Coupled-channel meson electroproduction analysis

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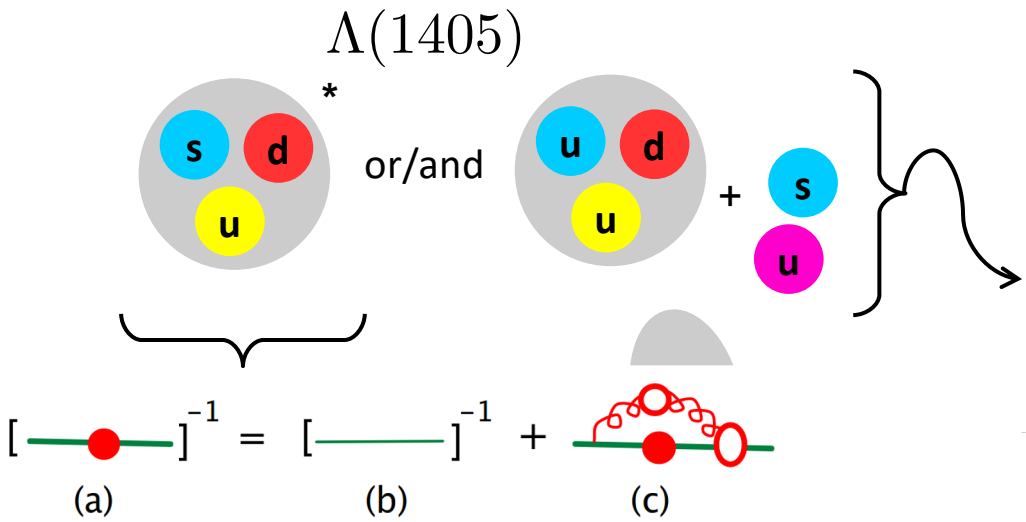
Degrees of freedom: Quarks or hadrons

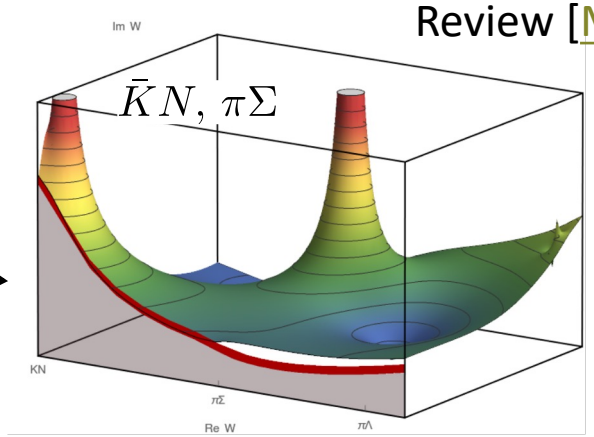
- Resonance review [Mai 2022]

QCD at low energies

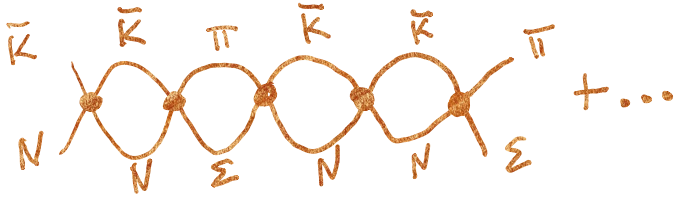
Non-perturbative dynamics	→ rich spectrum of excited states
How many states are there?	→ missing resonance problem (does it exist?)
What are they?	→ 2-quark/3-quark, hadron molecules, …

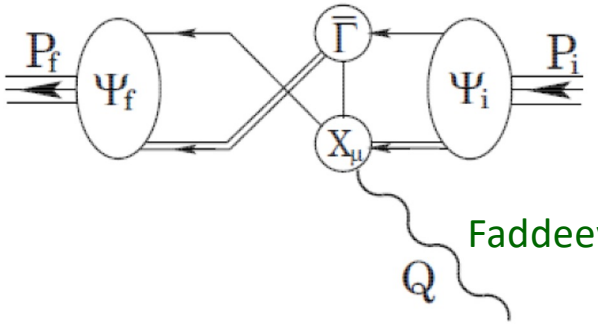


Review [Mai 2021]



Hadronic molecule

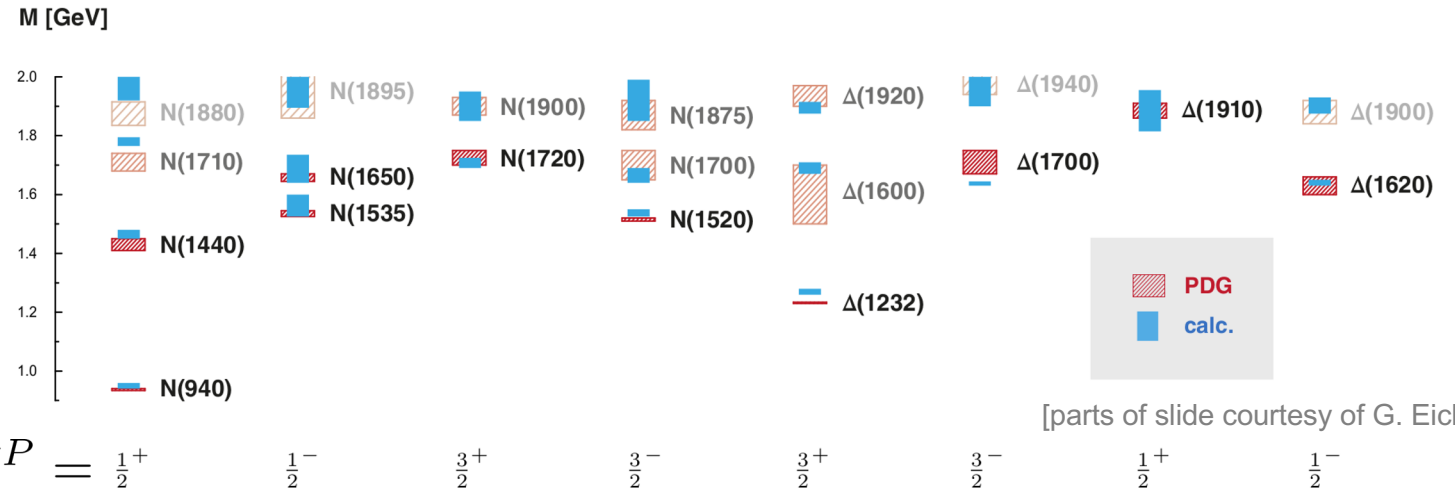




Faddeev Eq. / DSE (Binosi, Cloet, Chang, Eichman, Roberts,…);

Light baryons from diquark dynamics

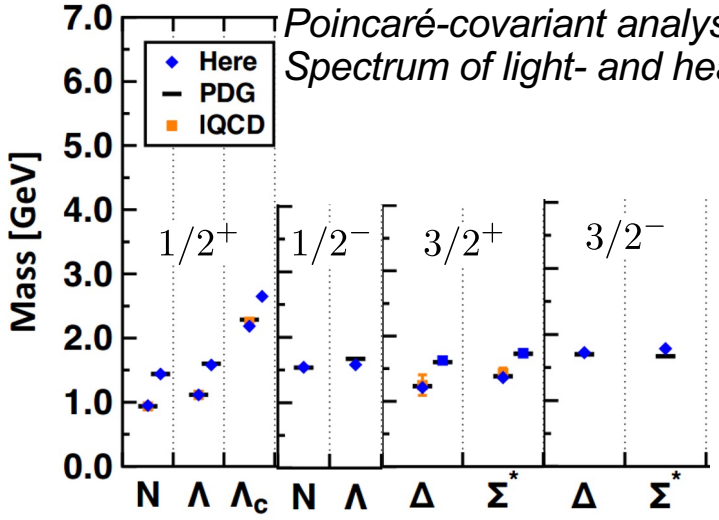
Quark-diquark with reduced pseudoscalar + vector diquarks: [Eichmann (2016)]

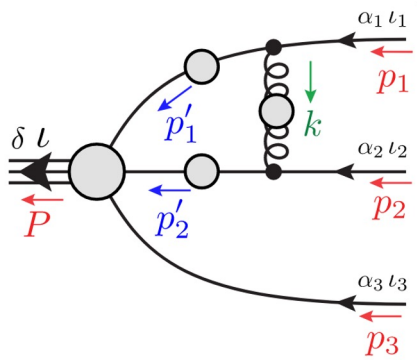


Not much of a missing resonance problem left

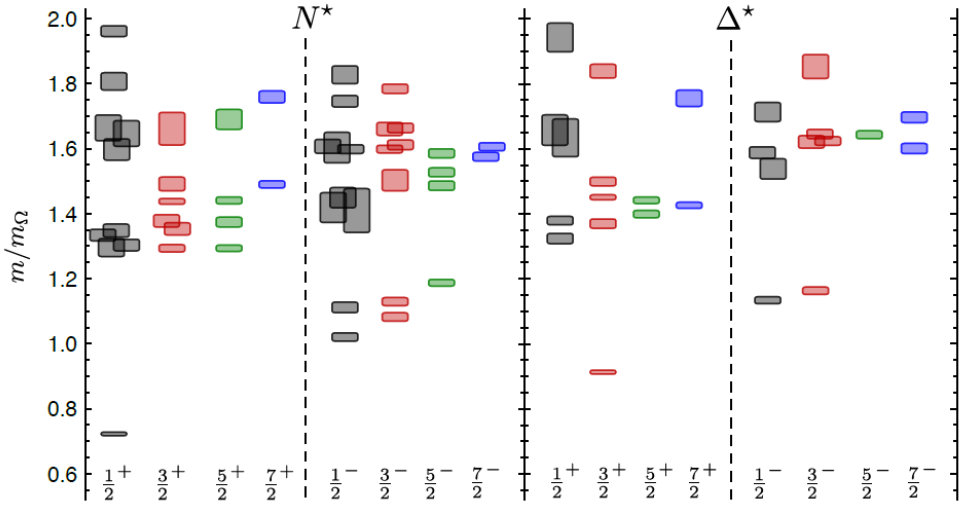
[parts of slide courtesy of G. Eichmann, Few Body 2018]

Poincaré-covariant analysis of heavy-quark baryons, [Qin 2018]
Spectrum of light- and heavy-baryons, [Qin 2019]



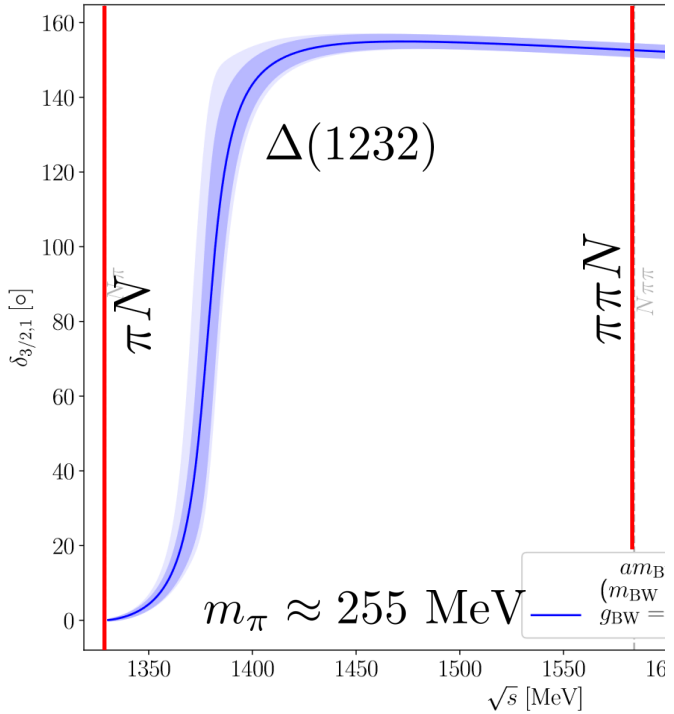


Lattice QCD for excited baryons



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

- Information on existence, width & properties of resonances requires
  - Meson-baryon interpolating operators
  - Detailed finite-volume analysis



[G. Silvi et. al., arXiv: 2101.00689]

See also: Bulava et al., [2208.03867]

How about $\pi\pi N$? 3B Dynamics?

Phenomenology of the baryon spectrum

Review by [Thiel, Afzal, Wunderlich 2022]

Dynamical coupled-channel approaches

[MD, M. Mai, J. Haidenbauer, T. Sato, upcoming review]

- ANL-Osaka (former: EBAC) [Kamano et al.]
- Dubna-Mainz-Taipei model [Tiator]
- Jülich-Bonn(-Washington) [Rönchen]
- …
- Characteristics:
  - Direct fit to data (pion & photon-induced)
  - Simultaneous fit to data of different final states
  - Integral scattering equation as needed for proper treatment of three-body channels ($\pi\pi N$) & inclusion of lefthand cut

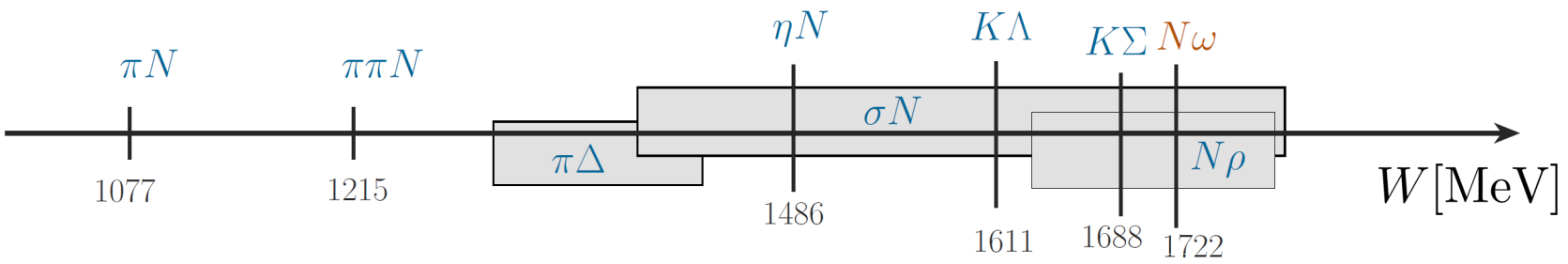
Note: Only a sub-class of analysis efforts; see, e.g., Bonn-Gatchina group K-matrix approach

JBW DCC approach (Jülich-Bonn-Washington)

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq\; q^2\; \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

- channels ν, μ, γ:



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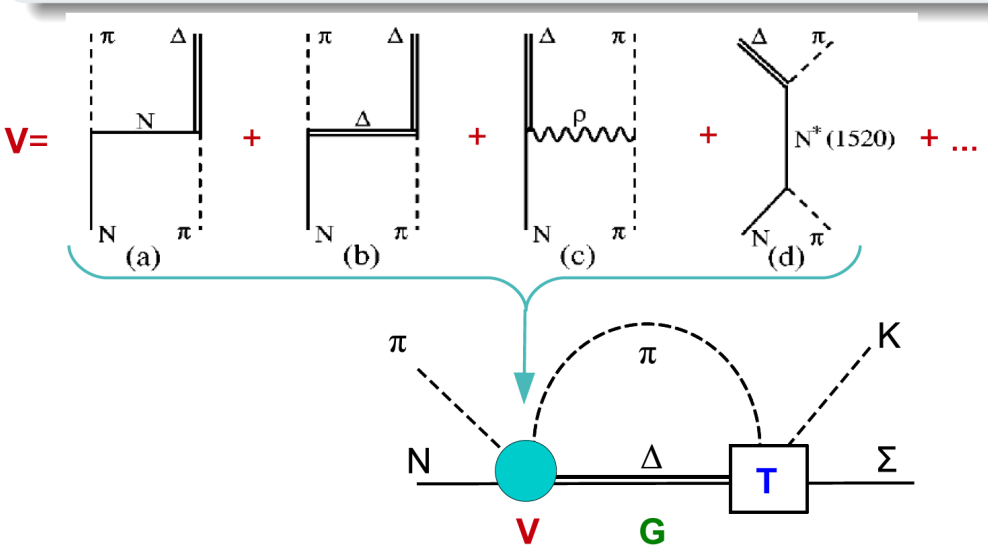


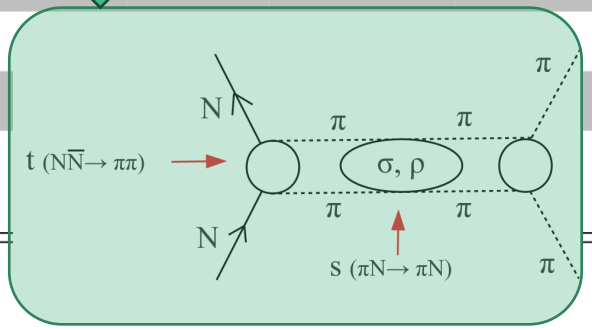
- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- contact terms

Transitions in s, t, and u-channels

- 21 s-channel exchanges (resonance)
- Contact terms
- t and u-channel exchanges:

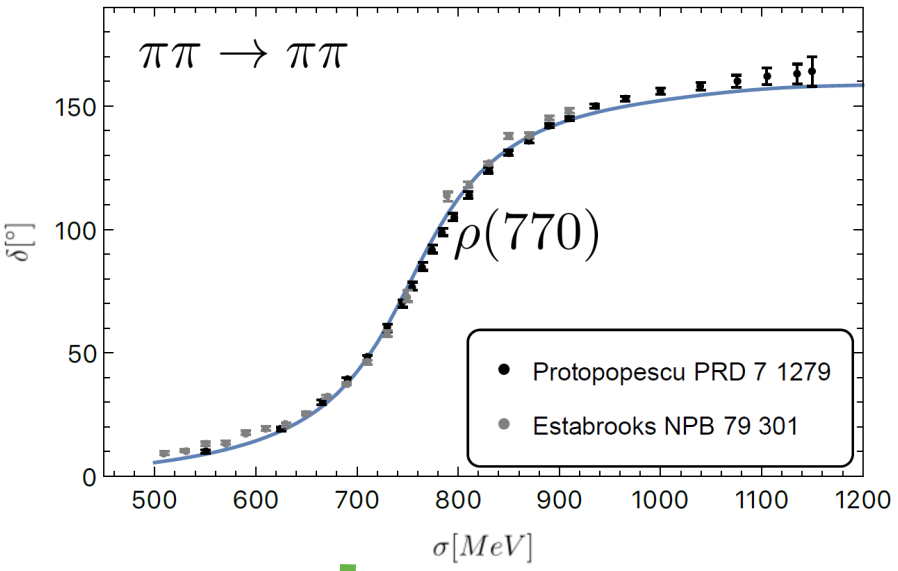
[Yu-Fei Wang 2022]

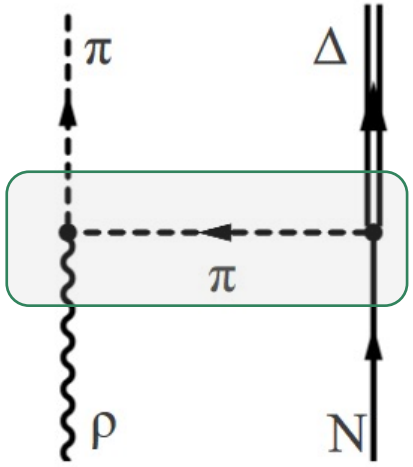
μ	πN	ηN	$K\Lambda$	$K\Sigma$	ωN	$\pi\Delta$	σN	ρN
πN	$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$, N, Δ	a_0, N	K^*, Σ, Σ^*	K^*, Λ, Σ, Σ^*	ρ, N	ρ, N, Δ	π, N	π, ω, a_1, N, Δ, C
ηN		N, f_0	K^*, Λ	K^*, Σ, Σ^*	ω, N			
$K\Lambda$			ω, f_0, ϕ, Ξ, Ξ^*	ρ, a_0, Ξ, Ξ^*	K, K^*, Λ			
$K\Sigma$				ρ, ω, ϕ, f_0, a_0, Ξ, Ξ^*	K, K^*, Σ, Σ^*			
ωN					σ, N			
$\pi\Delta$						ρ, N, Δ	π	π, N
σN							σ, N	
ρN								ρ, N, Δ, C

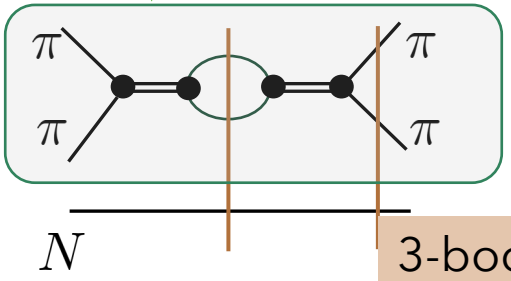


Three-body channels $\sigma N, \pi\Delta, \rho N$

- Resonant sub-channels
- Fit 2→2 amplitude to 2→ 2 scattering data
- Include as sub-channel in 3-body amplitude:
- 3-body unitarity: Requires, e.g.



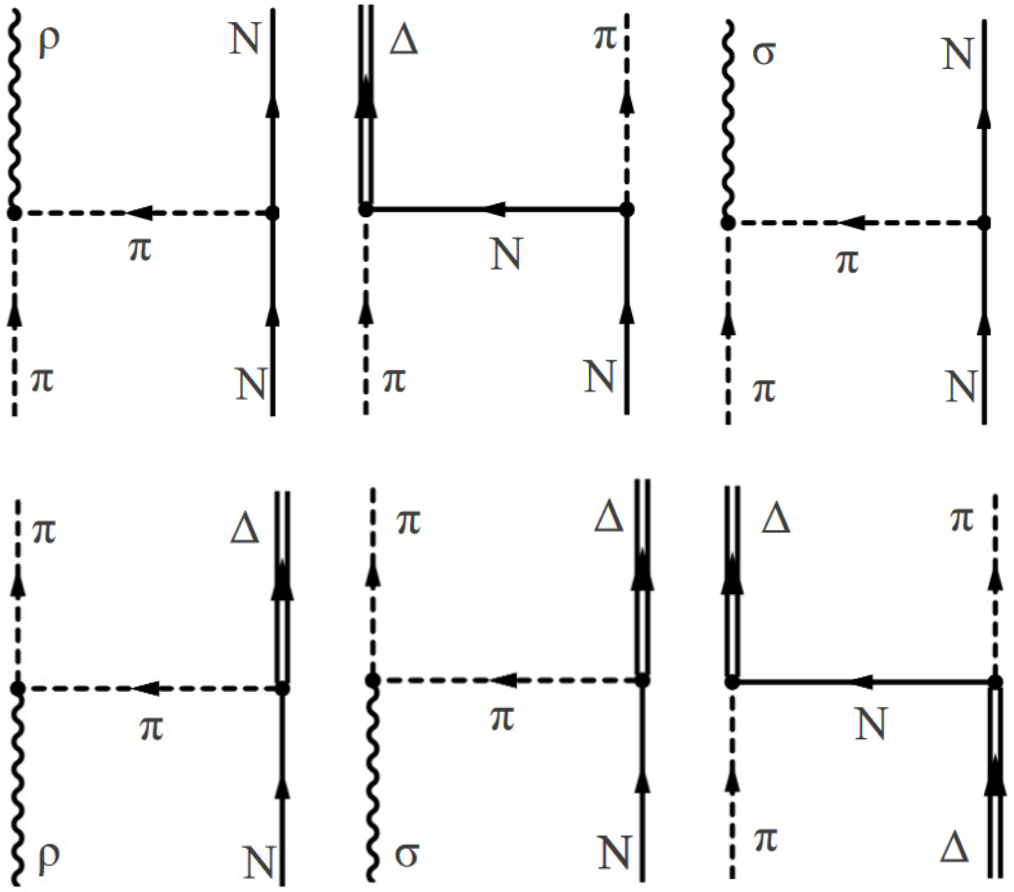




3-body cuts in
- Propagation
- exchange

2→ 3 and 3→3 body unitarity

- Unitarity requires certain transition amplitudes



2→3

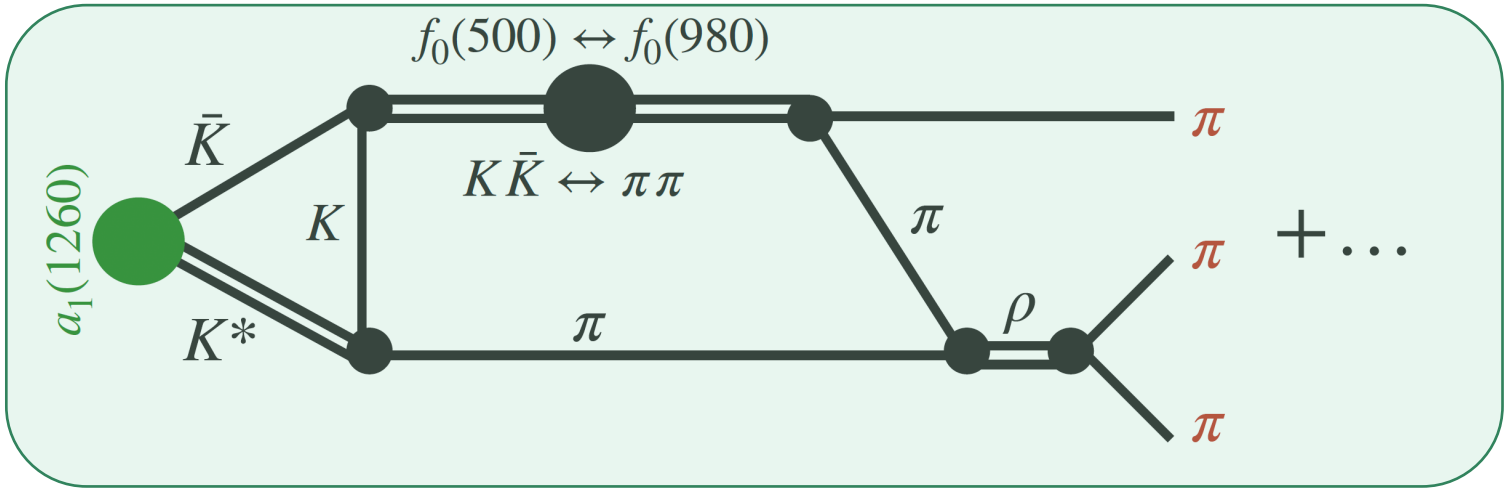
Unitarity requires formulation of scattering via integral equation
→ ANL-Osaka & JB(W)

3→3

Unitary amplitudes for meson analysis

[Y. Feng, F. Gil, R. Molina, M. Mai, V. Shastry, A. Szczepaniak, et al.]

- Talk tomorrow morning



EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

JB: Data base

- $\pi N \to X$: $>$ **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

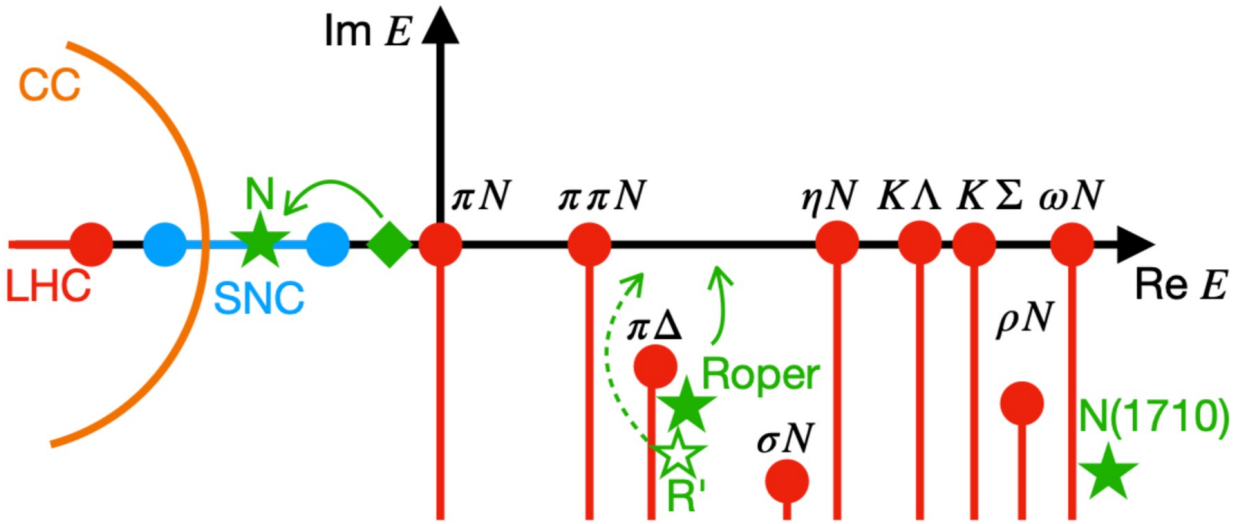
New: $\pi N \to \omega N$ [Yu-Fei Wang 2022]
New: Updated fit (**C. Schneider** talk at 10:15 am)

Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

New interface [https://jbw.phys.gwu.edu/]

Partial-Wave Analytic structure

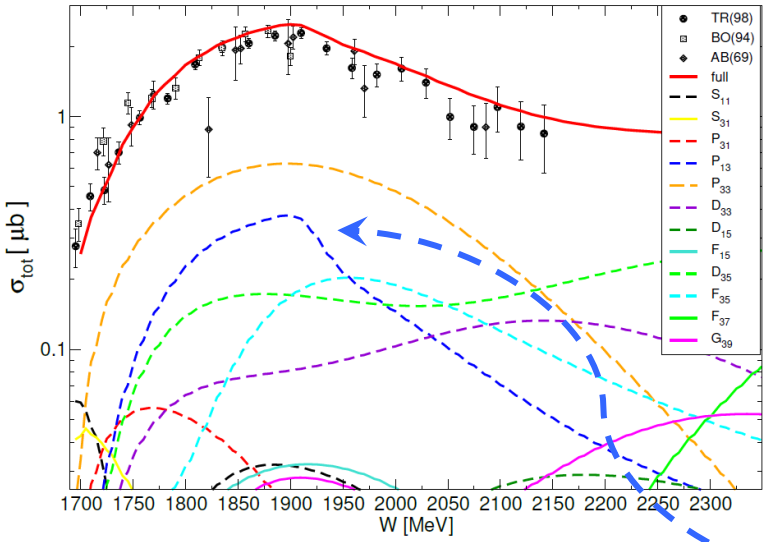
- Branch points indicate thresholds
- Partial-wave amplitudes have more cuts than plane-wave amplitude
- Example: The structure of the P11 amplitude

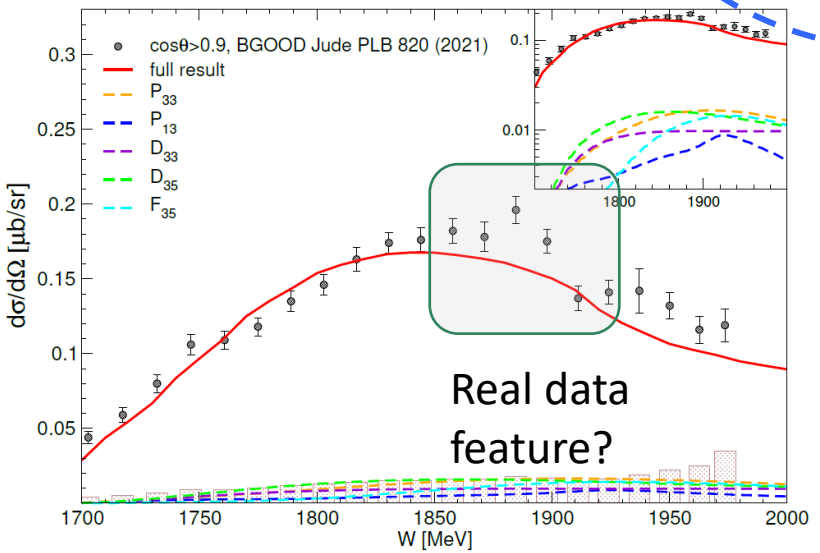


Resonances in $K\Sigma$ photoproduction

$\gamma p \to K^+\Sigma^0$

- [D. Roenchen et al. (EPJA 2022)]
- [Webpage all results]





Similarly: $K^0\Sigma^+$

- dominant partial waves: $I = 3/2$

Exception: P_{13} partial wave ($I = 1/2$):

$N(1720)\ 3/2^+$ ****	Re E_0 [MeV]	$-2\mathrm{Im}\ E_0$ [MeV]	$\frac{\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1726	185	5.9	82
2017	1689(4)	191(3)	0.6(0.4)	26(58)
PDG 2021	1675 ± 15	250^{+150}_{-100}	—	—

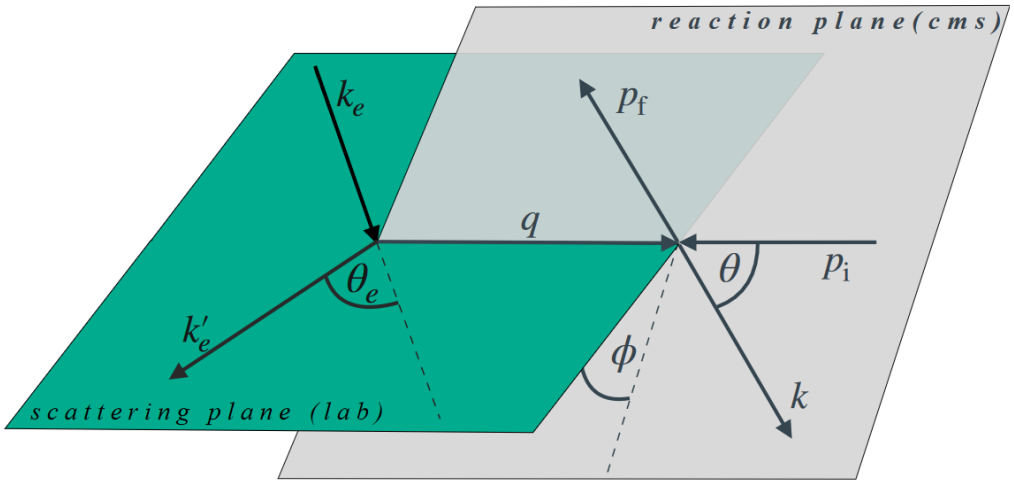
$N(1900)\ 3/2^+$ ****	Re E_0 [MeV]	$-2\mathrm{Im}\ E_0$ [MeV]	$\frac{\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1905	93	1.3	−40
2017	1923(2)	217(23)	10(7)	−34(74)
PDG 2021	1920±20	150±50	4±2	110±30

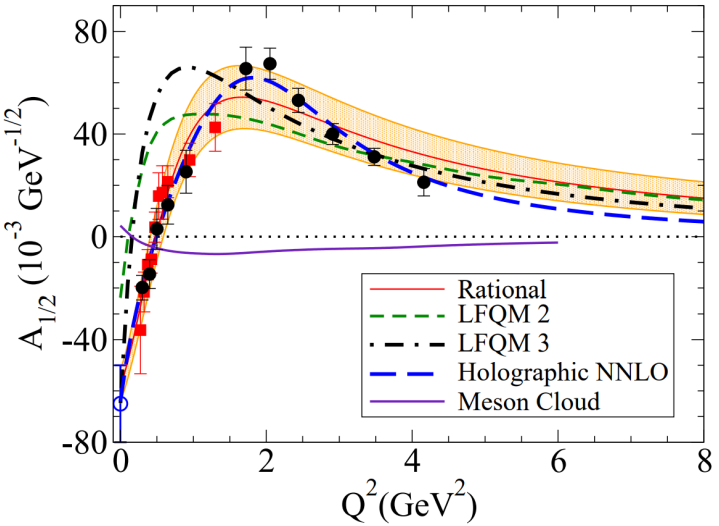
- drop in cross section ("cusp-like structure") due to $N(1900)3/2^+$

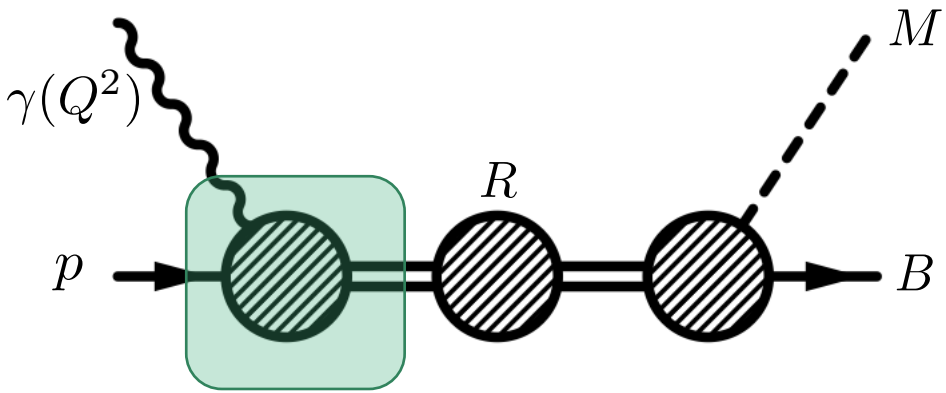
$N(1535)\ ½^-$ ****	Re E_0 [MeV]	$-2\mathrm{Im}\ E_0$ [MeV]
2022	1504(0)	74 (1)
2017	1495(2)	112(1)
PDG 2021	1510 ± 10	130 ± 20

New, wide dynamically generated states in $J^P=3/2^-$

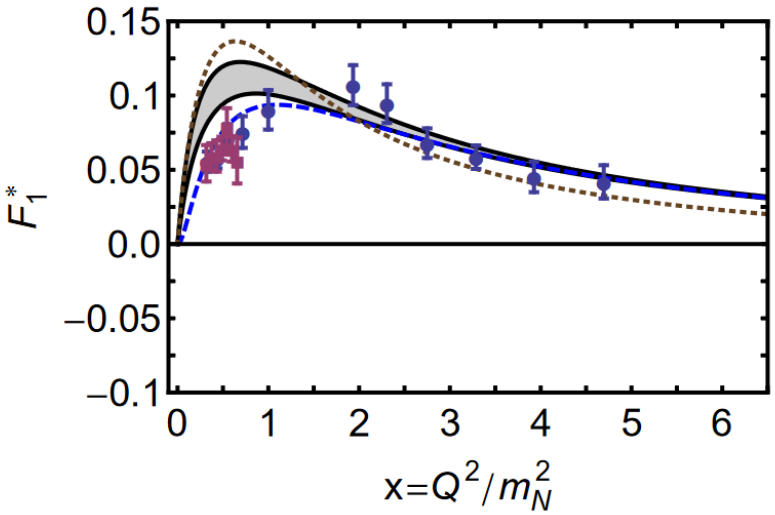
Electroproduction reveals resonance structure







Reaction-independent transition form factor at resonance pole



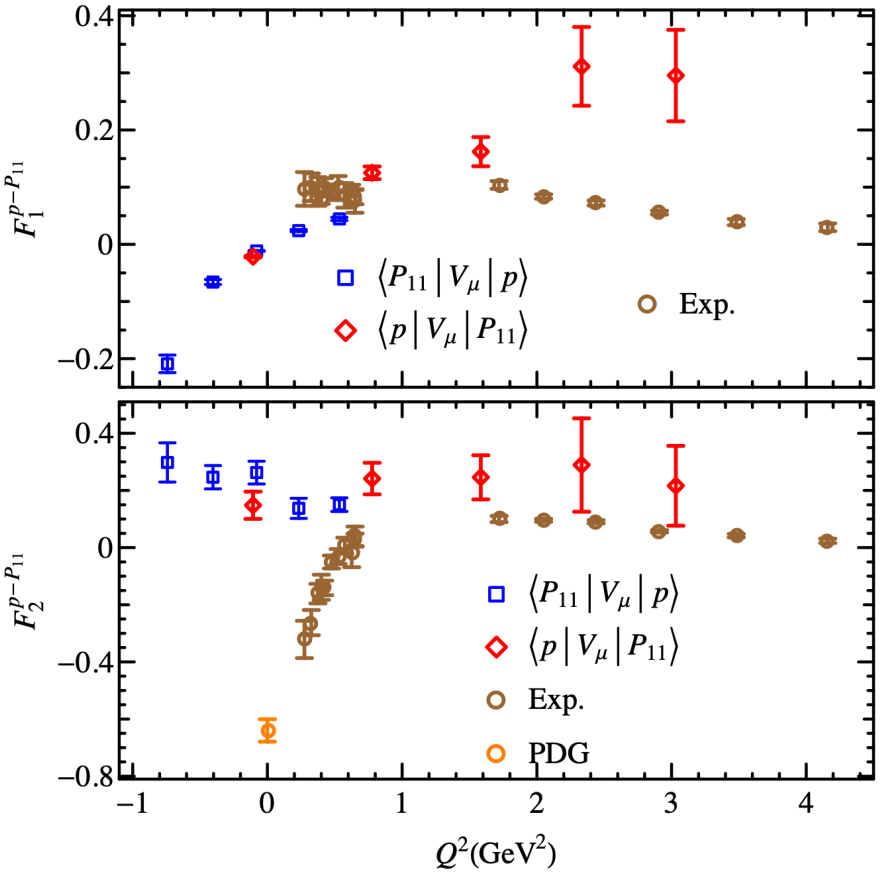
Proton-Roper Transition [Burkert] [Segovia]

Baryon TFFs from Lattice

Meson sector: See J. Dudek talk Tuesday

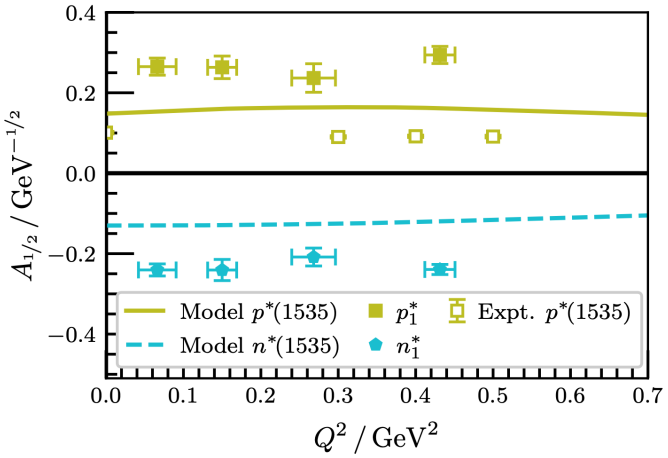
- Proton-Roper

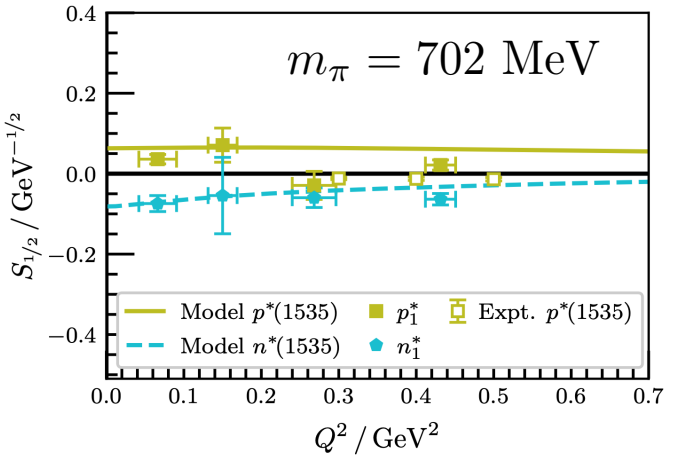
  [H.-W. Lin et al. (2008)]



- N(1535)1/2$^-$

  [F.-M. Stokes et al. (2024)]



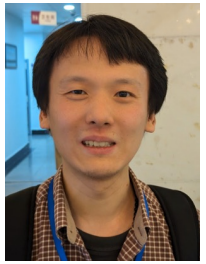


Electroproduction Analysis efforts

- **MAID**: electroproduction of pions, eta mesons, and kaons in separate approaches [Tiator 2007]
- **JM (JLab)** approach: single-pion analysis, double pion analysis [Mokeev, PRC 2023]
- **ANL-Osaka:** Single-pion electroproduction, using multi-channel model. [Kamano, Lee, Nakamura, Sato, 2016]
- **JBW**: simultaneous analysis of multiple electroproduction final states, using multi-channel model

J. Hergenrather

M. Mai



Yu-Fei Wang

Coupled-Channel Electroproduction

First coupled-channel electroproduction analysis of different final states (2020 -)

Formalism

- Photoproduction solution as constraint
- Constraints from (Pseudo)-threshold:

$$\begin{aligned}
(E^I_{l+}, L^I_{l+}) &\rightarrow k^l q^l \quad (l \geq 0) \\
(M^I_{l+}, M^I_{l-}) &\rightarrow k^l q^l \quad (l \geq 1) \\
(L^I_l) &\rightarrow kq \quad (l = 1) \\
(E^I_{l-}, L^I_{l-}) &\rightarrow k^{l-2} q^l (l \geq 2)
\end{aligned}$$

$$k = |\mathbf{k}| = \frac{\sqrt{\left((W - M_N)^2 + Q^2\right)\left((W + M_N)^2 + Q^2\right)}}{2W}$$

$$q = |\mathbf{q}| = \frac{\sqrt{\left((W - M_N)^2 - M_m^2\right)\left((W + M_N)^2 - M_m^2\right)}}{2W}$$

- Siegert's theorem

$$\frac{E_{l_+}}{L_{l_+}} \rightarrow 1, \qquad \frac{E_{l_-}}{L_{l_-}} \rightarrow \frac{-l}{l-1}$$

at pseudo-threshold

Amaldi, Fubini, Furlan,
Springer Tracts Mod. Phys. 83, 1 (1979)
Tiator, Few-body Systems 57, 1087 (2016)

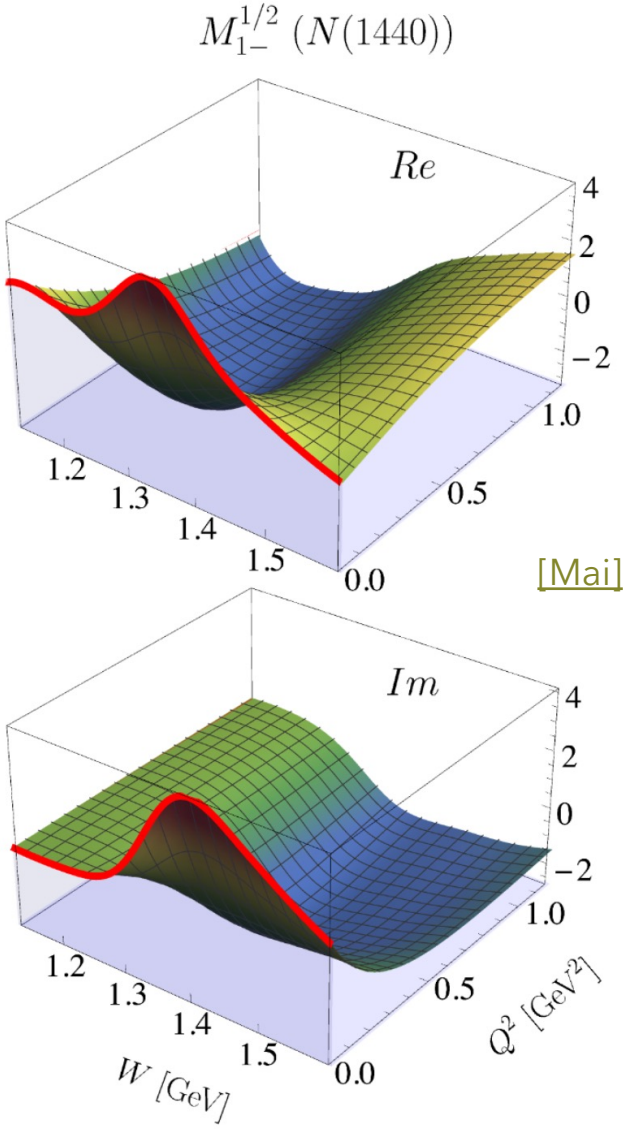
- Watson's theorem, multi-channel unitarity

$$M_{\mu\gamma^*}(q, W, Q^2) = V_{\mu\gamma^*}(q, W, Q^2) + \sum_\kappa \int dp p^2 T_{\mu\kappa}(q, p, W) G_\kappa(p, W) V_{\nu\gamma^*}(p, W, Q^2)$$

$$V_{\mu\gamma^*}(p, W, Q^2) = \alpha^{NP}_{\mu\gamma^*}(p, W, Q^2) + \sum_i \frac{\gamma^a_{\mu;i}(p)\gamma^c_{\gamma^*;i}(W, Q^2)}{W - m^b_i}$$

Resonances, background, channels have independent Q^2 dependence for flexible fit

Photoproduction boundary



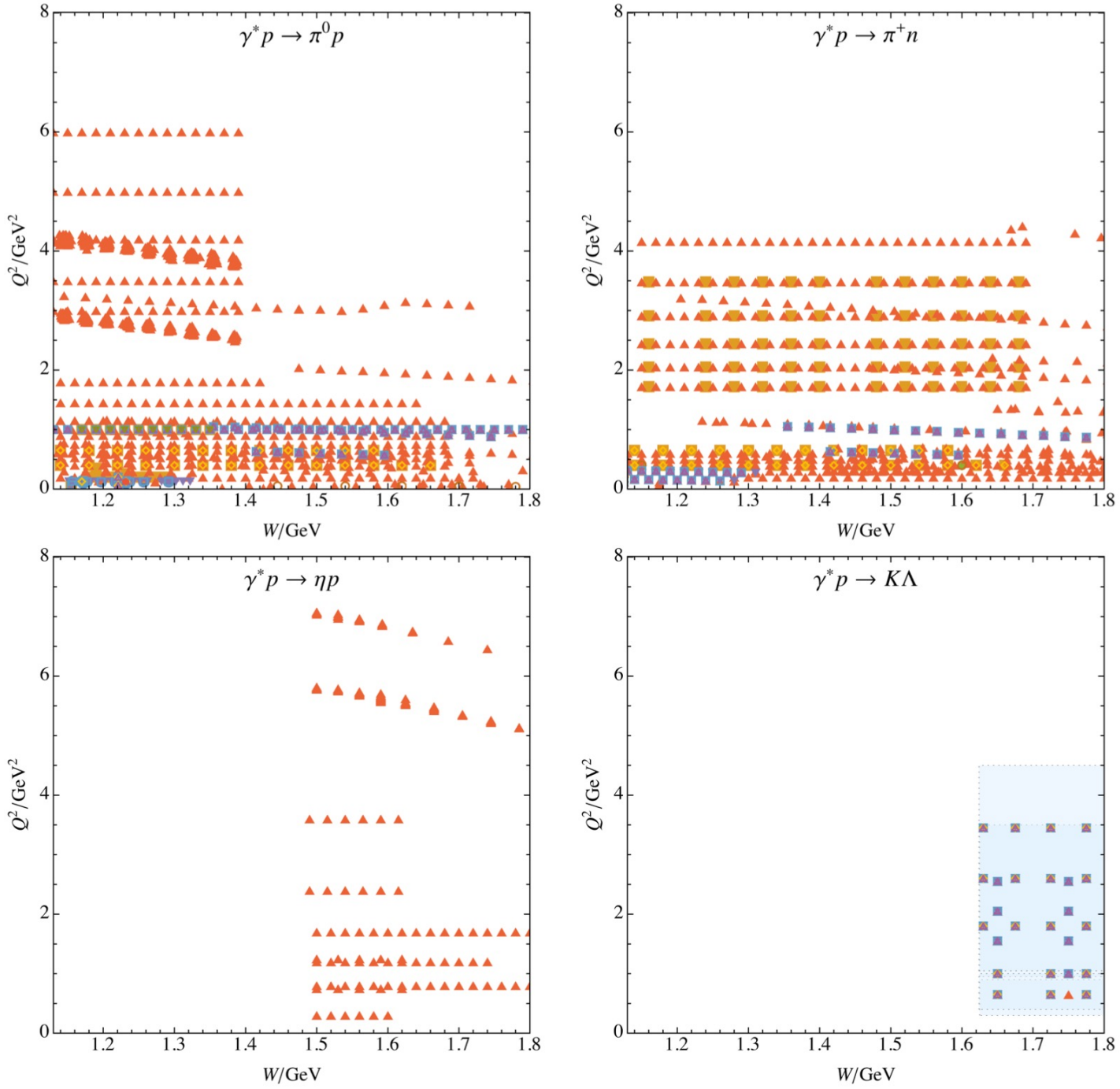
Photoproduction included as boundary

[Mai]

JBW Electroproduction data base

Type	$N^{\pi^0 p}_{\text{data}}$	$N^{\pi^+ n}_{\text{data}}$	$N^{\eta p}_{\text{data}}$	$N^{K\Lambda}_{\text{data}}$
ρ_{LT}	45	–	–	–
$\rho_{LT'}$	2768	5068	–	–
σ_L	–	2	–	–
$d\sigma/d\Omega$	48135	44266	3665	2055
$\sigma_T + \epsilon\sigma_L$	384	182	–	204
σ_T	30	2	–	–
σ_{LT}	373	138	–	204
$\sigma_{LT'}$	214	208	–	156
σ_{TT}	327	123	–	204
K_{D1}	1527	–	–	–
P_Y	–	2	–	–
Total	53804	49989	3665	2823

- Data base grown over decades with recent input mostly by CLAS, MAMI.
- Far from complete: Kinematic gaps & consistency issues. Need to combine information from different (W, Q^2) regions
- Need to combine information from simultaneous analysis of different final states ($\pi N/\eta N/K Y/\pi\pi N, \ldots$) to extract resonance helicity couplings


$\gamma^* p \to \pi^0 p$
$\gamma^* p \to \pi^+ n$
$\gamma^* p \to \eta p$
$\gamma^* p \to K\Lambda$
Q^2/GeV^2
W/GeV
0
2
4
6
8
1.2
1.3
1.4
1.5
1.6
1.7
1.8

Fit details: Weighted vs. unweighted χ^2

- Meson production data bases are heterogeneous:
  - A few polarization measurements with large error bars (small weight in χ^2)
  - Many cross section data with smaller error bars (large weight in χ^2)
  - … but those **few** polarization possess **great** power to discriminate solutions
- Introduce **weighted** vs. **unweighted** χ^2:

$$\chi^2_{\rm wt} = \sum_{j\in\{\pi^0 p,\pi^+ n,\eta p\}} \frac{N_{\rm all}}{3N_j} \sum_{i=1}^{N_j} \left(\frac{\mathcal{O}_{ji}^{\rm exp} - \mathcal{O}_{ji}}{\Delta_{ji}^{\rm stat} + \Delta_{ji}^{\rm syst}} \right)^2 .$$

$$\chi^2_{\rm reg} = \sum_{i=1}^{N_{\rm all}} \left(\frac{\mathcal{O}_{i}^{\rm exp} - \mathcal{O}_{i}}{\Delta_{i}^{\rm stat} + \Delta_{i}^{\rm syst}} \right)^2$$

- Quote results for both cases

Fit Strategies (πN)

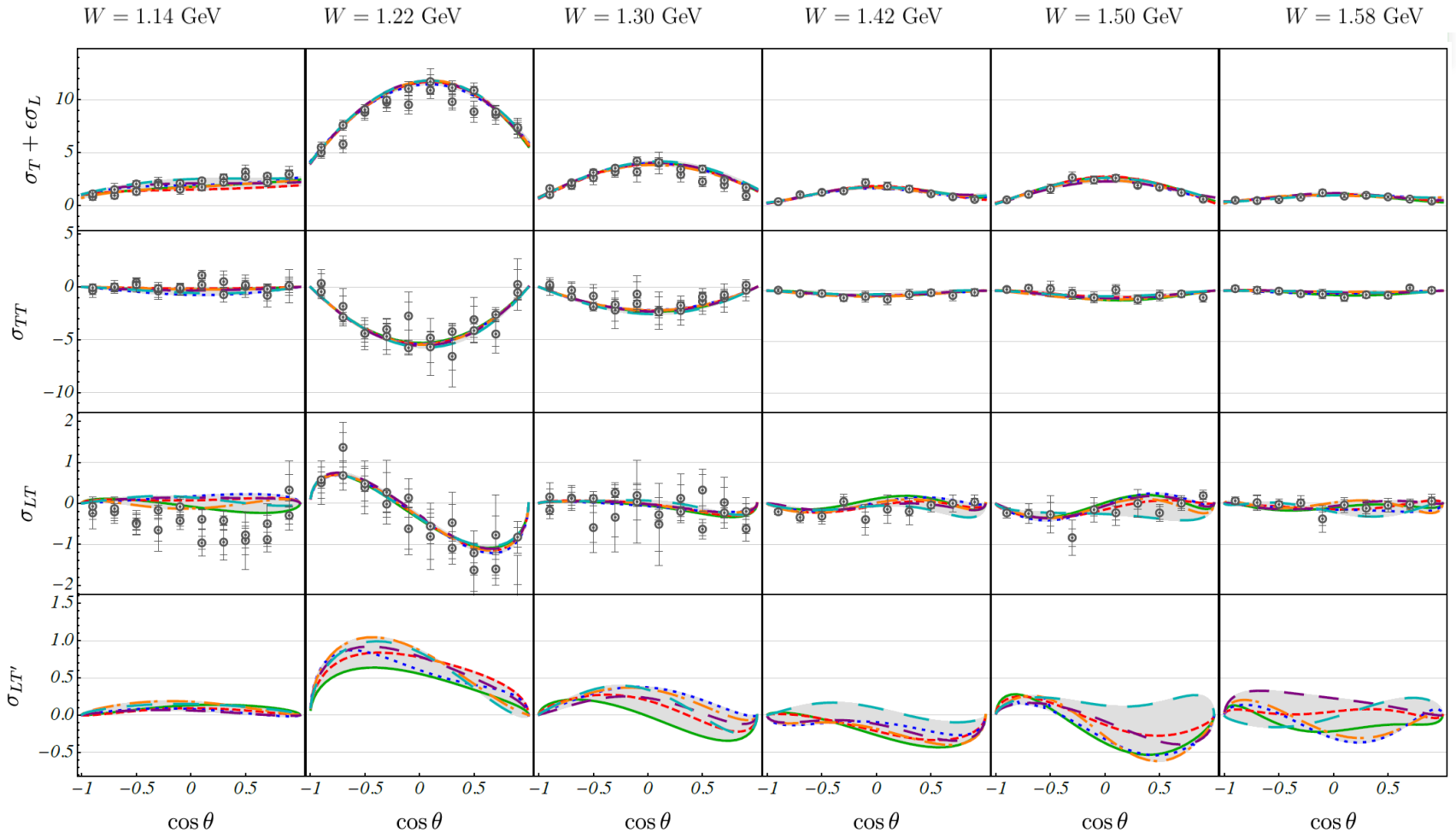
[M. Mai et al, 2021]

- Different fit strategies for $N \approx 85{,}000$ data in $\gamma^* N \to \pi N, \eta N$:
  - Sequential S → S+P → S+P+D waves;
  - Subsets of data until full data set reached
  - Simultaneous fit all parameters (209) set to zero without any (!) guidance
  - Extend data range from $0 < Q^2 < 4$ Gev^2 to $0 < Q^2 < 6$ Gev^2 to check for stability

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T + \epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		χ^2_{dof}
	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	1.77
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	1.69
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	1.81
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	1.78
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	1.81
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	1.78

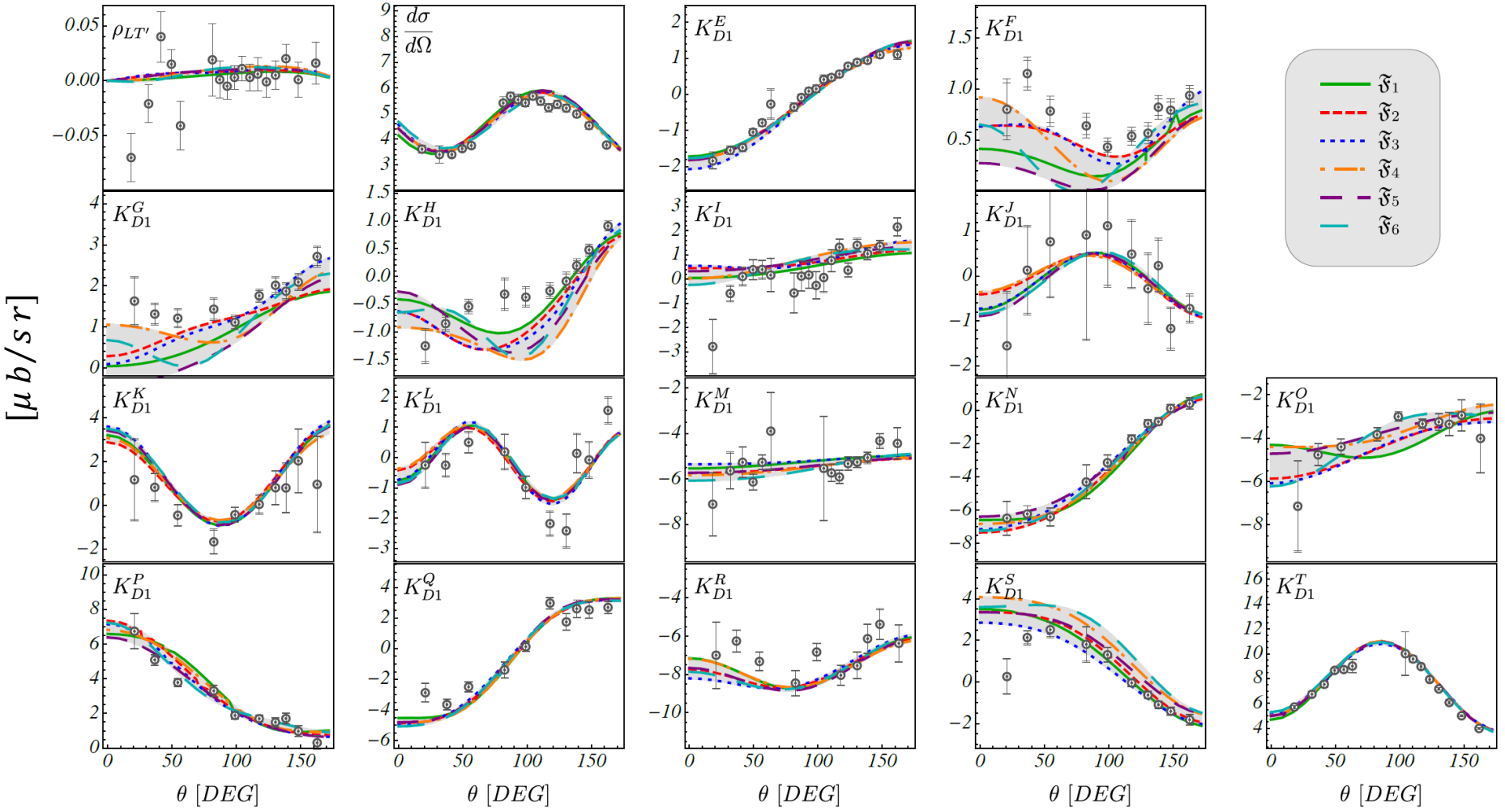
χ^2

Structure functions $\pi^0 p$ (not fitted)



Data: CLAS, Phys. Rev. C (2003) 0301012 [nucl-ex], Phys. Rev. Lett. (2002) 0110007 [hep-ex]

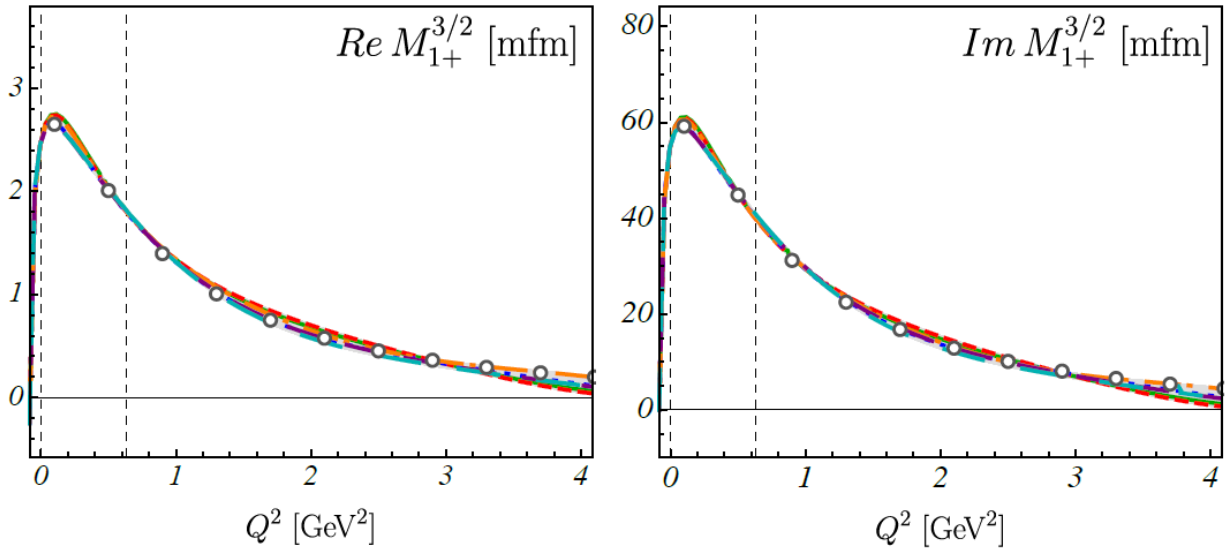
Description of Polarization Observables (πN)

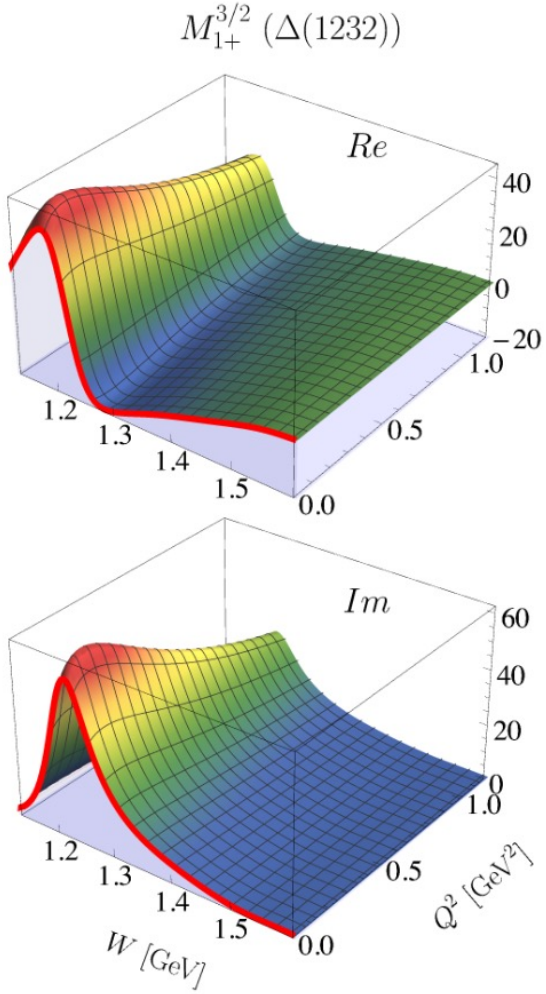


$\pi^0 p$, Q^2=1 GeV^2, W=1.23 GeV, ϕ=15^0

J. J. Kelly, Phys. Rev. Lett. 95 (2005).

Large Multipoles



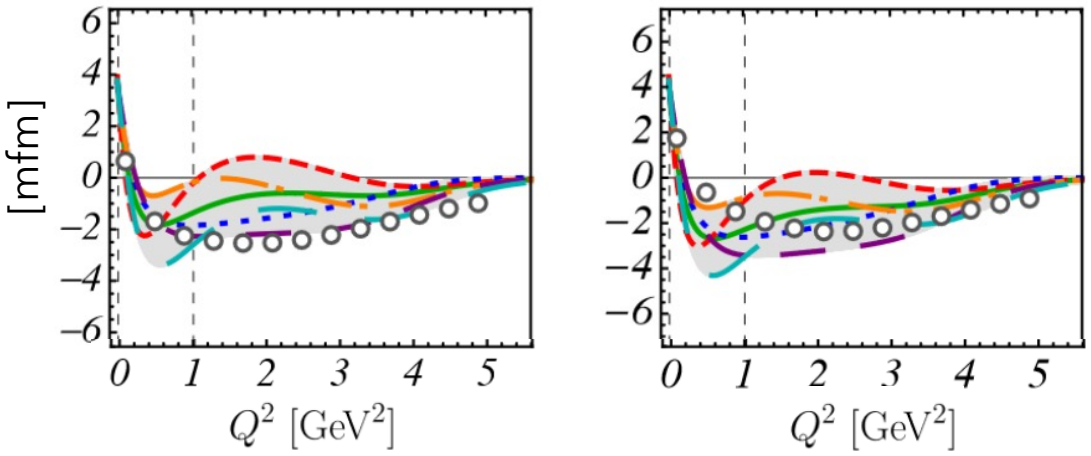


Fit strategies 1-6 together with MAID (open dots) for the magnetic multipole of the $\Delta(1232)$ Drechsel et al., EPJA (2007) 0710.0306 [nucl-th]

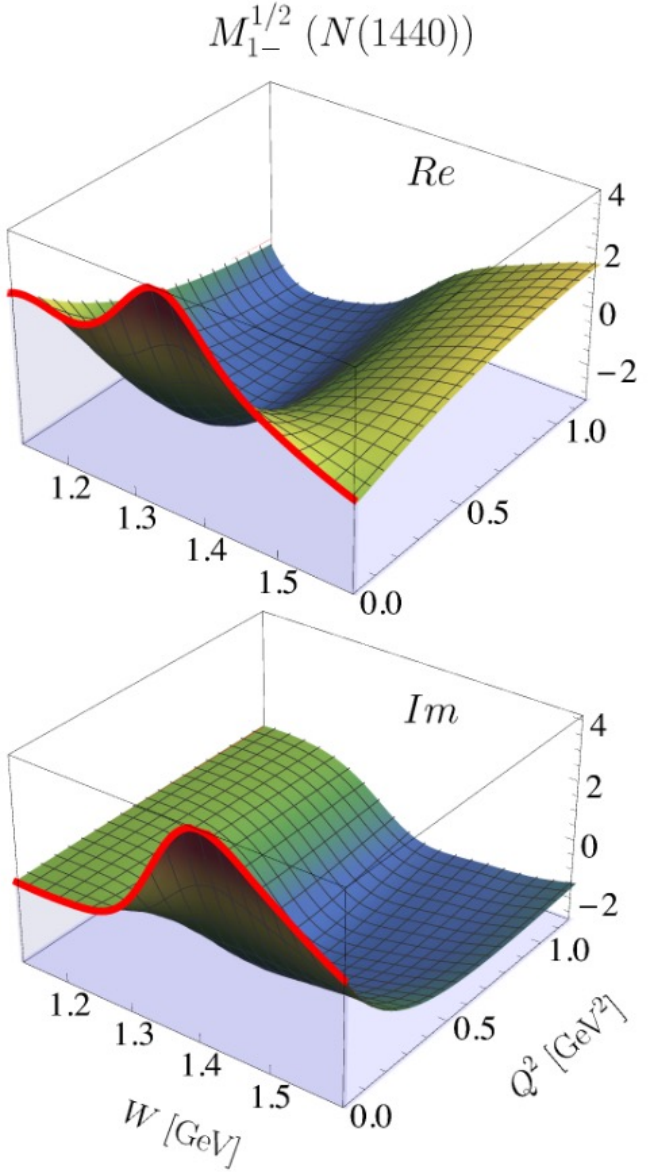
Prominent multipoles are well determined

(Strategy 1 only)

Roper Multipole



(W=1.38 GeV fixed)



(Strategy 1 only)

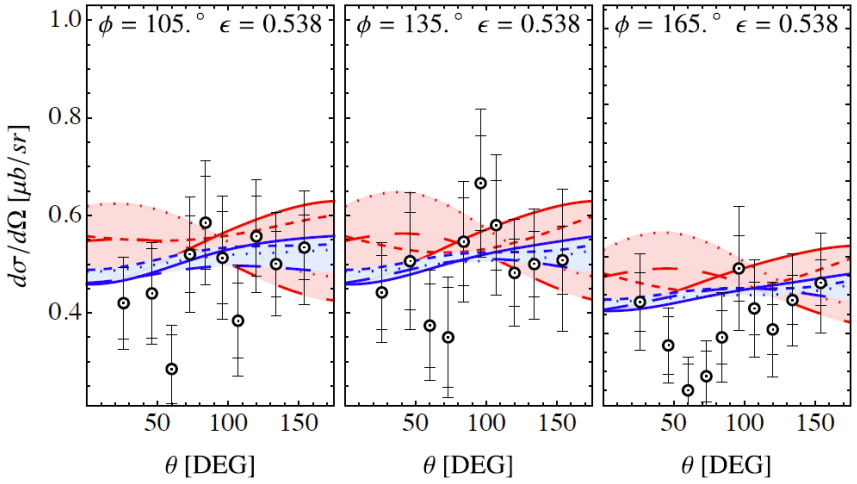
- Zero-transition (agrees with MAID)
- Extensive exploration of parameter space reveals ambiguities in PWA and reflects systematic uncertainties

η Electroproduction

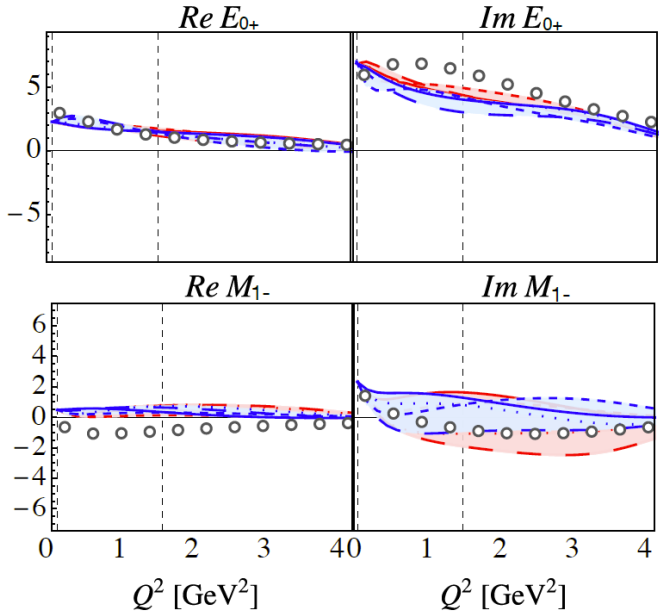
[M. Mai et al., PRC (2022)]

- $N_{data}^{\eta p} = 1,874$ (only $d\sigma/d\Omega$) (84,842 in total)
- kinematic range: $0 < Q^2 < 4$ GeV^2, $1.13 < W < 1.6$ GeV
- 8 different fit strategies: 4 with standard χ^2, 4 with weighted χ^2 to account for the smaller $N_{data}^{\eta p}$
  $\rightarrow$ better data description with weighted fit strategies:

Selected fit results: $\gamma^* p \rightarrow \eta p$ at $W = 1.5$ GeV, $Q^2 = 1.2$ GeV^2. Data: Denizli et al. (CLAS) PRC 76 (2007)



Selected multipoles at $W = 1535$ MeV



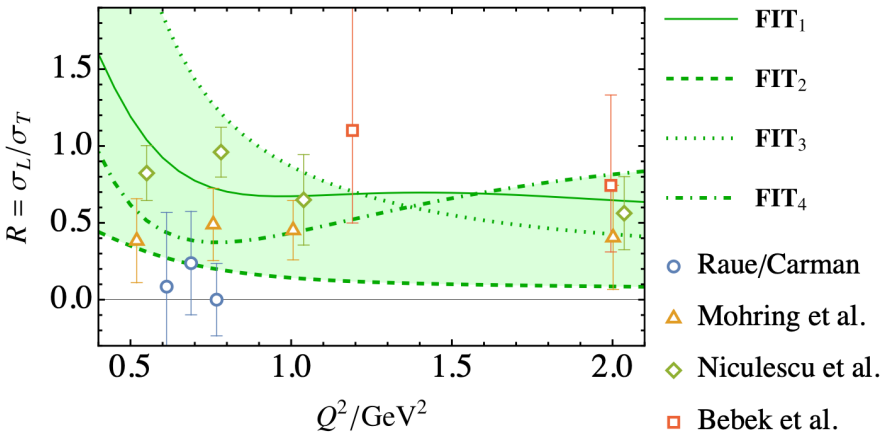
Dots: eta-MAID, arXiv:0110034

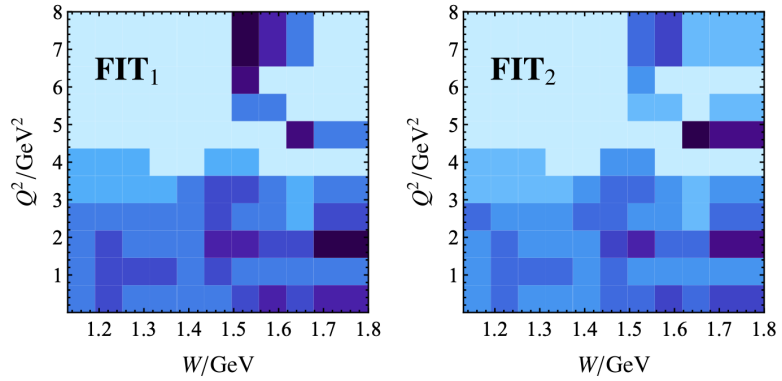
Upcoming: Polarization observables! Talk **Izzy Illari**

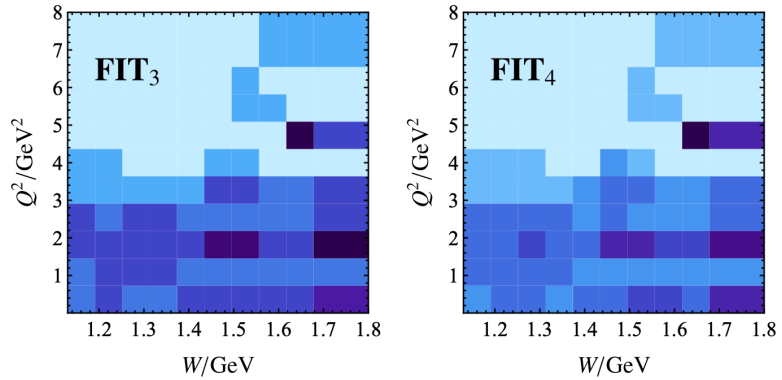
Kaon electroproduction

[M. Mai et al., EPJA 2023]

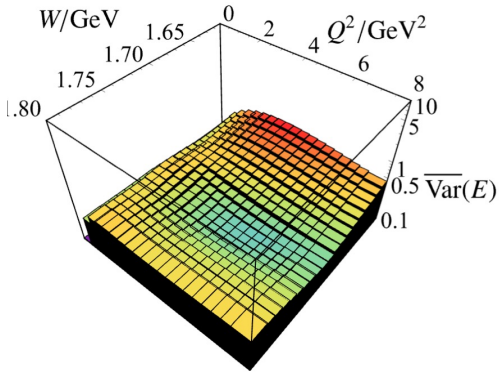
	$\chi^2_{\rm dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\rm wt,dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32

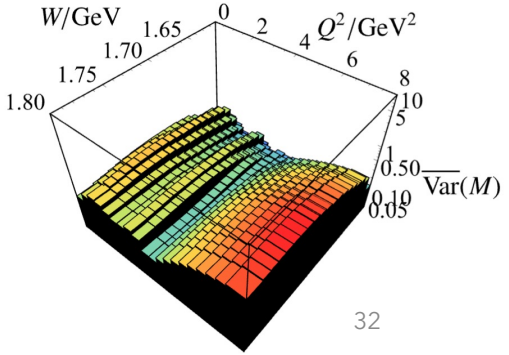






$$\overline{\rm Var}(W,Q^2)(X) := \sum_{\ell\pm} \frac{{\rm Var}\{|{\rm X}_{\ell\pm,{\rm i}}|\}}{{\rm Mean}\{|{\rm X}_{\ell\pm,{\rm i}}|\} + \varepsilon}$$

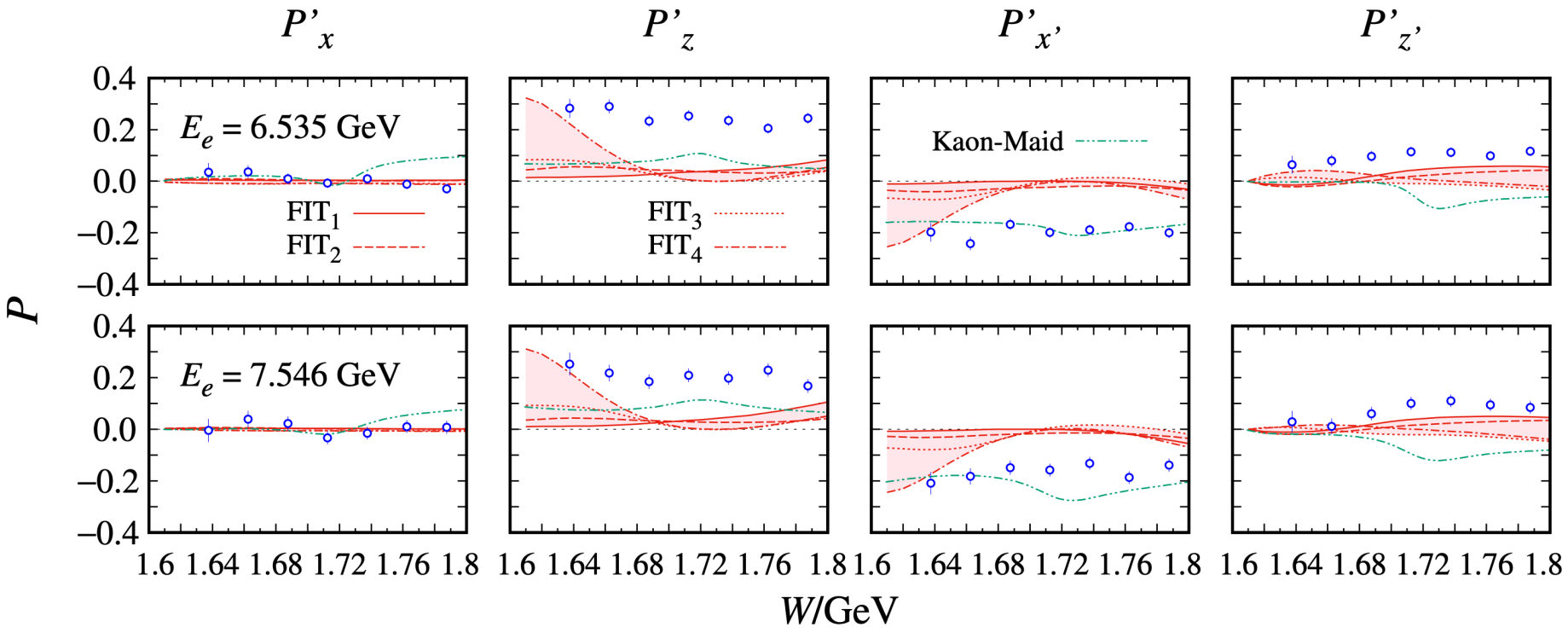




K Λ Challenges: Polarization measurements

- During analysis, first polarization data became available by CLAS:
- Predicted, not fitted

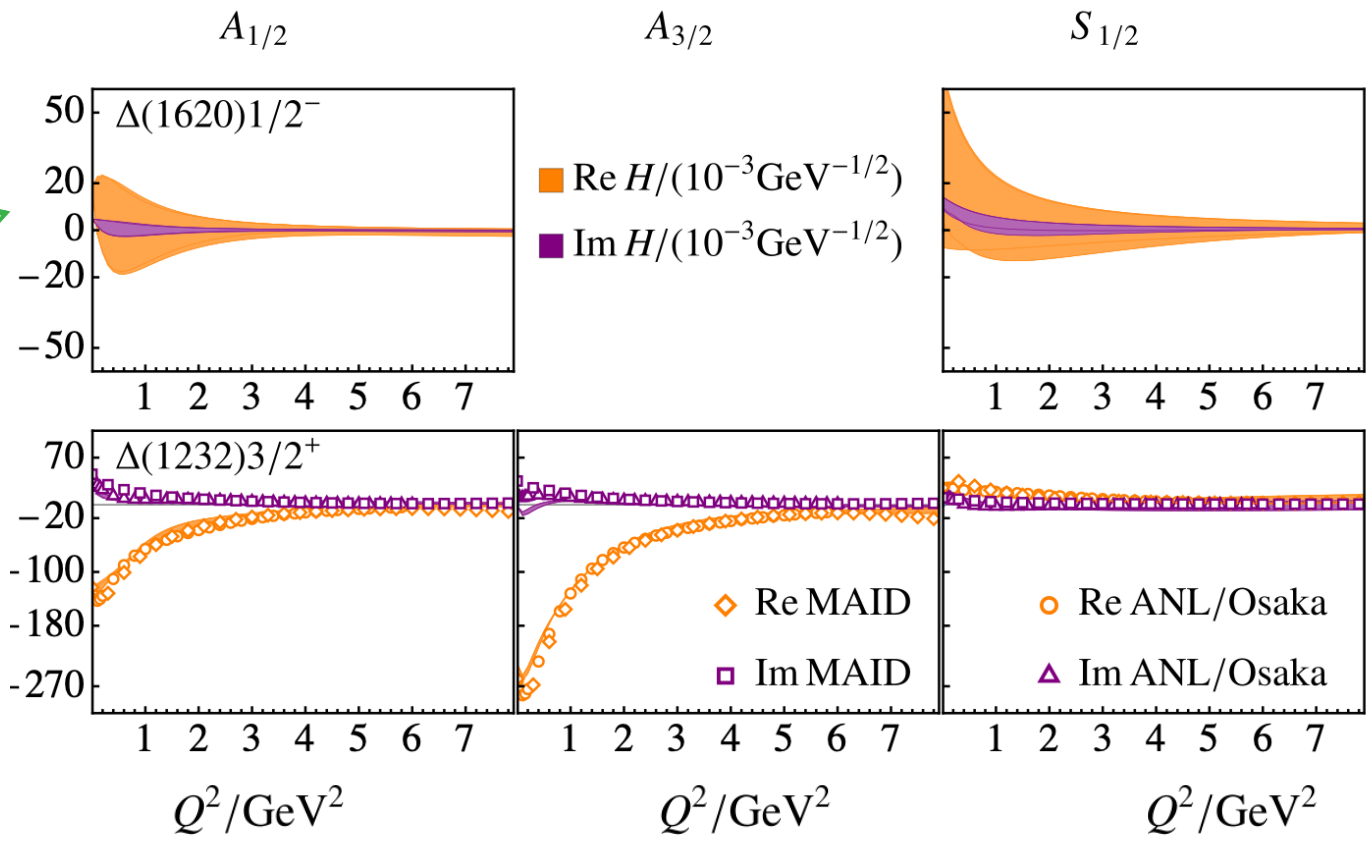
Beam-recoil polarization transfer [Carman et al. 2022]



Helicity Couplings I

[Yu-Fei Wang et al., 2024]

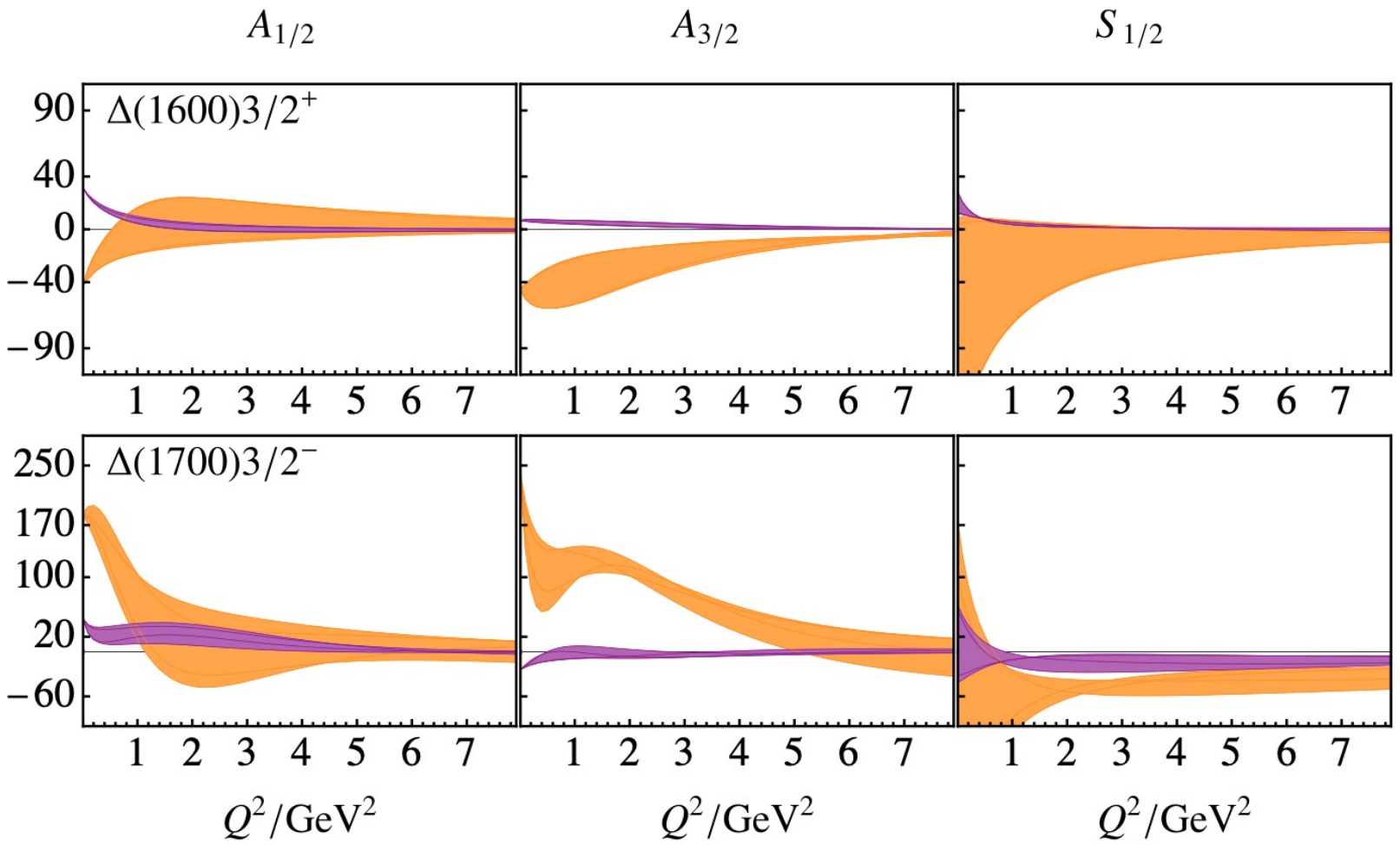
- Determined at resonance poles/ many for the first time



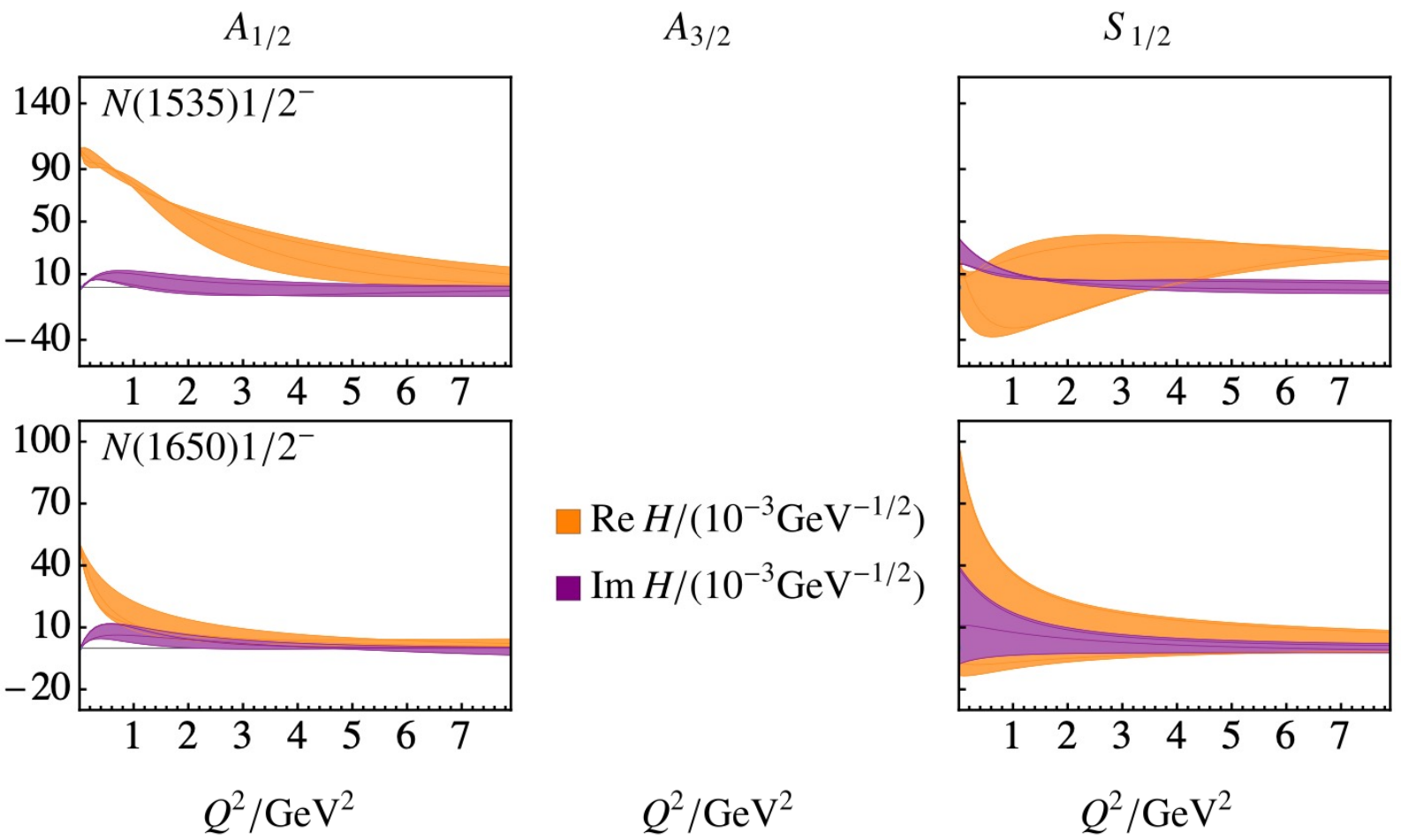
Compares qualitatively with [Mokeev et al., 2022]

Helicity Couplings II

[Yu-Fei Wang et al., 2024]



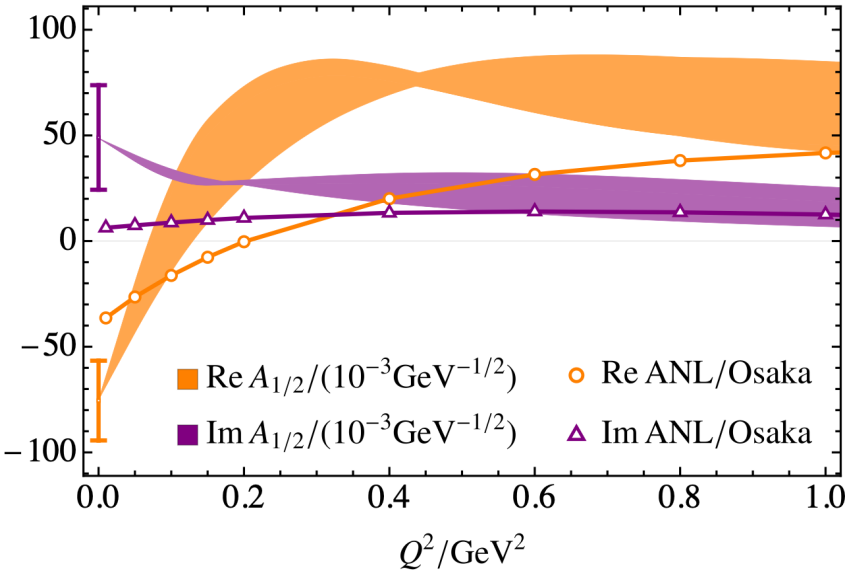
Helicity Couplings III



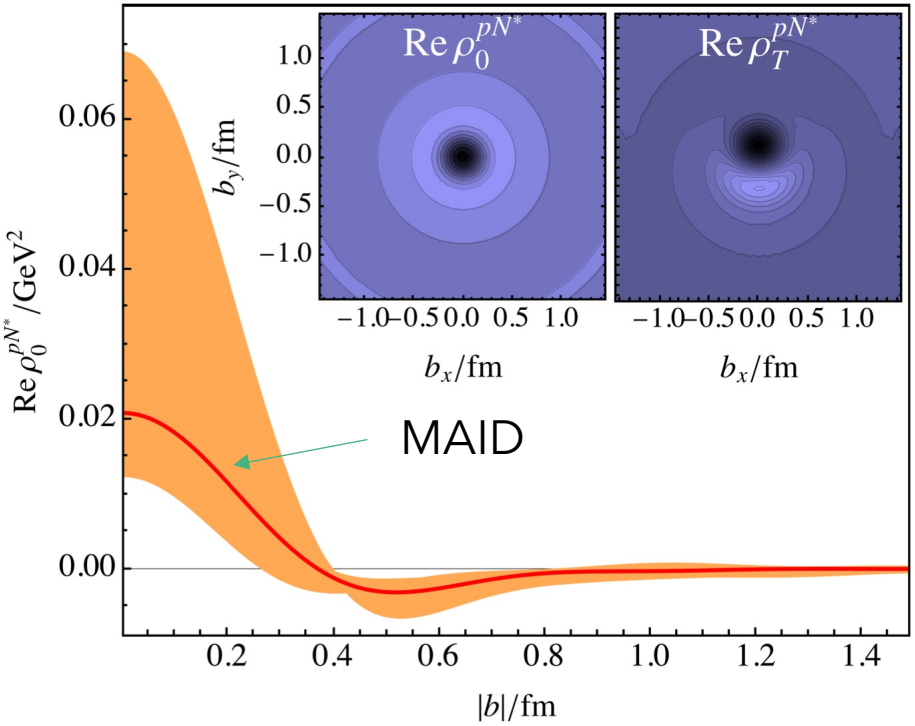
Results for the Roper resonance

Charge density structure

[approx./ following Tiator et al., (2009)]



$$\rho_0^{NN^*}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \times \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle,$$



Machine learning for partial-wave analysis

Exciting new developments: **Deep learning/neural networks** for

- Coupled-channel scattering [Sombillo, Ikeda, Sato, Hosaka, 2021]
- Resonance poles [Sombillo, Ikeda, Sato, Hosaka, 2022]
- Threshold structures [Sombillo, Ikeda, Sato, Hosaka, 2021]
- Near-threshold structures [Jpac, 2021]

Model selection for baryon analysis:

- How many partial waves are needed to describe a given data set? *Phys.Rev.C* 95 (2017) 1, 015203
- How many resonance are needed to describe a given data set? *Phys.Rev.D* 99 (2019) 1, 016001

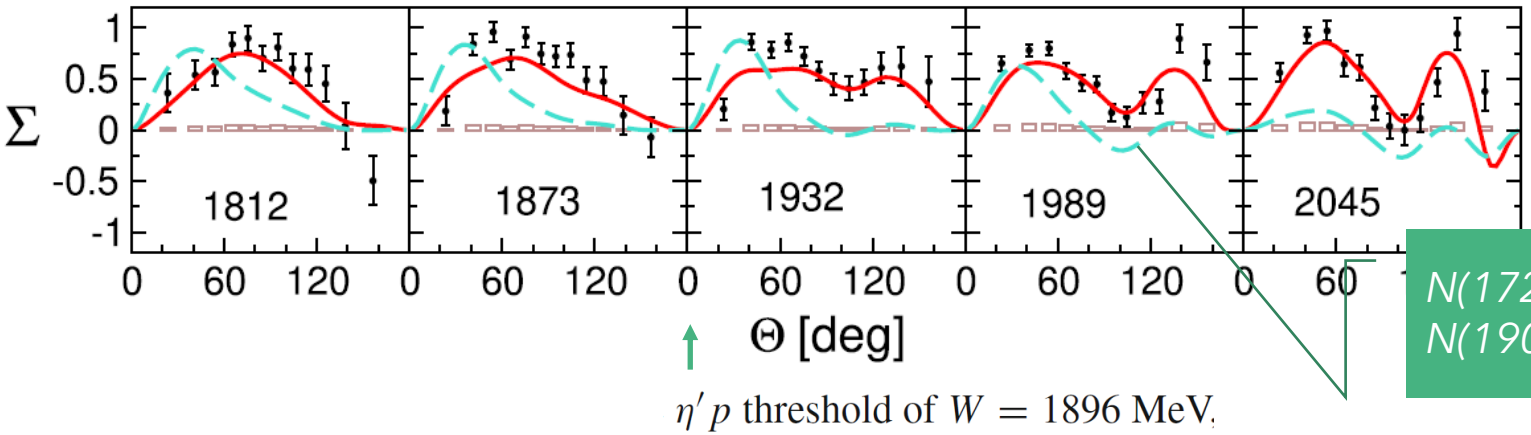
Summary

- Juelich-Bonn-Washington/JBW model: Phenomenology of excited baryons through coupled-channels, two- and three-body dynamics.
- Renewed effort to explore additional reaction channels in the last years:
  - $\gamma p \to K\Sigma$
  - $\pi N \to \omega N$
  - $\gamma^* p \to \pi N, \eta N, K\Lambda$ (Electroproduction)
- First global electroproduction analysis of different final states
- "Extensive" exploration of parameter space with good χ^2 (better than MAID) leads to *significant* variance of some multipoles.
- Many transition form factors at the pole extracted for the first time.
- Open questions:
  - How to get solid statistical statements out of a heterogeneous data base dominated by systematic errors?
  - Blinding spectroscopy

(spare slides)

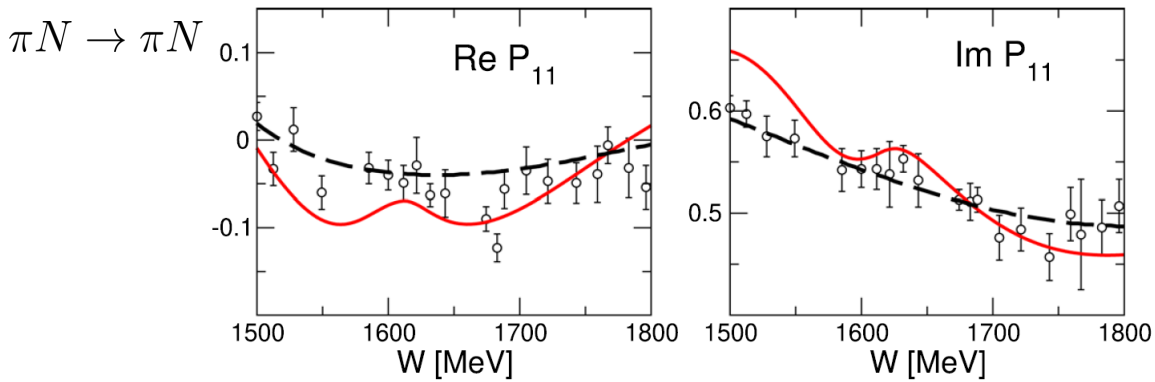
2022 Update in other reactions

- Beam asymmetry in η photoproduction (different W)



N(1720)3/2⁺ off
N(1900)3/2⁺ off

- N(1710)1/2+ returns with large ηN and KΛ branching ratios



SAID WI08 (fitted)

SES SAID WI08 (not fitted)

Parametrization - Details

- Dependence on virtuality: Channel and resonance-wise:

$$\tilde{F}_\mu(Q^2) = \tilde{F}_D(Q^2)\, e^{-\beta_\mu^0 Q^2/m^2}\; P^N(Q^2/m^2, \vec{\beta}_\mu)$$
$$\tilde{F}_i(Q^2) = \tilde{F}_D(Q^2)\, e^{-\delta_i^0 Q^2/m^2}\; P^N(Q^2/m^2, \vec{\delta}_i)$$

- Factorization:

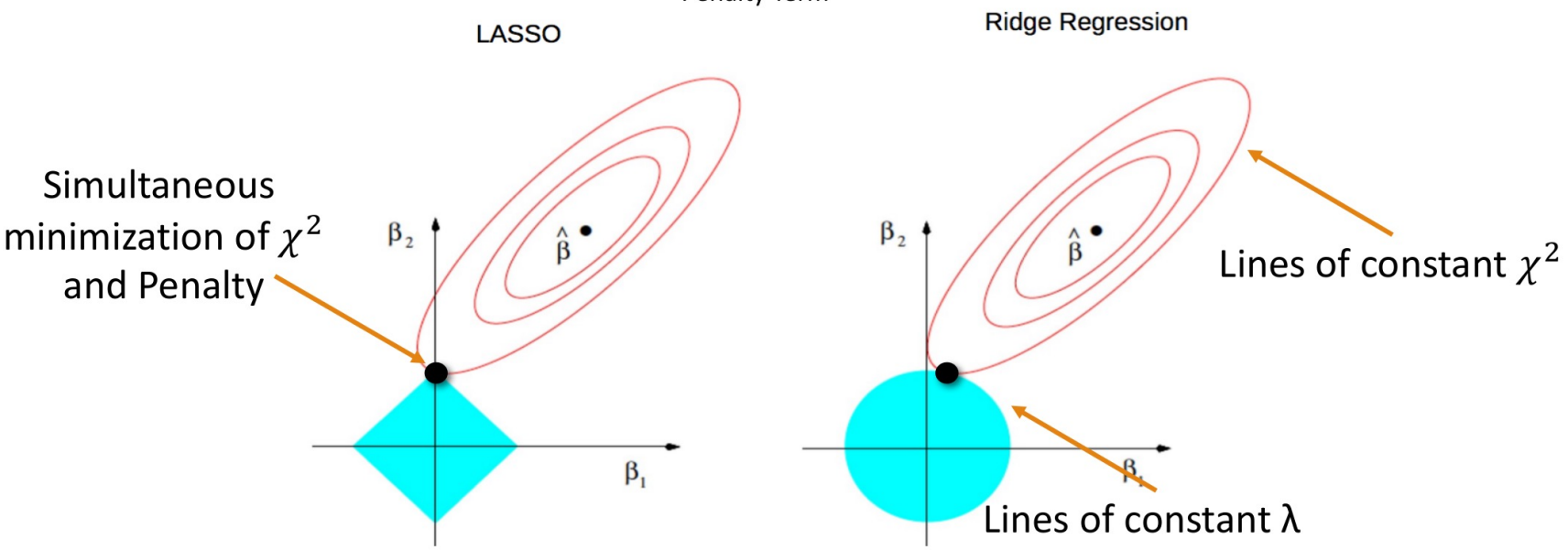
$$\alpha^{NP}_{\mu\gamma^*}(p, W, Q^2) = \tilde{F}_\mu(Q^2)\alpha^{NP}_{\mu\gamma}(p, W)$$
$$\gamma^c_{\gamma^*;i}(W, Q^2) = \tilde{F}_i(Q^2)\gamma^c_{\gamma;i}(W)$$

Least Absolute Shrinkage and Selection Operator (LASSO)

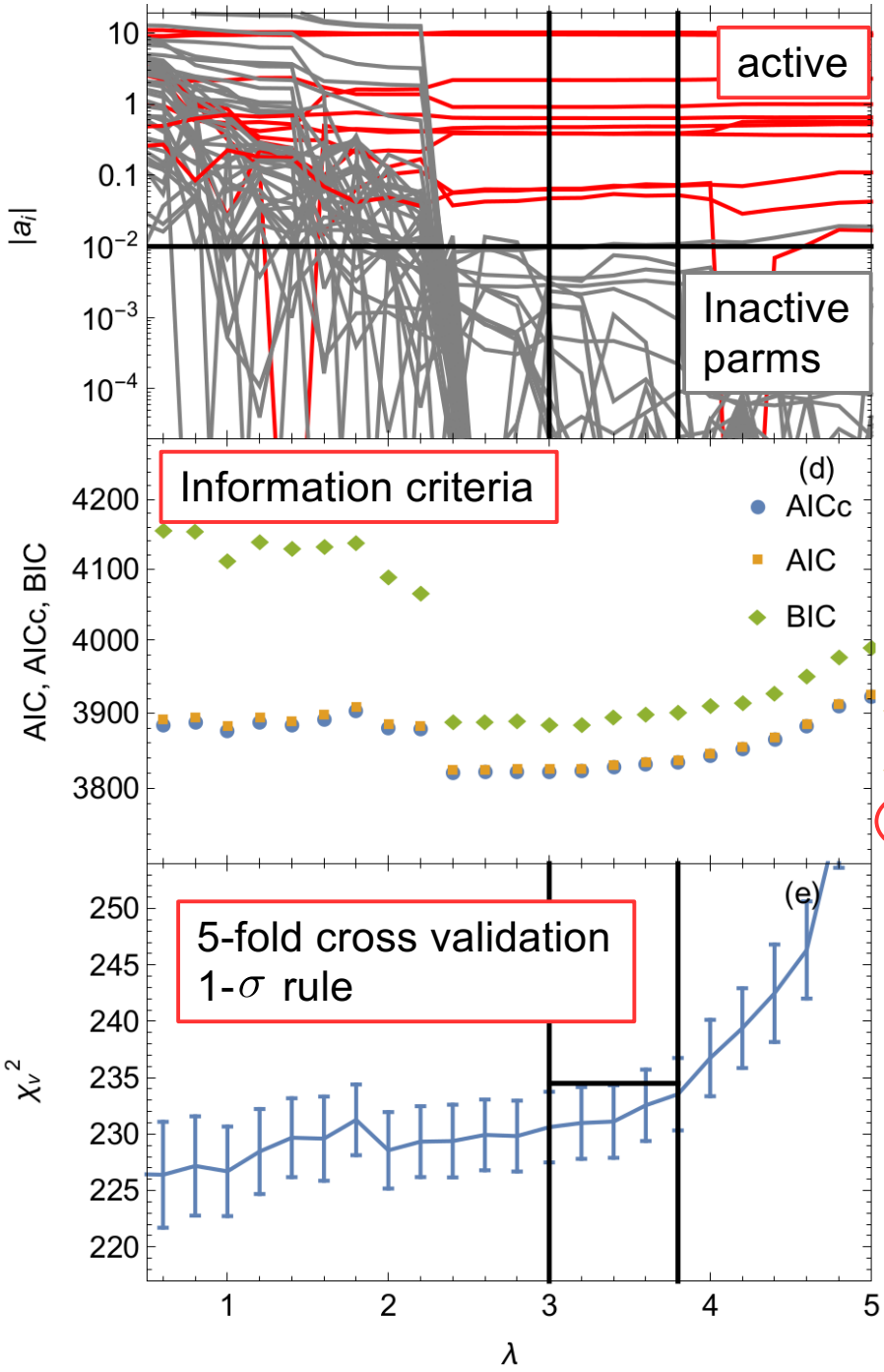
$$\sum_{i=1}^{n} \frac{(y_i - f(x_i, \beta_j))^2}{\sigma_i^2} + \lambda \sum_{j=1}^{m} |\beta_j|$$

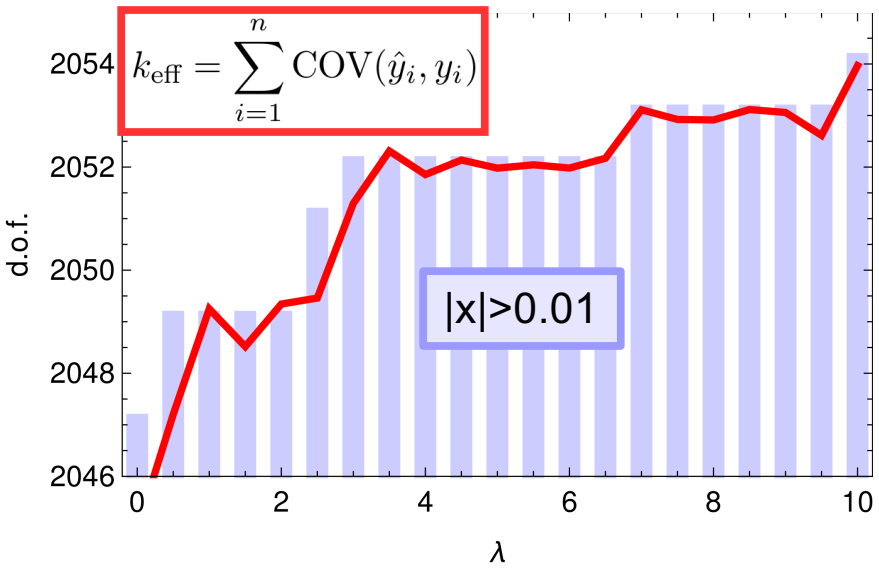
Normal X^2 — Penalty Term

$\hat{\beta}_i$: Best parameters without penalty
$\beta_i = 0$: Best parameters only penalty



See, e.g.: *The Elements of Statistical Learning*: Data Mining, Inference, and Prediction, T. Hastie, R. Tibshirani, J. Friedman, Springer 2009 second ed.





$k \to k_{\text{eff}}$ subsequently

$$AIC = -2 \max \log(L(\hat{\theta}|data)) + 2k = \chi^2 + 2k$$

$$AIC_c = AIC + \frac{2k(k+1)}{n-k-1}$$

$$BIC = -2 \max \log(L(\hat{\theta}|data)) + 2log(n) = \chi^2 + k\, log(n)$$

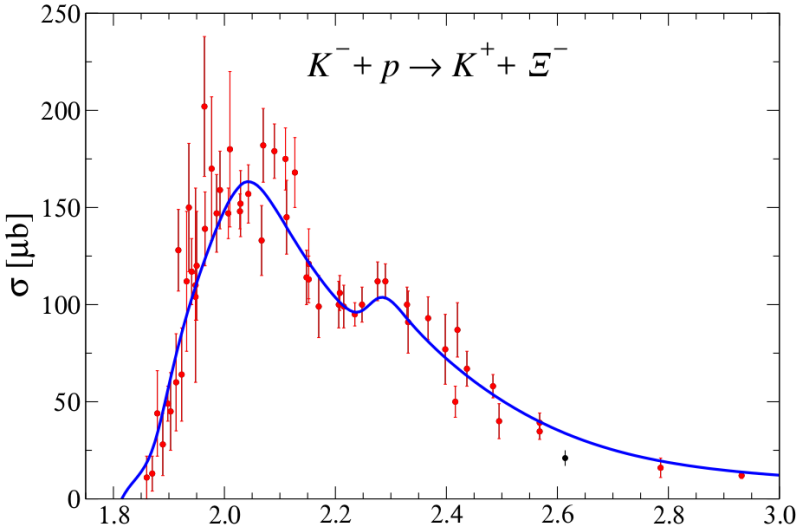
Close relation to Bayesian model comparison (here: $n \gg k$)

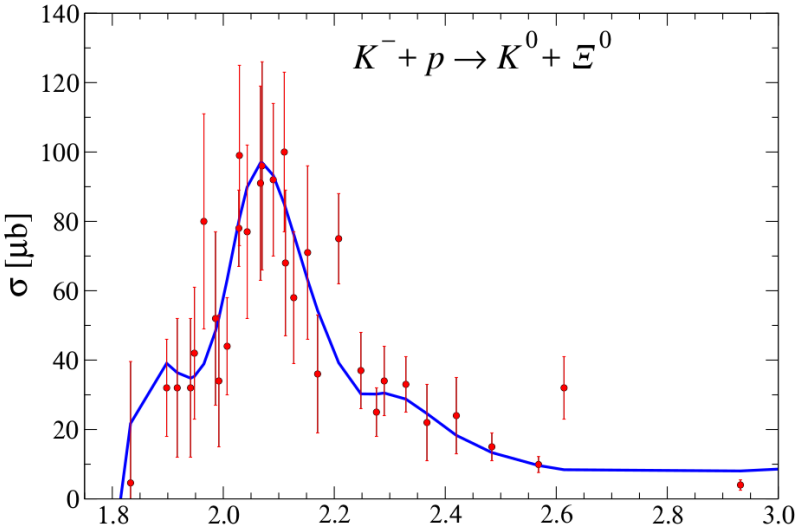
Determine the simplest model:

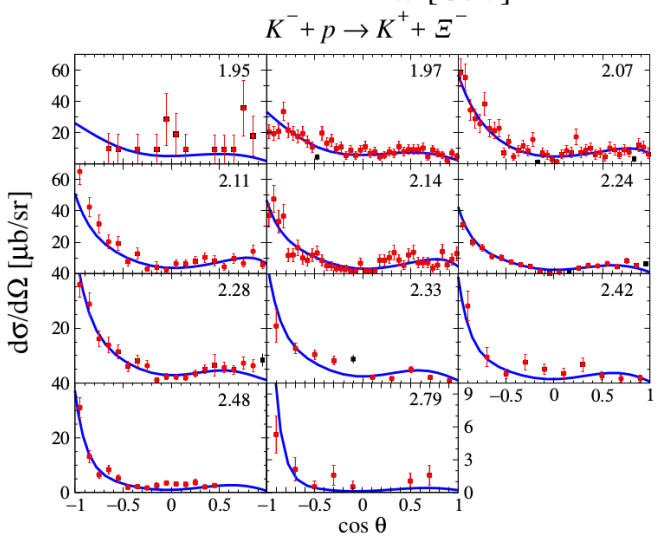
- Cross validation
- Information criteria

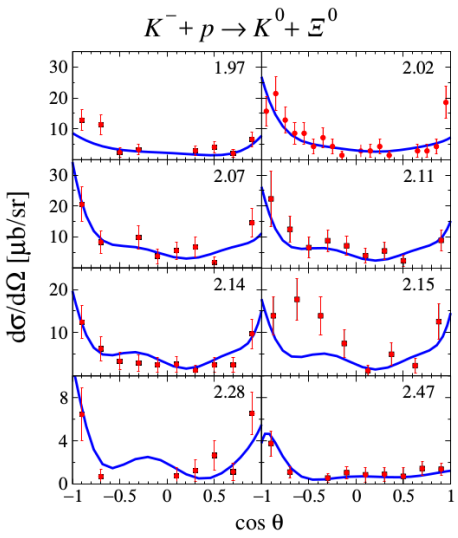
LASSO for the reaction $K^- p \to K \Xi$

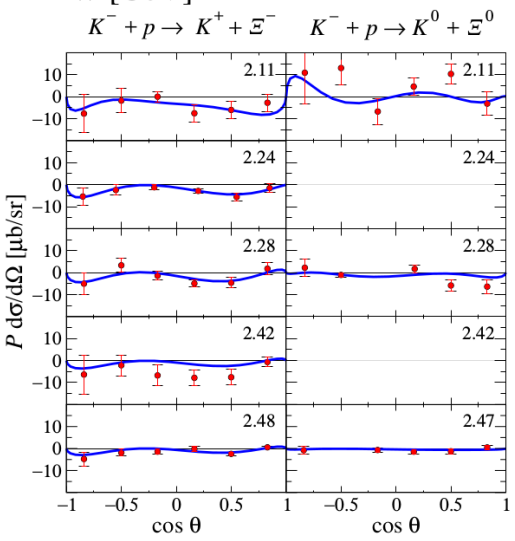
[Landay, Mai, M.D., Nakayama et al., 2018]





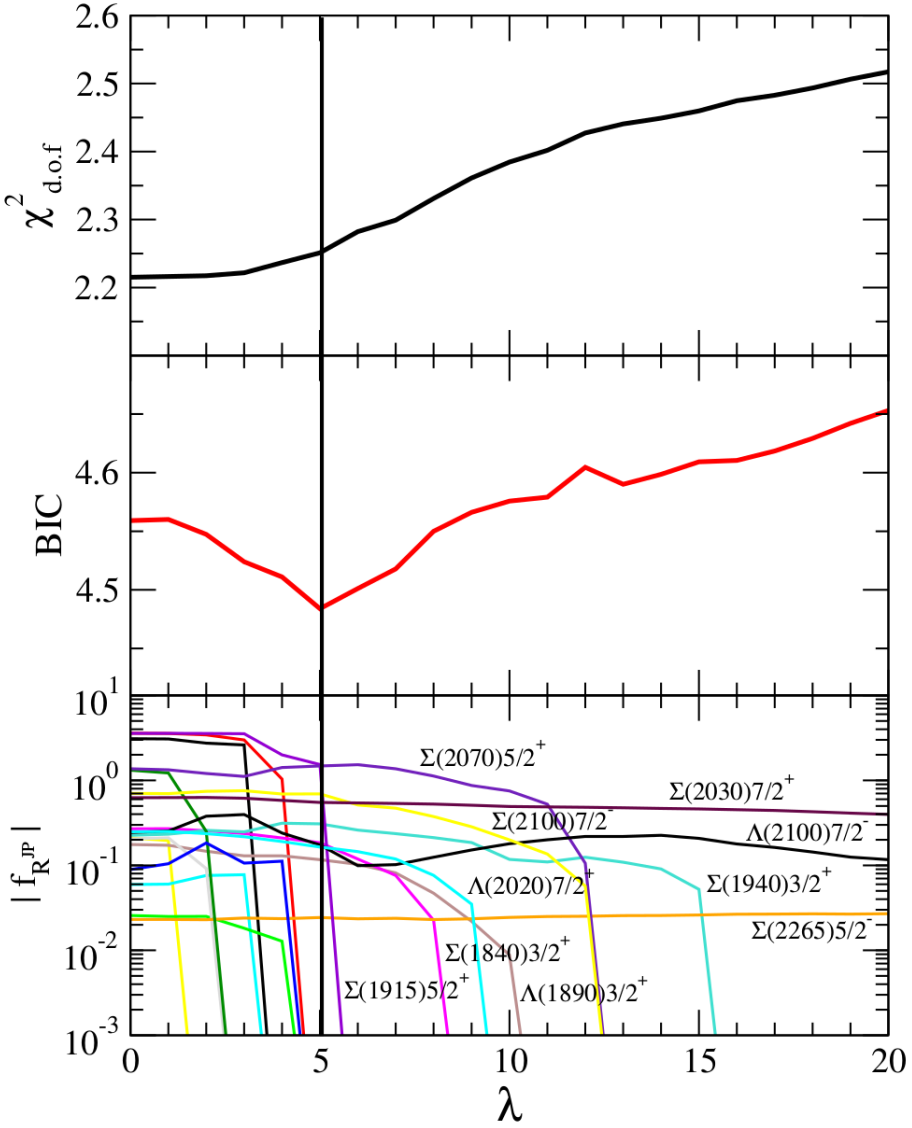






LASSO regularization

[Landay, Mai, M.D., Nakayama et al., 2018]



- Self-consistent data pruning for outliers using smoothness as criterion (10 out of 448 removed)
- All 21 resonance candidates from PDG
- Masses and widths fixed to PDG values
- Only Couping constants fitted
- 10 out of 21 resonances automatically selected by LASSO.

See also J. Oh's talk on this conference on Cascade production