

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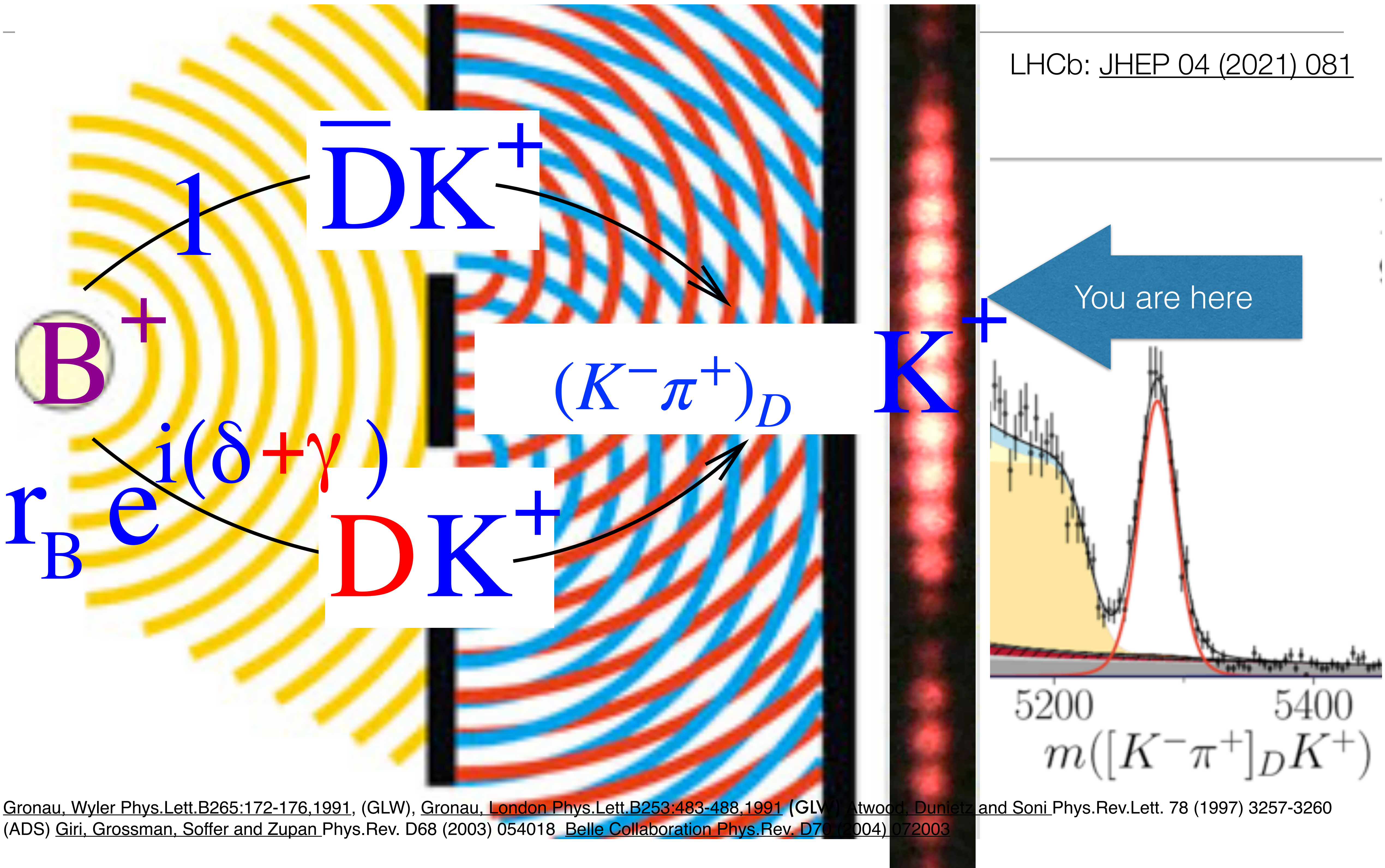

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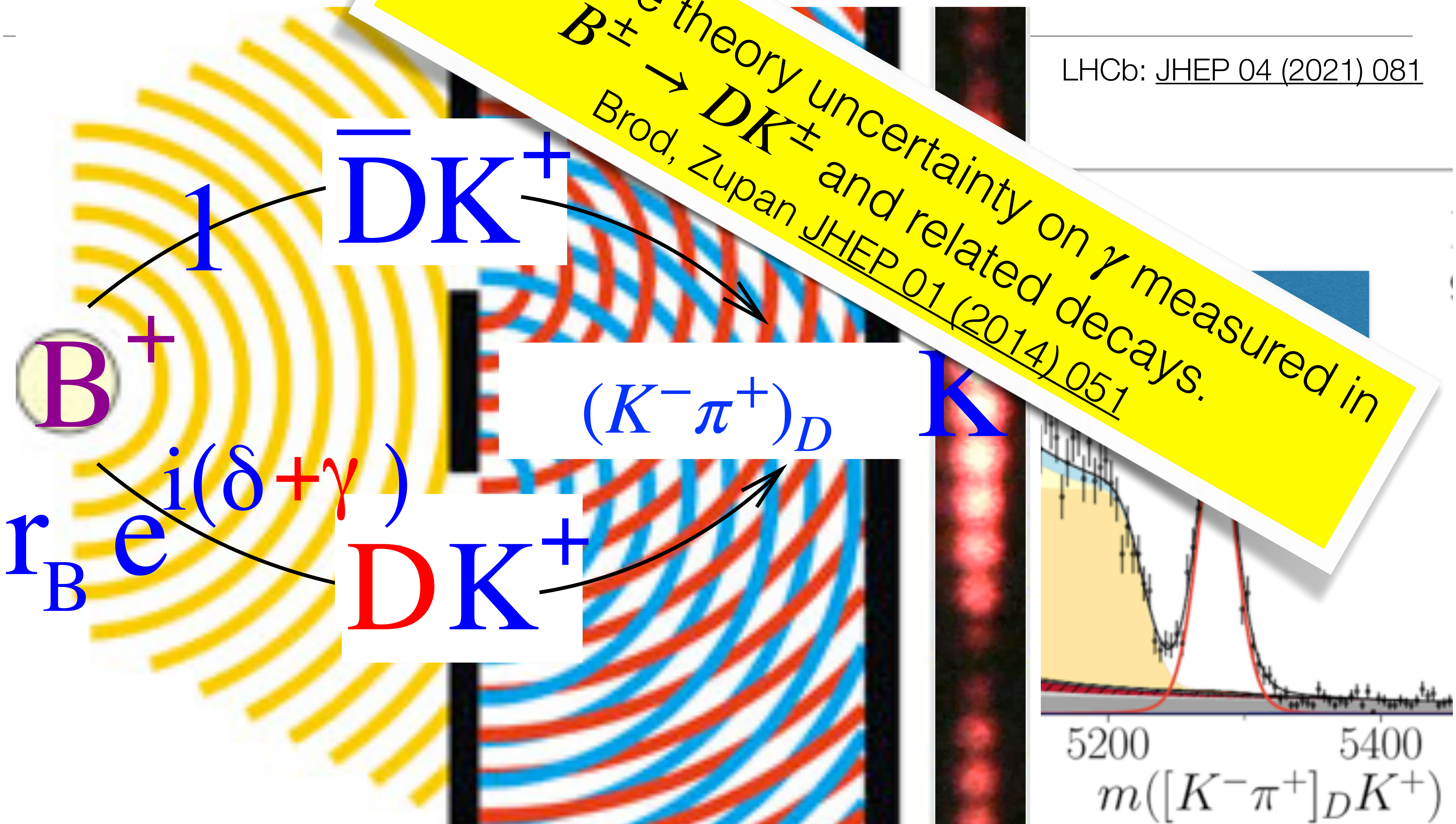
CP violation is ... ce effect



Negligible theory uncertainty on γ measured in $B^\pm \to DK^\pm$ and related decays.
Brod, Zupan JHEP 01 (2014) 051

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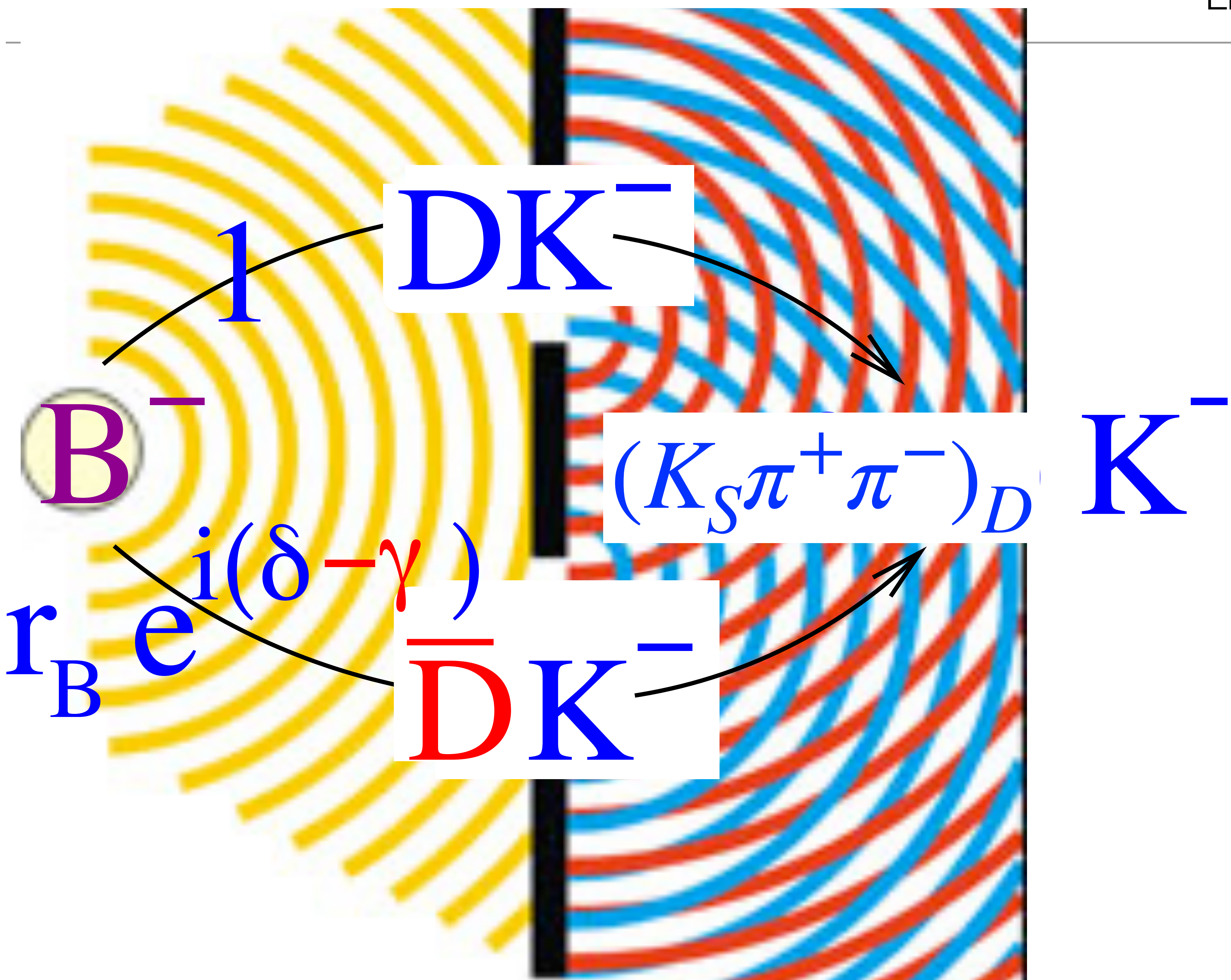
CP violation is an interference effect

LHCb: JHEP 02 (2021) 169



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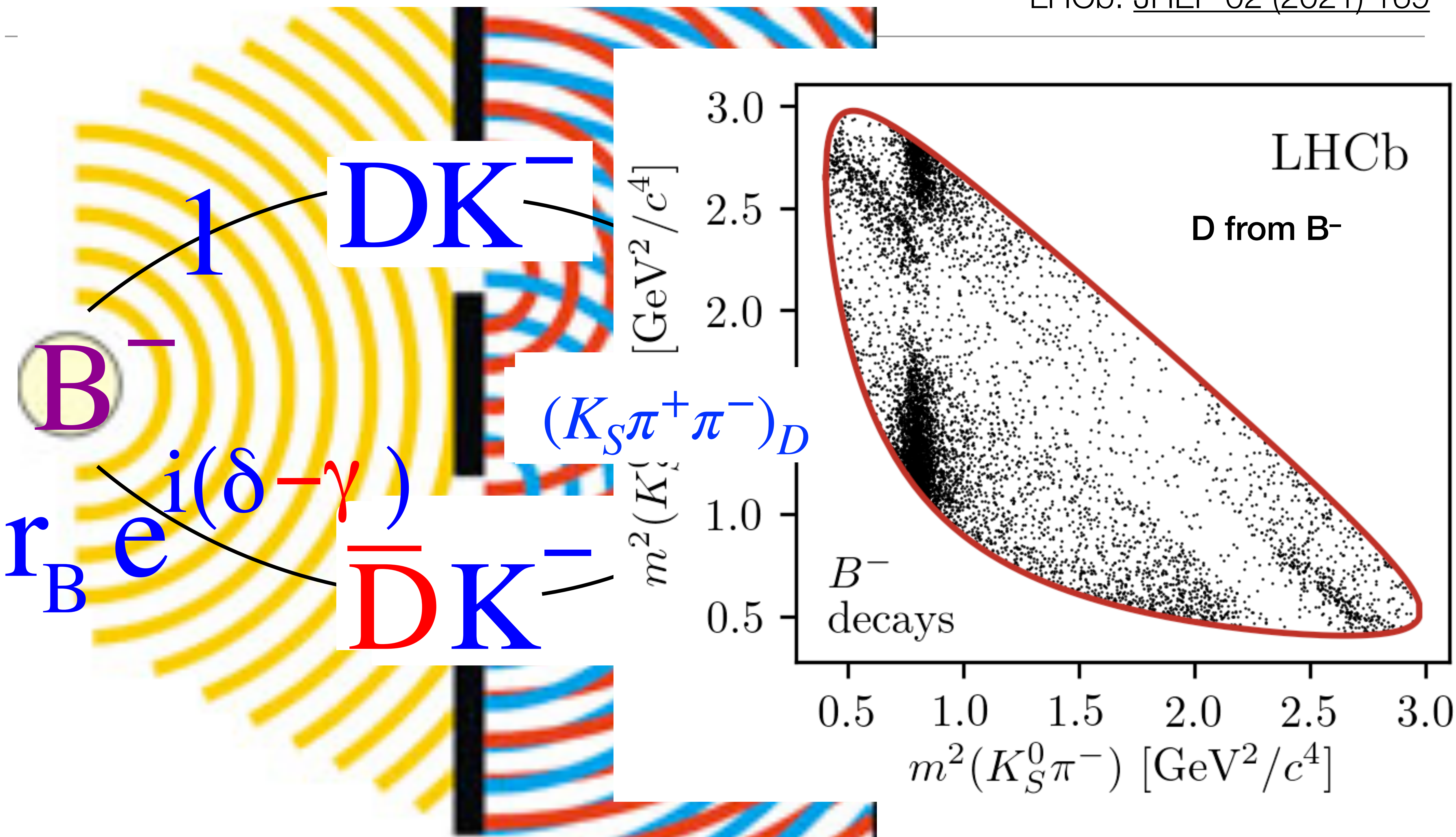

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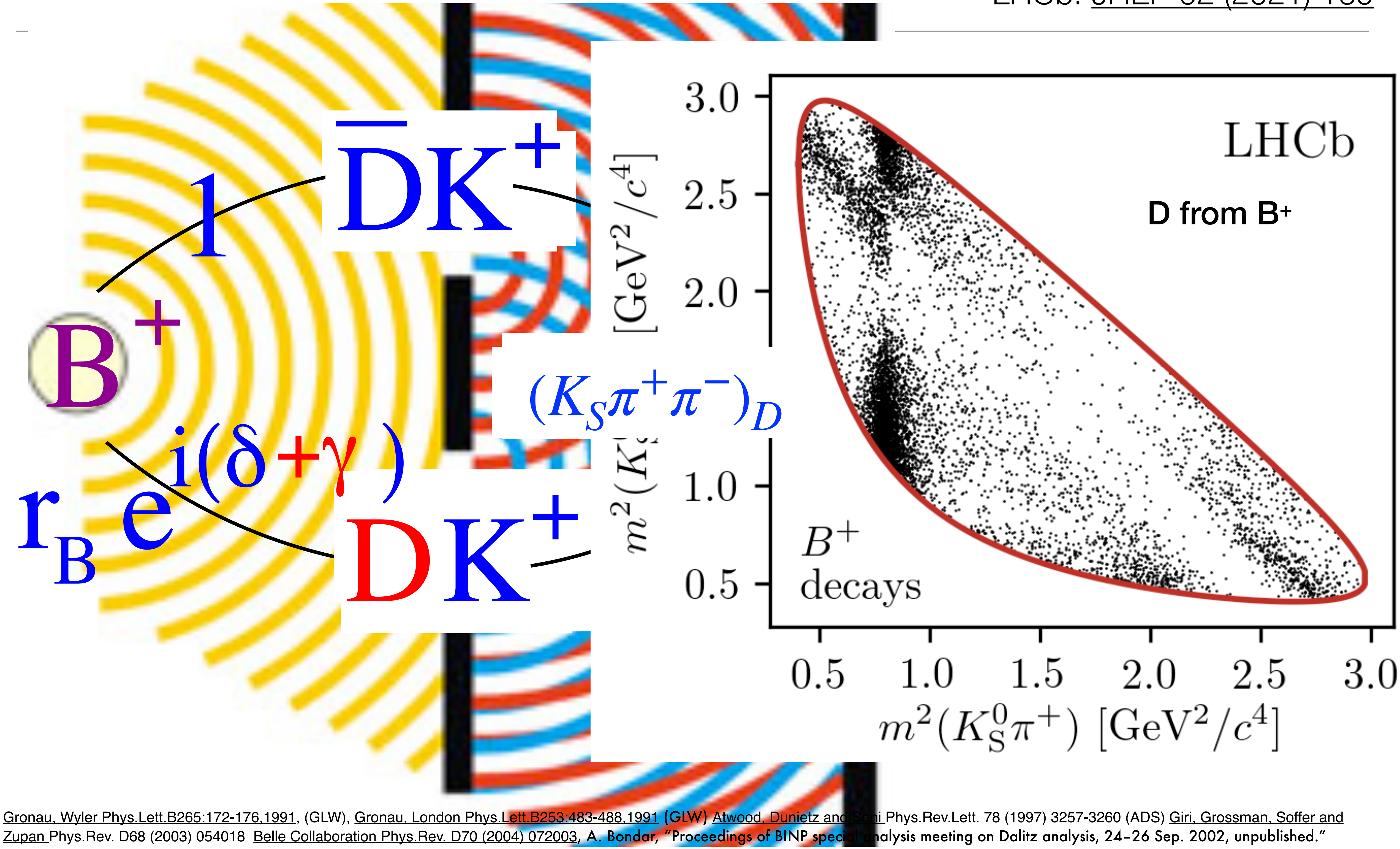

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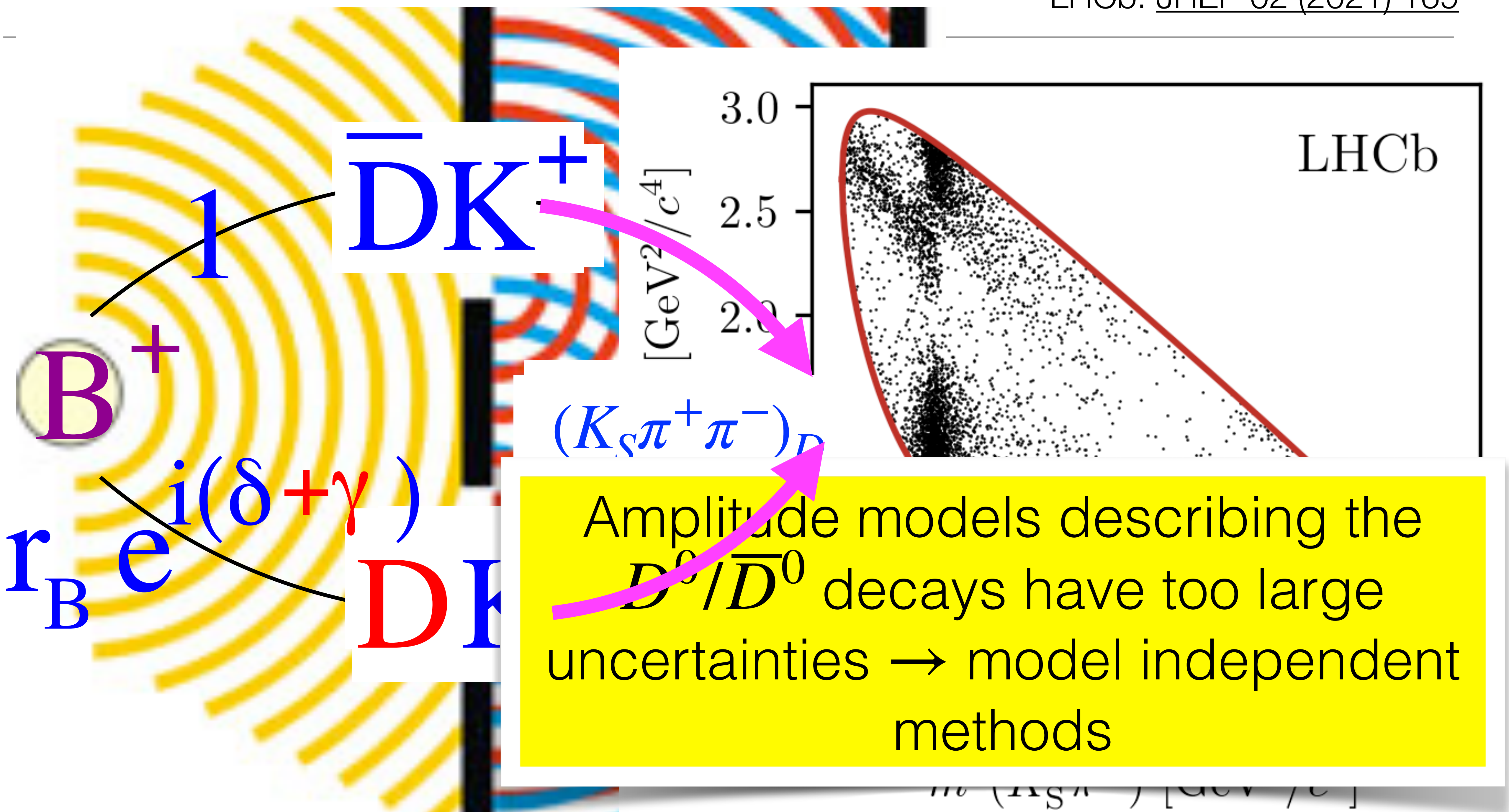

Amplitude models describing the $D^0/\overline{D}^0$ decays have too large uncertainties → model independent methods

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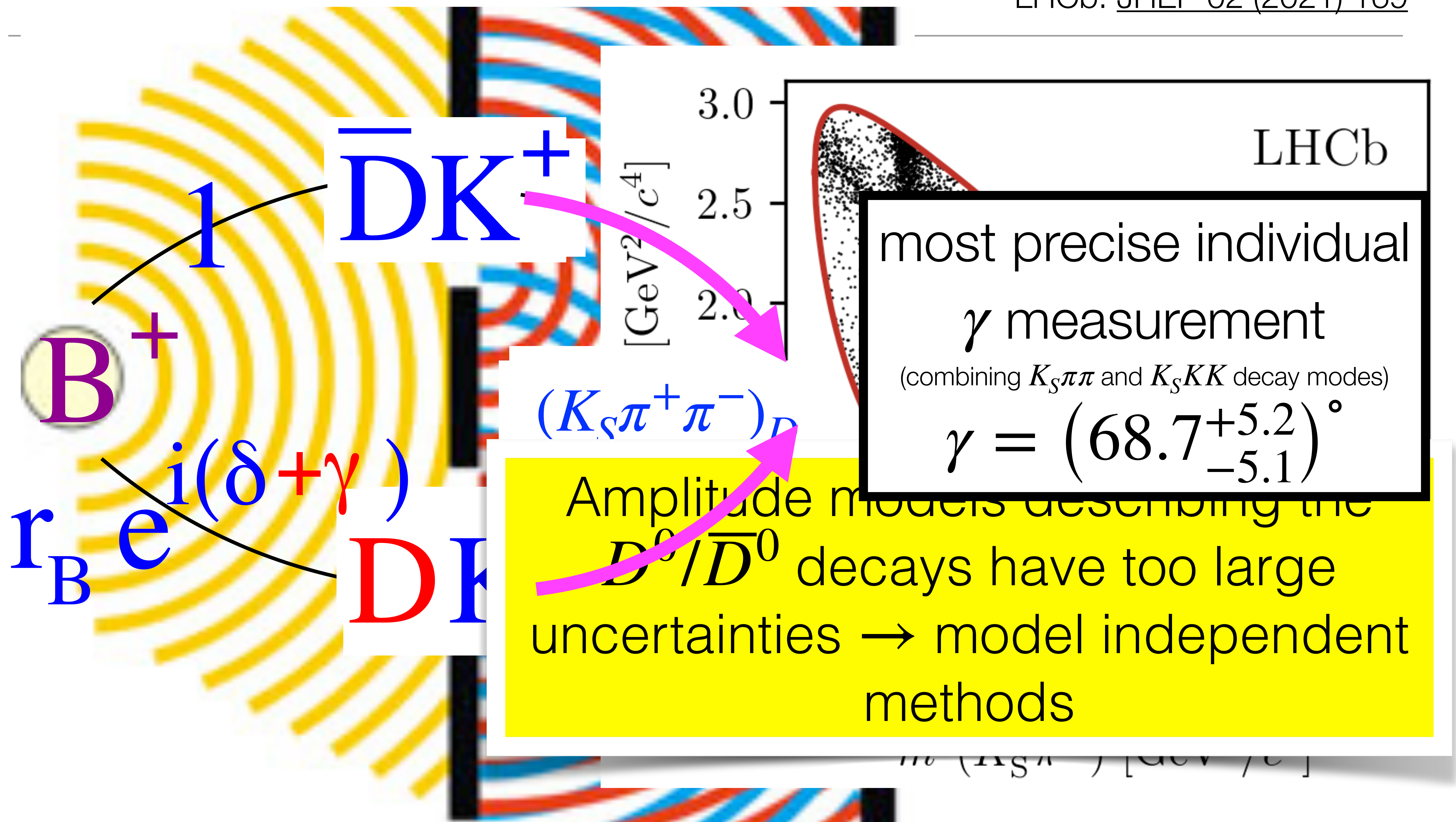

most precise individual γ measurement

(combining $K_S\pi\pi$ and K_SKK decay modes)

$$\gamma = \left(68.7^{+5.2}_{-5.1}\right)^\circ$$

Amplitude models describing the $D^0/\overline{D}^0$ decays have too large uncertainties → model independent methods

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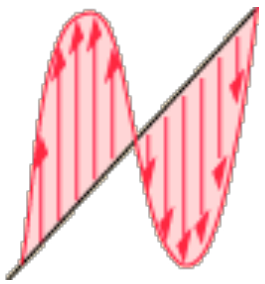
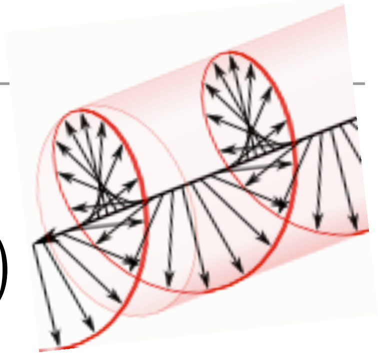
CLEO-c/BES III's unique data provide additional information on D mesons.

analogous to linear and circular polarised light

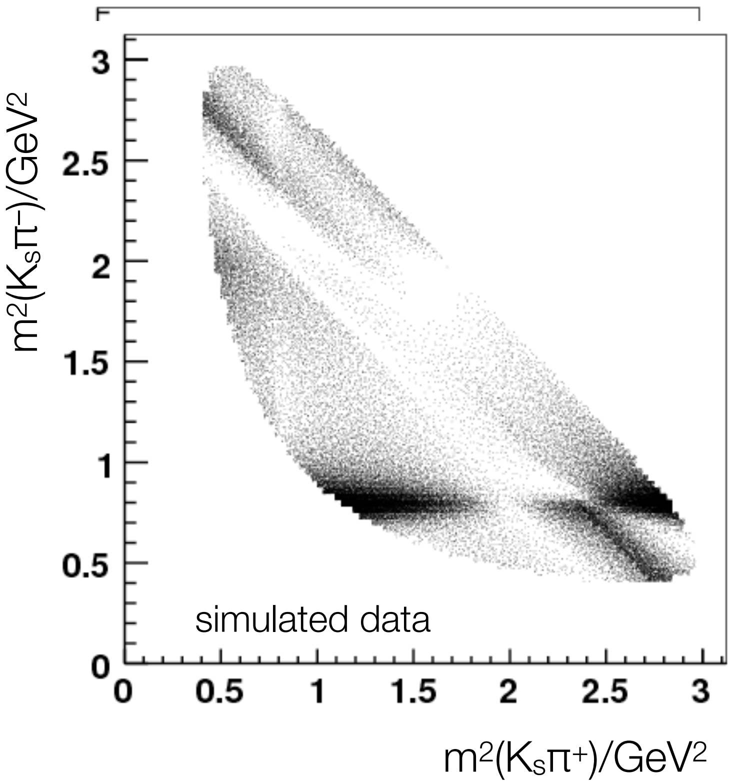
$|D\rangle$

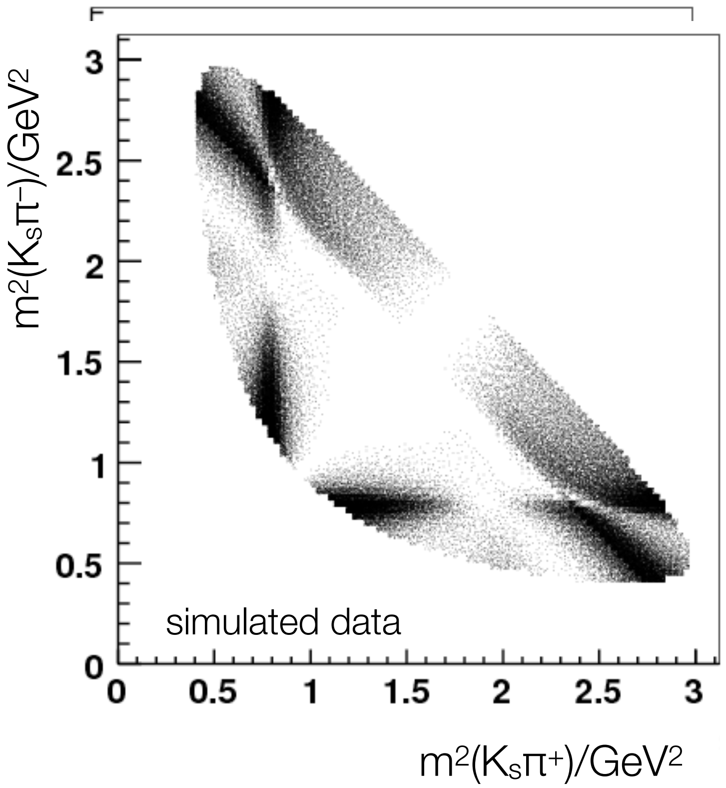


$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |\overline{D}\rangle\right)$$



$\rightarrow K_s\pi^+\pi^-$ $\rightarrow K_s\pi^+\pi^-$



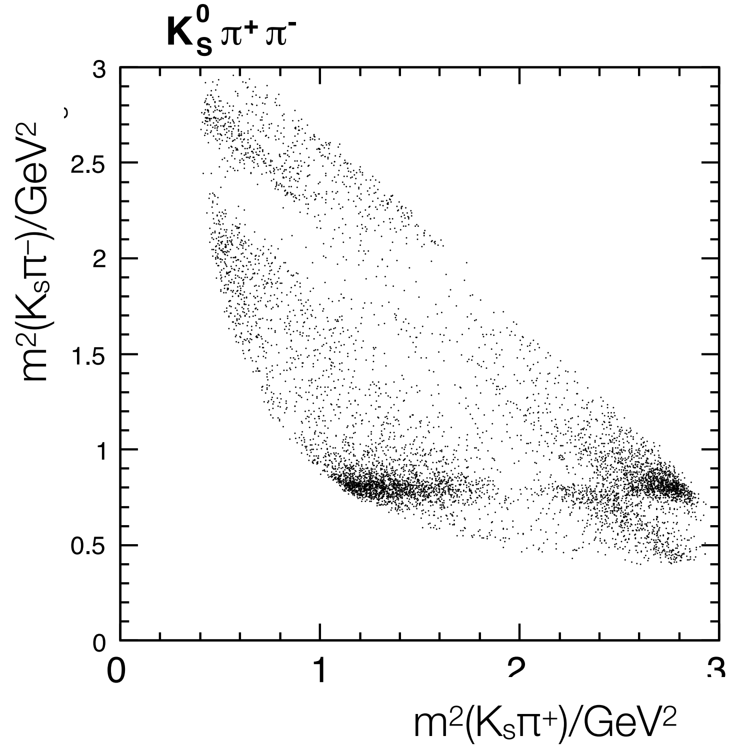


Two measurements for each point in Dalitz space - can extract magnitude and phase!

CLEO-c/BES III's unique data provide additional information on D mesons.

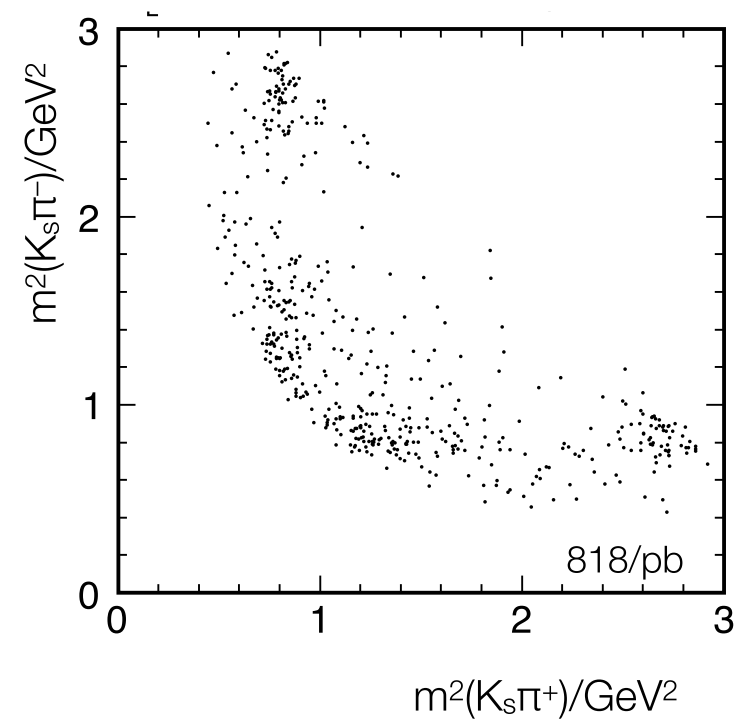
$$|D\rangle$$

$$\rightarrow K_s \pi^+ \pi^-$$



$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}} \left(|D\rangle - |\overline{D}\rangle \right)$$

$$\rightarrow K_s \pi^+ \pi^-$$



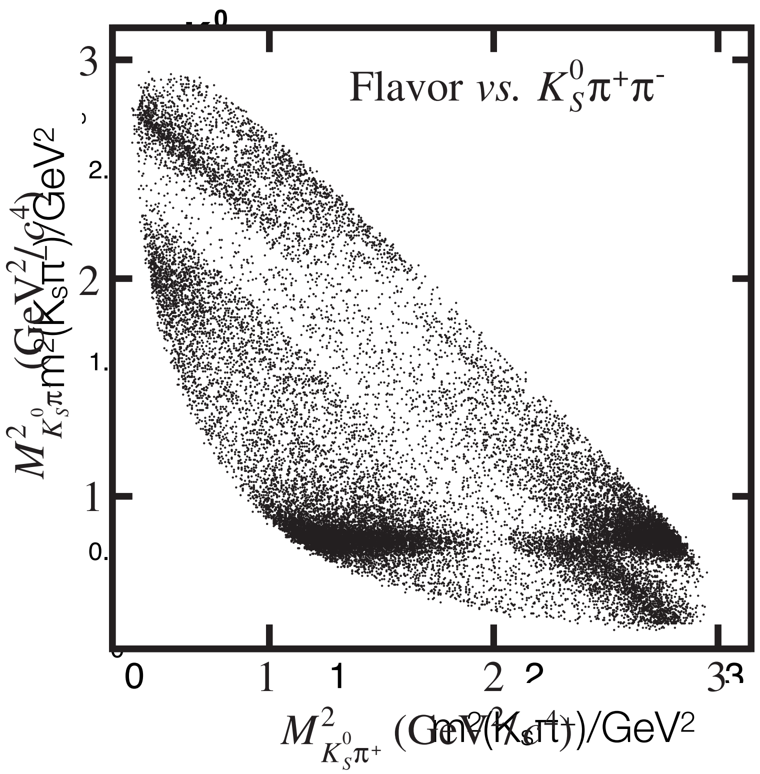
Two measurements for each point in Dalitz space - can extract magnitude and phase!

BESIII: PRL 124 (2020) 24, 241802

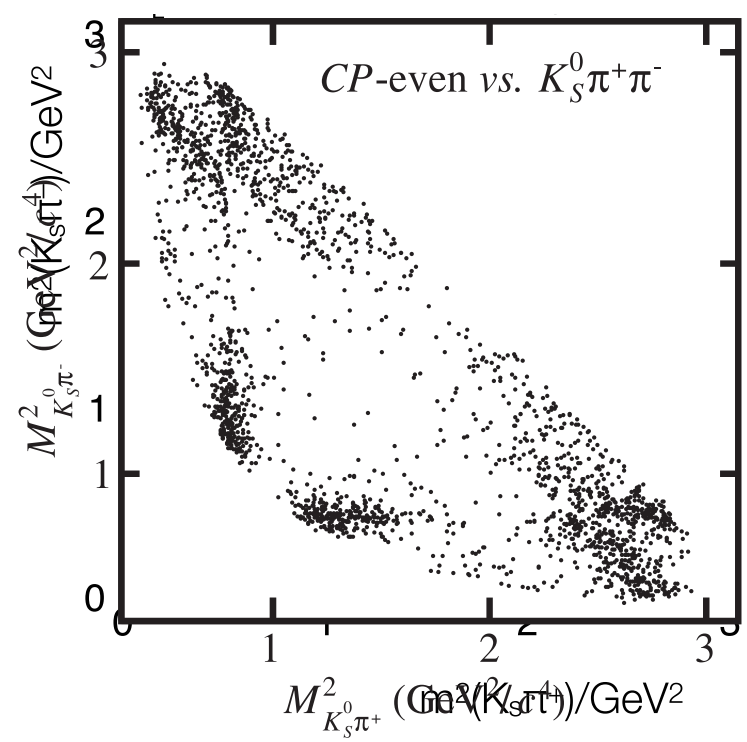
CLEO-c: Phys. Rev. D80, 032002 (2009), updated in Phys.Rev. D82 (2010) 112006

CLEO-c/BES III's unique data provide additional information on D mesons.

$$|D\rangle \rightarrow K_s\pi^+\pi^-$$



$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |\overline{D}\rangle\right) \rightarrow K_s\pi^+\pi^-$$

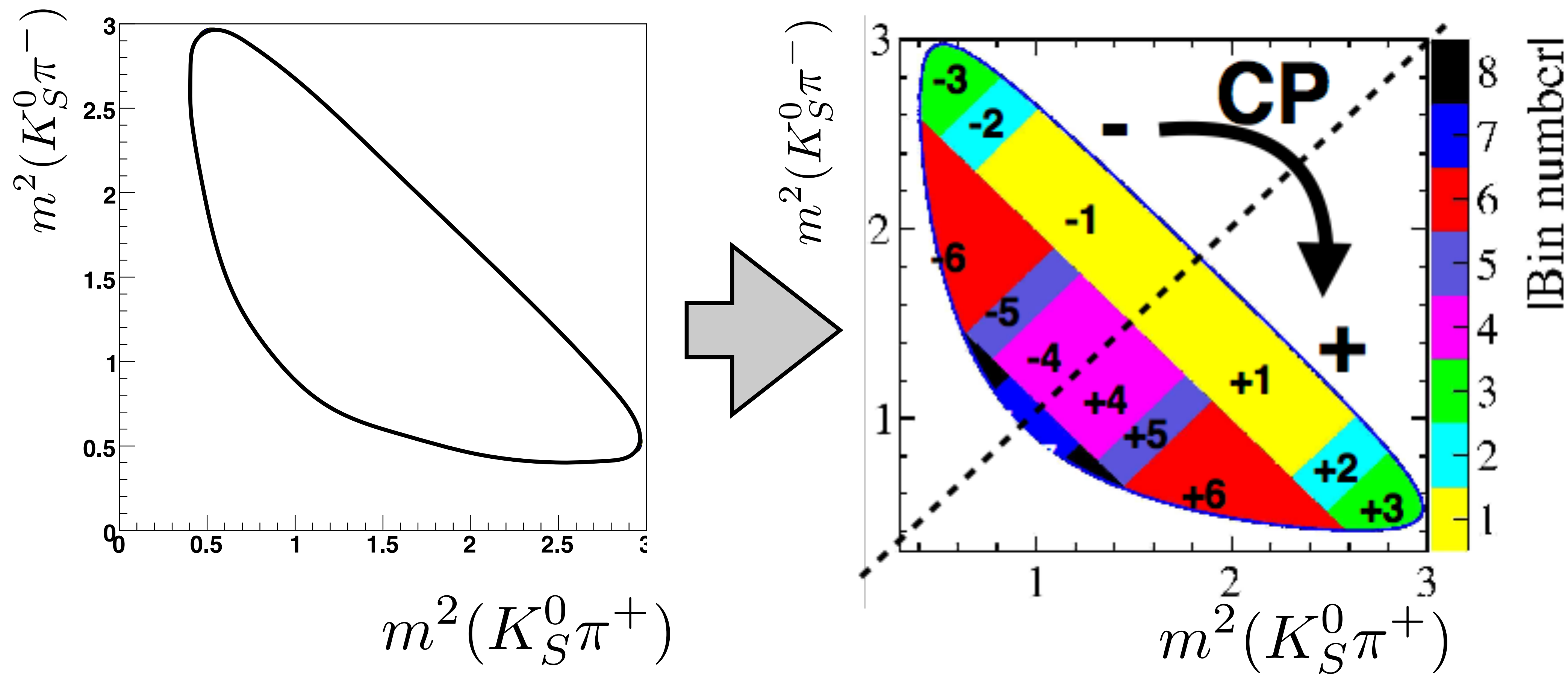


Two measurements for each point in Dalitz space - can extract magnitude and phase!

BESIII: PRL 124 (2020) 24, 241802

CLEO-c: Phys. Rev. D80, 032002 (2009), updated in Phys.Rev. D82 (2010) 112006

Model-independent, binned approach

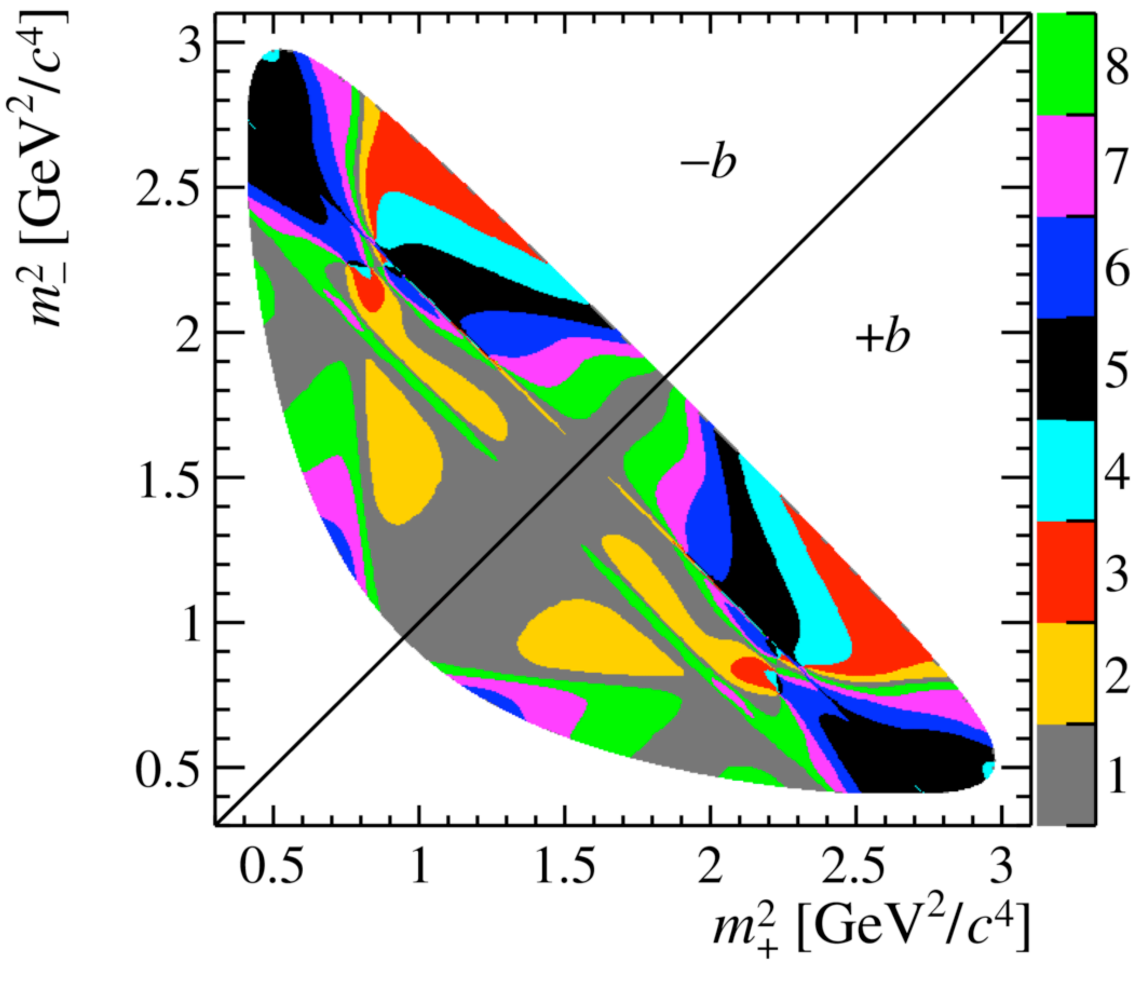


One complex number per bin-pair, $c_i + \mathrm{i}s_i$, contains the key information uniquely accessible at CLEO-c/BES III, which is related to the phases the D^0 and $\overline{D}^0$ decay amplitudes.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Measurements of c_i, s_i at BES III

Model-informed, optimised binning



Measurements of c_i, s_i at BES III

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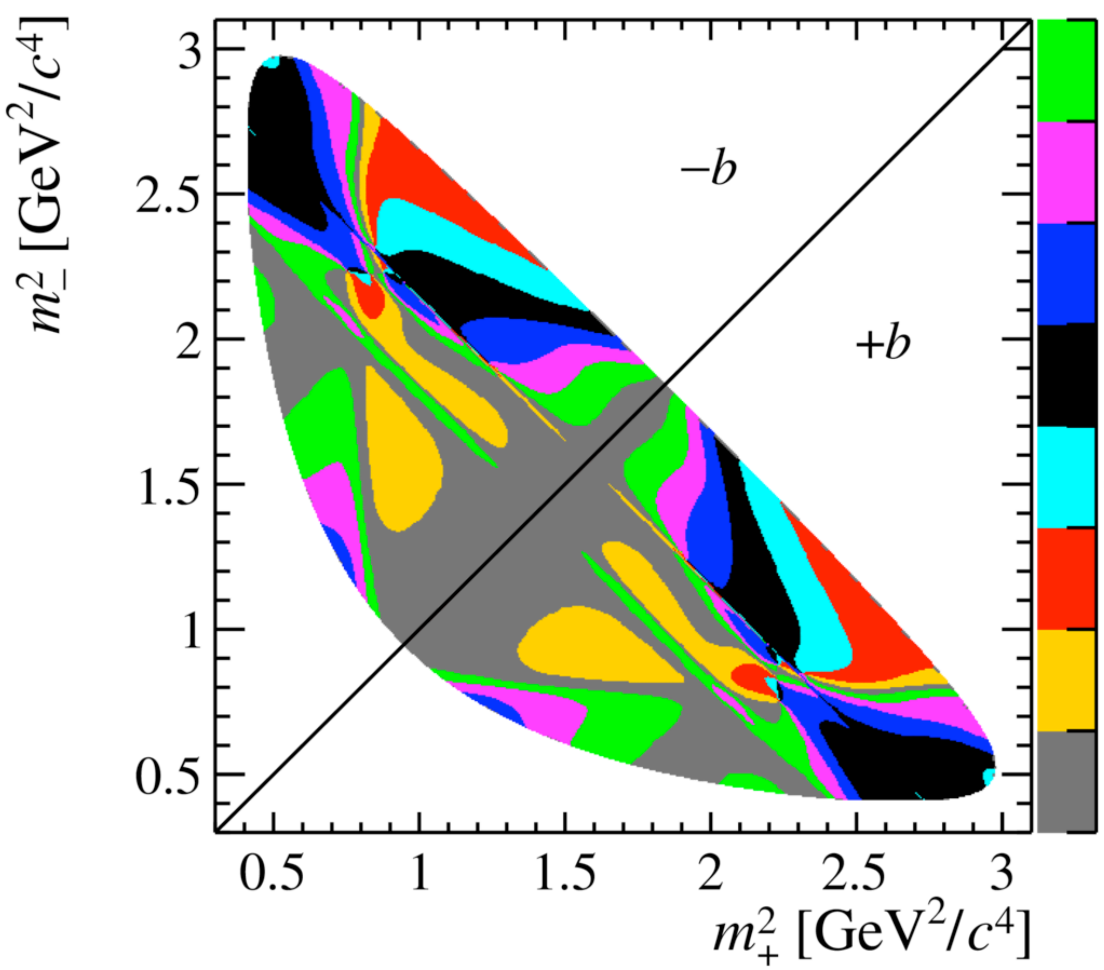
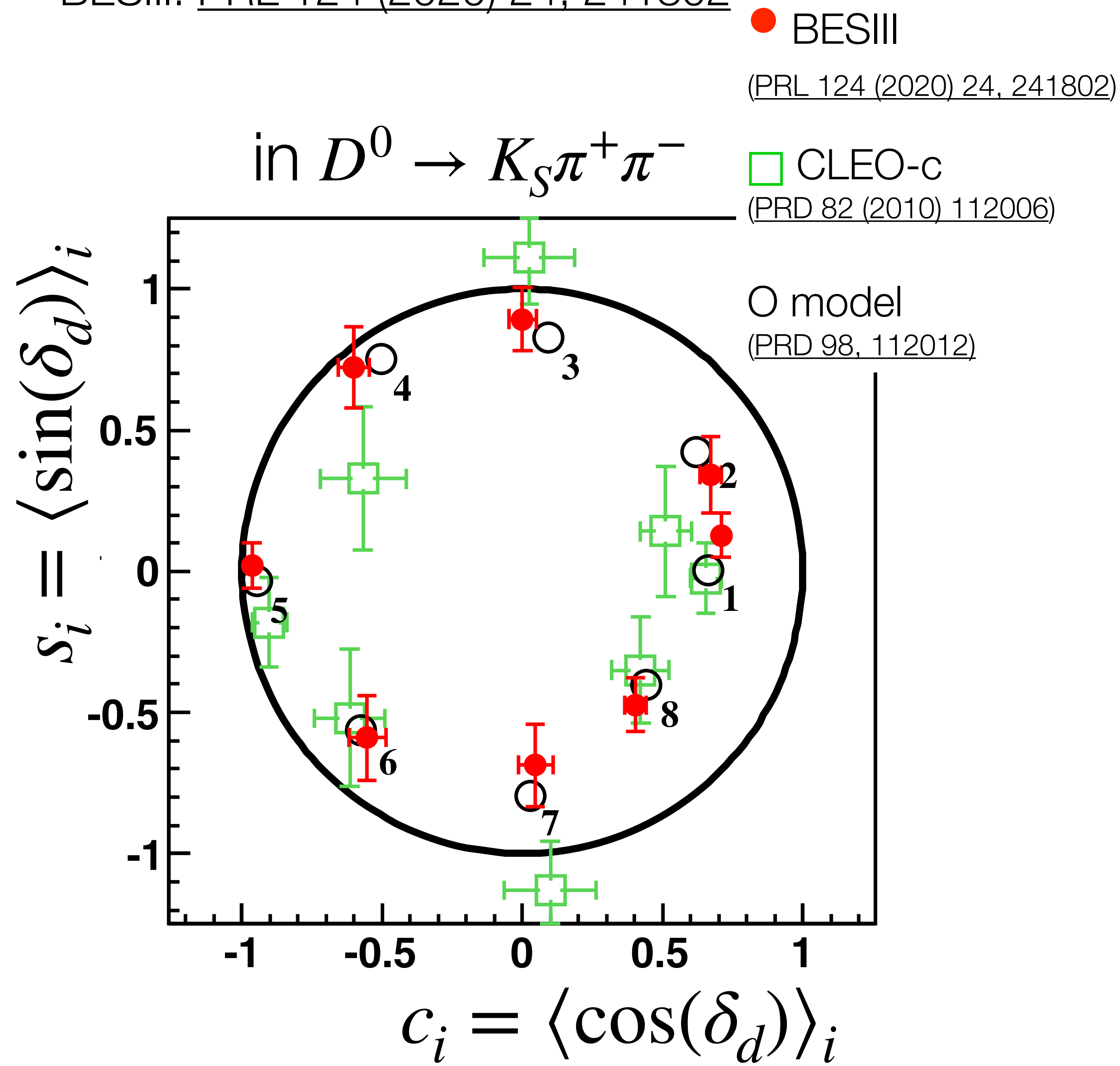


BESIII: PRL 124 (2020) 24, 241802

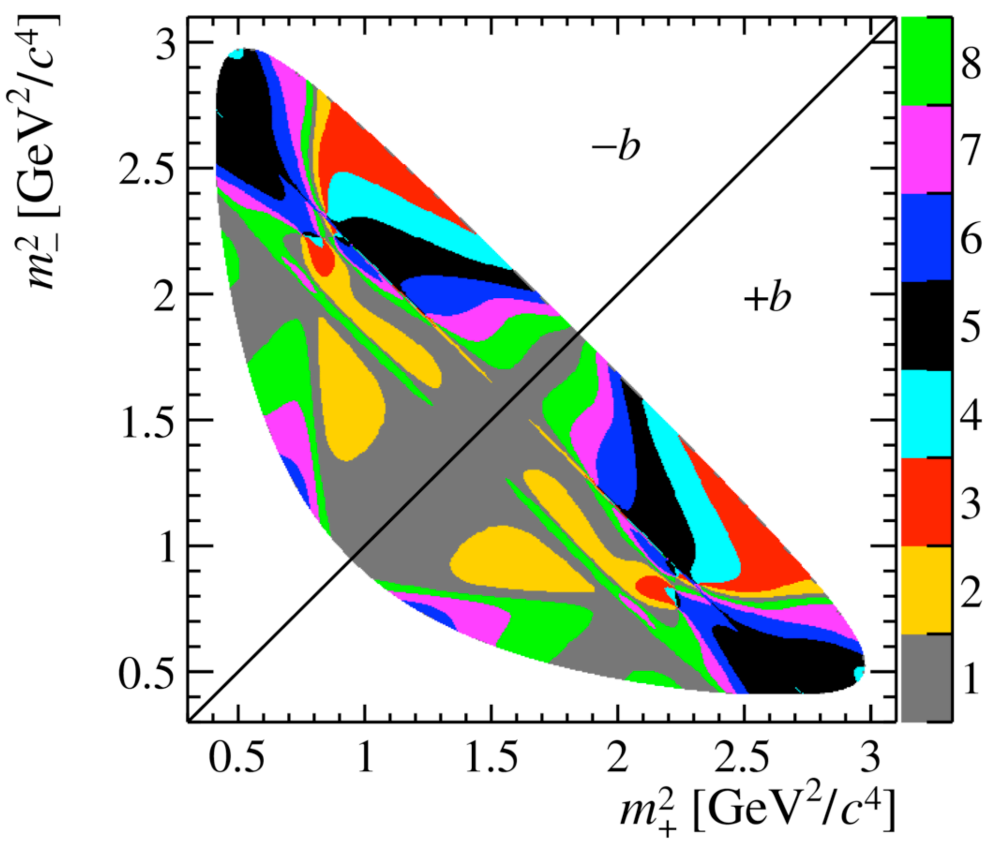
in $D^0 \to K_S\pi^+\pi^-$



New unbinned method

JHEP 09 (2023) 007

Carefully optimised binning



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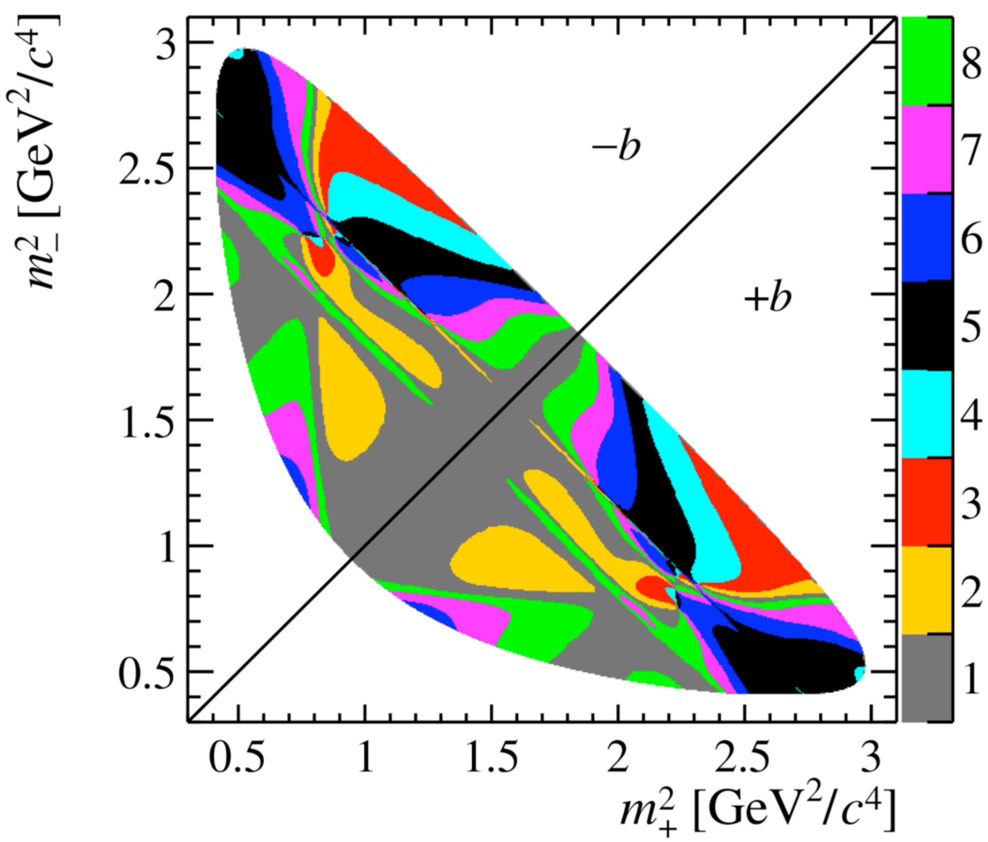

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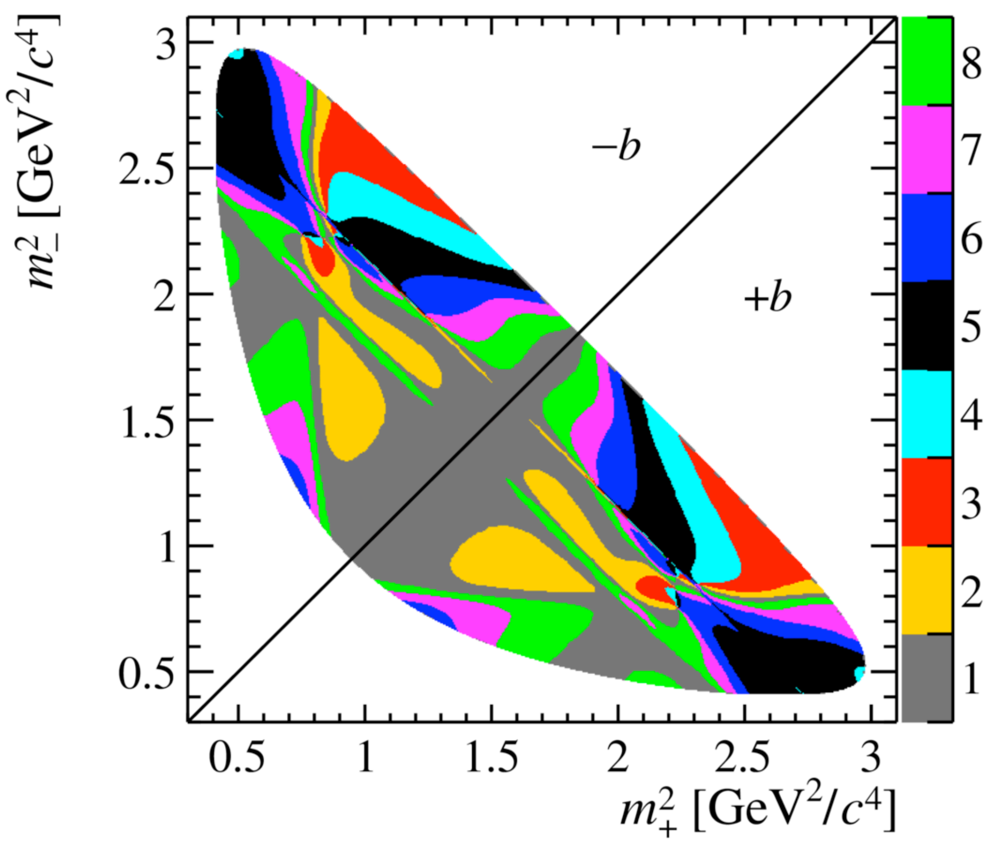
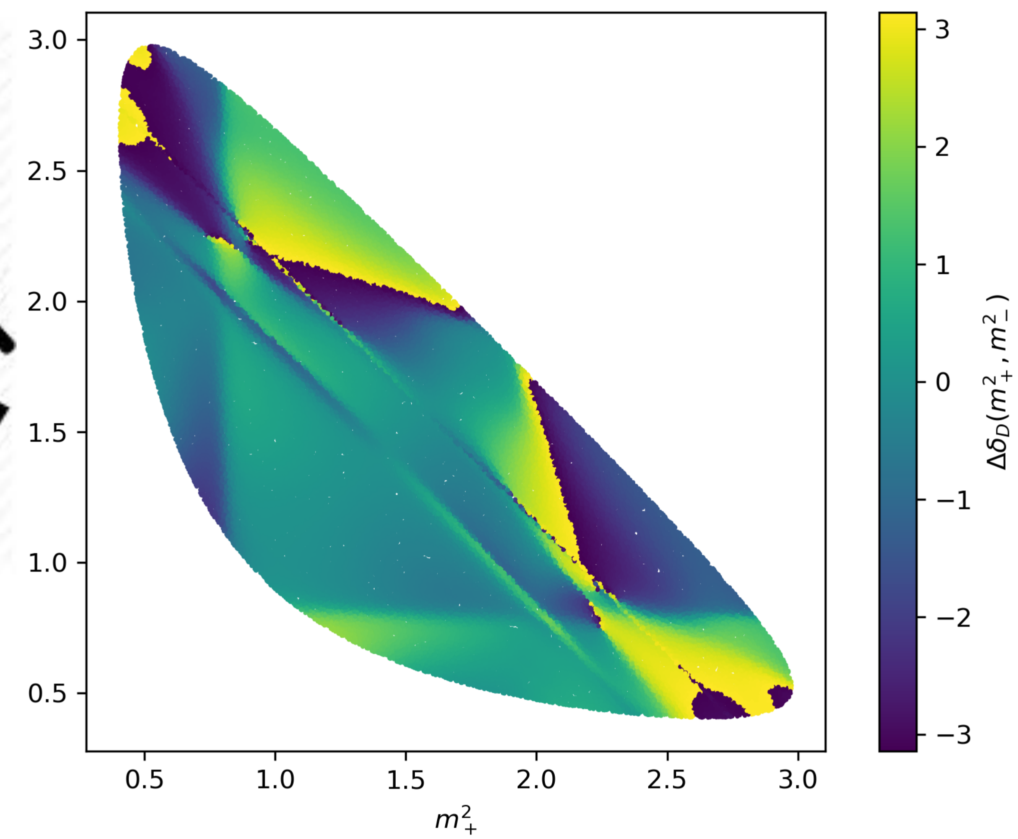


New, unbinned model-independent method



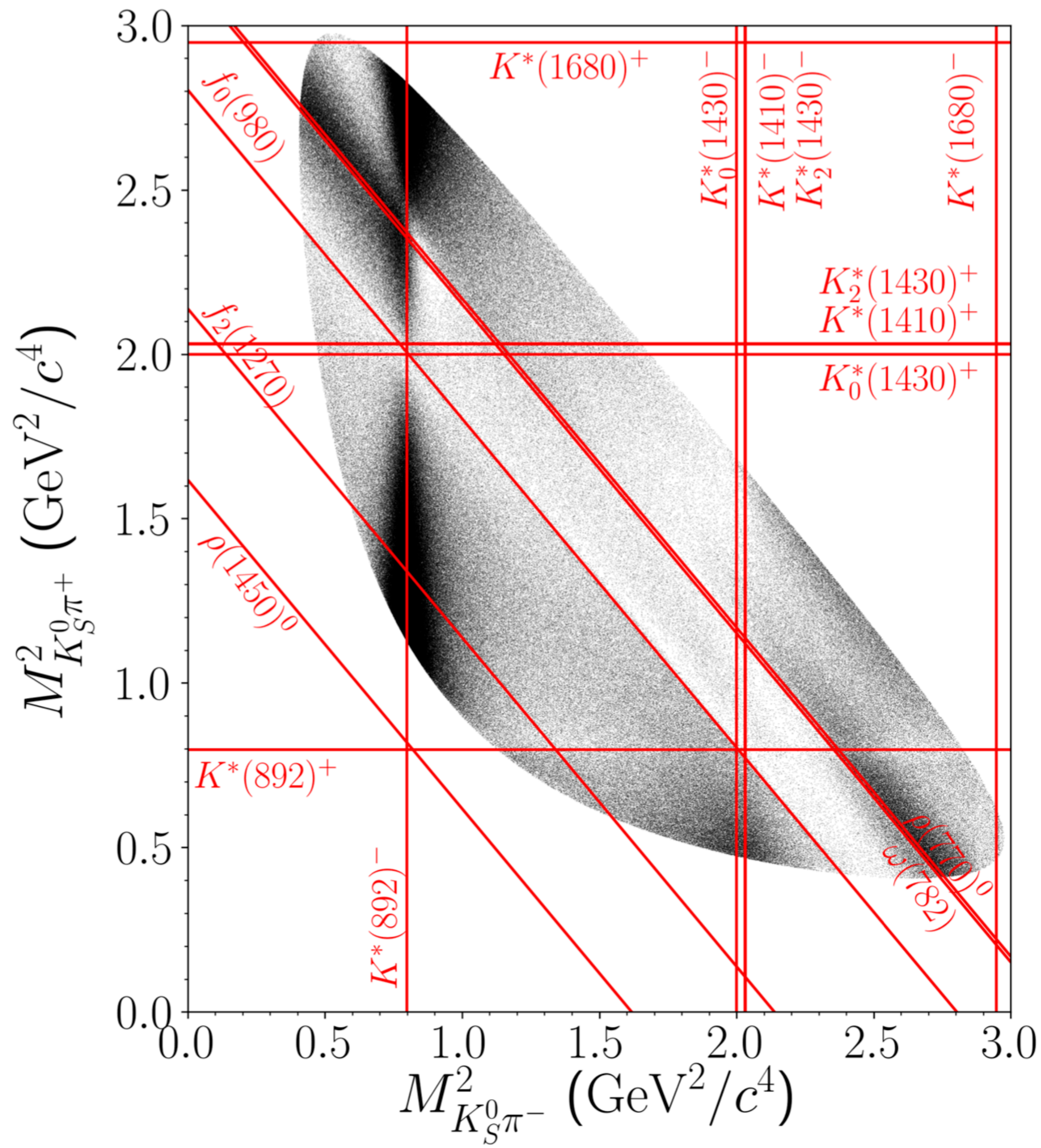
Other unbinned methods exist: Poluektov, Eur.Phys.J.C 78 (2018) 2, 121; Backus et al, arXiv:2211.05133. In contrast to these and the binned method, we do not do any integration, averaging or projection from 2D to 1D, and therefore do not suffer the associated information loss.

BaBar & BELLE $D^0 \to K_S\pi^+\pi^-$ amplitude analysis

PRD 98 (2018) 11, 112012

Best statistical precision on γ is achieved with an unbinned model-*dependent* method. So let's have a look at those models.

1.2M signal events (BELLE)
94% signal purity



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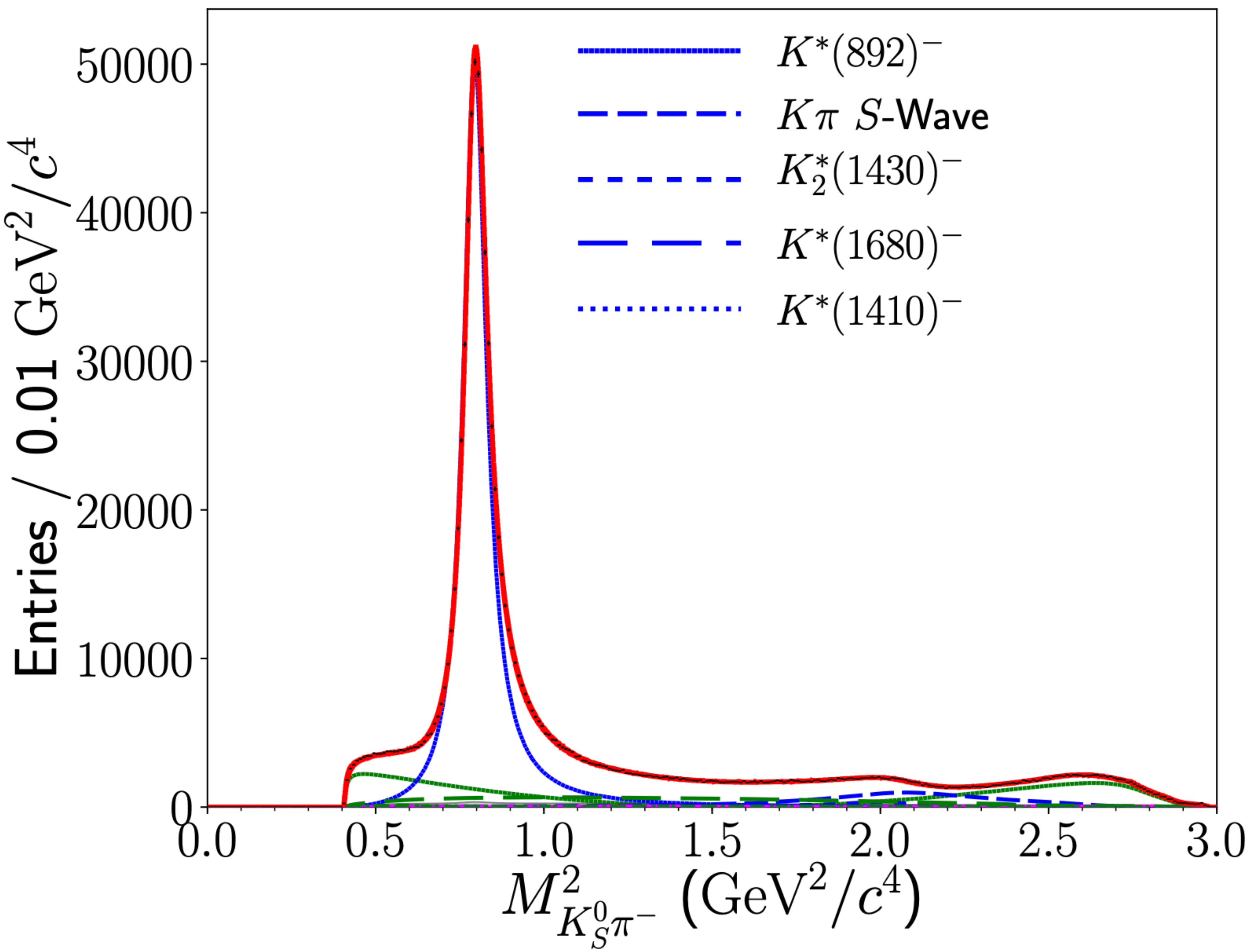
PRD 98 (2018) 11, 112012



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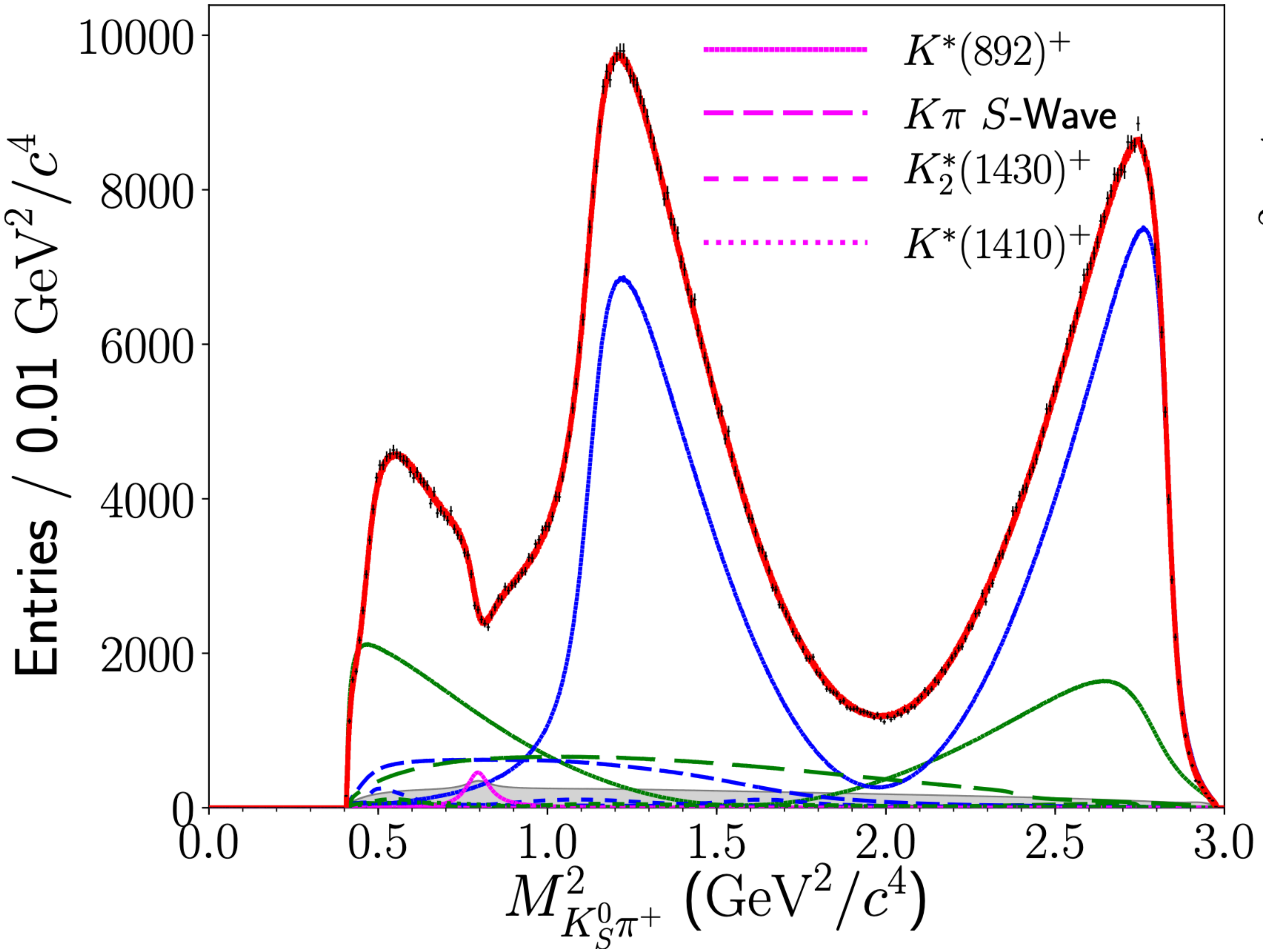

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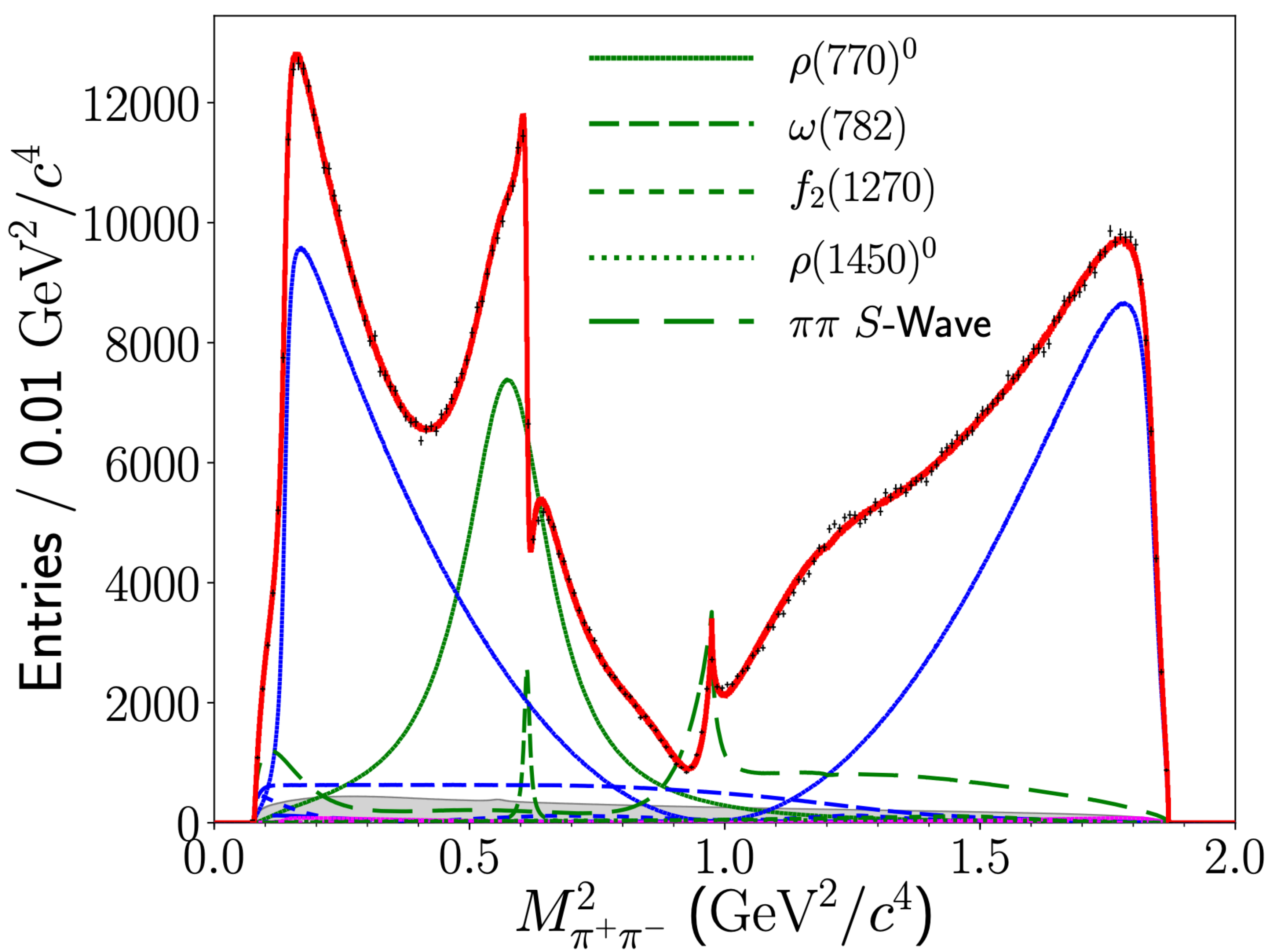
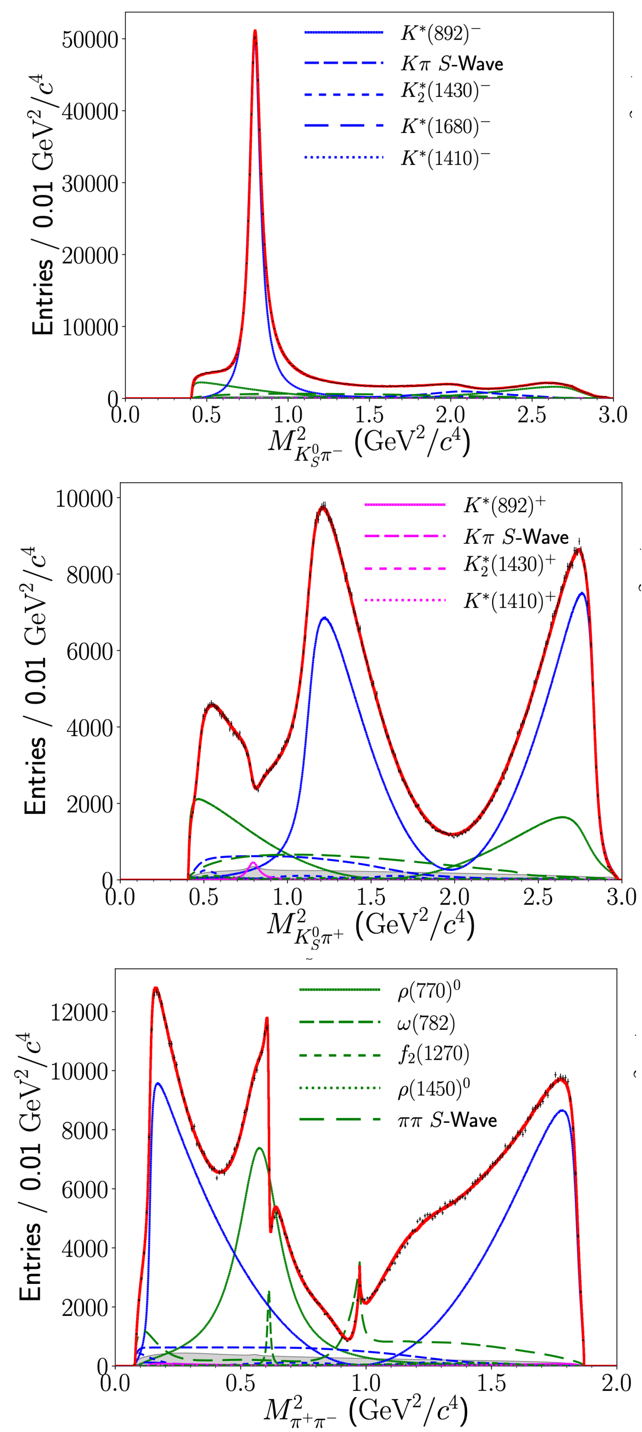
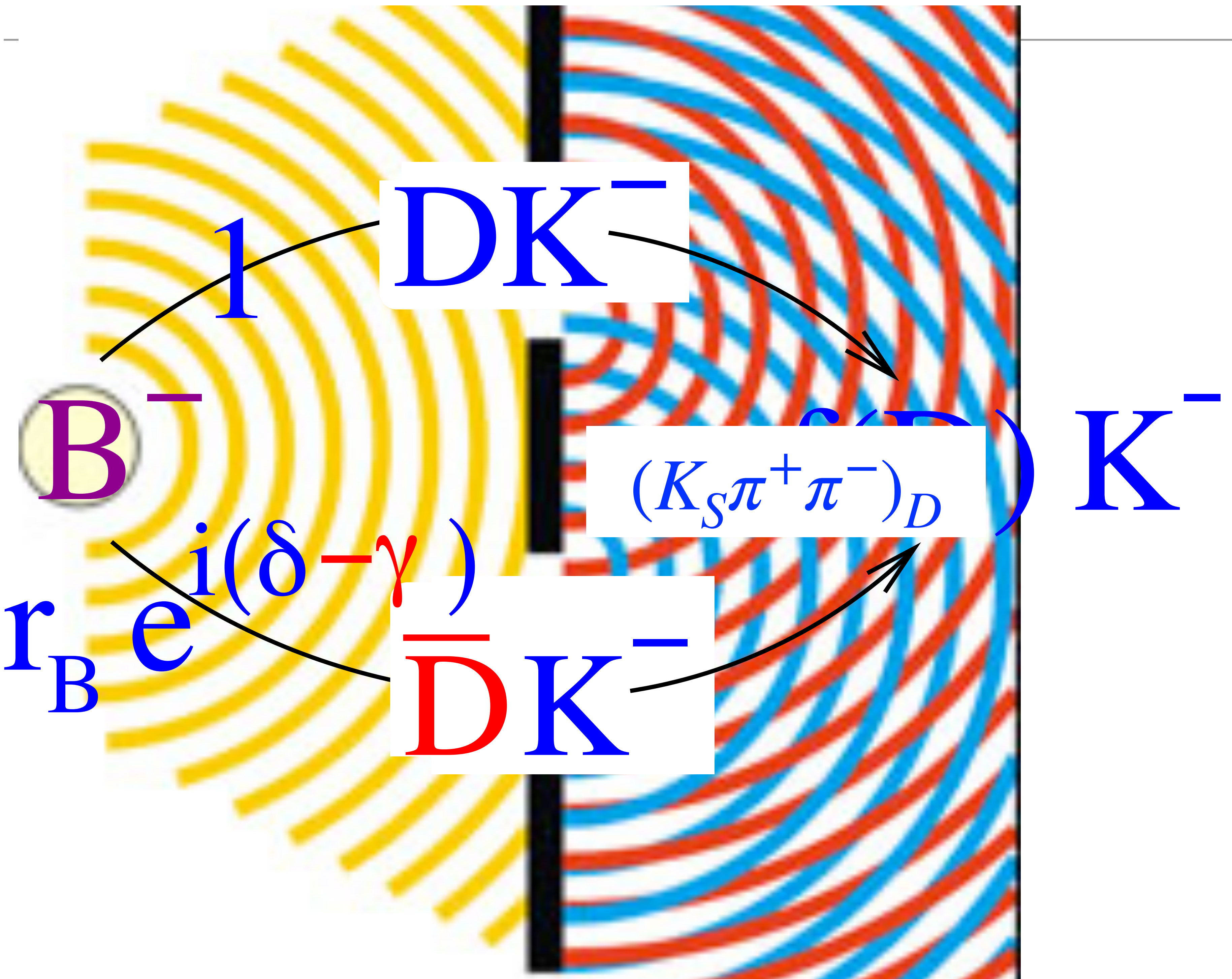


Model-independent phase correction

- The magnitudes of amplitude models are OK and can be verified on data.
- Violation of unitarity and analyticity in models destroys link between magnitude and phase - the models' phases are uncertain.
- Idea: Keep models' magnitudes, but correct phases in model-independent way.



CP violation is an interference effect



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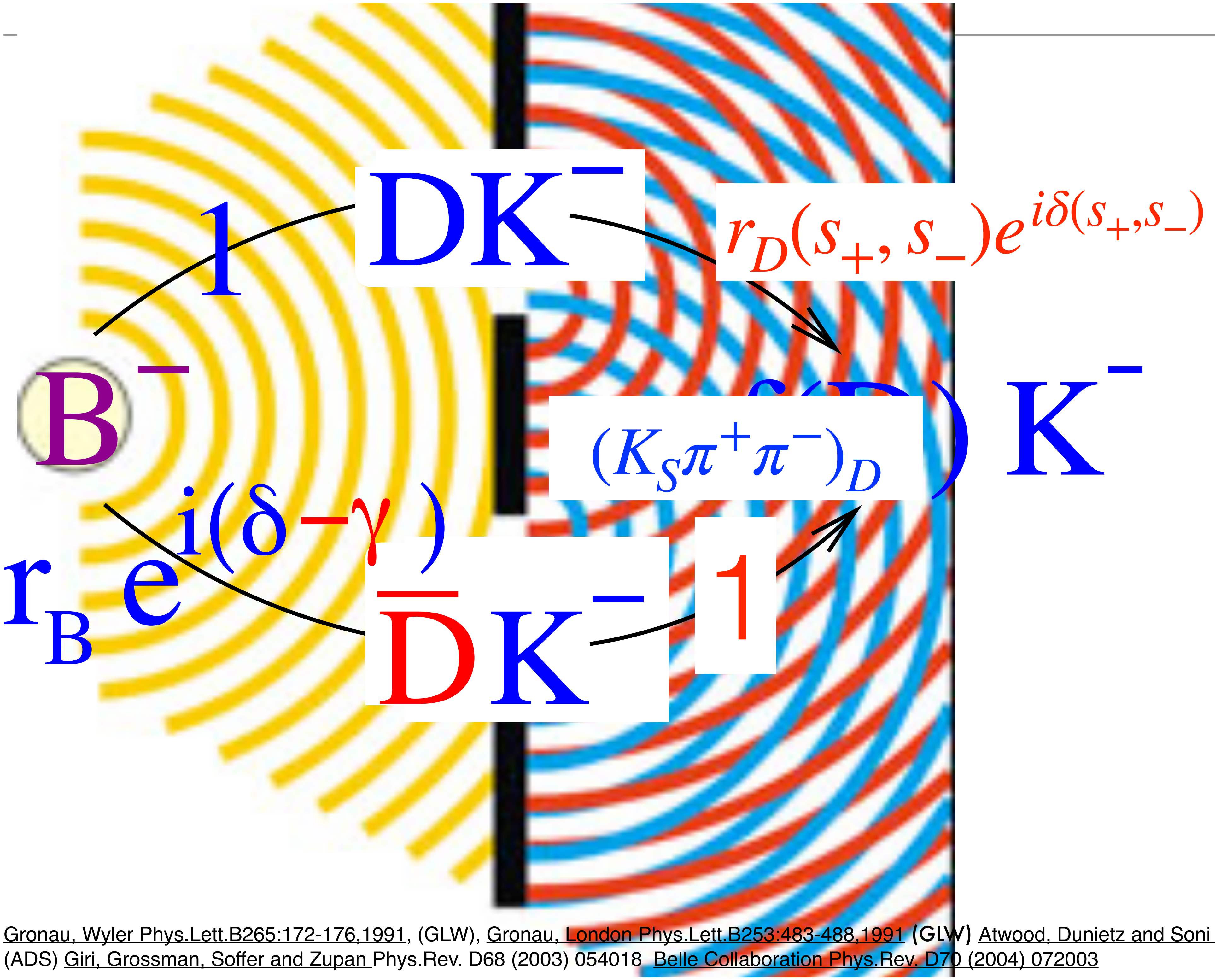

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Correct phase differences

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In all relevant decay rates, at the charm threshold and in B decays, phases enter through interference terms between D^0 and $\overline{D}^0$ decay amplitudes, e.g. for D^0 from B^-

$$\Gamma^-(s_+, s_-) \propto r_D^2(s_+, s_-) + r_B^2 + 2r_D(s_+, s_-)r_B \cos\left(\delta_B - \gamma - \delta_D(s_+, s_-)\right)$$

Or CP-tagged BESIII charm decays:

$$\Gamma^{CP}(s_+, s_-) \propto r_D^2(s_+, s_-) + 1 \pm 2r_D(s_+, s_-)\cos\left(\delta_D(s_+, s_-)\right)$$

We correct this term, the phase difference of the D^0 and $\overline{D}^0$ decay amplitudes to the same phase space point:

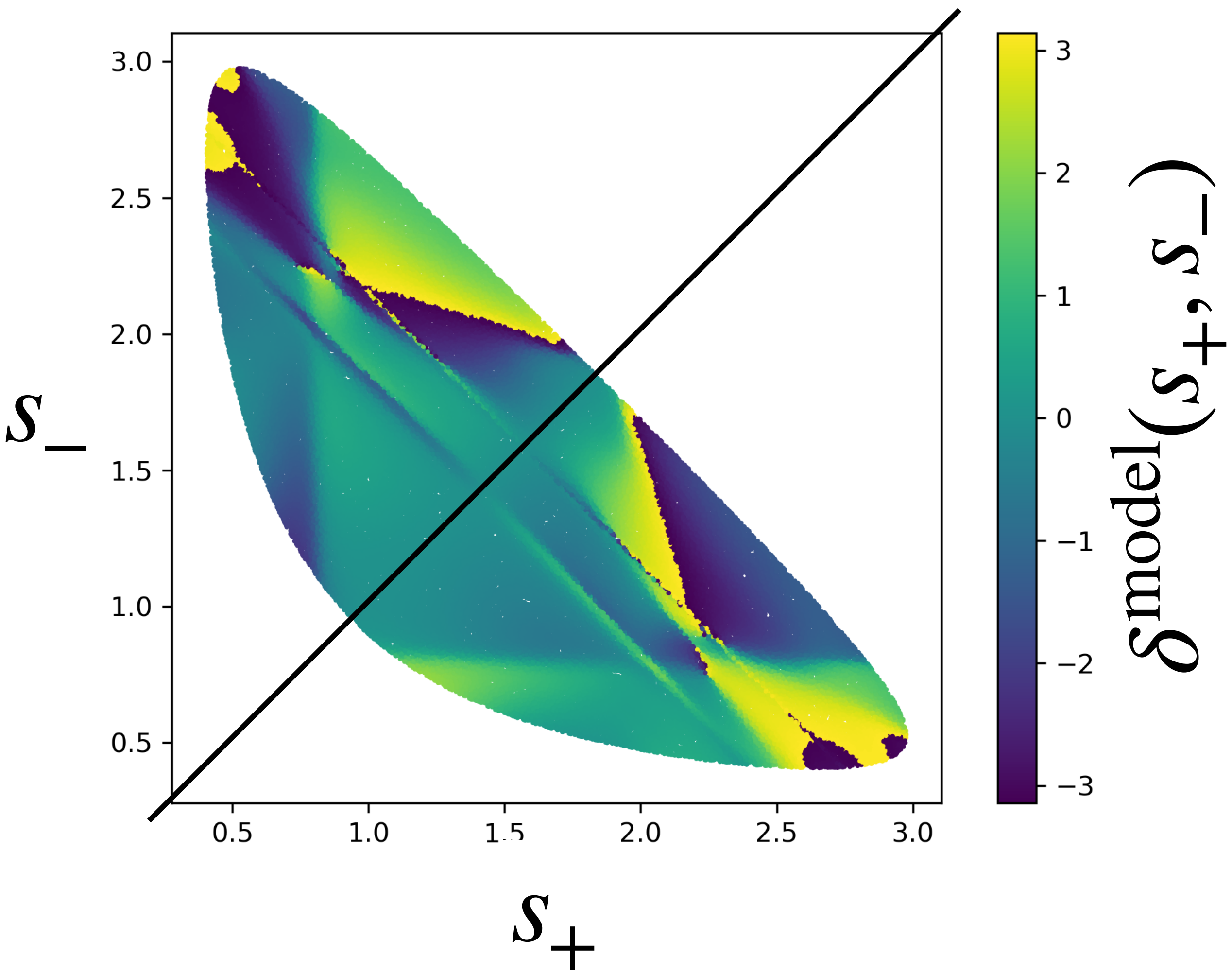
$$\delta_D = \delta_D^{\text{model}} + \delta_D^{\text{corr}}$$

Idea: Generic parametrisation of deviation of phase from model-prediction

JHEP 09 (2023) 007

$$\delta(s_+, s_-) = \delta^{\text{model}}(s_+, s_-) + \delta^{\text{corr}}(s_+, s_-)$$

$\delta^{\text{corr}}(s_+, s_-)$ = polynomial in s_+, s_-, determined in simultaneous fit to $B^\pm \to DK^\pm$ and $\psi(3770) \to D\overline{D}$ data



Symmetry:

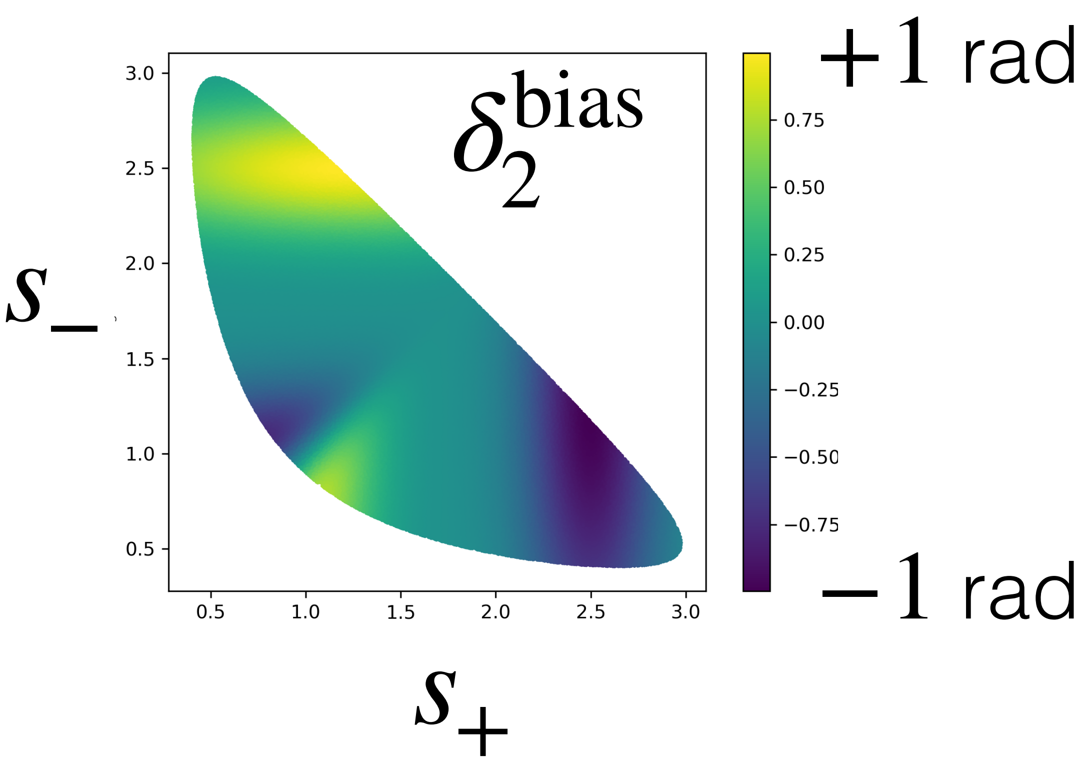
$$\delta(s_+, s_-) = -\delta(s_-, s_+)$$

$$\delta^{\text{corr}}(s_+, s_-) = -\delta^{\text{corr}}(s_-, s_+)$$

Testing the method

JHEP 09 (2023) 007

Generate Babar & BELLE amplitude model with modified phase difference $\delta(s_+, s_-)$



This is a sum of two gaussian bias functions

Fit starts from un-modified model - will it be able to find $\delta^{\text{corr}} \approx \delta_2^{\text{bias}}$ to a sufficient approximation?

Single fit, different-order correction polynomials

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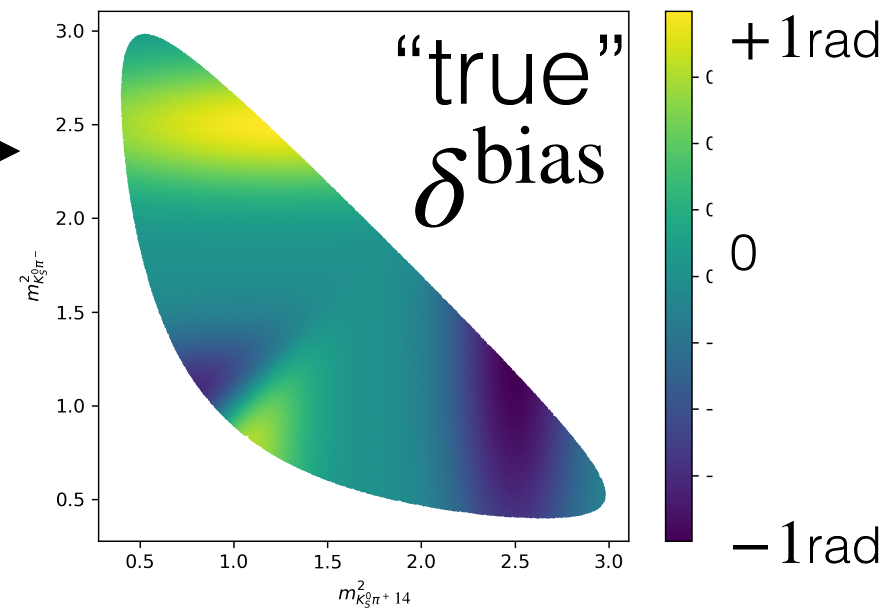
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



Order	$\Delta x_+ \cdot 100$	$\Delta y_+ \cdot 100$	$\Delta x_- \cdot 100$	$\Delta y_- \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$

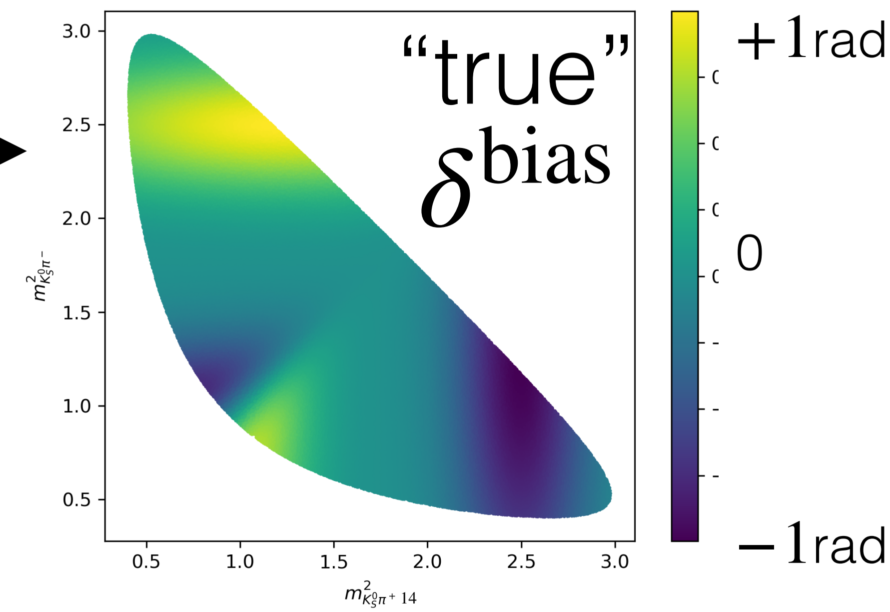
LHCb yields as in JHEP 02 (2021) 169
BESIII yields as in: PRL 124 (2020) 24, 241802, PRD 101 (2020) 11200

Single fit, different-order correction polynomials

JHEP 09 (2023) 007

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation



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1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
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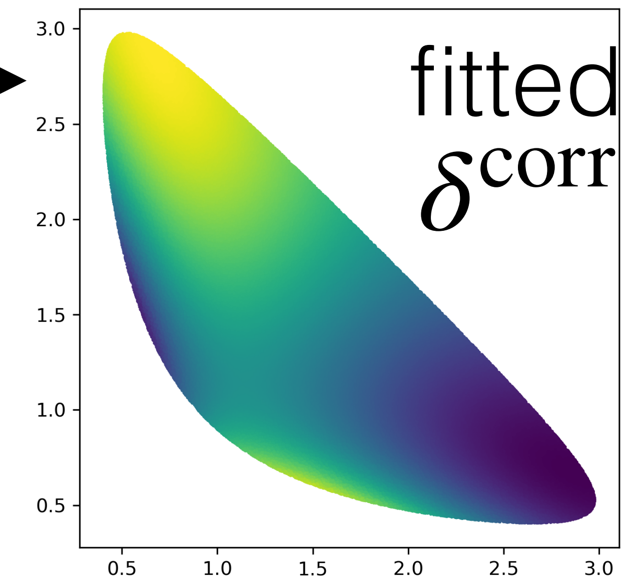


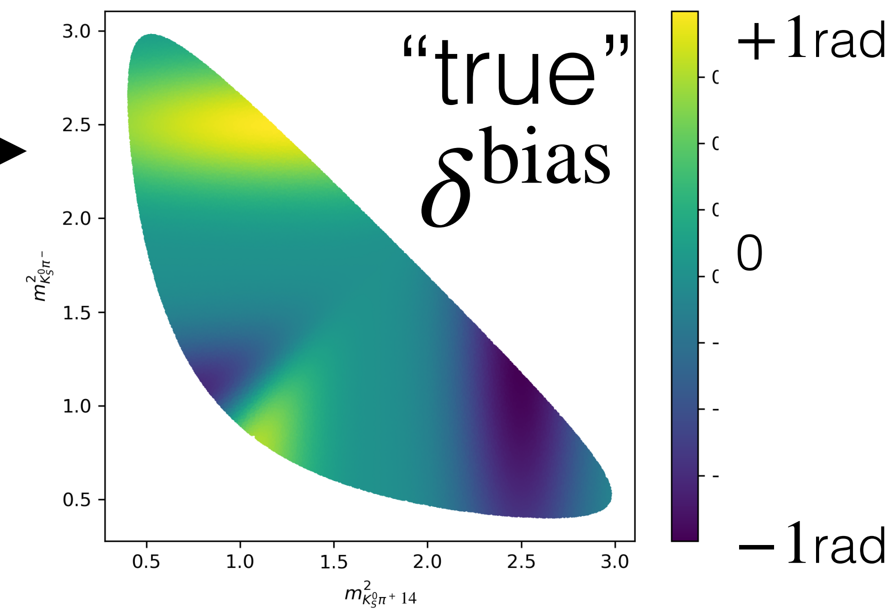
LHCb yields as in JHEP 02 (2021) 169
BESIII yields as in: PRL 124 (2020) 24, 241802, PRD 101 (2020) 11200

Single fit, different-order correction polynomials

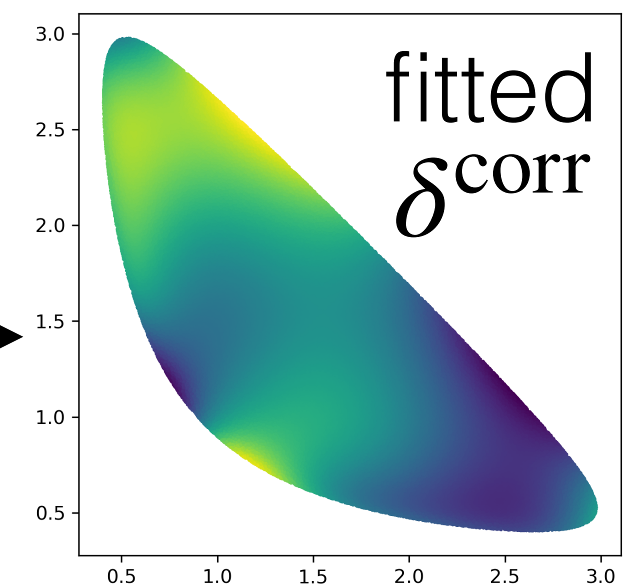
JHEP 09 (2023) 007

$$x_\pm = r_B \cos(\delta_B \pm \gamma)$$
$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



Order	$\Delta x_+ \cdot 100$	$\Delta y_+ \cdot 100$	$\Delta x_- \cdot 100$	$\Delta y_- \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$



LHCb yields as in JHEP 02 (2021) 169
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Single fit, different-order correction polynomials

[JHEP 09 (2023) 007]

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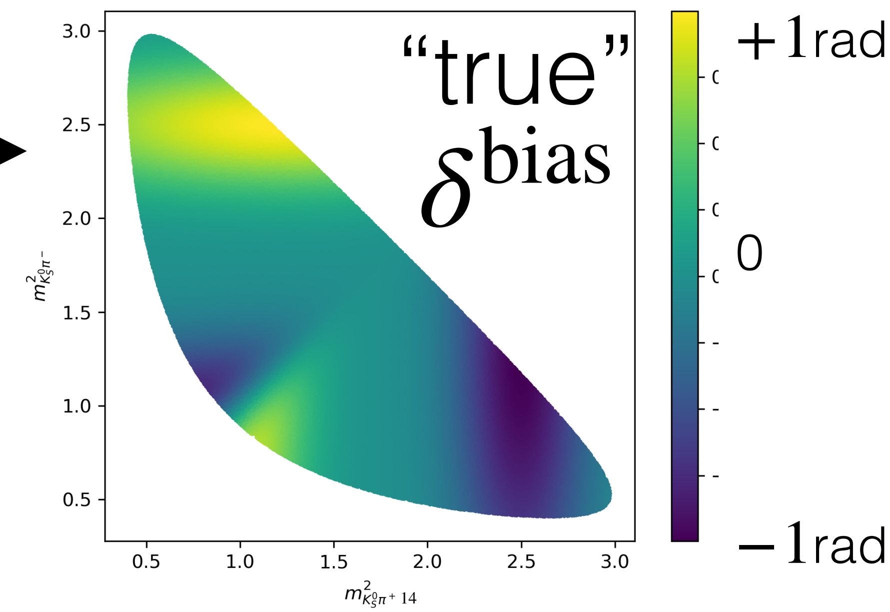
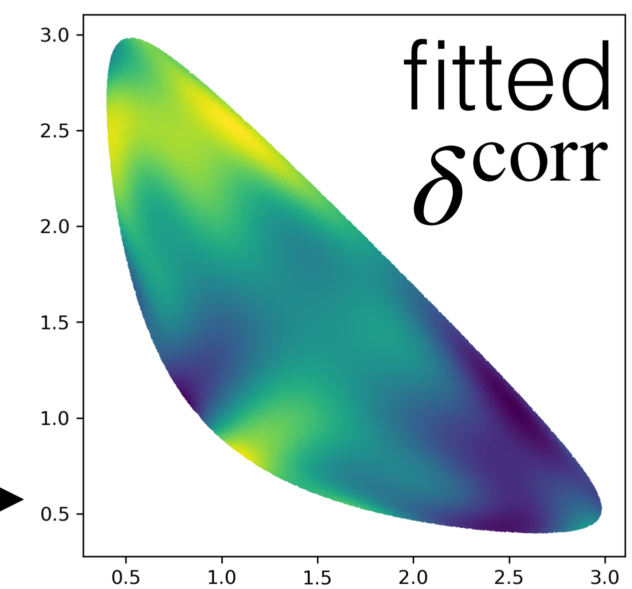
Deviation of δ from model in event generation →



Order	$\Delta x_+ \cdot 100$	$\Delta y_+ \cdot 100$	$\Delta x_- \cdot 100$	$\Delta y_- \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	**$+0.3 \pm 0.8$**	**$+0.5 \pm 1.0$**	**-0.9 ± 0.8**	**$+0.7 \pm 1.0$**
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$

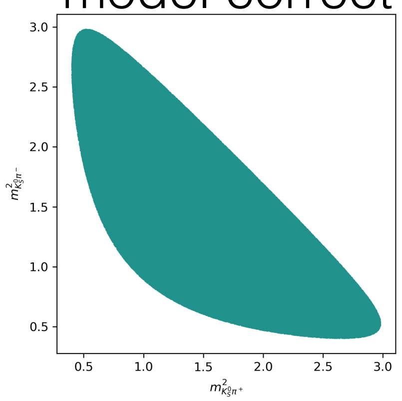


LHCb yields as in [JHEP 02 (2021) 169]
BESIII yields as in: [PRL 124 (2020) 24, 241802], [PRD 101 (2020) 11200]

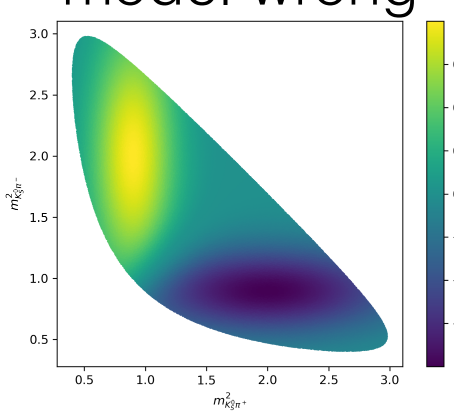
Pull studies, 100 fits

JHEP 09 (2023) 007

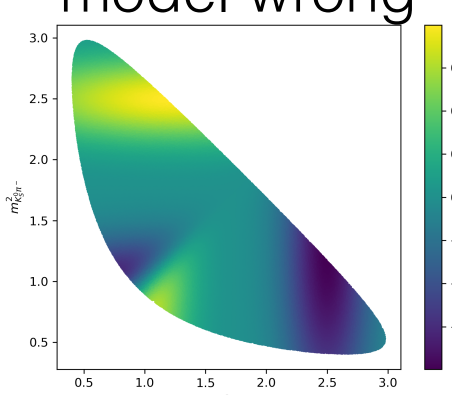
model correct



model wrong



model wrong



pull: $\frac{\text{fit} - \text{input}}{\sigma}$

Table shows (mean) ± (standard deviation) of pull distribution. Expect: 0 ± 1

Method	$\frac{\Delta x_+}{\sigma_{x_+}}$	$\frac{\Delta y_+}{\sigma_{y_+}}$	$\frac{\Delta x_-}{\sigma_{x_-}}$	$\frac{\Delta y_-}{\sigma_{y_-}}$
QMI	-0.12 ± 0.82	$+0.07 \pm 1.01$	-0.12 ± 1.11	-0.02 ± 1.06
MD	-0.08 ± 0.82	$+0.13 \pm 1.01$	-0.07 ± 1.06	-0.12 ± 1.01
QMI	$+0.22 \pm 0.89$	-0.07 ± 0.93	$+0.16 \pm 1.08$	$+0.07 \pm 1.28$
MD	$+1.10 \pm 0.85$	$+3.42 \pm 1.03$	$+0.36 \pm 1.03$	-3.85 ± 1.21
QMI	$+0.17 \pm 0.90$	$+0.07 \pm 0.94$	$+0.02 \pm 0.99$	$+0.13 \pm 1.01$
MD	$+2.04 \pm 0.87$	$+1.07 \pm 0.95$	-0.93 ± 1.16	-1.81 ± 1.24

Uncertainty on mean: ± 0.1, on standard deviation: ± 0.07

Precision on γ with $B^+ \to DK^+, D \to K_S\pi\pi$

Using *our* implementation of the model, no background, no detector effects.

JHEP 09 (2023) 007

		$\sigma_\gamma(^\circ)$		
Lumi		**new QMI**	**Model-dependent**	**8 bins, fixed* ci, si**
1xLHCb	**1xBESIII**	4.2	4.2	5.1
1xLHCb	**10xBESIII**	4.2		
100xLHCb	**1xBESIII**	0.45	0.42	0.52
100xLHCb	**10xBESIII**	0.43		

(average error reported in 100 pseudo experiments)

*) additional uncertainty on binned γ fit due to finite BESIII data for 1xBESIII: 1.2°
PRD 101 (2020) 11200

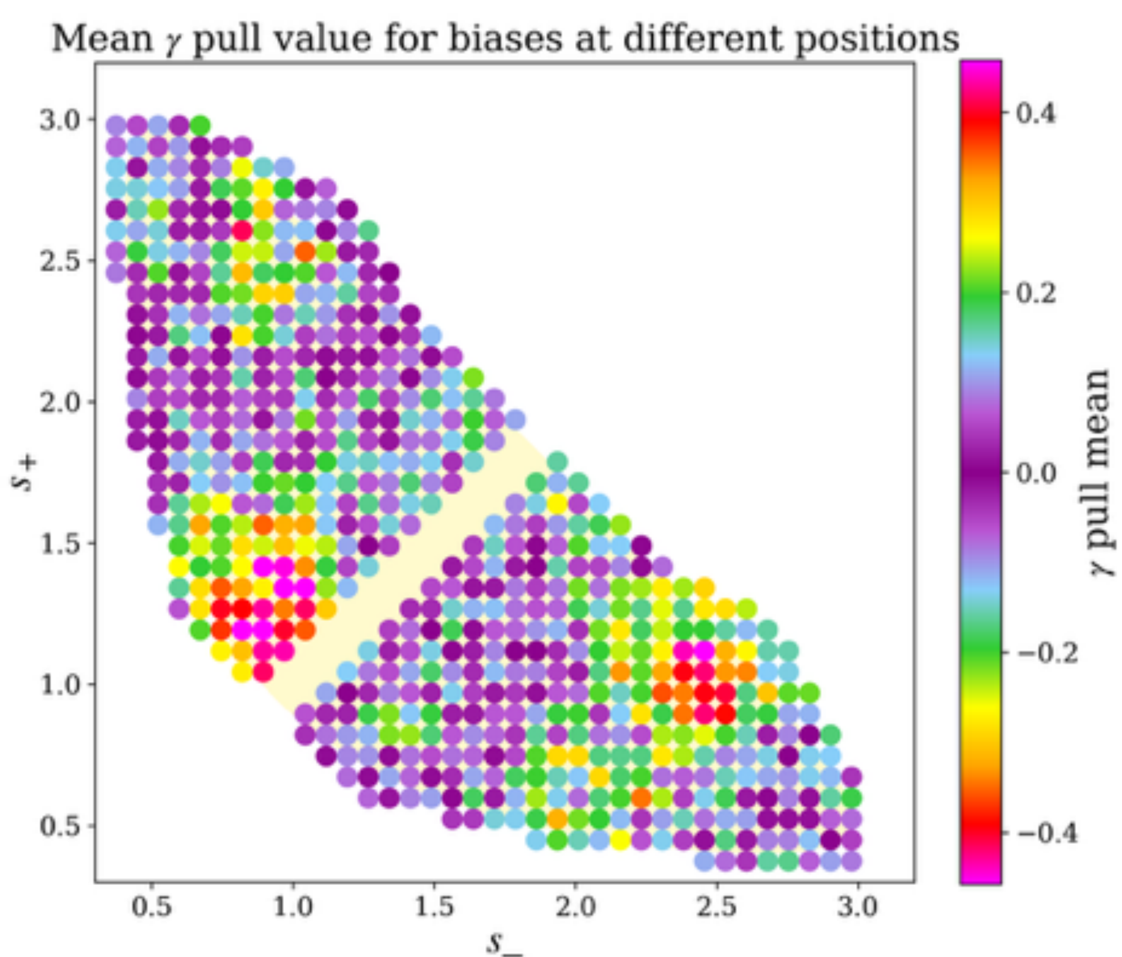
(BTW, ultimate precision on γ achieved through combining multiple decay modes)

To which order do we have to expand the polynomial?

To test this, are trying to find phase modifications to the model that affects γ but not (too much) the c_i and s_i measured by BESIII (i.e. "maximum damage with minimal signature").

Damage a phase shift around a given point does to γ:

We'll use this to construct nasty discrepancies between "true" and "assumed" model

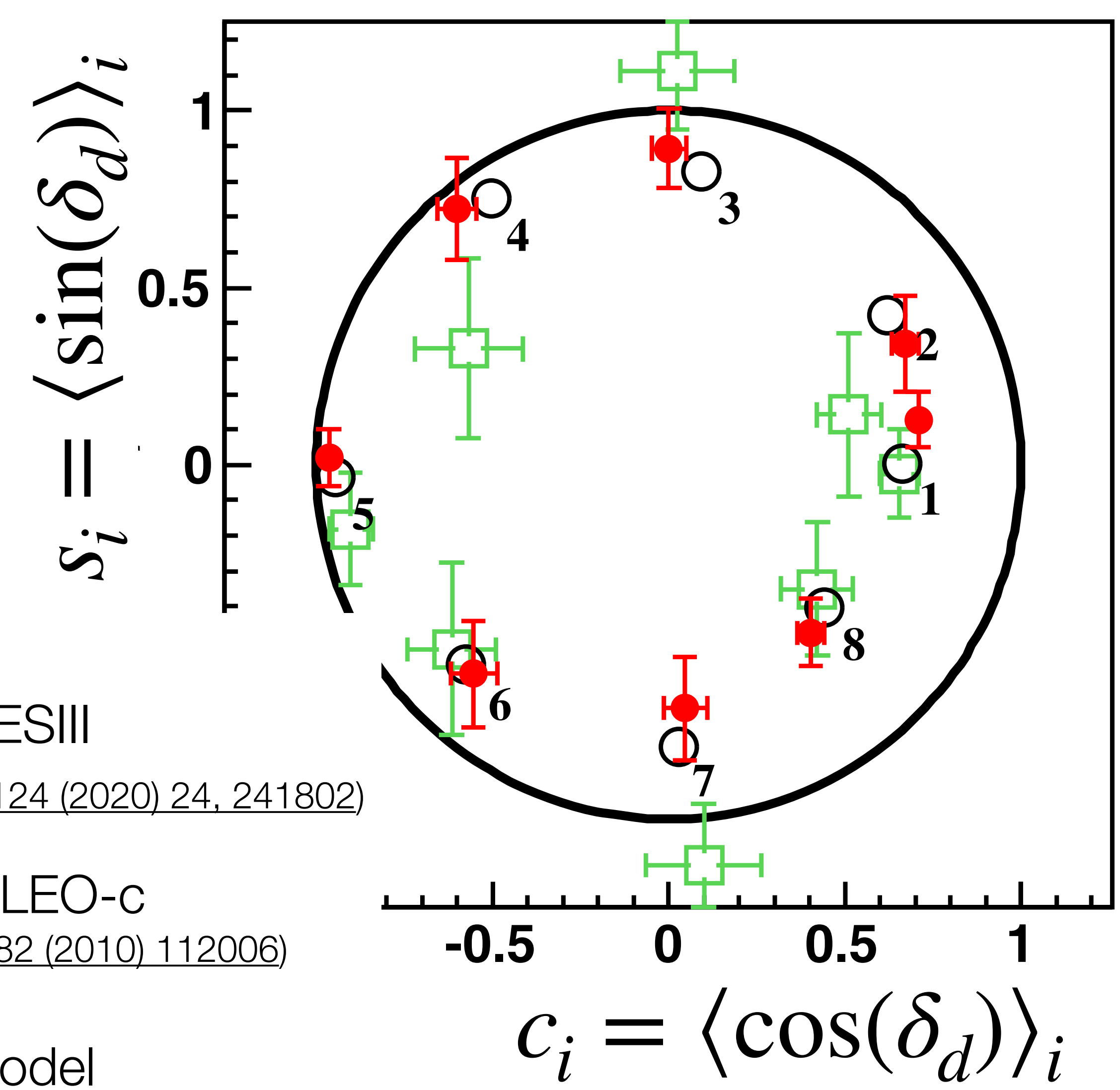


To which order do we have to expand the polynomial?

We tune our phase modifications such that the χ^2 between c_i and s_i calculated from “assumed” and “phase-modified” model corresponds to 5σ deviation (assuming current BESIII uncertainties)

BESIII: PRL 124 (2020) 24, 241802

in $D^0 \to K_S\pi^+\pi^-$

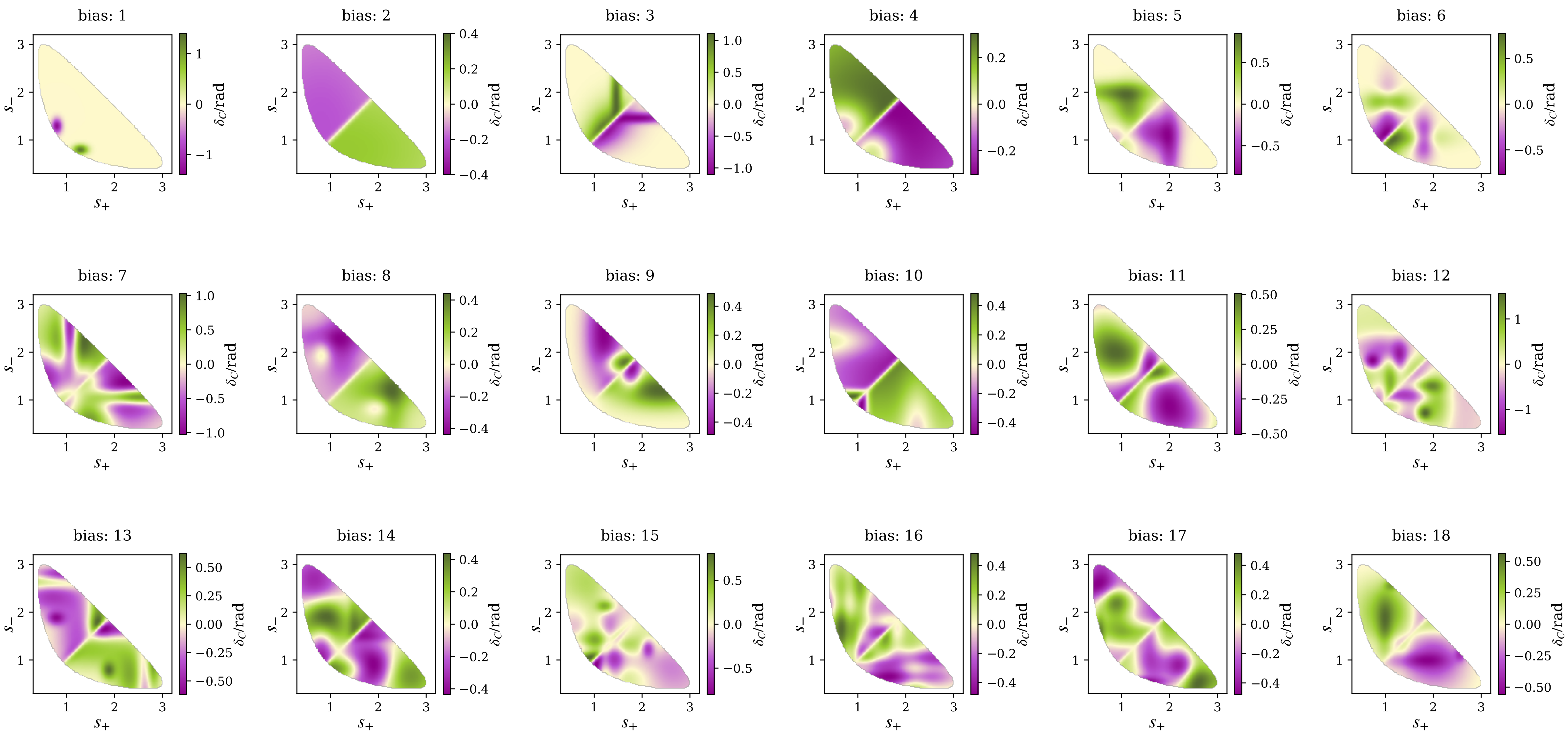


● BESIII
(PRL 124 (2020) 24, 241802)

□ CLEO-c
(PRD 82 (2010) 112006)

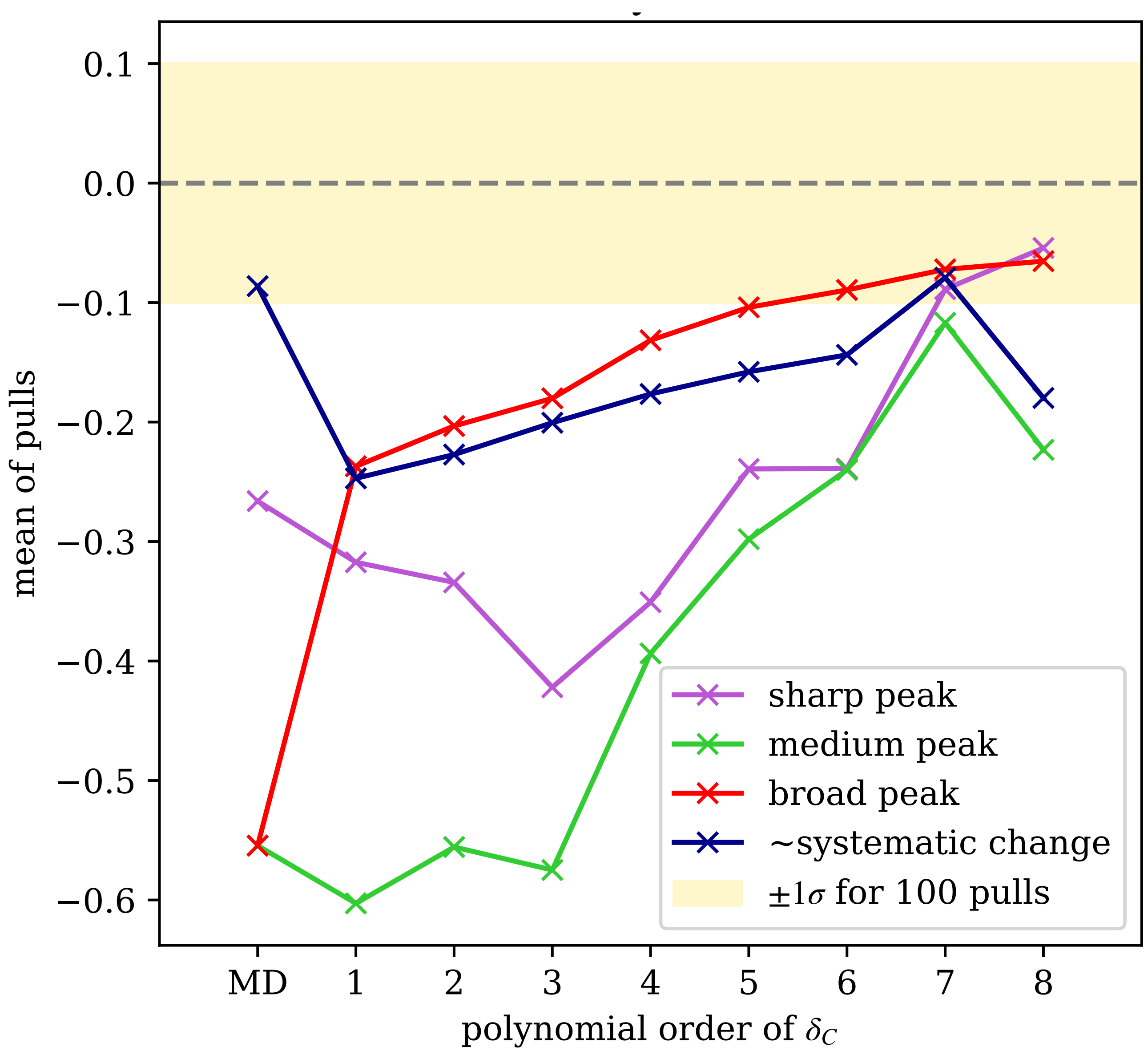
O model
(PRD 98, 112012)

Test a bunch of damaging phase corrections



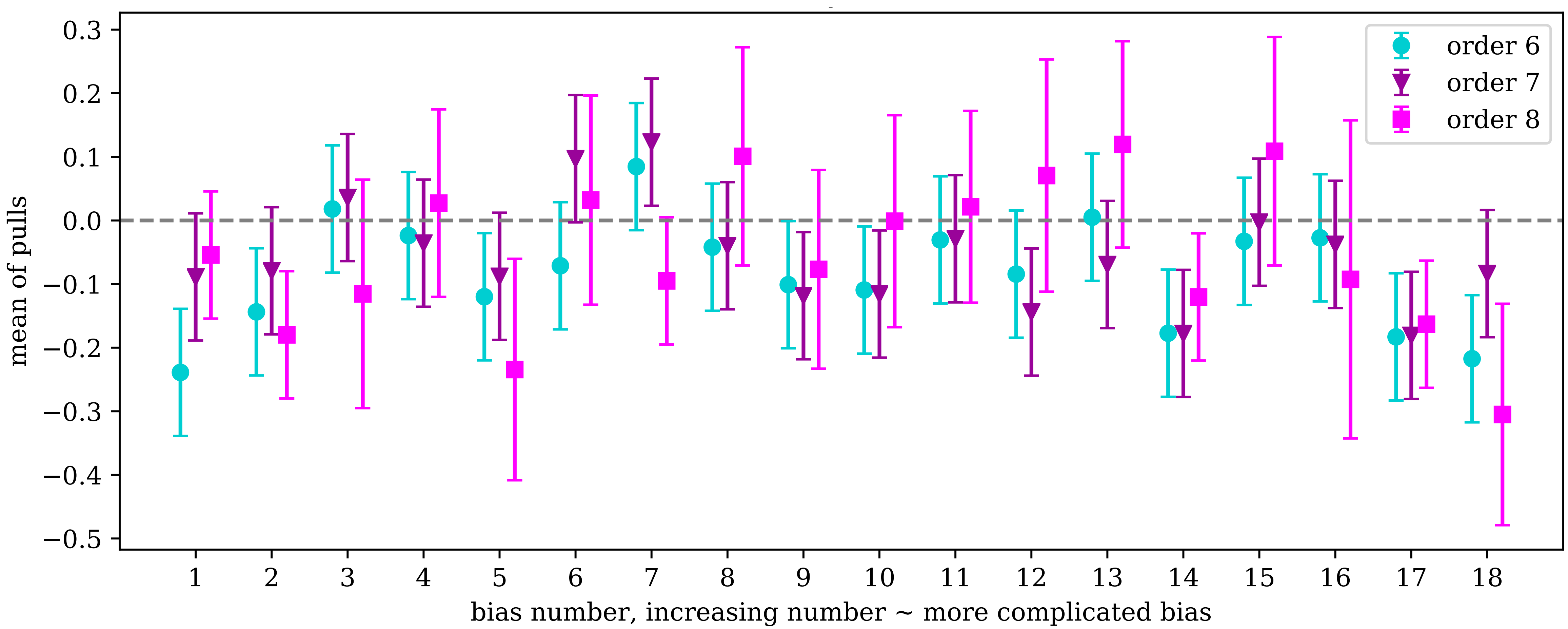
Pulls from 100 toy experiments, 4 phase-modifications, correction polynomial of 0th, 1st, …, 8th order

Plot shows mean of $\left\langle \dfrac{\gamma^{fit} - \gamma^{input}}{\sigma(\gamma)} \right\rangle$, averaged over 100 simulated experiments



Pull results for γ for all models.

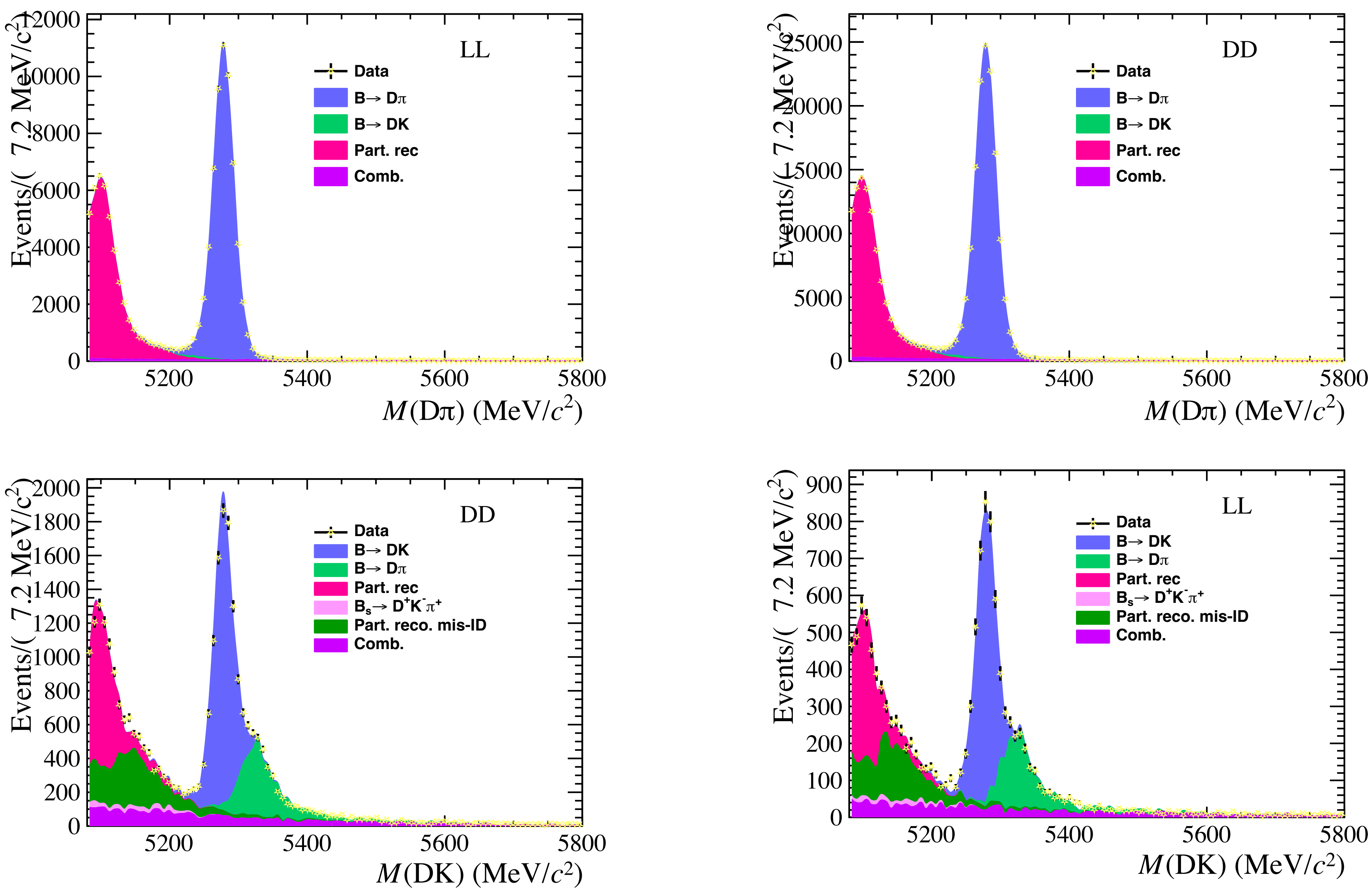
Pull results for γ for the different models, correction polynomials of order 6, 7, 8.



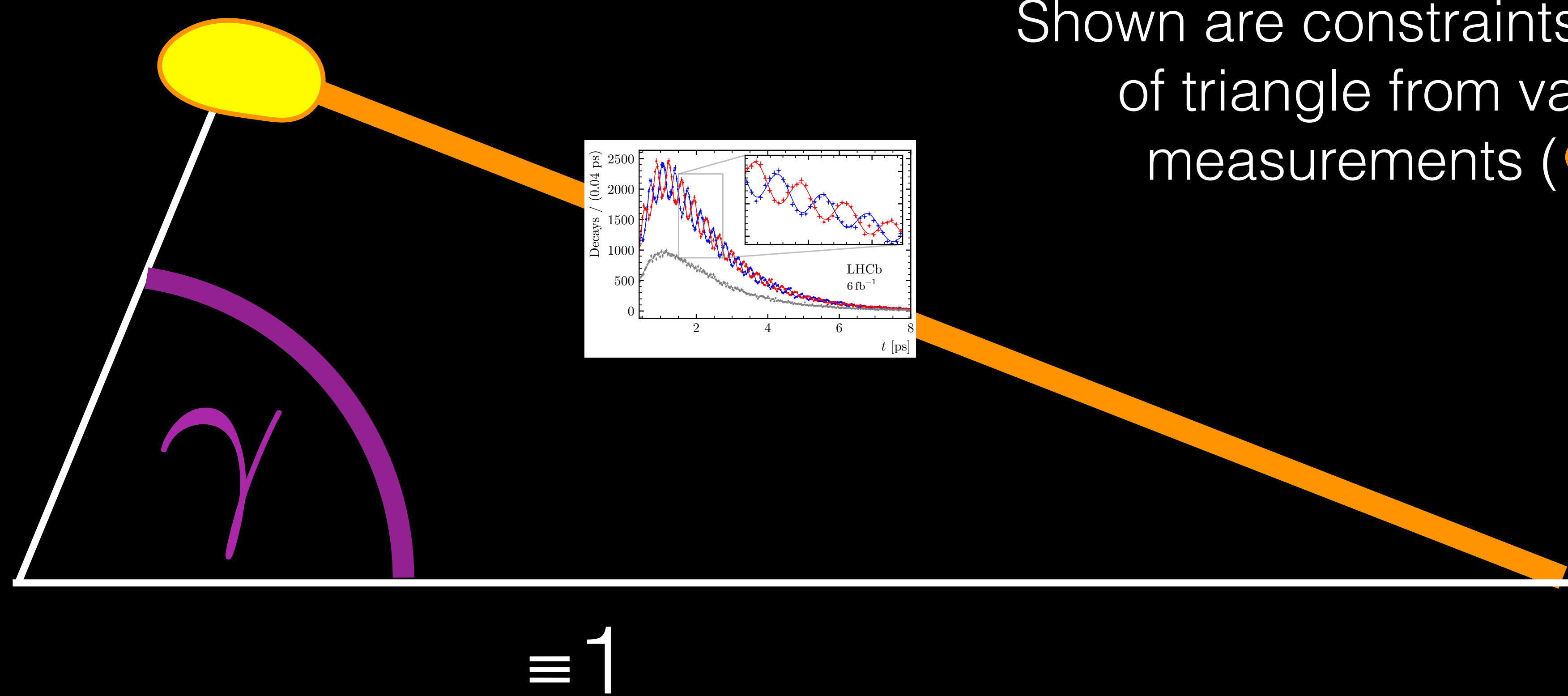
While γ is looking good, some other parameters do show some bias for 2 or 3 of the phase-modification scenarios - still needs to be understood.

Next steps: adding more realism

Toy data for $B^{\pm} \to DK^{\pm}$ and control-channel $B^{\pm} \to D\pi^{\pm}$, including realistic backgrounds

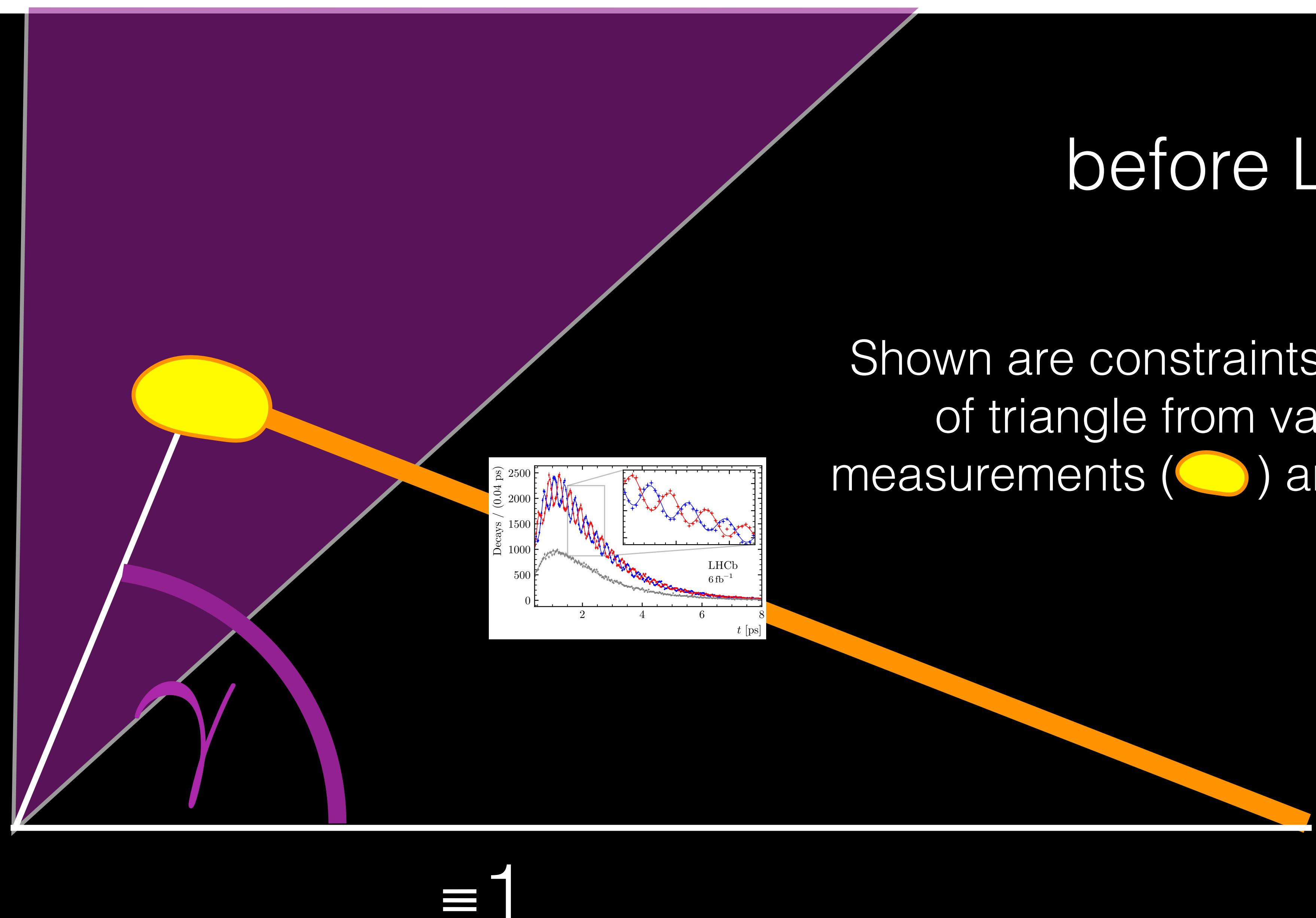


Unitarity triangle



Shown are constraints on apex of triangle from various measurements ( )

Past

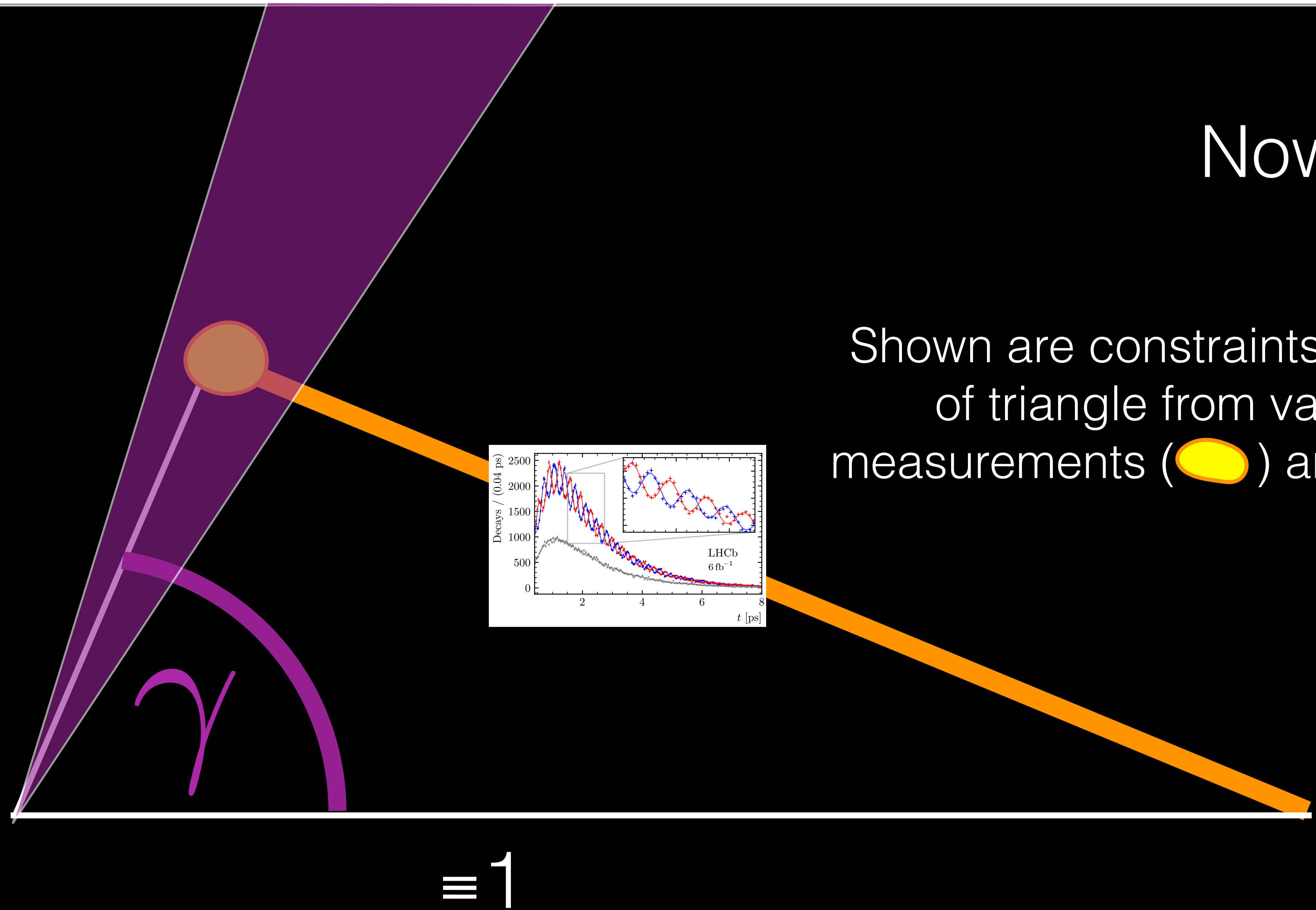


before LHCb

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

Present

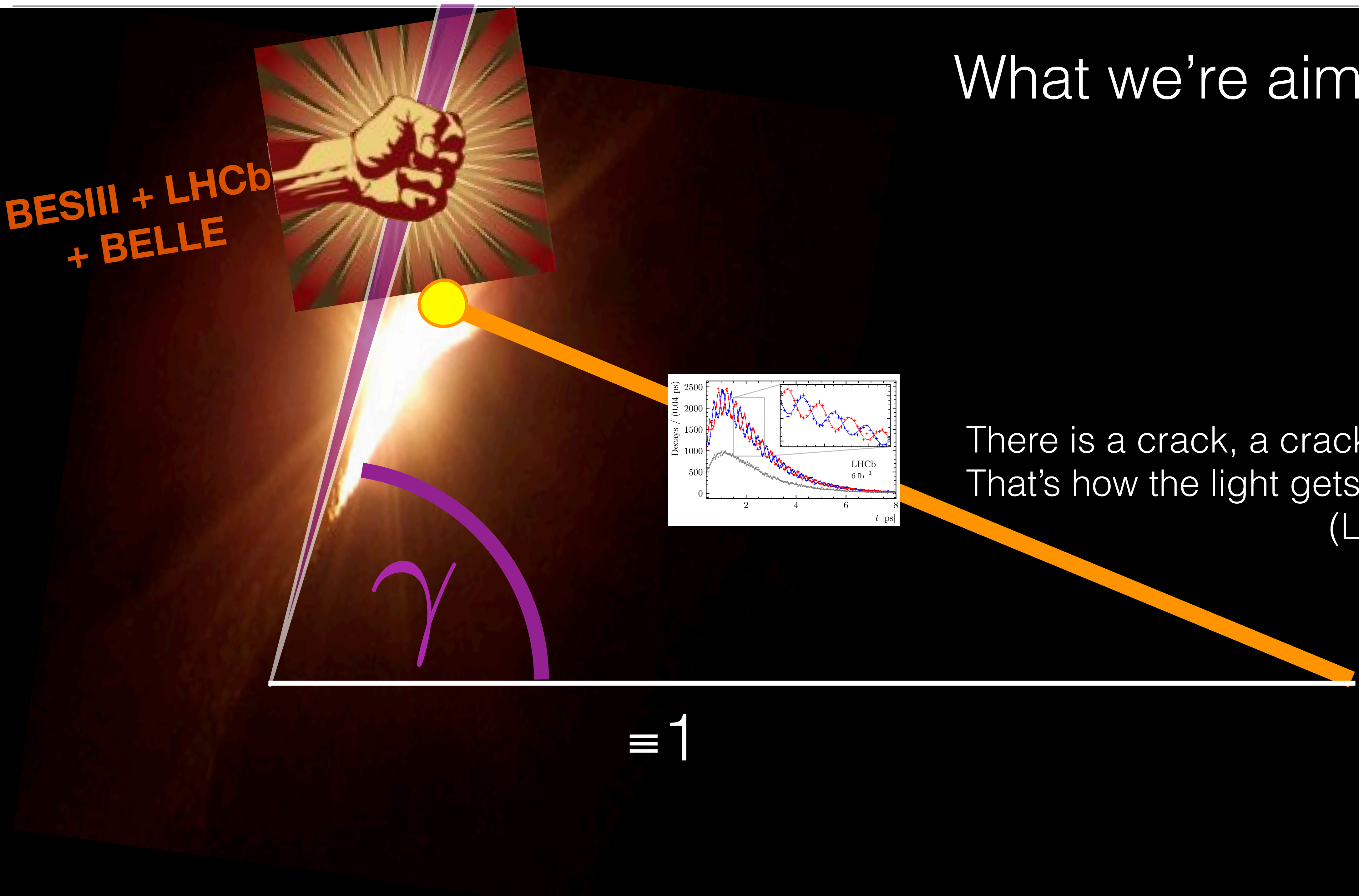
Now



Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

Outlook

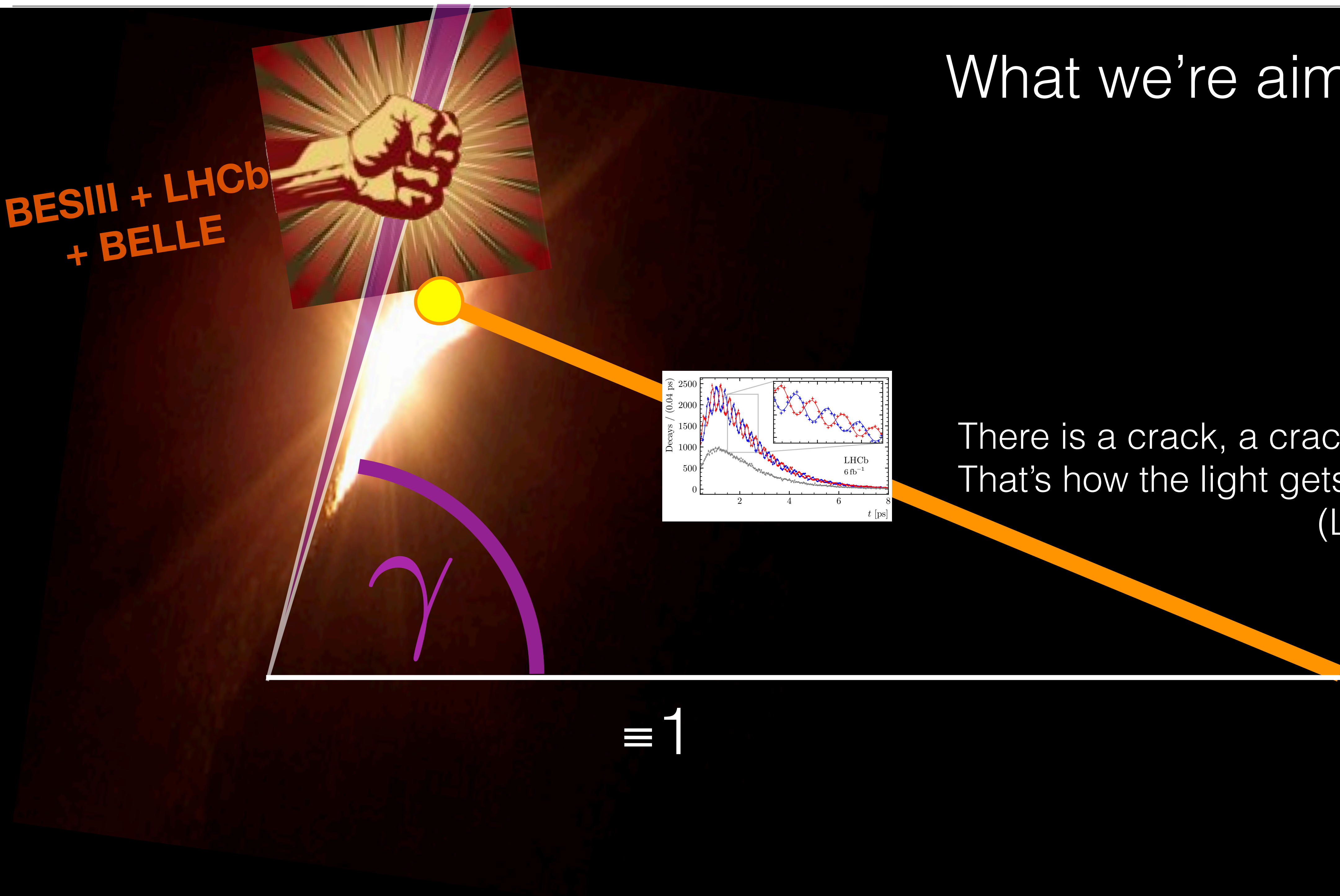
What we're aiming for



There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

Outlook & Summary I

What we're aiming for



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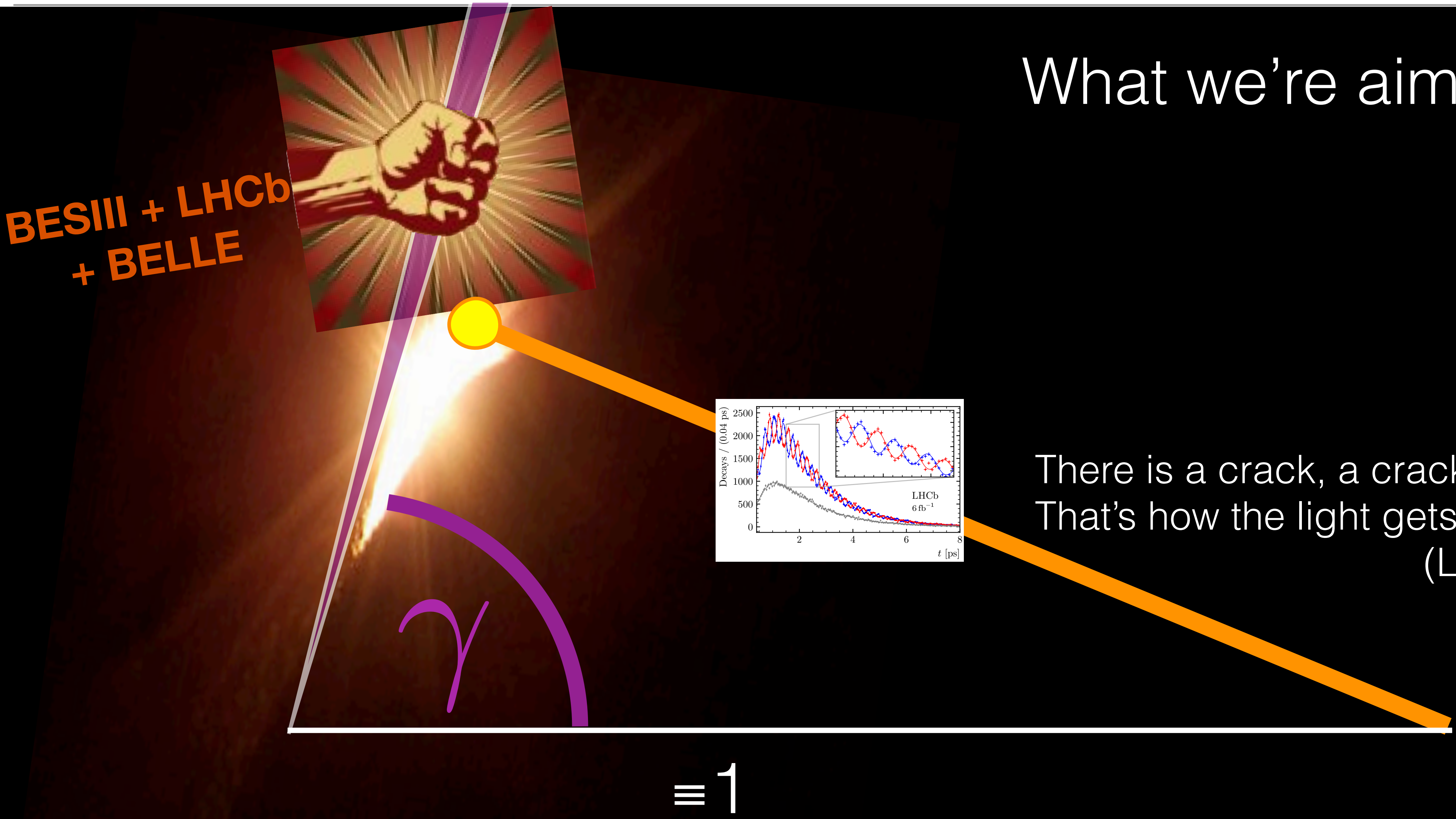
Outlook & Summary I

What we're aiming for

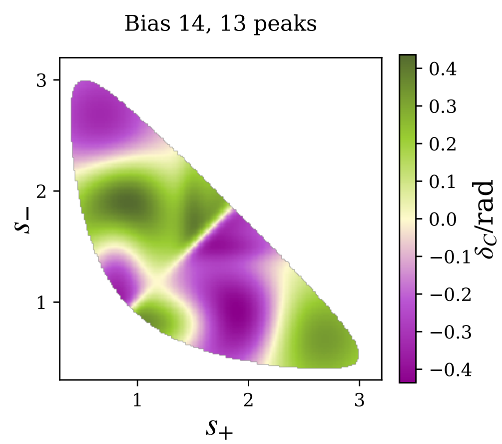


There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

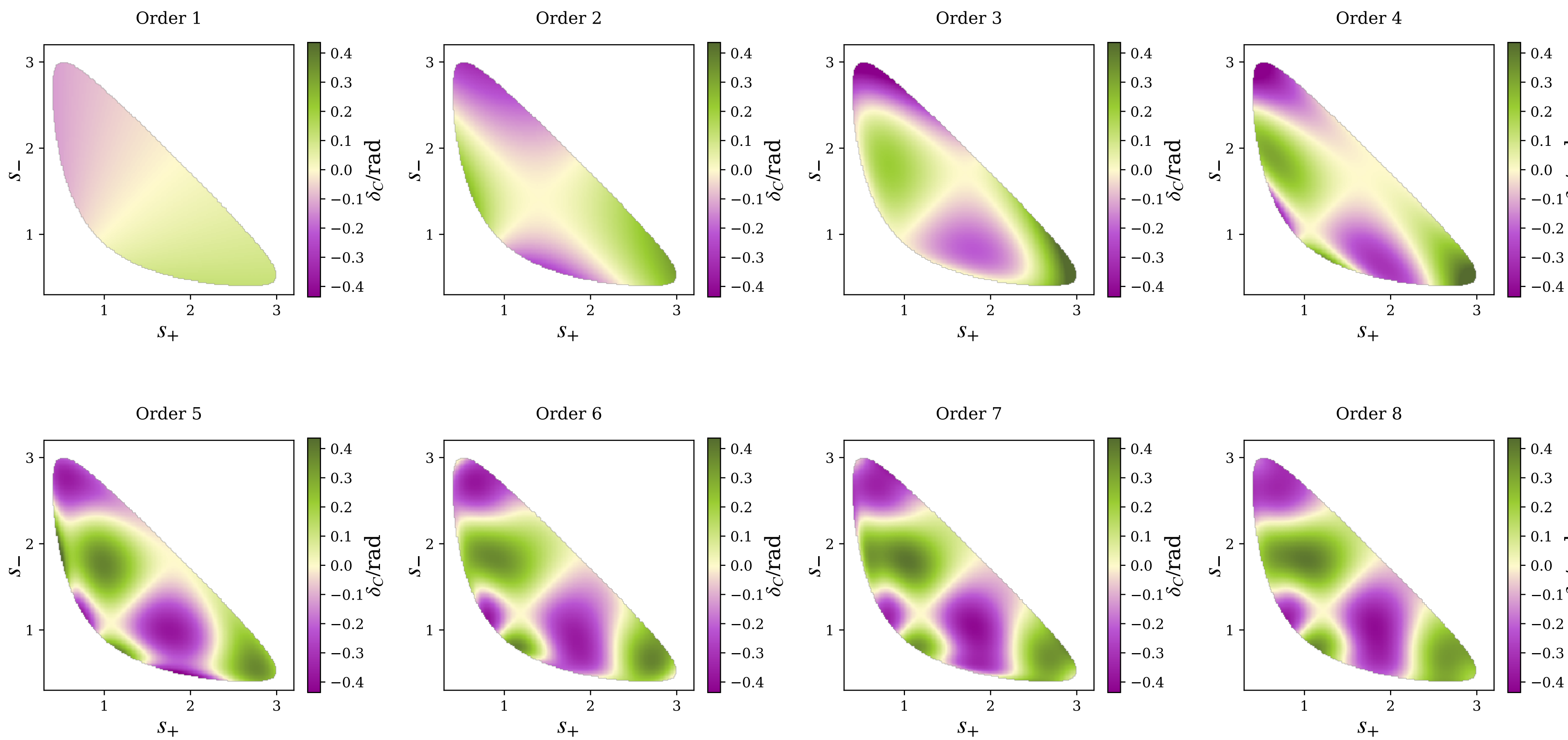
New unbinned method (JHEP 09 (2023) 007) makes optimal use of the information contained in $\psi(3770) \to D\overline{D}$ and $B^{\pm} \to DK^{\pm}$ data for a better precision on γ.

Summary II

Input phase modification



Fitted phase modification order by order



This should also be useful for studying the amplitude model itself!

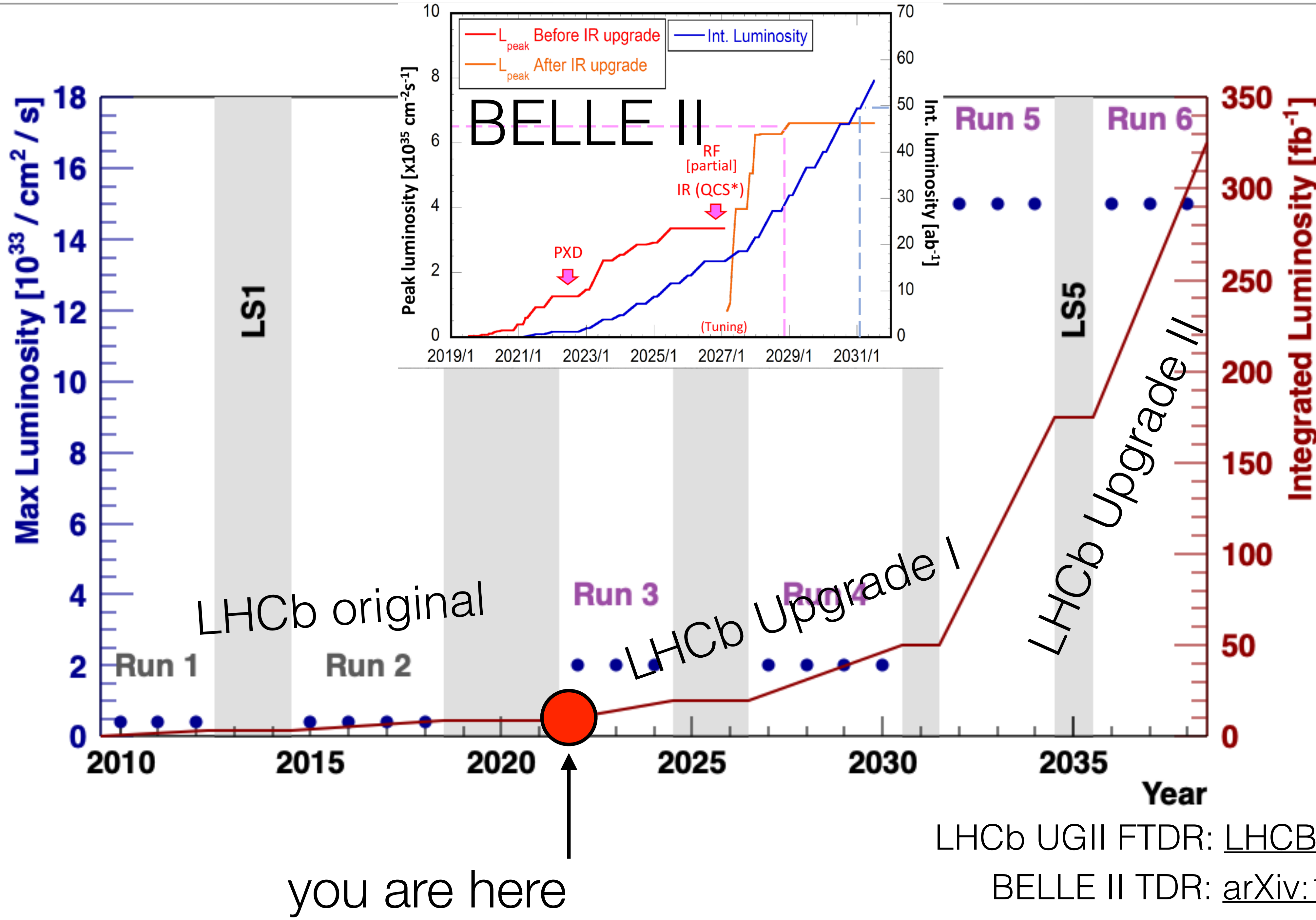
PS:

Our method assumes that the magnitude of BaBar/Belle amplitude model is (sufficiently) accurate. This implies we need to be able to implement that model *exactly*. Implementing others' models is notoriously difficult.

Misha will explain how to make this better in the future.

The End

The future

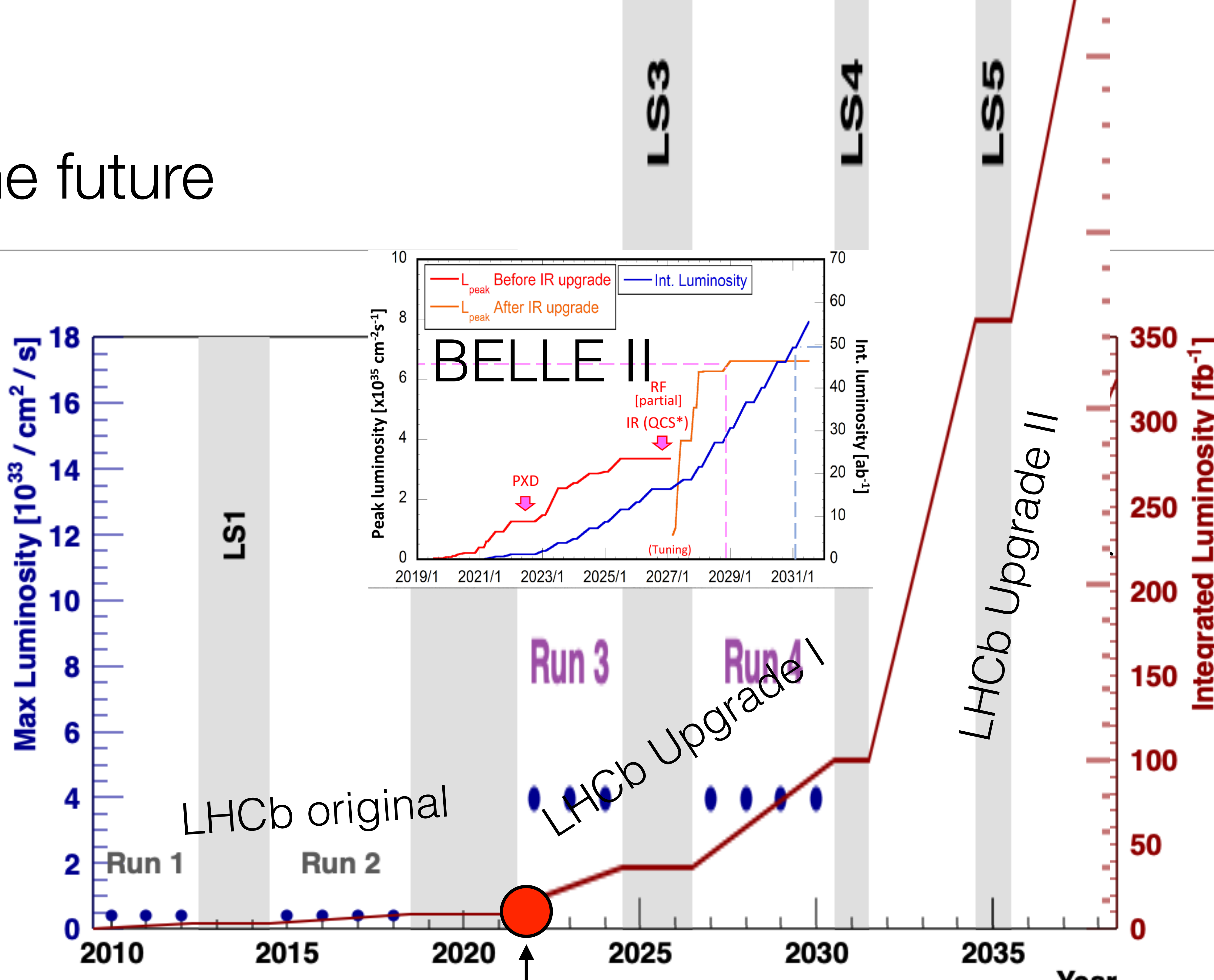


LHCb UGII FTDR: LHCB-TDR-023

BELLE II TDR: arXiv:1011.0352

LHCb UGII physics: arXiv:1808.08865

The future



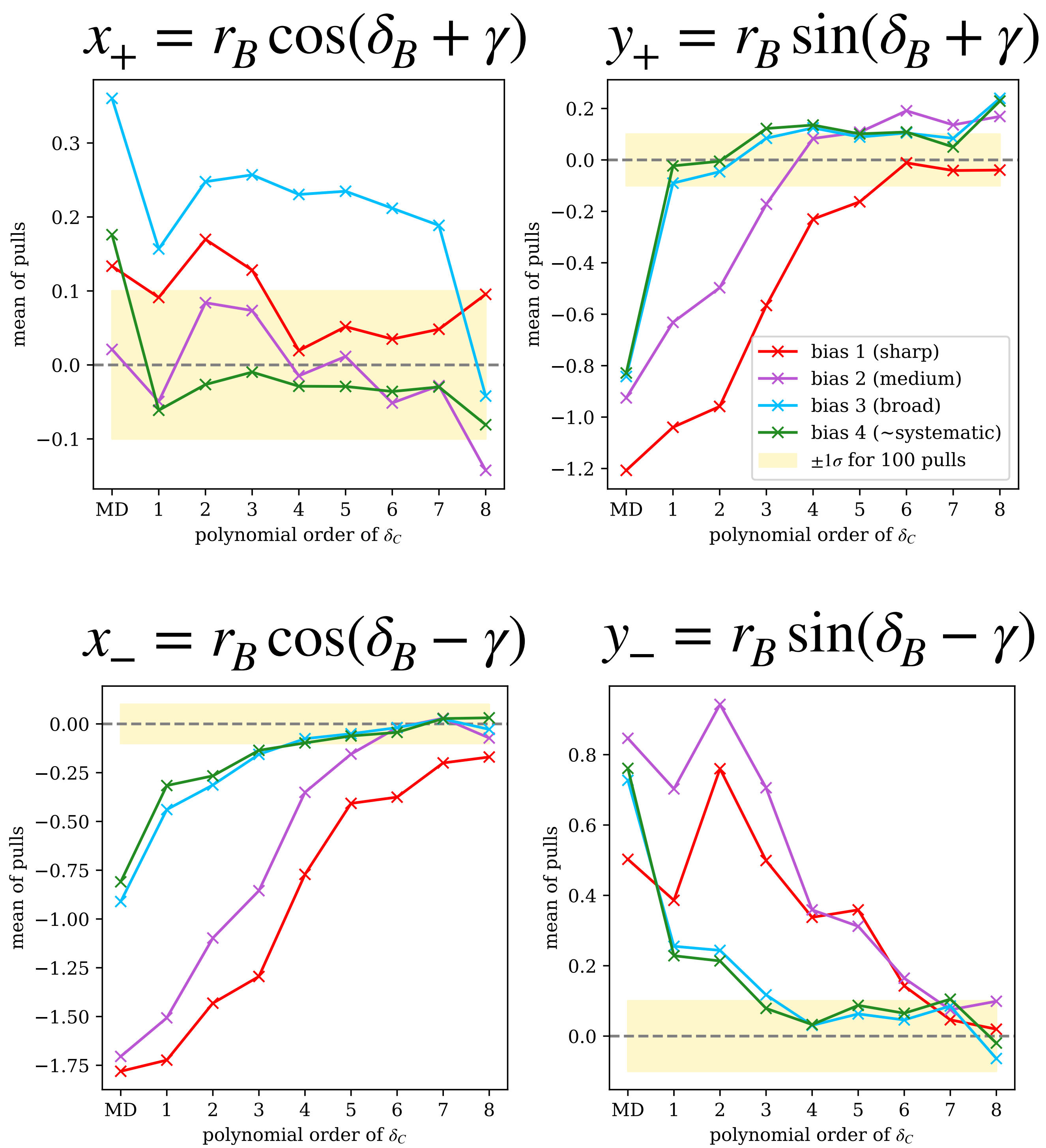
you are here

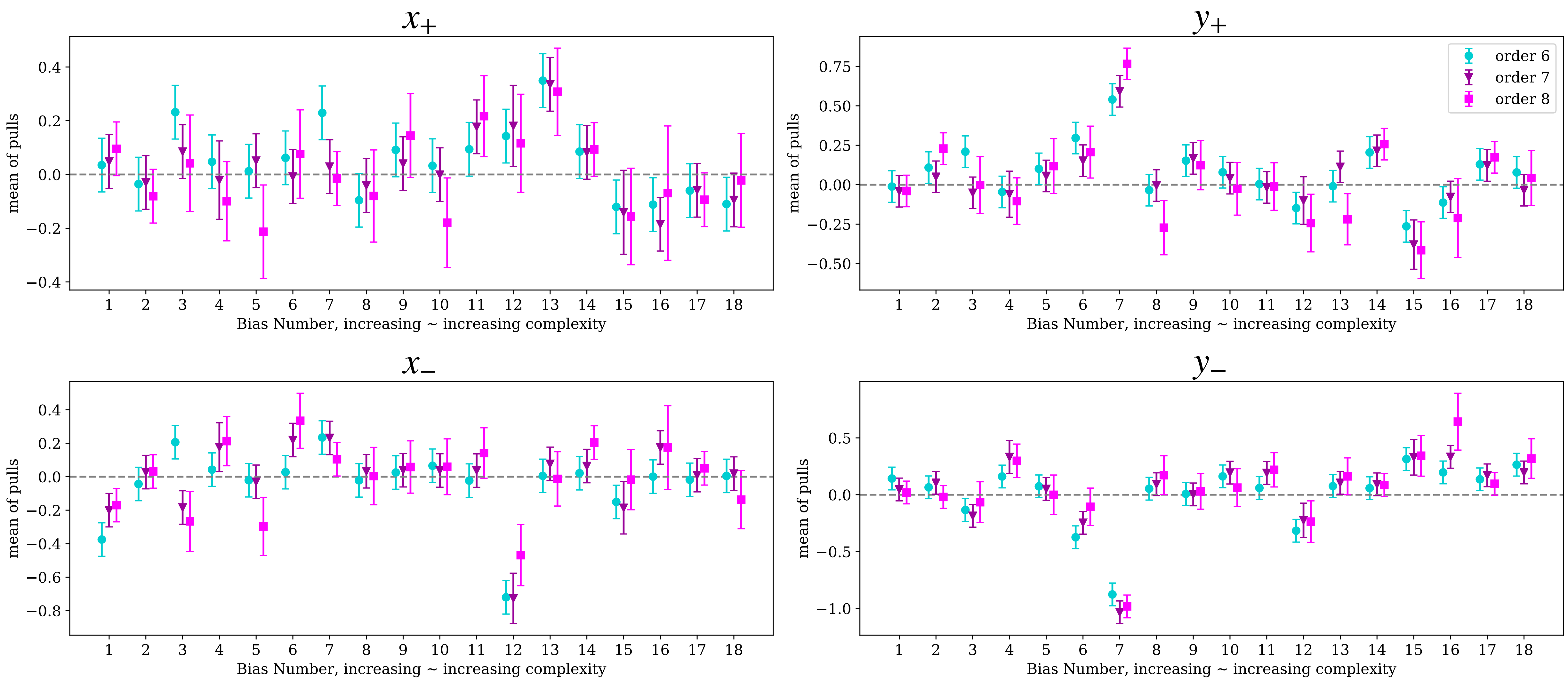
LHCb UGII FTDR: LHCB-TDR-023

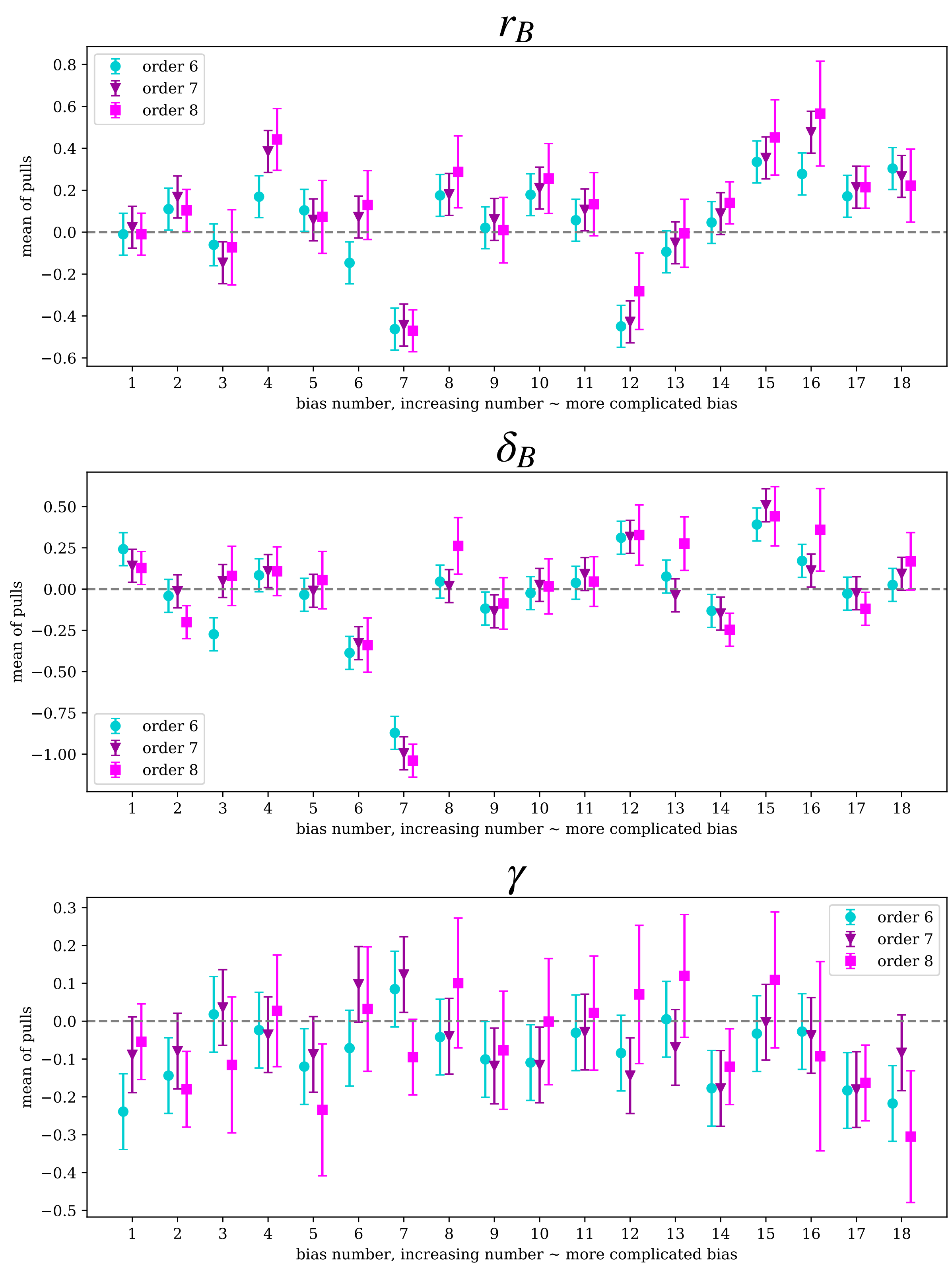
BELLE II TDR: arXiv:1011.0352

LHCb UGII physics: arXiv:1808.08865

Pulls from 100 toy experiments, 4 phase-modifications, correction polynomial of 0th, 1st, …, 8th order



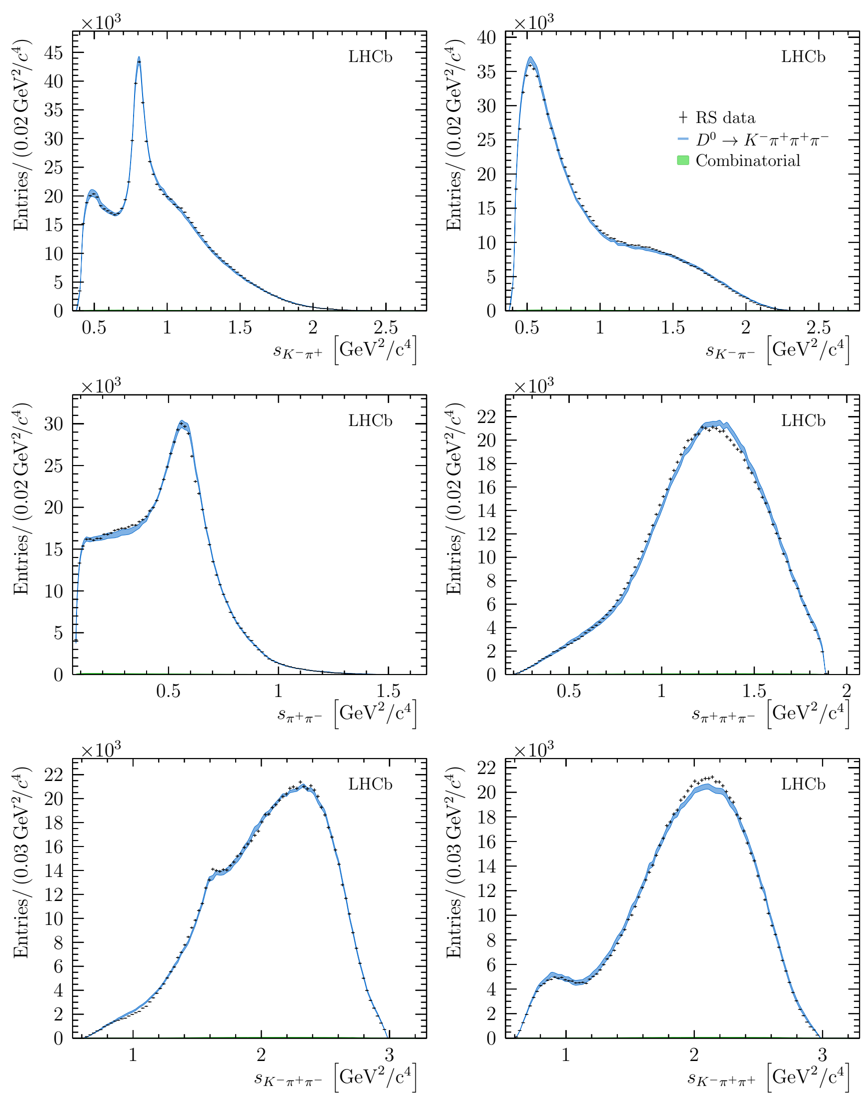

x_+
y_+
x_-
y_-
mean of pulls
Bias Number, increasing ~ increasing complexity
order 6
order 7
order 8

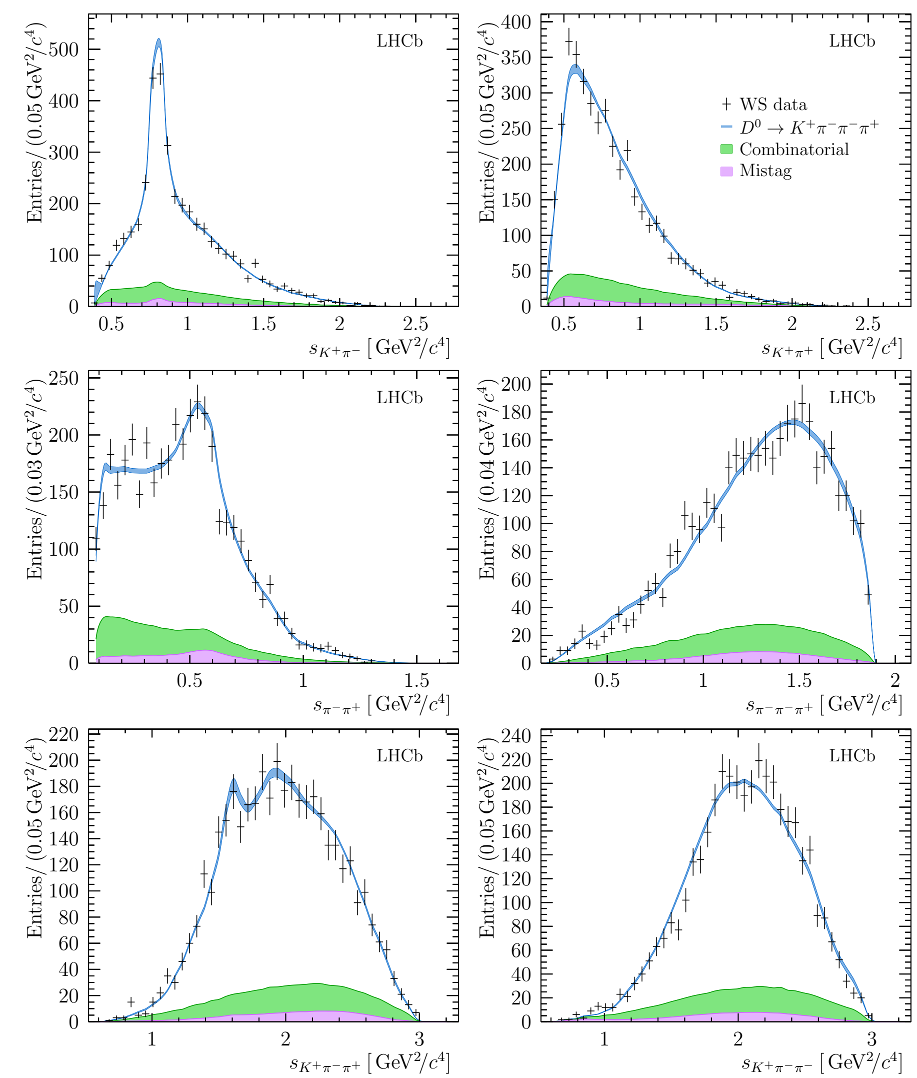

r_B
δ_B
γ
mean of pulls
bias number, increasing number ~ more complicated bias
order 6
order 7
order 8

$B^- \to DK^-, D \to K^+\pi^-\pi^+\pi^-$

LHCb: Eur.Phys.J. C78 (2018) no.6, 443

Step 1: Two amplitude models
$D^0 \to K^+\pi^-\pi^+\pi^-$ and $\overline{D}^0 \to K^+\pi^-\pi^+\pi^-$

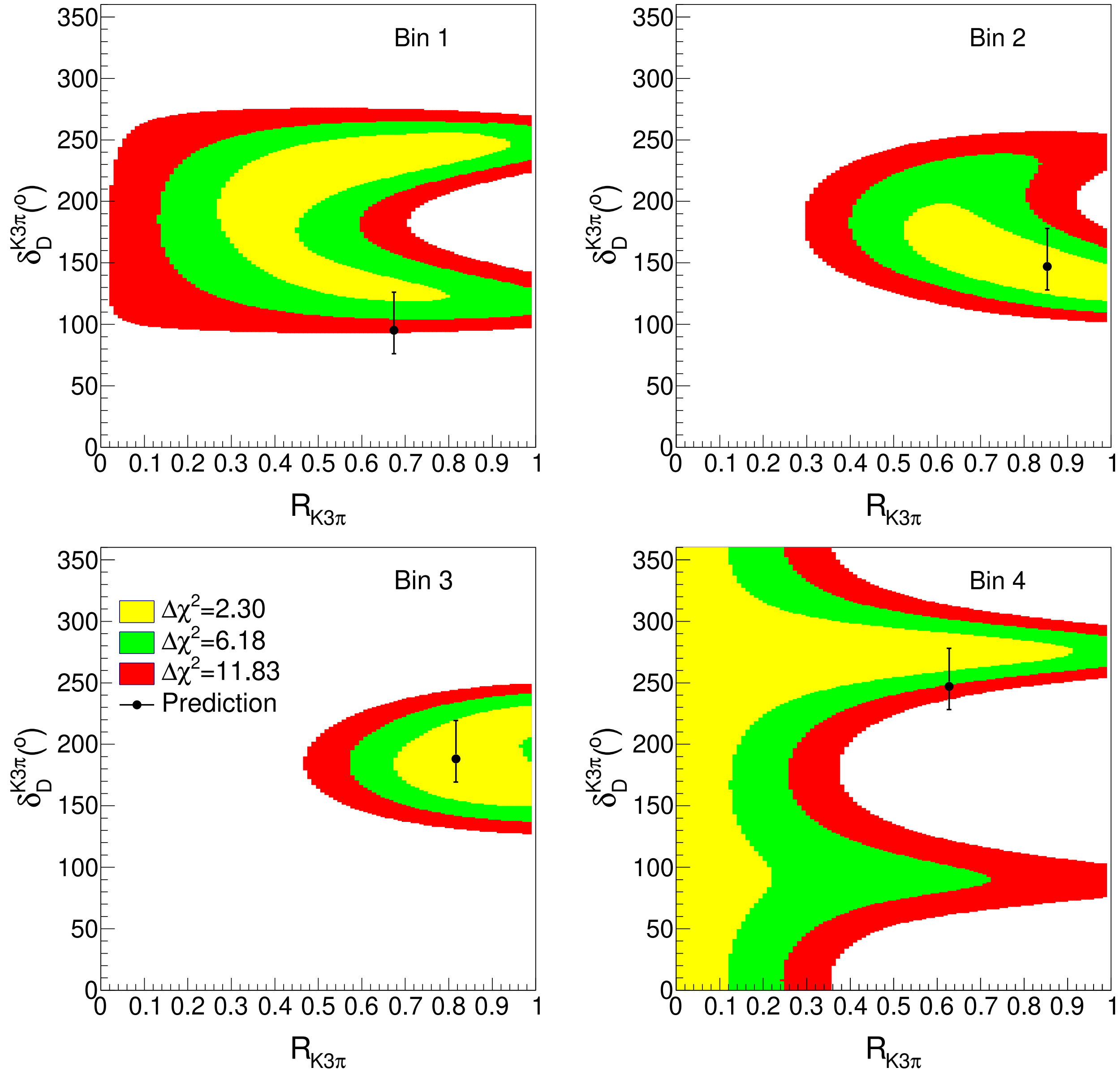




$B^- \to DK^-, D \to K^+\pi^-\pi^+\pi^-$

BESIII: JHEP 05 (2021) 164

Step 2 Model-informed binning, analysis of correlated $D, \overline{D}$ pairs in bins at BES III



$$Re^{-i\delta_D} = c_i + is_i$$

Coherence factor: Atwood & Soni PRD 68 (2003) 033003

1st (unbinned) measurement at CLEO-c: PRD 80 (2009) 031105

5-D binning: S Harnew, JR JHEP 03 (2015) 169

Binning based on latest model: Evans, Malde, Libby, Wilkinson: PLB 802 (2020) 135188

$B^- \to DK^-, D \to K^+\pi^-\pi^+\pi^-$ LHCb: arXiv:2209.03692 (2022)

Step 3: 2nd most precise γ individual measurement

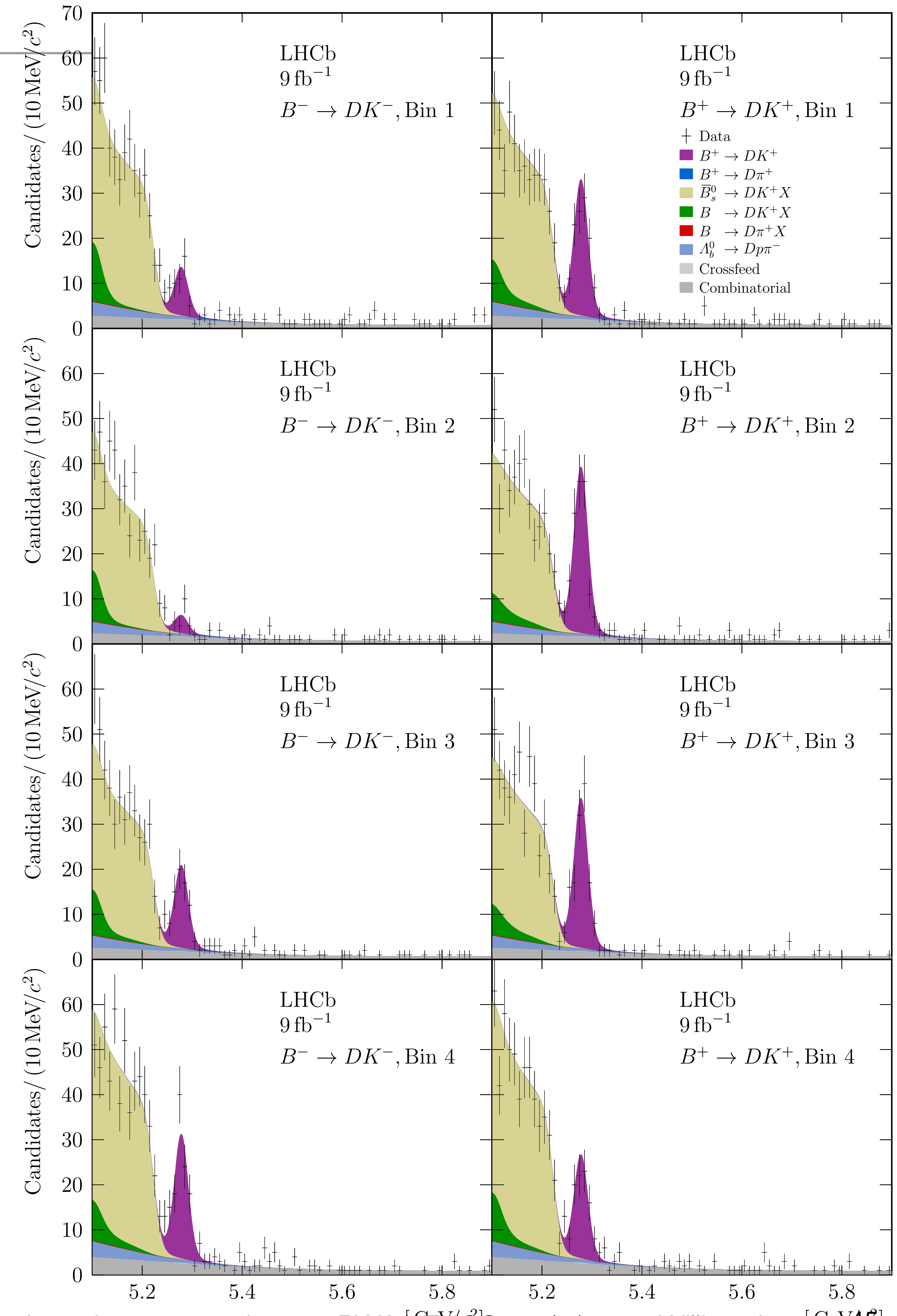
$$\gamma = \left(54.8 \, {}^{+6.0}_{-5.8} \, {}^{+0.6}_{-0.6} \, {}^{+6.7}_{-4.3}\right)^\circ$$

statistical

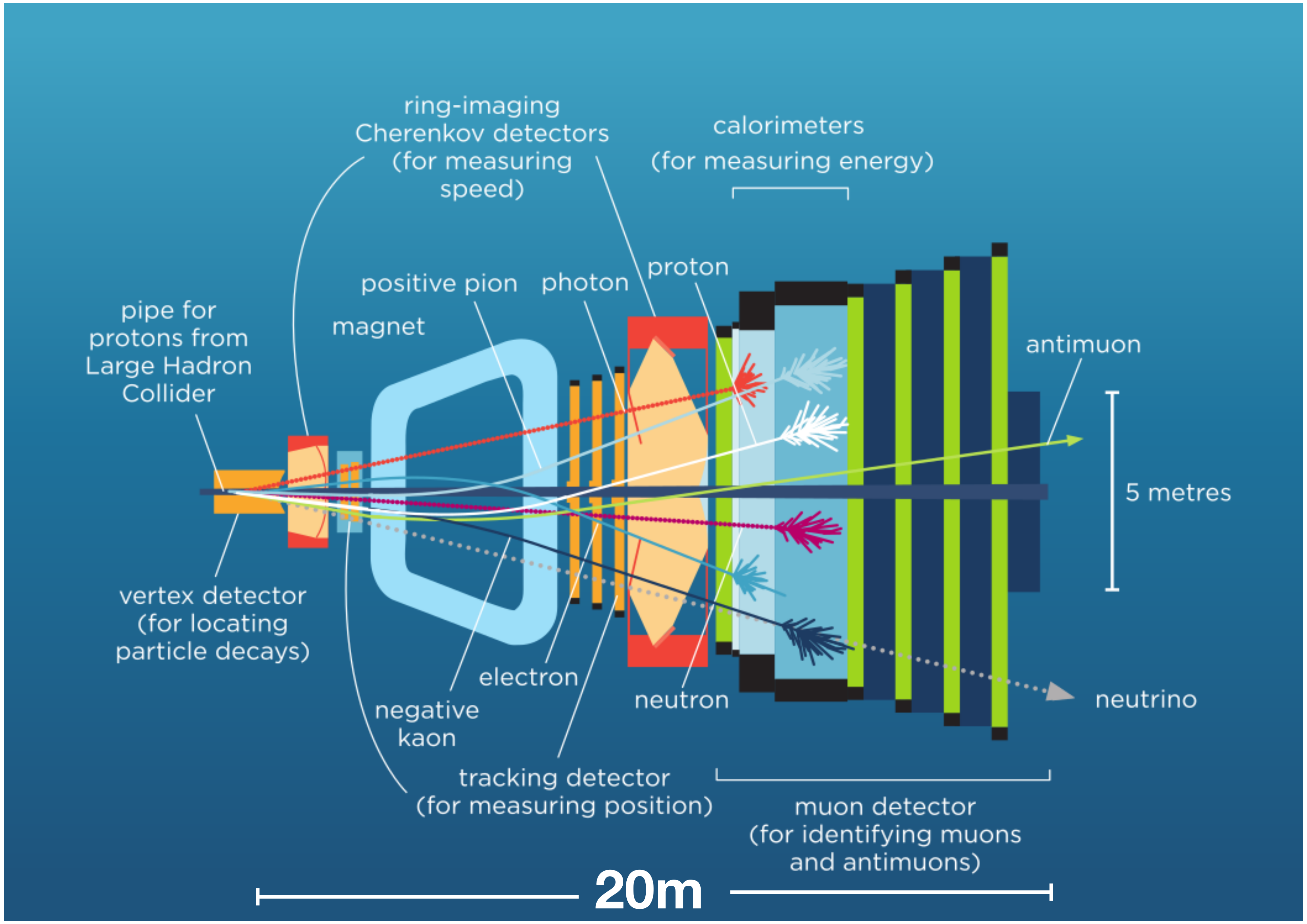
systematic

input from charm threshold

hope for more BES III data at $\psi(3770)$



The LHCb Detector



Comparing methods

JHEP 09 (2023) 007

LHCb	$\sigma_{x_+} \cdot 10^2$		$\sigma_{y_+} \cdot 10^2$		$\sigma_{x_-} \cdot 10^2$		$\sigma_{y_-} \cdot 10^2$		σ_γ $(^\circ)$	
Lumi	MD	bin	MD	bin	MD	bin	MD	bin	MD	bin
$\times 1$	0.780	0.886	1.081	1.482	0.878	1.189	0.939	1.328	4.23	5.09
$\times 100$	0.078	0.089	0.108	0.149	0.088	0.118	0.093	0.134	0.42	0.52

Lumi scenario: LHCb	BES III	$\sigma_{x_+} \cdot 10^2$	$\sigma_{y_+} \cdot 10^2$	$\sigma_{x_-} \cdot 10^2$	$\sigma_{y_-} \cdot 10^2$	σ_γ $(^\circ)$
$\times 1$	$\times 1$	0.780	1.091	0.877	0.945	4.21
$\times 1$	$\times 10$	0.773	1.062	0.866	0.924	4.18
$\times 100$	$\times 1$	0.079	0.122	0.090	0.104	0.45
$\times 100$	$\times 10$	0.078	0.115	0.089	0.099	0.43

Parameter counting

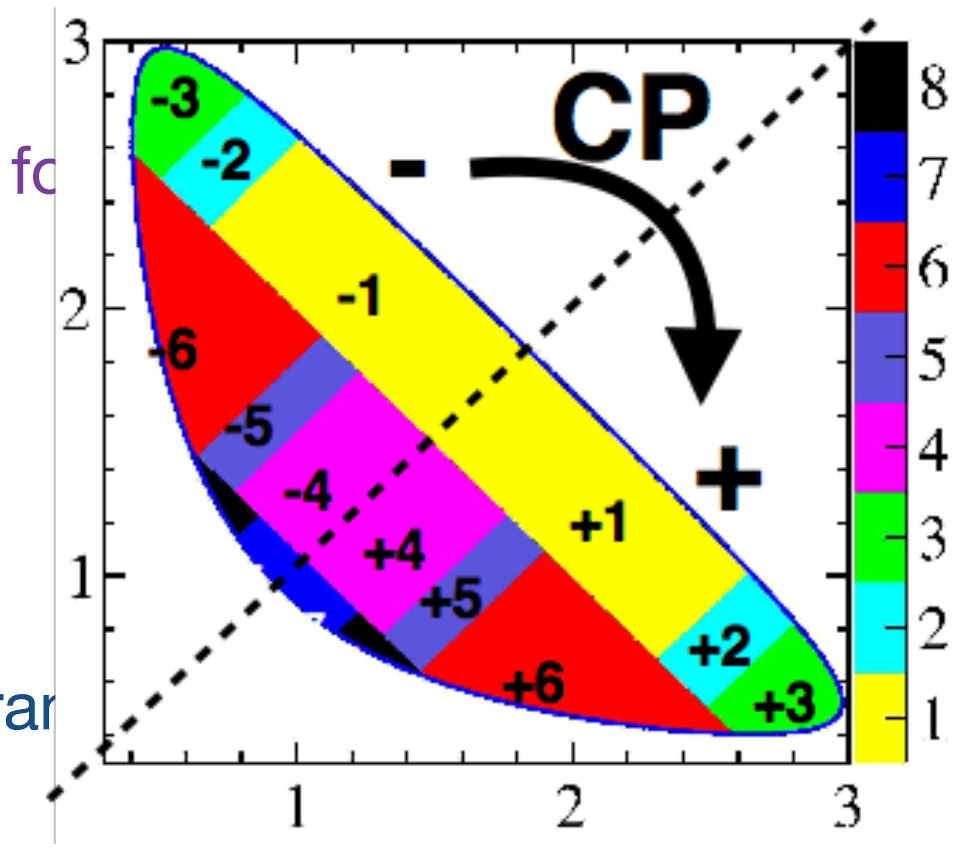
Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$

- Parameter counting:
  Number of bins:
  Dividing Dalitz plot into N bin pairs gives 4N bins (2N fo

  Number of parameters:
  3 global (r_B, δ_B, γ)
  2 N $(c_i, s_i$ one per each bin pair)

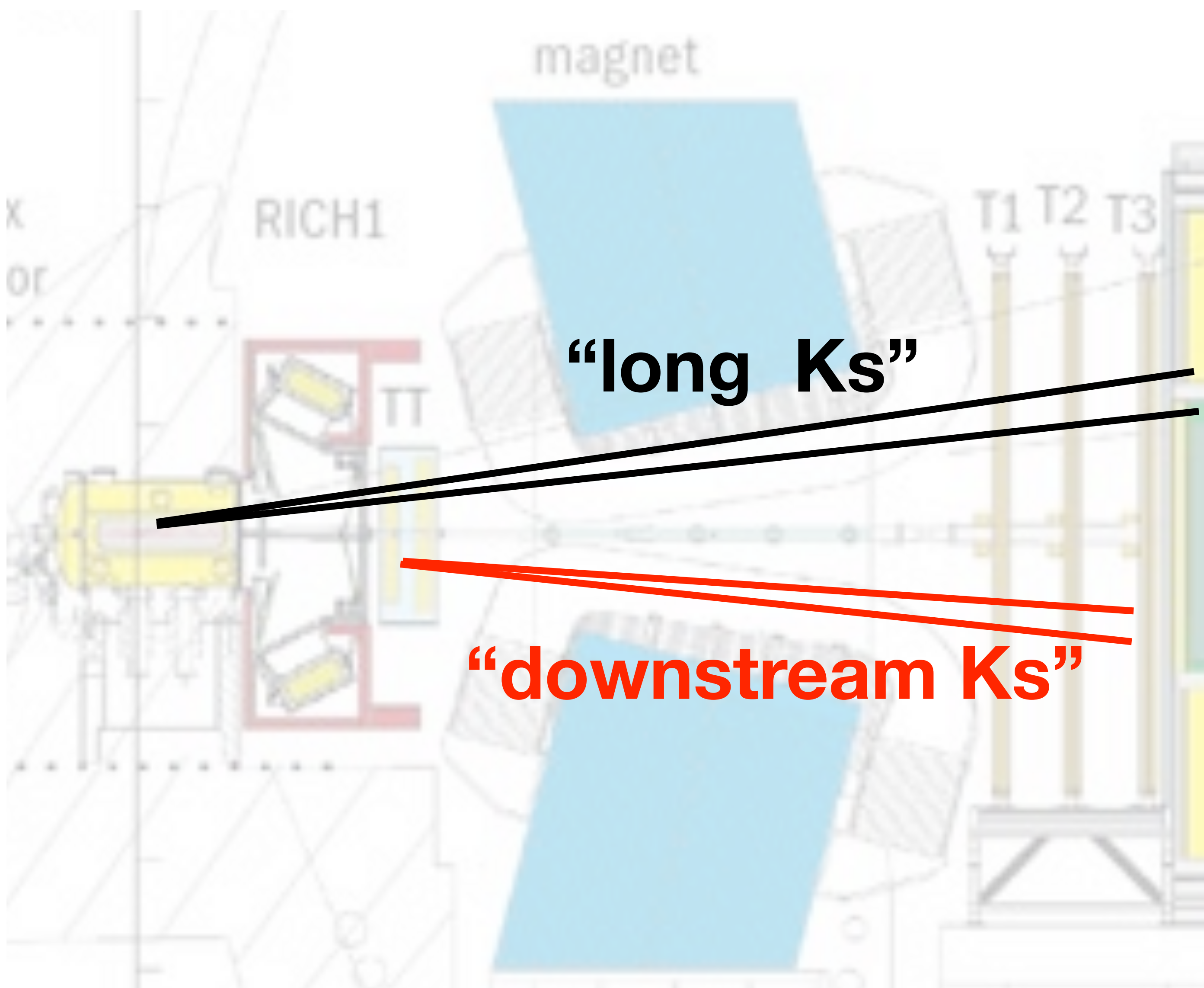
  Theoretically, if 2N+3 $\leq$ 4N (i.e. $N \geq 2$), can fit all parar
  from B decays.



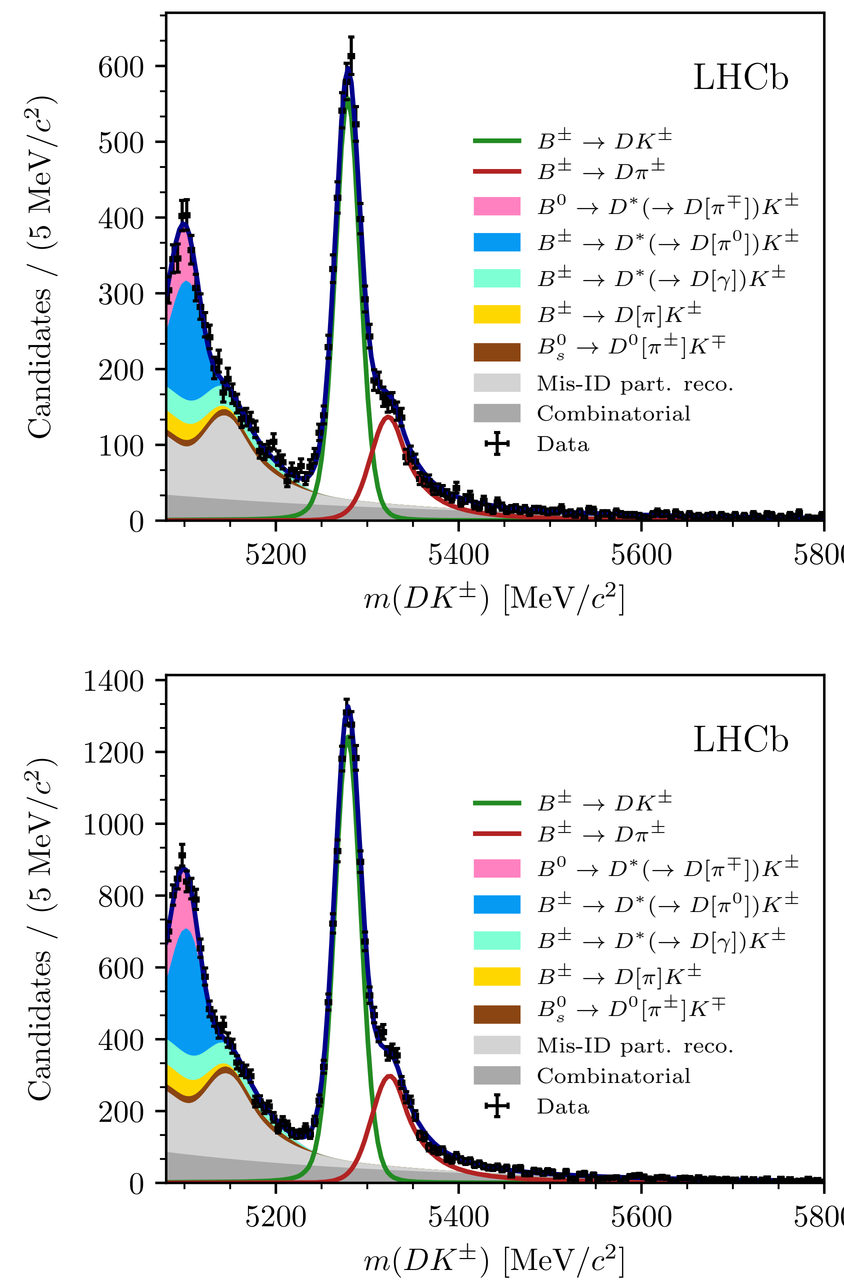
- In practice, to achieve good precision on γ, input from threshold to constrain c_i, s_i is absolutely critical.

$B^{\pm} \to DK^{\pm}, D \to K_S\pi^+\pi^-$ at LHCb

LHCb: JHEP 02 (2021) 169



$B^{\pm} \to DK^{\pm}$



12.5k signal events

Input to LHCb γ combination

LHCb-CONF-2022-003

JHEP 12 (2021) 141

LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^{\pm} \to Dh^{\pm}$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^{\pm} \to Dh^{\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[30]	Run 1	As before
$B^{\pm} \to Dh^{\pm}$	$D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	[18]	Run 1&2	**New**
$B^{\pm} \to Dh^{\pm}$	$D \to h^+h^-\pi^0$	[19]	Run 1&2	**Updated**
$B^{\pm} \to Dh^{\pm}$	$D \to K^0_S h^+h^-$	[31]	Run 1&2	As before
$B^{\pm} \to Dh^{\pm}$	$D \to K^0_S K^{\pm}\pi^{\mp}$	[32]	Run 1&2	As before
$B^{\pm} \to D^*h^{\pm}$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^{\pm} \to DK^{*\pm}$	$D \to h^+h^-$	[33]	Run 1&2(*)	As before
$B^{\pm} \to DK^{*\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[33]	Run 1&2(*)	As before
$B^{\pm} \to Dh^{\pm}\pi^+\pi^-$	$D \to h^+h^-$	[34]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to h^+h^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to h^+\pi^-\pi^+\pi^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	[36]	Run 1	As before
$B^0 \to D^{\mp}\pi^{\pm}$	$D^+ \to K^-\pi^+\pi^+$	[37]	Run 1	As before
$B^0_s \to D^{\mp}_s K^{\pm}$	$D^+_s \to h^+h^-\pi^+$	[38]	Run 1	As before
$B^0_s \to D^{\mp}_s K^{\pm}\pi^+\pi^-$	$D^+_s \to h^+h^-\pi^+$	[39]	Run 1&2	As before
D decay	Observable(s)	Ref.	Dataset	Status since Ref. [14]
$D^0 \to h^+h^-$	ΔA_{CP}	[24, 40, 41]	Run 1&2	As before
$D^0 \to K^+K^-$	$A_{CP}(K^+K^-)$	[16, 24, 25]	Run 2	**New**
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[42]	Run 1	As before
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[15]	Run 2	**New**
$D^0 \to h^+h^-$	ΔY	[43–46]	Run 1&2	As before
$D^0 \to K^+\pi^-$ (Single Tag)	$R^{\pm}$, $(x'^{\pm})^2$, $y'^{\pm}$	[47]	Run 1	As before
$D^0 \to K^+\pi^-$ (Double Tag)	$R^{\pm}$, $(x'^{\pm})^2$, $y'^{\pm}$	[48]	Run 1&2(*)	As before
$D^0 \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	$(x^2+y^2)/4$	[49]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x, y	[50]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[51]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[52]	Run 2	As before
$D^0 \to K^0_S\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_{CP}, Δx, Δy	[17]	Run 2	**New**

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
$B^{\pm} \to DK^{*\pm}$	$\kappa_{B^{\pm}}^{DK^{*\pm}}$	LHCb	[33]	As before
$B^0 \to DK^{*0}$	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[53]	As before
$B^0 \to D^{\mp}\pi^{\pm}$	β	HFLAV	[13]	As before
$B^0_s \to D^{\mp}_s K^{\pm}(\pi\pi)$	ϕ_s	HFLAV	[13]	As before
$D \to K^+\pi^-$	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[27]	**New**
$D \to K^+\pi^-$	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[28]	**New**
$D \to h^+h^-\pi^0$	$F^+_{\pi\pi\pi^0}$, $F^+_{KK\pi^0}$	CLEO-c	[54]	As before
$D \to \pi^+\pi^-\pi^+\pi^-$	$F^+_{4\pi}$	CLEO-c+BESIII	[26, 54]	**Updated**
$D \to K^+\pi^-\pi^0$	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[55–57]	As before
$D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
$D \to K^0_S K^{\pm}\pi^{\mp}$	$r_D^{K^0_S K\pi}$, $\delta_D^{K^0_S K\pi}$, $\kappa_D^{K^0_S K\pi}$	CLEO	[58]	As before
$D \to K^0_S K^{\pm}\pi^{\mp}$	$r_D^{K^0_S K\pi}$	LHCb	[59]	As before

Input to LHCb γ combination

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to D^*h^\pm$	$D \to h$			
$B^\pm \to DK^{*\pm}$	$D \to h$			
$B^\pm \to DK^{*\pm}$	$D \to h$			
$B^\pm \to Dh^\pm\pi^+\pi^-$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to K$			
$B^0 \to D^\mp\pi^\pm$	$D^+ \to$			
$B_s^0 \to D_s^\mp K^\pm$	$D_s^+ \to$			
$B_s^0 \to D_s^\mp K^\pm\pi^+\pi^-$	$D_s^+ \to$			

D decay	Observ
$D^0 \to h^+h^-$	ΔA_{CP}
$D^0 \to K^+K^-$	$A_{CP}(K$
$D^0 \to h^+h^-$	$y_{CP} -$
$D^0 \to h^+h^-$	$y_{CP} -$
$D^0 \to h^+h^-$	ΔY
$D^0 \to K^+\pi^-$ (Single Tag)	$R^\pm$, $(x$
$D^0 \to K^+\pi^-$ (Double Tag)	$R^\pm$, $(x$
$D^0 \to K^\pm\pi^\mp\pi^+\pi^-$	$(x^2 + y$
$D^0 \to K_S^0\pi^+\pi^-$	x, y
$D^0 \to K_S^0\pi^+\pi^-$	x_{CP}, y_C
$D^0 \to K_S^0\pi^+\pi^-$	x_{CP}, y_C
$D^0 \to K_S^0\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_C

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
		LHCb	[33]	As before
		LHCb	[53]	As before
		HFLAV	[13]	As before
		HFLAV	[13]	As before
		CLEO-c	[27]	**New**
	$\delta_D^{K\pi}$	BESIII	[28]	**New**
		CLEO-c	[54]	As before
		CLEO-c+BESIII	[26, 54]	**Updated**
		CLEO-c+LHCb+BESIII	[55–57]	As before
		CLEO-c+LHCb+BESIII	[49, 55–57]	As before
		CLEO	[58]	As before
		LHCb	[59]	As before

Too much for one slide to be readable. Key point: It's a lot. And there's more, e.g.

BES III measurements on $D \to$

$K_S\pi^+\pi^-\pi^0$ arXiv:2305.03975 (2023)

$K^+K^-\pi^+\pi^-$ PRD 107 (2023) 3, 032009

LHCb measurements with

$B^\pm \to DK^\pm,\ D \to$

$K^+K^-\pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-$

arXiv:2301.10328 (2023)

LHCb γ combination

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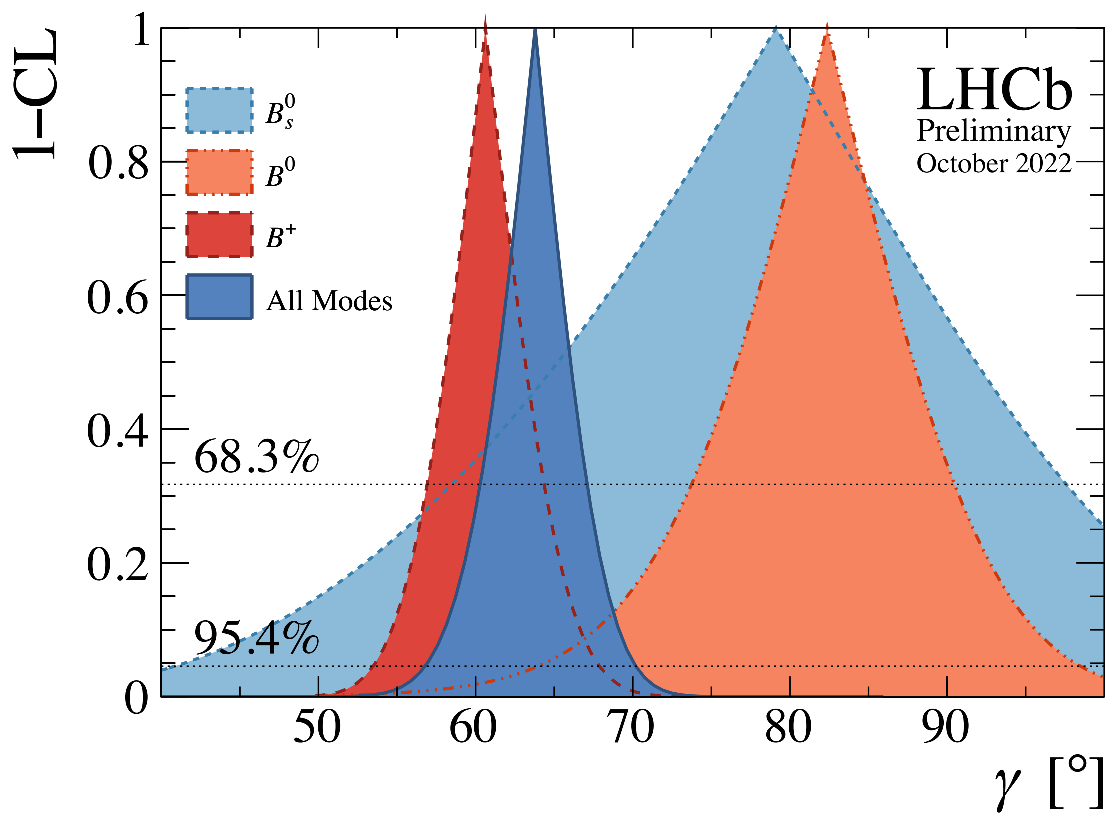
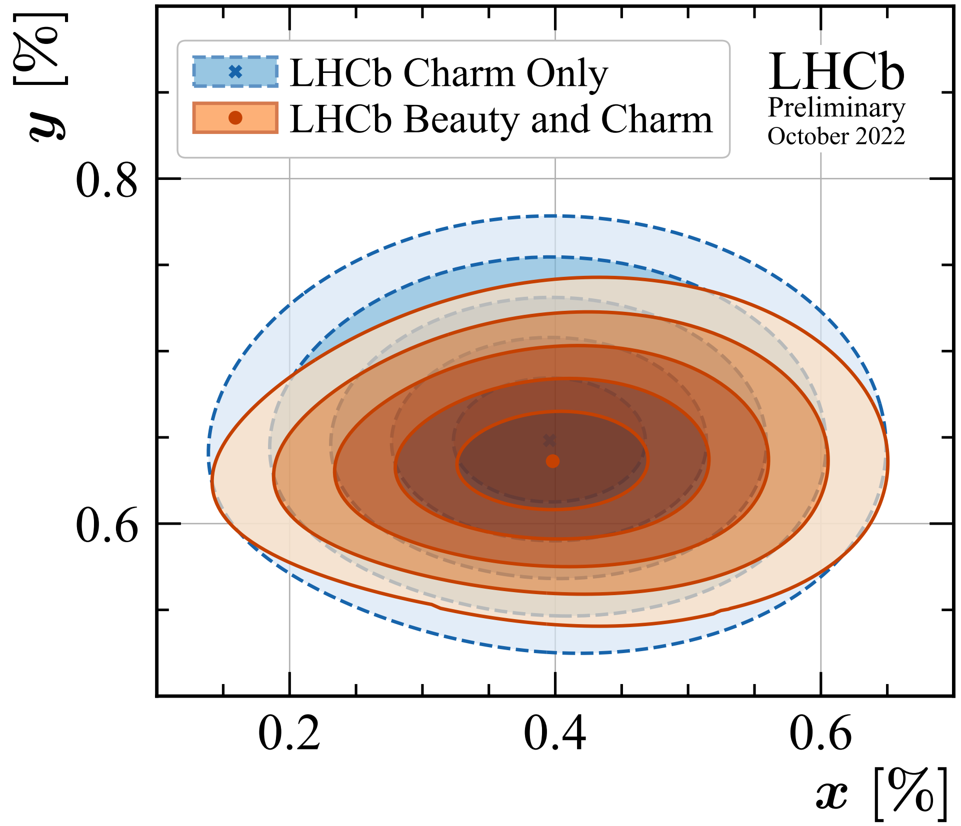
impact on charm mixing





$$\gamma = 63.8^{\circ}{}^{+3.5^{\circ}}_{-3.7^{\circ}}$$

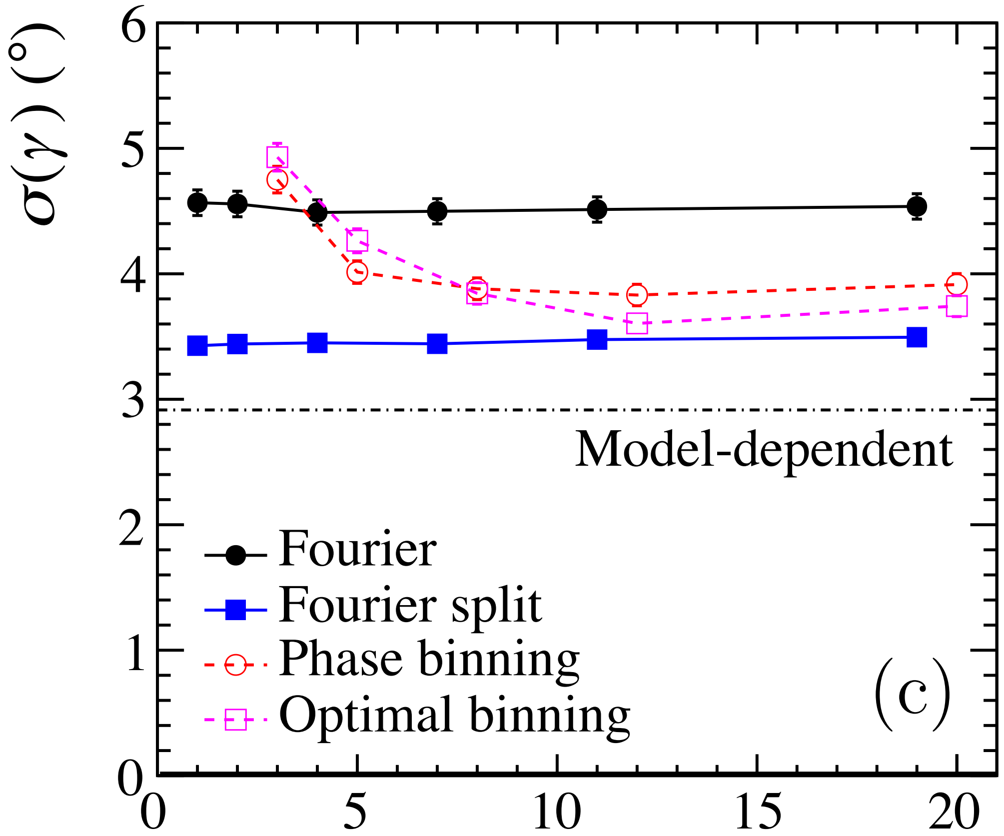
$$x = \left(3.98^{+0.50}_{-0.49}\right) \cdot 10^{-3}$$

$$y = \left(6.36^{+0.20}_{-0.19}\right) \cdot 10^{-3}$$

Unbinned model-independent method.

Anton Poluektov: Eur.Phys.J.C 78 (2018) 2, 121

- Project 2-D Dalitz plot onto 1D.
- Use amplitude model to associate each point in phase space to a phase difference δ^{model}.
- $c_i, s_i \to C(\delta^{\text{model}}), S(\delta^{\text{model}})$, functions C, S parameterised in a generic way (Fourier series)



Precision using various approaches, with 2×10^4 $B^\pm \to DK^\pm$ events and 10^4 $D\overline{D}$ events

Expect precision between binned and model-dependent approach.

Other unbinned methods exists

- Anton Poluektov: Eur.Phys.J.C 78 (2018) 2, 121. Projects 2-D Dalitz plot onto 1D. Achieves precision between binned and model-dependent approach.

- Jeffrey V. Backus et al, arXiv:2211.05133, integrate over the 2-D Dalitz plot in an unbinned way. Get a precision of ~5° for similar data set sizes we use, however, comparison is difficult due to different assumptions on the values of γ and δ_B, and implementation differences in amplitude model.

- In contrast to these methods and the binned method, we do not do any integration, averaging or projection and therefore do not suffer the associated information loss.

Input to LHCb γ combination

LHCb-CONF-2022-003

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^{\pm} \to Dh^{\pm}$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^{\pm} \to Dh^{\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[30]	Run 1	As before
$B^{\pm} \to Dh^{\pm}$	$D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	[18]	Run 1&2	**New**
$B^{\pm} \to Dh^{\pm}$	$D \to h^+h^-\pi^0$	[19]	Run 1&2	**Updated**
$B^{\pm} \to Dh^{\pm}$	$D \to K^0_S h^+h^-$	[31]	Run 1&2	As before
$B^{\pm} \to Dh^{\pm}$	$D \to K^0_S K^{\pm}\pi^{\mp}$	[32]	Run 1&2	As before
$B^{\pm} \to D^*h^{\pm}$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^{\pm} \to DK^{*\pm}$	$D \to h^+h^-$	[33]	Run 1&2(*)	As before
$B^{\pm} \to DK^{*\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[33]	Run 1&2(*)	As before
$B^{\pm} \to Dh^{\pm}\pi^+\pi^-$	$D \to h^+h^-$	[34]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to h^+h^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to h^+\pi^-\pi^+\pi^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	[36]	Run 1	As before
$B^0 \to D^{\mp}\pi^{\pm}$	$D^+ \to K^-\pi^+\pi^+$	[37]	Run 1	As before
$B^0_s \to D_s^{\mp}K^{\pm}$	$D_s^+ \to h^+h^-\pi^+$	[38]	Run 1	As before
$B^0_s \to D_s^{\mp}K^{\pm}\pi^+\pi^-$	$D_s^+ \to h^+h^-\pi^+$	[39]	Run 1&2	As before

D decay	Observable(s)	Ref.	Dataset	Status since Ref. [14]
$D^0 \to h^+h^-$	ΔA_{CP}	[24, 40, 41]	Run 1&2	As before
$D^0 \to K^+K^-$	$A_{CP}(K^+K^-)$	[16, 24, 25]	Run 2	**New**
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[42]	Run 1	As before
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[15]	Run 2	**New**
$D^0 \to h^+h^-$	ΔY	[43–46]	Run 1&2	As before
$D^0 \to K^+\pi^-$ (Single Tag)	$R^{\pm}$, $(x'^{\pm})^2$, $y'^{\pm}$	[47]	Run 1	As before
$D^0 \to K^+\pi^-$ (Double Tag)	$R^{\pm}$, $(x'^{\pm})^2$, $y'^{\pm}$	[48]	Run 1&2(*)	As before
$D^0 \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	$(x^2+y^2)/4$	[49]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x, y	[50]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[51]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[52]	Run 2	As before
$D^0 \to K^0_S\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_{CP}, Δx, Δy	[17]	Run 2	**New**

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
$B^{\pm} \to DK^{*\pm}$	$\kappa_{B^{\pm}}^{DK^{*\pm}}$	LHCb	[33]	As before
$B^0 \to DK^{*0}$	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[53]	As before
$B^0 \to D^{\mp}\pi^{\pm}$	β	HFLAV	[13]	As before
$B^0_s \to D_s^{\mp}K^{\pm}(\pi\pi)$	ϕ_s	HFLAV	[13]	As before
$D \to K^+\pi^-$	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[27]	**New**
$D \to K^+\pi^-$	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[28]	**New**
$D \to h^+h^-\pi^0$	$F^+_{\pi\pi\pi^0}$, $F^+_{KK\pi^0}$	CLEO-c	[54]	As before
$D \to \pi^+\pi^-\pi^+\pi^-$	$F^+_{4\pi}$	CLEO-c+BESIII	[26, 54]	**Updated**
$D \to K^+\pi^-\pi^0$	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[55–57]	As before
$D \to K^{\pm}\pi^{\mp}\pi^+\pi^-$	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
$D \to K^0_S K^{\pm}\pi^{\mp}$	$r_D^{K^0_S K\pi}$, $\delta_D^{K^0_S K\pi}$, $\kappa_D^{K^0_S K\pi}$	CLEO	[58]	As before
$D \to K^0_S K^{\pm}\pi^{\mp}$	$r_D^{K^0_S K\pi}$	LHCb	[59]	As before

$$\gamma = 63.8^{\circ}{}^{+3.5^{\circ}}_{-3.7^{\circ}}$$

Input to LHCb γ combination

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^\pm \to$				
$B^0 \to$				
$B^0 \to$				
$B^0 \to$				
$B^0 \to$				
$B^0_s \to$				
$B^0_s \to$				

D deca…				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				

Too much for one slide to be readable. Key point: It's a lot. And there's more, e.g.

BES III measurements on $D \to$
$K_S\pi^+\pi^-\pi^0$ arXiv:2305.03975 (2023)
$K^+K^-\pi^+\pi^-$ PRD 107 (2023) 3, 032009

LHCb measurements with
$B^\pm \to DK^\pm,\ D \to K^+K^-\pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-$
arXiv:2301.10328 (2023)

BES III and others

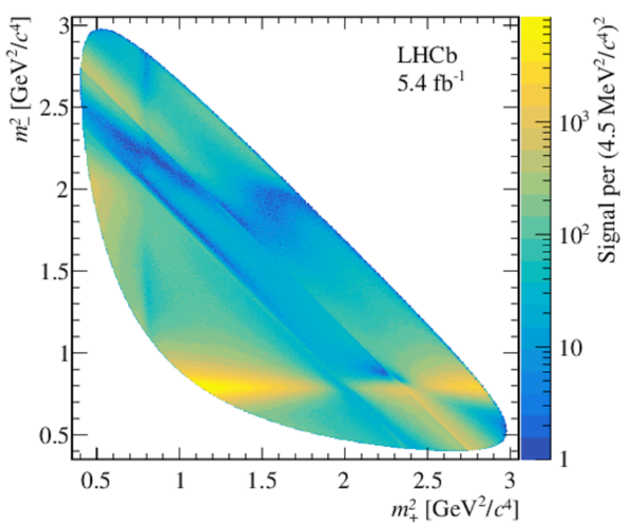
Decay	Parameters	Source	Ref.	Status since Ref. [14]
	$\kappa_{B^\pm}^{DK^{*\pm}}$	LHCb	[33]	As before
	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[53]	As before
	β	HFLAV	[13]	As before
…π)	ϕ_s	HFLAV	[13]	As before
	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[27]	**New**
	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[28]	**New**
	$F_{\pi\pi\pi^0}^+$, $F_{KK\pi^0}^+$	CLEO-c	[54]	As before
	$F_{4\pi}^+$	CLEO-c+BESIII	[26, 54]	**Updated**
	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[55–57]	As before
	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
	$r_D^{K^0_SK\pi}$, $\delta_D^{K^0_SK\pi}$, $\kappa_D^{K^0_SK\pi}$	CLEO	[58]	As before
	$r_D^{K^0_SK\pi}$	LHCb	[59]	As before

$$\gamma = 63.8^{\circ}{}^{+3.5^\circ}_{-3.7^\circ}$$

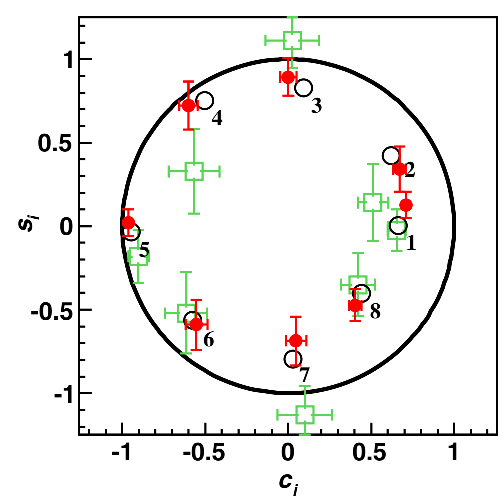
LHCb model-independent mixing with $D^0 \to K_S\pi^+\pi^-$

Same BES III input also critical for charm mixing

first observation of non-zero $x = \frac{\Delta m}{\Gamma}$, i.e. of a mass difference between the two charm mass eigenstates.



$+$



$$x = (3.98^{+0.56}_{-0.54}) \times 10^{-3},$$
$$y = (\ 4.6^{+1.5}_{-1.4}\ ) \times 10^{-3},$$
$$|q/p| = 0.996 \pm 0.052,$$
$$\phi = 0.056^{+0.047}_{-0.051}.$$

3.1×10^7

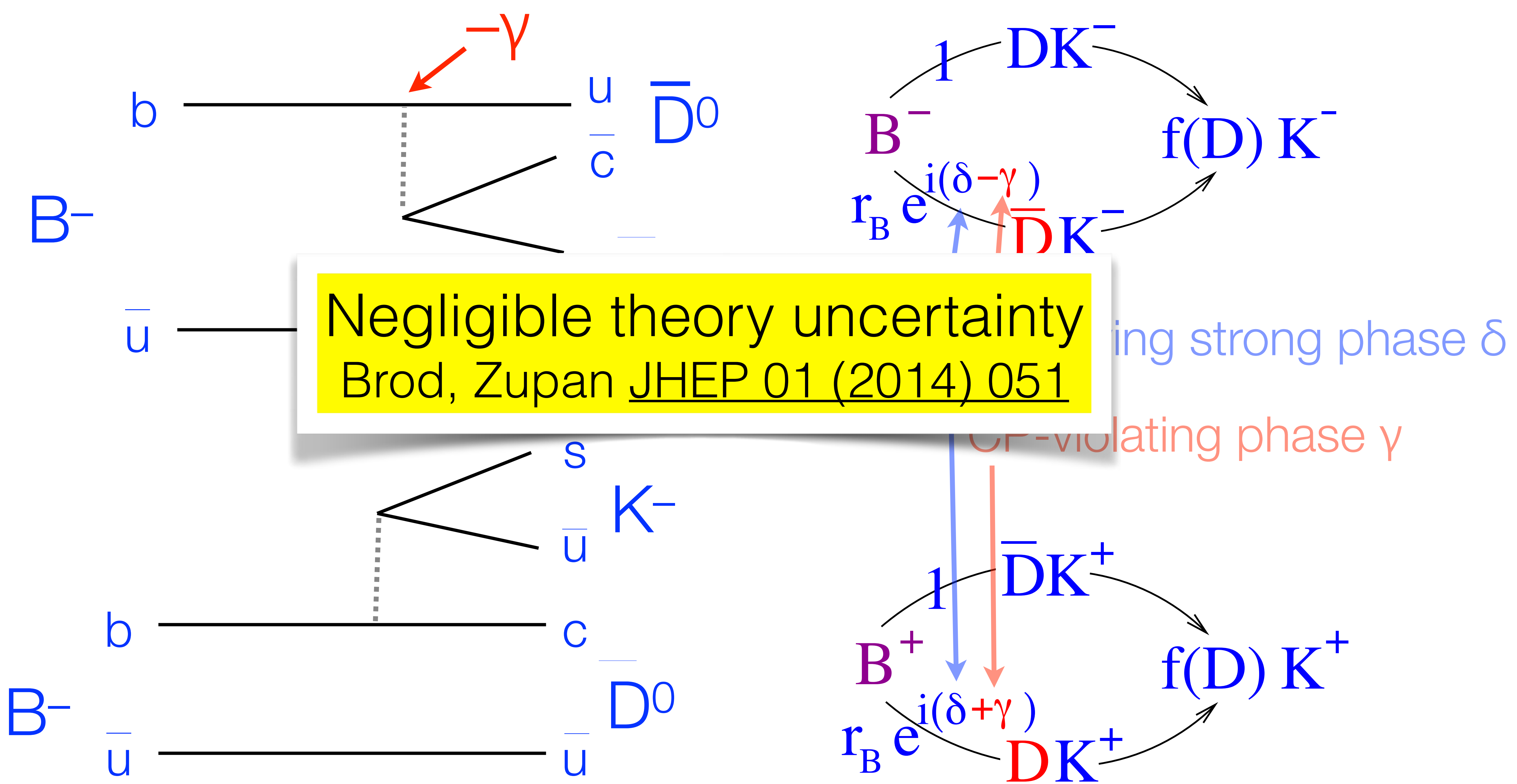
$D^0 \to K^0_S\pi^+\pi^-$

BESIII: PRL 124 (2020) 24, 241802

LHCb: PRL 127 (2021) 11, 111801

Method: Phys.Rev. D99 (2019) no.1, 012007

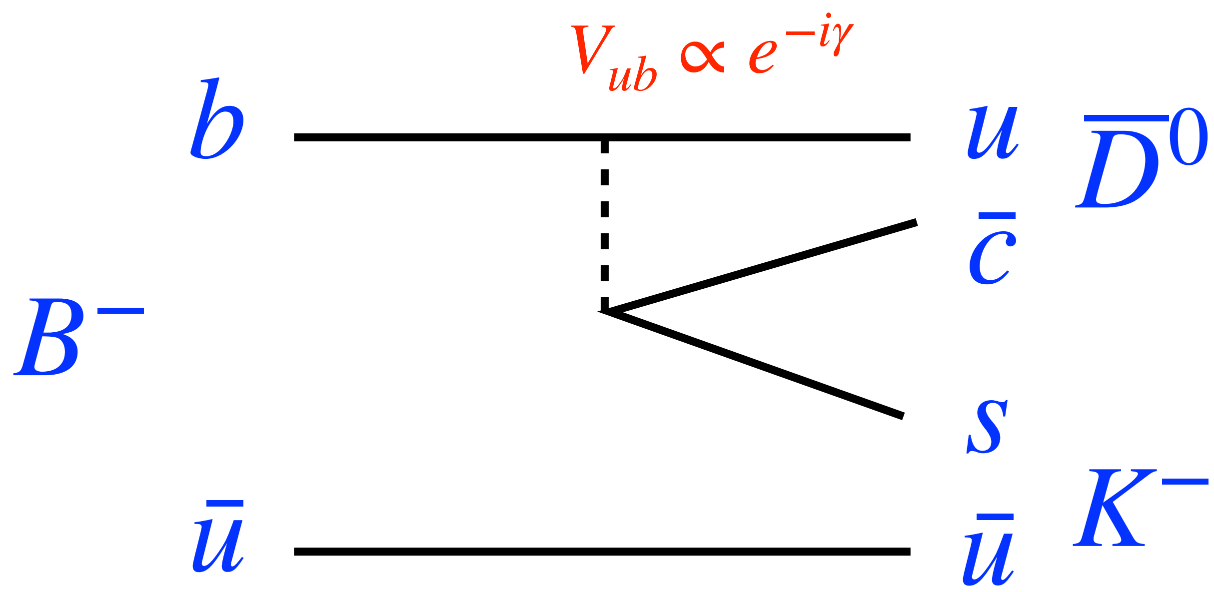
$B^{\pm} \to DK^{\pm}$



Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

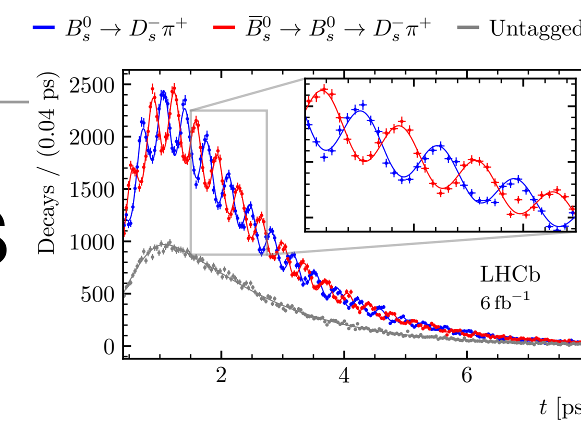
Trees and loops

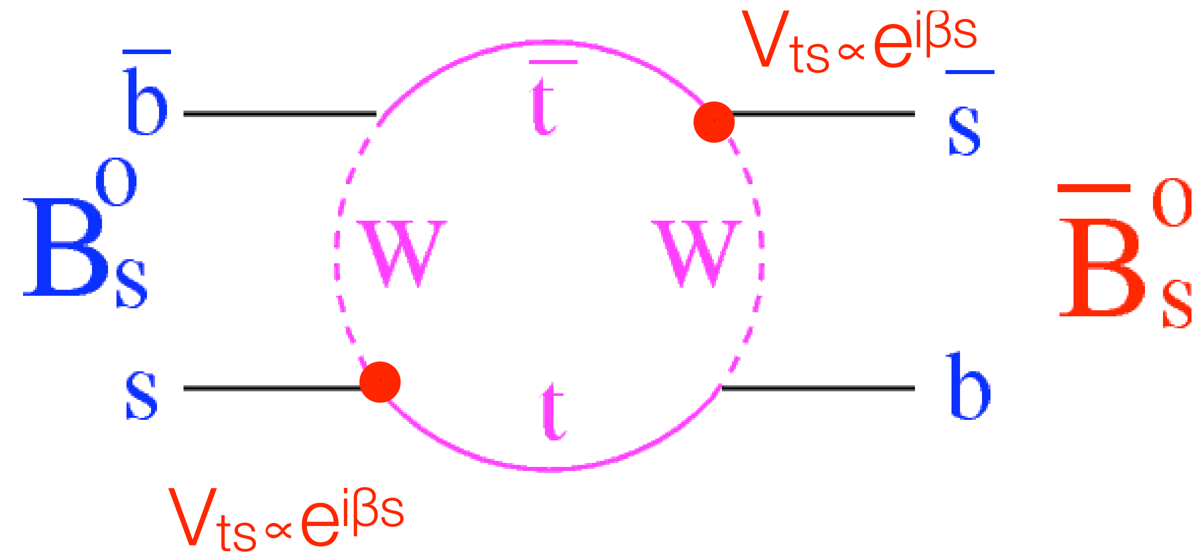
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5



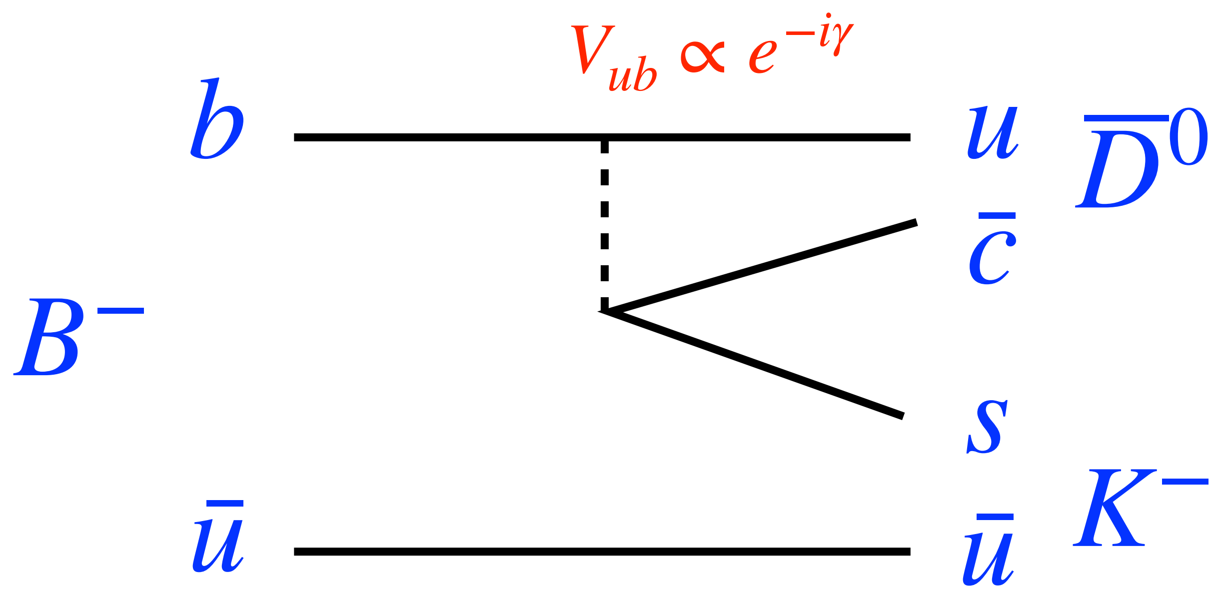


"New Physics" in trees generally seen as less likely than in loops. (Lenz et al find however that there is room for NP in trees that could affect γ by several degrees.) In any case: would like to compare tees and loops.

JHEP 06 (2014) 040, JHEP 07 (2020) 177

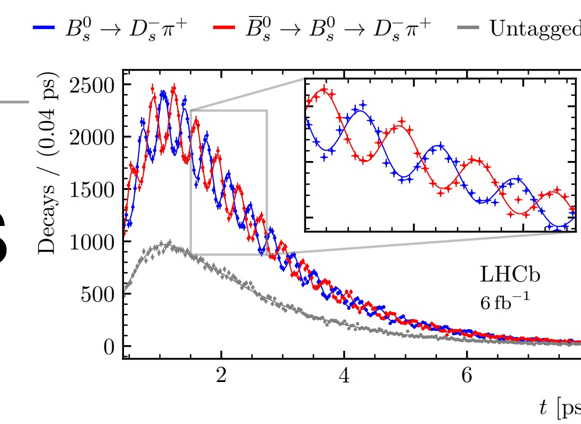
Trees and loops

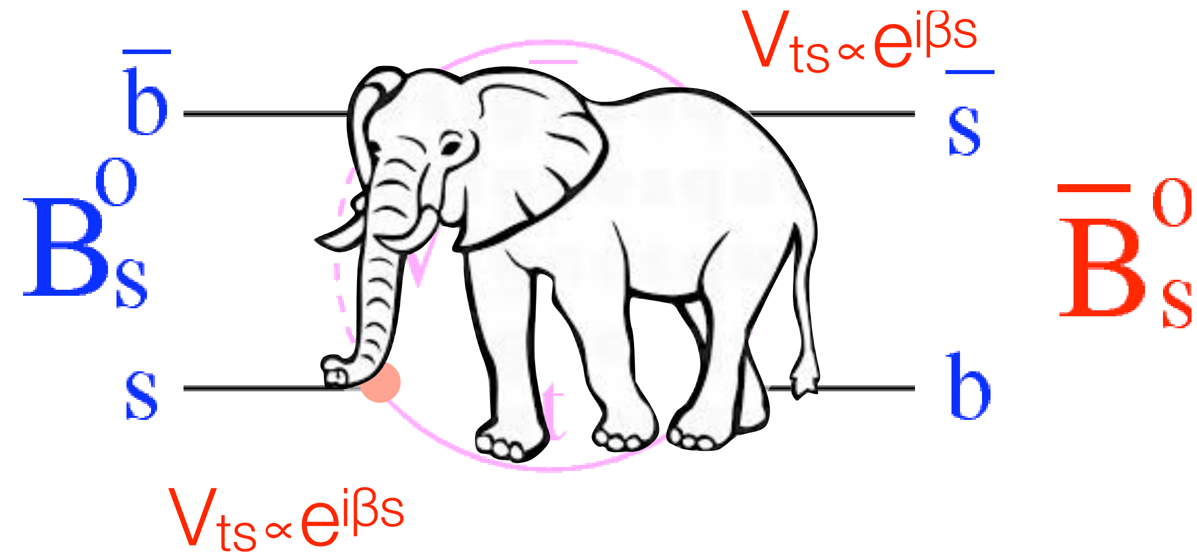
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5





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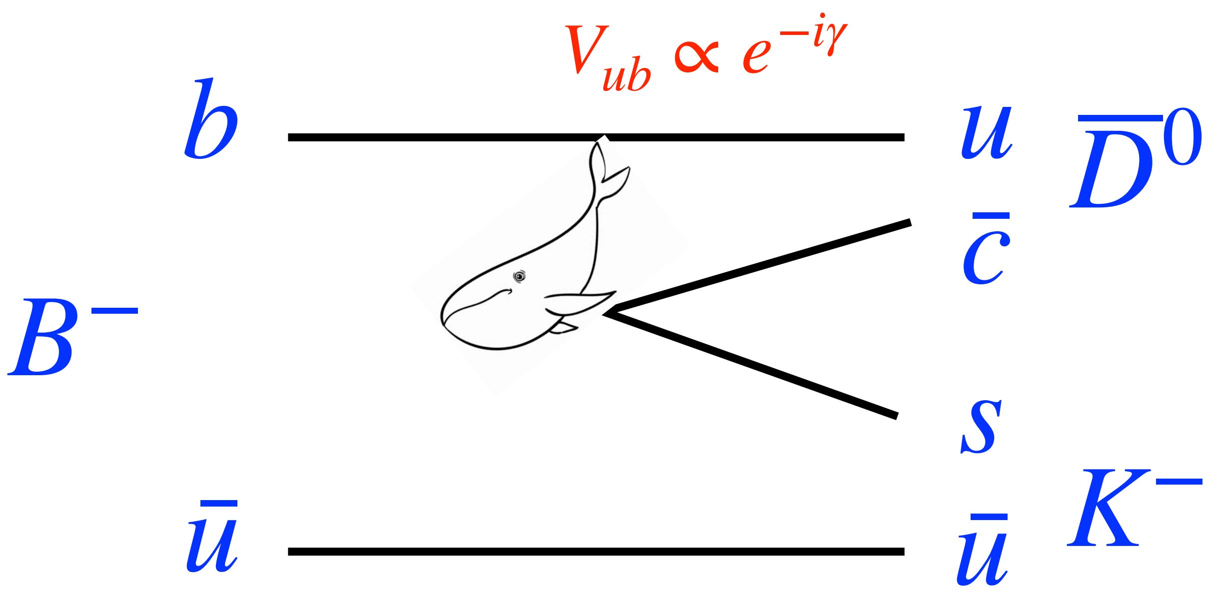
JHEP 06 (2014) 040, JHEP 07 (2020) 177

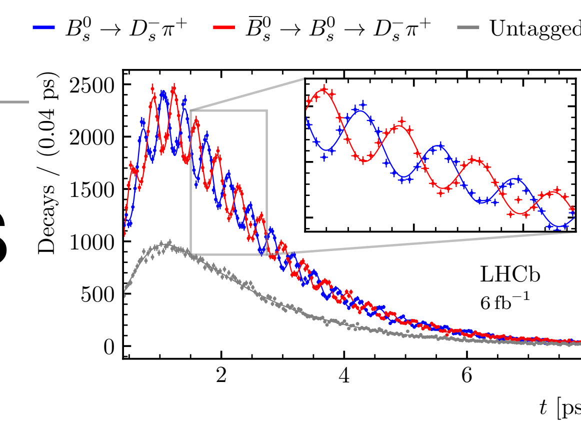
Trees and loops

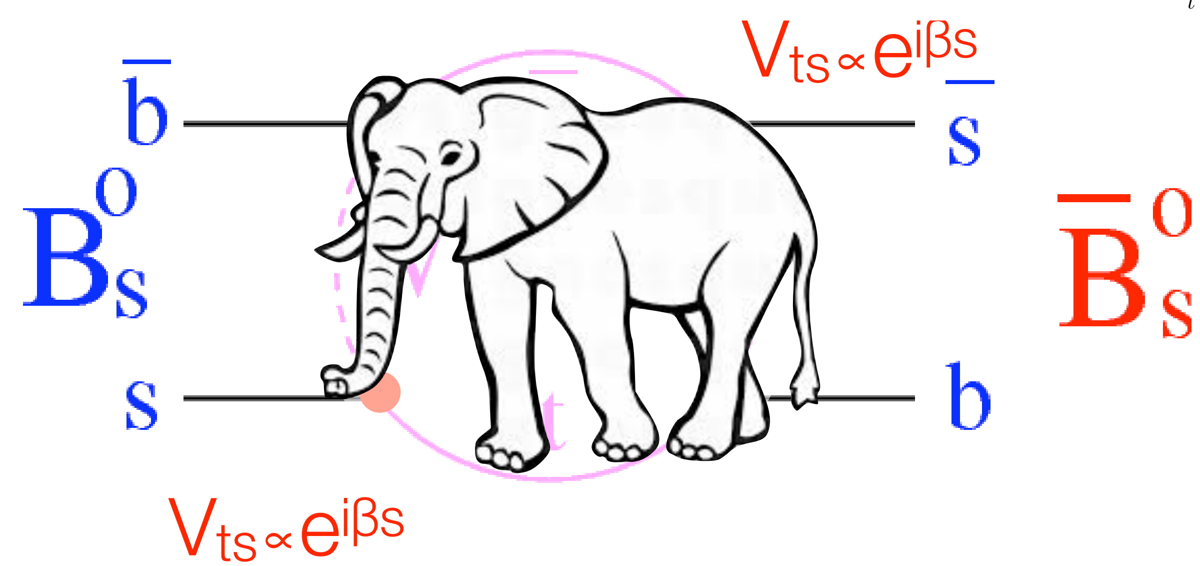
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5

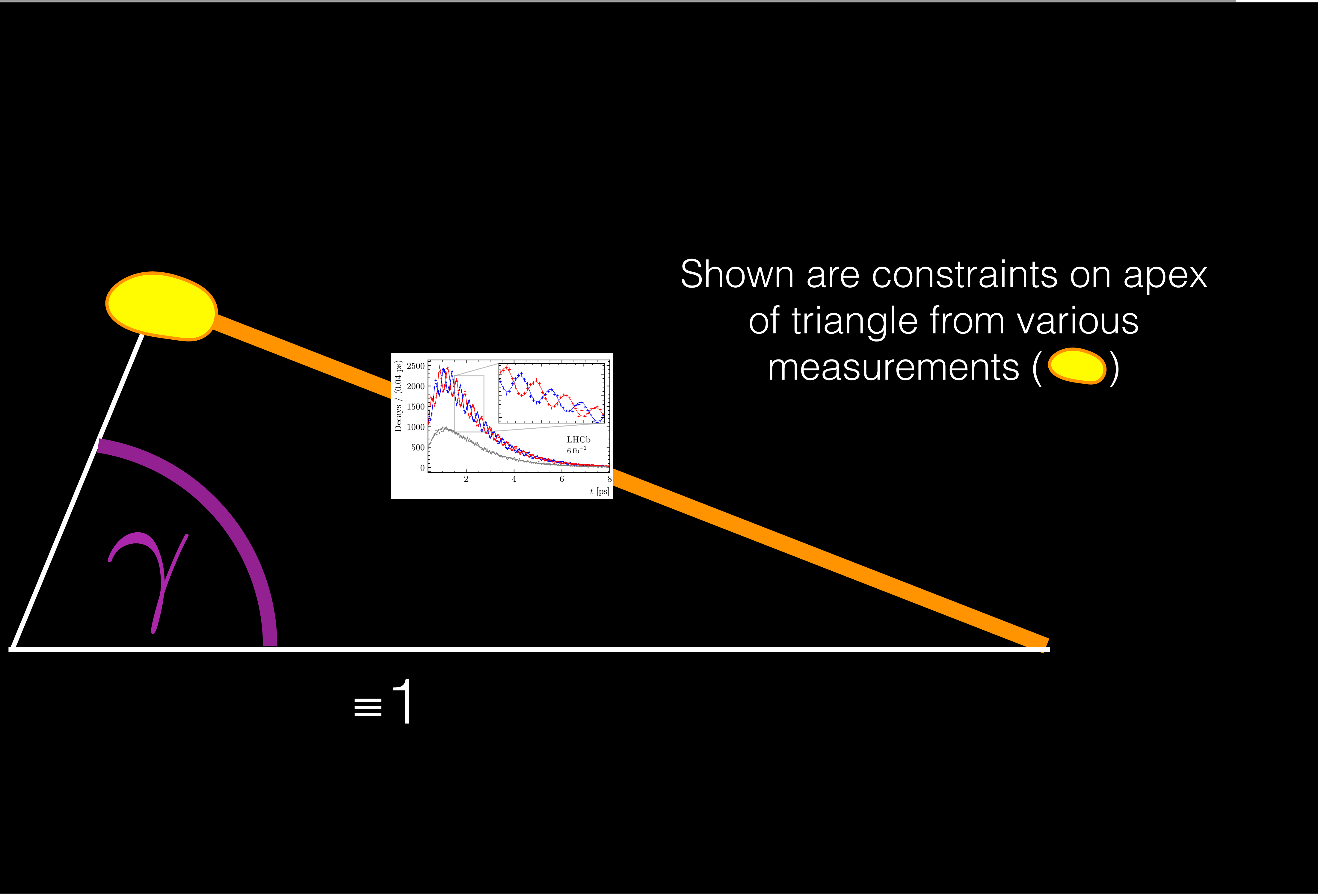




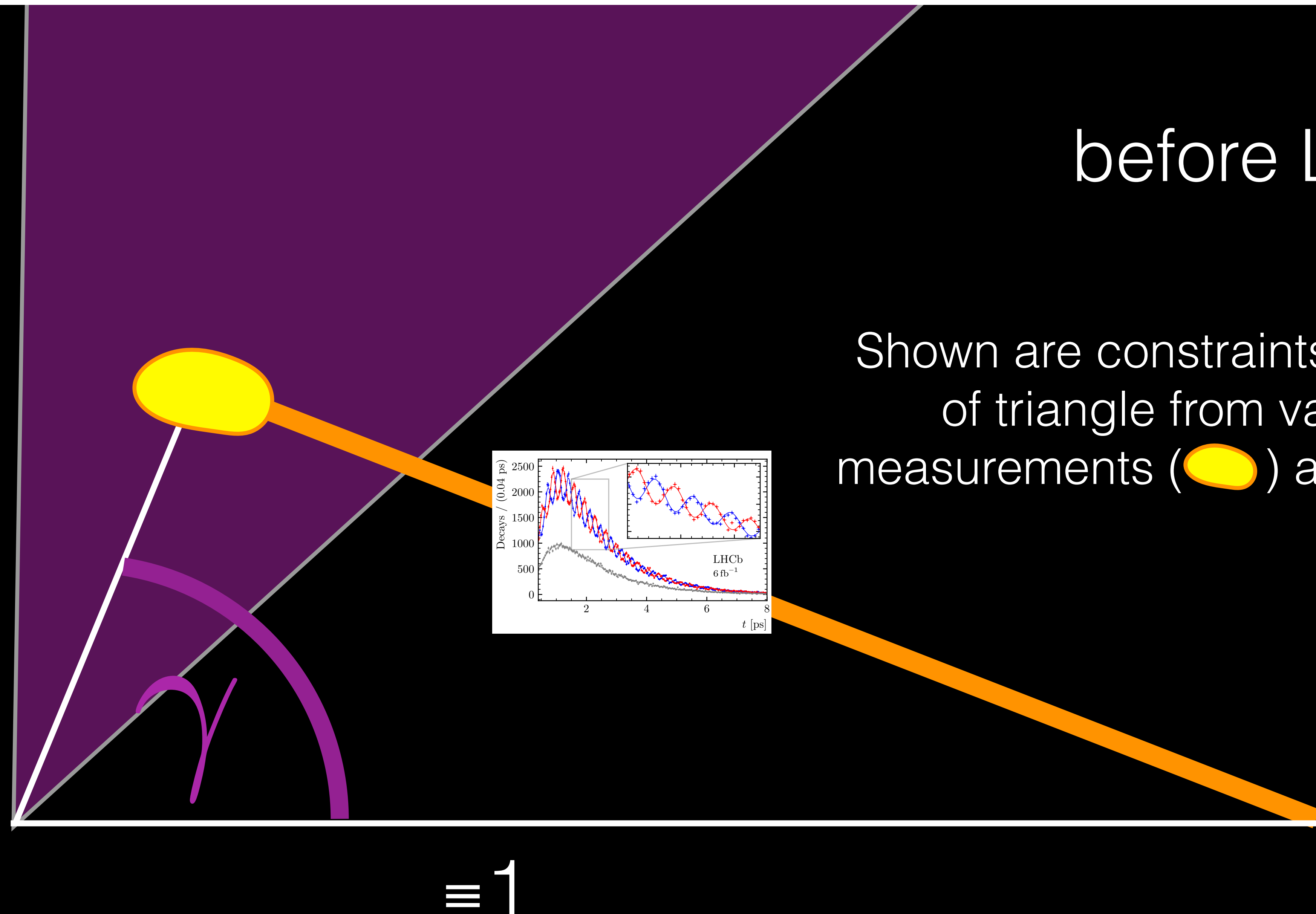
"New Physics" in trees generally seen as less likely than in loops. (Lenz et al find however that there is room for NP in trees that could affect γ by several degrees.) In any case: would like to compare tees and loops.

JHEP 06 (2014) 040, JHEP 07 (2020) 177

Unitarity triangle
geometric representation of Standard Model constraints



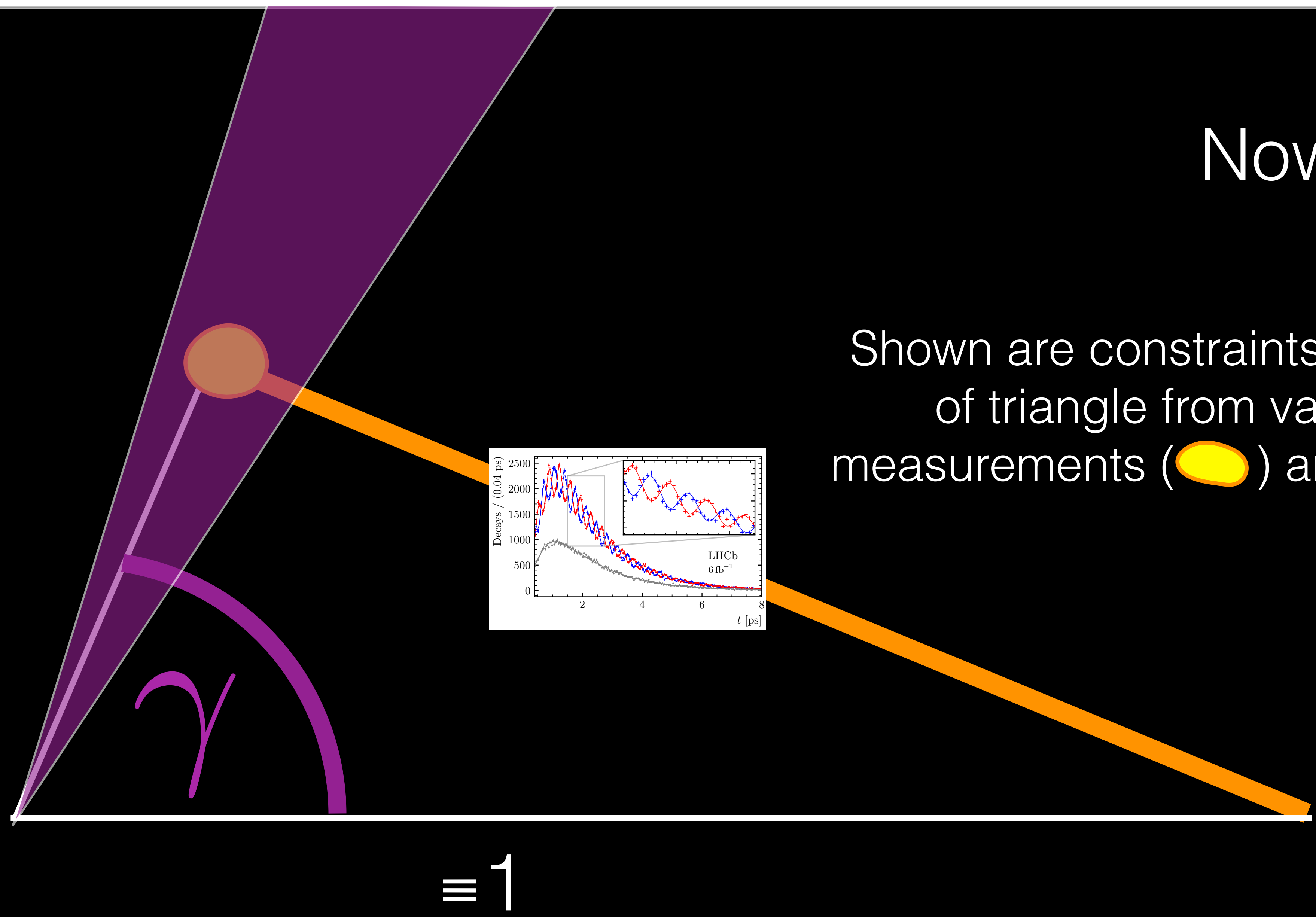
Unitarity triangle



before LHCb

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

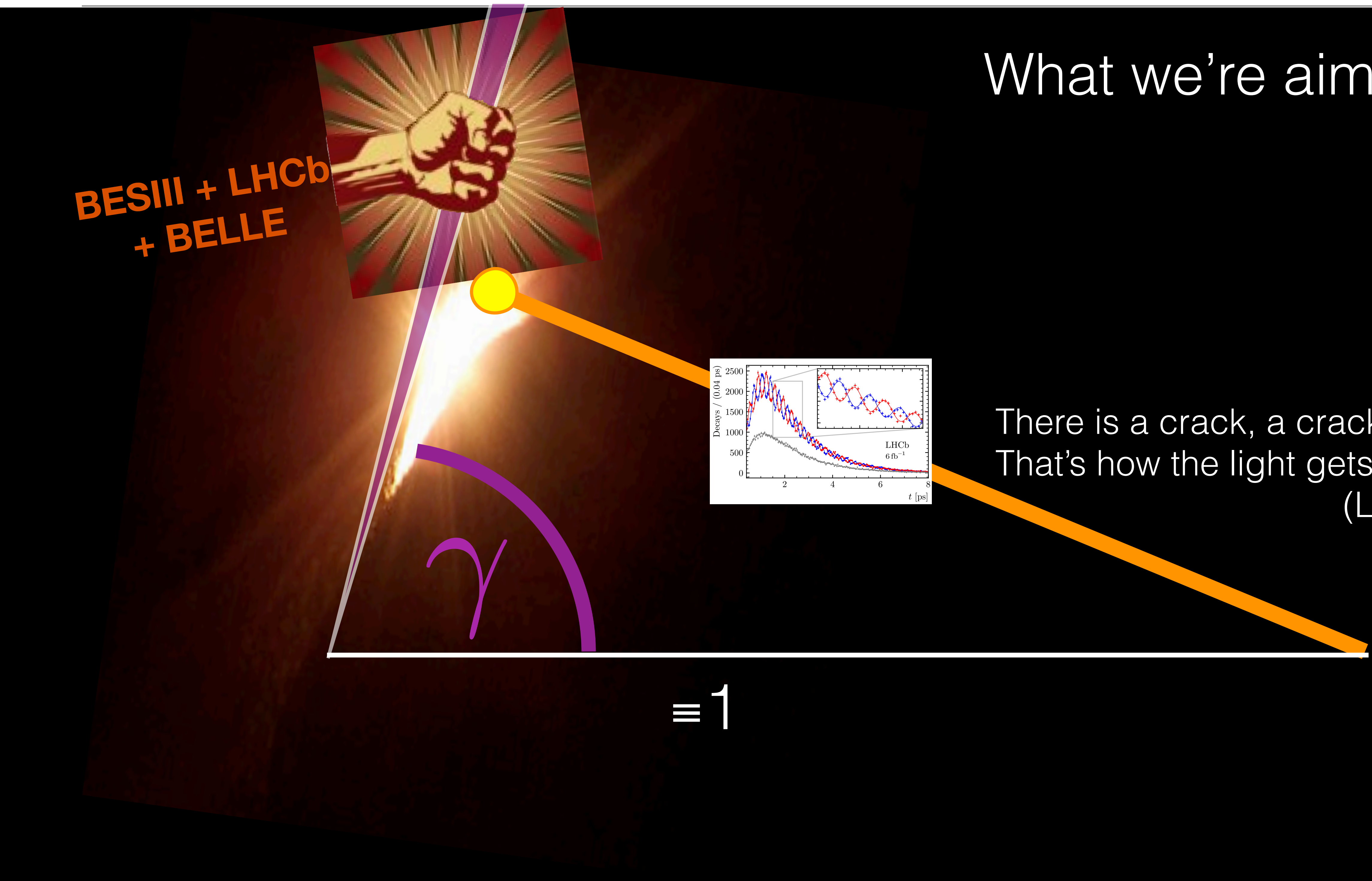
Unitarity triangle



Now

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

Unitarity triangle



What we're aiming for

There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

Measurment of γ with LHCb & BES III data with model-independent binned method.

BESIII: PRL 124 (2020) 24, 241802

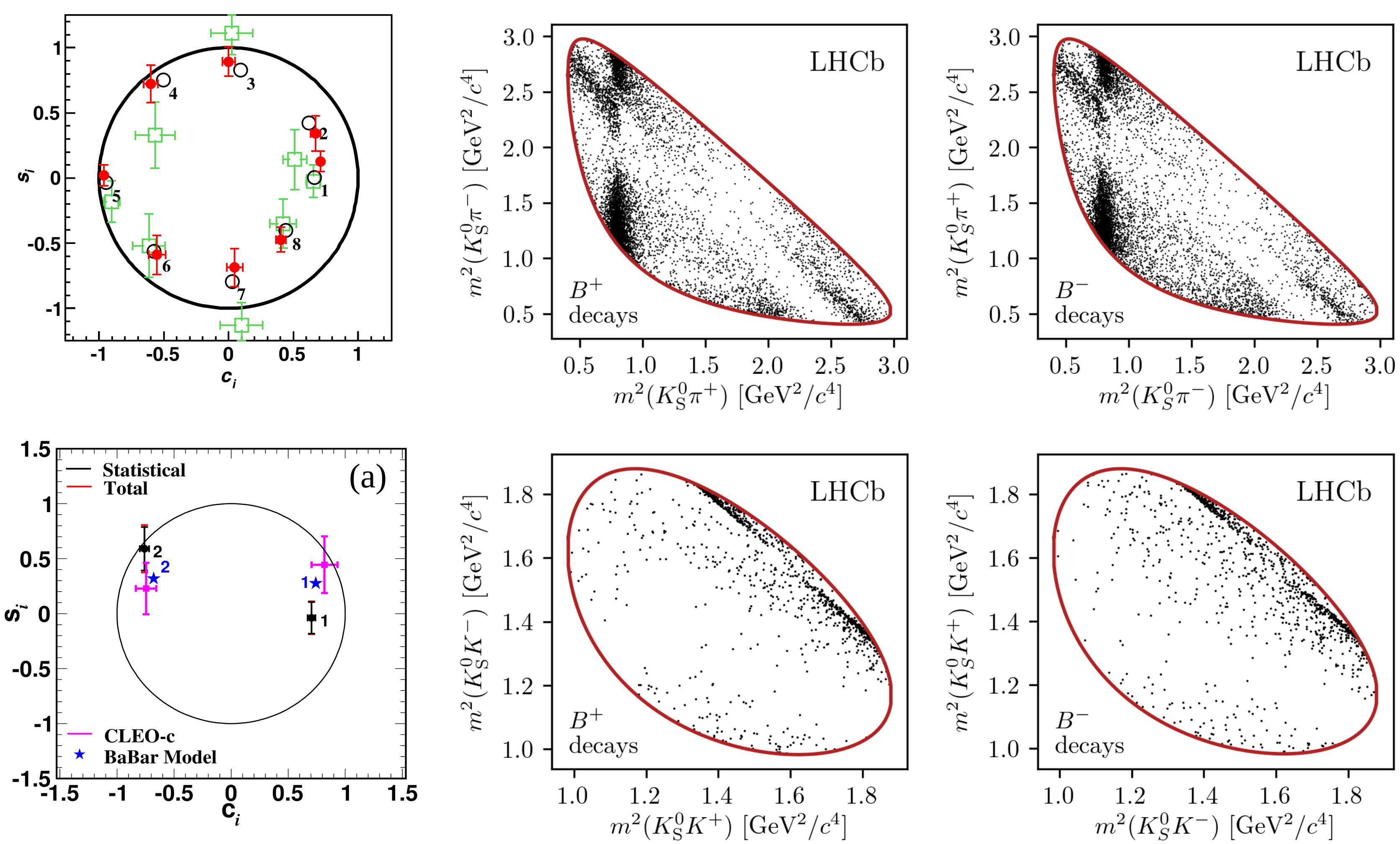
BESIII: PRD 102 (2020) 5, 052008

LHCb: JHEP 02 (2021) 169

$\psi'' \to D\overline{D}$ $B^+ \to DK^+$ $B^- \to DK^-$

$D \to K_S \pi^+ \pi^-$

$D \to K_S K^+ K^-$



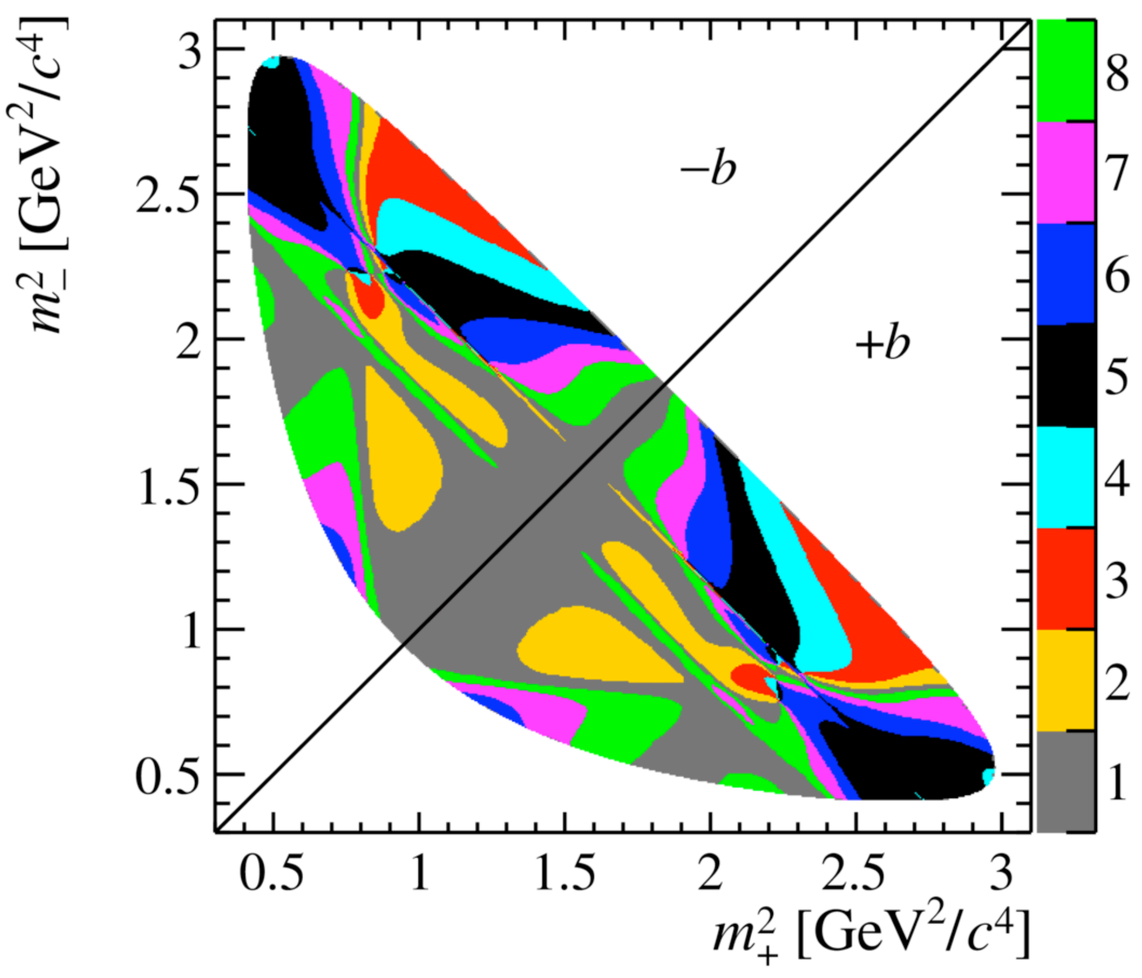
$$\gamma = \left(68.7^{+5.2}_{-5.1}\right)^\circ$$

best individual measurement of γ

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 **(GLW)** Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

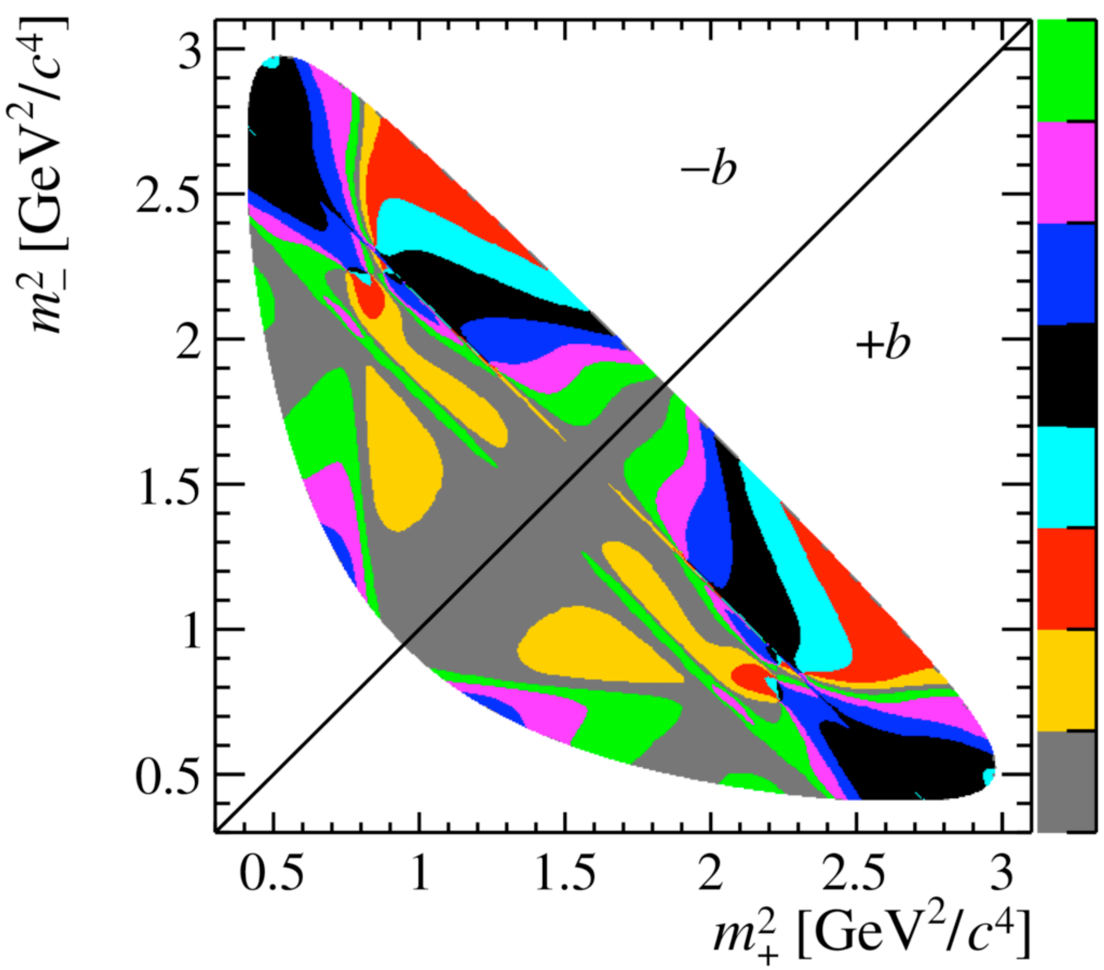
Measurements of c_i, s_i at BES III

Model-informed, optimised binning



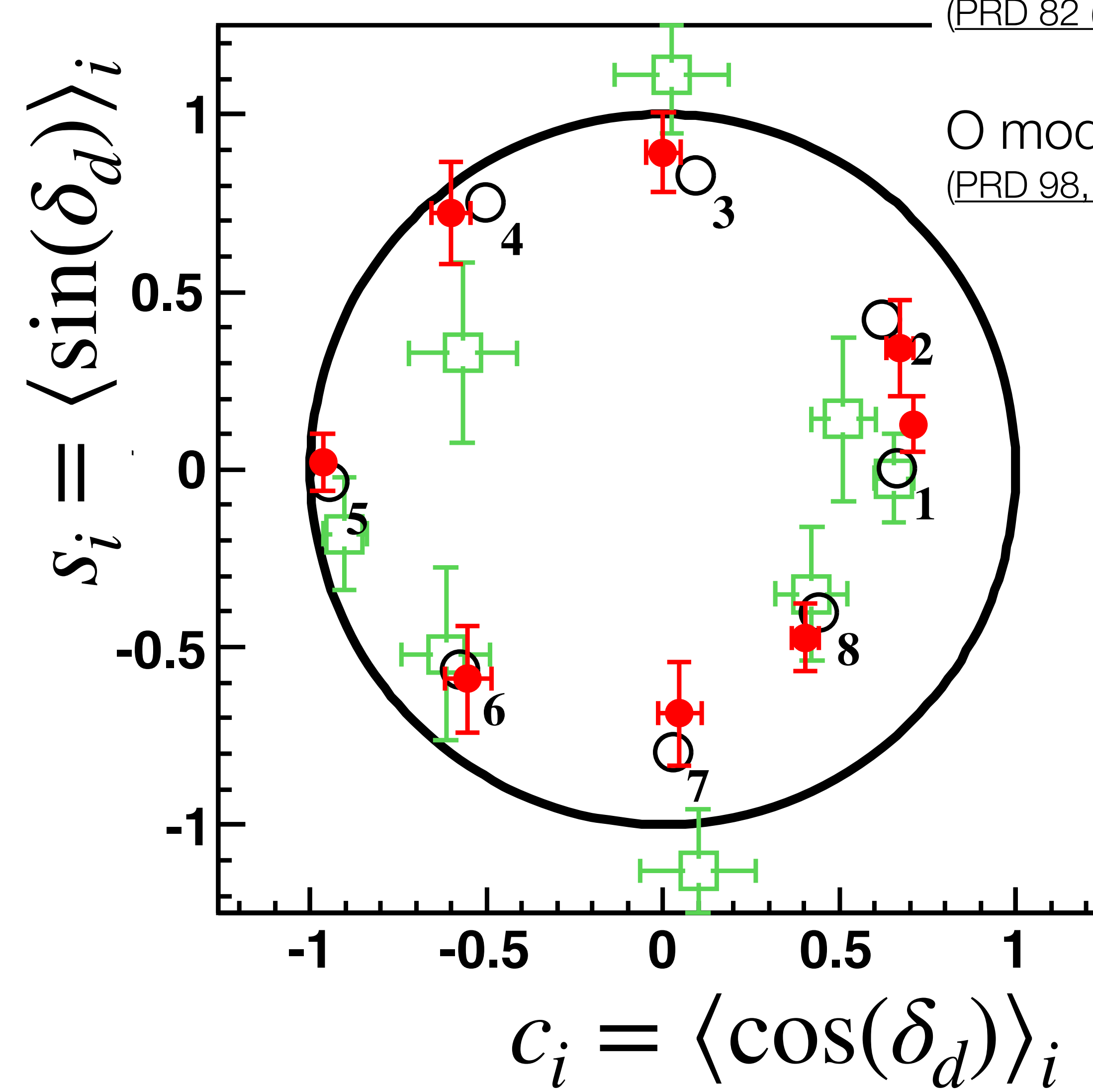
Measurements of c_i, s_i at BES III

Model-informed, optimised binning



BESIII: PRL 124 (2020) 24, 241802

in $D^0 \to K_S\pi^+\pi^-$



● BESIII (PRL 124 (2020) 24, 241802)

□ CLEO-c (PRD 82 (2010) 112006)

O model (PRD 98, 112012)

Model independent, binned γ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binned decay rate:

$$\Gamma\left(B^{\pm} \to D(K_s\pi^+\pi^-)K^{\pm}\right)_i =$$

$$\mathcal{T}_i + r_B^2\mathcal{T}_{-i} + 2r_B\sqrt{\mathcal{T}_i\mathcal{T}_{-i}}\left\{c_i \cos\left(\delta \pm \gamma\right) + s_i \sin\left(\delta \pm \gamma\right)\right\}$$

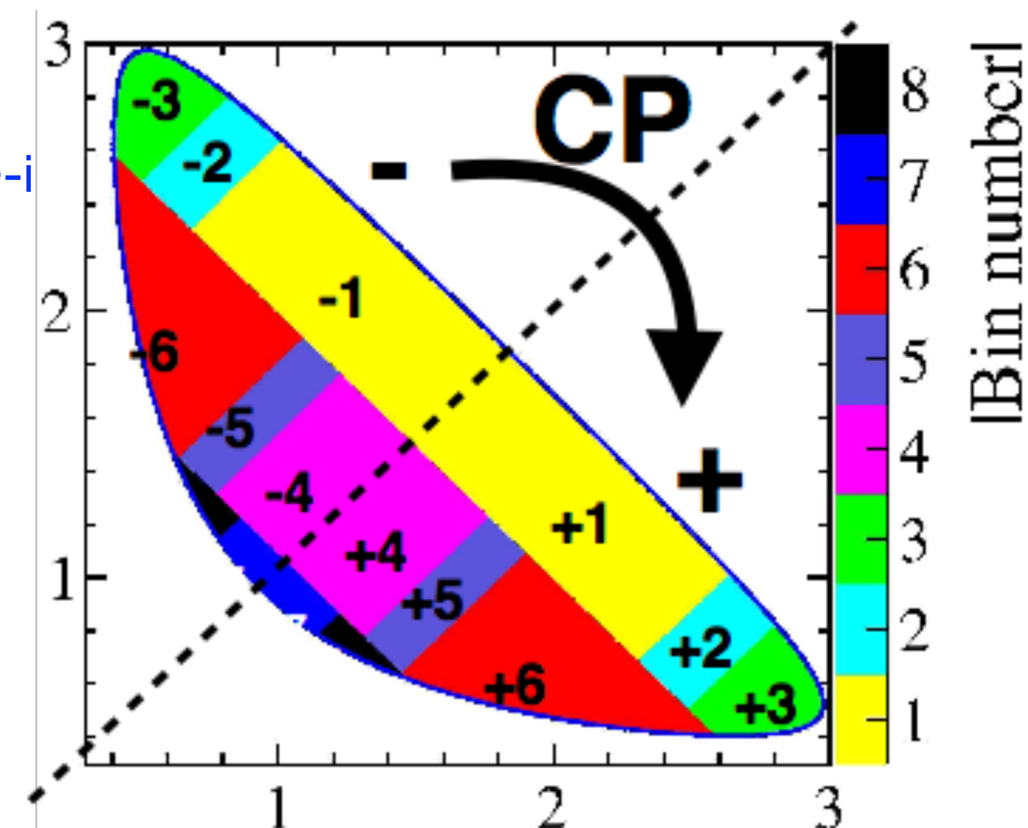
$\mathcal{T}_i$ known from flavour-specifc D decays

(weighted) average of $\cos(\delta_D)$ and $\sin(\delta_D)$ over bin i, where δ_D = phase difference between D→Ksππ and Dbar→Ksππ

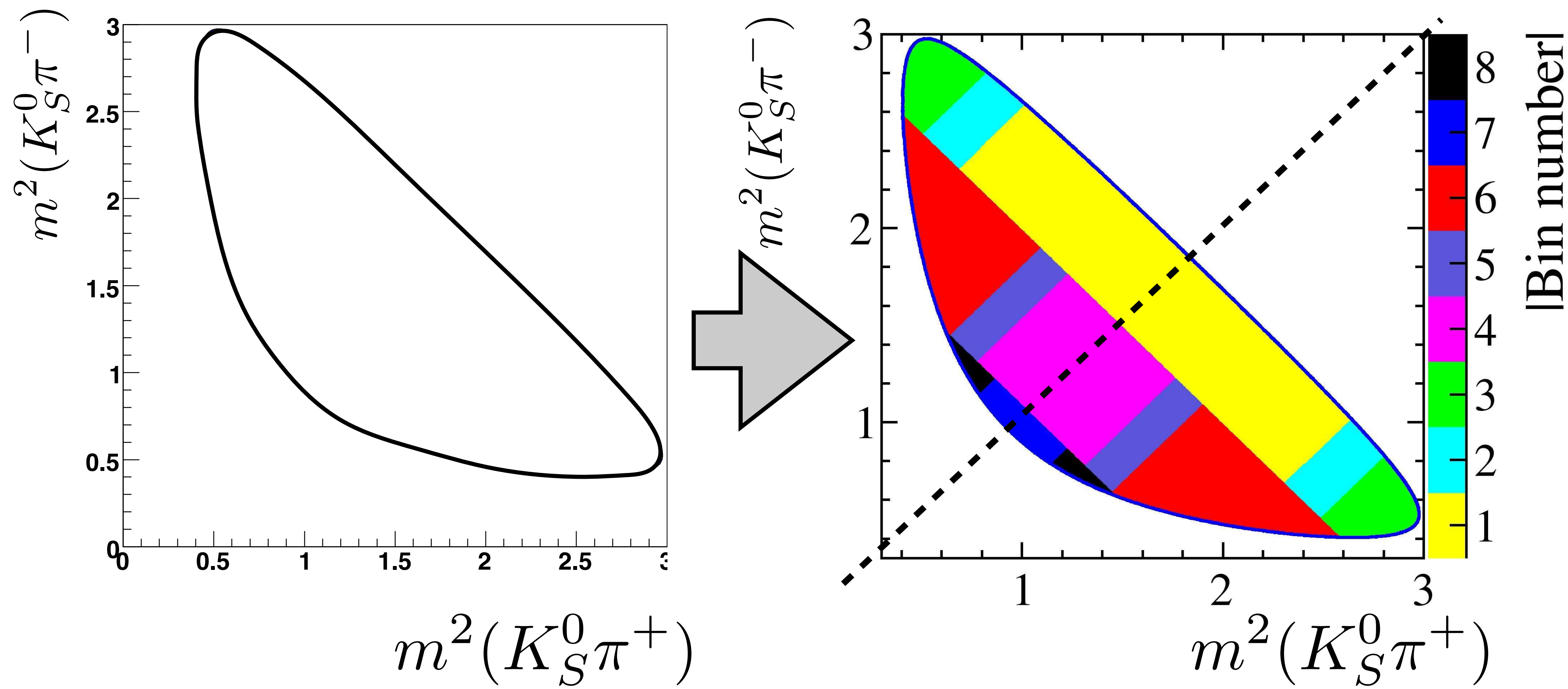
- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$

- Distribution sensitive to c_i, s_i, r_B, δ and γ.

- c_i, s_i, measured at charm threshold.



Model-independent, binned approach

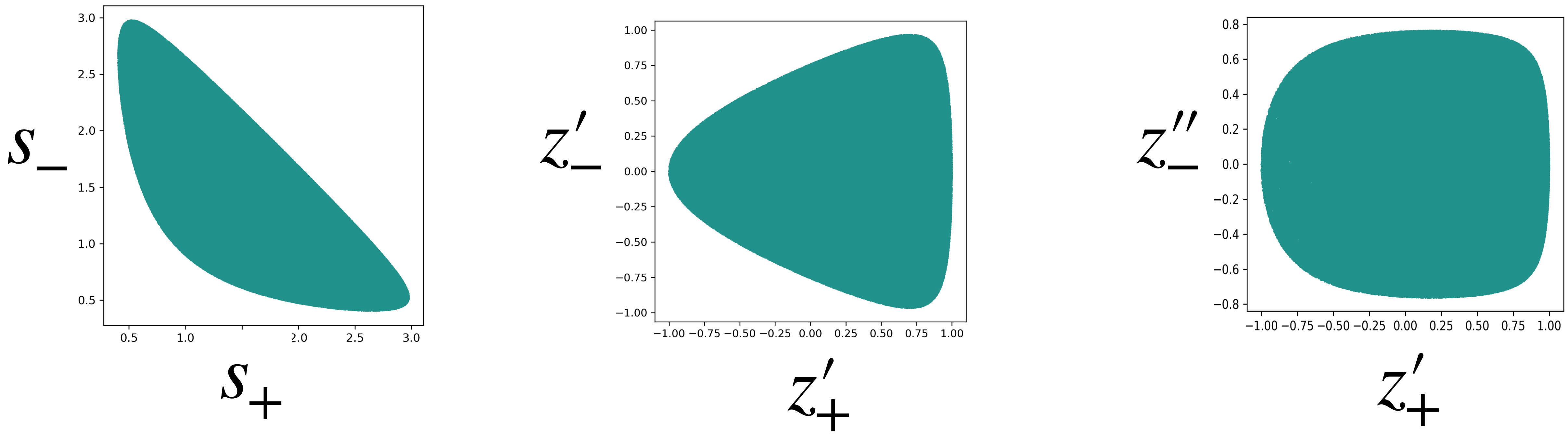


One complex number per bin-pair, $c_i + \mathbf{i}s_i$, contains the key information uniquely accessible at CLEO-c/BES III, which is related to the phases the D^0 and $\overline{D}^0$ decay amplitudes.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Rotate and stretch

JHEP 09 (2023) 007



fit parameters

ith order Legendre polynomials

(2j+1)th order Legendre polynomials

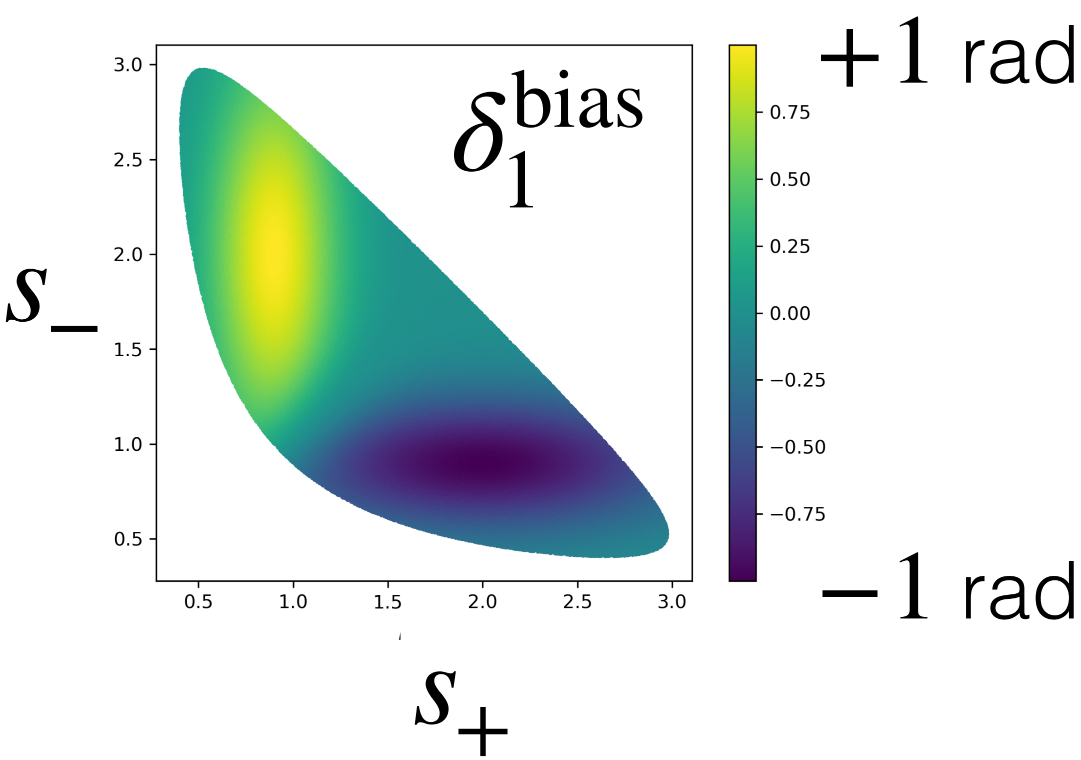
$$\delta^{\text{corr}} = \sum_{i=0}^{N} \sum_{j=0}^{\frac{N-i}{2}} C_{i,2j+1}\, P_i(z'_+)\, P_{2j+1}(z''_-)$$

only odd powers of $z''_- \propto s_- - s_+$, ensures that $\delta^{\text{corr}}(s_+, s_-) = -\,\delta^{\text{corr}}(s_-, s_+)$

Testing the method

JHEP 09 (2023) 007

Generate Babar & BELLE amplitude model with modified phase difference $\delta(s_+, s_-)$



$$\delta_1^{\text{bias}} = \text{erf}\left(\frac{s_+ - s_-}{\varepsilon}\right) g(s_+, s_-)$$

where g is a 2-D Gaussian, mirror reflected at $s_+ = s_-$

Fit starts from un-modified model - will it be able to find $\delta^{\text{corr}} \approx \delta_1^{\text{bias}}$ to a sufficient approximation?