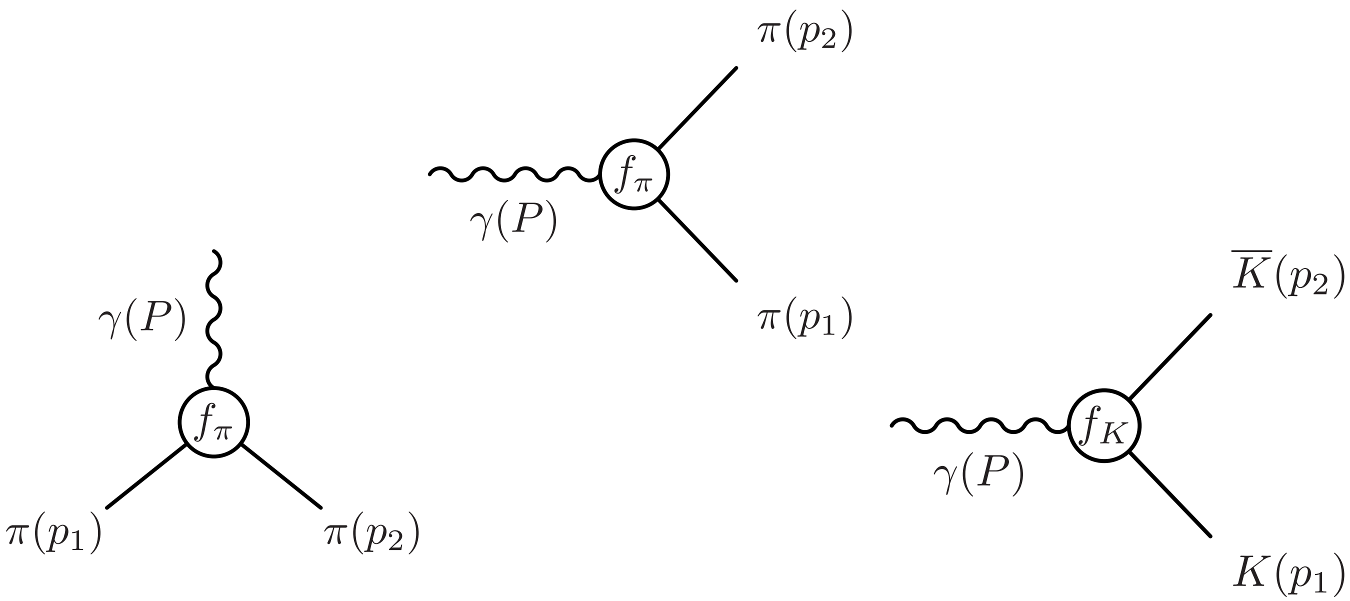
Pseudoscalar form factors from lattice QCD



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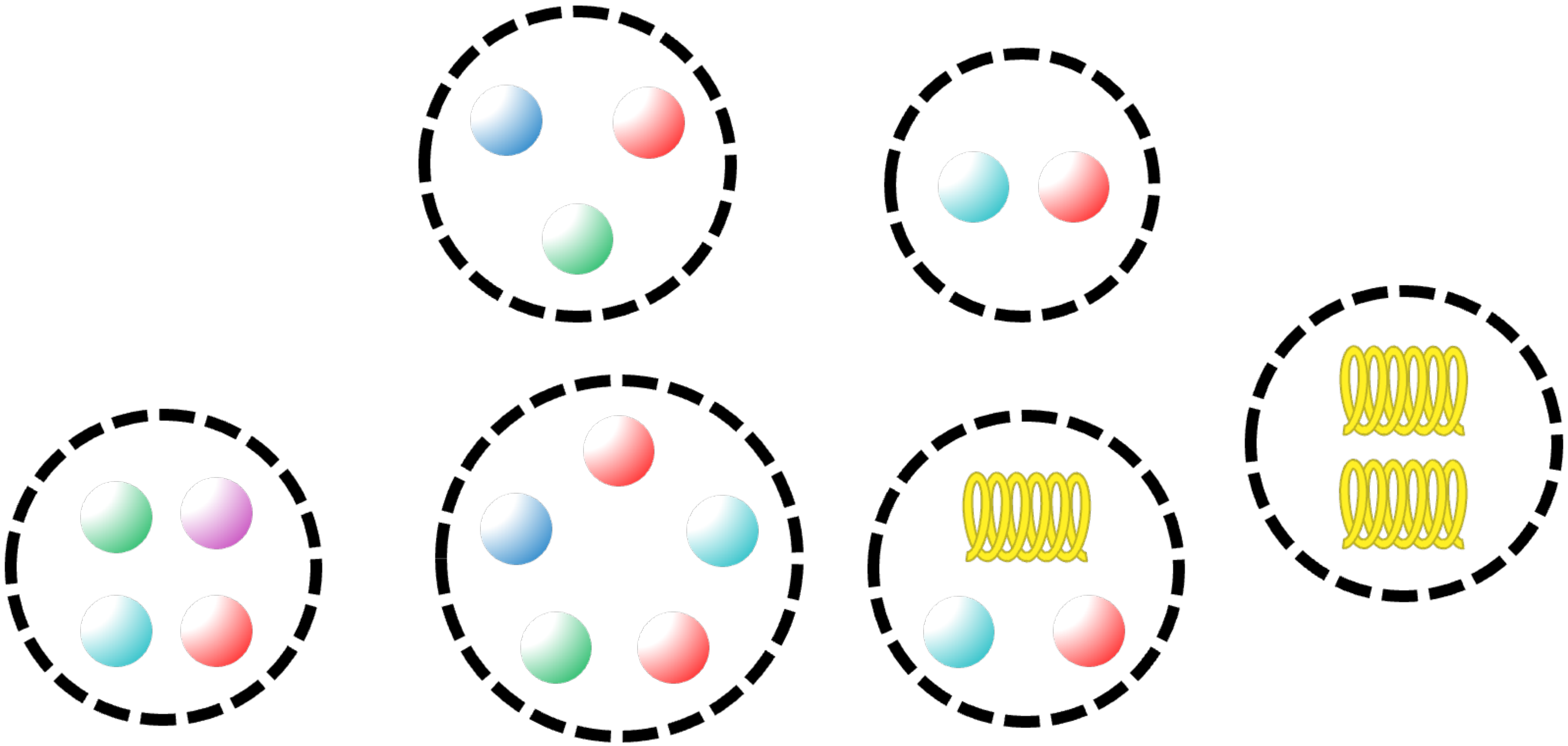
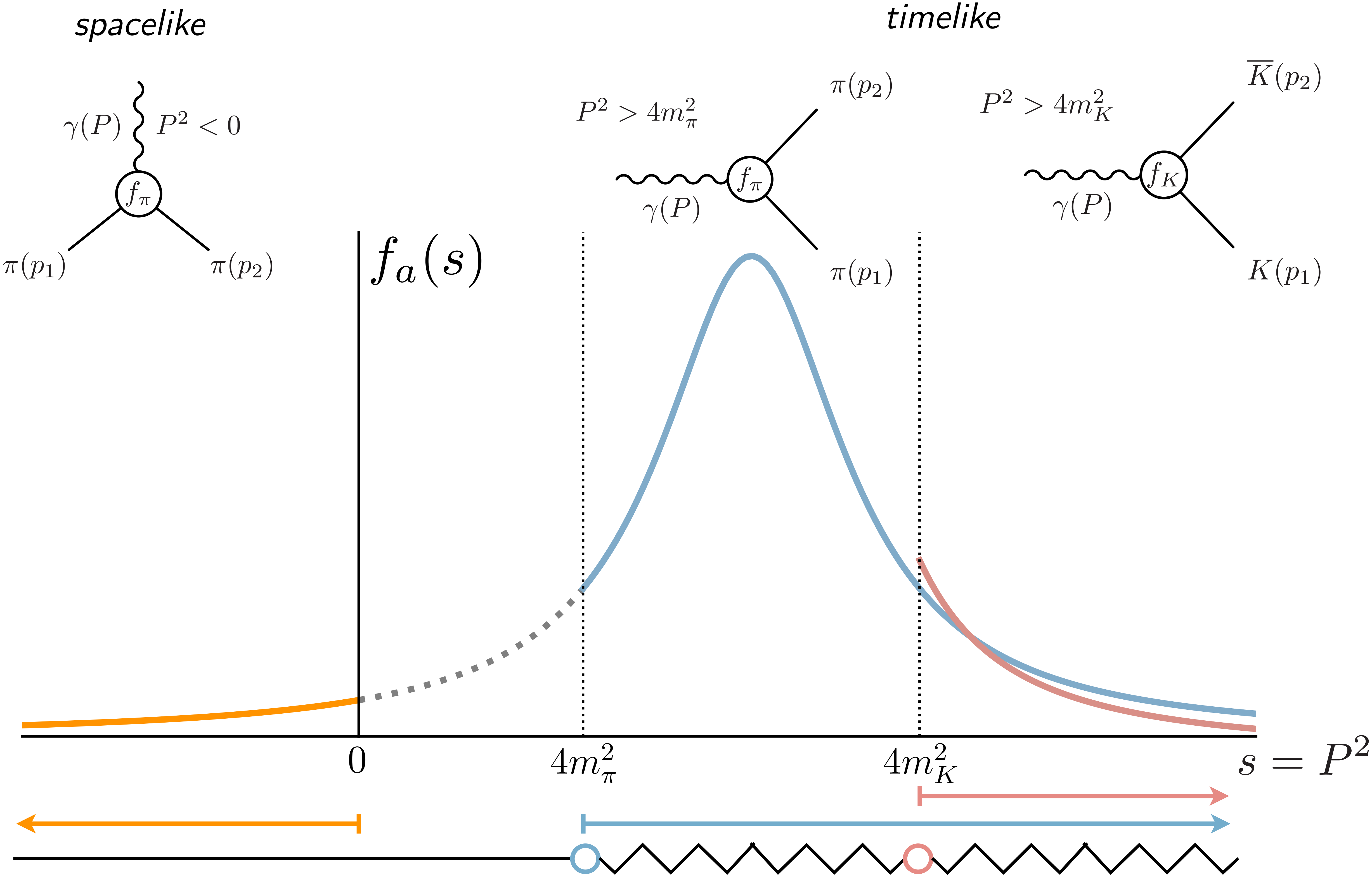
In collaboration with J. Dudek

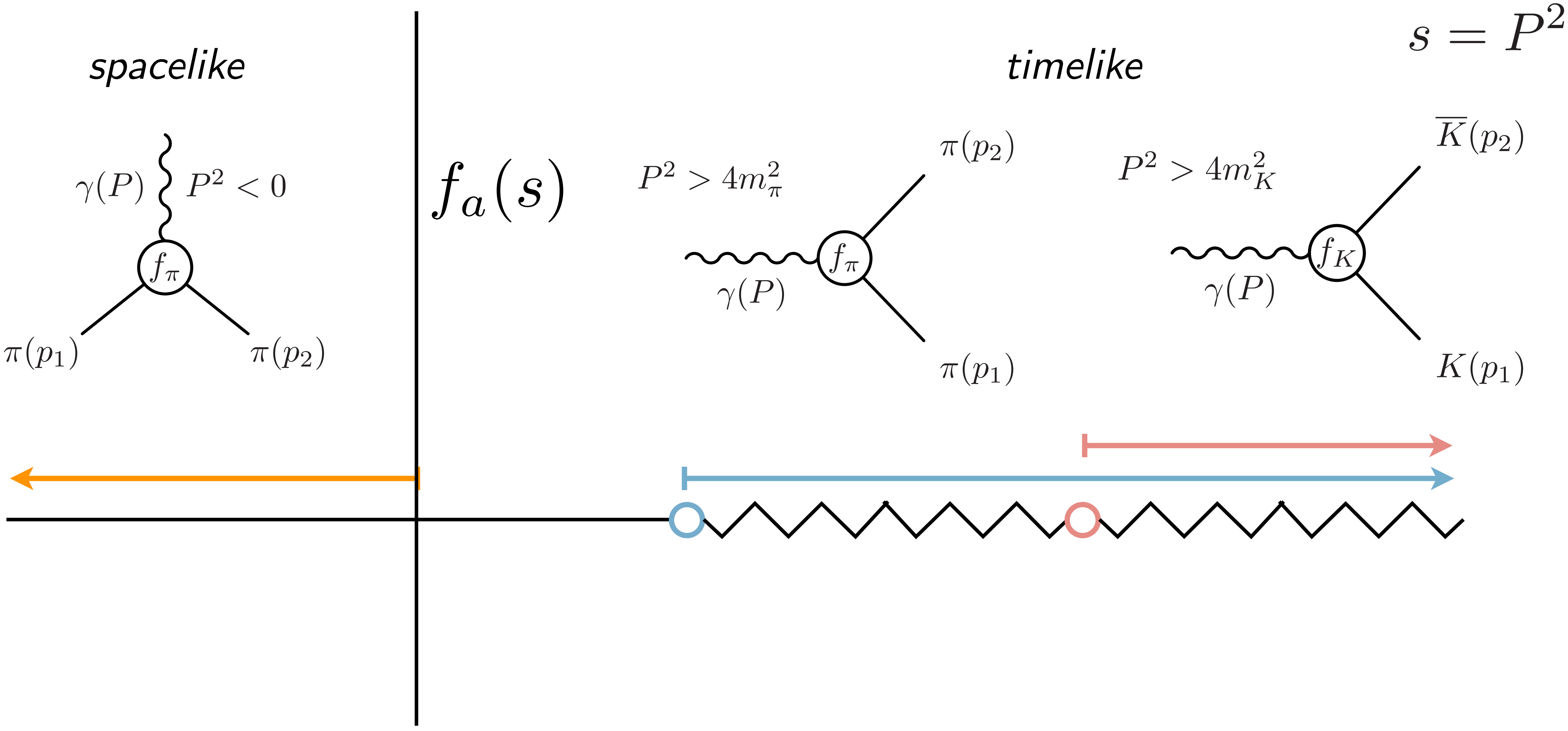
Why form factors for spectroscopy?



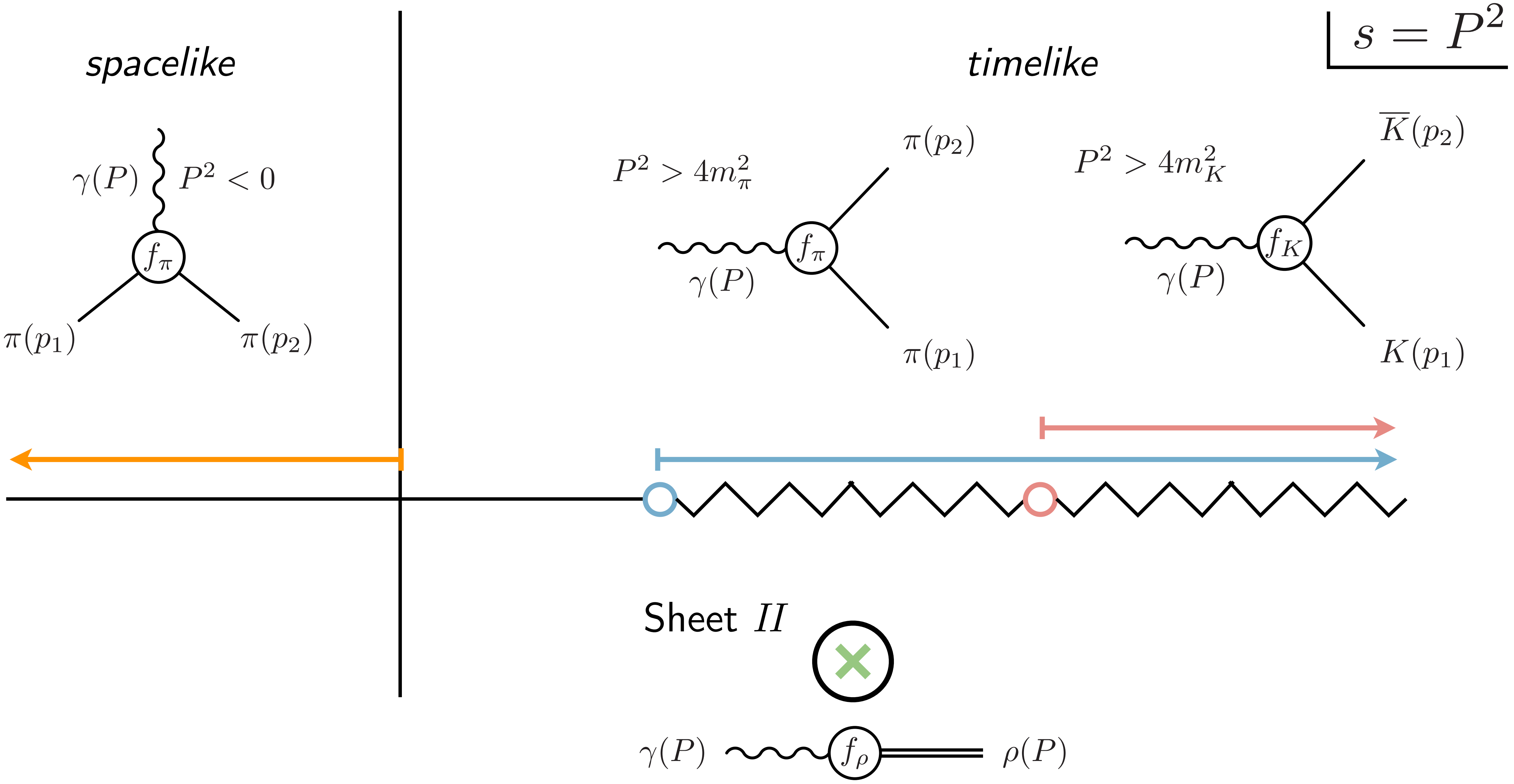
1. Internal structure from virtuality dependence.
2. Production/decay mechanisms to guide experiments.
3. Connection to EW processes, e.g. a_μ.

Pseudoscalar form factors



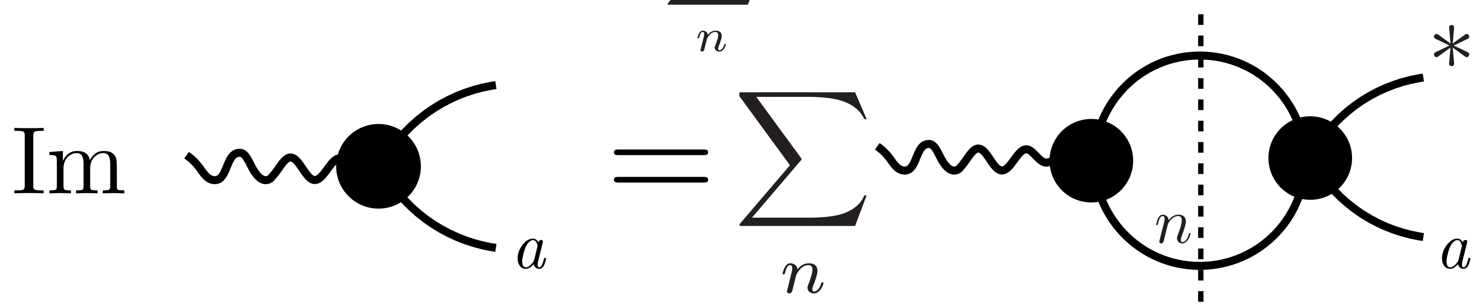

$s = P^2$
spacelike
timelike
$\gamma(P)$
$P^2 < 0$
f_π
$\pi(p_1)$
$\pi(p_2)$
$f_a(s)$
$P^2 > 4m_\pi^2$
$\pi(p_2)$
f_π
$\gamma(P)$
$\pi(p_1)$
$P^2 > 4m_K^2$
$\overline{K}(p_2)$
f_K
$\gamma(P)$
$K(p_1)$

Analytic structure



Watson's theorem and unitarity

$$\mathrm{Im} f_a = \sum_n f_n \rho_n \mathcal{M}^*_{na}$$



Elastic solution

“K-matrix” representation

$$f = \mathcal{M} \times \mathcal{A}$$

$$f = \frac{\mathcal{A}}{\mathcal{K}^{-1} - i\rho}$$

Omnès representation

$$f = \Omega \times \mathcal{F}$$

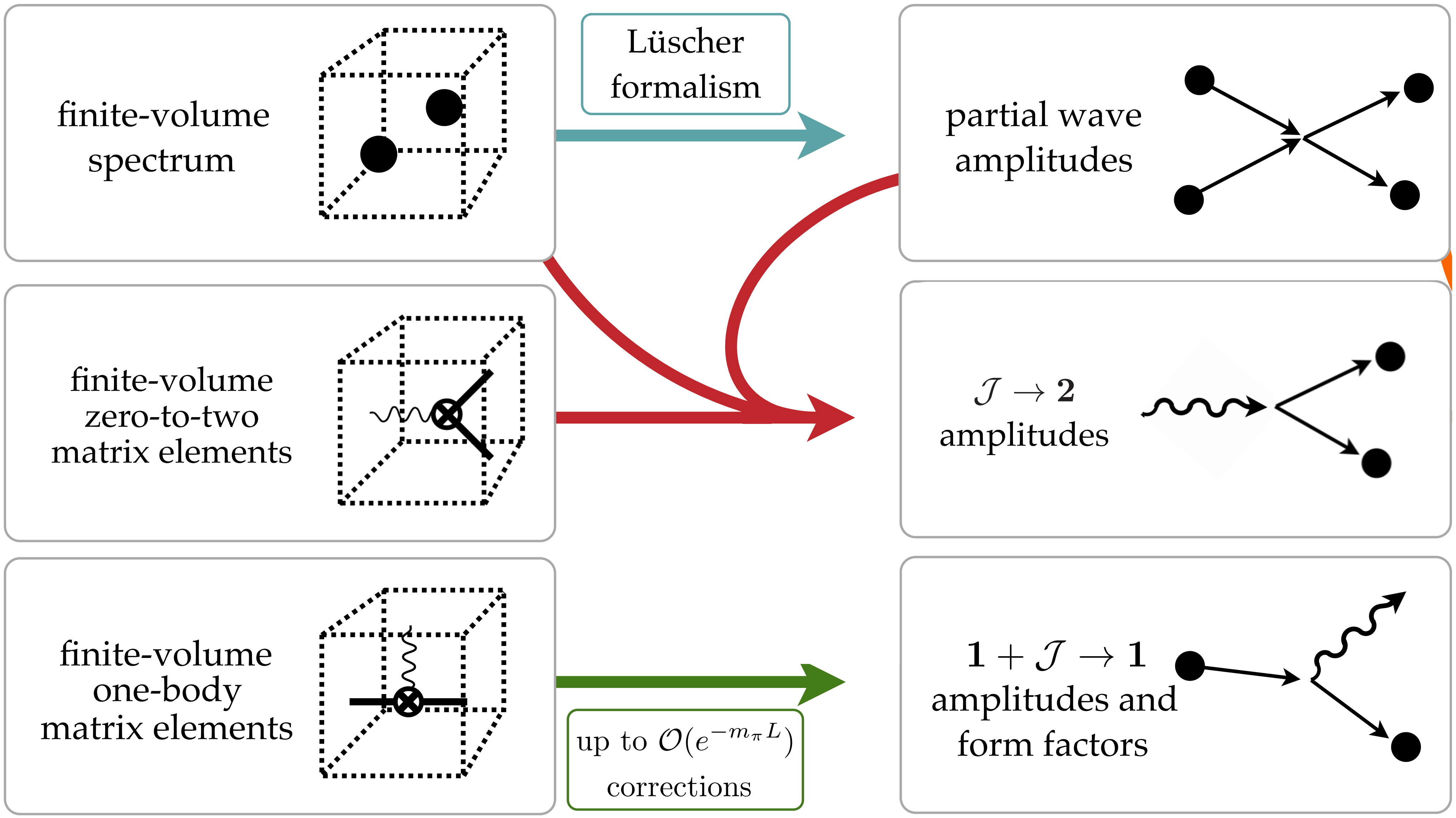
$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} s' \frac{\delta(s')}{s'(s'-s)} \right)$$

Dispersive parameterization

Lattice QCD and timelike form factors

See also talk

J. Dudek@Tue-16:15



timelike

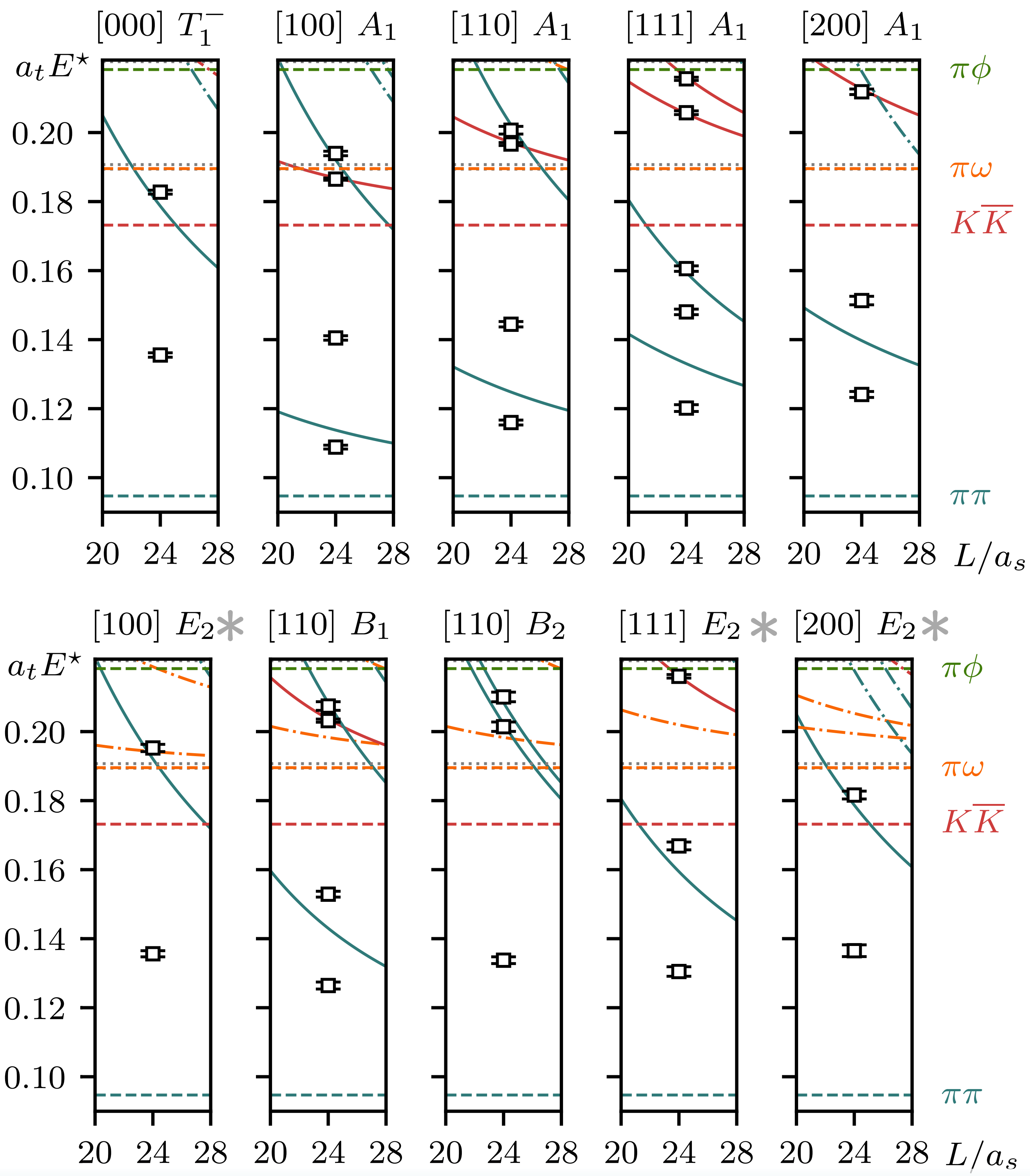
spacelike

Finite volume spectrum

- 17 elastic levels
- 15 above KK thr

✦ 25 $\pi\pi$-like levels

✦ 7 KK-like levels

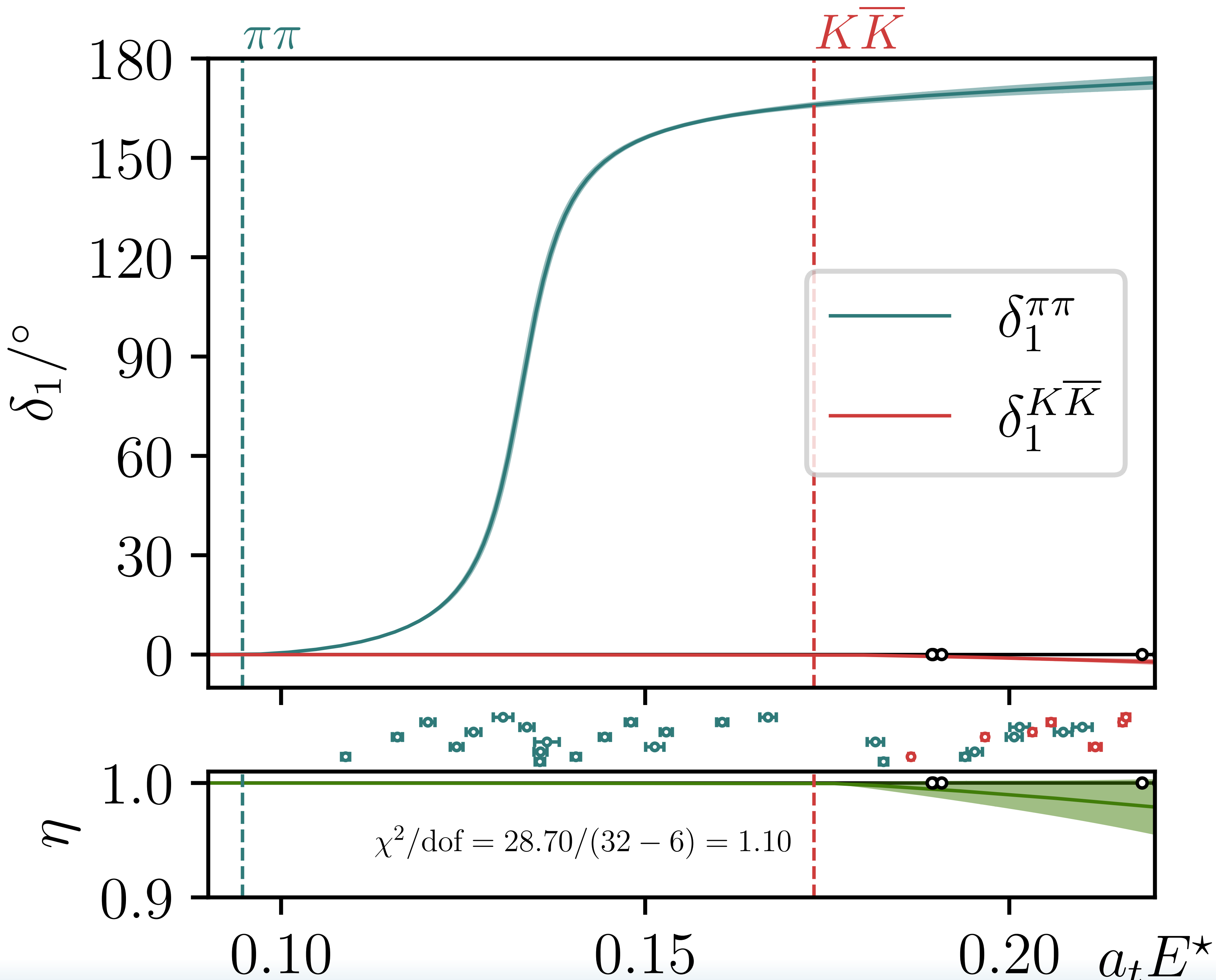


*without $\omega\pi$ operators

Scattering coupled channel fit

$$K_{ab} = \frac{g_a g_b}{-s + m_r^2} + \gamma_{ab} \qquad \mathcal{M}^{-1}_{ab} = \frac{1}{2k_a^\star} K^{-1}_{ab} \frac{1}{2k_b^\star} - i\rho_{\mathrm{CM},ab}$$

Relative momentum in cm frame



ρ resonance

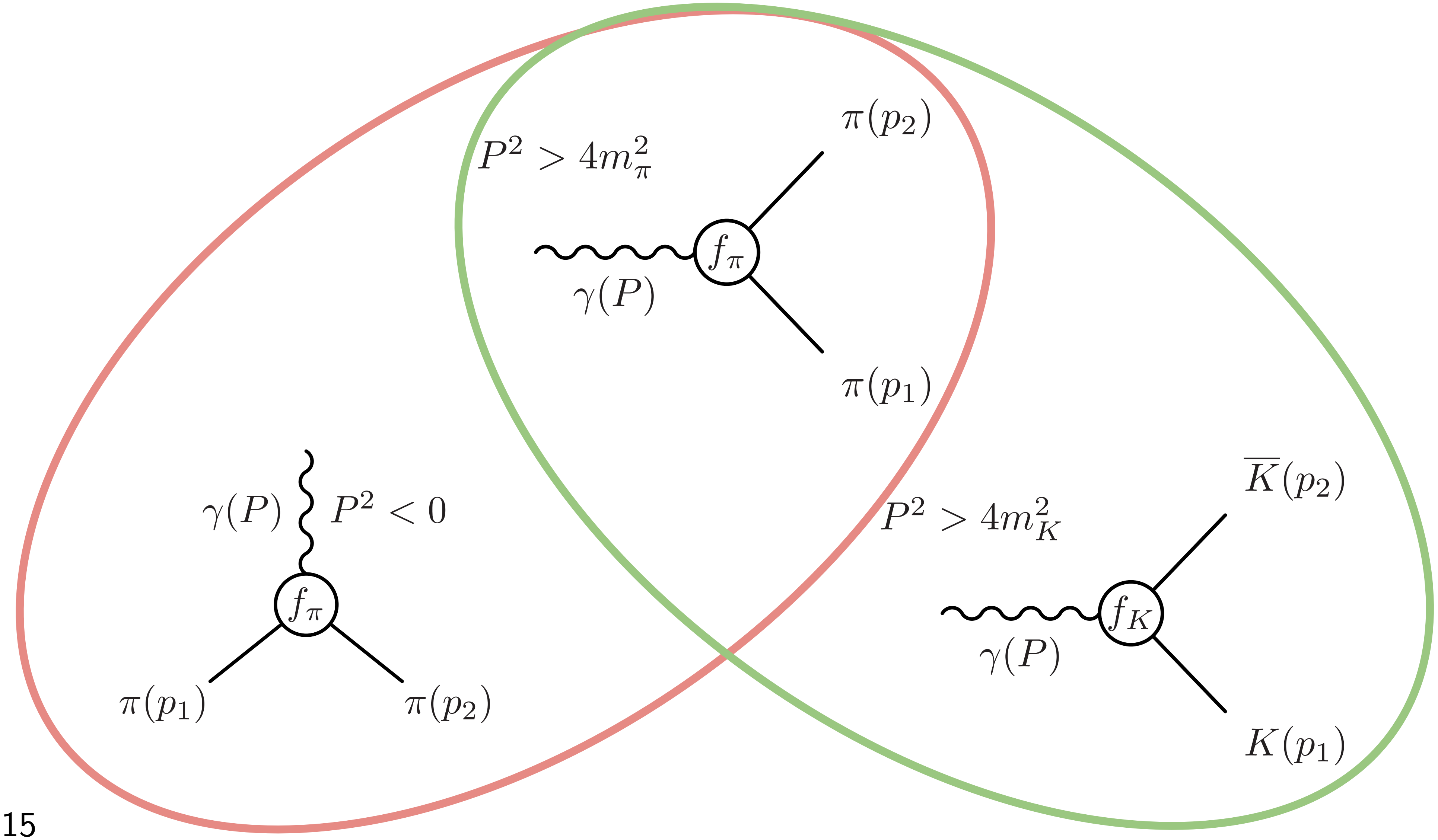
$$\mathcal{M}_{ab}(s \sim s_R) \sim \frac{c_a\, c_b}{s_R - s}$$

$$\mathrm{Re}(\sqrt{s_R}) = 797 \pm 2.6\ \mathrm{MeV}$$
$$\mathrm{Im}(\sqrt{s_R})/2 = 28.5 \pm 1\ \mathrm{MeV}$$

$$\left| \frac{c_{\pi\pi}}{k^\star_{\pi\pi}(s_R)} \right| = 6.41 \pm 0.13$$

$$\left| \frac{c_{K\overline{K}}}{k^\star_{K\overline{K}}(s_R)} \right| = 2.4 \pm 4.0$$

Beyond the *elastic* timelike extraction



See also talk

A. Rodas@Fri-16:15

Dispersive parameterization

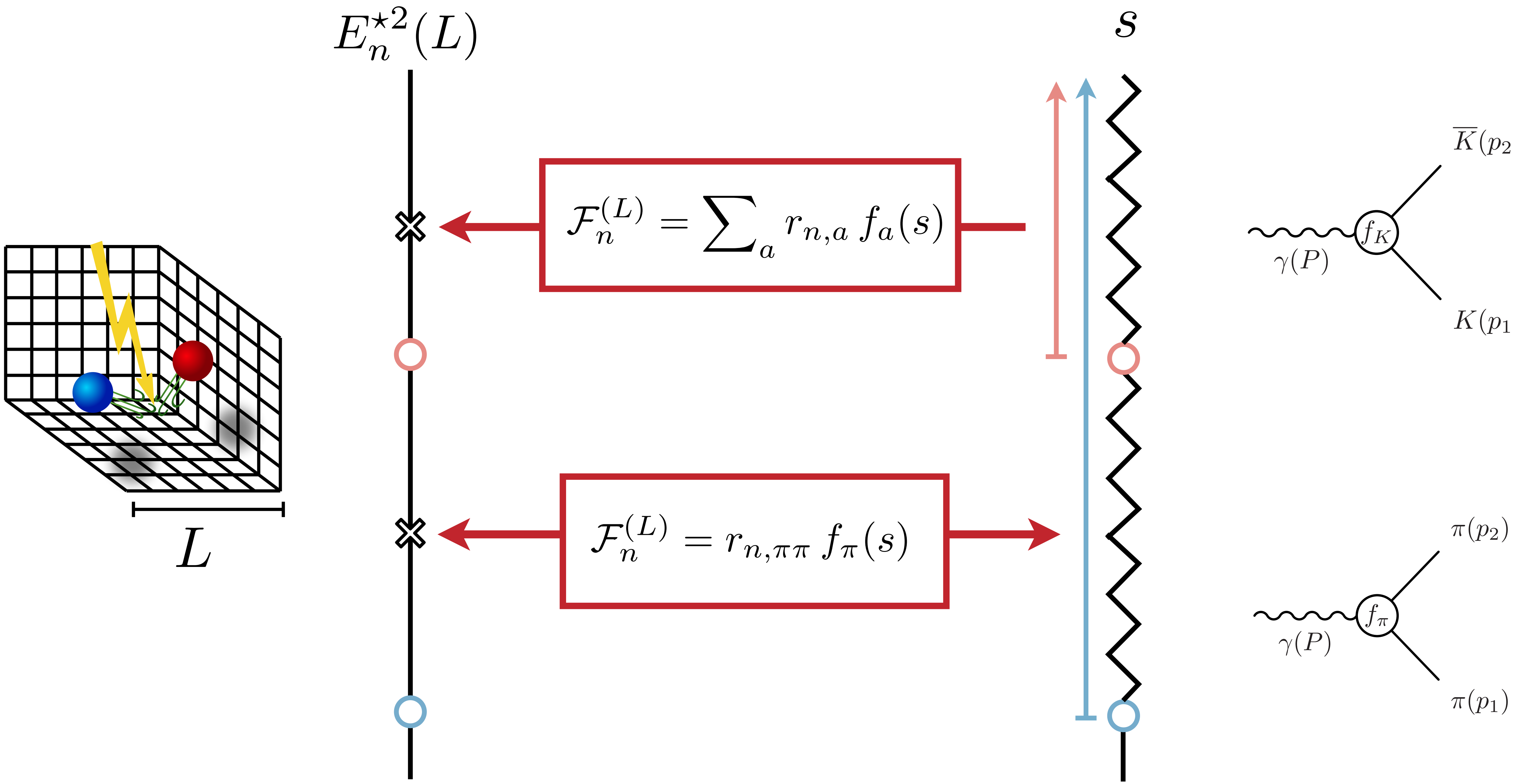
- Average out systematic effects.
- Analytic representation constrained over a wide energy range.

Coupled-channel analysis

- Test of the *transition* c.c. formalism.
- First K timelike FF from LQCD.

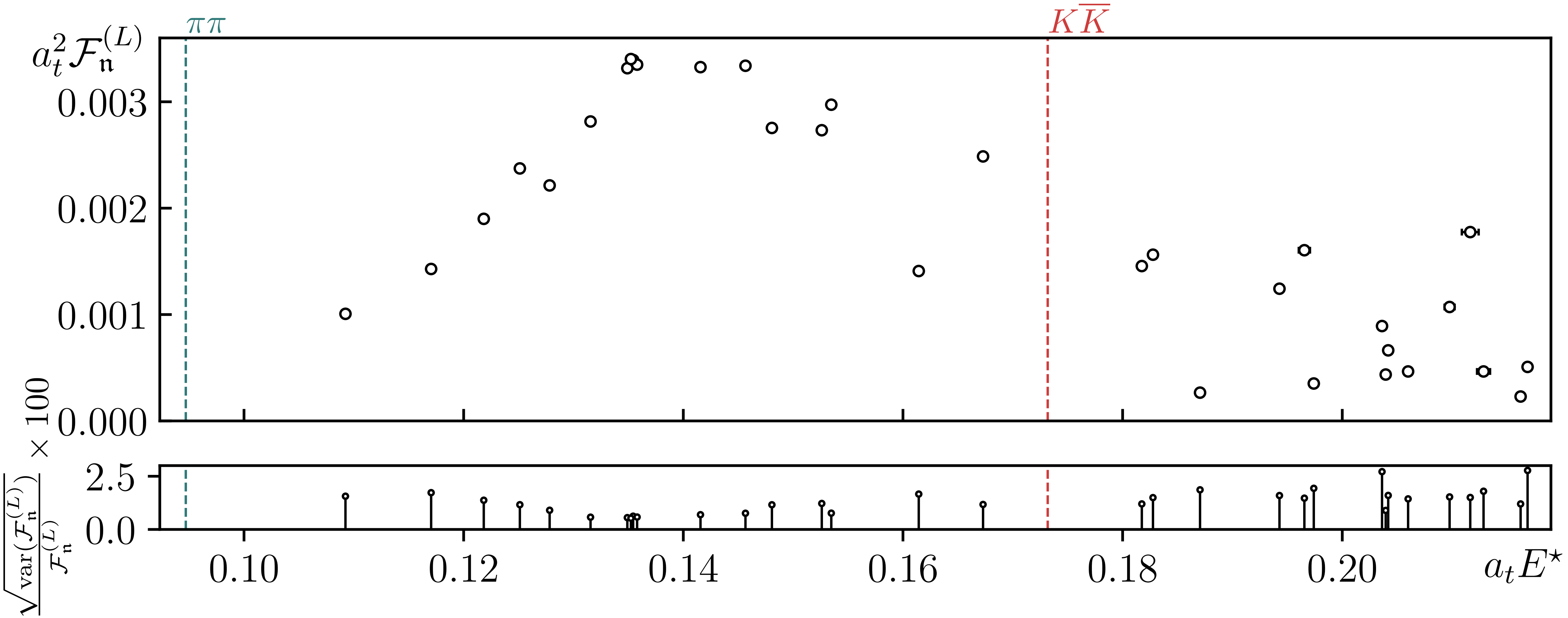
Pair production

FV correction: $r_{n,a}$



$\langle 0|\mathcal{J}|E_n^\star\rangle_L \propto \mathcal{F}_n^{(L)}$ $\qquad$ $\langle 0|\mathcal{J}|P,a\rangle \propto f_a$

Finite volume matrix elements

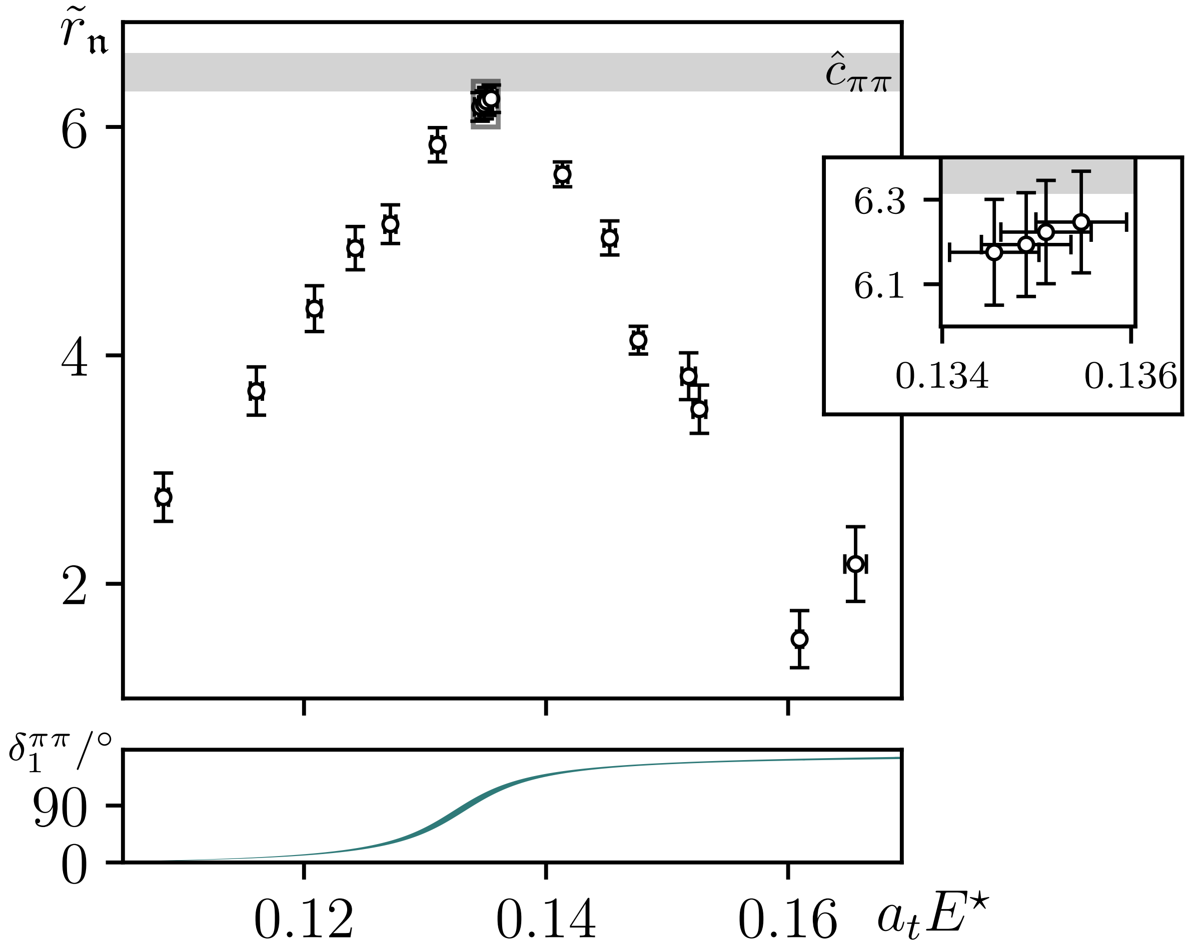


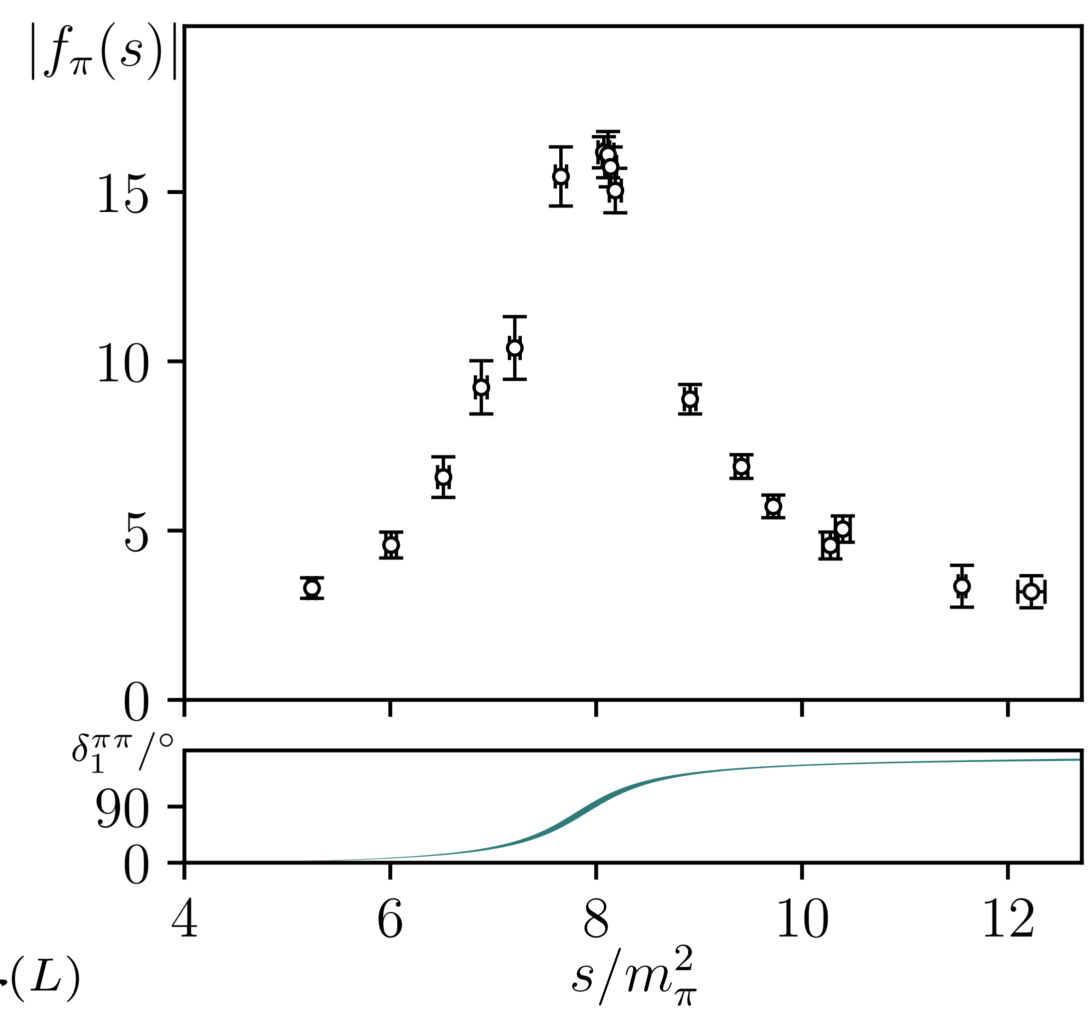
Finite volume correction (elastic region)

$$\mu_0'^{\star}\, w_0\, w_0^{\intercal} = \frac{\partial}{\partial E^{\star}}\left(\mathcal{M}^{-1} + F\right)$$

LL factor: $\tilde{r}_{n,a} = \frac{1}{k_a^{\star}} w_{0,a} \sqrt{\frac{2E_n^{\star}}{-\mu_0'^{\star}}}$

$$f_{\pi} = \frac{\mathcal{M}_{\pi\pi,\pi\pi}}{k_{\pi\pi}^{\star 2}} \times \frac{\mathcal{F}_n^{(L)}}{\tilde{r}_{n,\pi\pi}}$$





In the narrow width limit

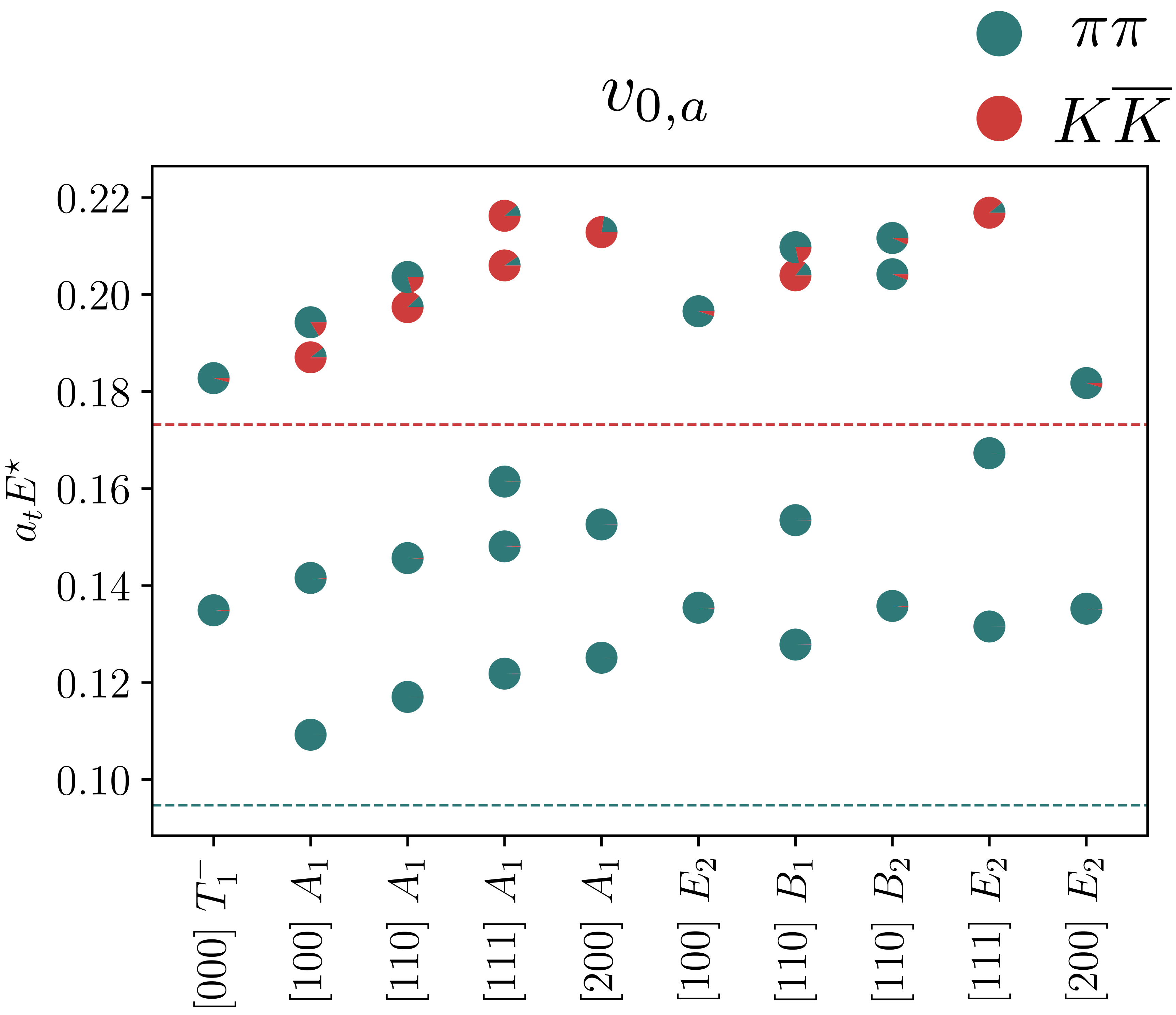
$$\tilde{r}_{\mathfrak{n},a} \approx \frac{c_a}{k_a^{\star}} = \hat{c}_a\,, \quad f_a \approx \frac{\hat{c}_a\, \mathcal{F}_n^{(L)}}{s_R - s}$$

Finite volume correction (coupled channel)

$$\lambda_0'^{\star}\, v_0\, v_0^{\intercal} = \frac{\partial}{\partial E^{\star}}\left(\mathcal{M} + F^{-1}\right)$$

- ✦ 25 $\pi\pi$-like levels
- ✦ 7 KK-like levels

$$\mathcal{F}^{(L)} \propto \sum_a v_{0,a} f_a$$



Finite volume correction

$$\lambda_0'^{\star}\, v_0\, v_0^{\intercal} = \frac{\partial}{\partial E^{\star}}\left(\mathcal{M} + F^{-1}\right)$$

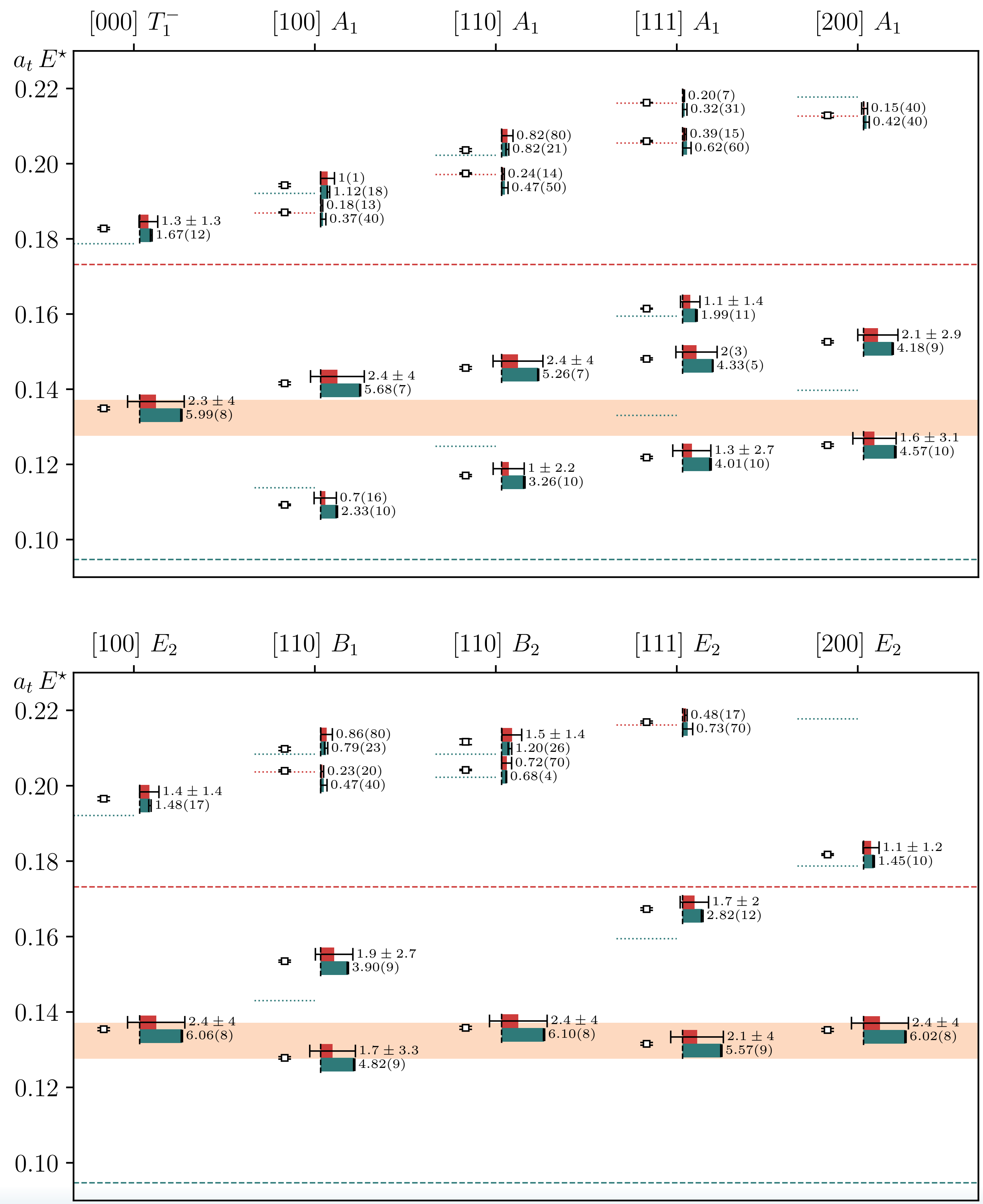
$$\mu_0'^{\star}\, w_0\, w_0^{\intercal} = \frac{\partial}{\partial E^{\star}}\left(\mathcal{M}^{-1} + F\right)$$

In the narrow width limit

$$\tilde{r}_{\mathfrak{n},a} \approx \frac{c_a}{k_a^{\star}}$$

$$\mathcal{F}^{(L)} = \sum_a \tilde{r}_a \mathcal{F}_a$$

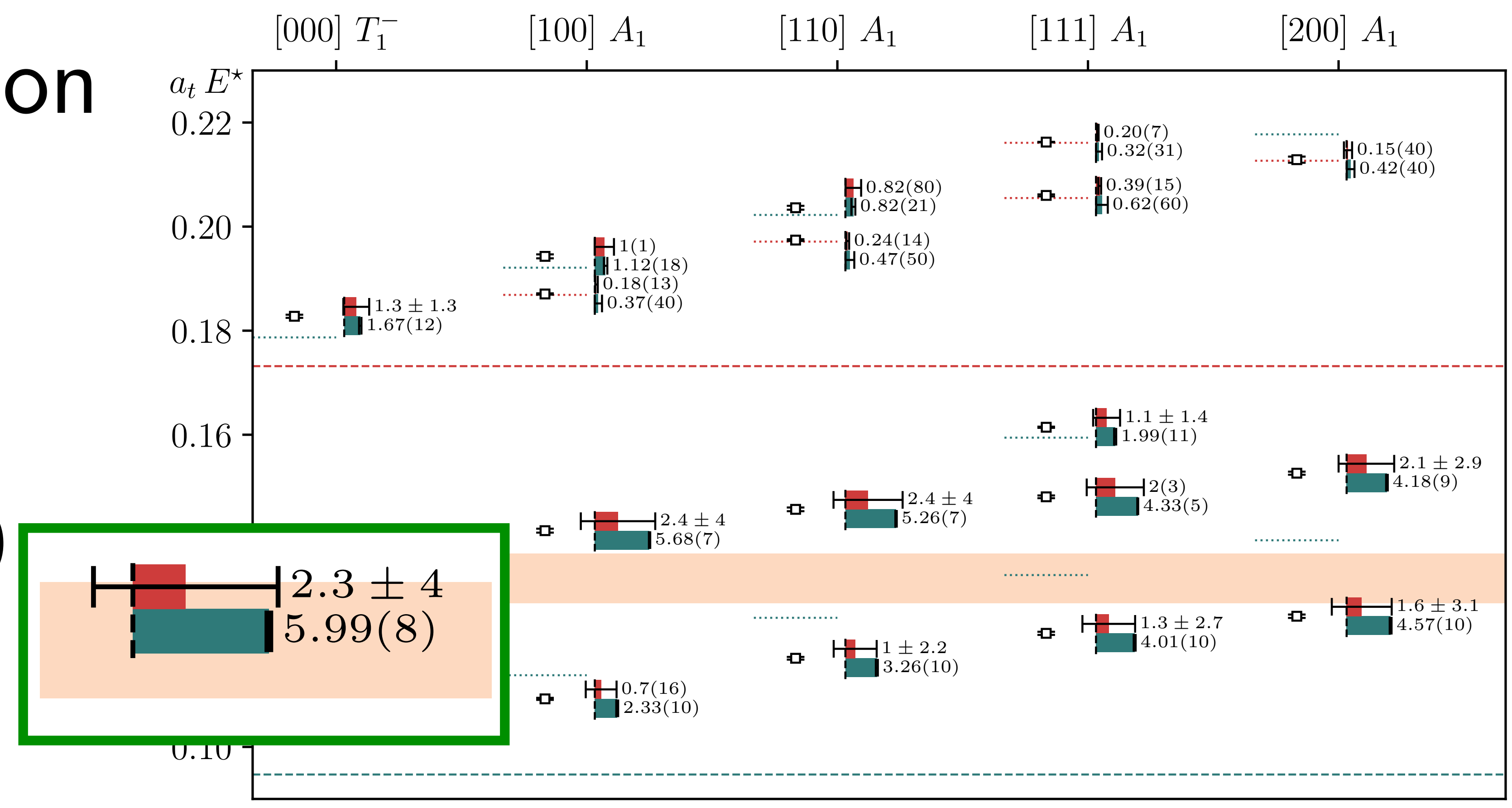
$$\mathcal{F}^{(L)} \propto \sum_a v_{0,a} f_a$$



Finite volume correction

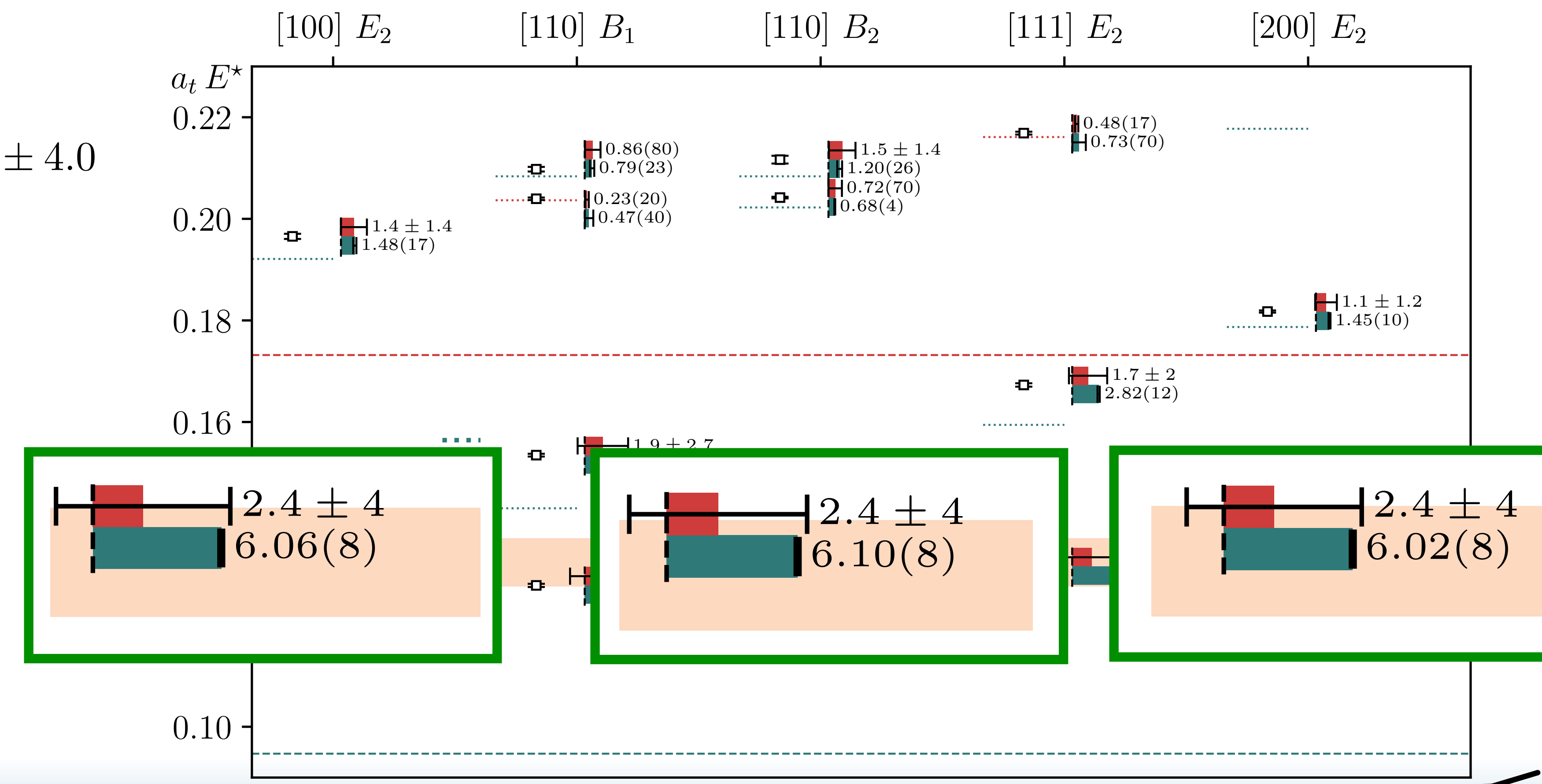
$$\lambda_0'^{\star}\, v_0\, v_0^{\intercal} = \frac{\partial}{\partial E^\star}\left(\mathcal{M} + F^{-1}\right)$$

$$\mu_0'^{\star}\, w_0\, w_0^{\intercal} = \frac{\partial}{\partial E^\star}\left(\mathcal{M}^{-1} + F\right)$$



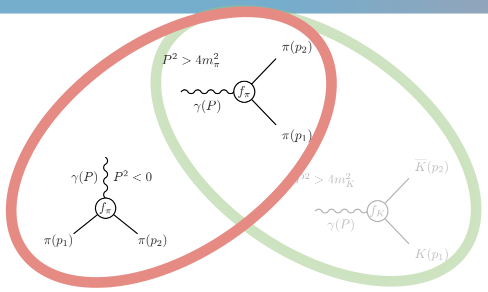
In the narrow width limit $\quad \tilde{r}_{\mathfrak{n},a} \approx \dfrac{c_a}{k_a^\star}$

$$\left|\frac{c_{\pi\pi}}{k^\star_{\pi\pi}(s_R)}\right| = 6.41 \pm 0.13 \qquad \left|\frac{c_{K\overline{K}}}{k^\star_{K\overline{K}}(s_R)}\right| = 2.4 \pm 4.0$$

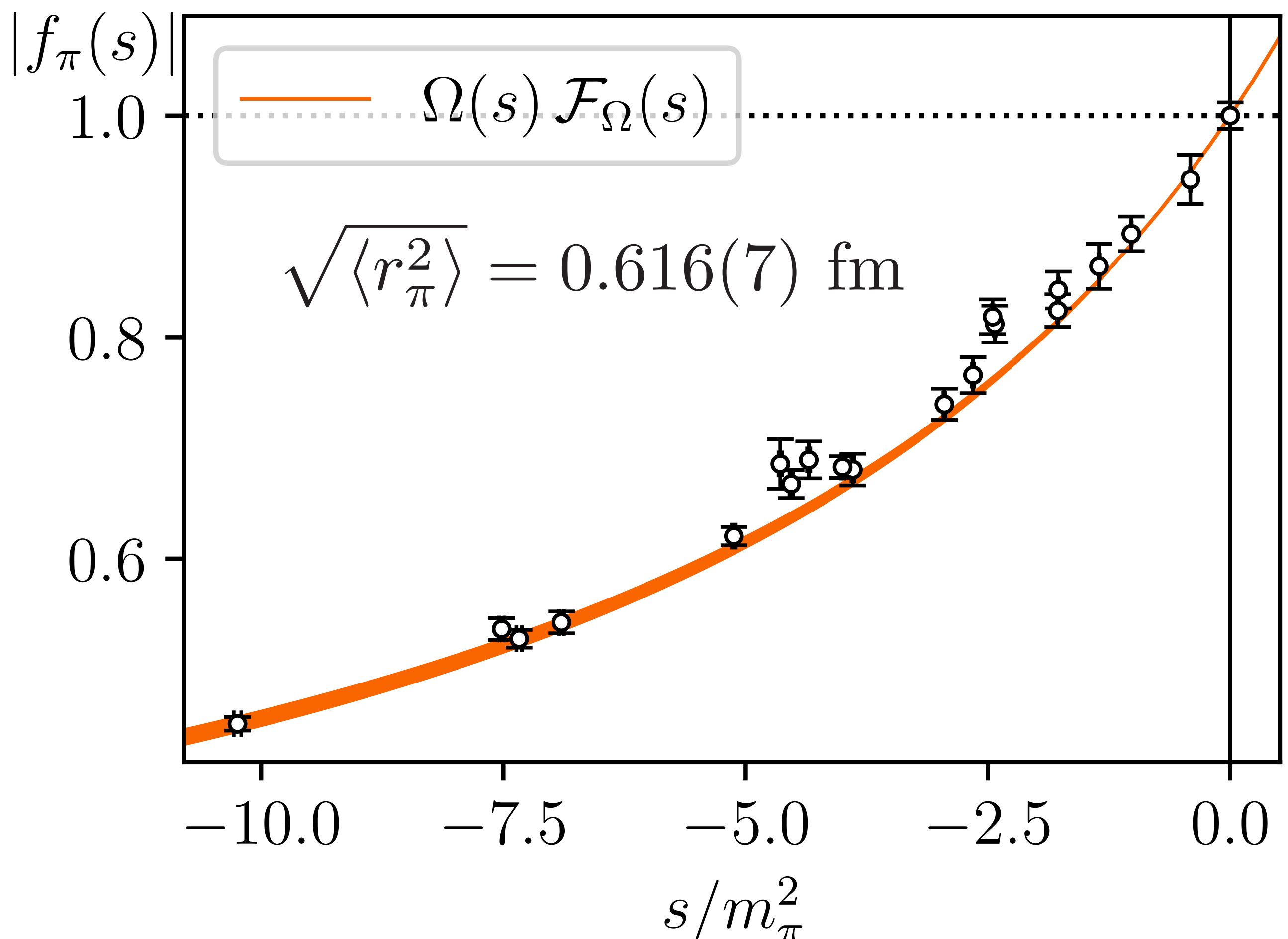


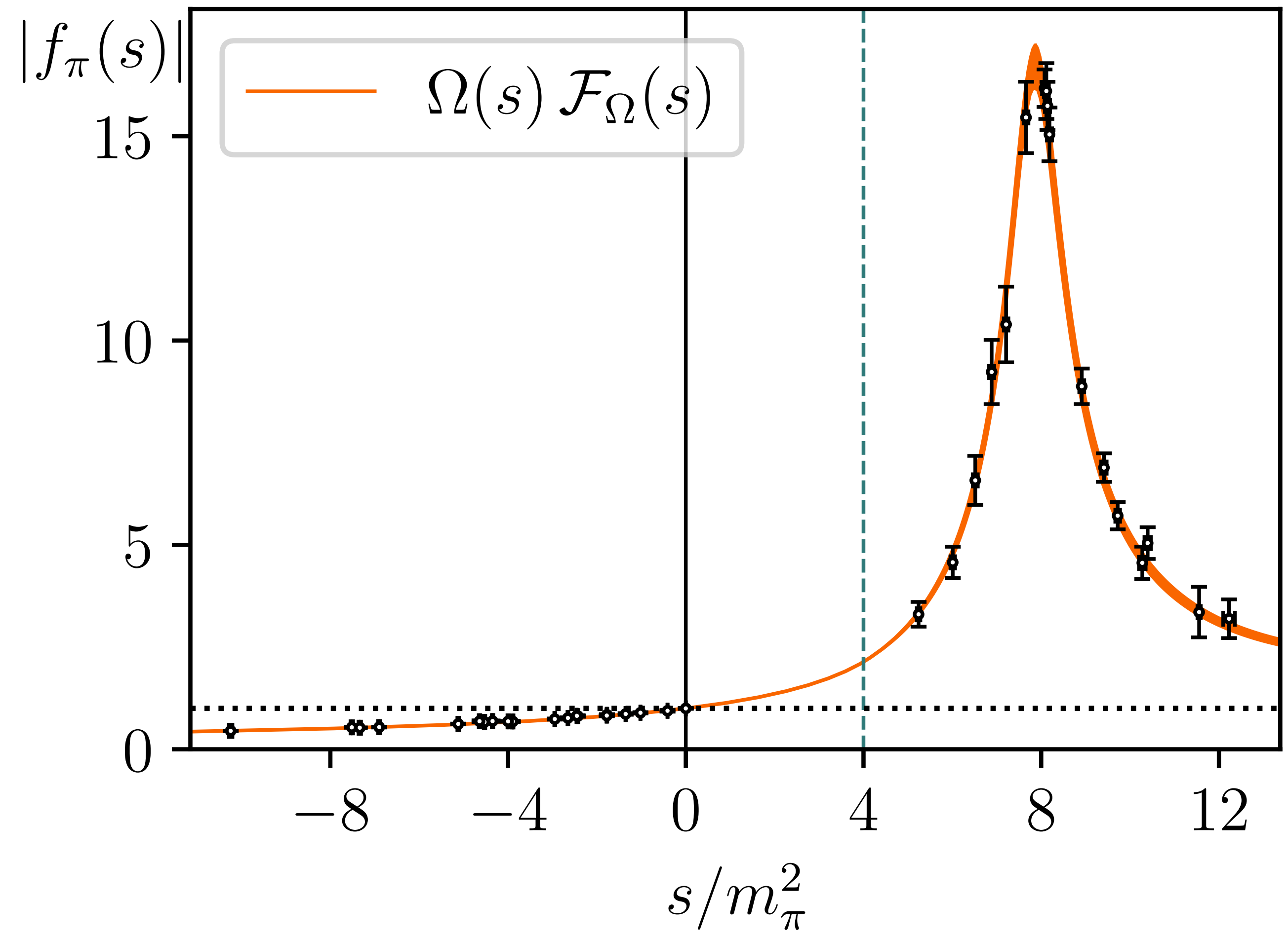
$$\mathcal{F}^{(L)} = \sum_a \tilde{r}_a \mathcal{F}_a$$

$$\mathcal{F}^{(L)} \propto \sum_a v_{0,a} f_a$$



Dispersive fit: spacelike + elastic



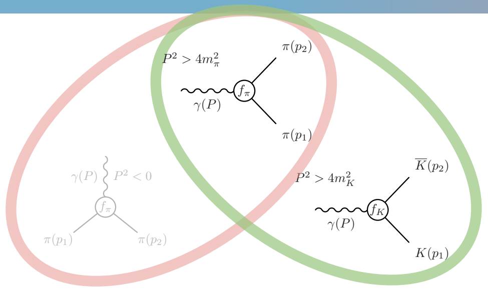


$\chi^2/n_{\mathrm{dof}} = 85.4/(37-2) = 2.44$

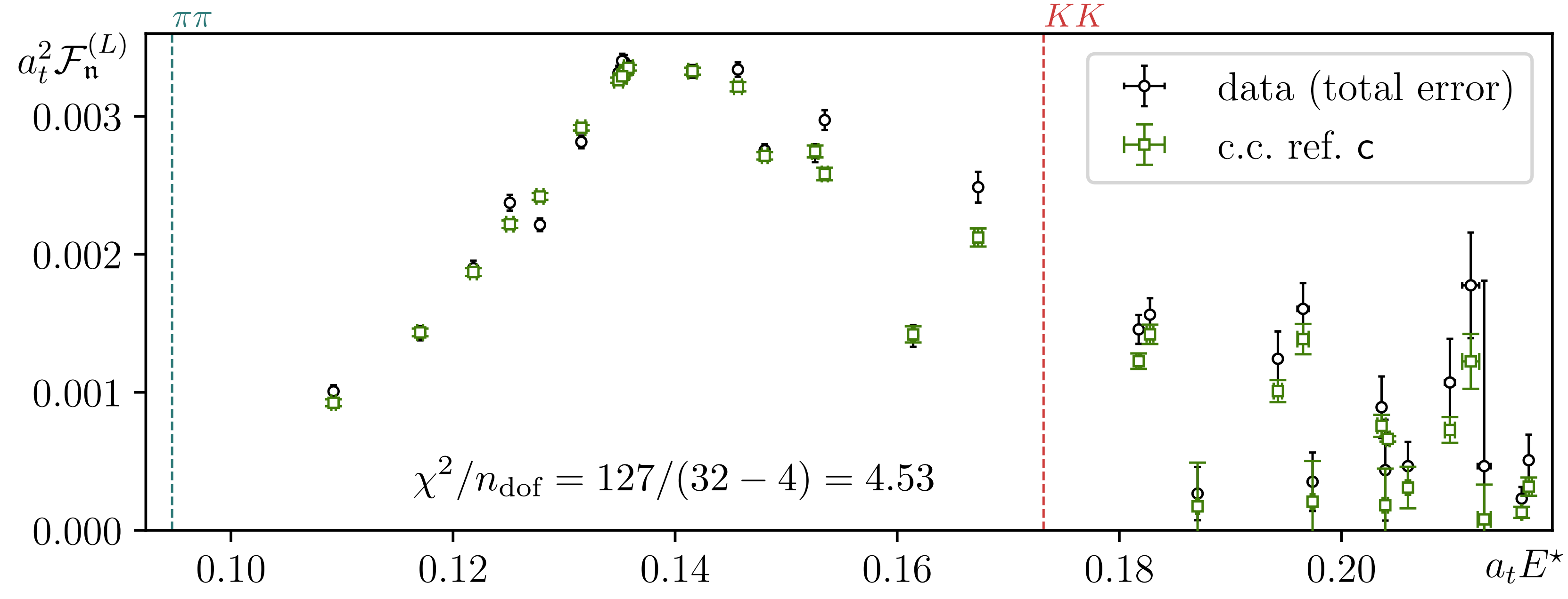
$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)}\right) \qquad \mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n\left(z_c(s)^n - z_c(0)^n\right)$$

High energy ps behavior inspired by [2]

$$z_c(s) = \frac{\sqrt{s_c - s_0} - \sqrt{s_c - s}}{\sqrt{s_c - s_0} + \sqrt{s_c - s}}$$

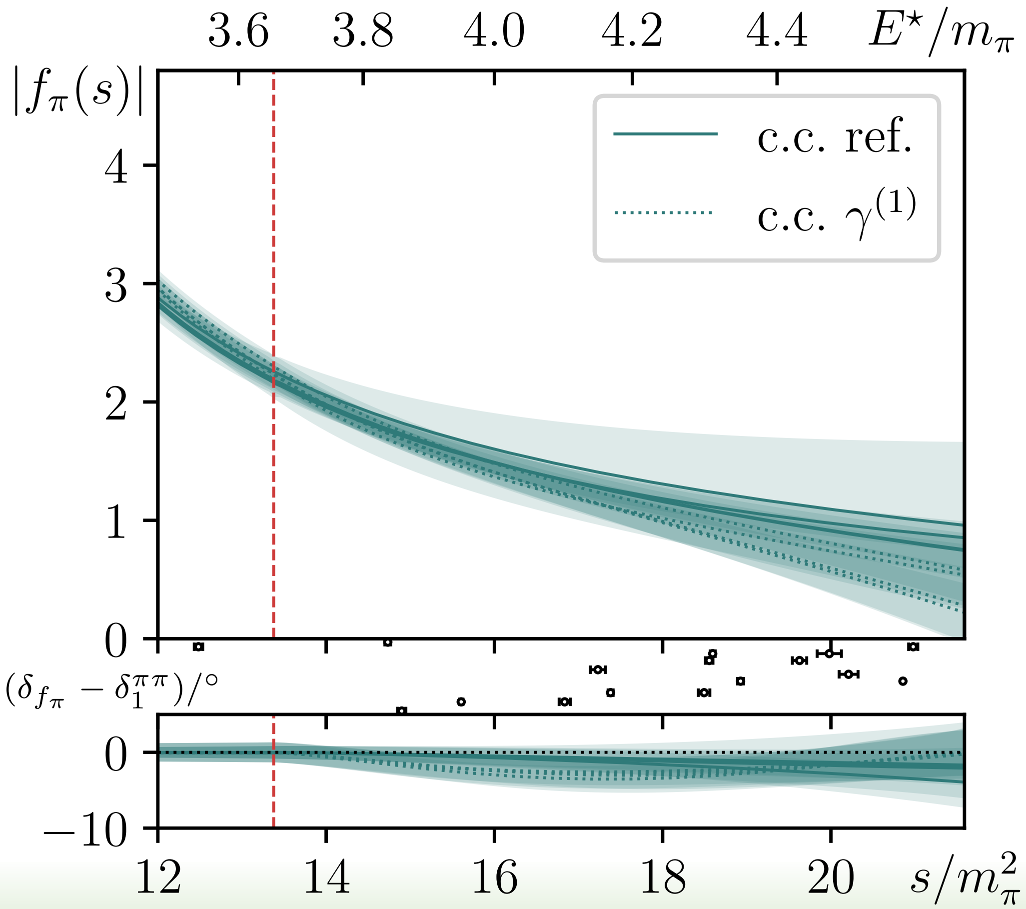


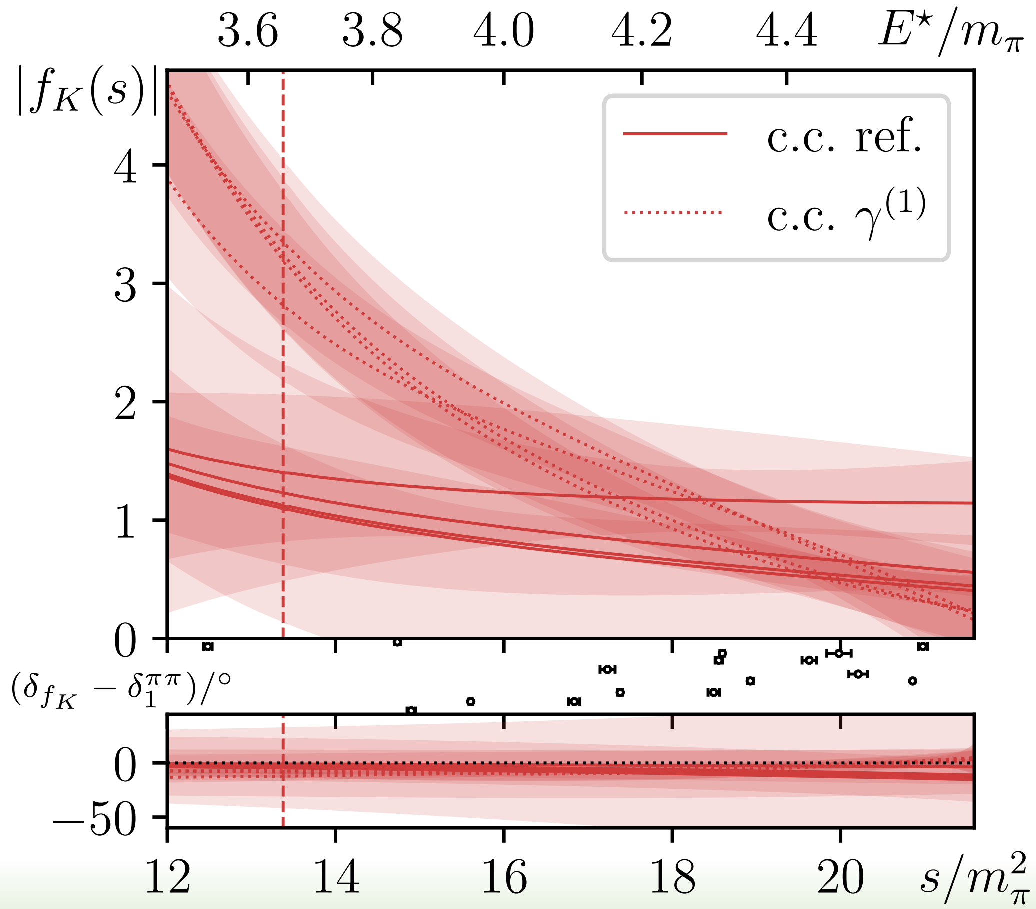
Coupled channel fit



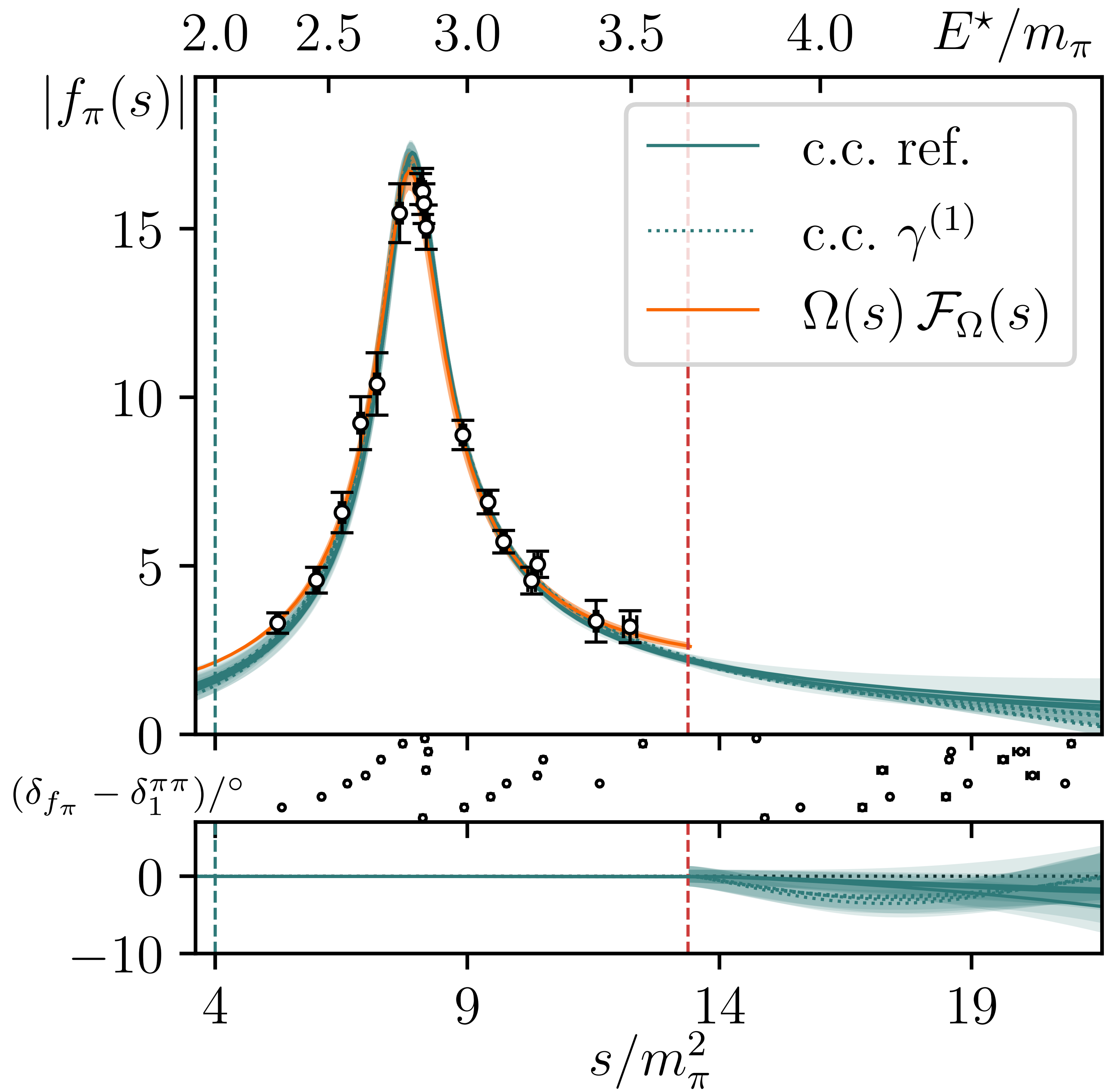
$$f_a = \frac{1}{k_a^\star}\mathcal{M}_{ab}\frac{1}{k_b^\star}\mathcal{F}_b$$

$$\mathcal{F}_a(s) = \sum_n h_{n,a}\, s^n$$

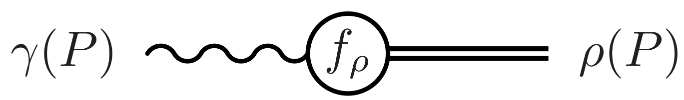




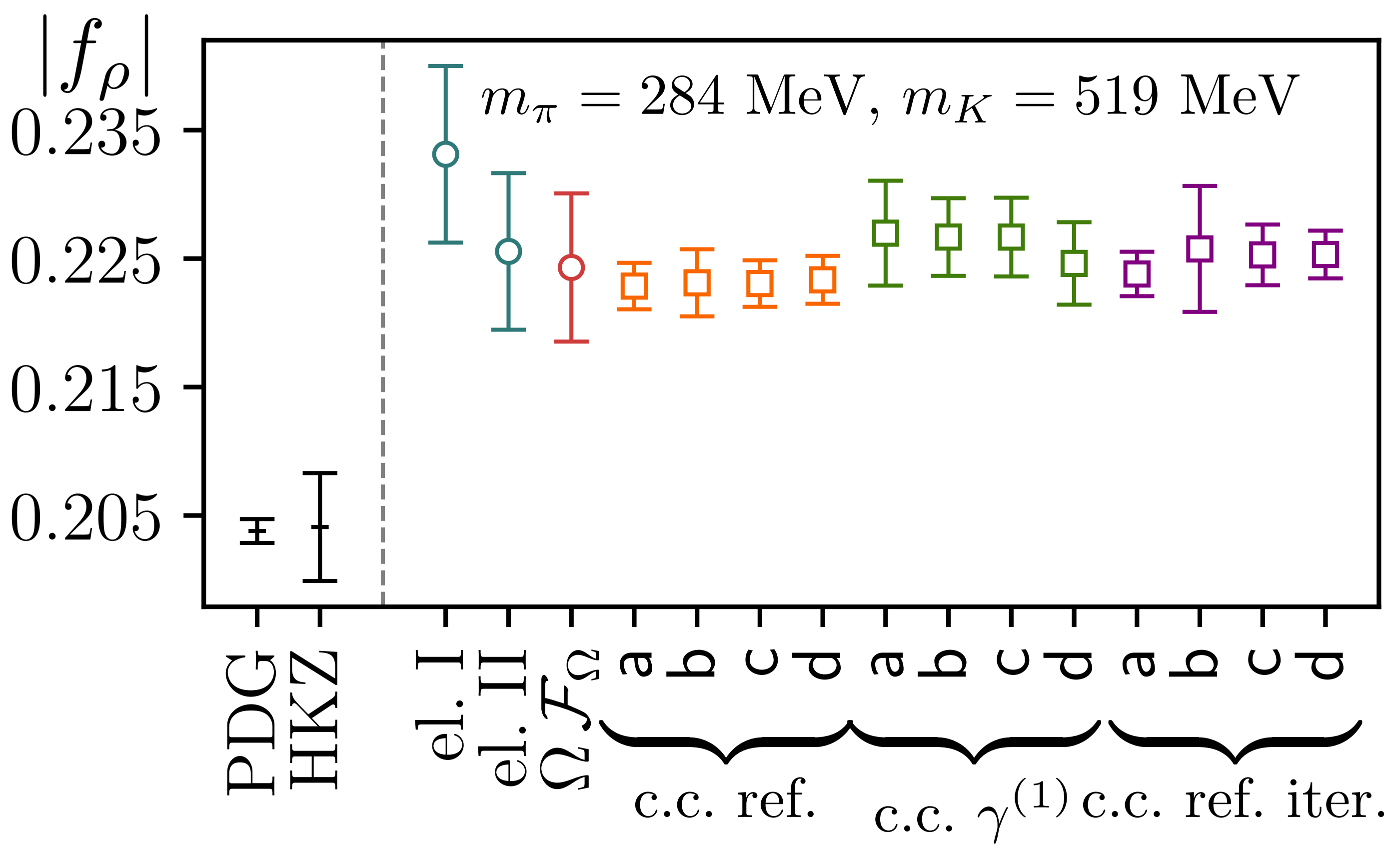
Comparison dispersive vs. c.c.



Vector meson decay constant

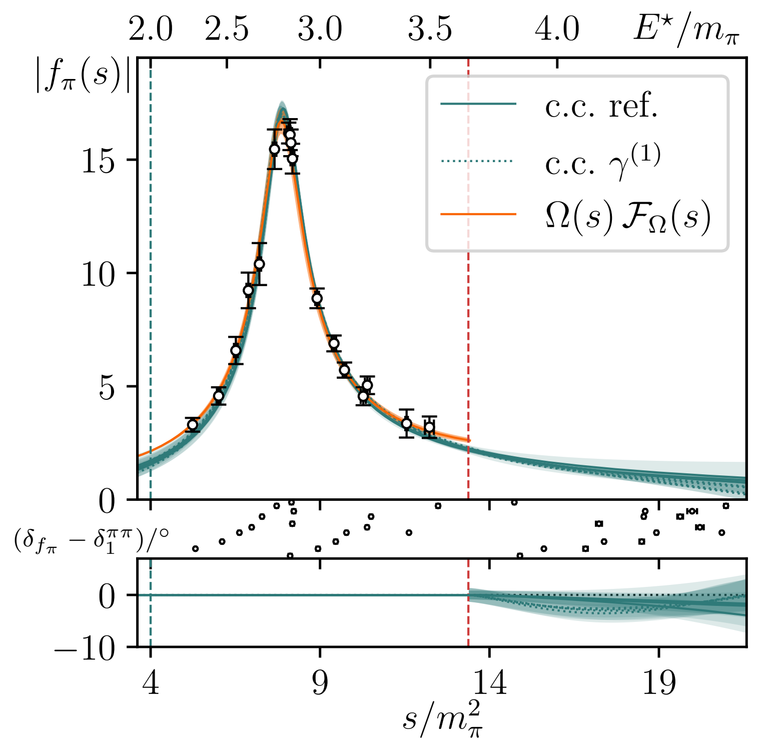


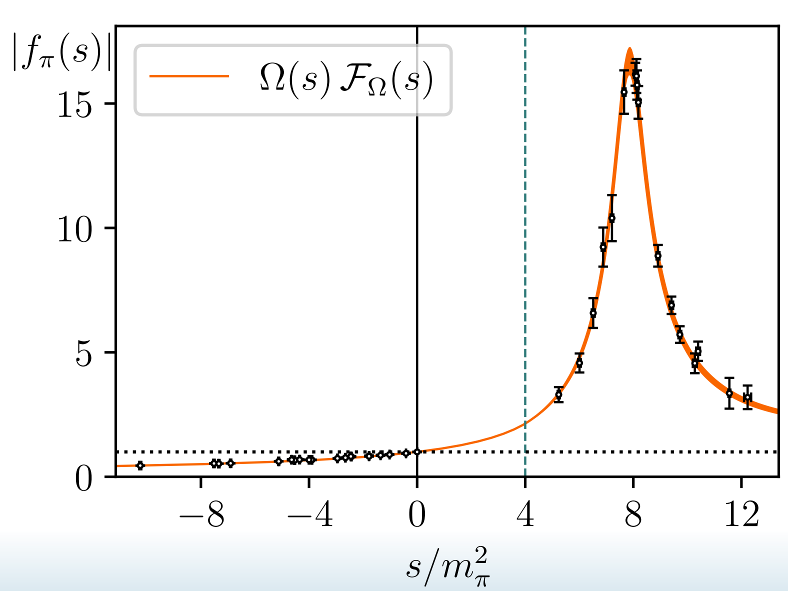
$$\langle \rho(\vec{P}, m) | \mathcal{J}^\mu | 0 \rangle = \epsilon^{\mu *}(P, m)\, f_\rho\, m_R^2$$

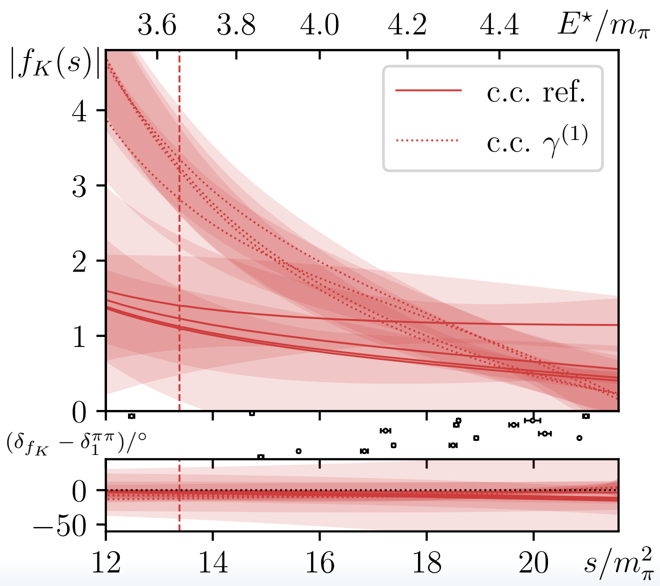


Summary and outlook

- Isovector p-wave coupled channel $\pi\pi/KK$
- Pair production amplitude
  - Spacelike and timelike region
  - Coupled channel region
- We demonstrate the feasibility of future analysis on other channels.



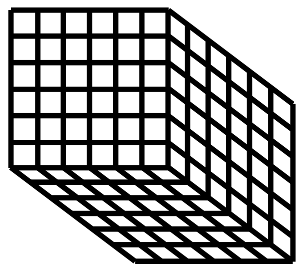




Back up slides

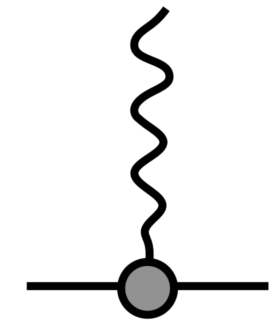
Scattering and pair-production $J^P(I^G)$=1⁻(1⁺)

$$m_\pi L \approx 4$$

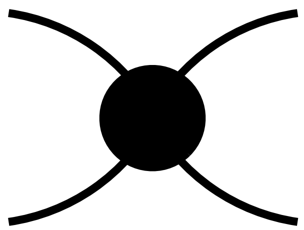


$L^3 \times T = 24^3 \times 256$

	$a_t m$	m/MeV
π	0.0474	284
K	0.0866	519

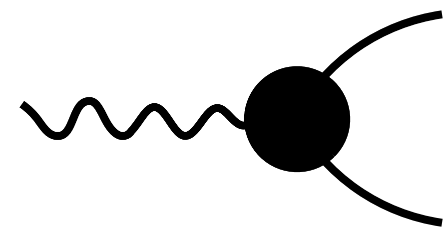


For renormalization of the current



$$C_{ij}(t) = \left\langle O_i(t) O_j^\dagger(0) \right\rangle$$

- Two-meson-*like* operators
- $q\bar{q}$-*like* operators



$$\lim_{t\to\infty} \frac{\left\langle \mathcal{J}(t)\Omega_n^\dagger(0)\right\rangle}{\left\langle \Omega_n(t)\Omega_n^\dagger(0)\right\rangle} \propto \mathcal{H}_L$$

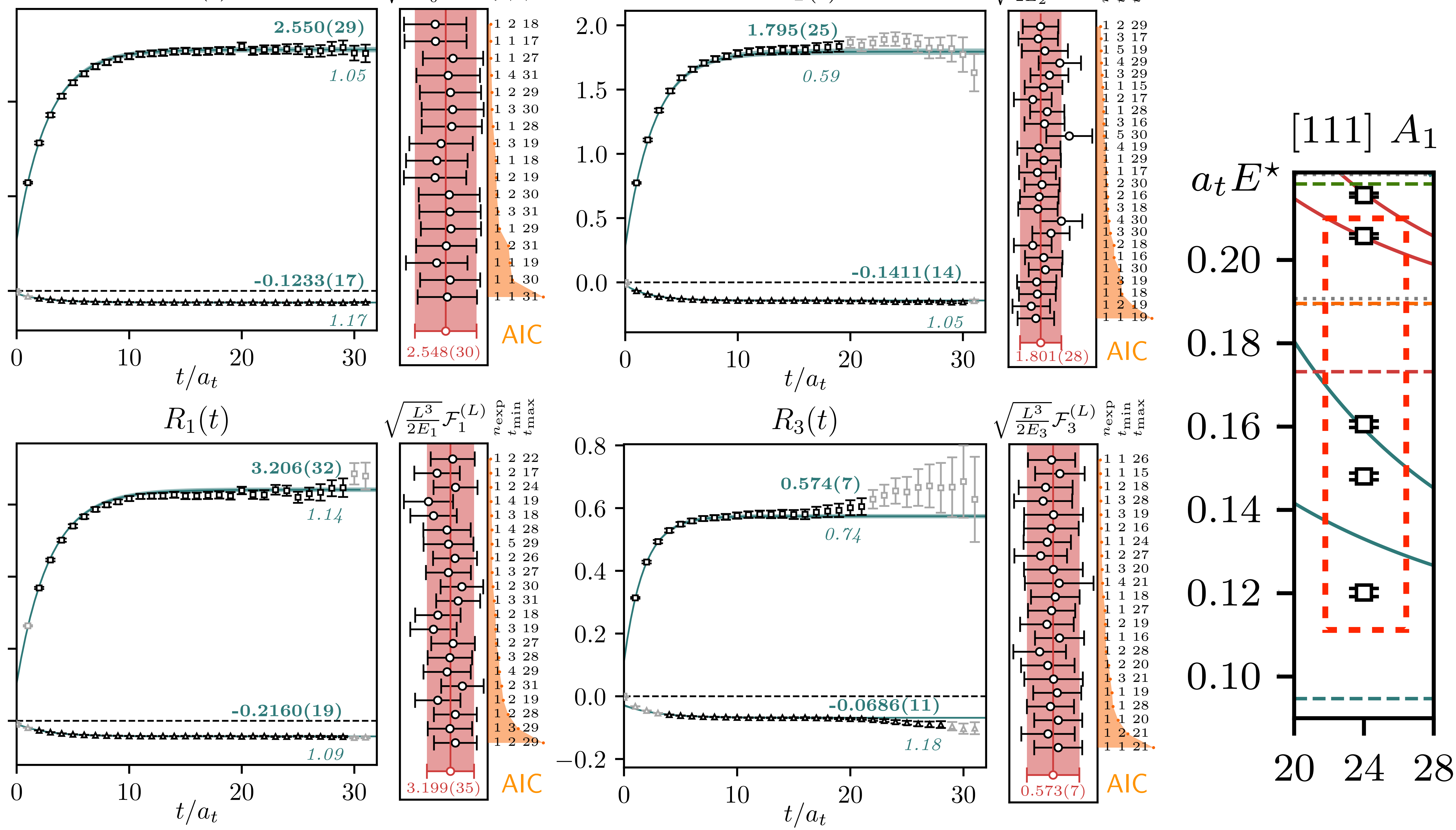
- Optimized operators $\Omega_n \propto v_j^n O_j$

1. Generalized eigenvalue problem λ_n , v_j^n
2. Extract energies $\lambda_n(t-t_0) = e^{-E_n(t-t_0)}$
3. Minimize QC $\det(\mathcal{M}^{-1} + F) = 0$

1. Extract invariant $\mathcal{H}_L^\mu = K^\mu \mathcal{F}_n^{(L)}$
2. FV correction $\mu_0'^\star \, w_0 \, w_0^\intercal = \dfrac{\partial}{\partial E^\star}(\mathcal{M}^{-1} + F)$

Finite volume matrix elements

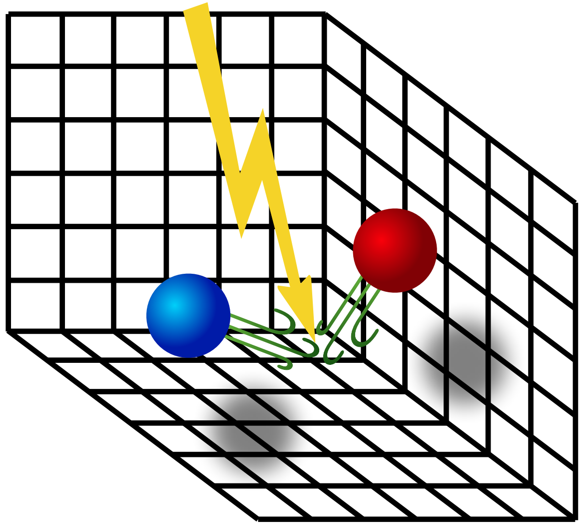
$$R_n(t) = \frac{\sqrt{2E_n}}{K} \frac{\langle \mathcal{J}(t)\Omega_n^\dagger(0)\rangle}{\langle \Omega_n(t)\Omega_n^\dagger(0)\rangle}$$

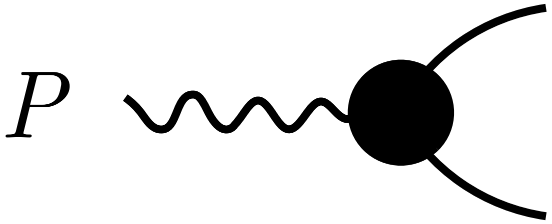


anisotropy

$$\langle 0|\mathcal{J}^{\rho}_{\text{impro}}|\mathfrak{n}\rangle = \langle 0|\rho|\mathfrak{n}\rangle - \frac{1}{4}(1-\xi)a_t E_n \langle 0|\rho_2|\mathfrak{n}\rangle$$

Pair production





Finite-volume matrix elements

Infinite-volume production

$$\mathcal{H}_L^{\mu} = K^{\mu}\,\mathcal{F}_n^{(L)}$$

$$\mathcal{H}_a^{\mu}(P) = K^{\mu}\,k_a^{\star}\,f_a(s)$$ No a sum

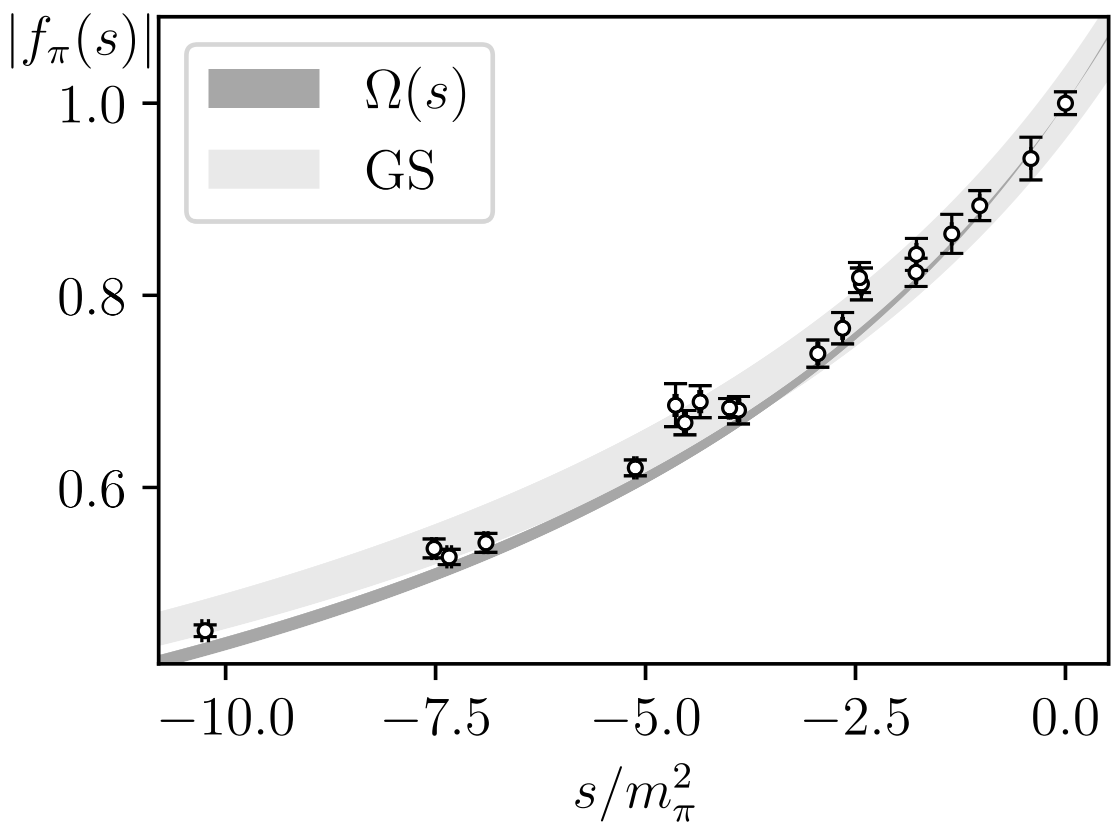
$$\lim_{t\to\infty}\frac{\sqrt{2E_n}}{K}\frac{\langle \mathcal{J}(t)\Omega_n^{\dagger}(0)\rangle}{\left\langle \Omega_n(t)\Omega_n^{\dagger}(0)\right\rangle} = \sqrt{\frac{L^3}{2E_n}}\mathcal{F}_n^{(L)}$$

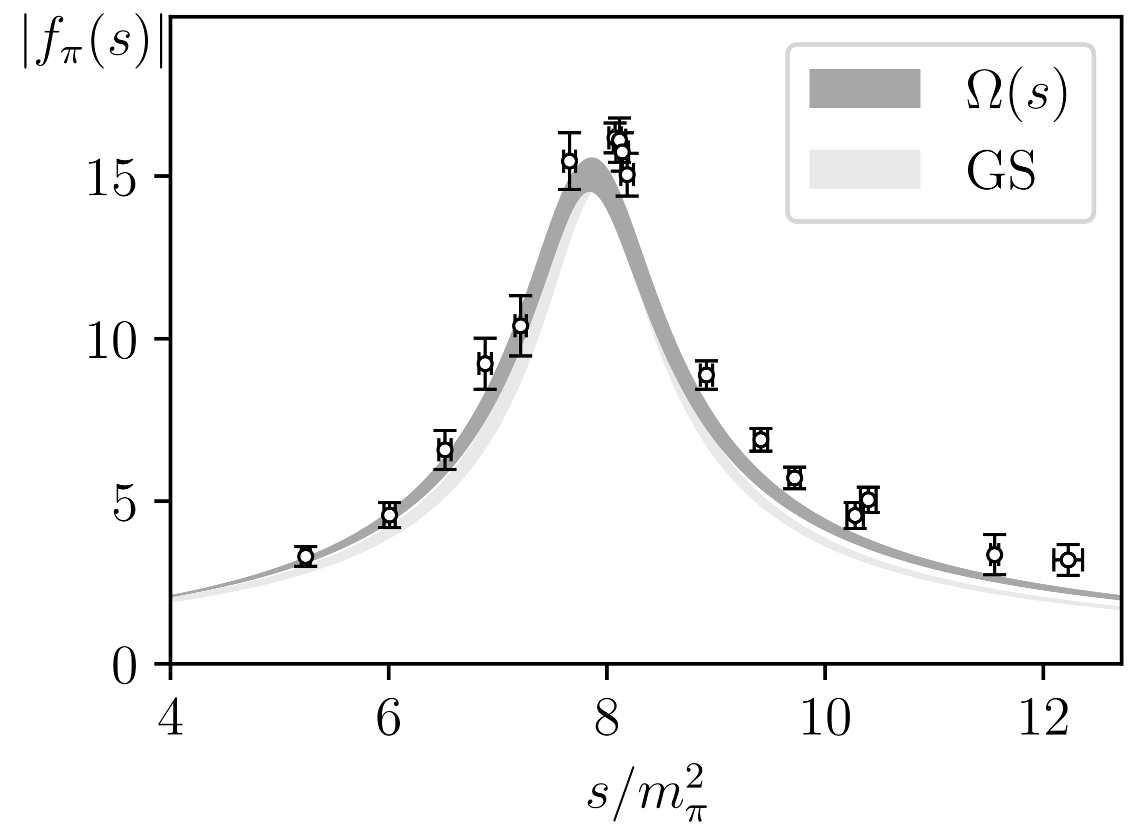
$$f_a = \frac{1}{k_a^{\star}}\mathcal{M}_{ab}\frac{1}{k_b^{\star}}\mathcal{F}_b$$

LL factor: $\tilde{r}_{n,a} = \frac{1}{k_a^{\star}} w_{0,a}\sqrt{\frac{2E_n^{\star}}{-\mu_0'^{\star}}}$ No a sum

$$\mathcal{F}_n^{(L)} = \sum_a \tilde{r}_{n,a}\,\mathcal{F}_a$$

Omnes and GS vs data





Pion form factor parameterizations

$$\mathcal{M}_{ab}^{-1} = \frac{1}{2k_a^\star} K_{ab}^{-1} \frac{1}{2k_b^\star} - i\rho_{\mathrm{CM},ab}$$

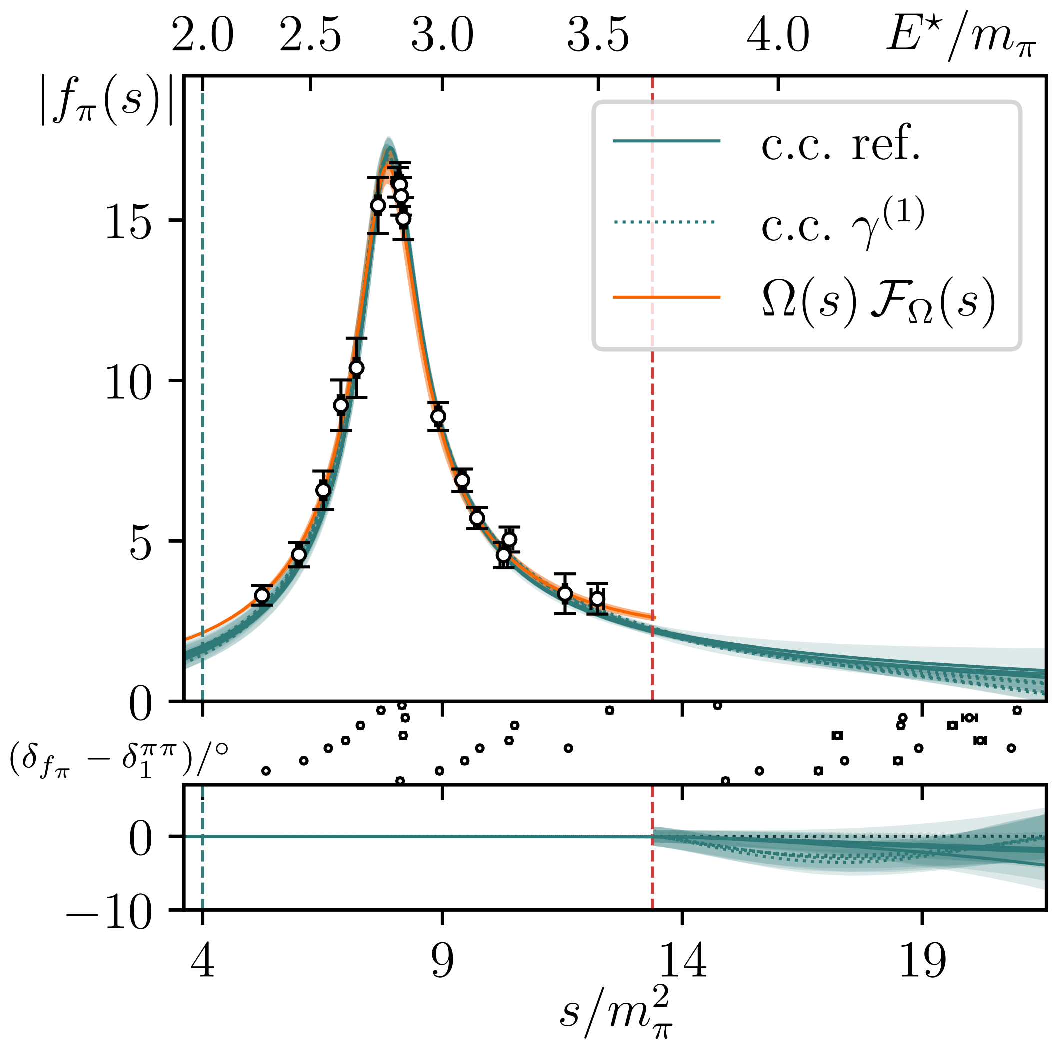
c.c. ref $\quad K_{ab} = \dfrac{g_a g_b}{-s + m_r^2} + \gamma_{ab}$

c.c. $\gamma^{(1)}$ $\quad K_{ab} = \dfrac{g_a\, g_b}{-s + m_r^2} + \gamma_{ab}^{(0)} + \gamma_{ab}^{(1)} s$

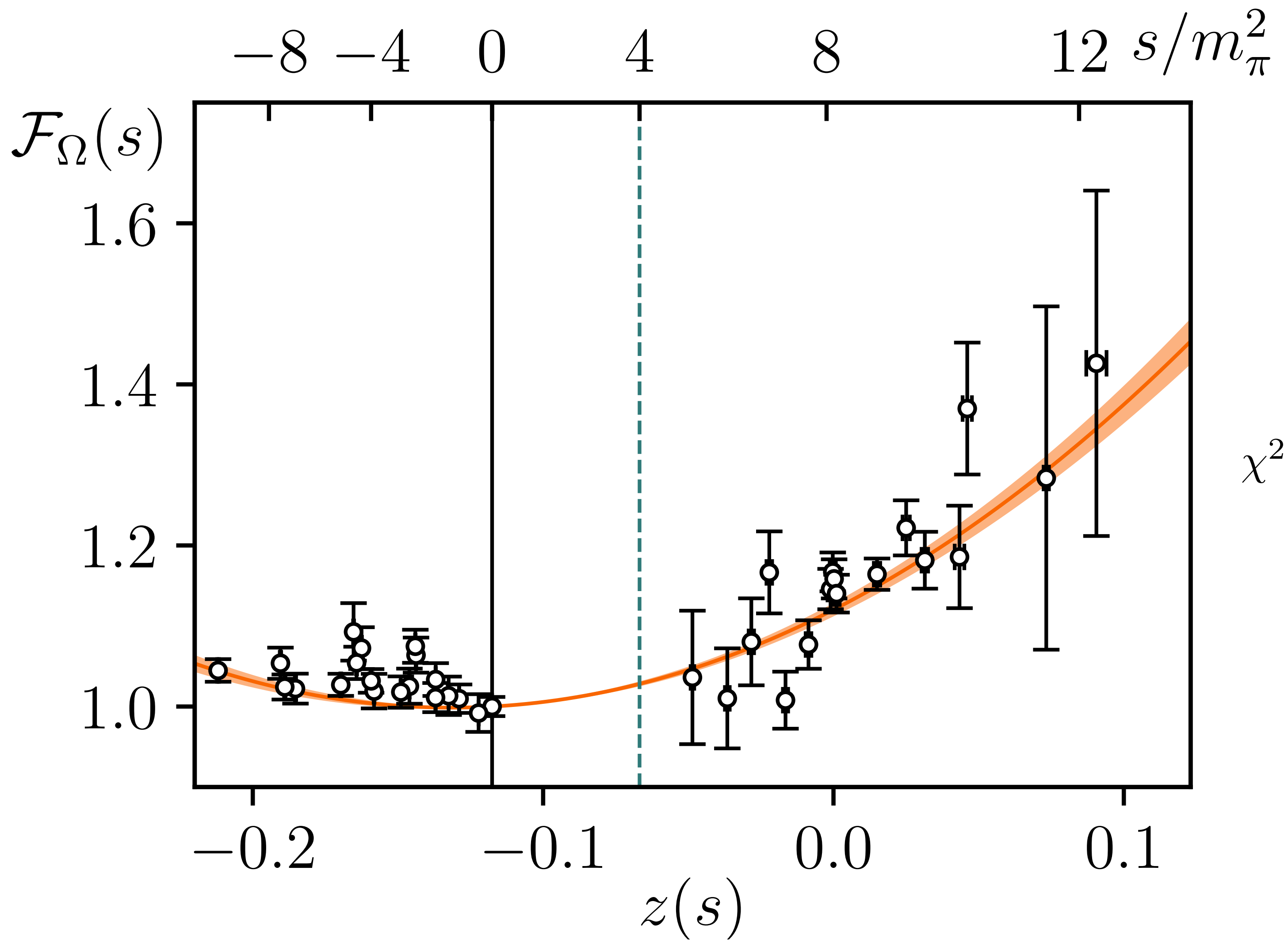
Elastic: $\quad K = \dfrac{g^2}{m^2 - s} + \gamma$

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n (z_c(s)^n - z_c(0)^n)$$



Fit Omnès



$\chi^2/n_{\text{dof}} = 85.4/(37-2) = 2.44$

$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n (z_c(s)^n - z_c(0)^n) \qquad z_c(s) = \frac{\sqrt{s_c - s_0} - \sqrt{s_c - s}}{\sqrt{s_c - s_0} + \sqrt{s_c - s}}$$

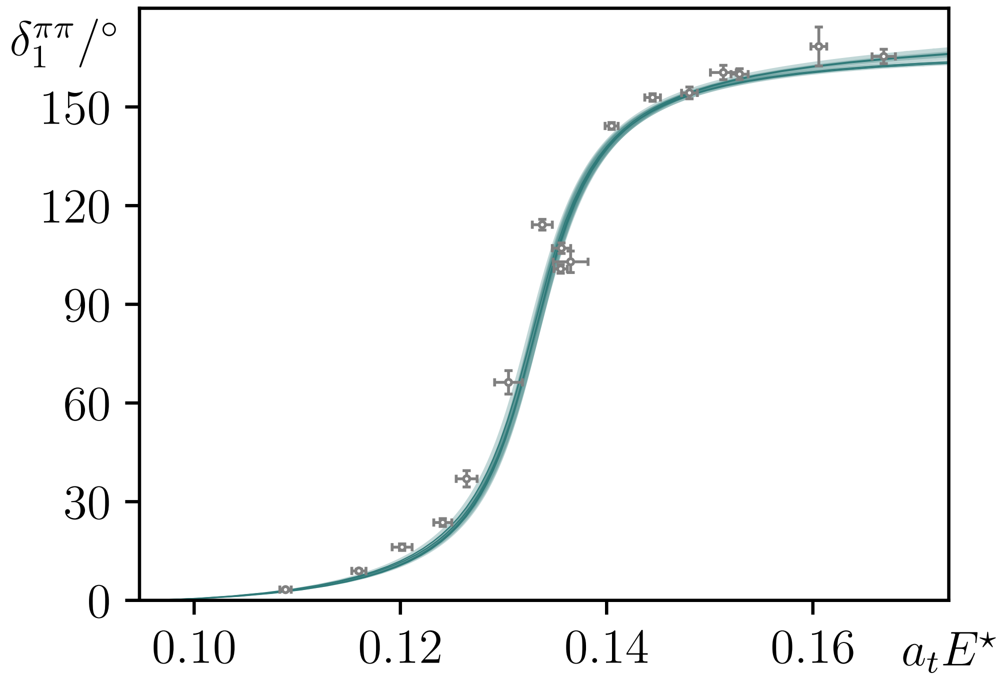
Elastic region

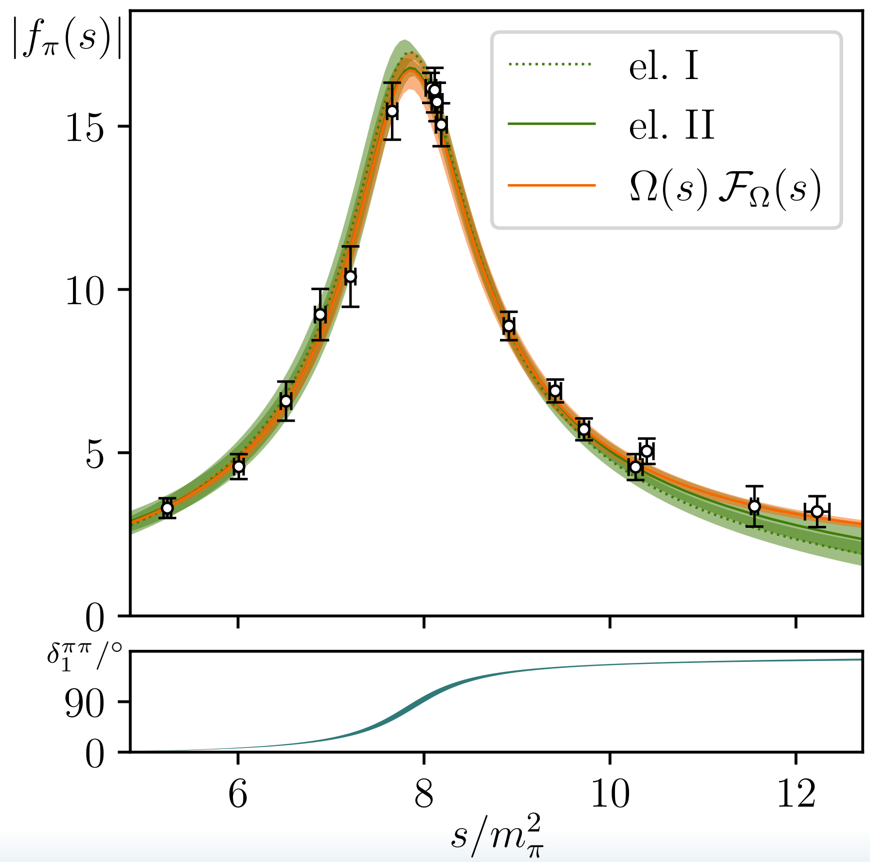
Parameterization	N_{pars}	χ^2/N_{dof}
Relativistic Breit-Wigner	2	1.23
$\mathcal{K} = \frac{g^2}{m^2-s}$ (Gounaris-Sakurai)	2	1.29
$\mathcal{K} = \frac{g^2}{m^2-s} + \gamma$ (With $-i\rho$ phase space)	3	1.23
$\mathcal{K} = \frac{g^2}{m^2-s} + \gamma$	3	1.21

$$\mathcal{M}(s) = \frac{16\pi}{\frac{1}{(2k^\star)^2}\mathcal{K}^{-1}(s) - I_{\text{CM}}(s)}$$

$$f_\pi(s) = \frac{1}{k^{\star\,2}_{\pi\pi}}\mathcal{M}(s)\,\mathcal{F}(s)$$

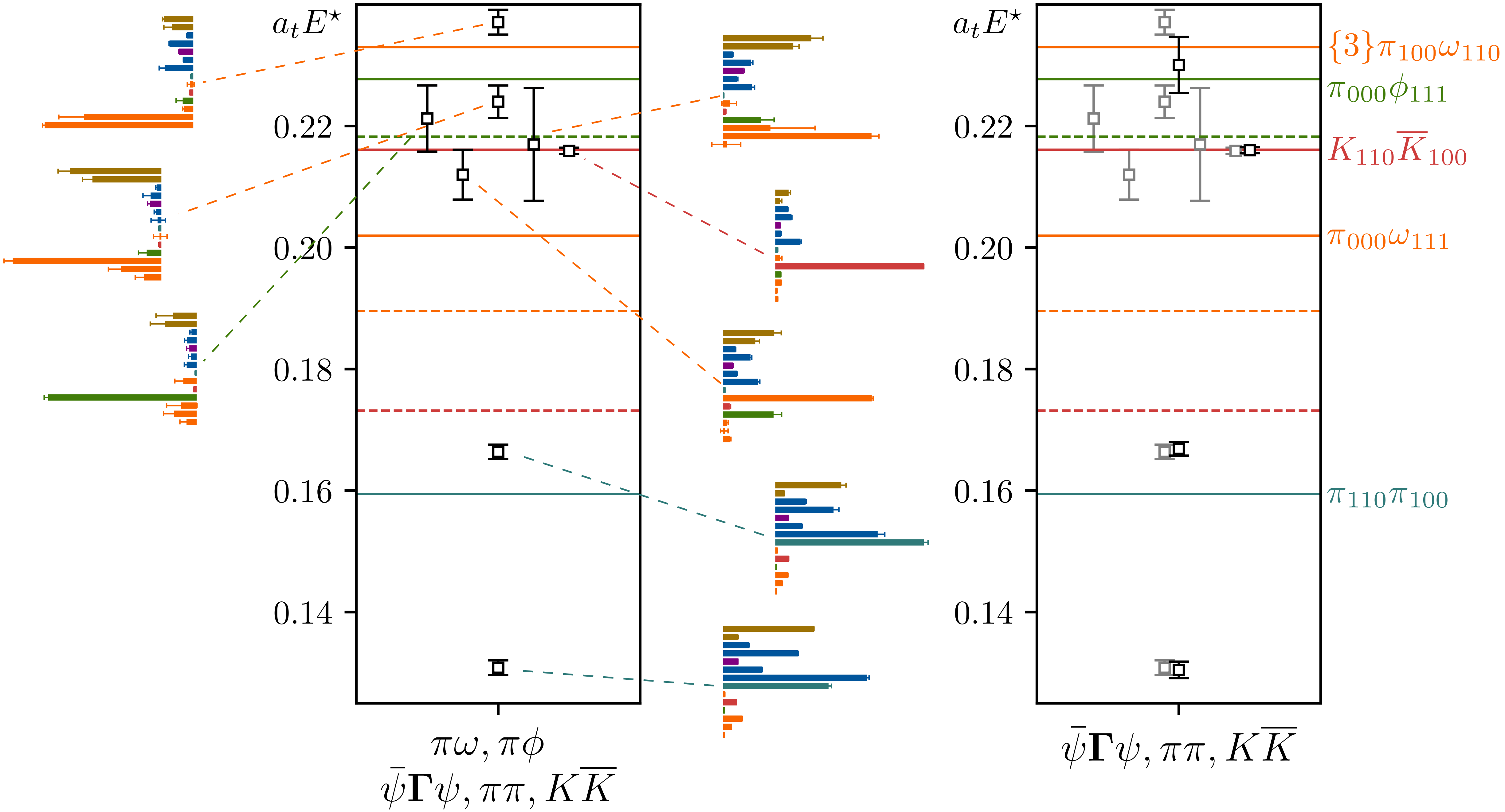
$$\mathcal{F}(s)/m_\pi^2 = \sum_{n=0} c_n \cdot \left(\frac{s-s_0}{s_0}\right)^n$$



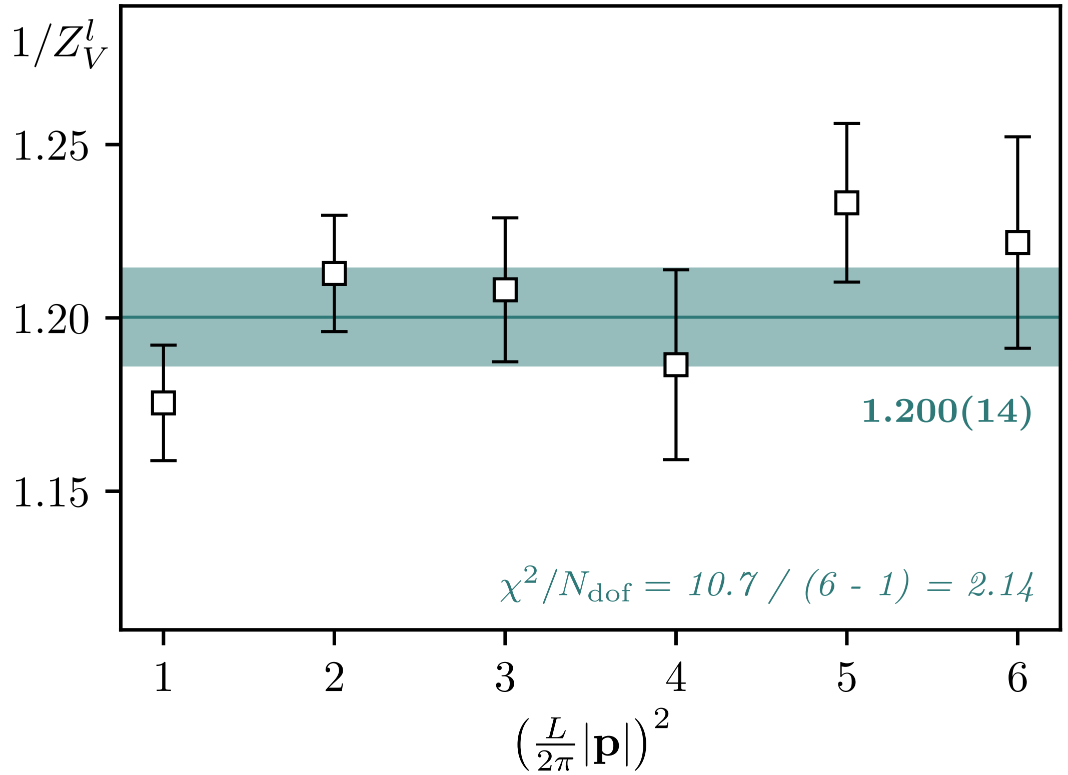


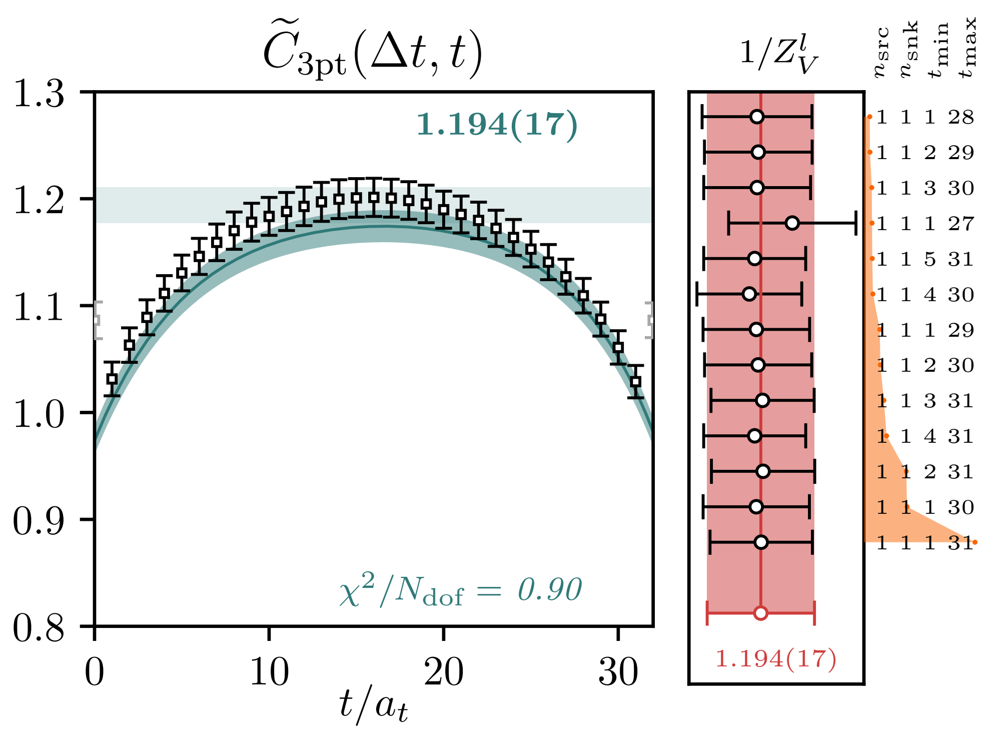
Spectrum operator dependence

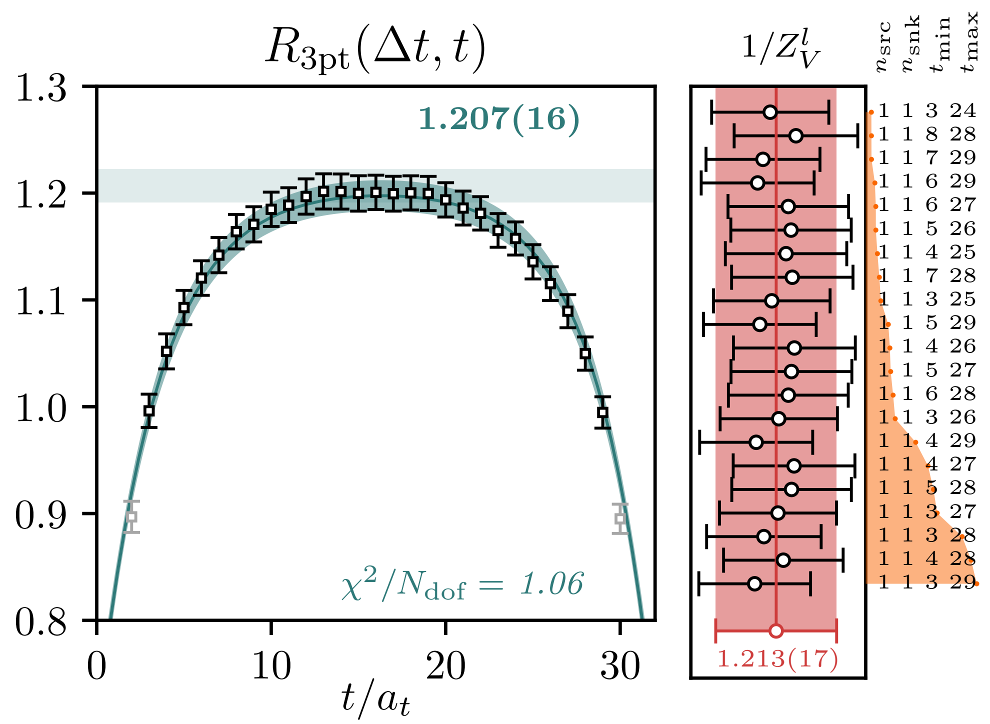
$[111]\,E_2$



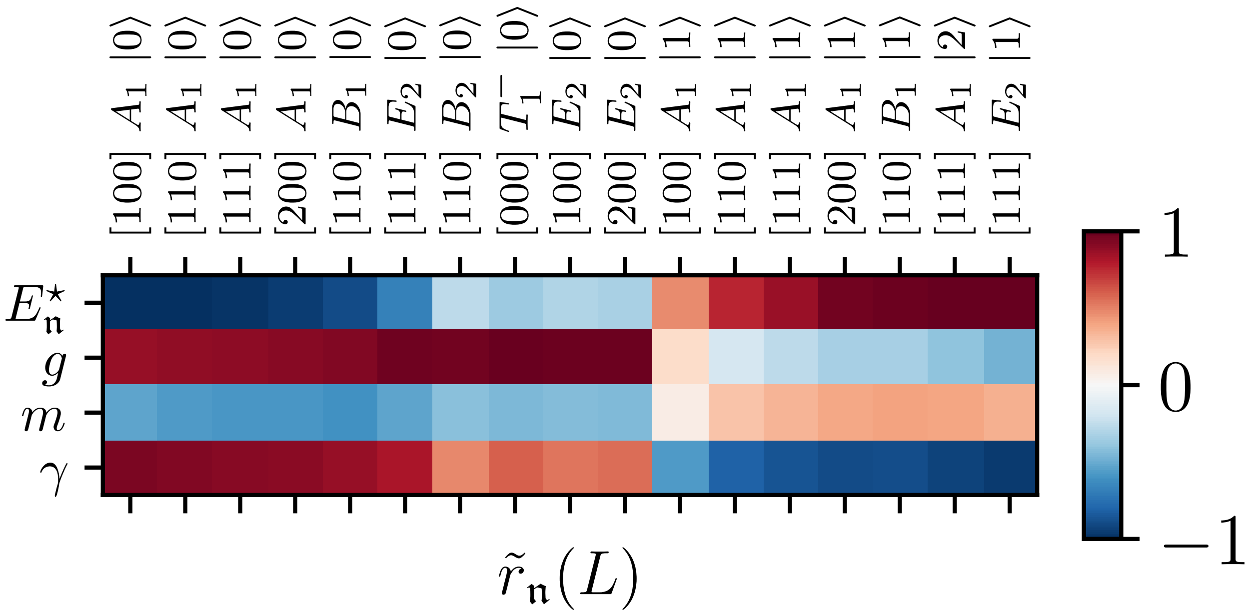
Current renormalization

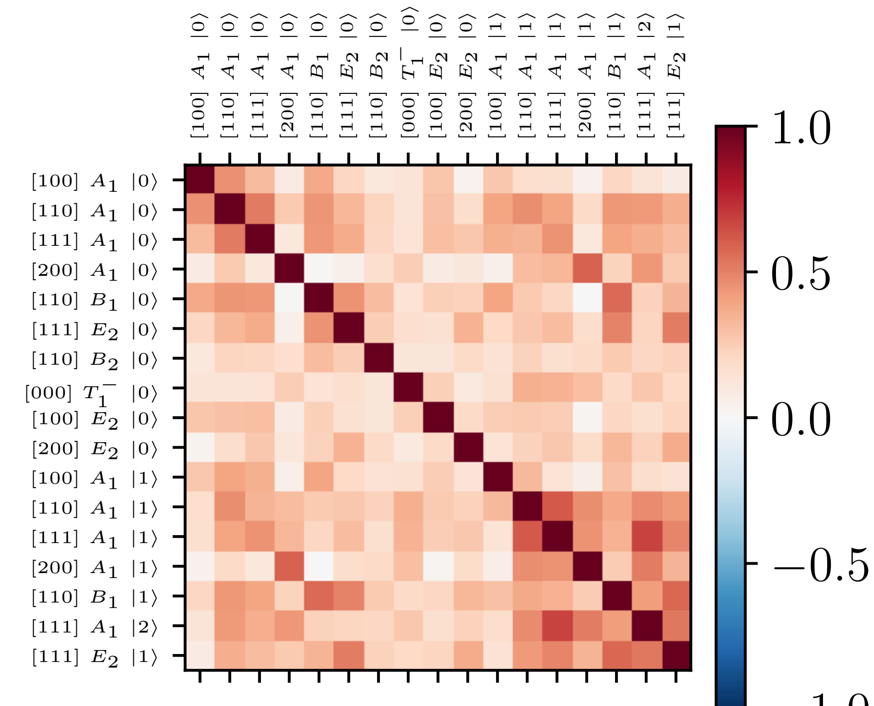






Correlation/Covariance LL factor



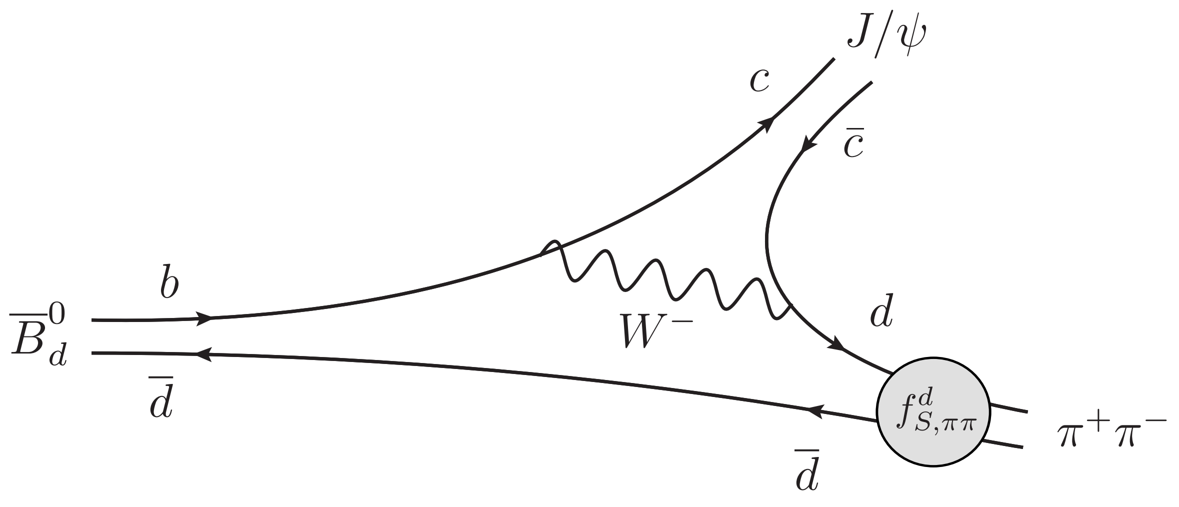


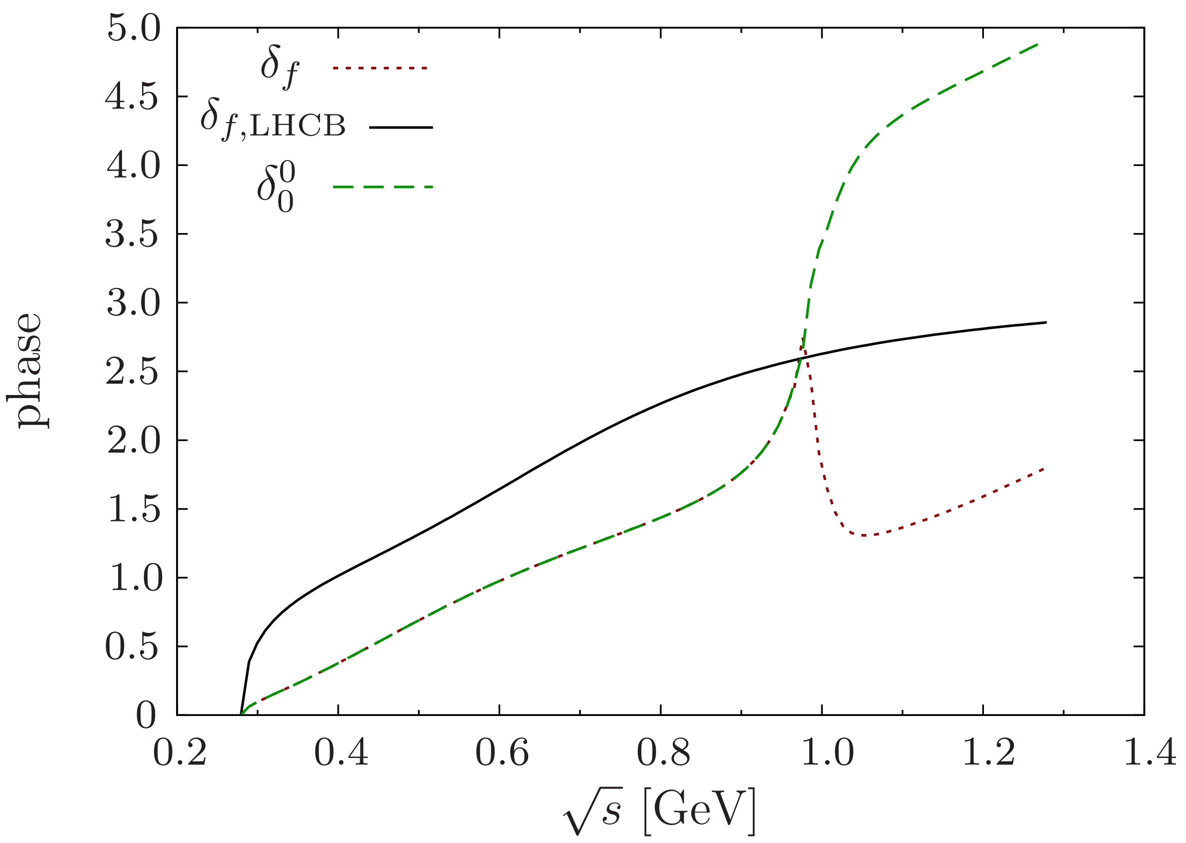
Correlation among energy levels

$$K = \frac{g^2}{m^2 - s} + \gamma$$

Coupled channel example: *S*-wave, isoscalar form factors

$$f_a = \mathcal{M}_{a,\pi\pi}\mathcal{F}_{\pi\pi} + \mathcal{M}_{a,K\overline{K}}\mathcal{F}_{K\overline{K}}$$





S-wave phaseshift and form factor phase comparison, adapted from [1]

The inelasticity of the $f_0(980)$ has a significant role in the coupled channel region.