Dispersive determination of the σ resonance from lattice QCD

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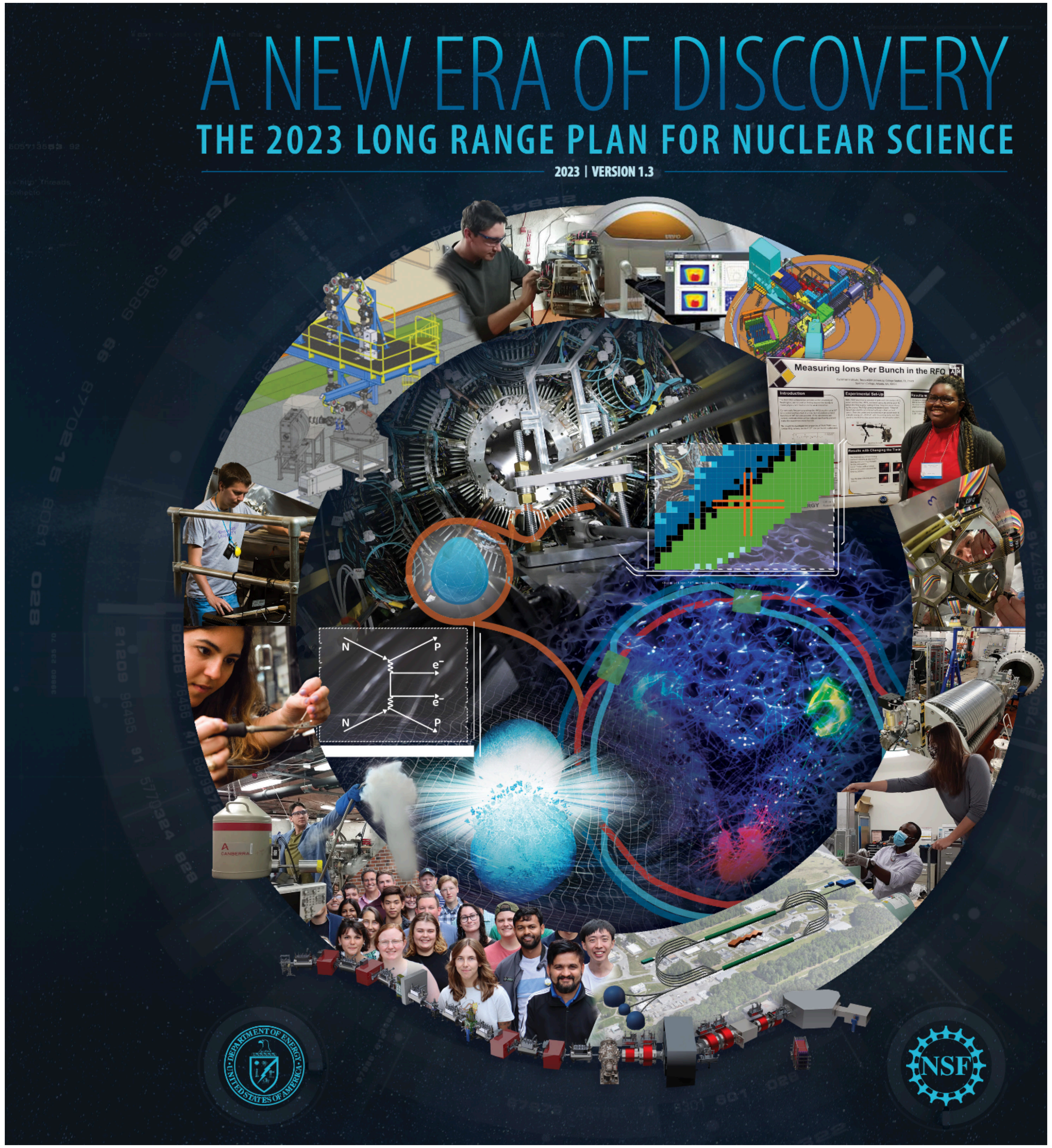
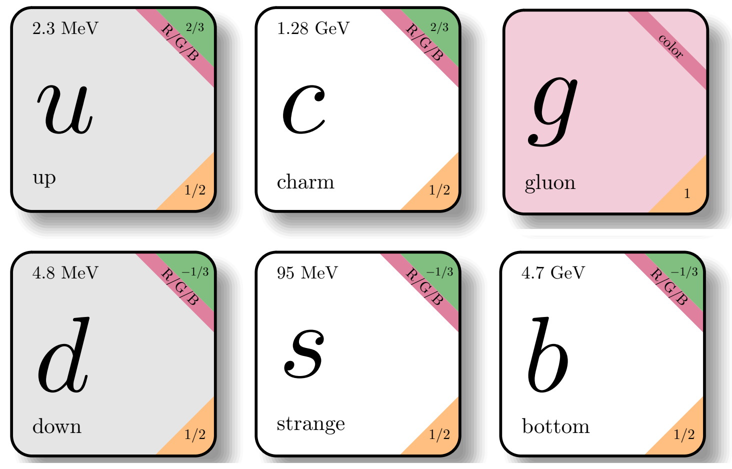
hadspec

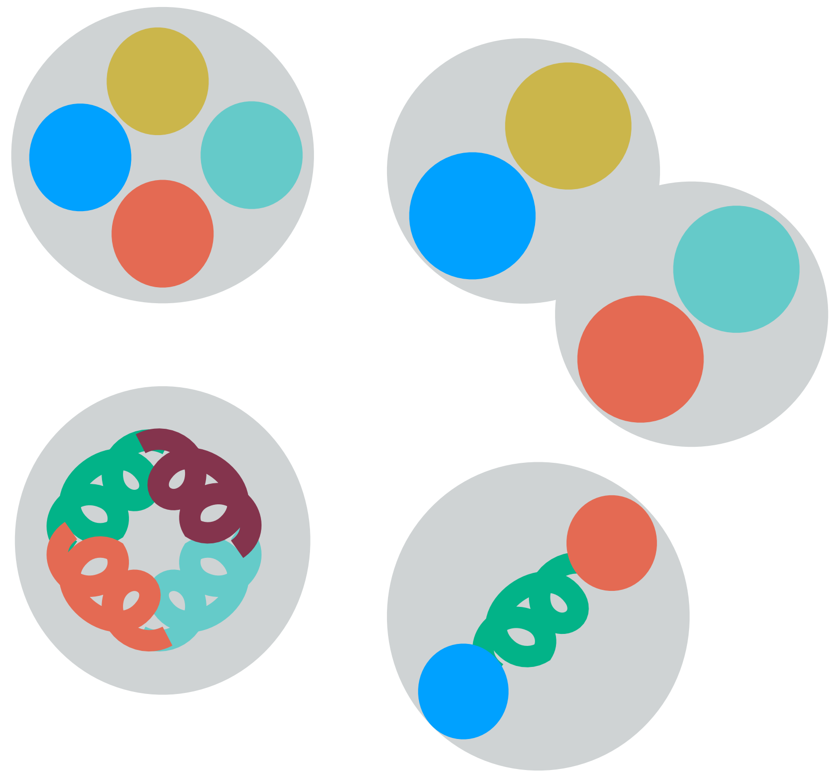
ExoHad
Exotic Hadrons Topical Collaboration

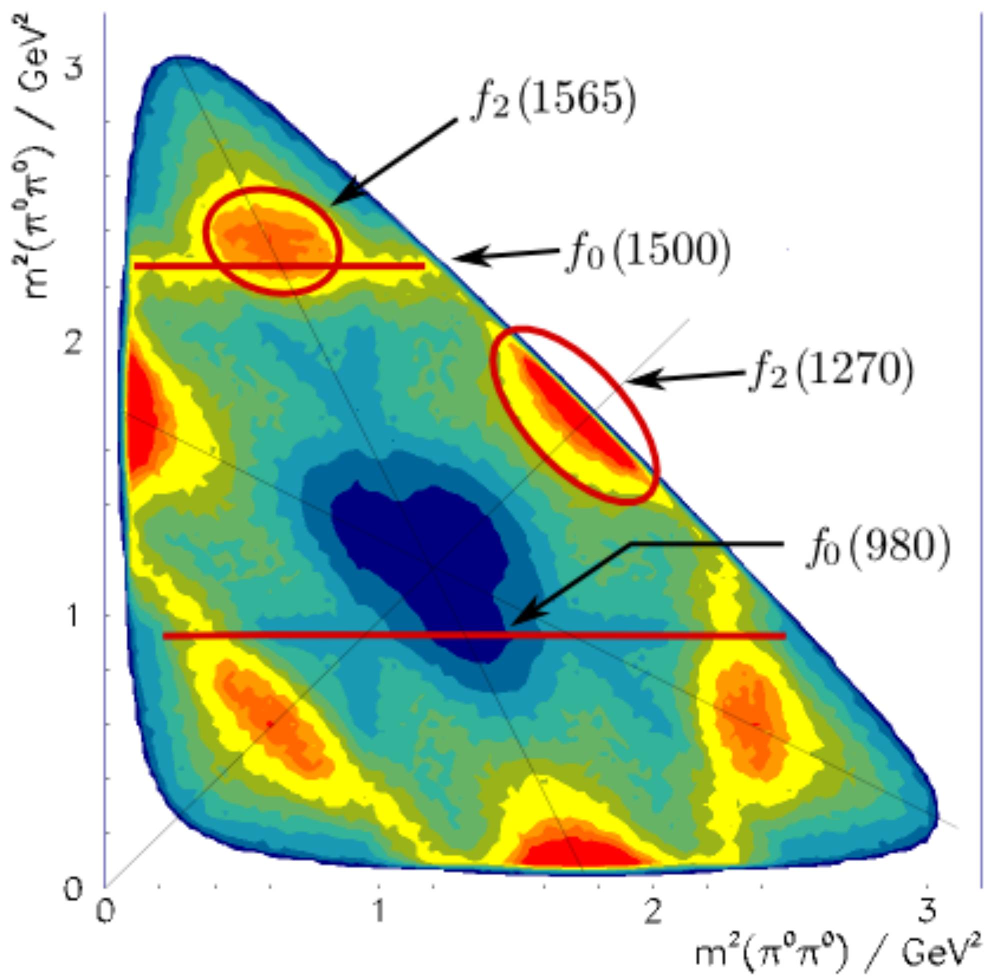


"hadron spectroscopy explores the possible bound combinations of quarks and gluons allowed by the interactions of QCD"

QCD







Understanding QCD spectrum

How do quark and gluons combine inside unstable hadrons?

- [ ] **Determine the spectrum**

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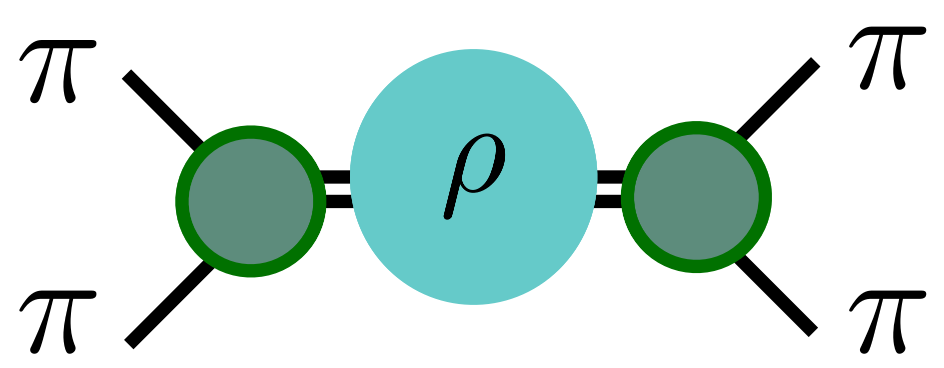


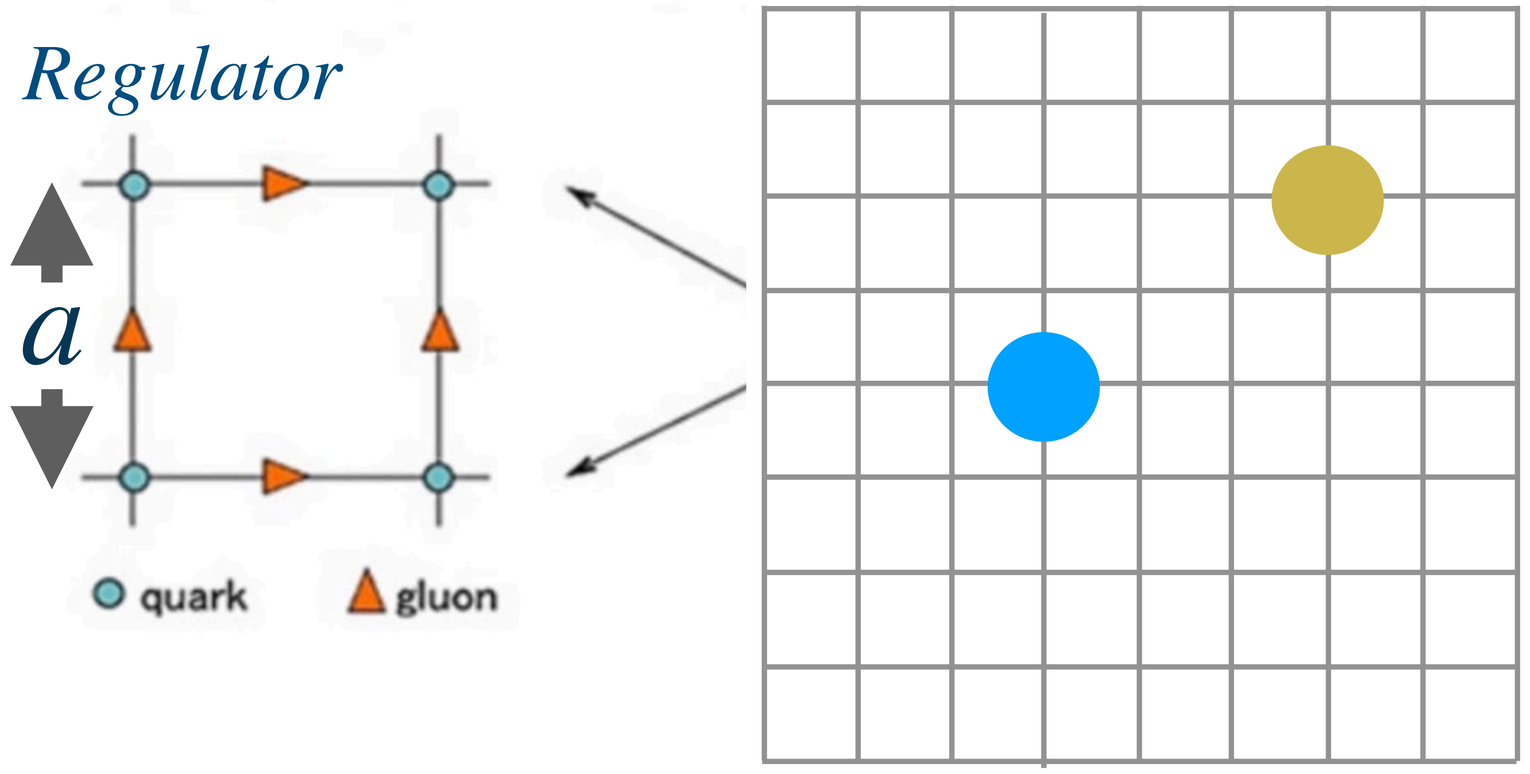
How do we extract this particle?

Lattice QCD

Dudek, Jackura, Ortega-Gama's (and others) talks for more/better details

Discretized, euclidean spacetime $$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

$$-iS = -i\int d^3x dt \mathcal{L} \rightarrow -\int d^3x dt \mathcal{L}_\mathrm{E} = -S_\mathrm{E}$$

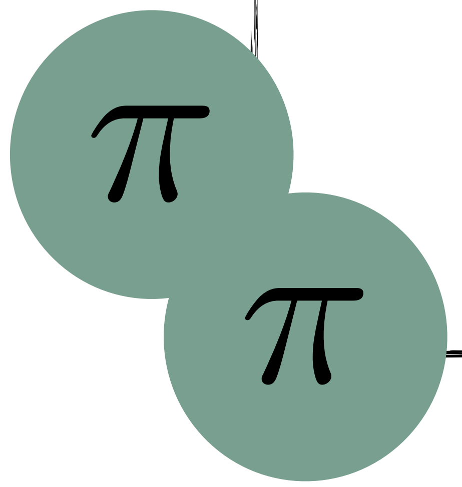


Numerical, Montecarlo sampling of our gluon fields

$$\langle O\rangle = \lim_{N\rightarrow\infty}\frac{1}{N}\sum_{n=1}^{N} O\left[U_n\right]$$

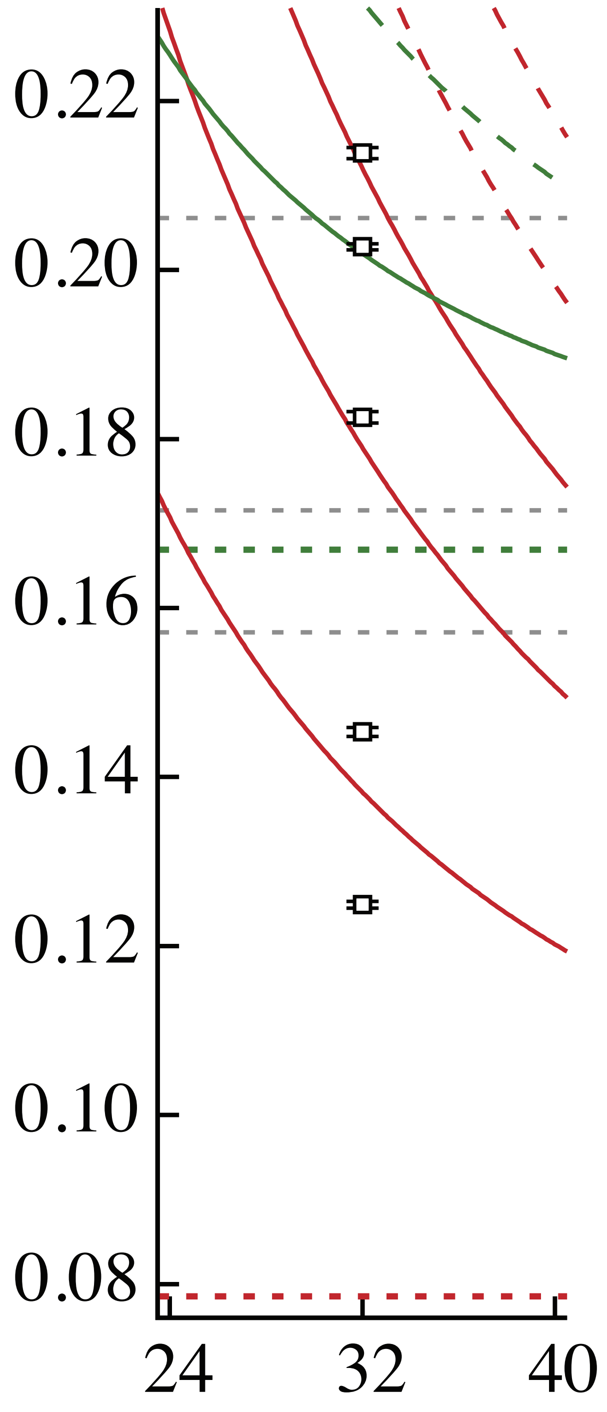
States Time evolution $|\psi(t)\rangle = e^{-Ht}|\psi(0)\rangle$

$$\langle O_f(t)O_i^\dagger(0)\rangle \sim \sum_\mathrm{n} e^{-E_\mathrm{n}t}\langle 0|O_\mathrm{f}(0)|\mathrm{n}\rangle\langle \mathrm{n}|O_\mathrm{i}^\dagger(0)|0\rangle$$

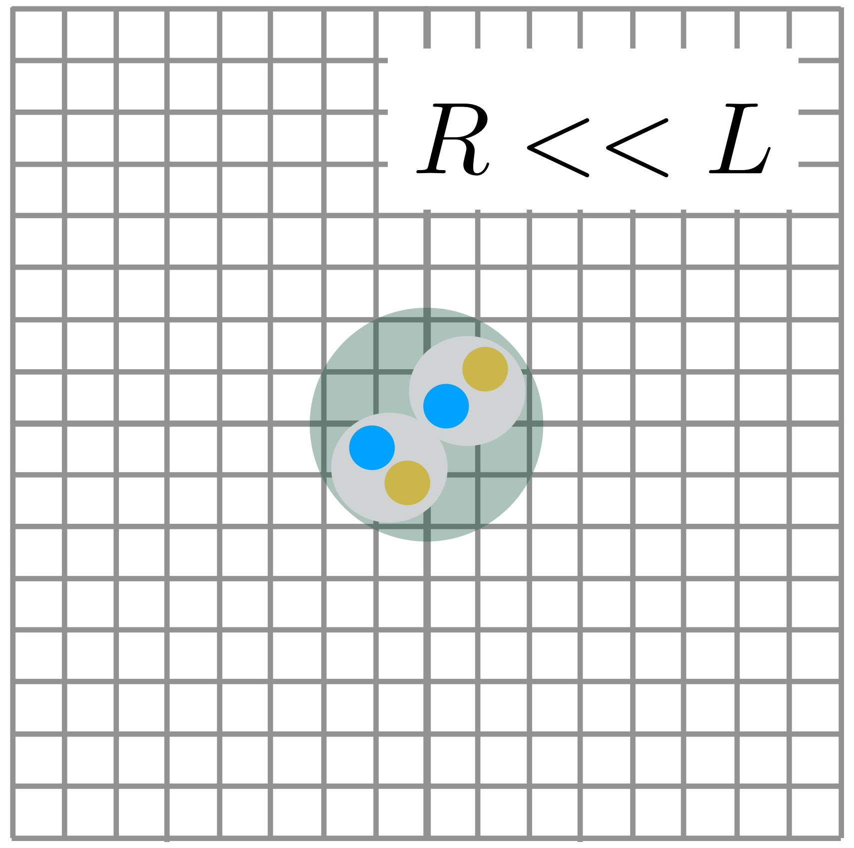


Desired energies

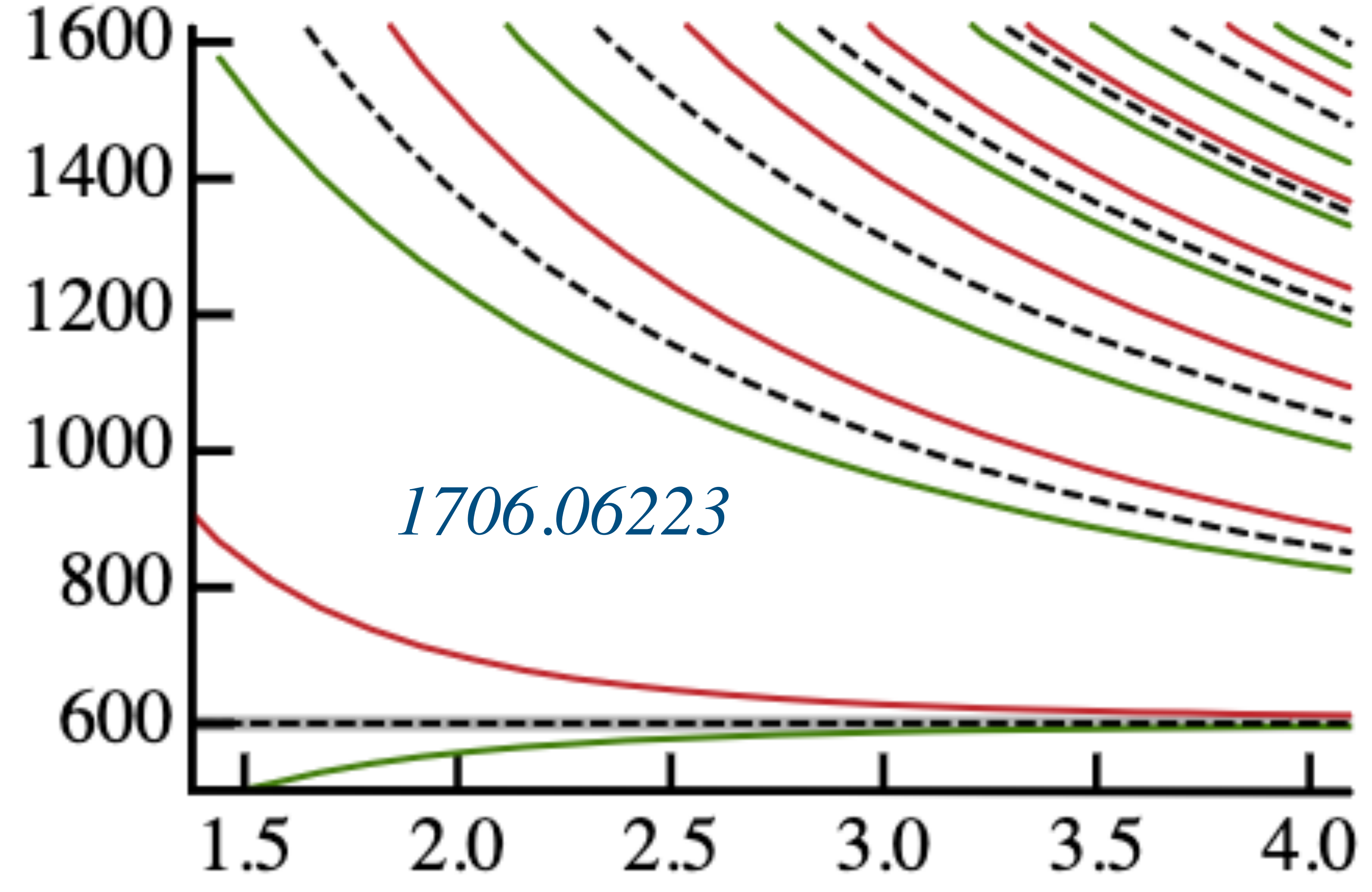
E_n



Lattice QCD

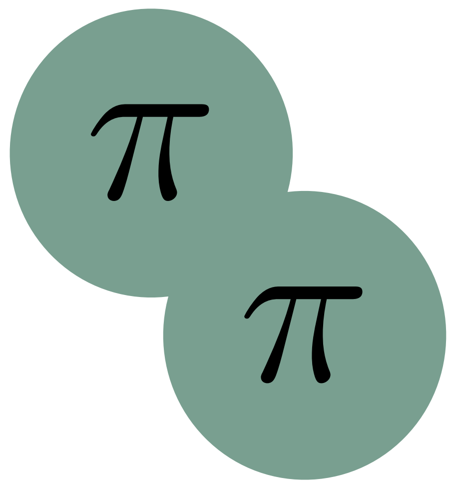


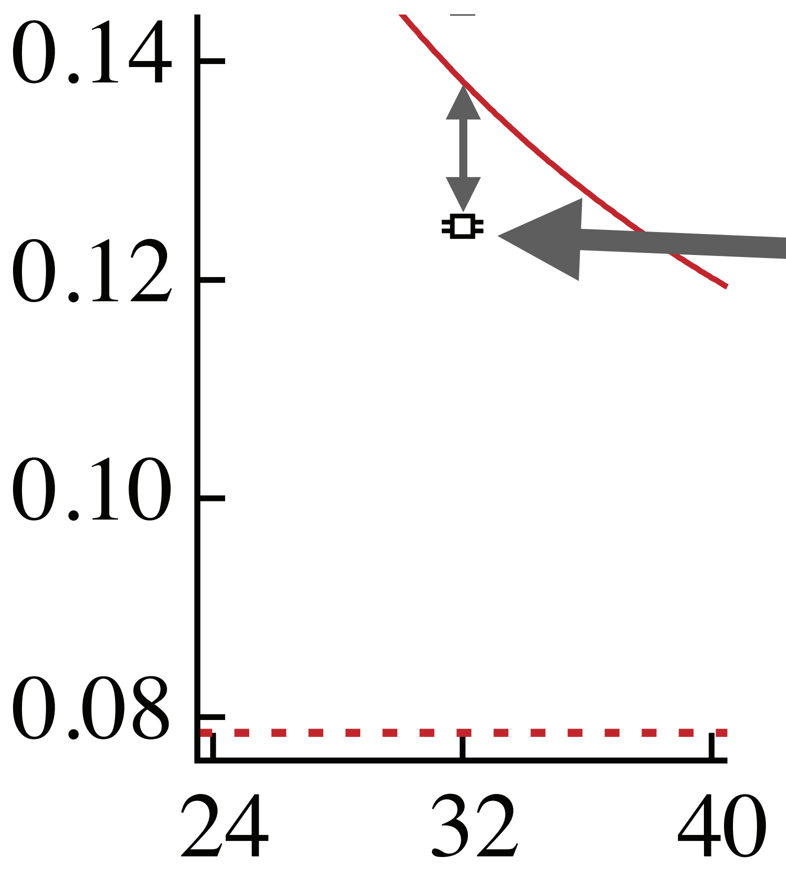
Attractive(Repulsive) interactions reduce(increase) the energy of the system

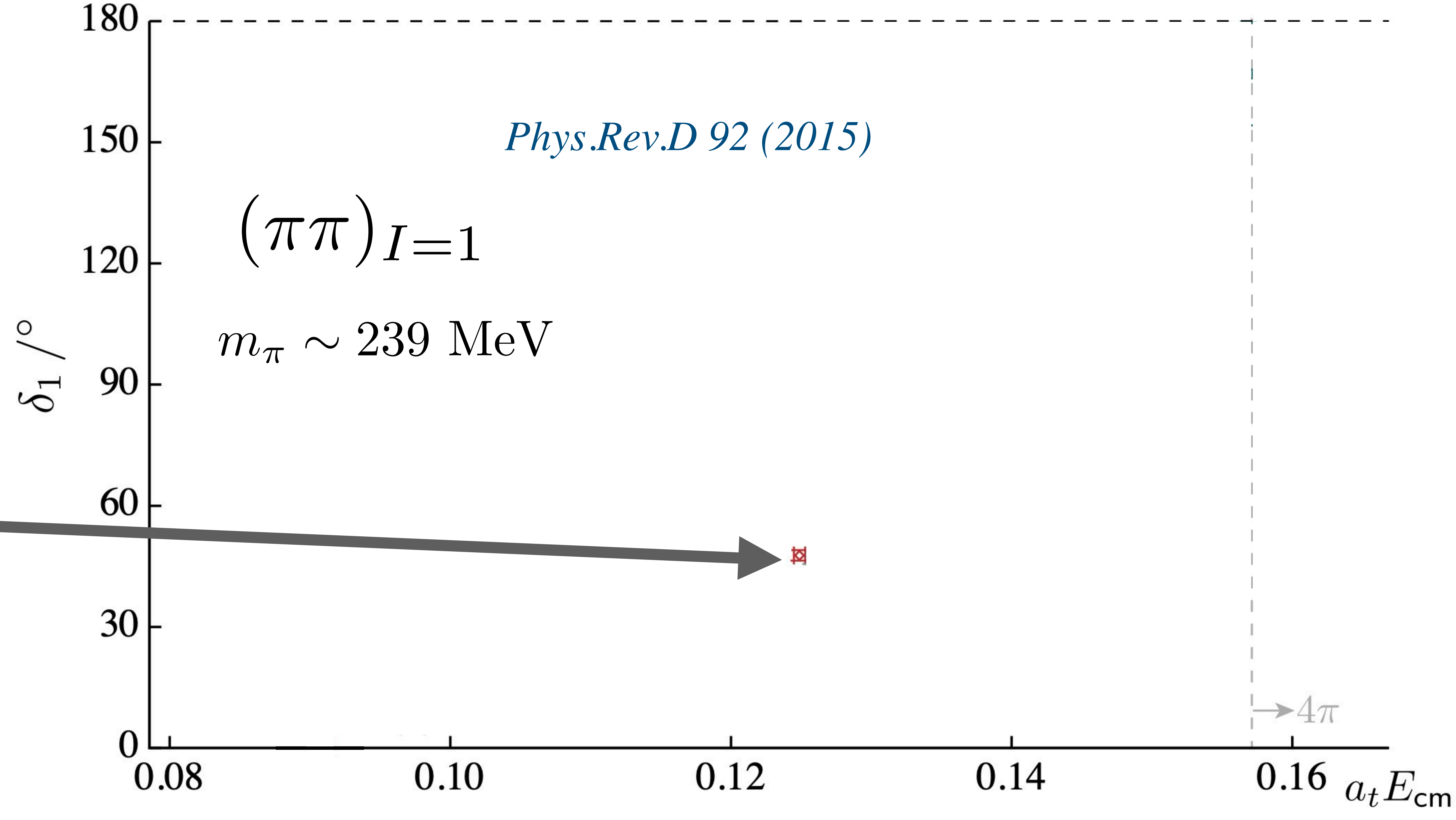


Every level corresponds to one "data" point

Lüscher, Nucl. Phys. B 354 (1991)







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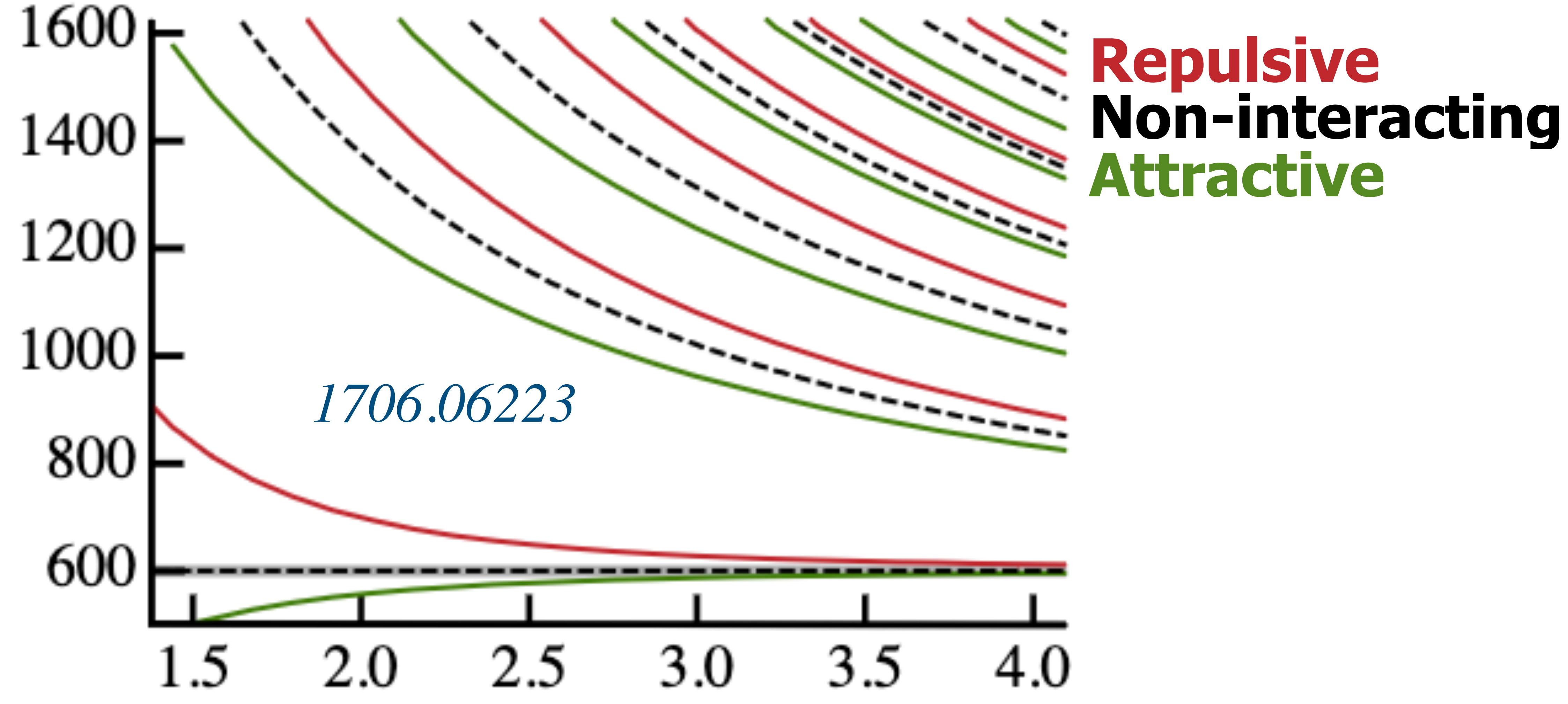

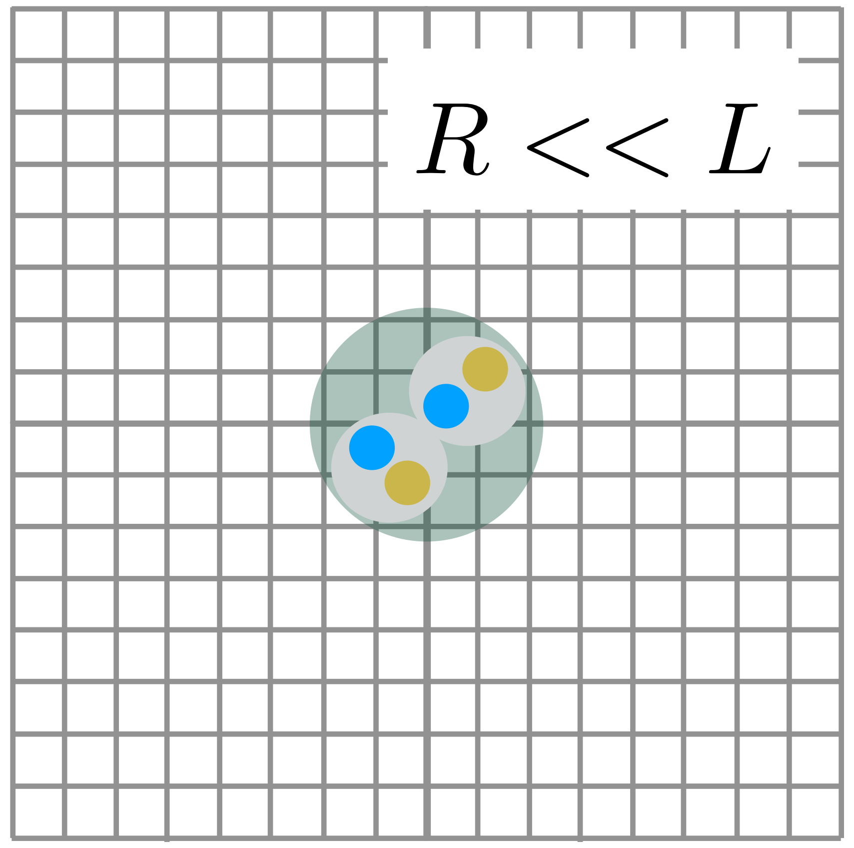

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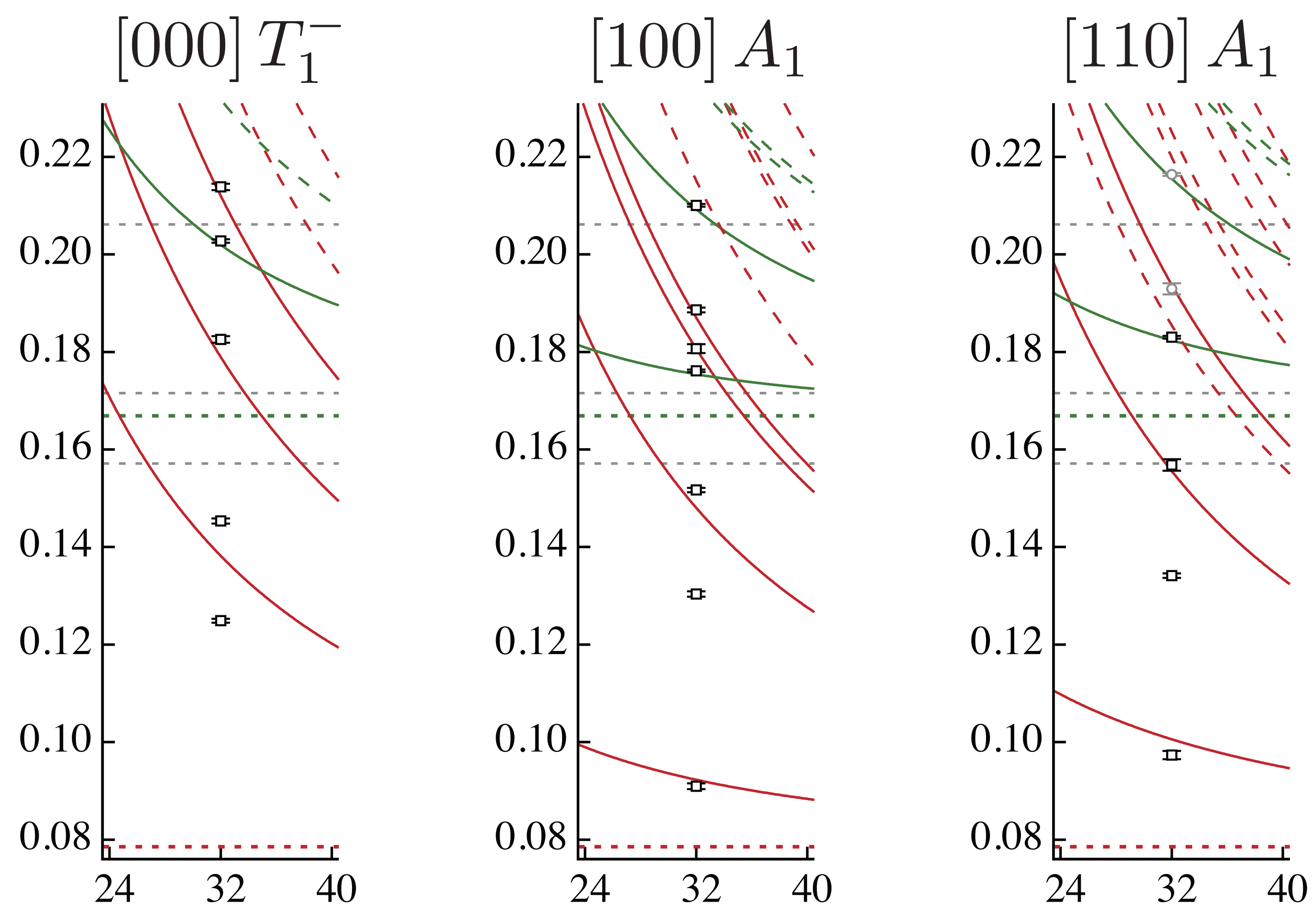
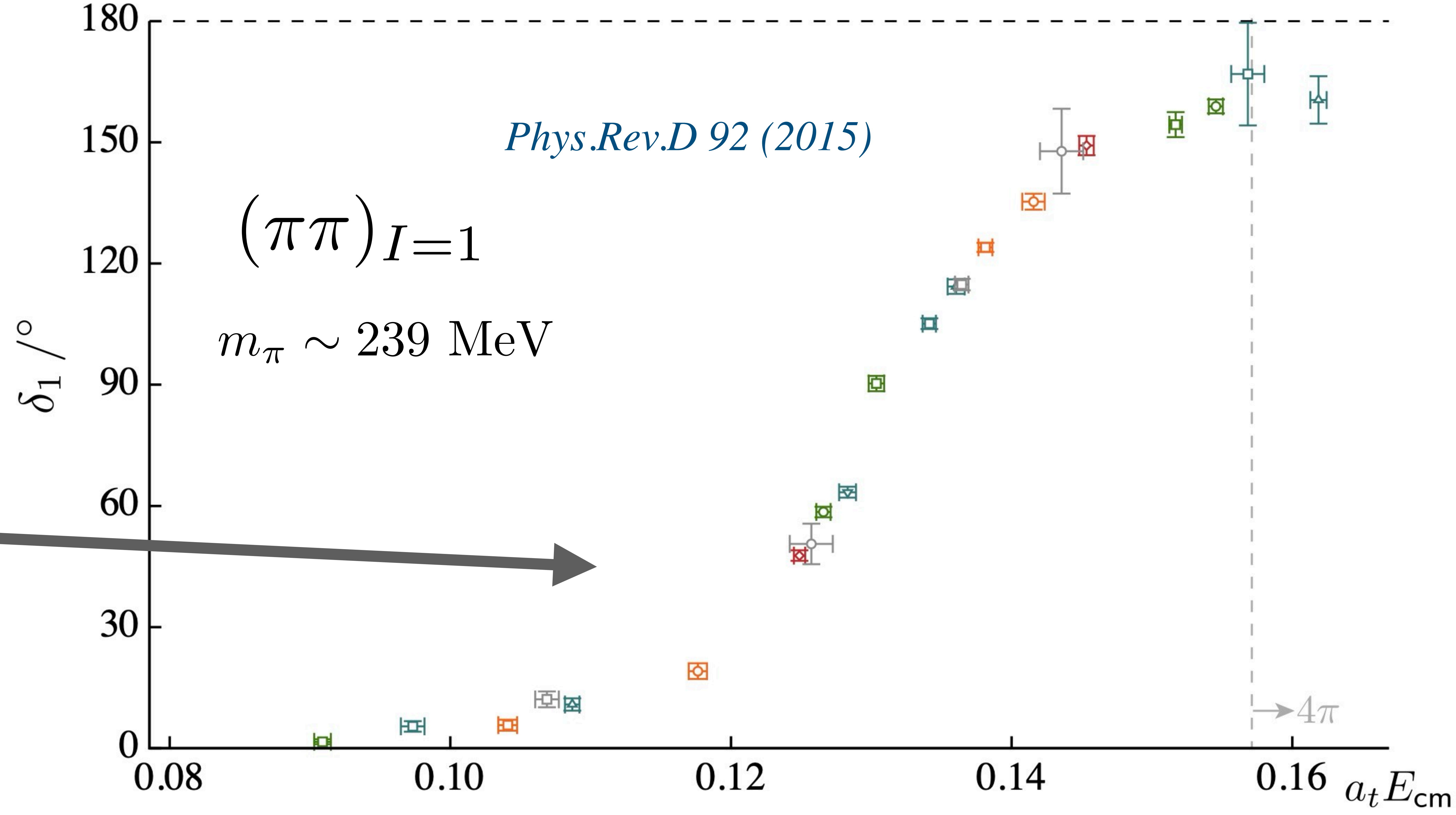




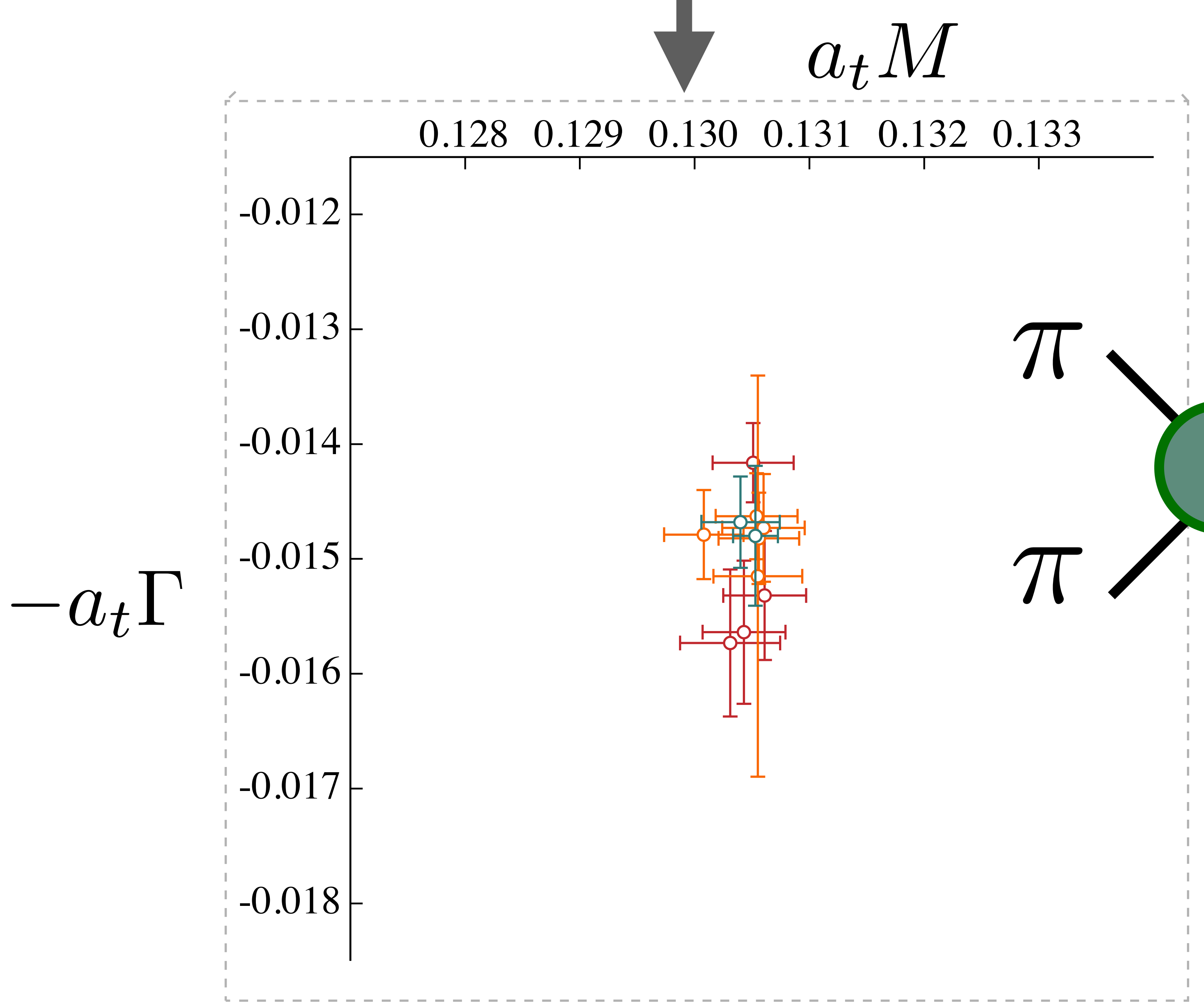
Lattice QCD

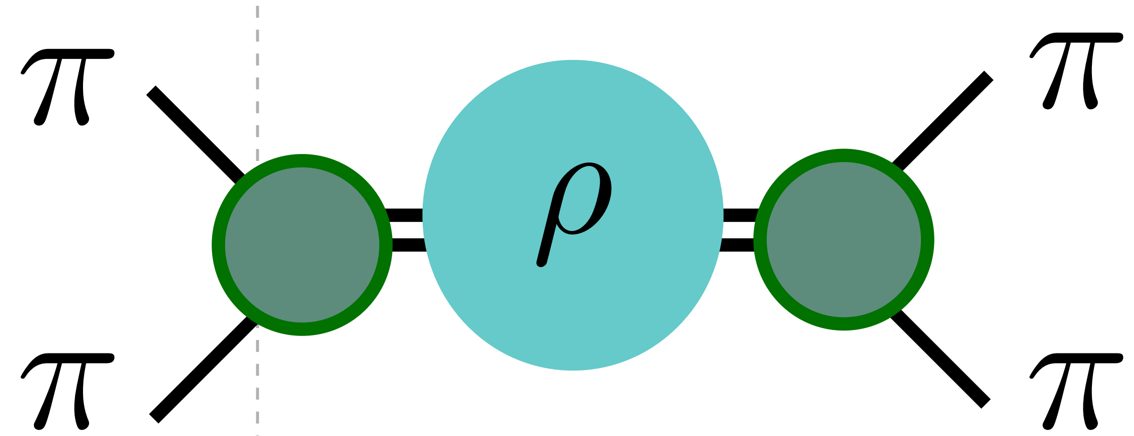
This amplitude can be easily modeled

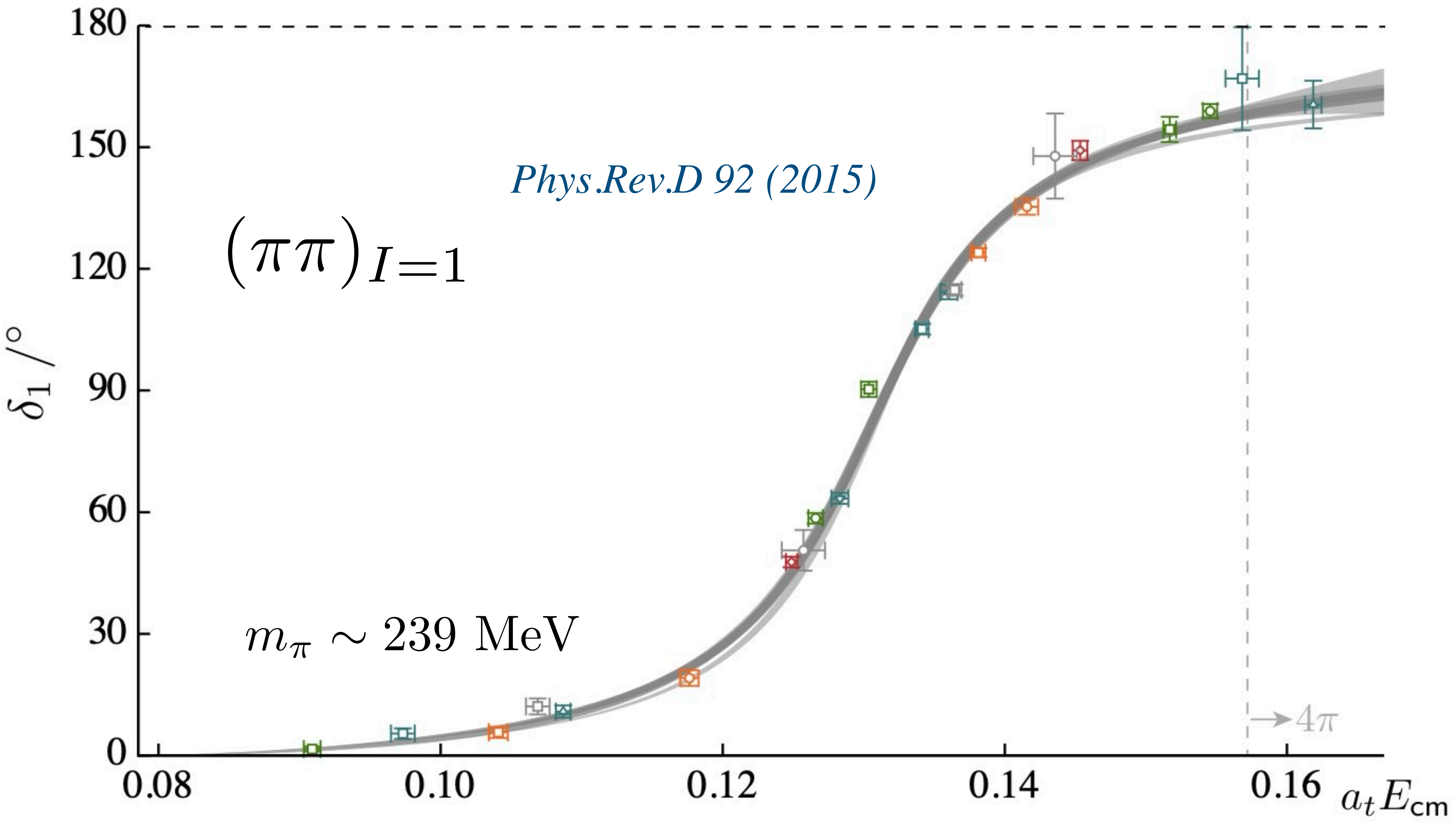
$$t_\ell^I(s) \simeq \frac{\sqrt{s}\Gamma}{M^2 - s - i\sqrt{s}\Gamma}$$

We can use many more models

Pole at $\sqrt{s_p} \sim (M - i\Gamma/2)$







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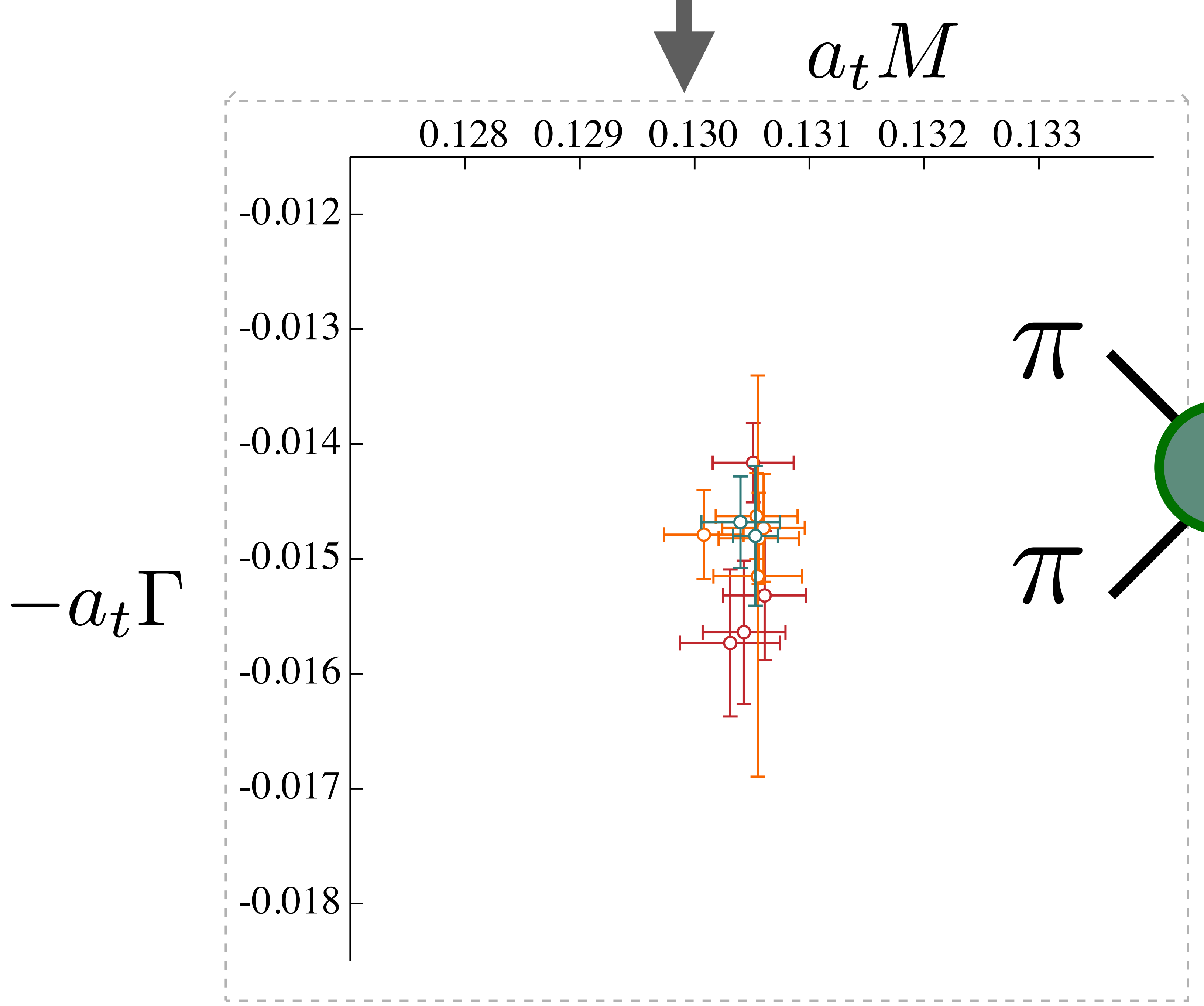
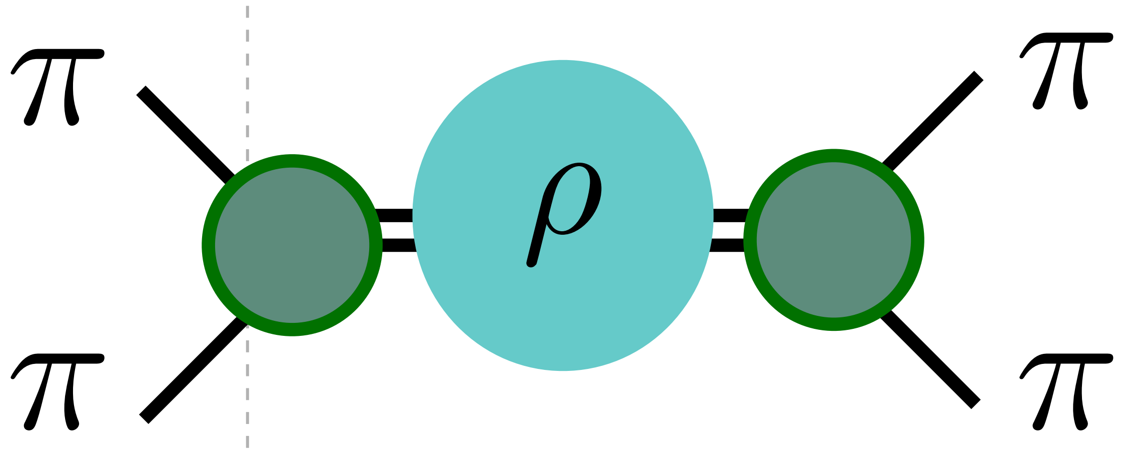
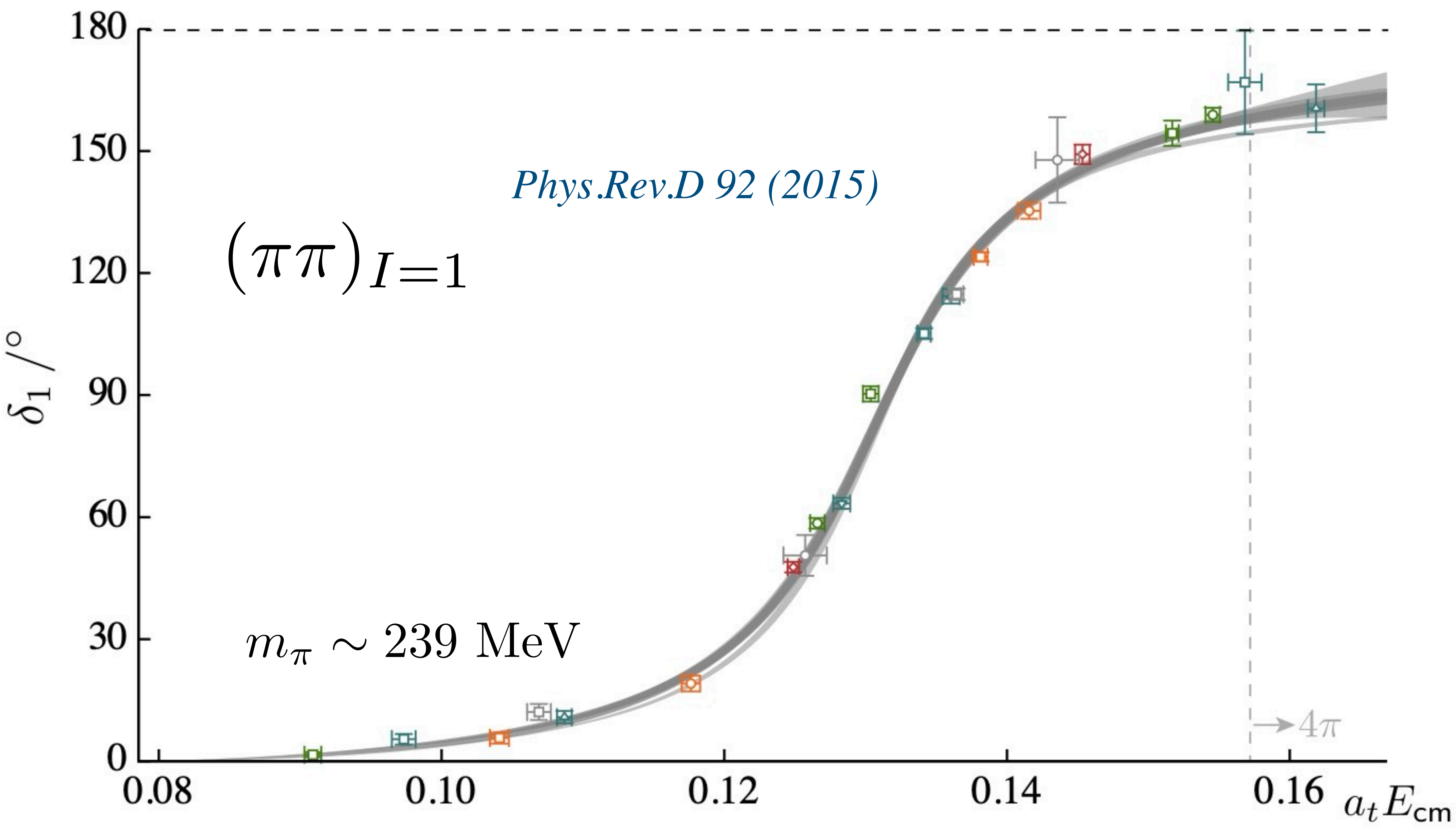
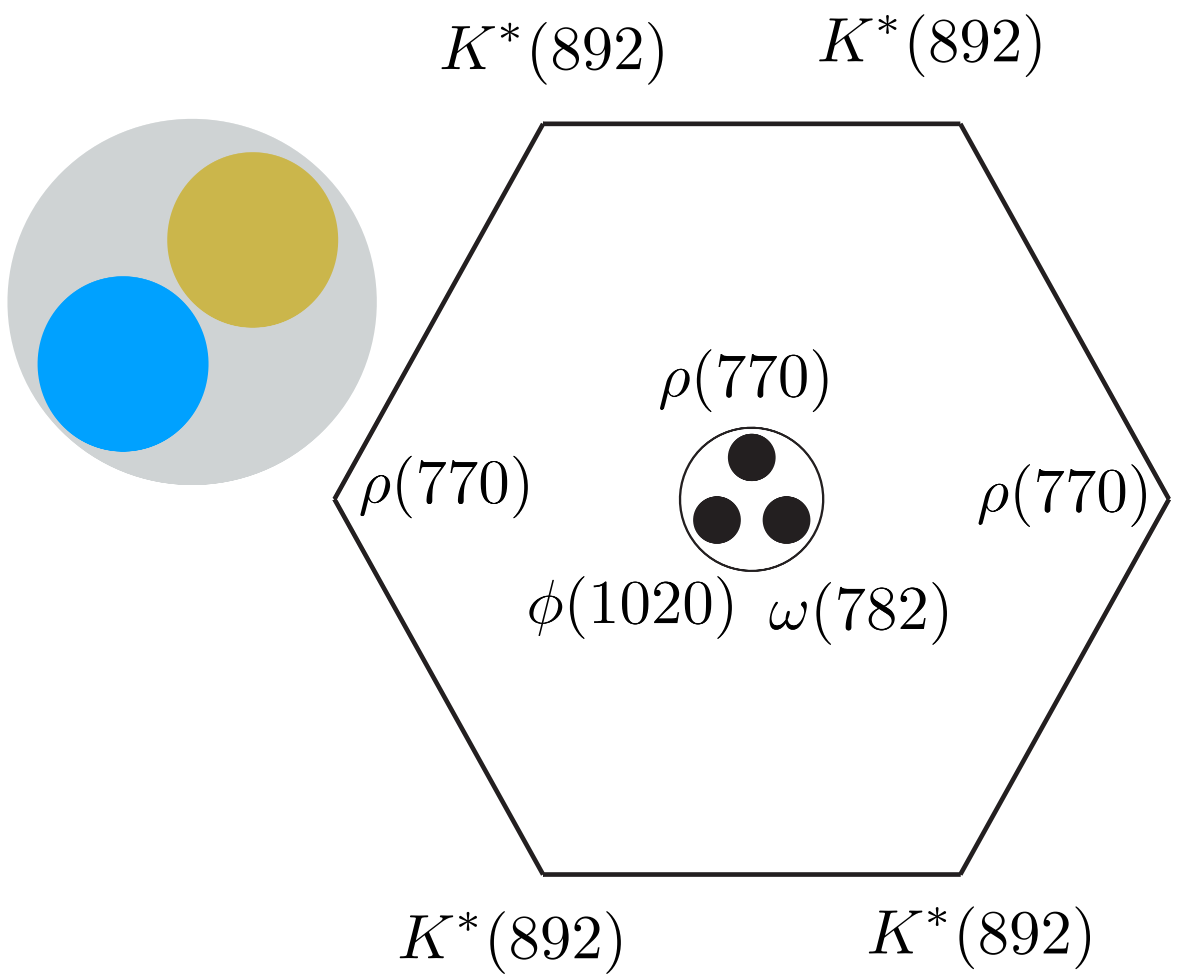






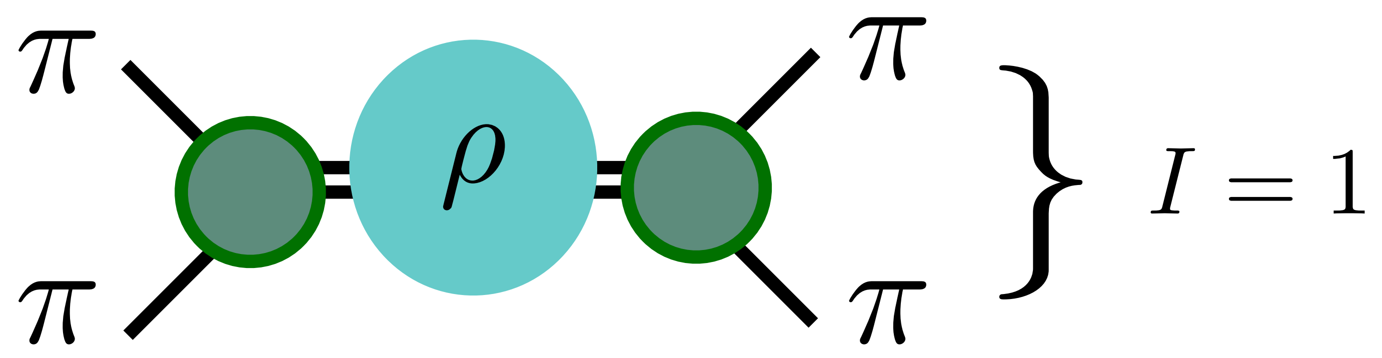
The ρ is an ordinary, narrow, isolated resonance



Understanding QCD spectrum

How do quark and gluons combine inside unstable hadrons?

☑ **Determine the spectrum**



Varying pion masses!!

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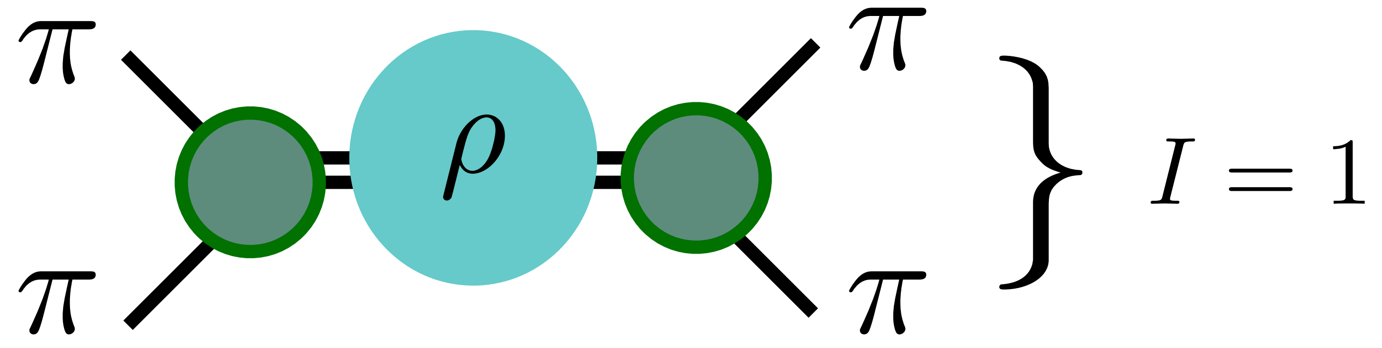
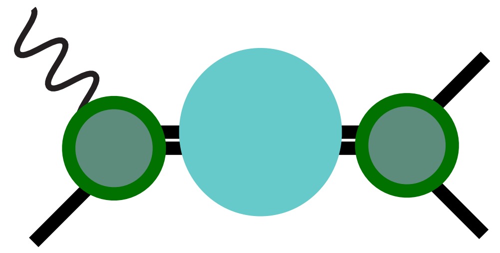
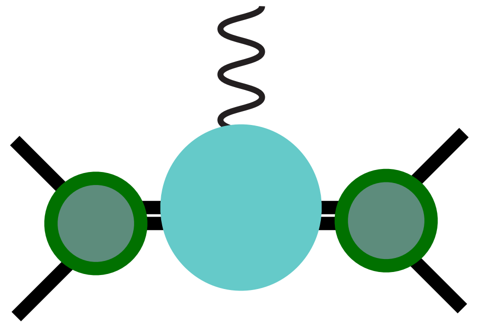
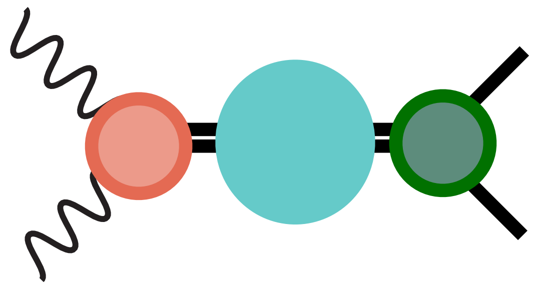


Varying pion masses!!

- ☑ **"Understand" the spectrum**

"Photoproduction"  *1502.04314*

Resonance "form factors"  *1509.08507*

"Photon" fusion  *2210.08051*

Understanding QCD spectrum

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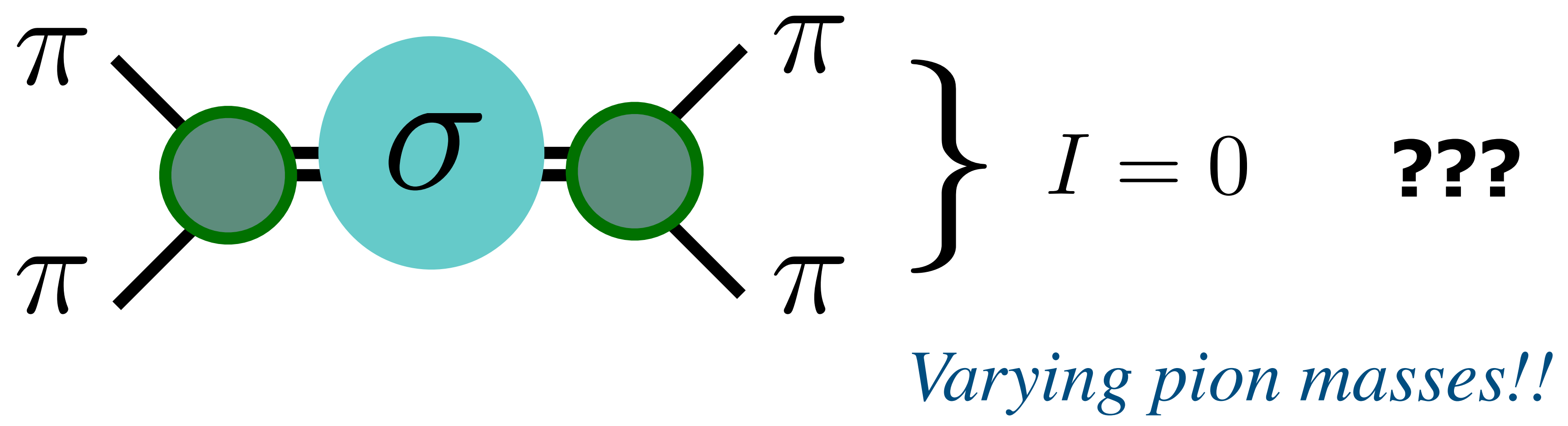
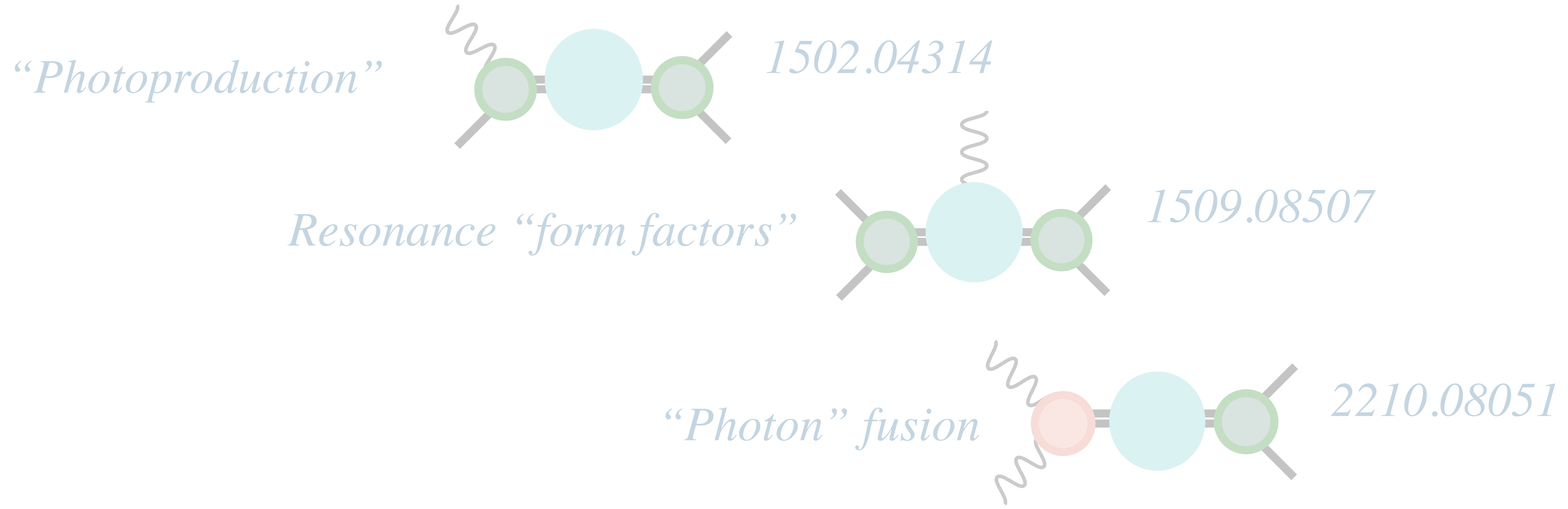


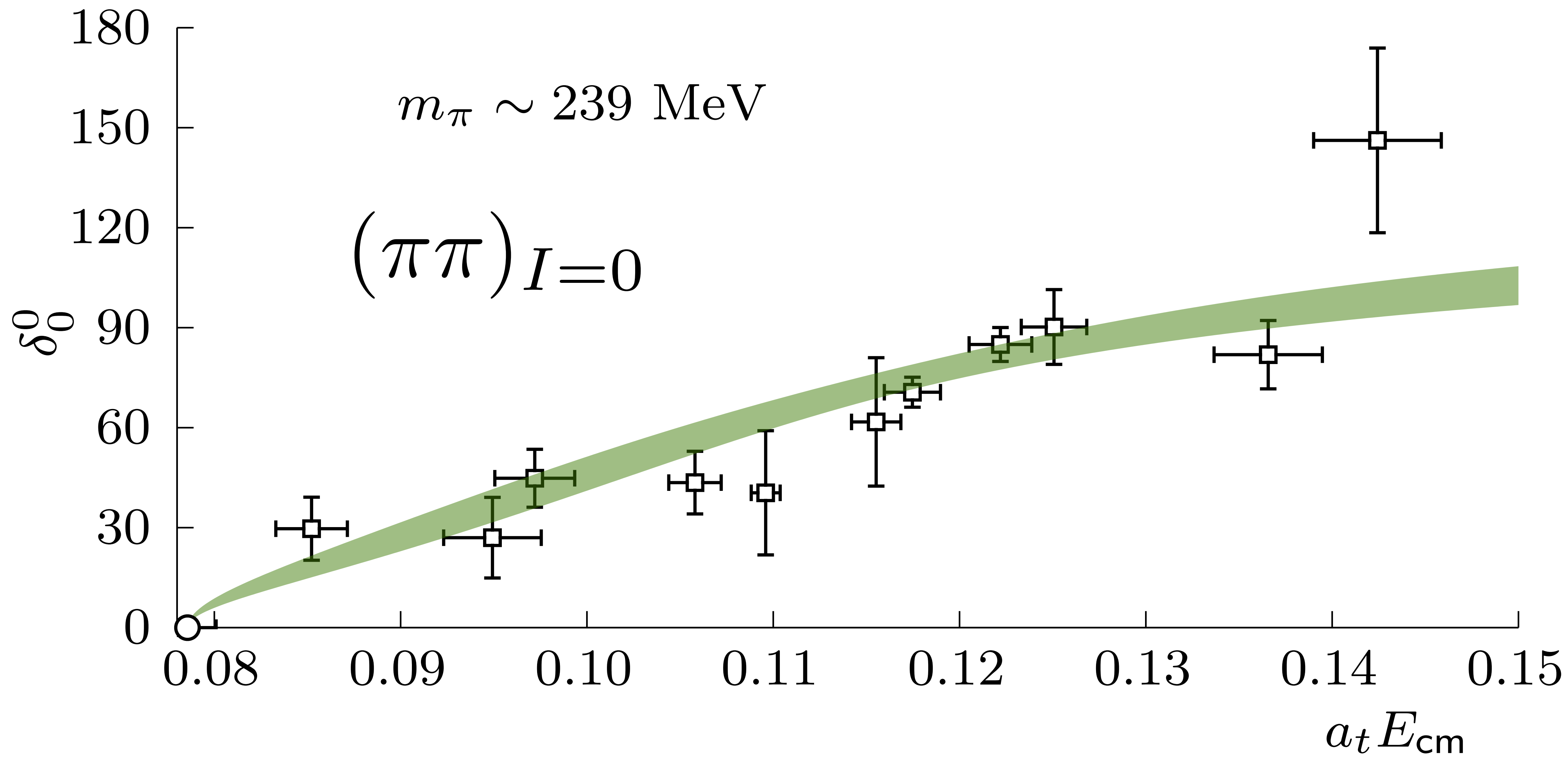
- ☐ **"Understand" the spectrum**



Light Scalars: the σ

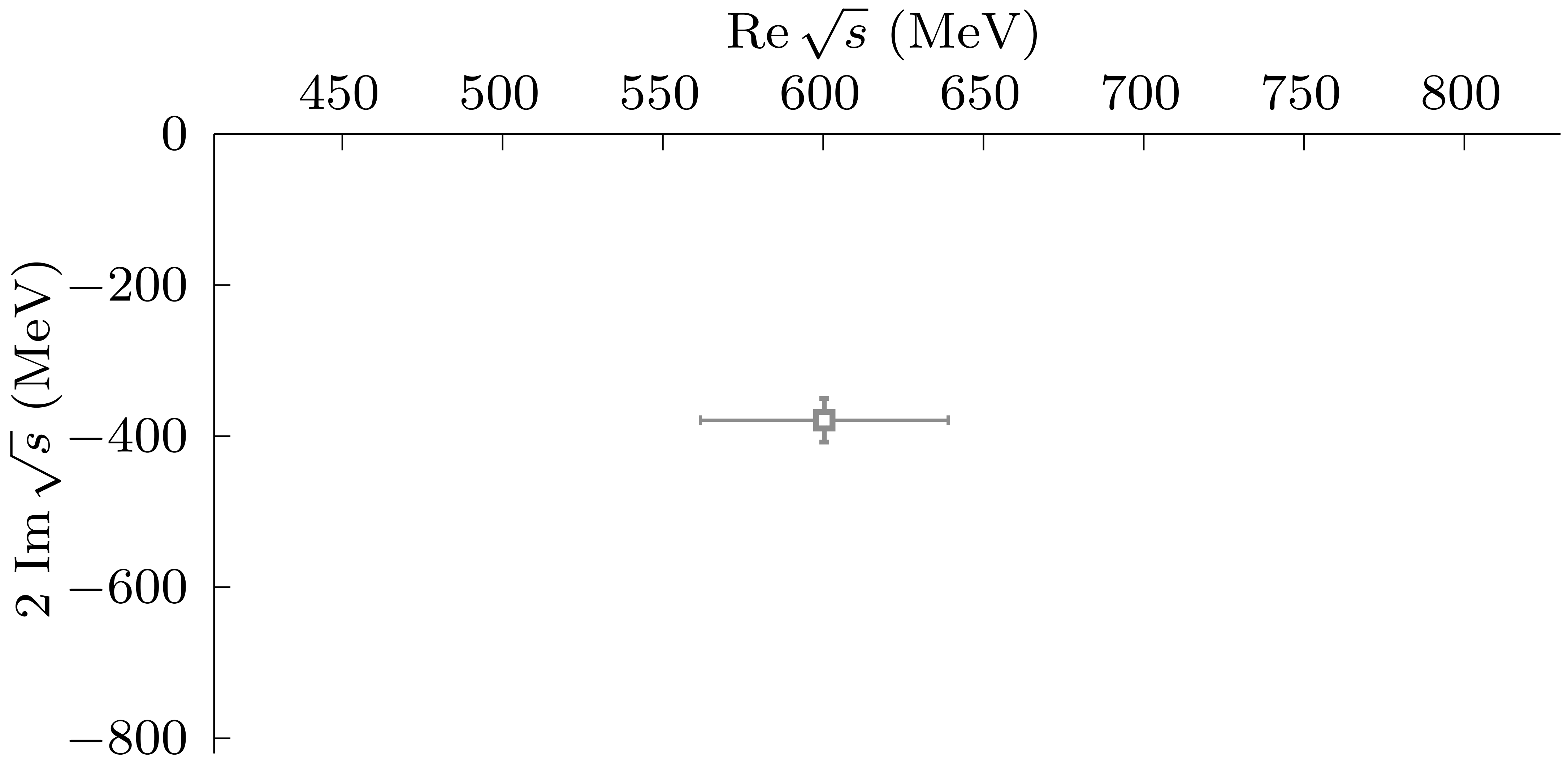
Data is precise !!

$t_0^0(s)\ (S0)$



σ **pole positions**

$$t_\ell^I(s) = \frac{e^{i\delta_\ell^I(s)} \sin \delta_\ell^I(s)}{\rho(s)}$$



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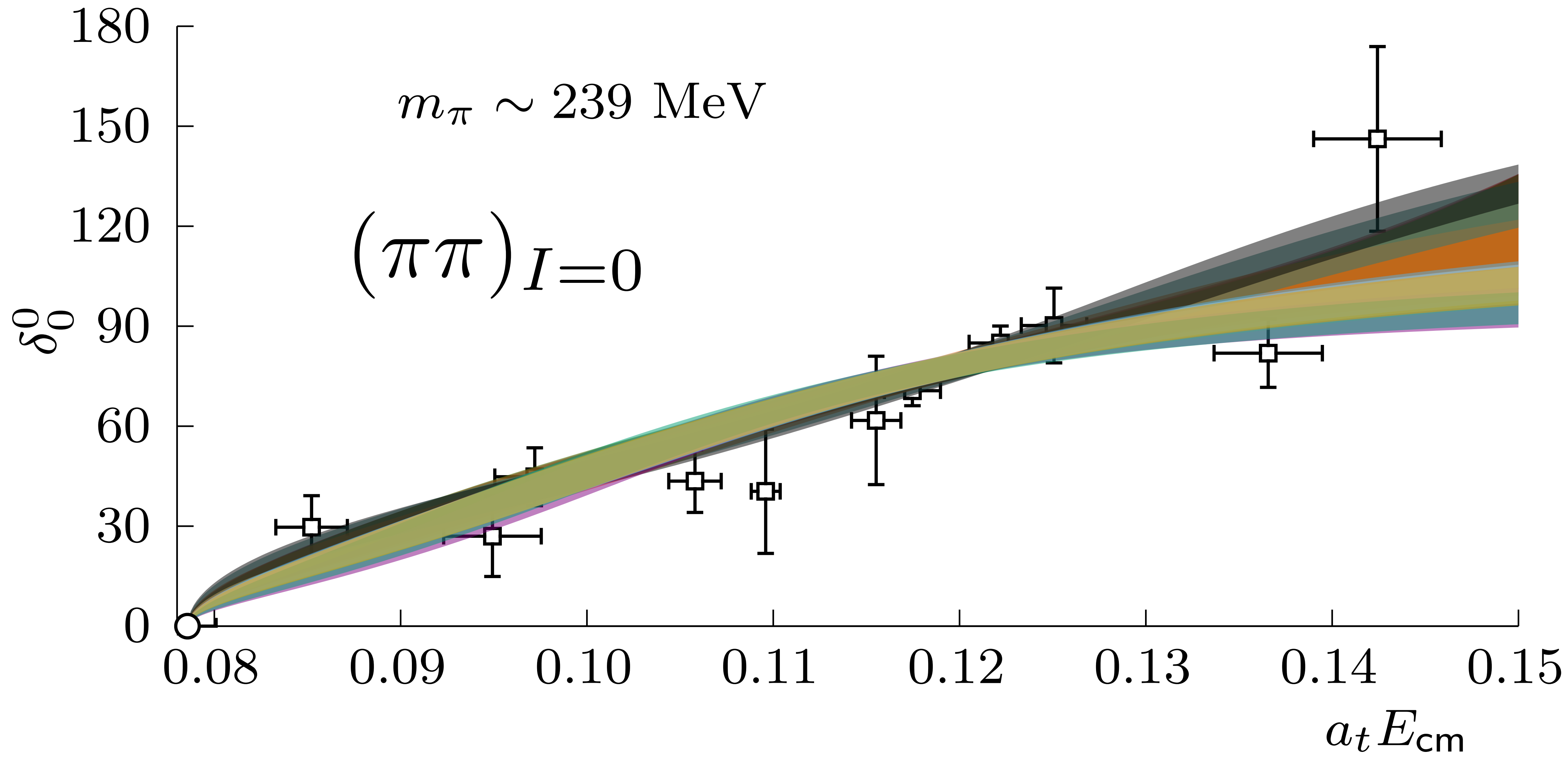

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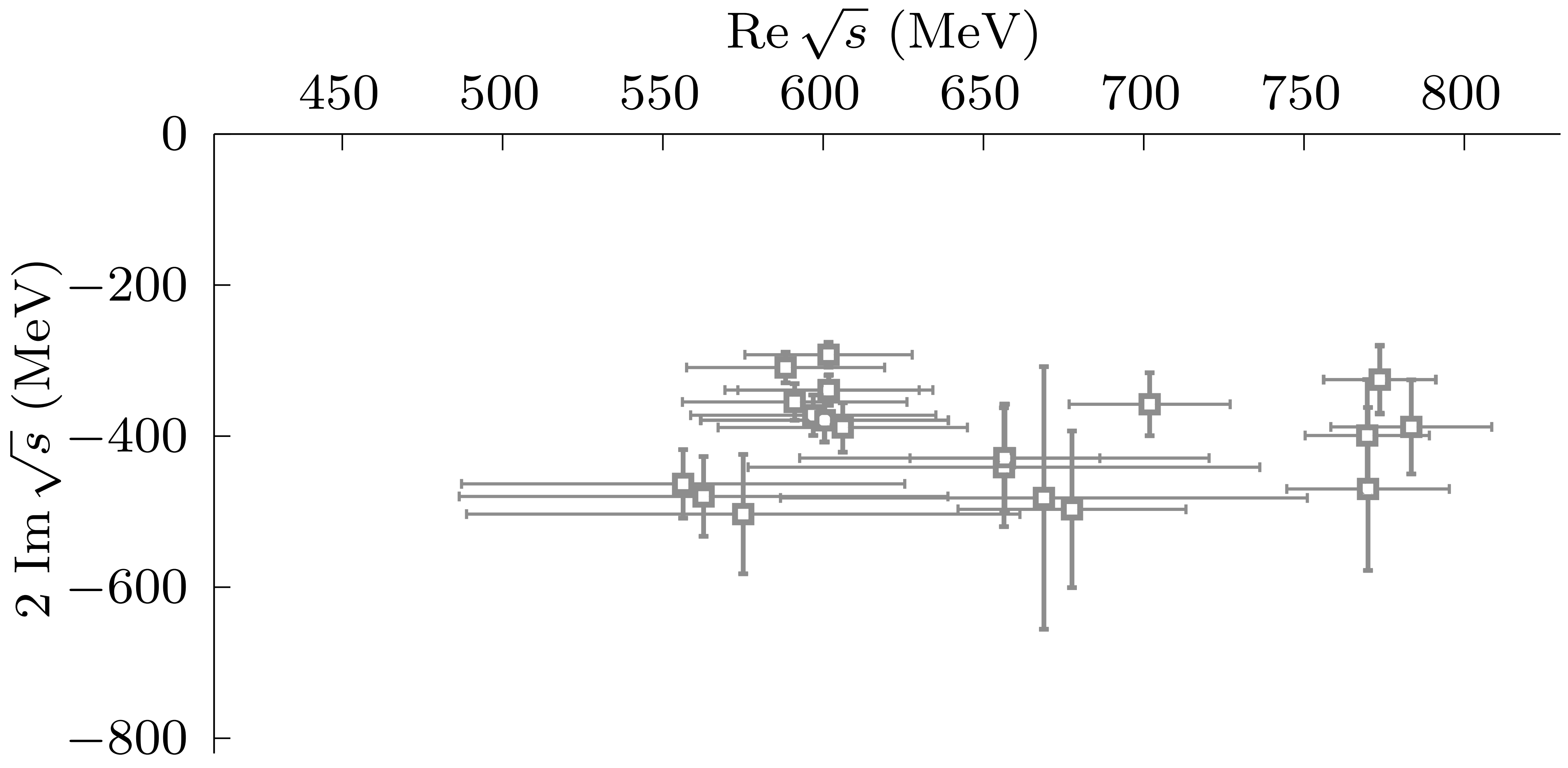
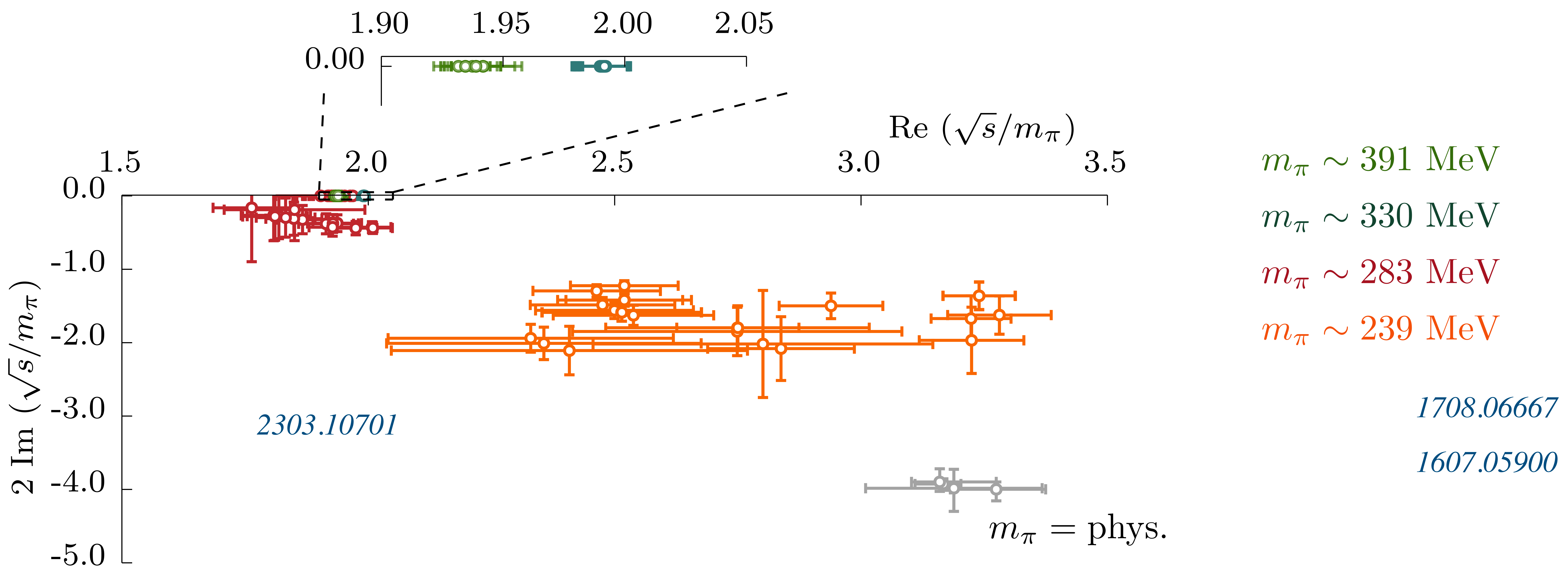


VERY large model (systematic) spread!!

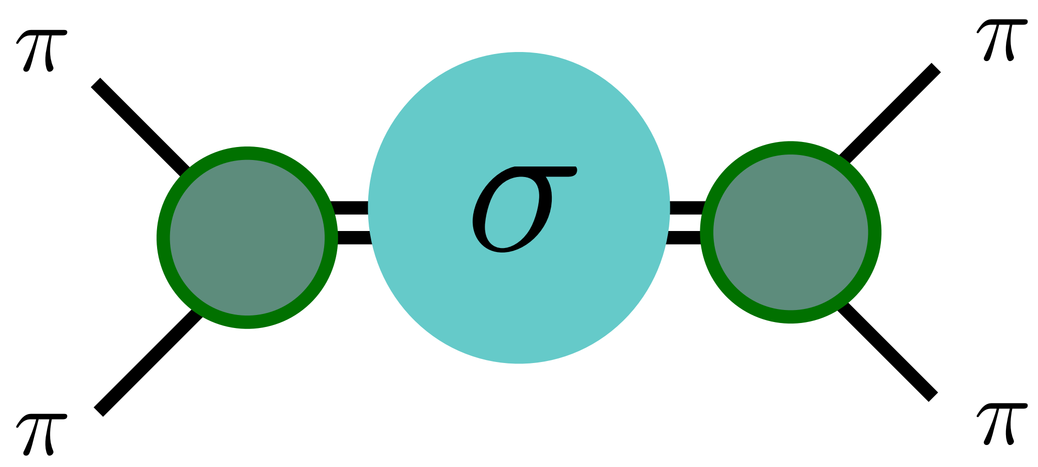
Light Scalars: the σ

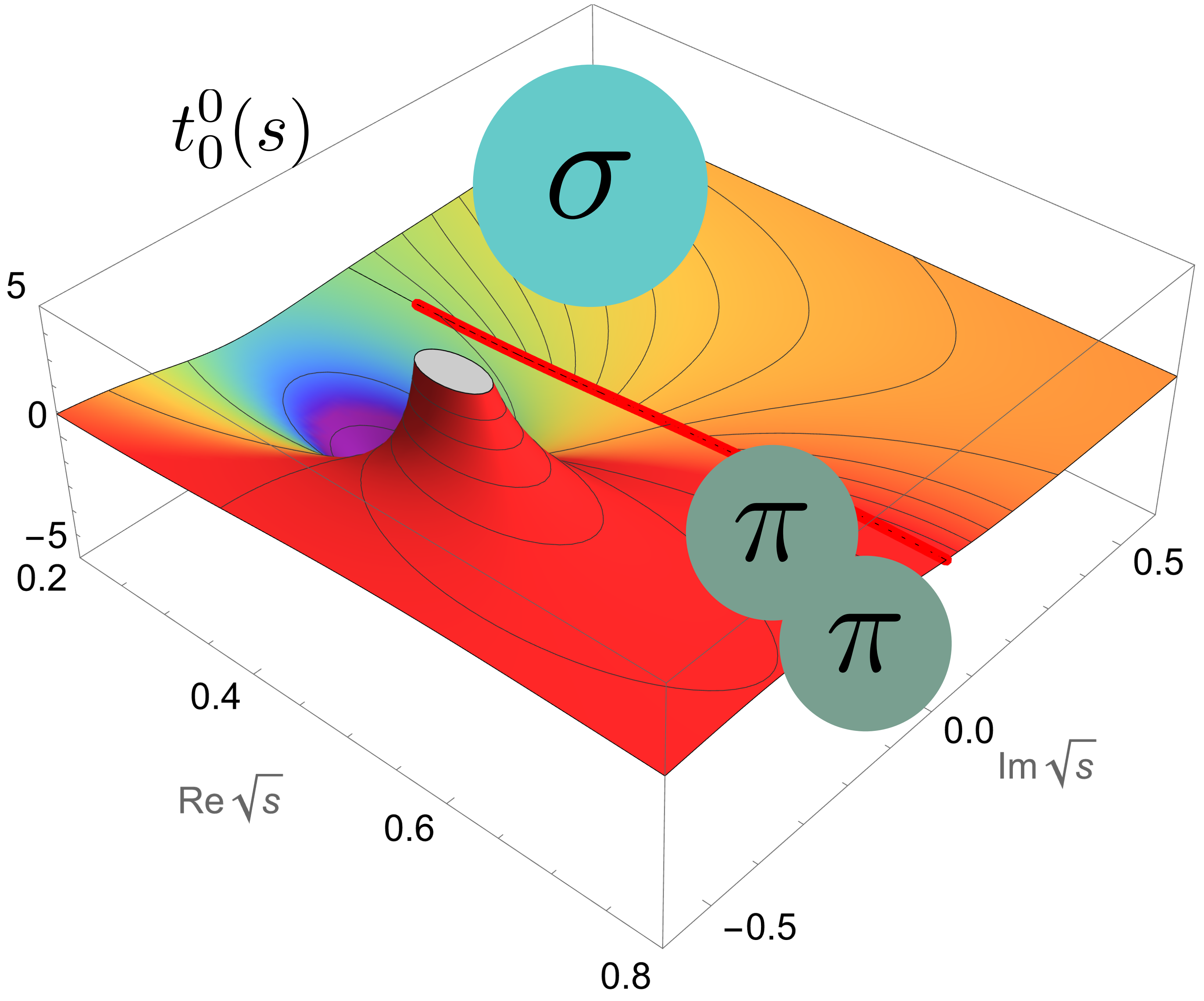
Total error becomes really large when the state is a resonance



Light Scalars: the σ

Lightest resonance in QCD



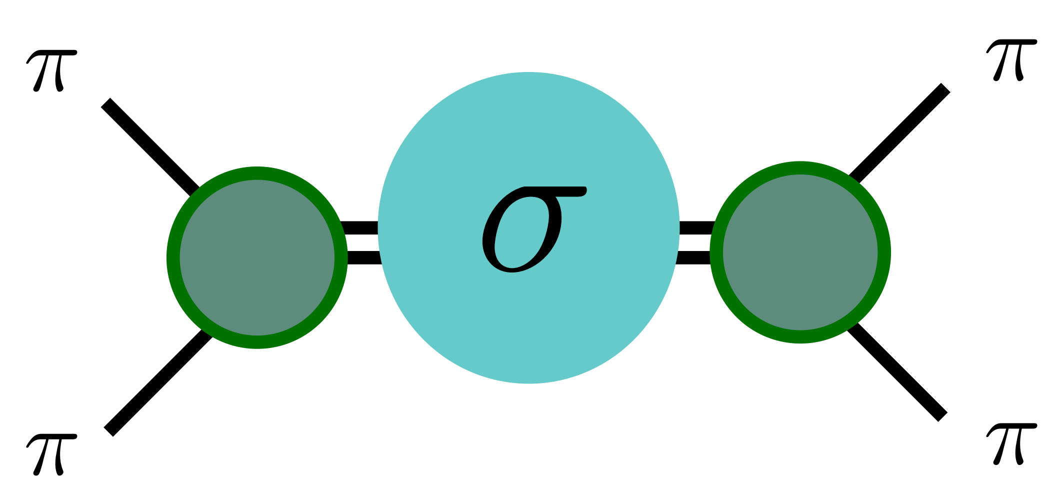


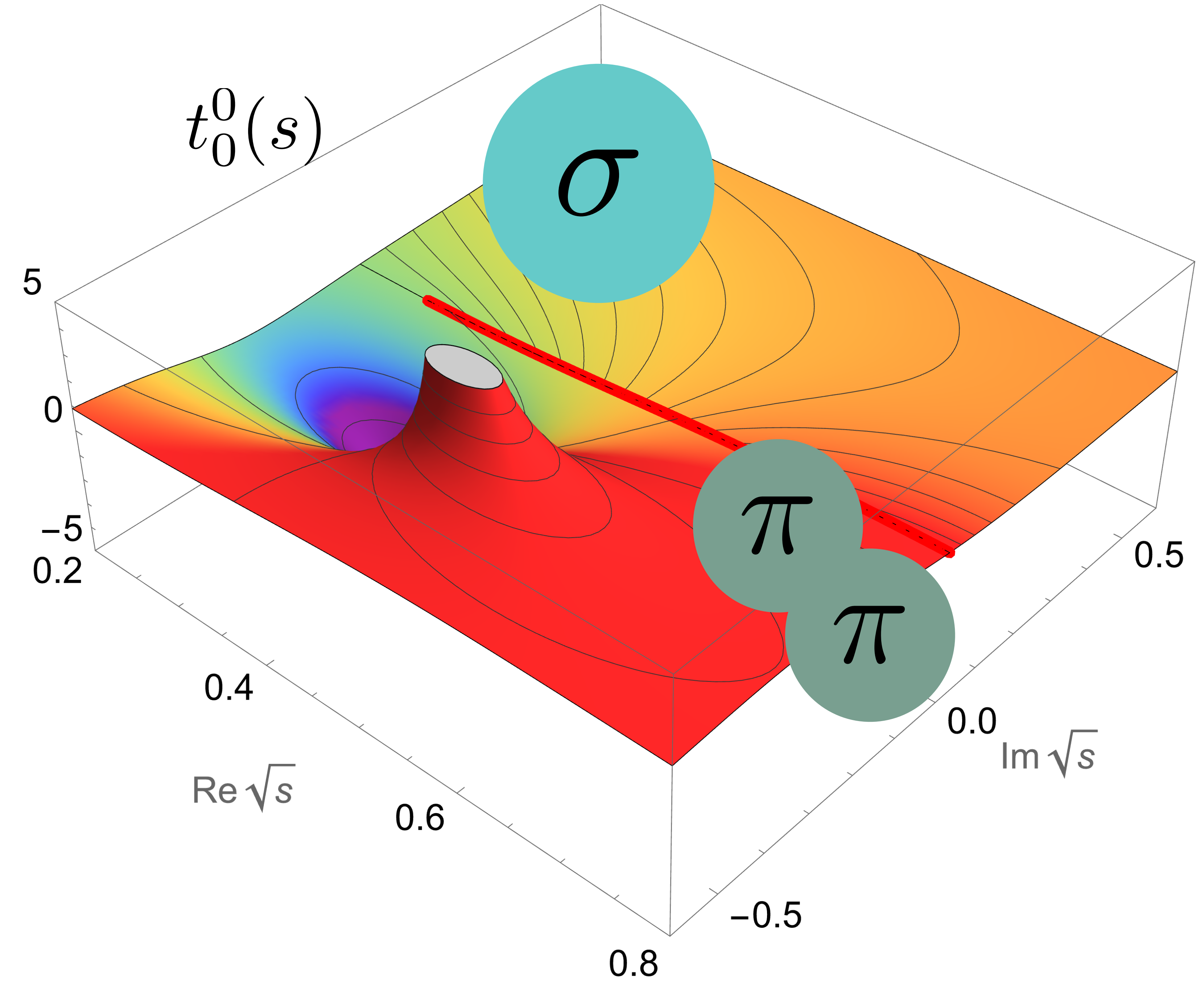
Extremely broad → extremely short-lived

Correlated with chiral symmetry-breaking phenomena (Adler zero)

Not well-understood → new observables ??

Light Scalars: the σ





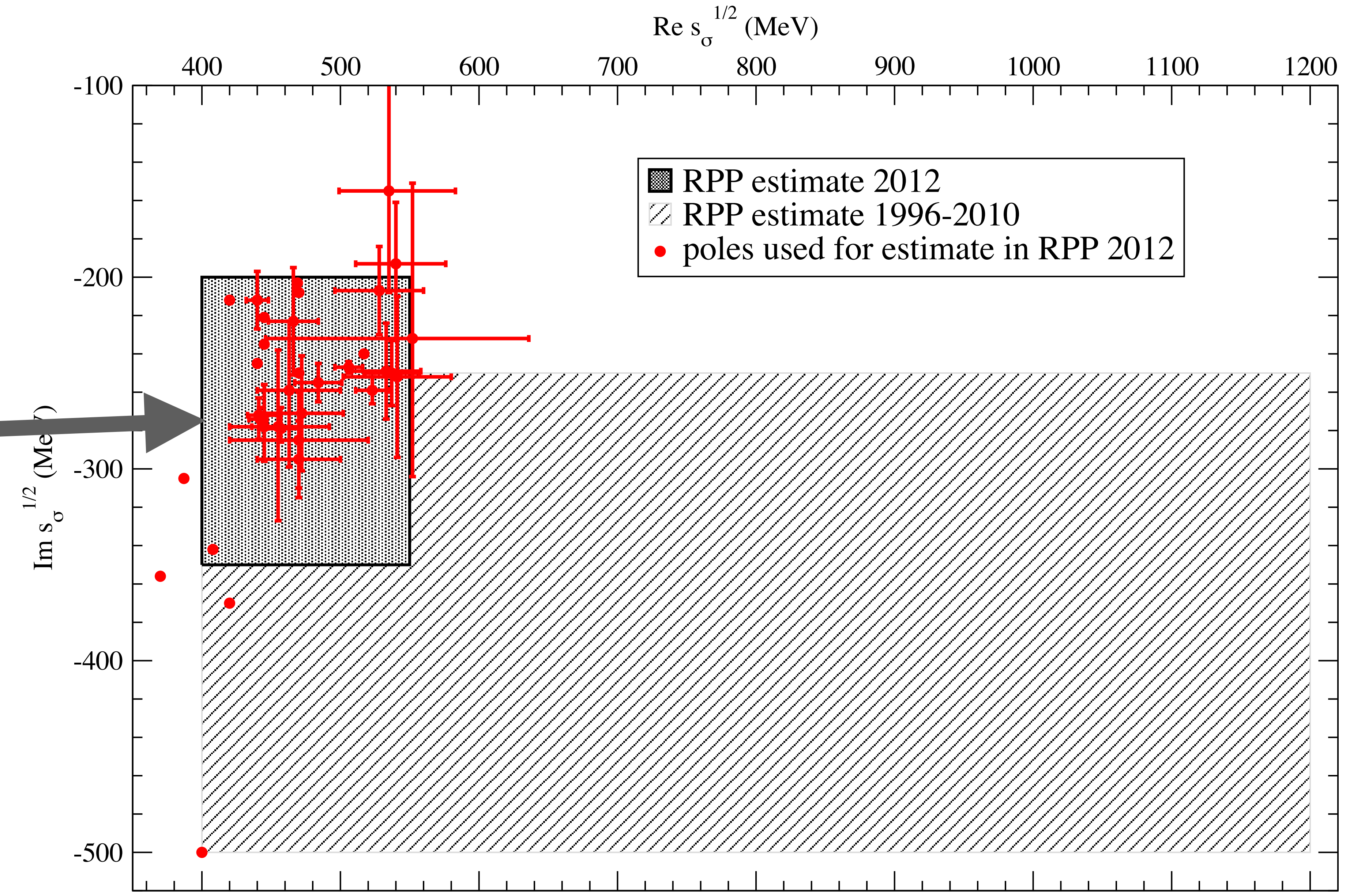
Lightest resonance in QCD

Extremely broad → extremely short-lived

Correlated with chiral symmetry-breaking phenomena (Adler zero)

Not well-understood → new observables ??

Very challenging experimental extraction



What happens for Lattice QCD ??

S-matrix

Basic principles that scattering amplitudes must preserve (more general than QCD)!

- [x] **Probability is conserved →Unitarity** $\text{Im}t_\ell^I(s) = \rho(s)|t_\ell^I(s)|^2$

- [ ] **Causality → analyticity**

- [ ] **Particle-antiparticle relation → crossing symmetry**

S-matrix

Basic principles that scattering amplitudes must preserve (more general than QCD)!

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- ☐ **Causality → analyticity**
- ☐ **Particle-antiparticle relation → crossing symmetry**

In practice, most analyses only apply the first one

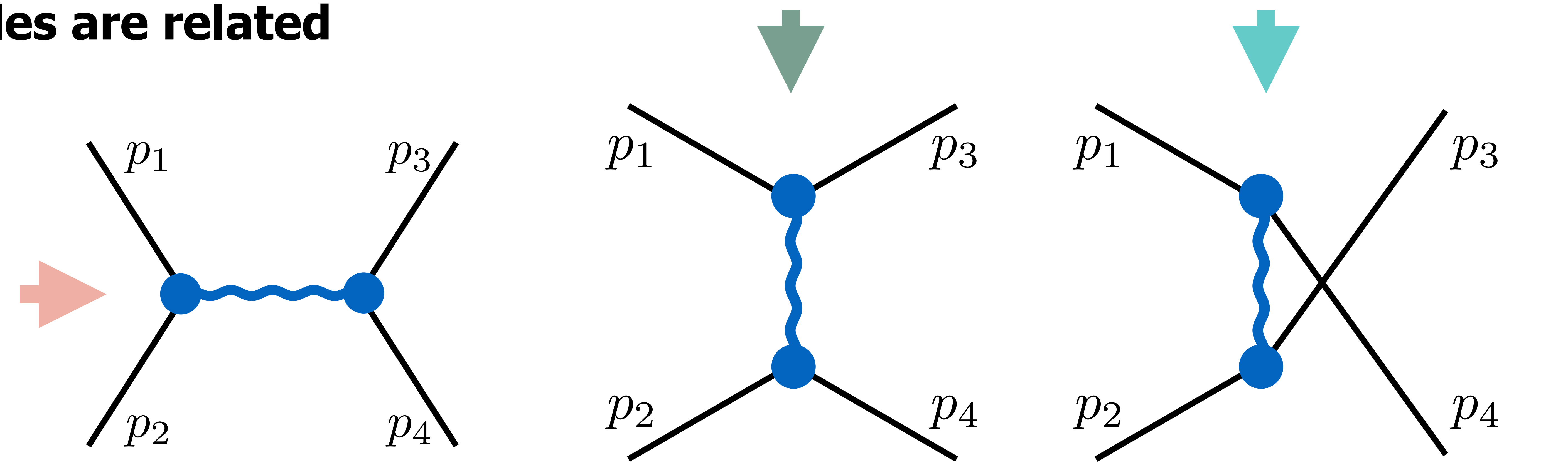
Crossing

Particles and anti-particles are related

s-channel

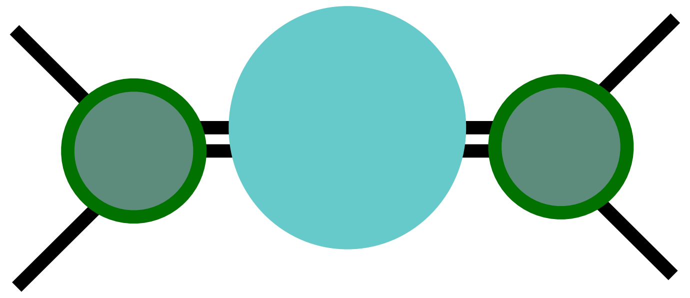
t-channel

u-channel



$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

Example, for $\pi\pi$, we can relate all amplitudes through a single function T



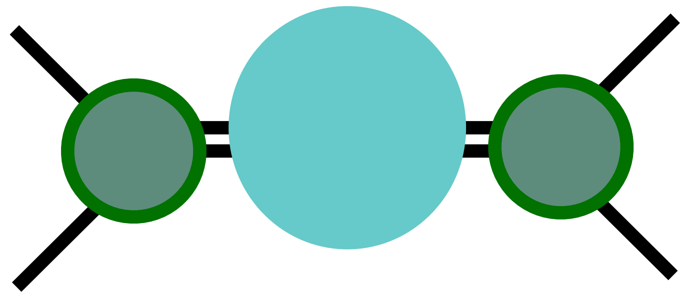
$$(\pi\pi)_{I=0} \to \ T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$
$$(\pi\pi)_{I=1} \to \ T^{I=1}(s,t,u) = T(t,s,u) - T(u,t,s)$$
$$(\pi\pi)_{I=2} \to \ T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)$$

More cumbersome for partial waves $T^I(s,t,u) = 32\pi \sum_\ell (2\ell+1) t^I_\ell(s) P_\ell\left(\cos\theta_s\right)$

$$t^I_\ell(s) = \tfrac{1}{64\pi} \int_{-1}^{1} \mathrm{d}\cos\theta_s \, T^I(s,t,u) \, P_\ell(\cos\theta_s)$$

Crossing

Example, for $\pi\pi$, we can relate all amplitudes through a single function T



$$(\pi\pi)_{I=0} \to T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$

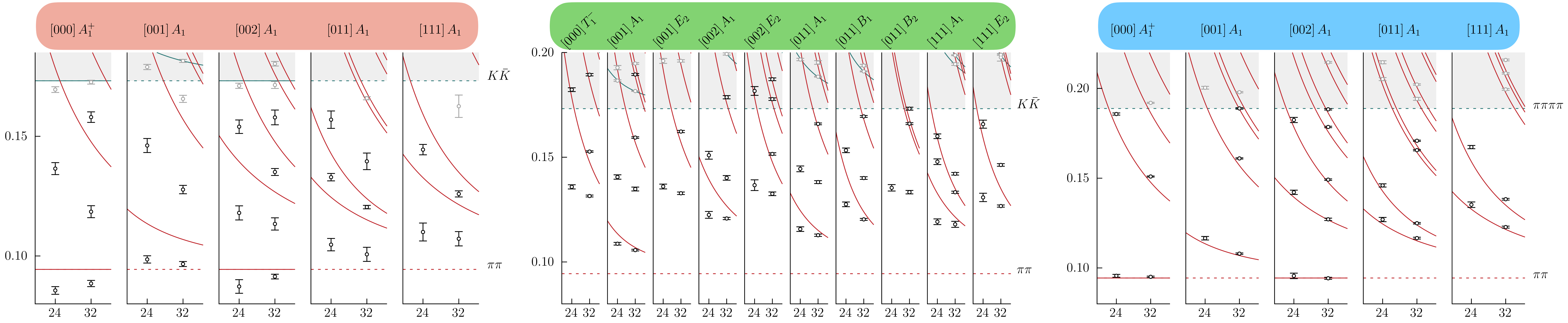
$$(\pi\pi)_{I=1} \to T^{I=1}(s,t,u) = T(t,s,u) - T(u,t,s)$$

$$(\pi\pi)_{I=2} \to T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)$$

Lattice QCD gives us information on well-defined isospin. We can combine it to build T

$m_\pi \sim 283$ MeV

$(\pi\pi)_{I=0}$ $(\pi\pi)_{I=1}$ $(\pi\pi)_{I=2}$



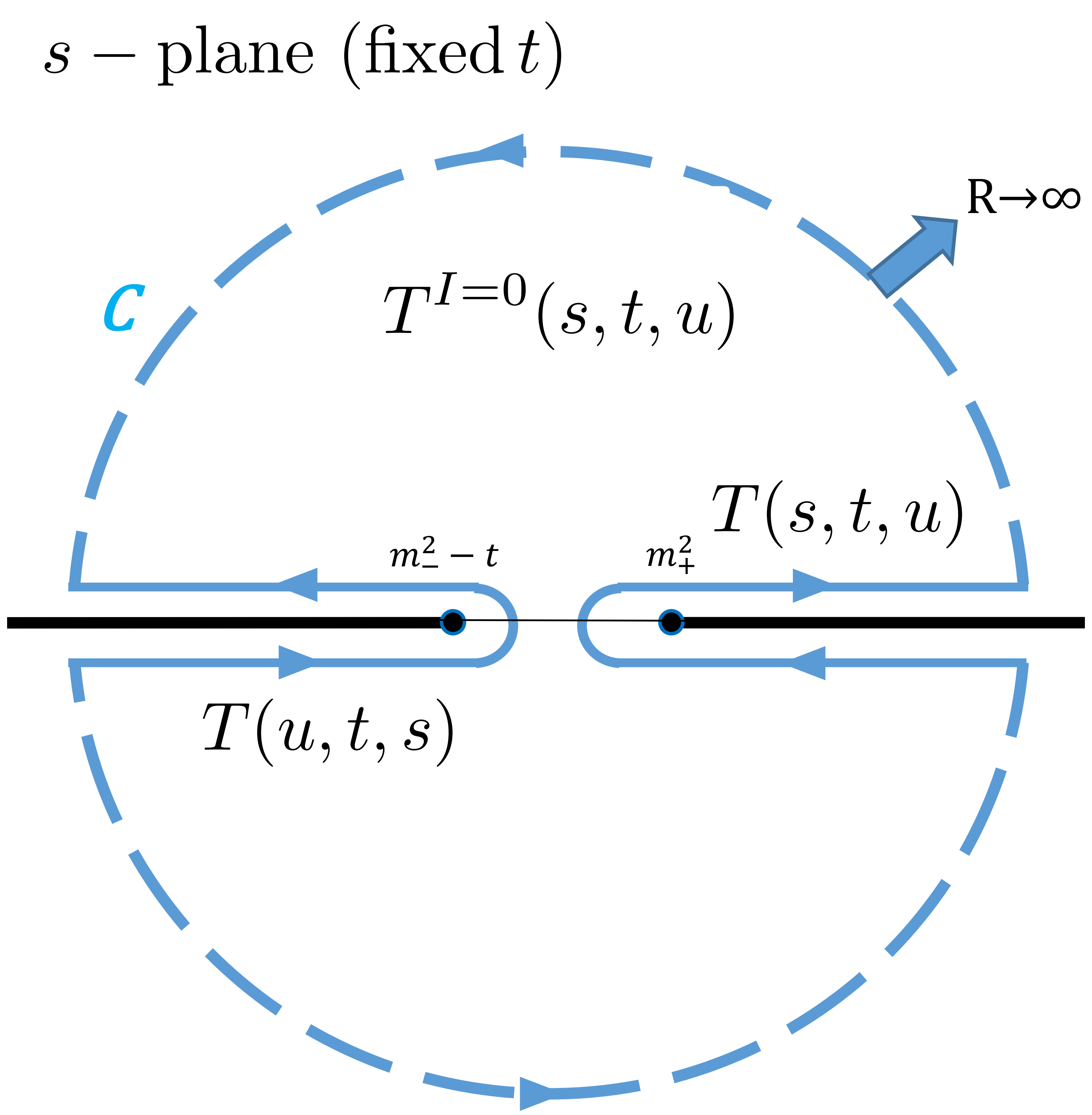
Dispersion relations

Hoferichter's talk

Cauchy theorem over contour C

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s}ds'$$

+ maybe subtractions



How is this useful?? → "hooks" are given by

$\text{Im}\,T(s,t,u) \rightarrow$ **data**

Project the integral to get your dispersion relations (ex. Roy eqs.):

Roy Phys.Lett.B 36 (1971)

$$t_\ell^I(s) \rightarrow \tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s',s)\,\text{Im}\,t_{\ell'}^{I'}(s')$$

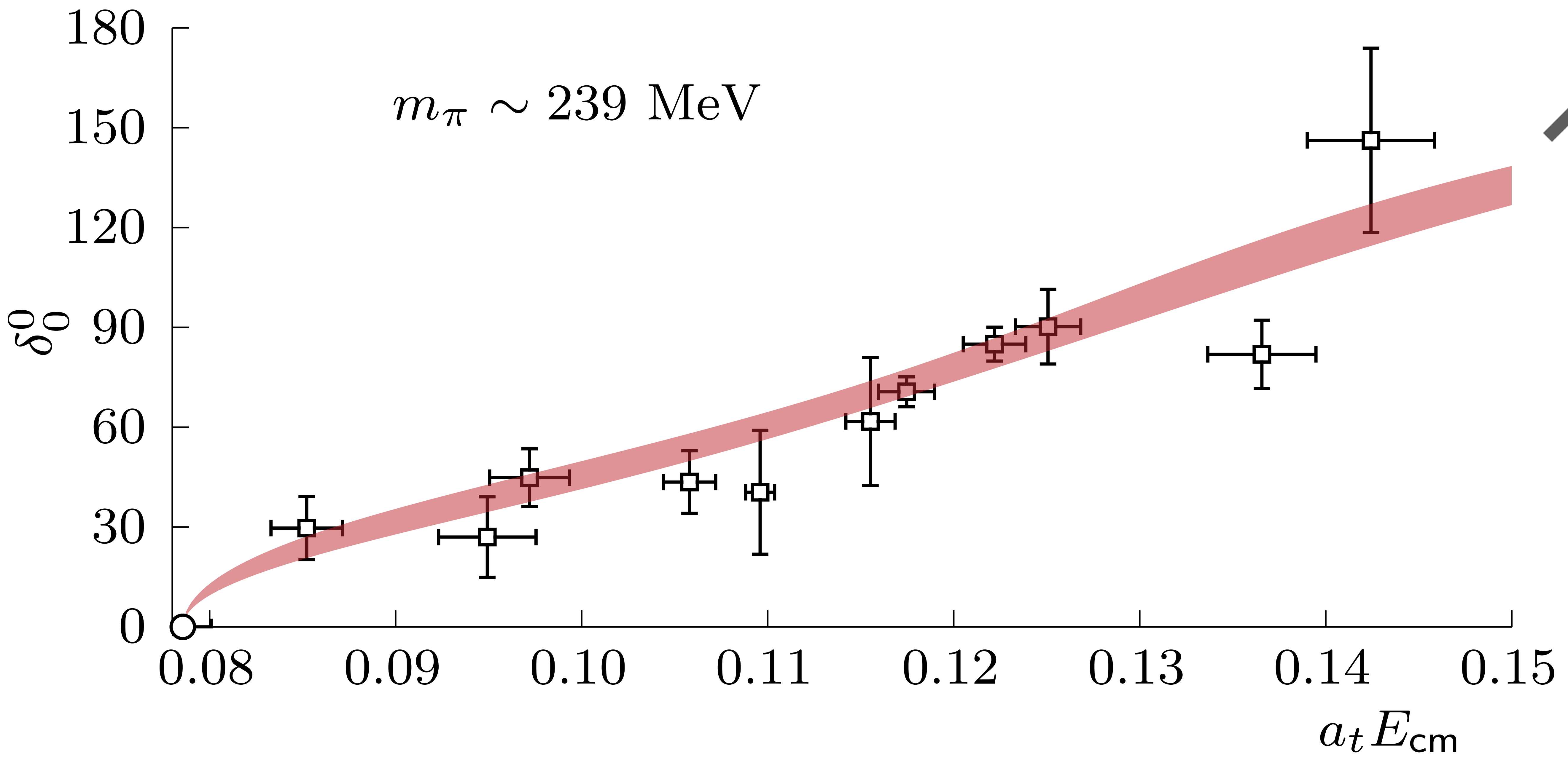
↑

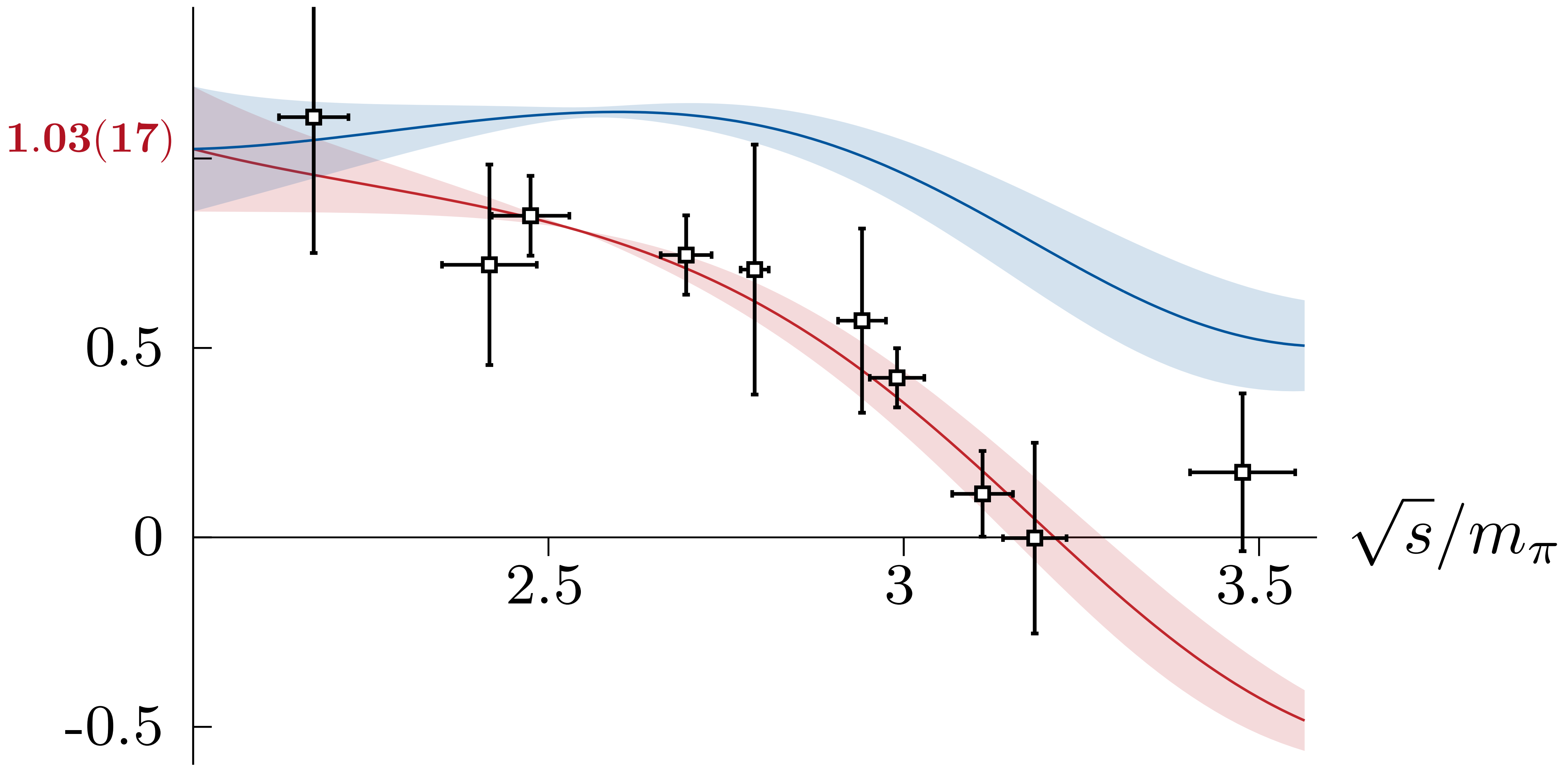
Sum over all isospins and partial waves (only a few ℓ contribute)

Select best combinations

Model 1

$$a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s', s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s') = \tilde{t}_0^0(s)$$

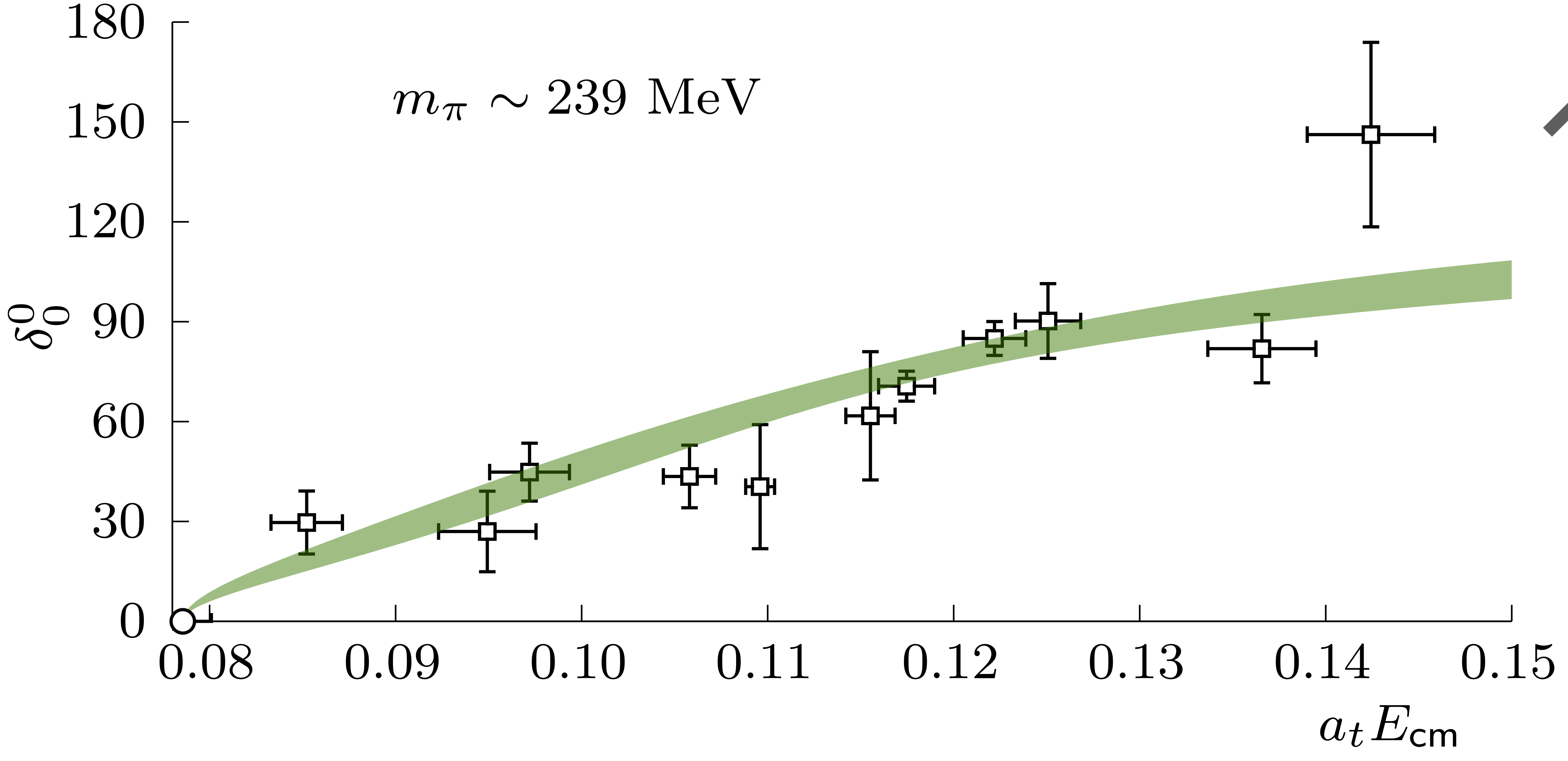


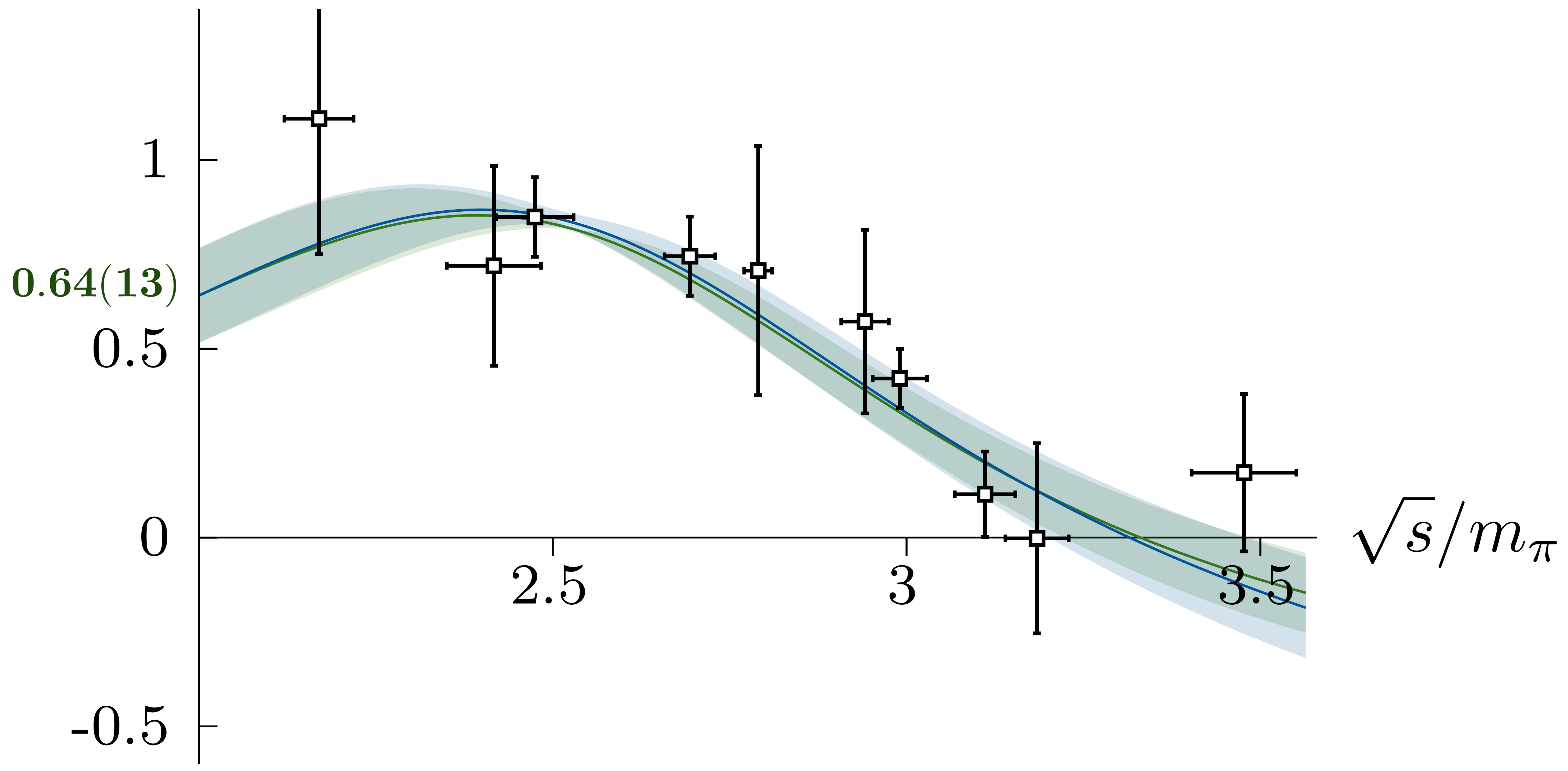


Select best combinations

Model 2

$$a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}\left(s', s\right) \mathrm{Im}\, t_{\ell'}^{I'}\left(s'\right) = \tilde{t}_0^0(s)$$





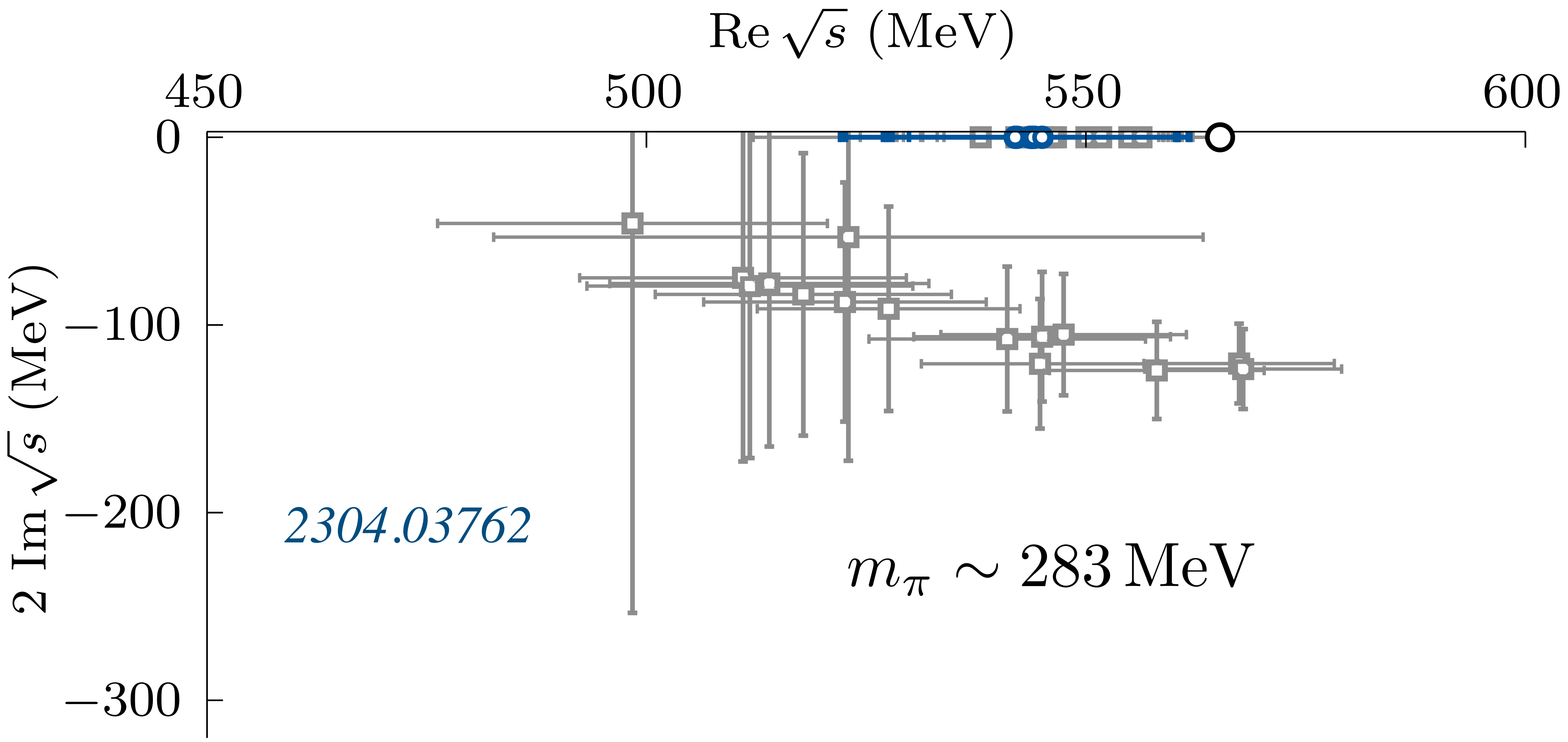
Outside the physical region

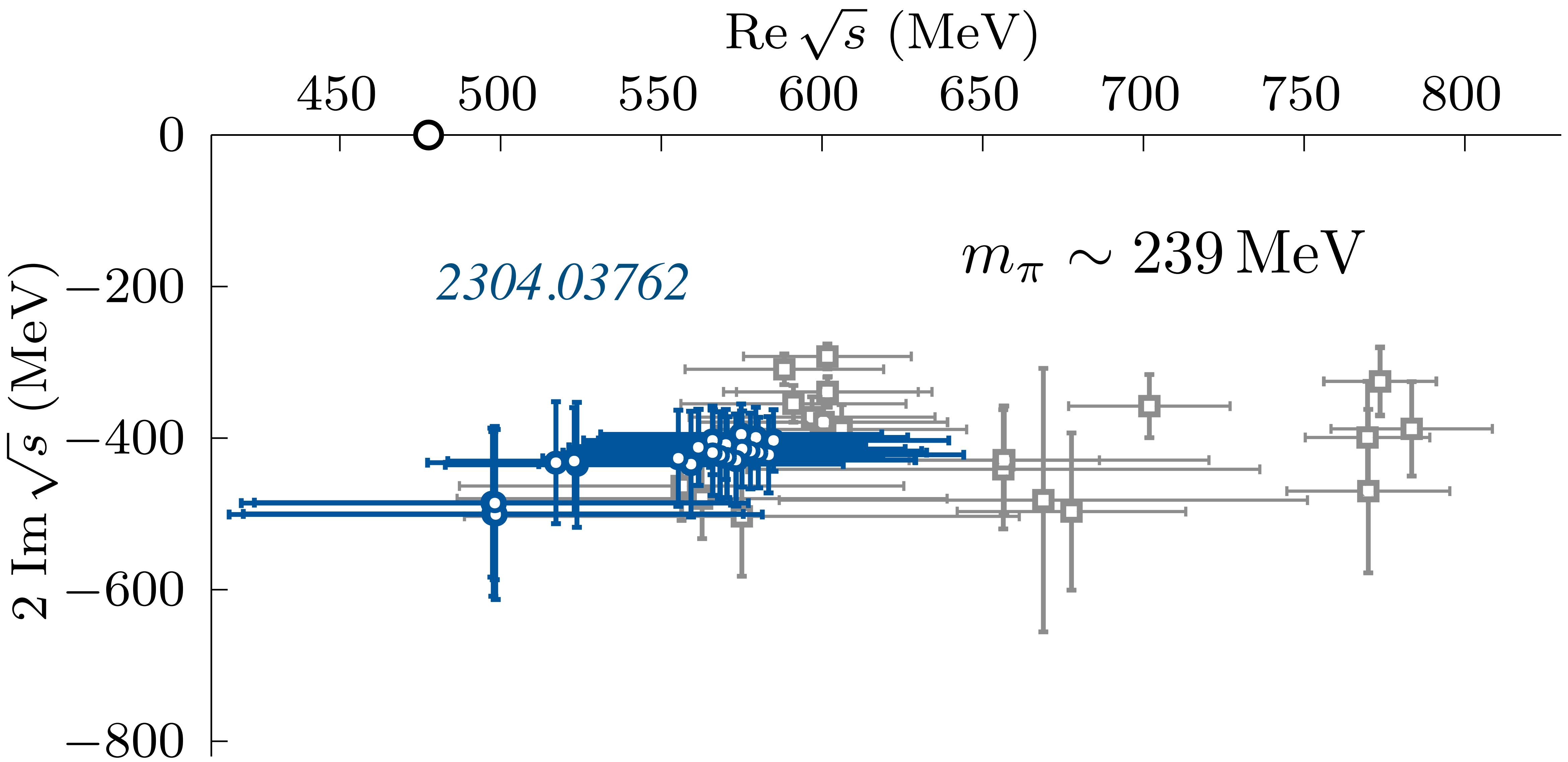
- Probability is conserved →Unitarity
- Causality → analyticity
- Particle-antiparticle relation → crossing symmetry

Systematic tension is drastically reduced!!

Ordinary analysis

Dispersive analysis



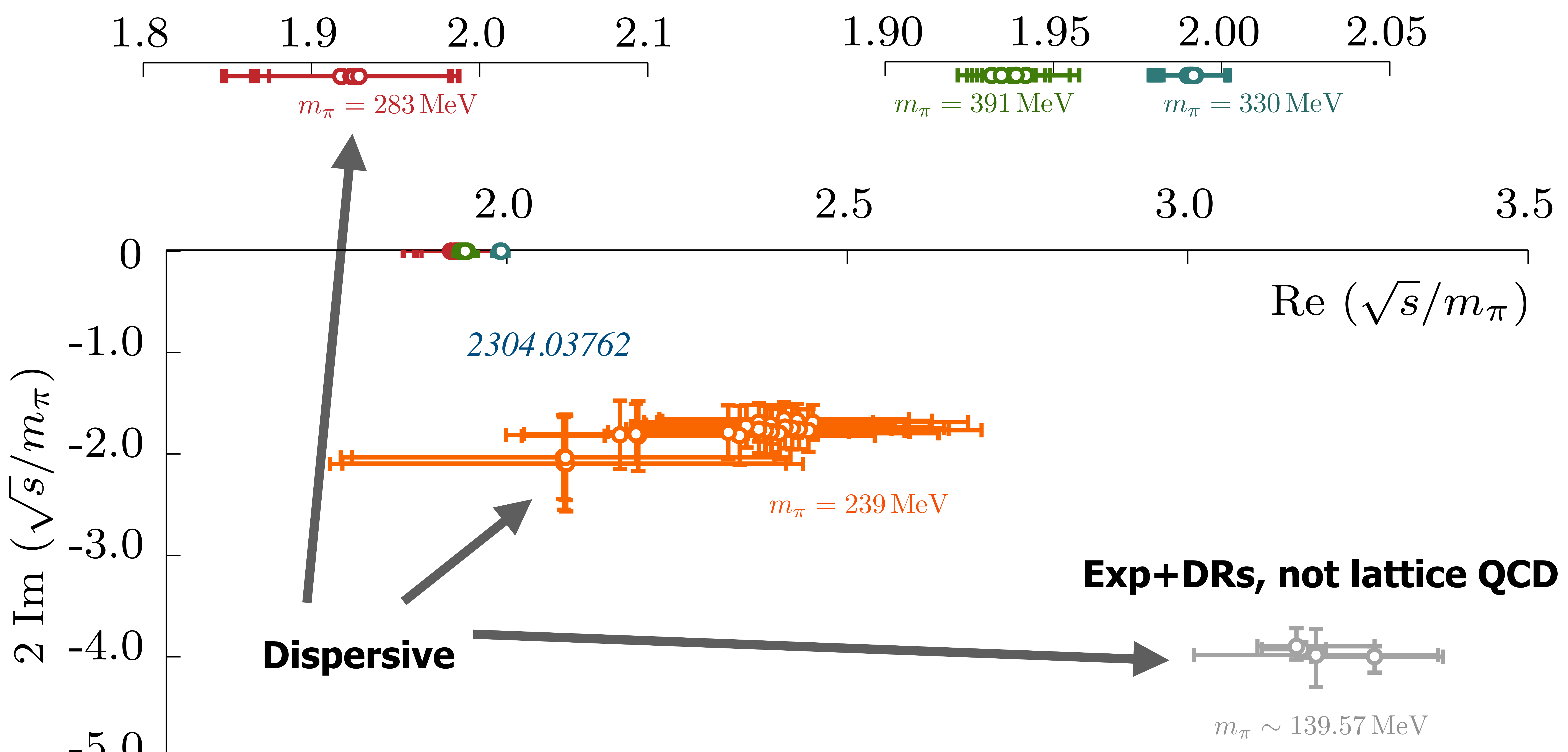


Outside the physical region

Various recent, dispersive determinations

Another dispersive approach

2303.02596



Summary and outlook

First-principles extraction of a broad resonance directly from QCD

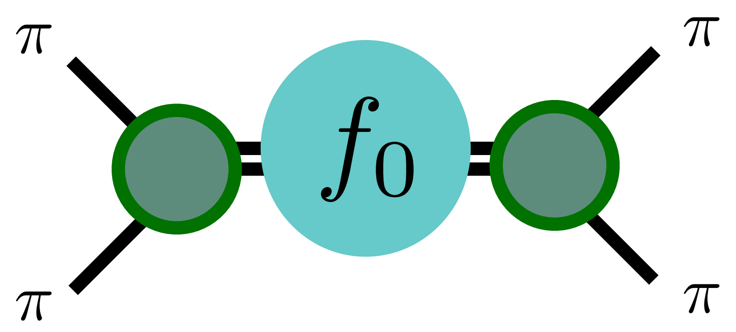
The lighter the π, the more relevant this approach is

(Much) Better constraints over scattering lengths

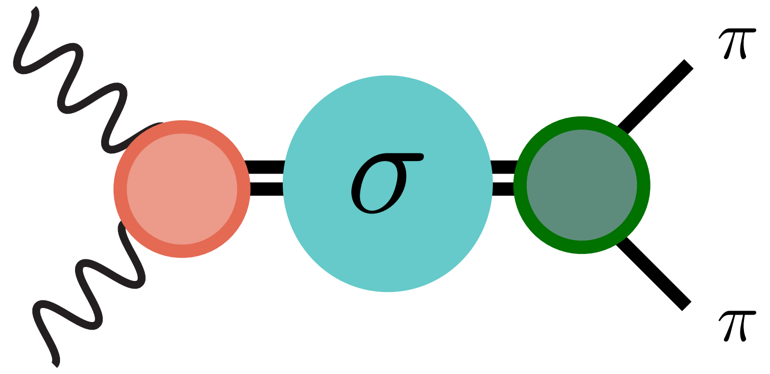
Future

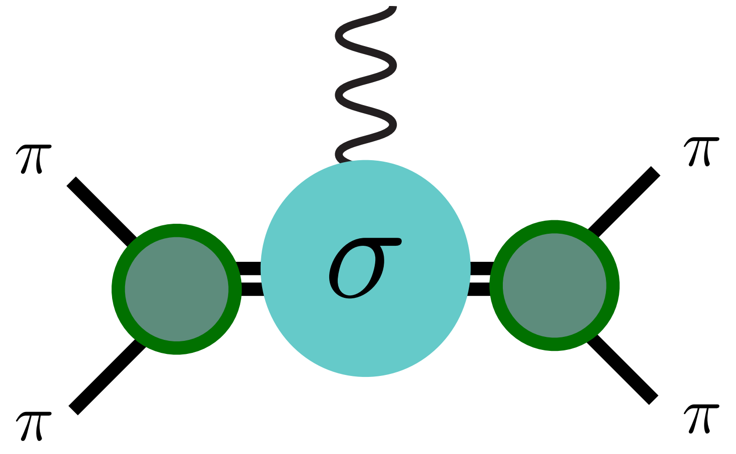
Study low-energy observables in more detail (Adler zeroes move away from real axis, for large m_π)

Extract the $f_0(980)$??



Study new observables ??





Spare slides

Permutations

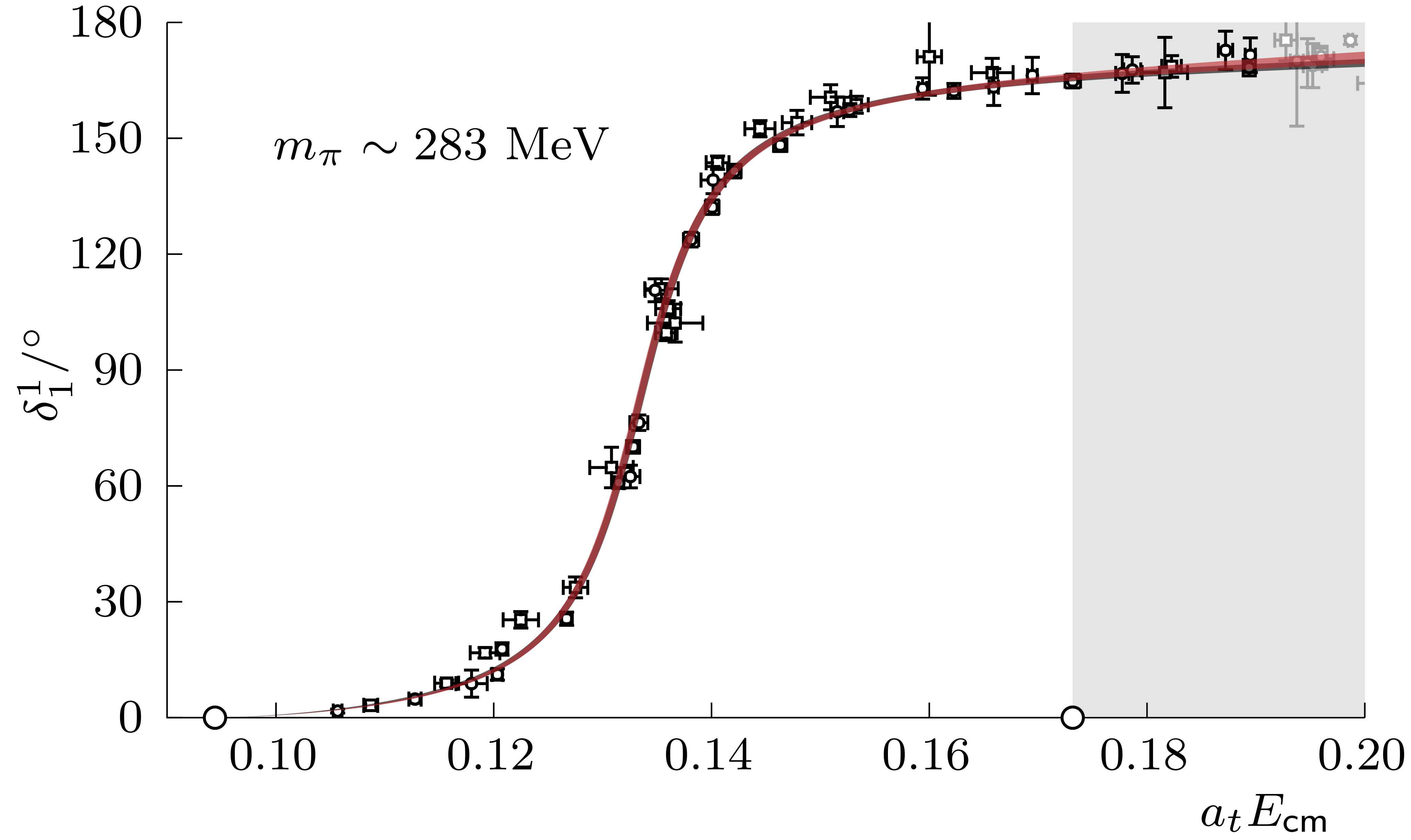
$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s', s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

For ℓ_{max} partial waves

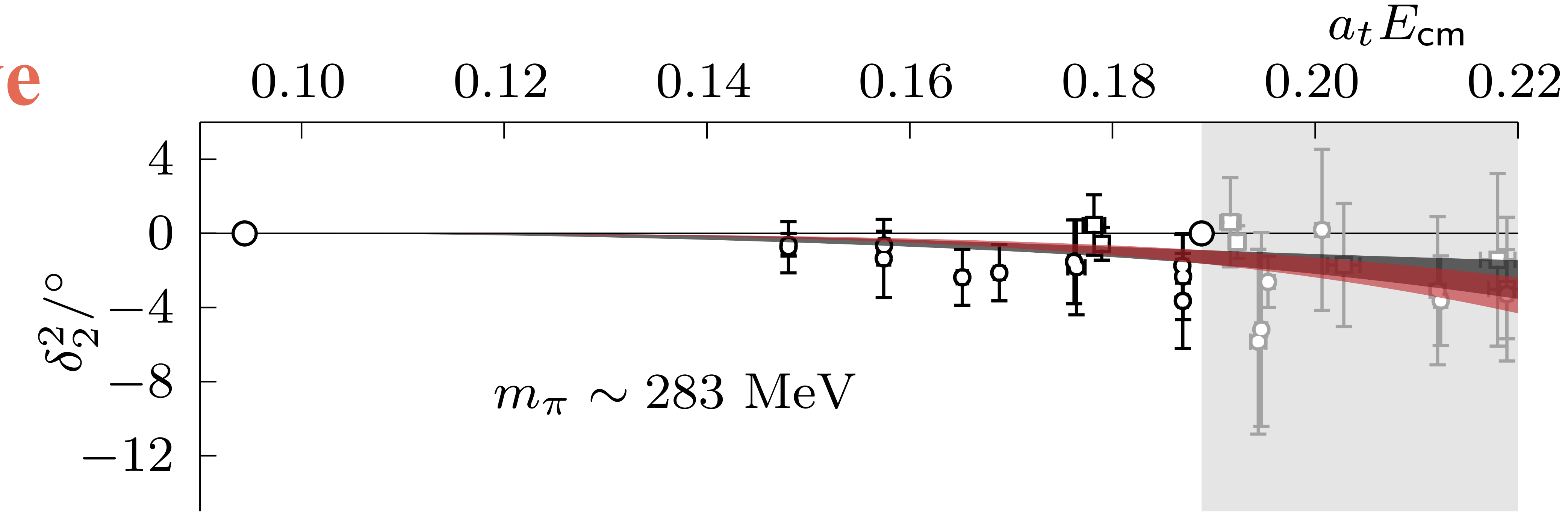
$$N_I \ell_{max} N_{params} \sim 10^5$$

We can fix most

$I = 1$ ***P*-wave**



$I = 2$ ***D*-wave**

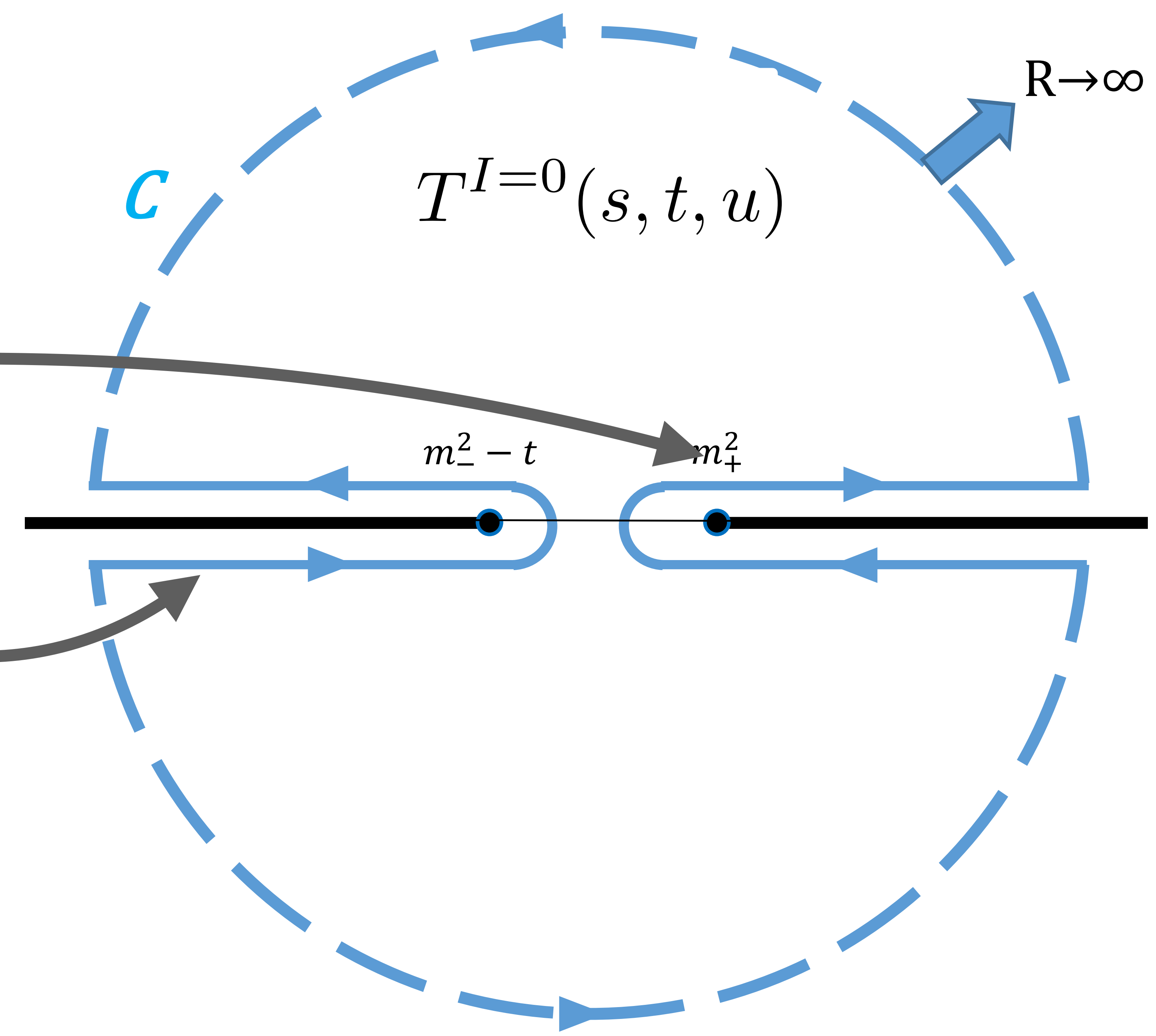


Crossing

Determines the analytic structure of the amplitudes

$T(s,t,u)$ **has a unitarity cut for** $s \geq 4m_\pi^2$

$$T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$



Cauchy theorem over contour C

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s}ds'$$

How is this useful?? → "hooks" are given by $\mathrm{Im}\,T(s,t,u) \rightarrow$ **direct+crossing data**

Project the integral to get your dispersion relations (ex. Roy eqs.):

Roy Phys.Lett.B 36 (1971)

$$t_\ell^I(s) \rightarrow \tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s',s)\,\mathrm{Im}\,t_{\ell'}^{I'}(s')$$

$$\tau_0^0(s)/m_\pi = \tfrac{1}{3}\left(a_0^0 + 5a_0^2\right) + \tfrac{1}{3}\left(2a_0^0 - 5a_0^2\right)\frac{s}{4m_\pi^2}$$

Outside the physical region

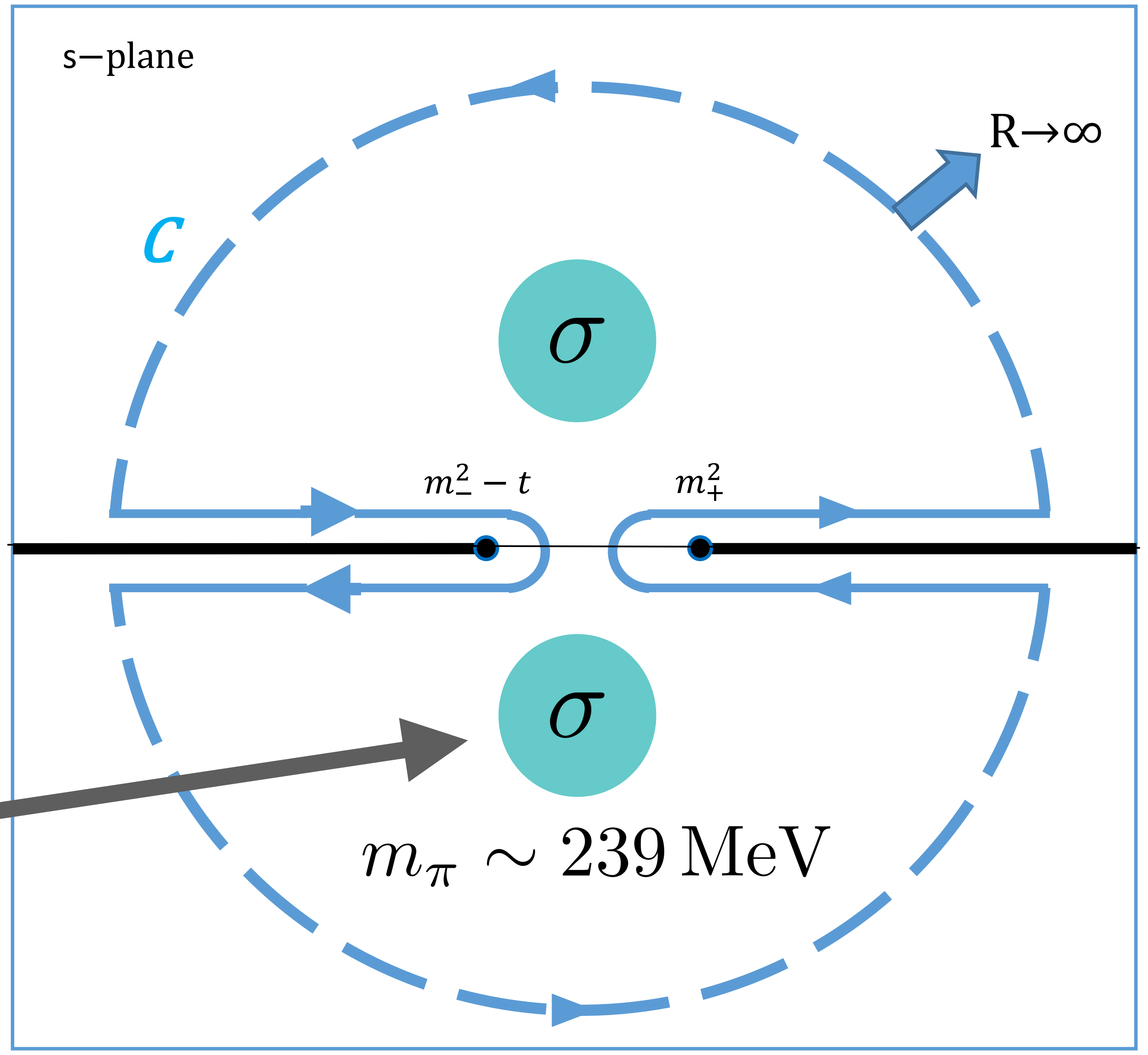
Both sides are good now, we can now apply Cauchy's theorem+crossing

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s}ds'$$

$$T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$

$$T^{I=1}(s,t,u) = T(t,s,u) - T(u,t,s)$$

$$T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)$$



Now, what happens here??

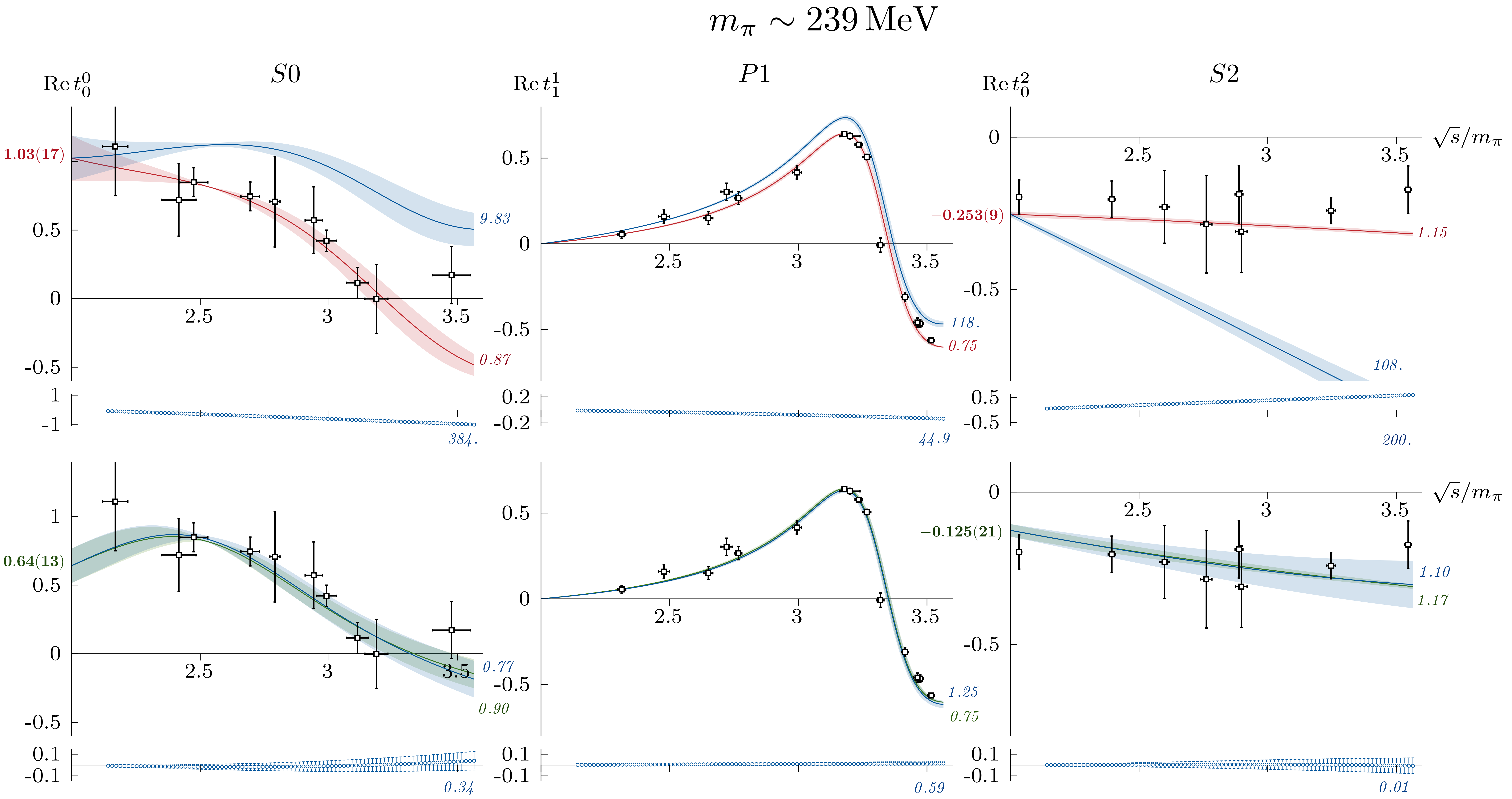
Tests: good vs bad

We select those models that respect the DRs

Fit combination 1

Dispersive output

Fit combination 2



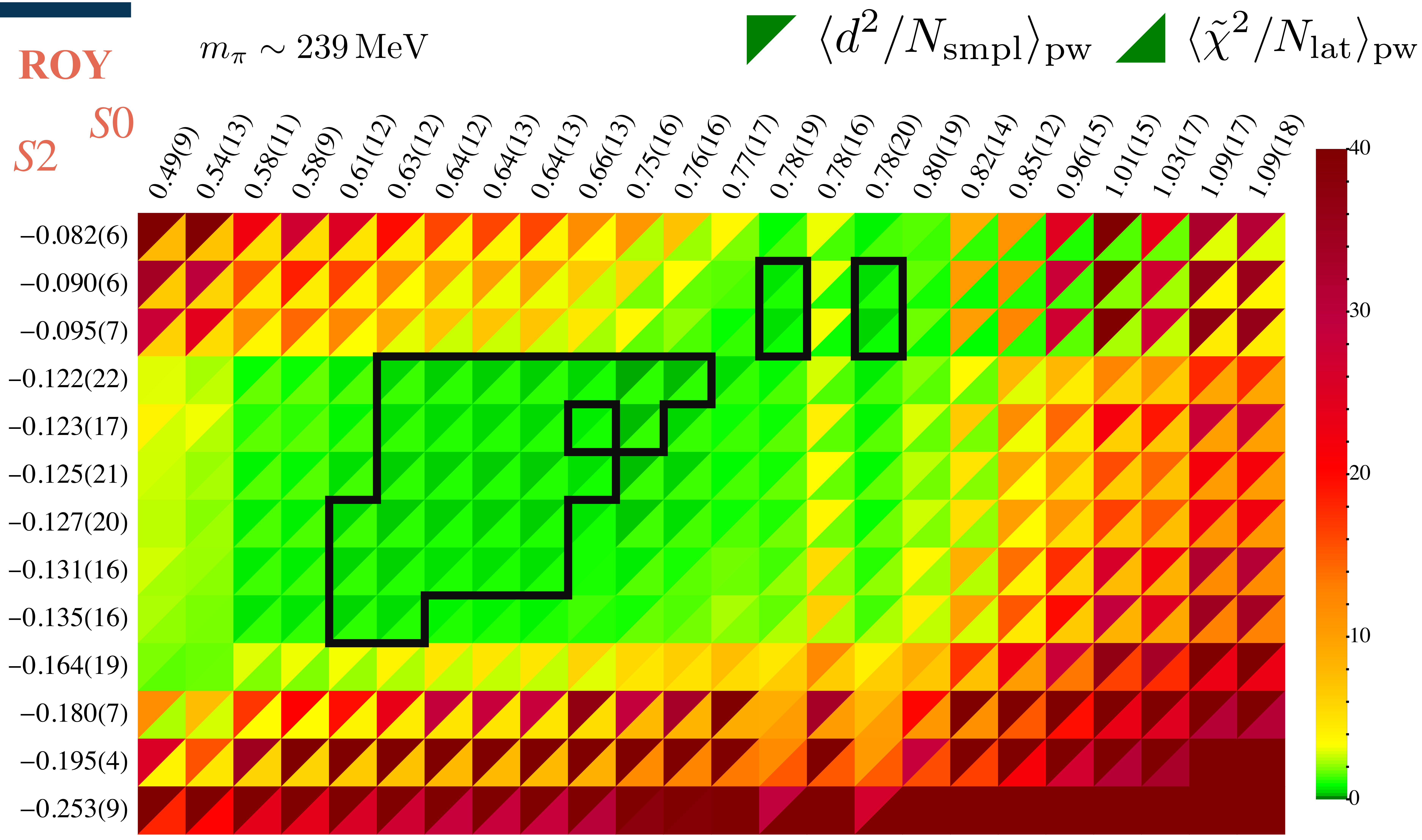
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Outside the physical region

Both sides are good now, we can now apply Cauchy's theorem+crossing

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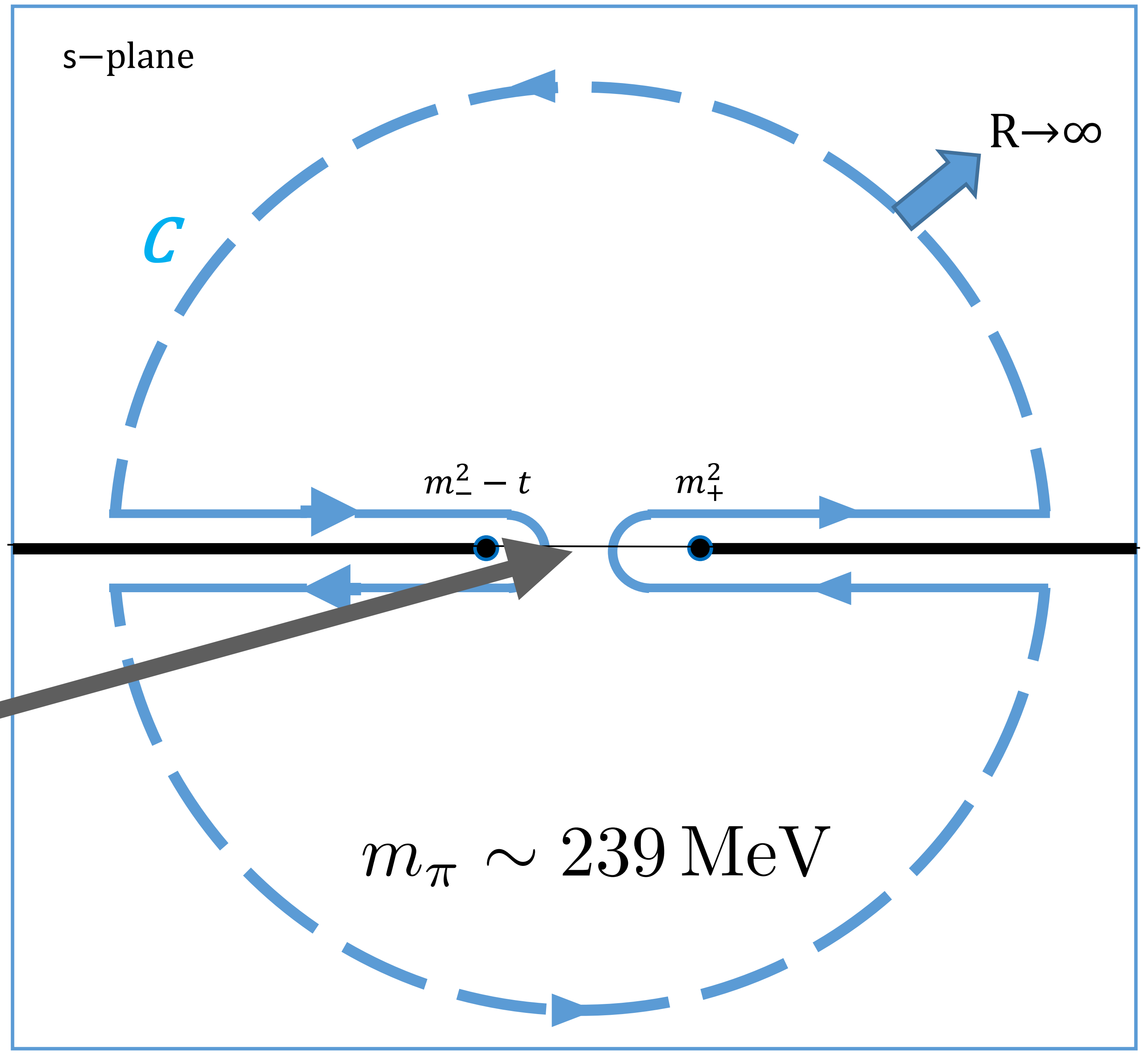
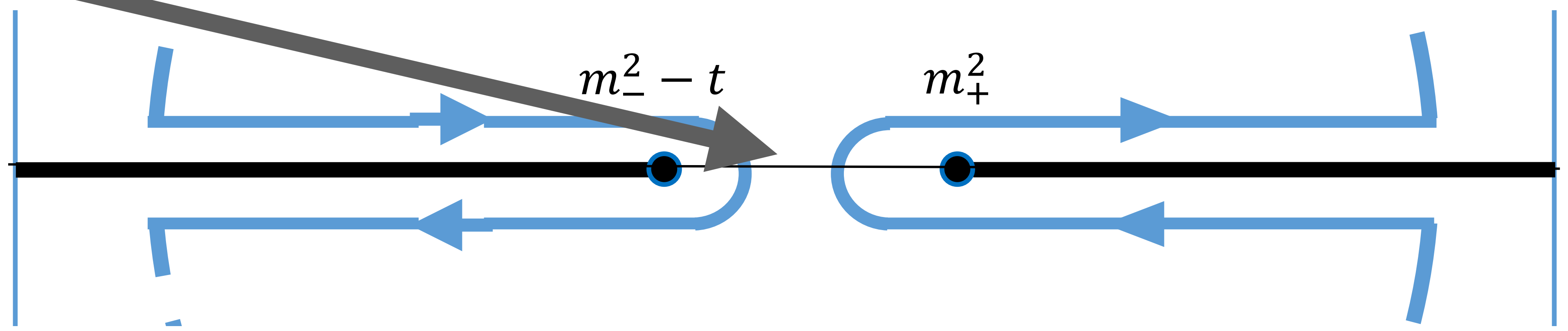


Now, what happens here??

Adler Zeroes

If $m_\pi \simeq 0$ **then** $T(s,t,u) \underset{s\sim 0}{\rightarrow} 0$

Adler, Phys.Rev. 137 (1965)



These zeroes appear on the S-waves and are considered directly linked to ChPT

ChPT predicts Adler zeroes for all pseudo-scalar scattering amplitudes at LO

$$s_{A,I=0} = m_\pi^2/2 \qquad\qquad s_{A,I=2} = 2m_\pi^2$$

In lattice QCD our $m_\pi \nsim 0$**, but we still use ChPT in most analyses**

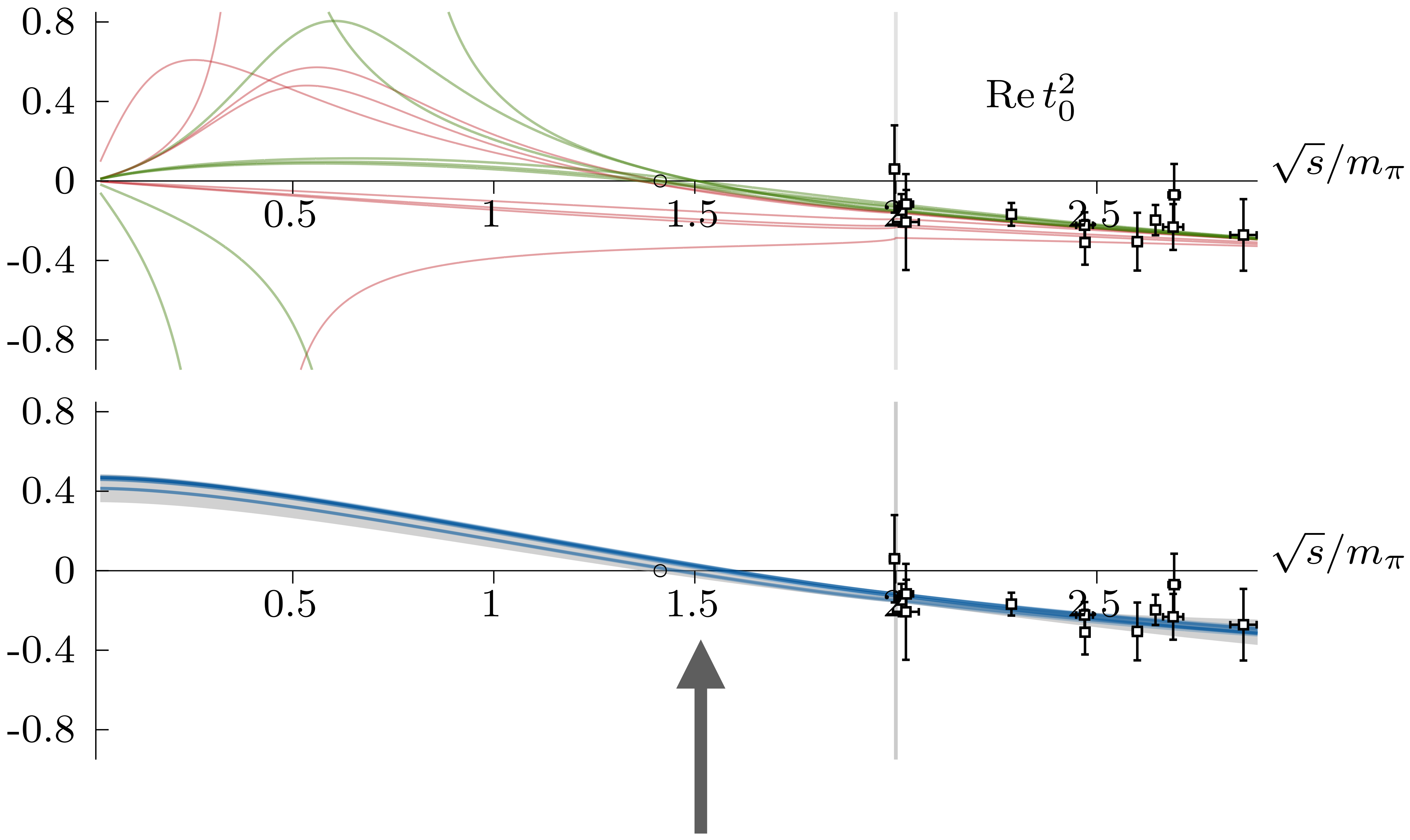
What can our DRs say about that?

Adler Zeroes

Very "stable" for $I = 2$ $\pi\pi$

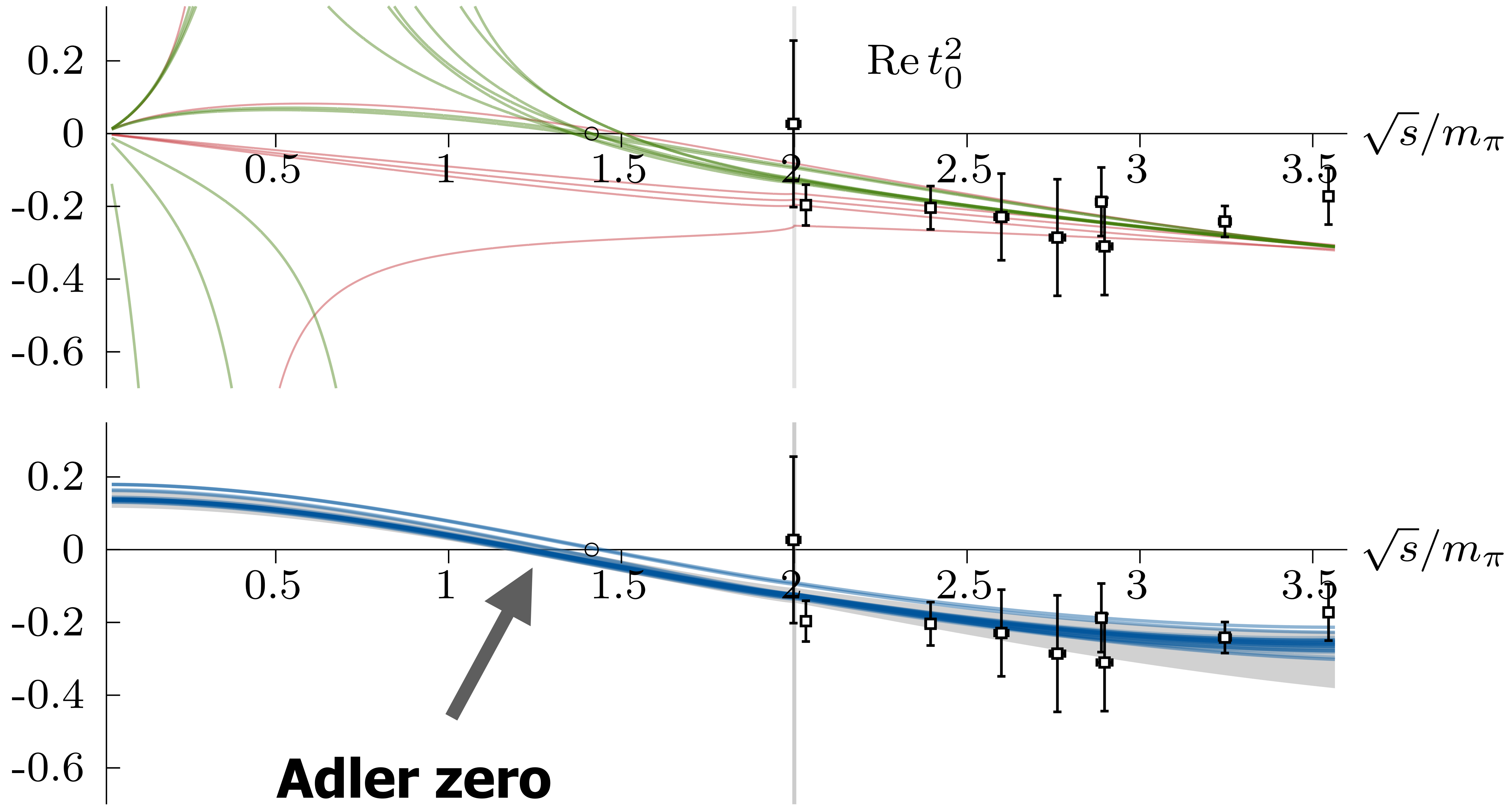
Even "bad" DRs produce Adler zeroes for I=2, close to the LO prediction $s_{A,I=2} = 2m_\pi^2$

$m_\pi \sim 283\,\mathrm{MeV}$



Adler zero

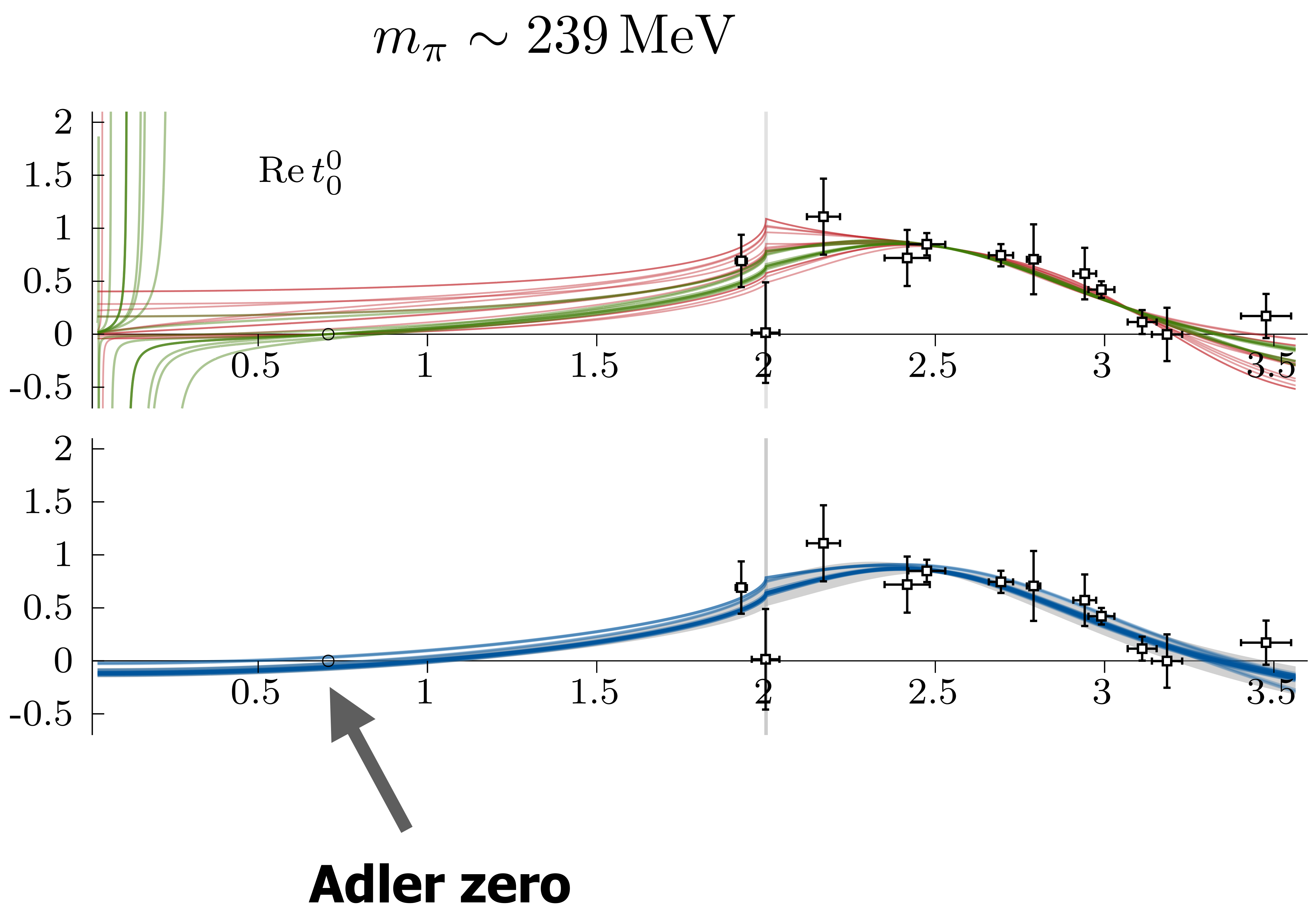
$m_\pi \sim 239\,\mathrm{MeV}$



Adler zero

Adler Zeroes

All good DRs produce an $I = 0$ $\pi\pi$ Adler zero for the lighter mass

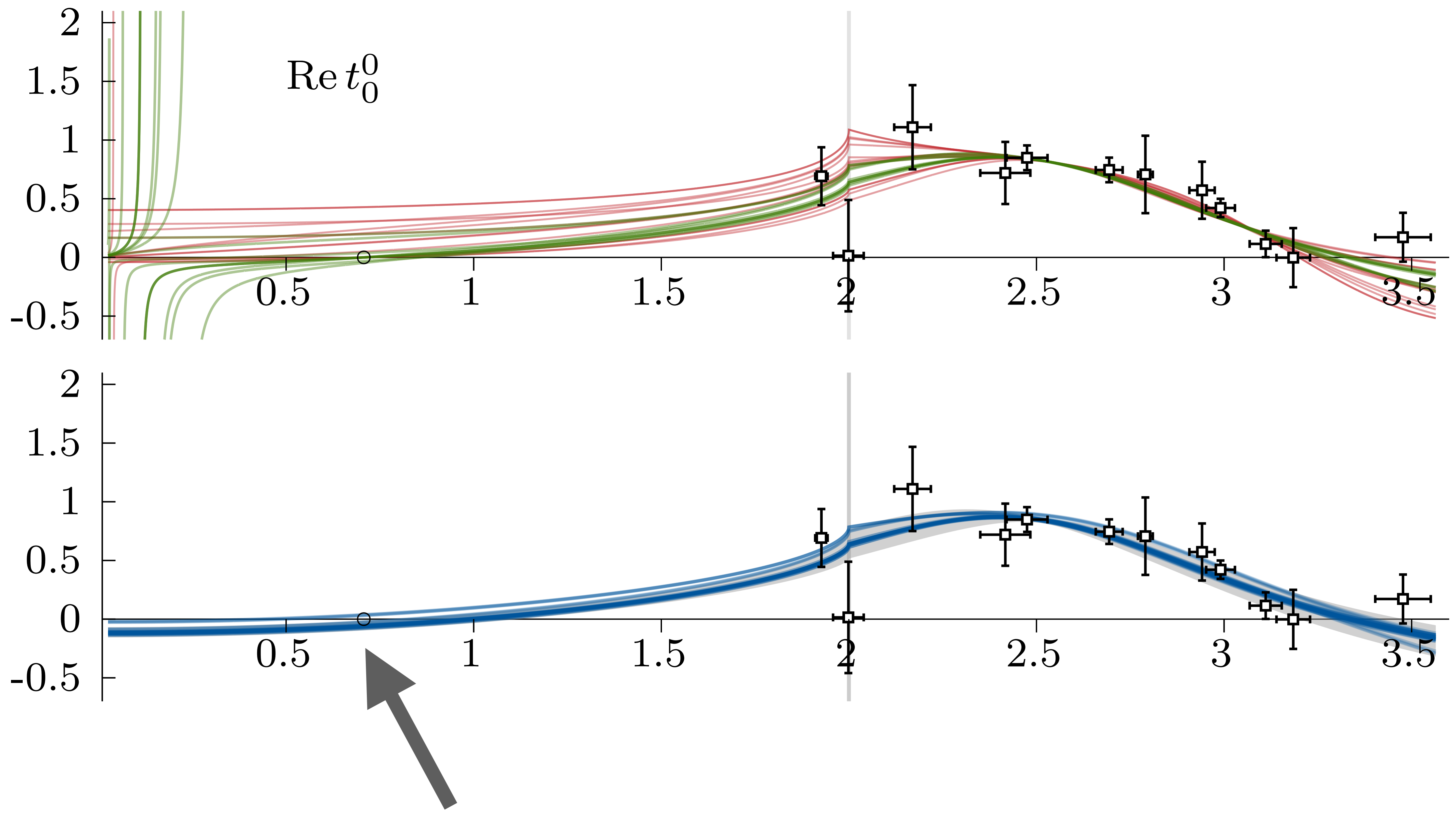


Adler Zeroes

All good DRs produce an $I = 0$ $\pi\pi$ Adler zero for the lighter mass

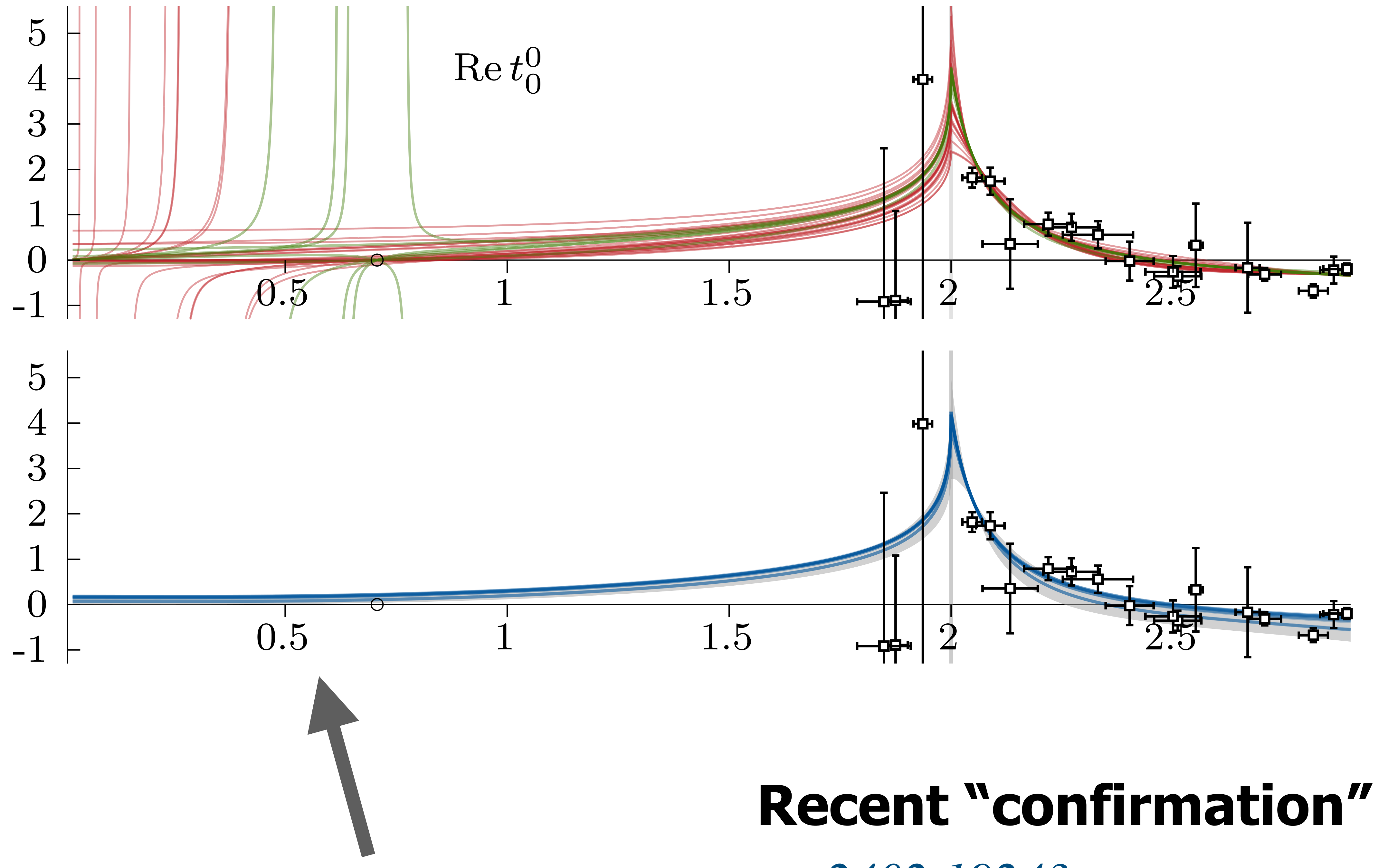
No good DR produces an $I = 0$ $\pi\pi$ Adler zero for the heavier mass

$m_\pi \sim 239\,\mathrm{MeV}$



Adler zero

$m_\pi \sim 283\,\mathrm{MeV}$



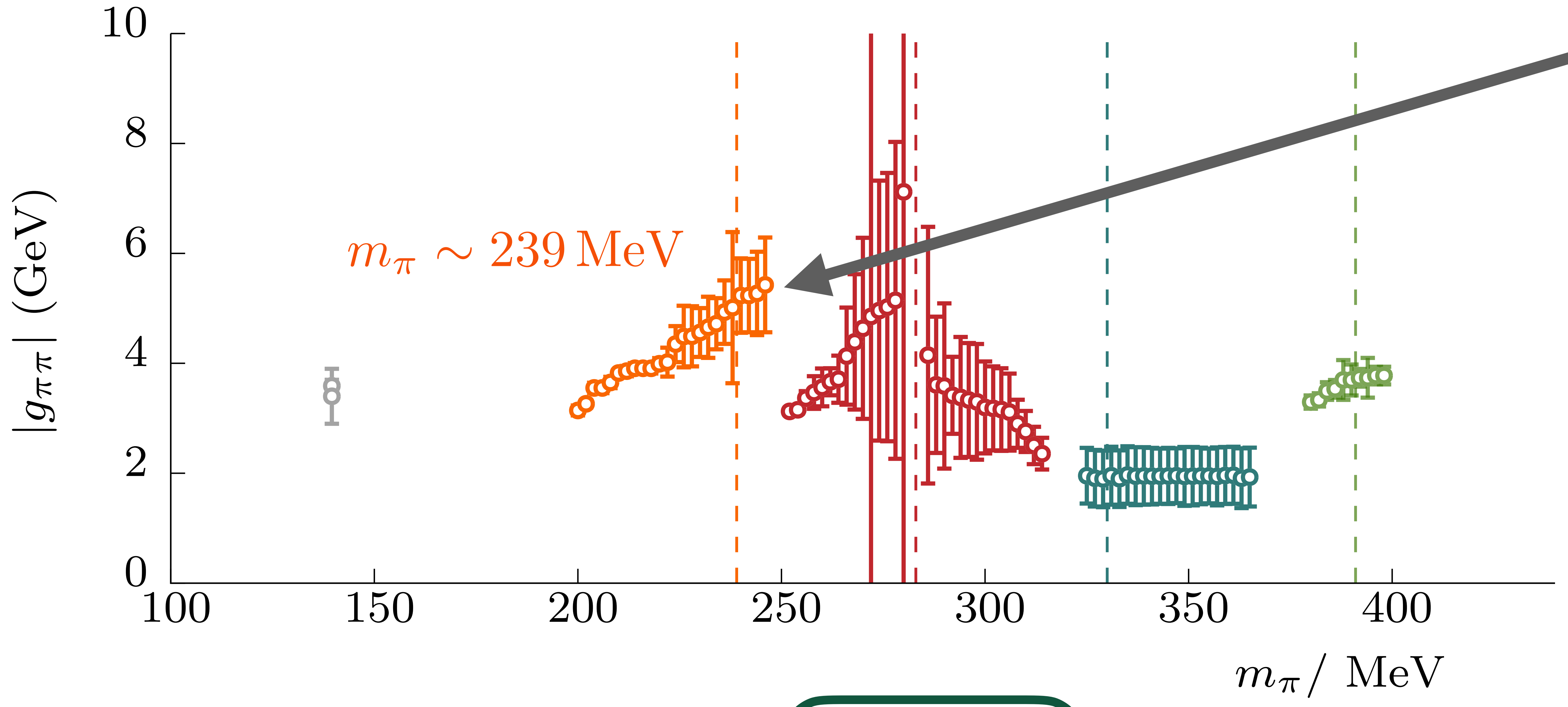
NO Adler zero

Recent "confirmation"

2402.19243

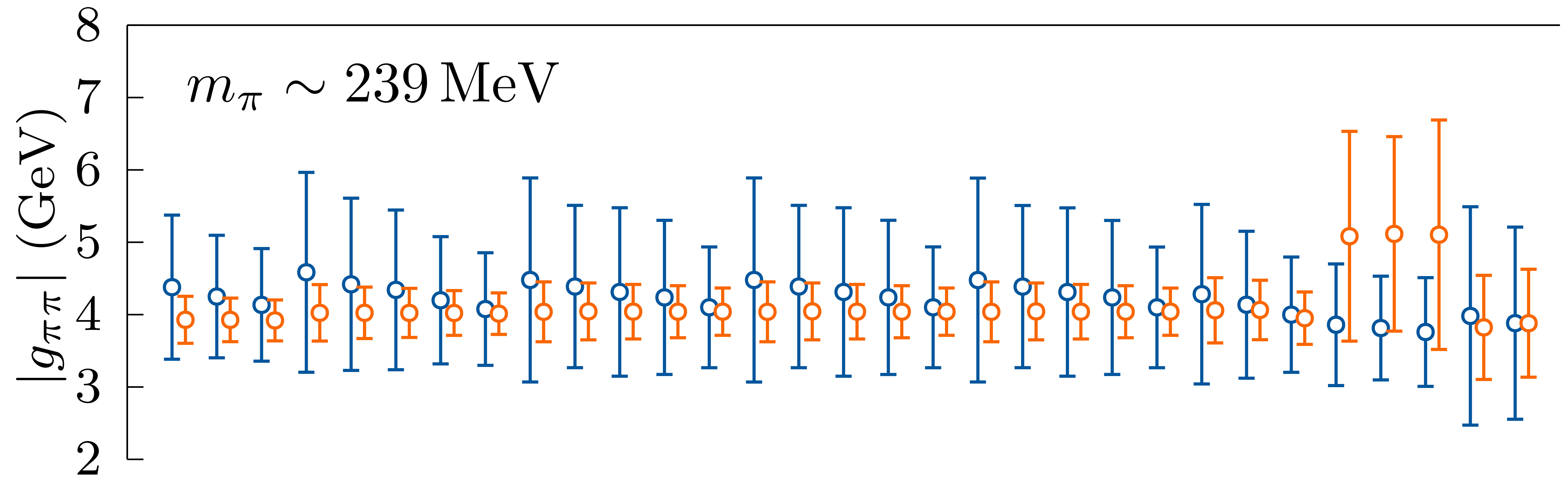
Couplings

Models: Large systematic dependence



2304.03762

DRs: No systematics



- Make $Fit \to In$ $DR \to Out$ compatible ⟷ Unitarity

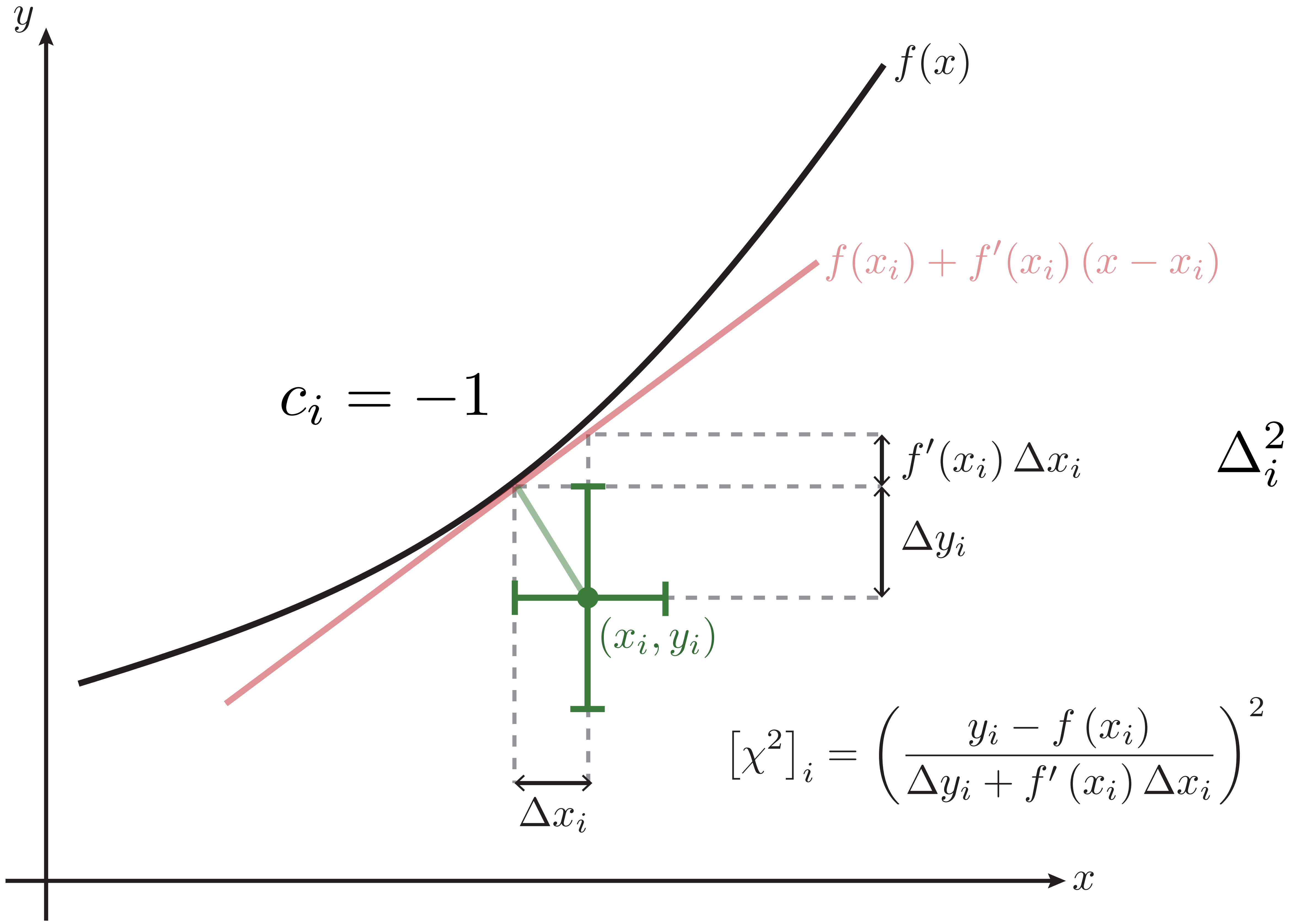
$$\left[d^2\right]_\ell^I \equiv \sum_{i=1}^{N_{\mathrm{smpl}}} \left(\frac{\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)}{\Delta\left[\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)\right]} \right)^2$$

- Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\mathrm{lat}}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i} \right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right)^{-1} \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j} \right)$$

Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i}\right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right)^{-1} \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j}\right)$$

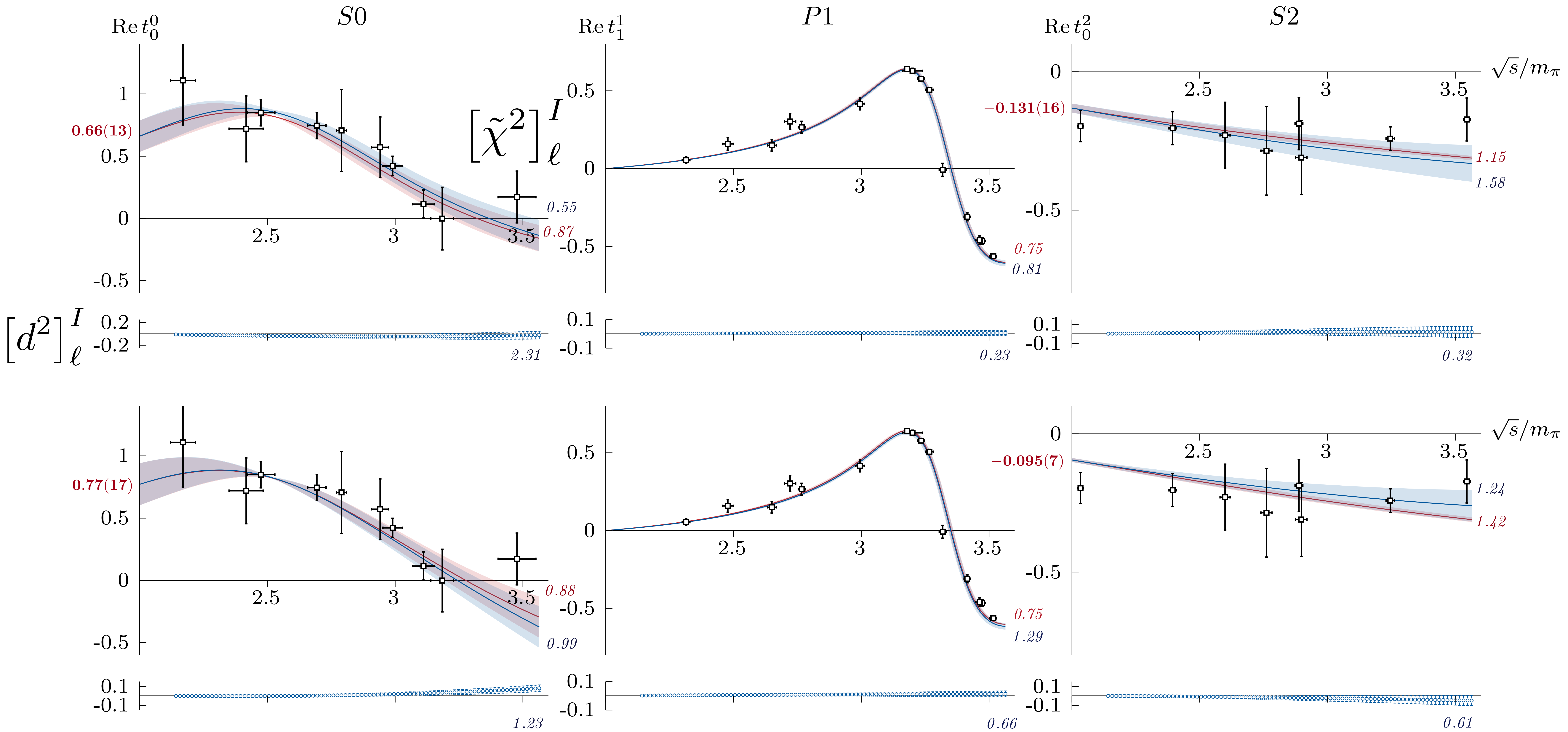


$$\Delta_i^2 = \begin{pmatrix} \Delta\mathfrak{f}_i & \frac{d\tilde{f}_\ell^I(s_i)}{dE_i}\Delta E_i \end{pmatrix} \begin{pmatrix} 1 & -c_i \\ -c_i & 1 \end{pmatrix} \begin{pmatrix} \Delta\tilde{f}_i \\ \frac{d\tilde{f}_\ell^I(s_i)}{dE_i}\Delta E_i \end{pmatrix}$$

Ok but not great

Visually, they describe the data and fit, but they are not perfect

$m_\pi \sim 239\,\mathrm{MeV}$



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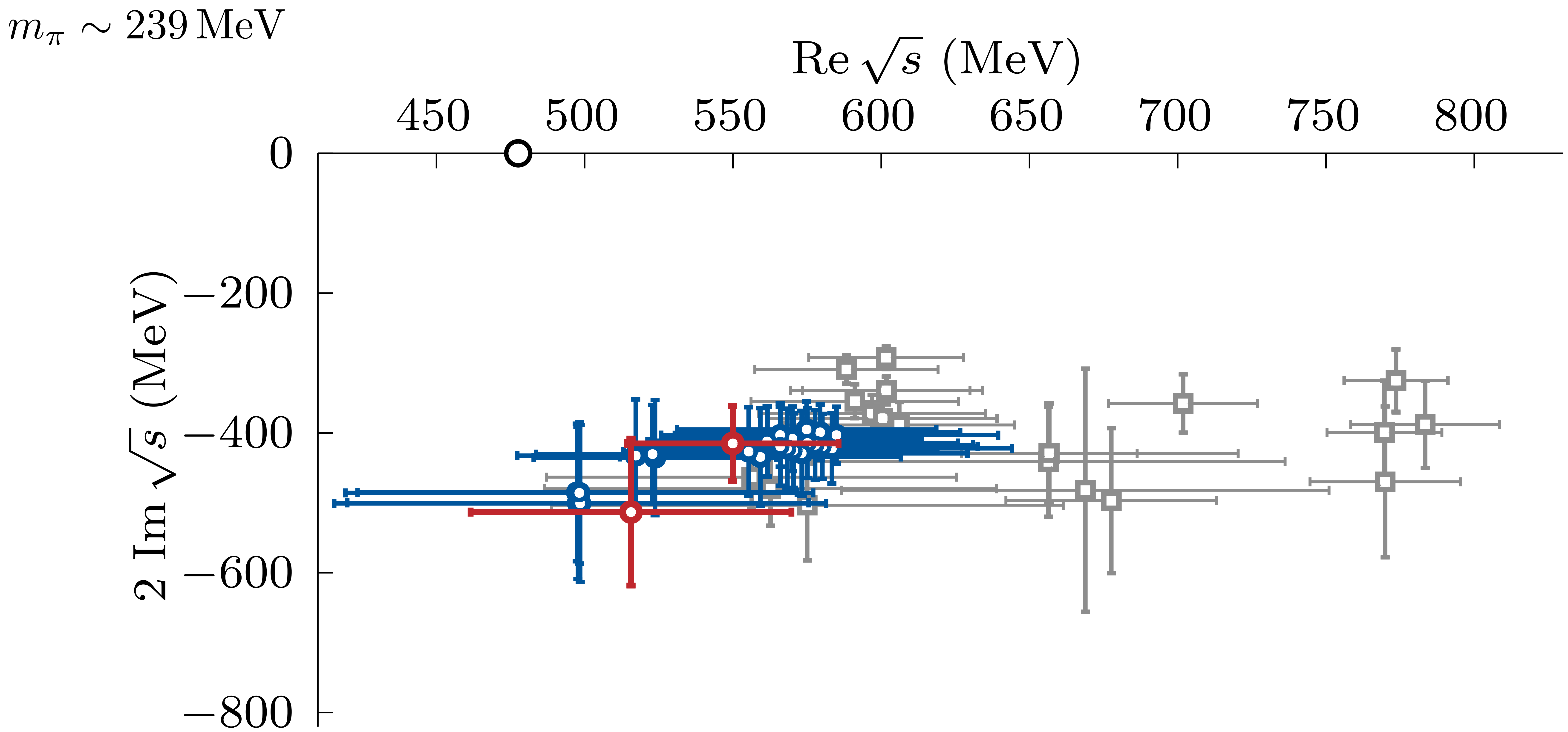
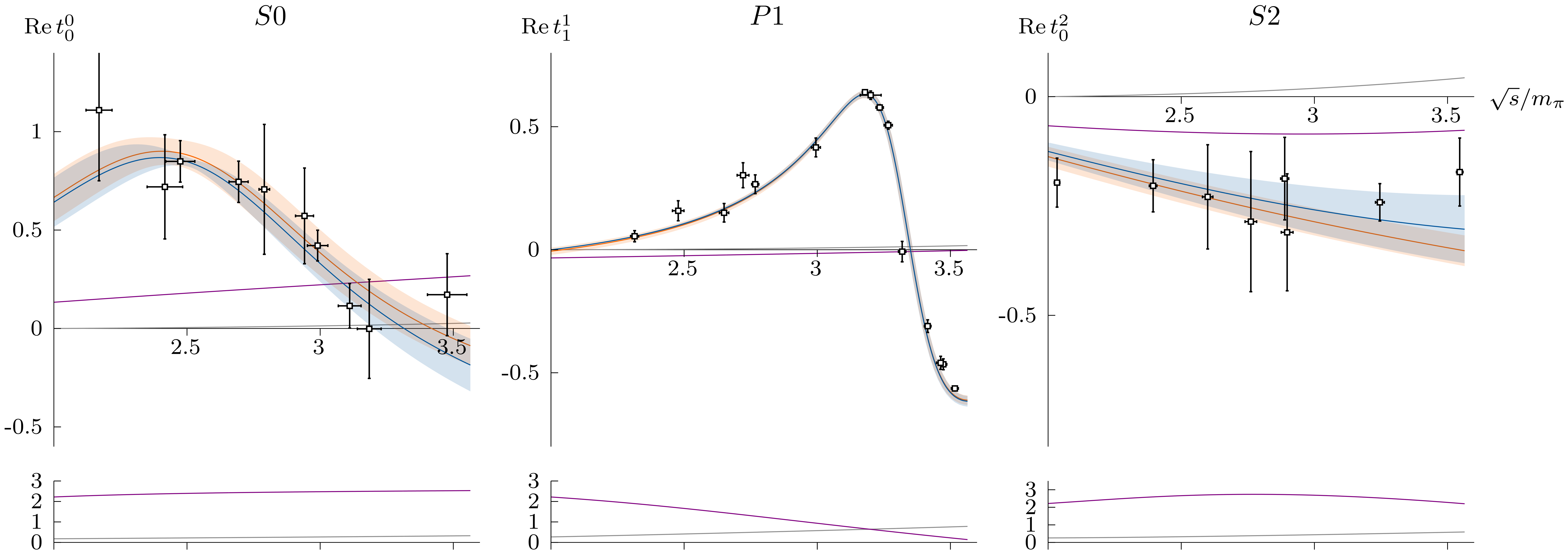


GKPY vs ROY

GKPY: Minimally subtracted → one less subtraction than ROY

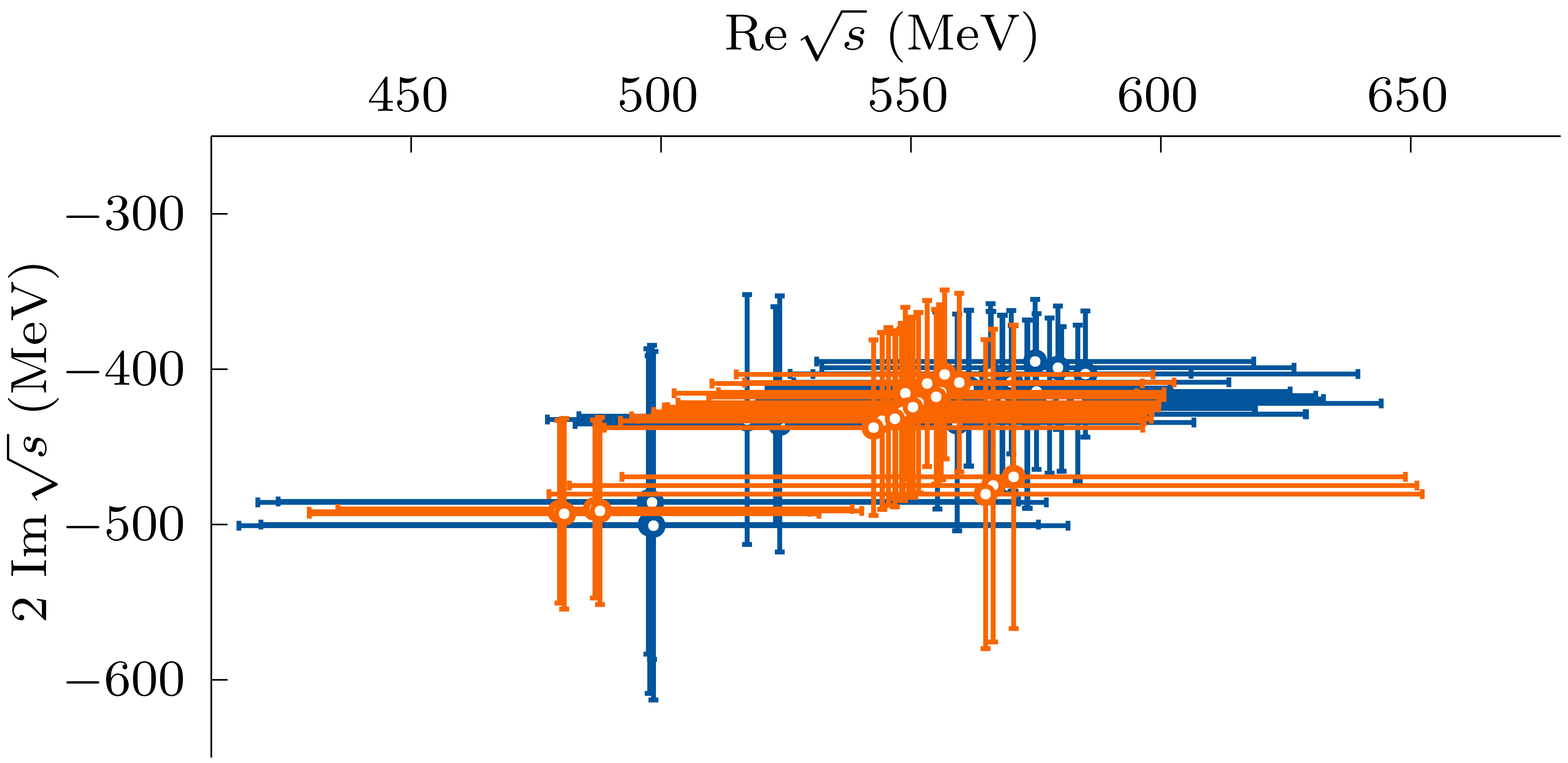
For our analysis, Regge contribution too large for d^2

$m_\pi \sim 239\,\mathrm{MeV}$

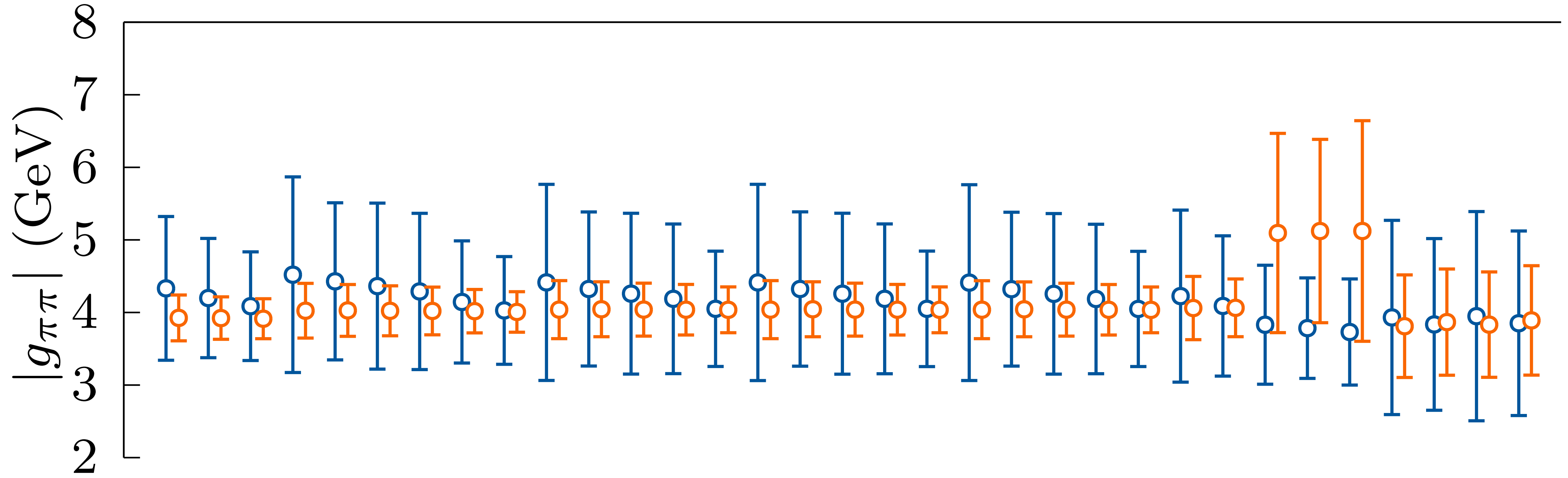


GKPY vs ROY

However, pole extraction is more accurate in most cases, particularly for the coupling



GKPY produces less than half the uncertainty in most cases



Scattering plane

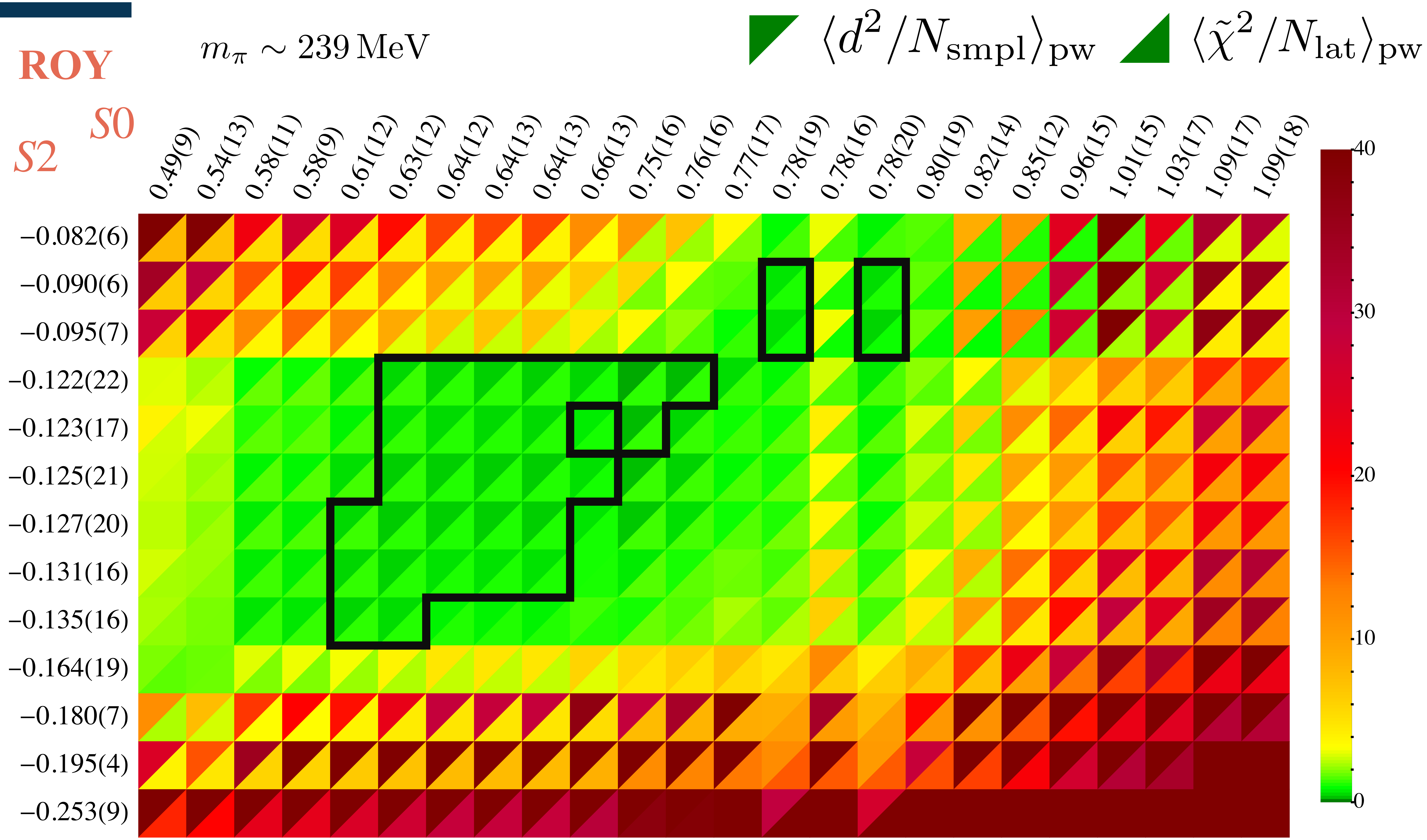
$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

Black

ROY

$m_\pi \sim 239\,\text{MeV}$

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

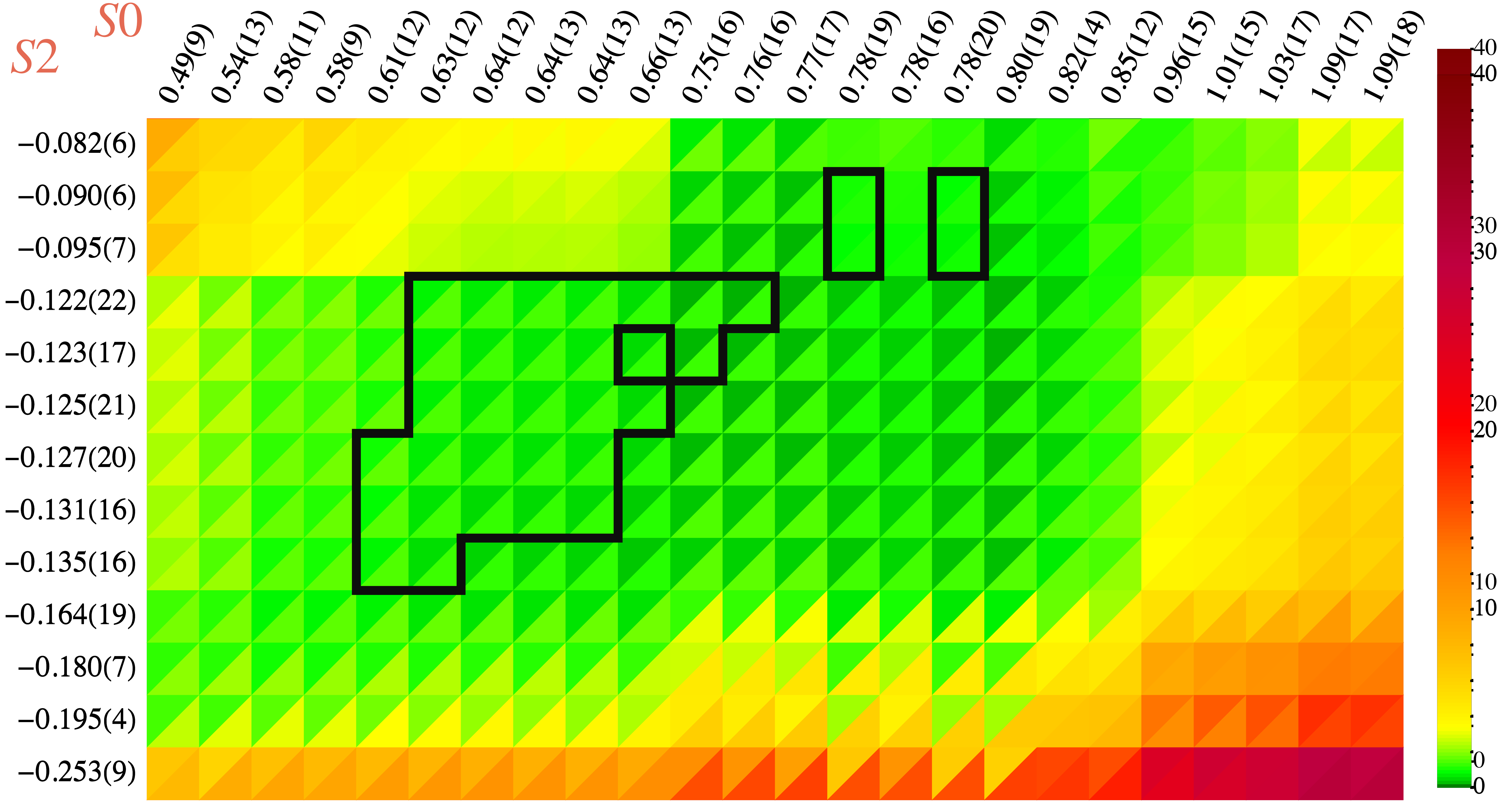
$\langle d^2/N_{\text{smpl}}\rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}}\rangle_{\text{pw}}$

Black GKPY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

S0

S2



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

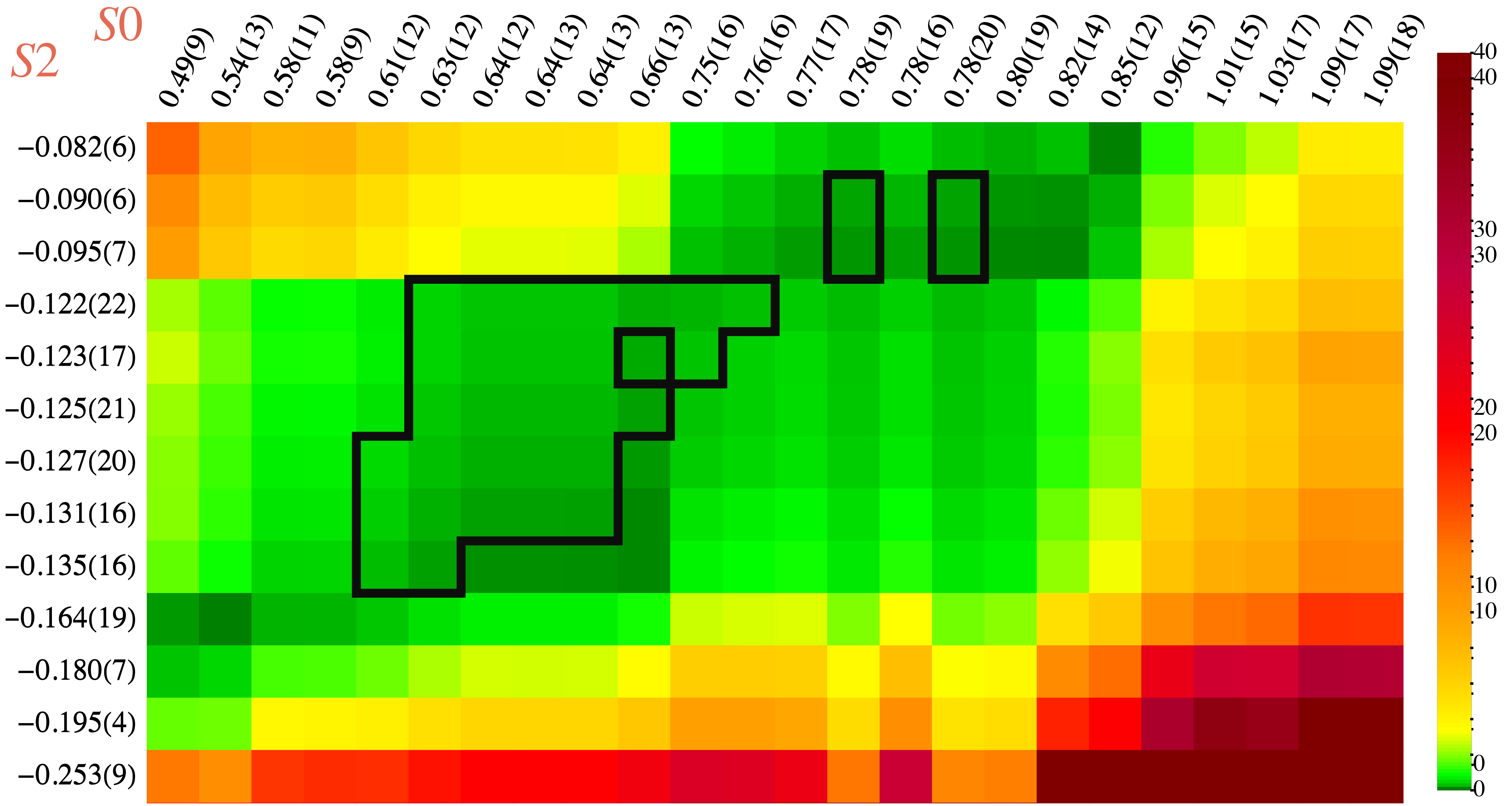
$m_\pi \sim 239\,\text{MeV}$

$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

Black

Olsson

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

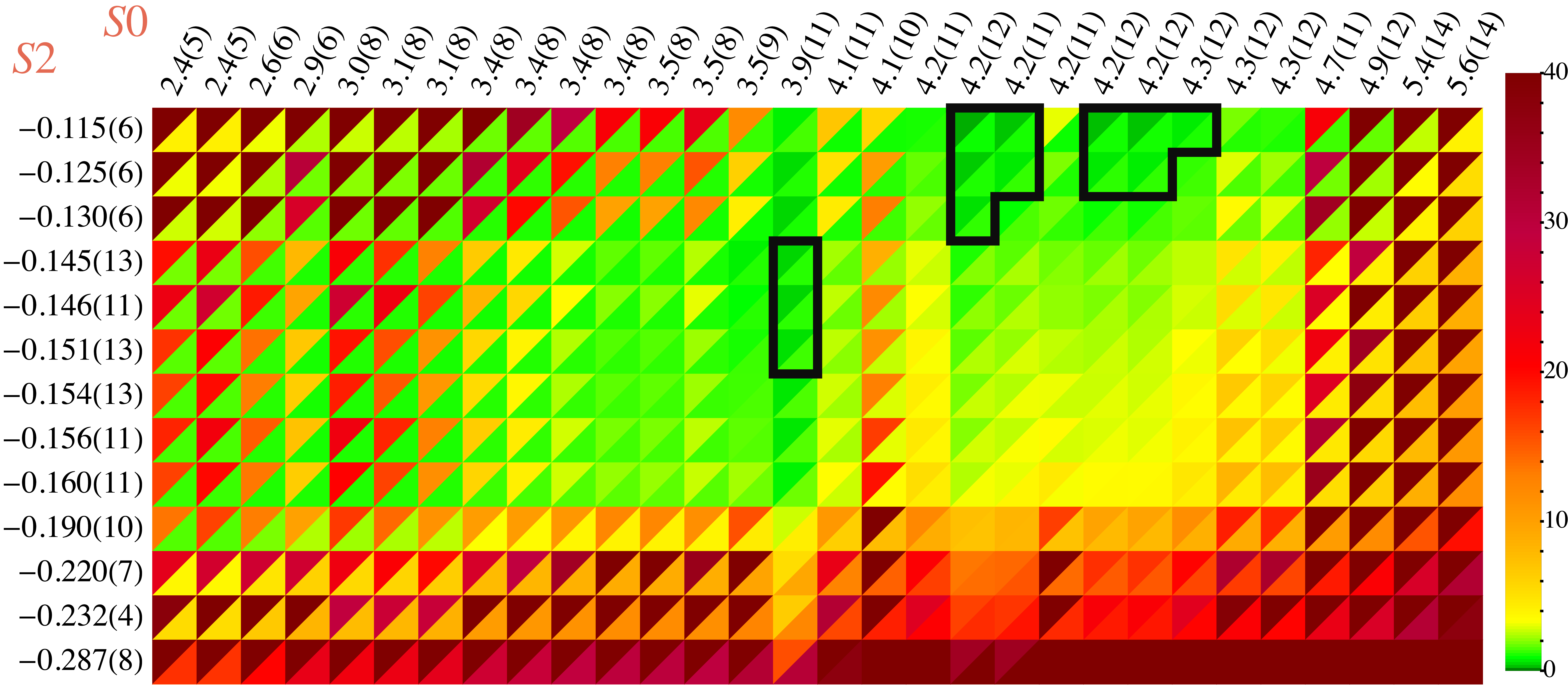
$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

Black ROY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



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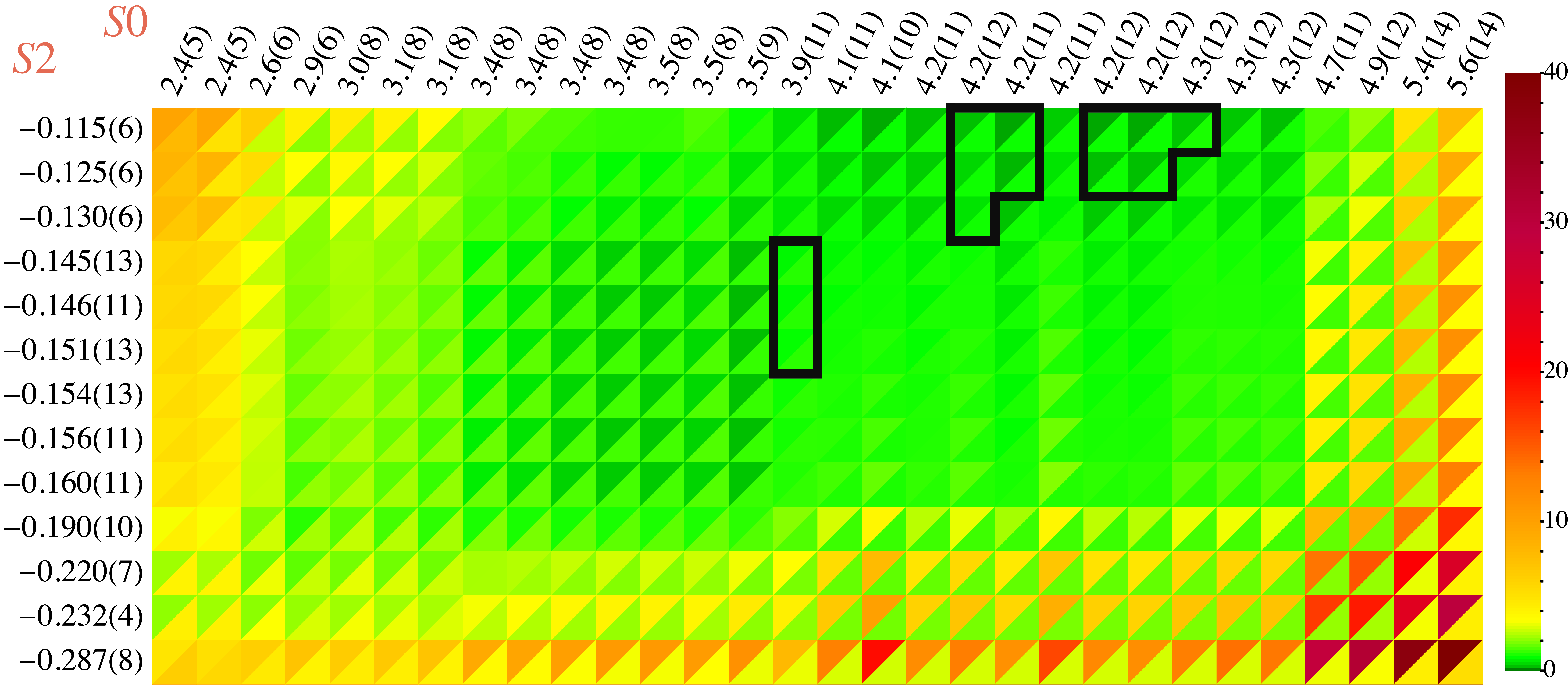
$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

Black **GKPY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

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Olsson

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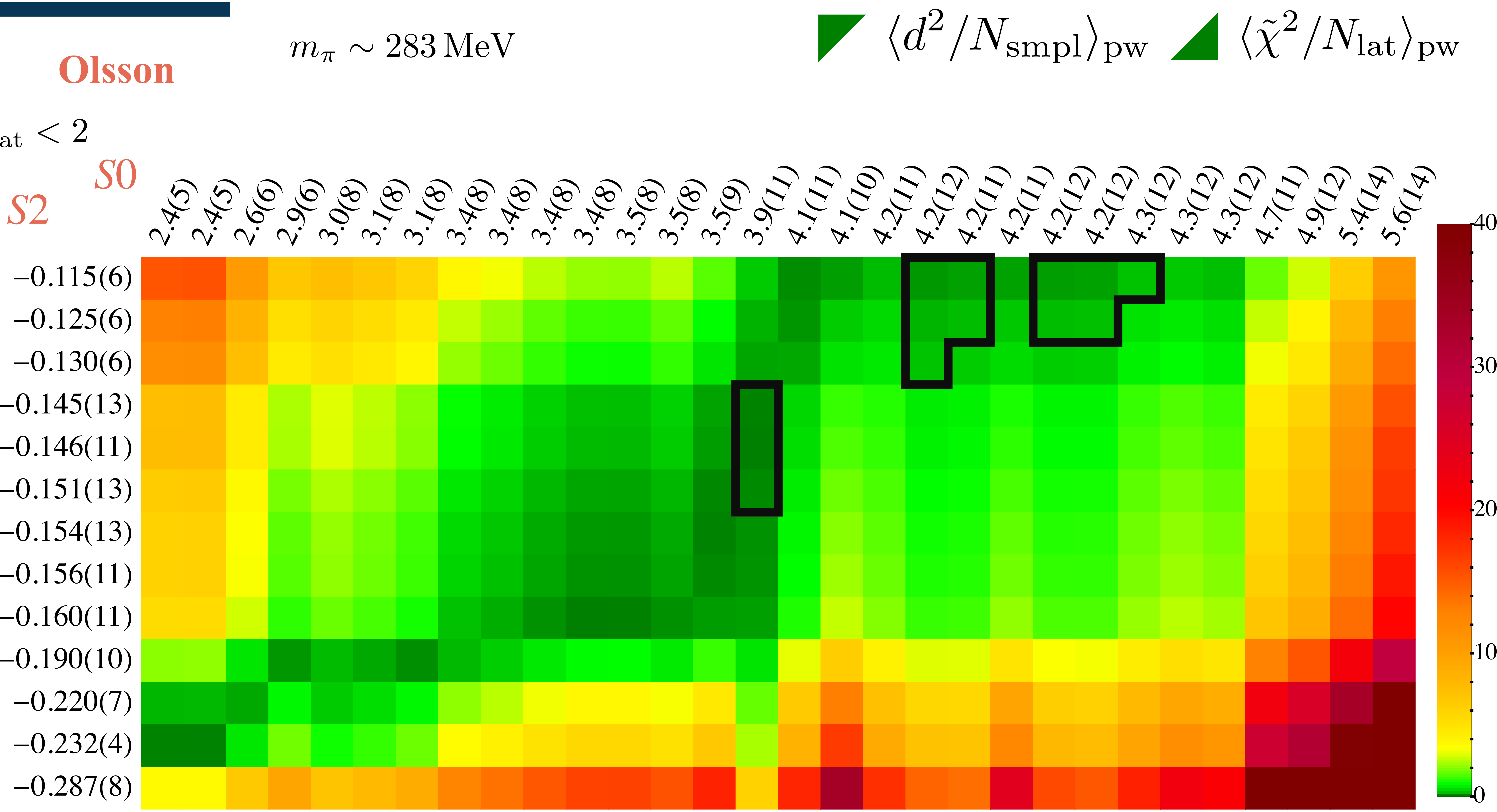


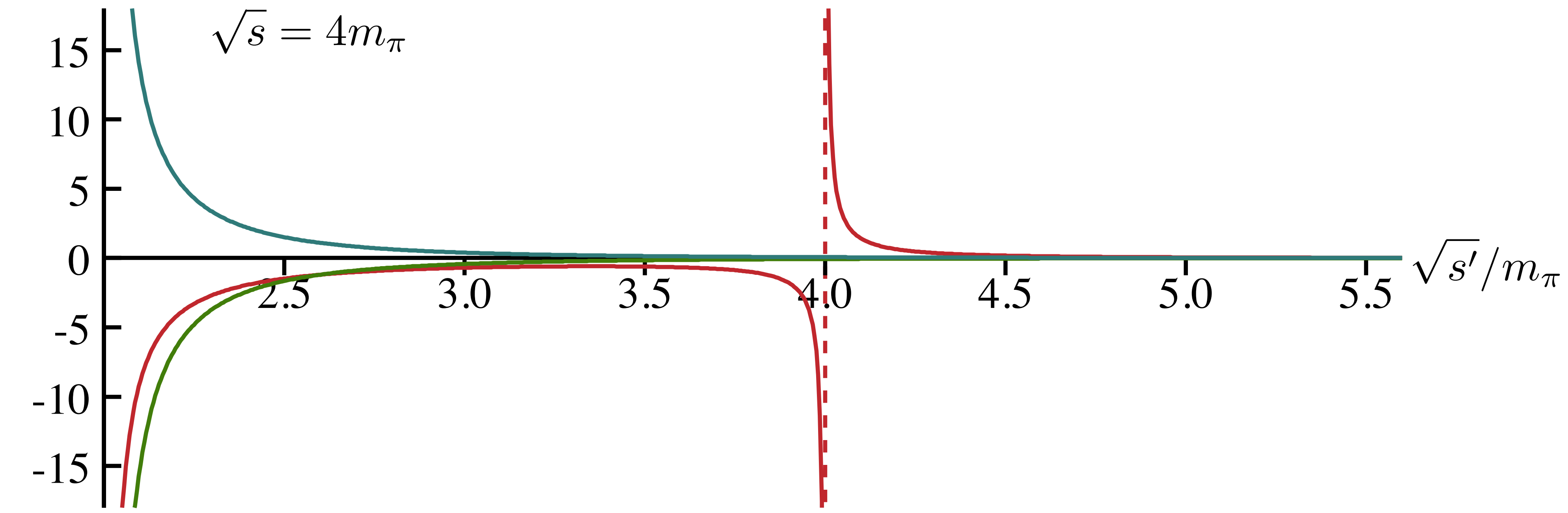
The good

Fit → *In*

DR → *Out*

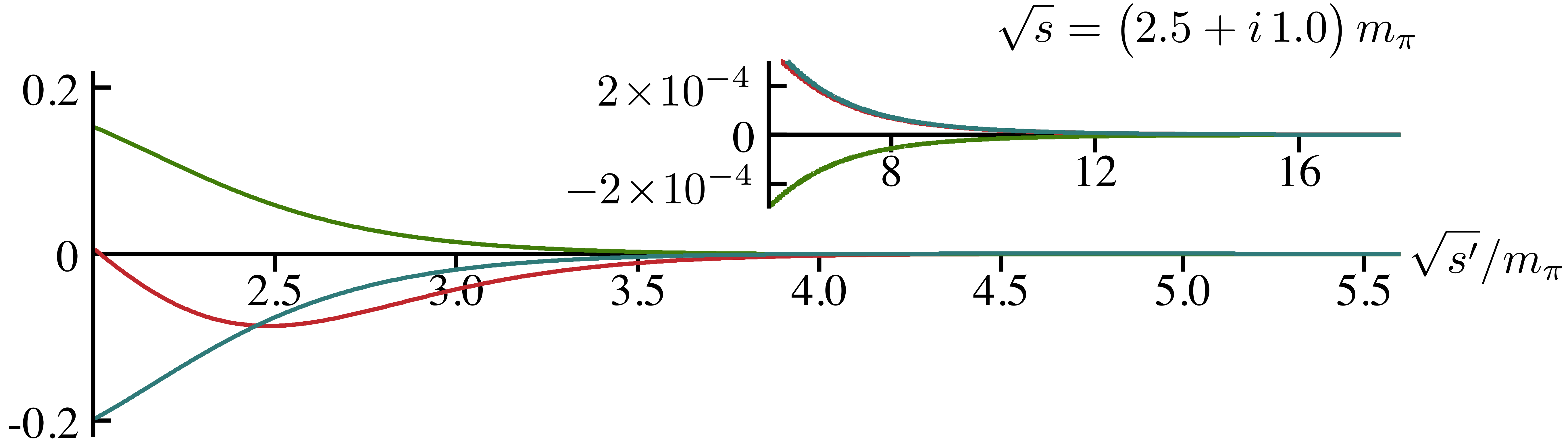
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Real s



Smeared over a large energy region

Complex s



An ϵ on the real axis → ϵ' in the complex plane

The bad

Not happening

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Partial waves

Extrapolated

$$\int_{4m_\pi^2}^{\infty} = \int_{4m_\pi^2}^{s_{max}} + \int_{s_{max}}^{\infty} = \int_{4m_\pi^2}^{s_{fit}} + \int_{s_{fit}}^{s_{max}} + \int_{s_{max}}^{\infty}$$

Regge

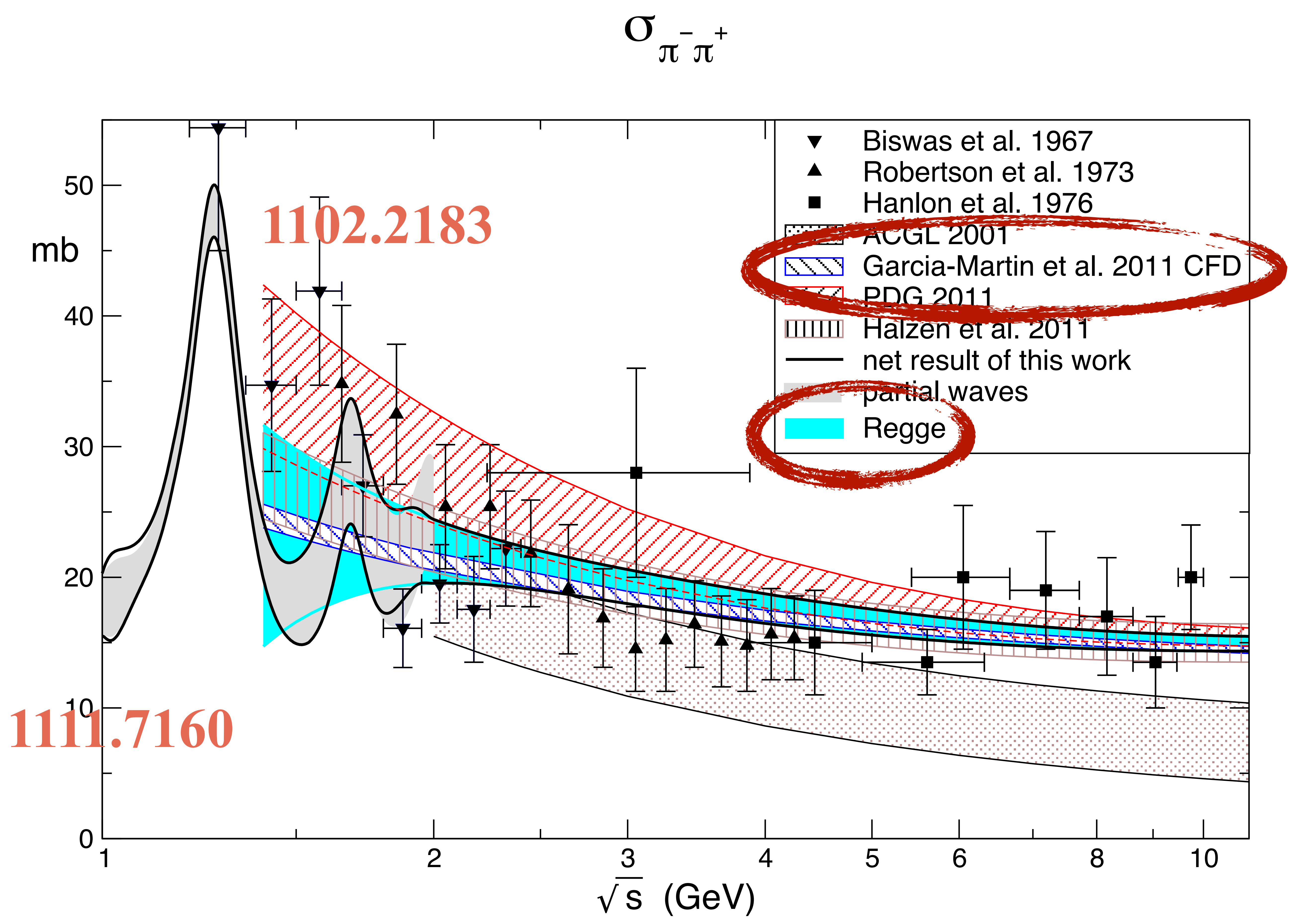
- Regge must be extrapolated from phys. m_π
- Regge is wrong below $a_t m_\pi \sim 0.22 - 0.25$

The Regge

Regge must be extrapolated from phys. m_π

$\mathbb{P}$ → gluon exchanges → constant over m_q

ρ, f_2 → resonances, not constant → $\lambda \sim \Gamma/M$

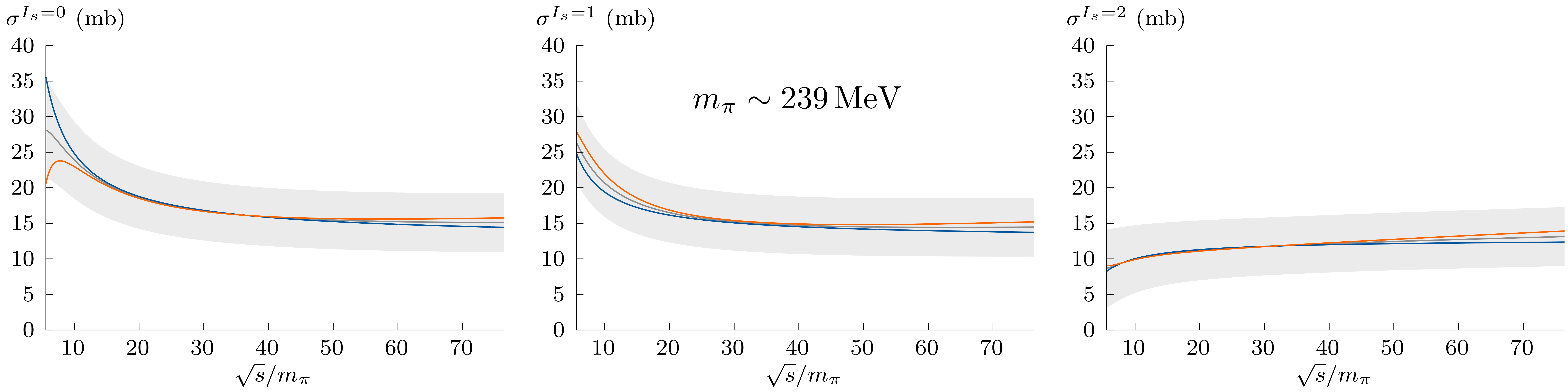


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

The Regge

Regge must be extrapolated from phys. m_π



Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

Pions on the lattice

Connected diagrams

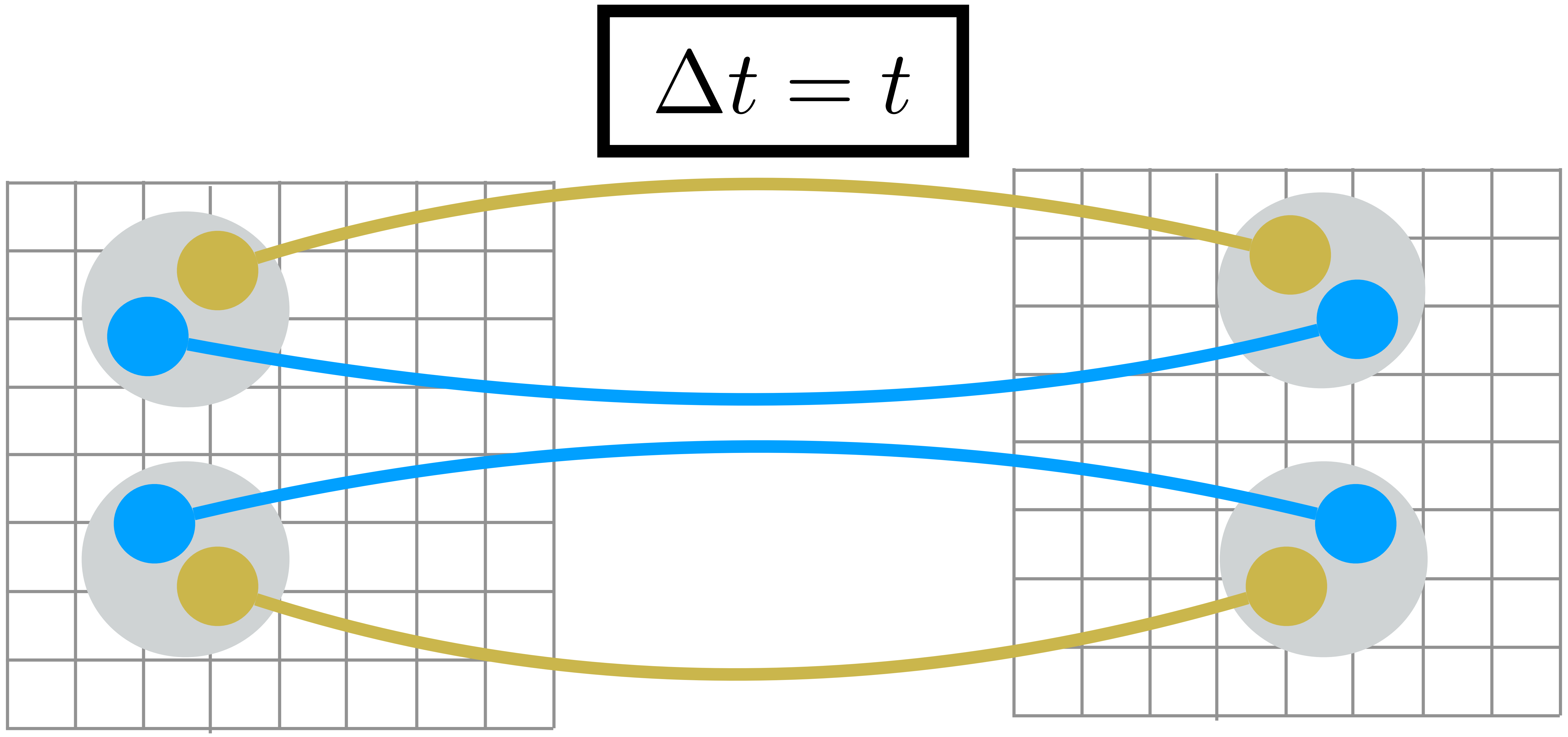
Actual lattices ($32^3 \times 256$)

$$\left[D^{-1}[U]\right]_{00,xt}$$

Size

$$4 \times 3 \times 4 \times 3 \times L^3 \times T$$

Around 10 GB per flavor



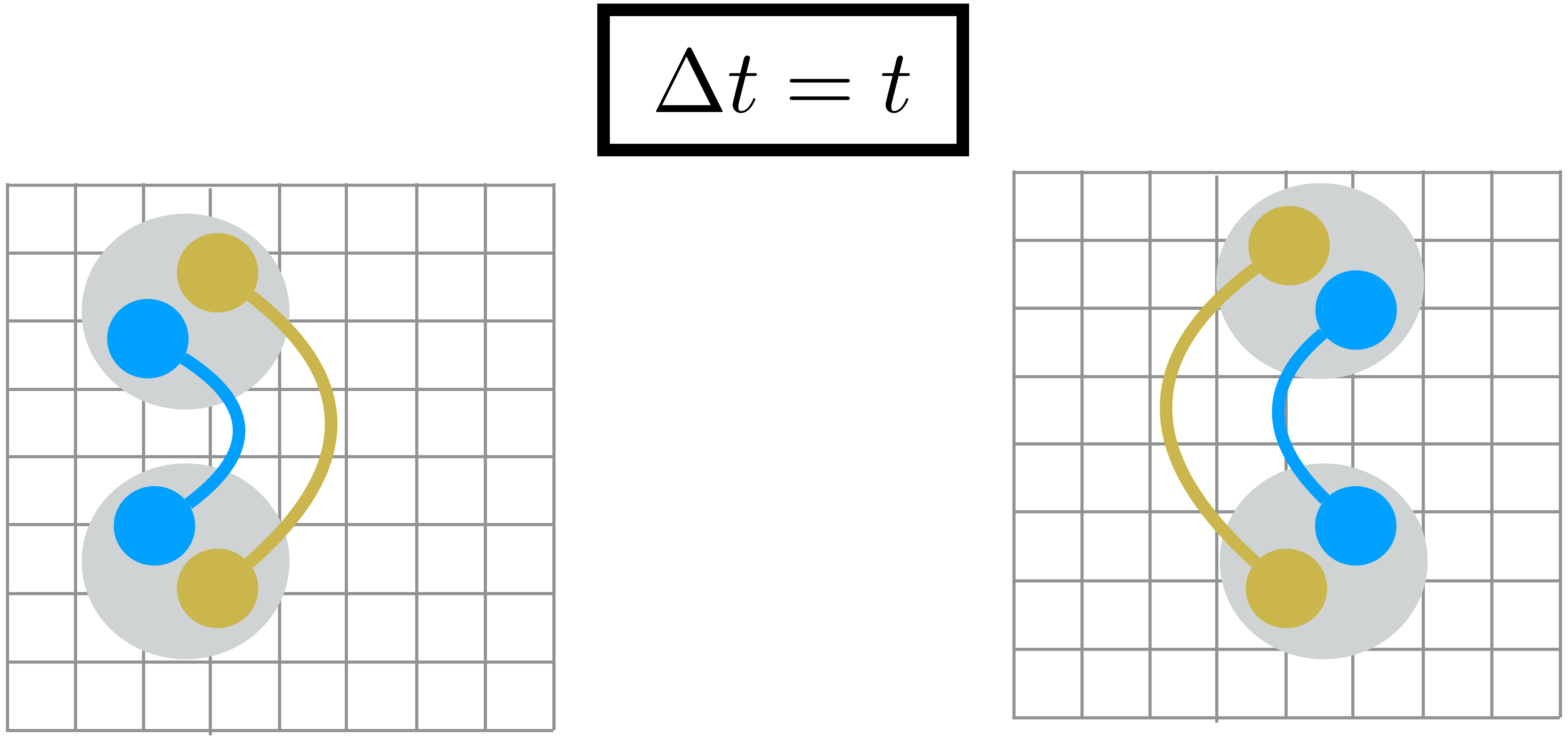
Disconnected diagrams

$$\left[D^{-1}[U]\right]_{x_i t_i, x_f t_f}$$

Size

$$(4 \times 3 \times L^3 \times T)^2$$

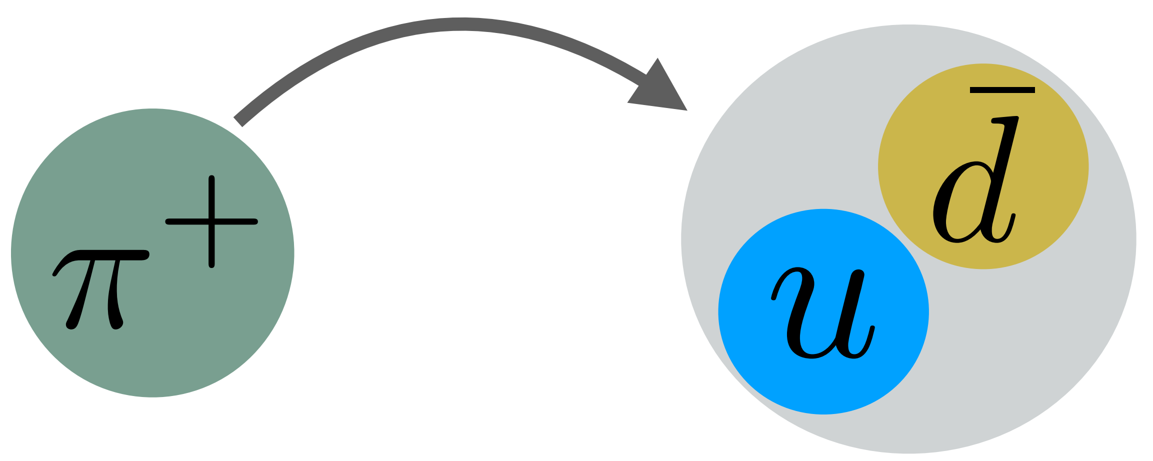
Around 80 PB per flavor



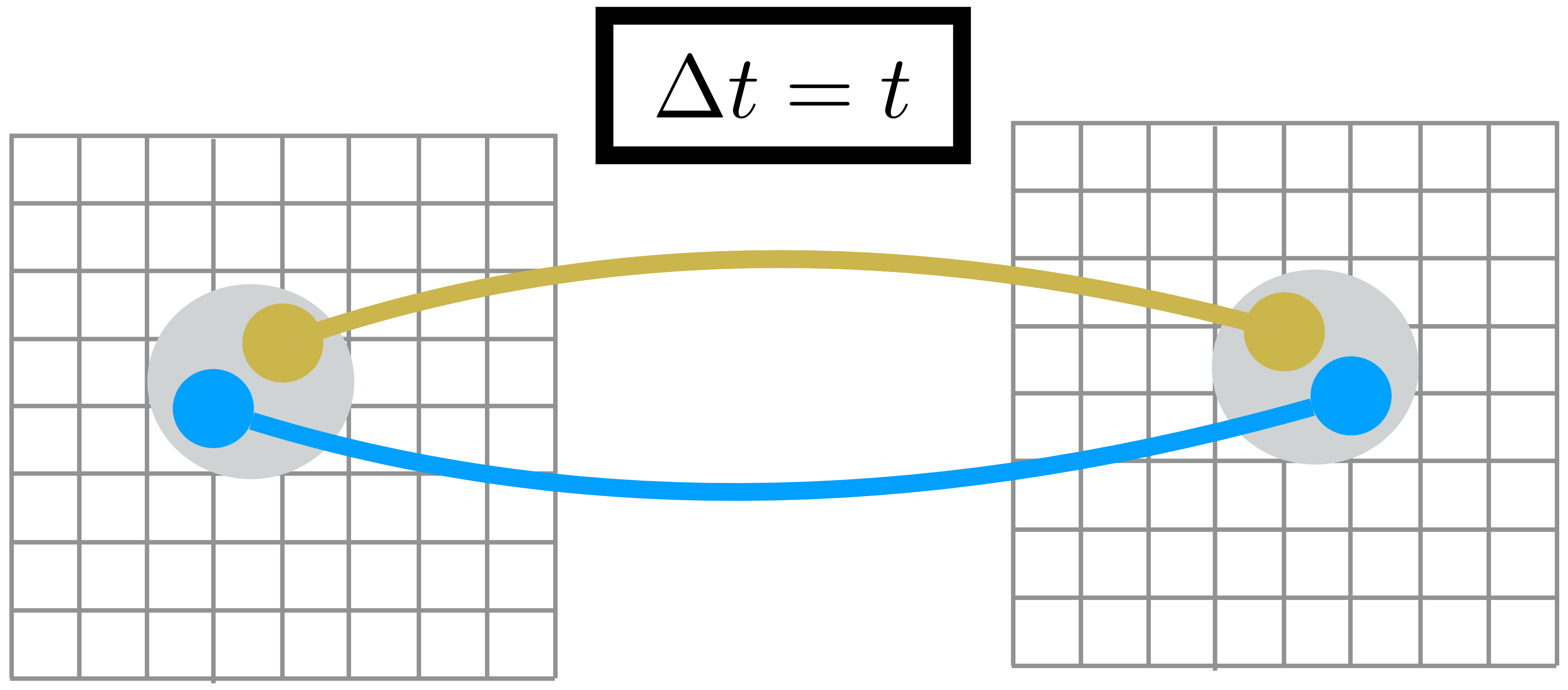
We use clever techniques to "solve" this

Pions on the lattice

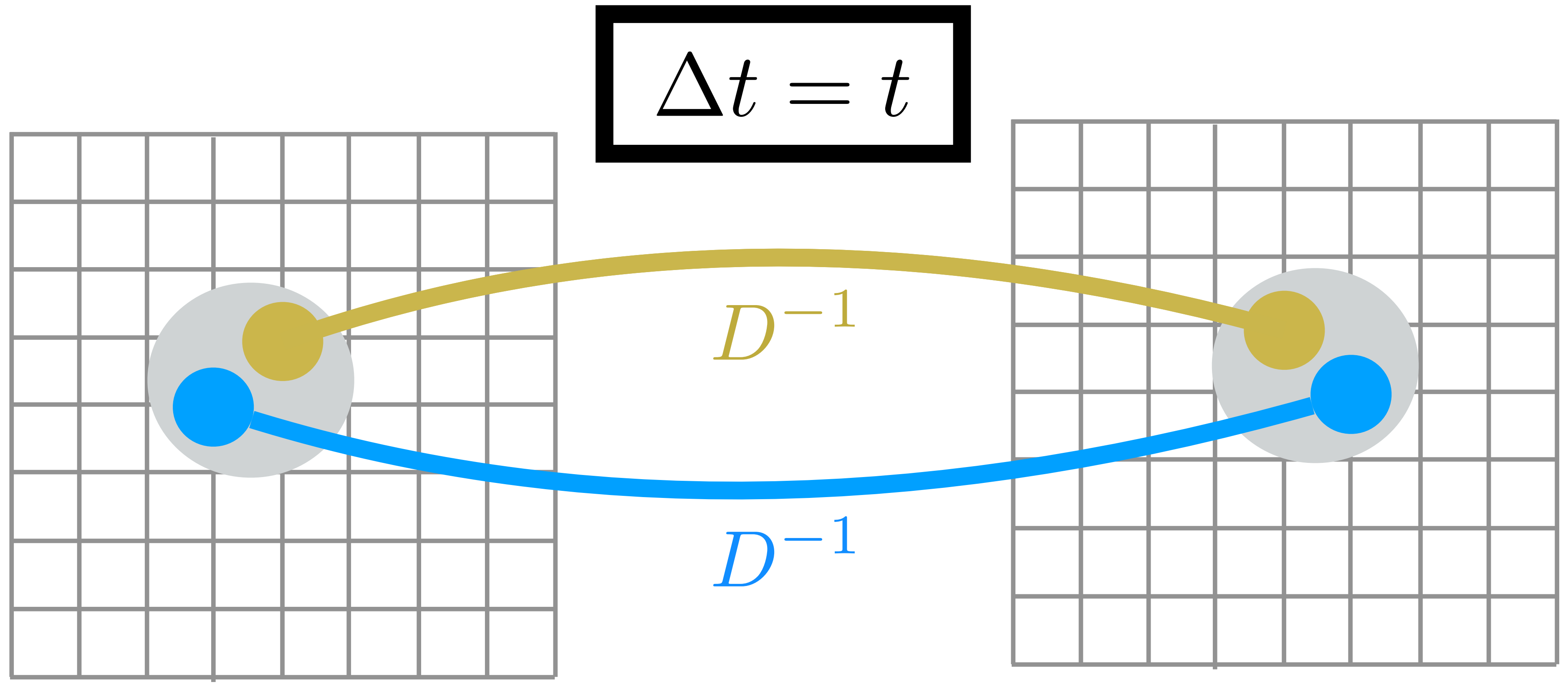
Easiest pion construction in terms of quarks



$$|\pi^+\rangle = |\bar{d}\gamma_5 u\rangle$$



Pions on the lattice



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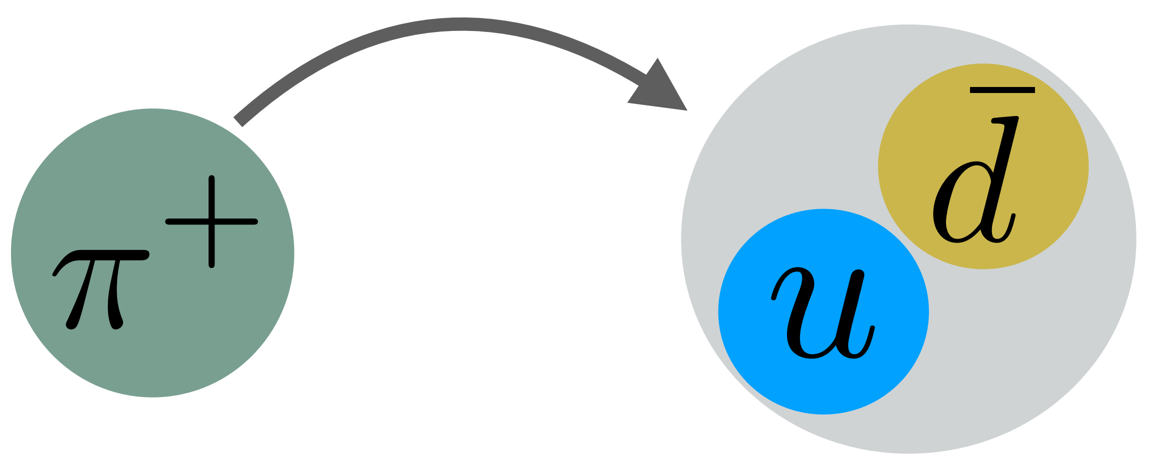


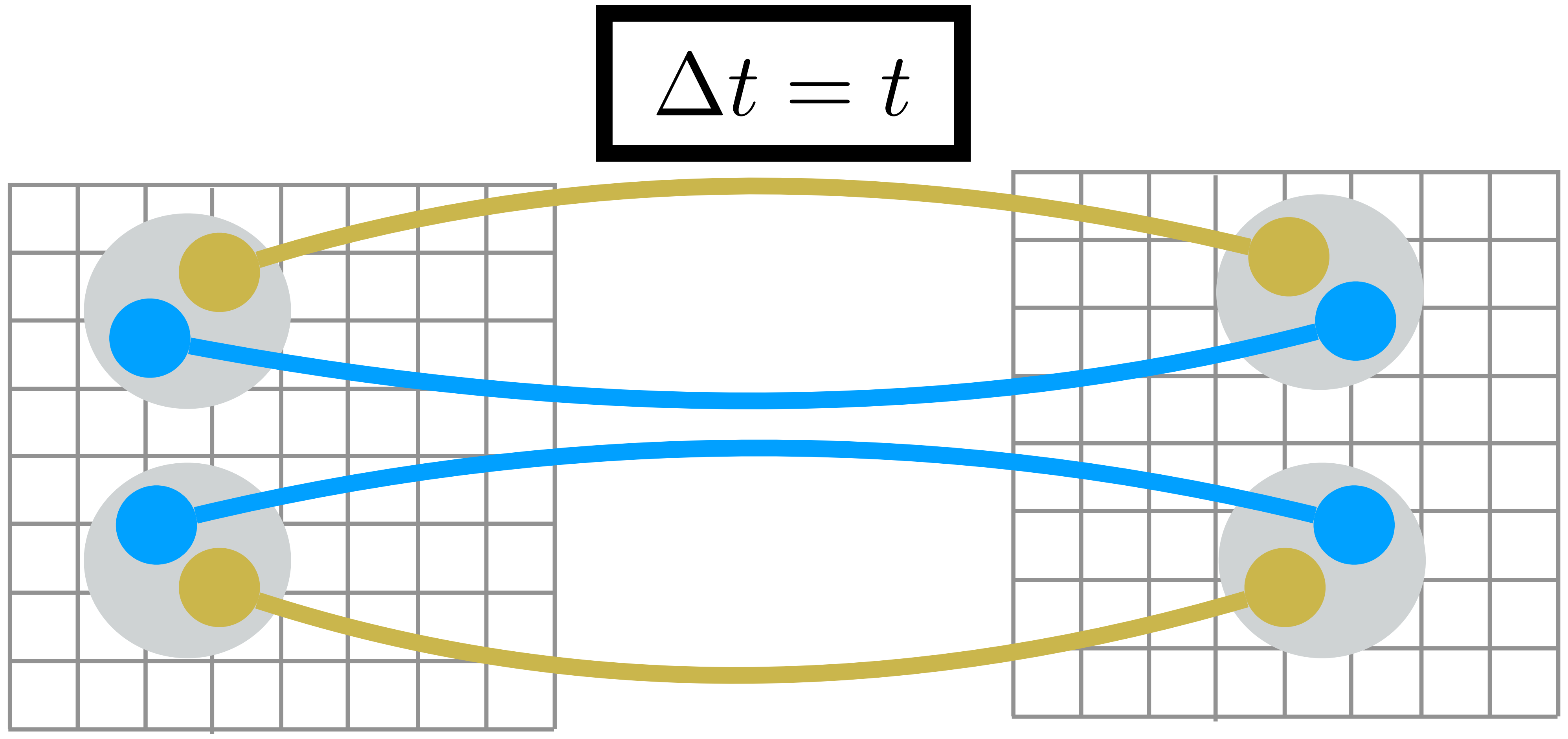
$$|\pi^+\rangle = |\bar{d}\gamma_5 u\rangle$$

What is the evolution? → contractions

$$\left\langle 0 \middle| \left(\bar{\psi}\gamma_5\psi\right)_{x,t} \left(\bar{\psi}\gamma_5\psi\right)_{0,0} \middle| 0 \right\rangle = -\,\mathrm{tr}\left(\left[D^{-1}[U]\right]_{00,xt} \gamma_5 \left[D^{-1}[U]\right]_{xt,00} \gamma_5 \right)$$

Point to all propagators

Pions on the lattice



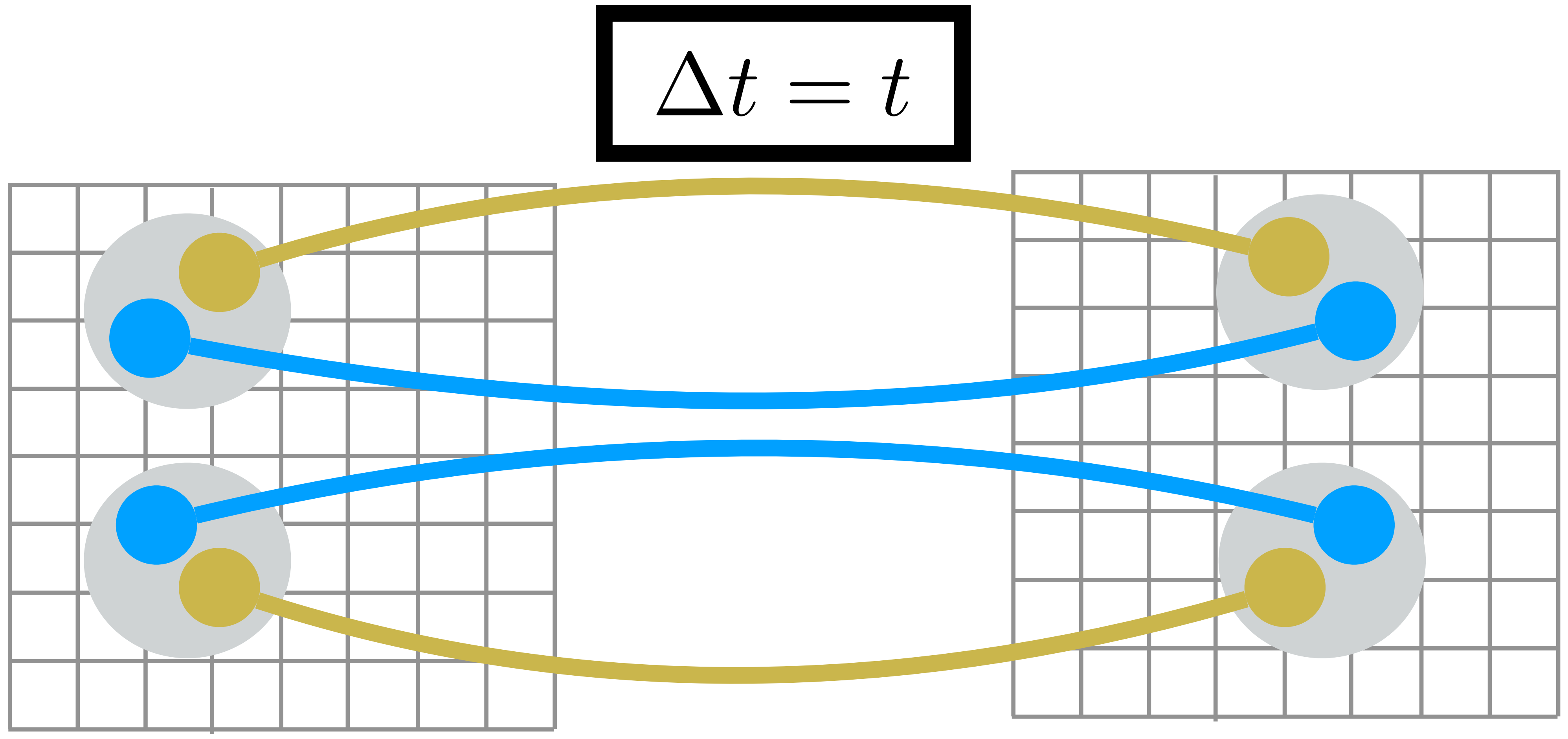
Lets study the temporal evolution of a pair of particles

$$C(t) \equiv \langle 0 | \mathcal{O}(t) \mathcal{O}^{\dagger}(0) | 0 \rangle$$

Basis

$$= \sum_n \langle 0 | \mathcal{O}(t) | n \rangle \langle n | \mathcal{O}^{\dagger}(0) | 0 \rangle$$

Pions on the lattice



Lets study the temporal evolution of a pair of particles

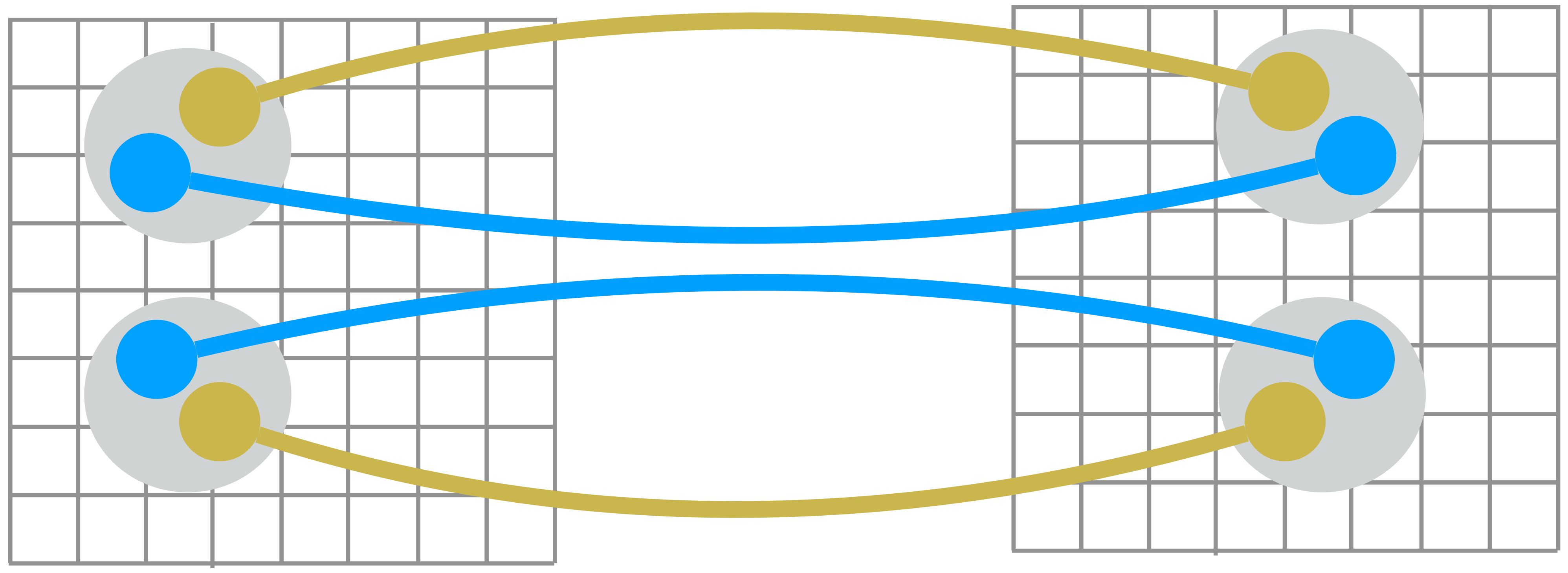
$$C(t) \equiv \left\langle 0 \left| \mathcal{O}(t)\, \mathcal{O}^\dagger(0) \right| 0 \right\rangle$$

Basis

$$= \sum_n \left\langle 0 \left| \mathcal{O}(t) \right| n \right\rangle \left\langle n \left| \mathcal{O}^\dagger(0) \right| 0 \right\rangle$$

$$e^{-iHt}$$

Pions on the lattice



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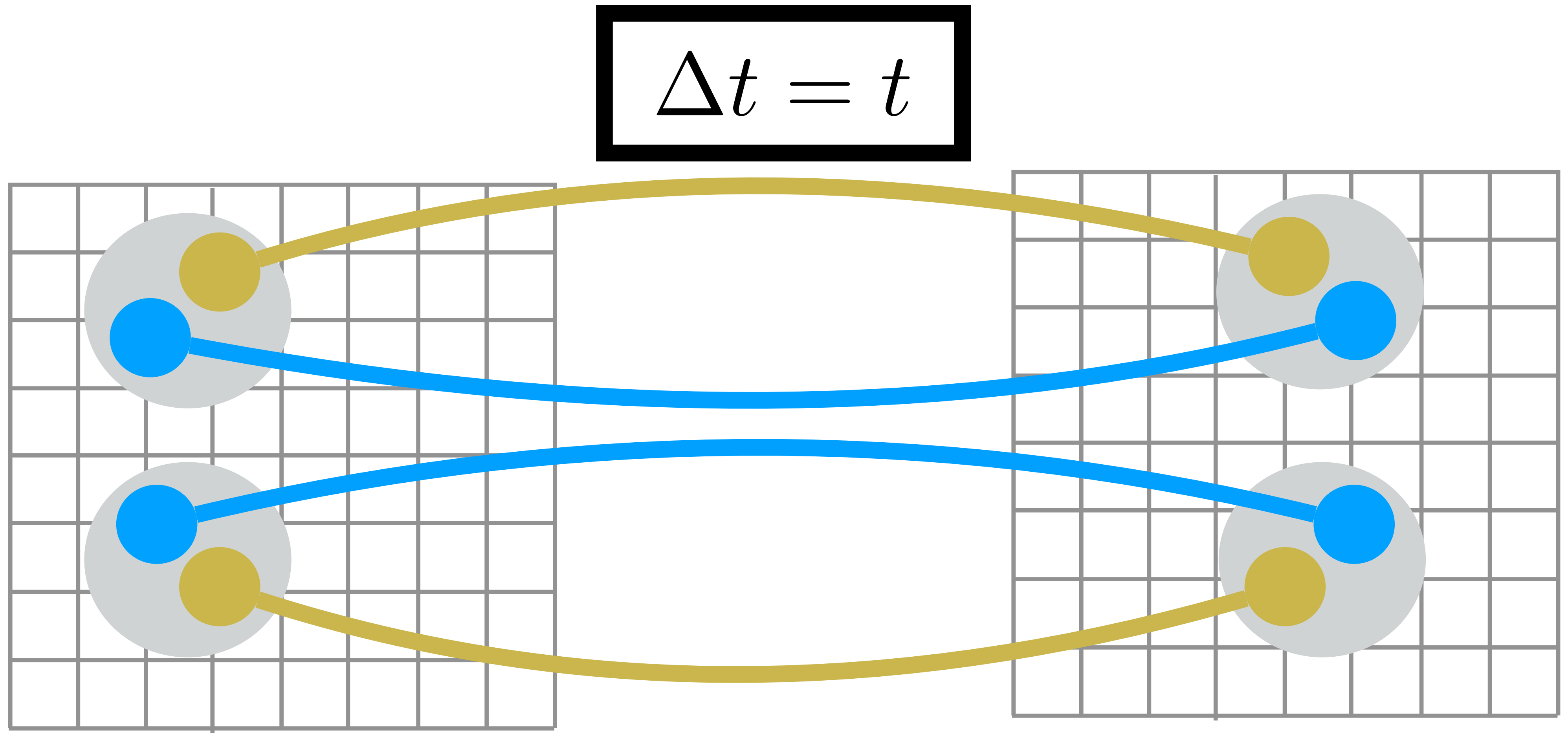

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We determine these energies from fitting the temporal evolution of the system

$$m_{\text{eff}} = log\left[\frac{C(t)}{C(t+1)}\right]$$