decayangle
a Tool for Wigner Rotations

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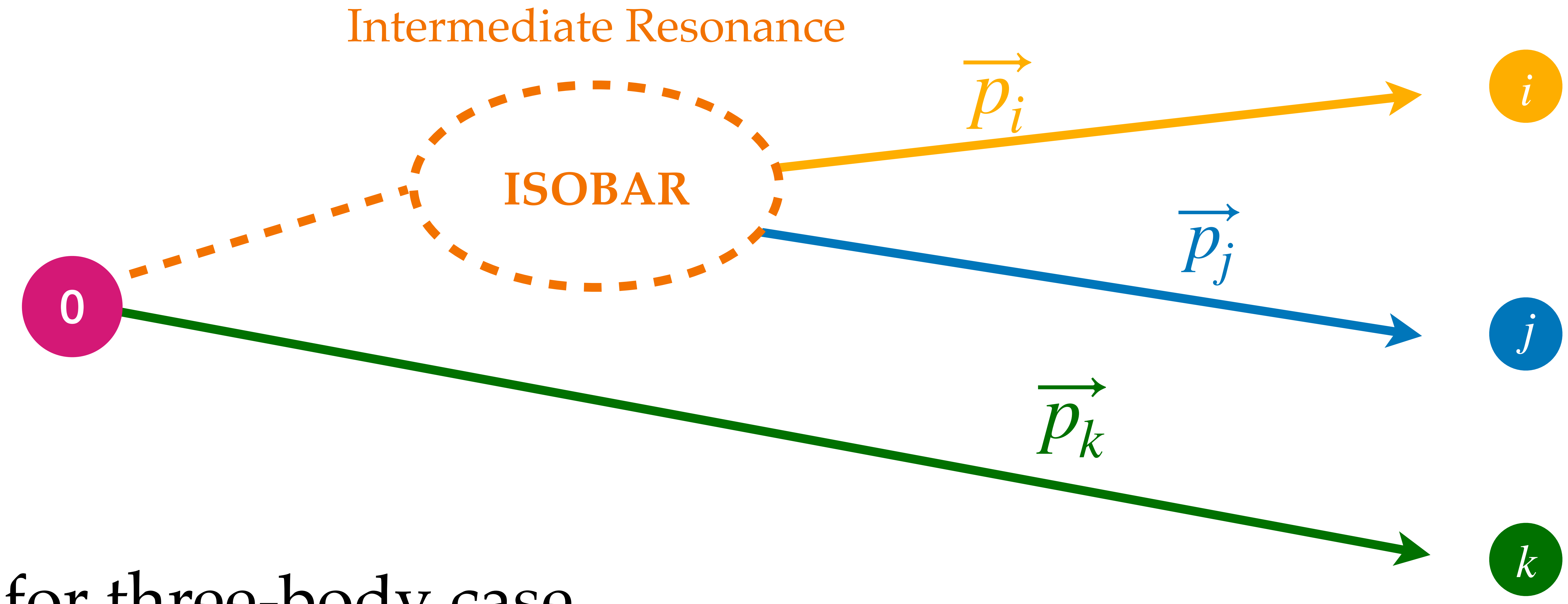
The Isobar Model in Helicity Formalism

Three-body decay as consecutive two-body decays

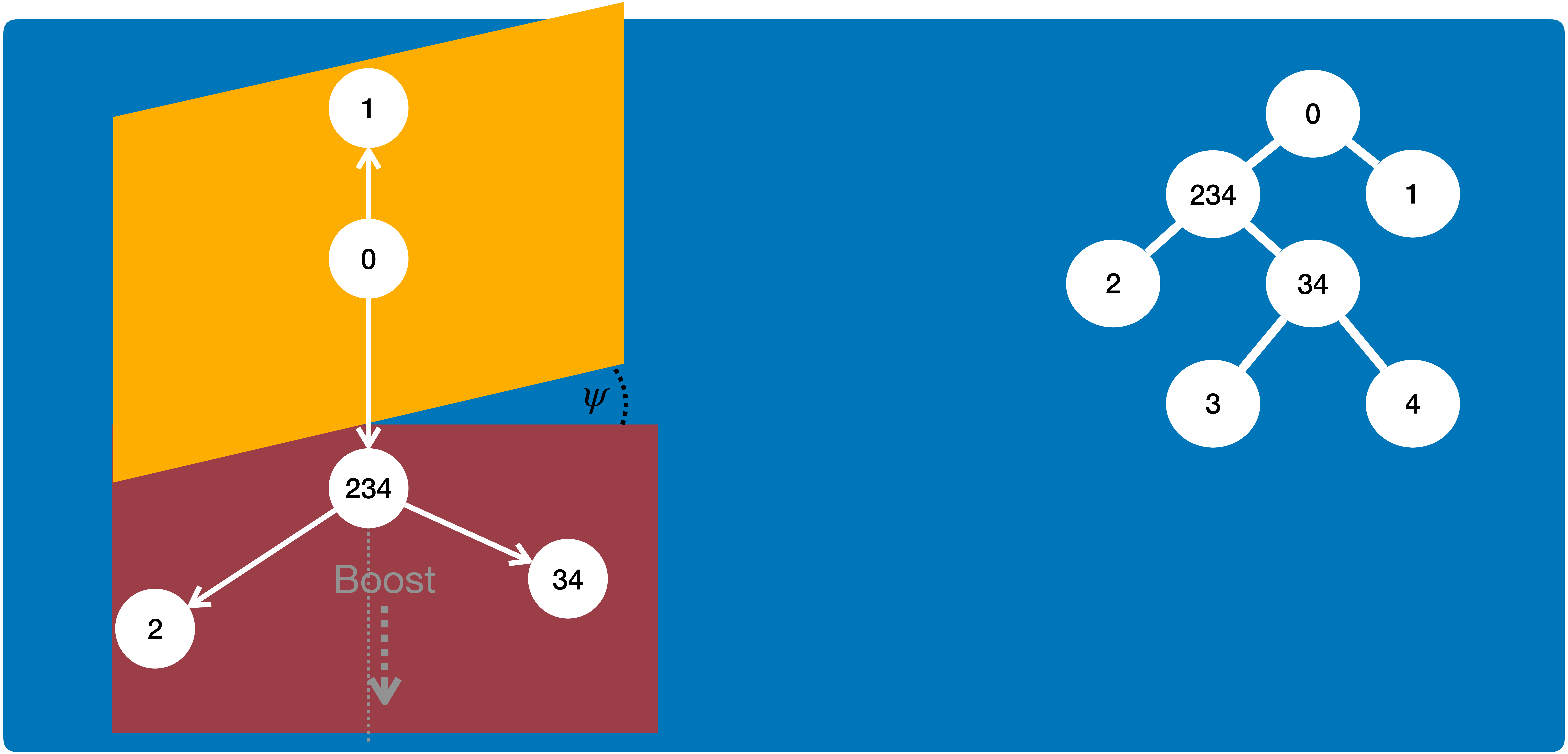


- Three distinct *topologies* for three-body case
- Strong interaction
  - → quick decay preserves coherence
  - → decays via different Isobars can not be separated

Helicity Amplitudes

Initial Decay	Isobar Decay
$H^{0\to(ij),k}$	$H^{(ij)\to i,j}$

Multiple Topologies and Spin



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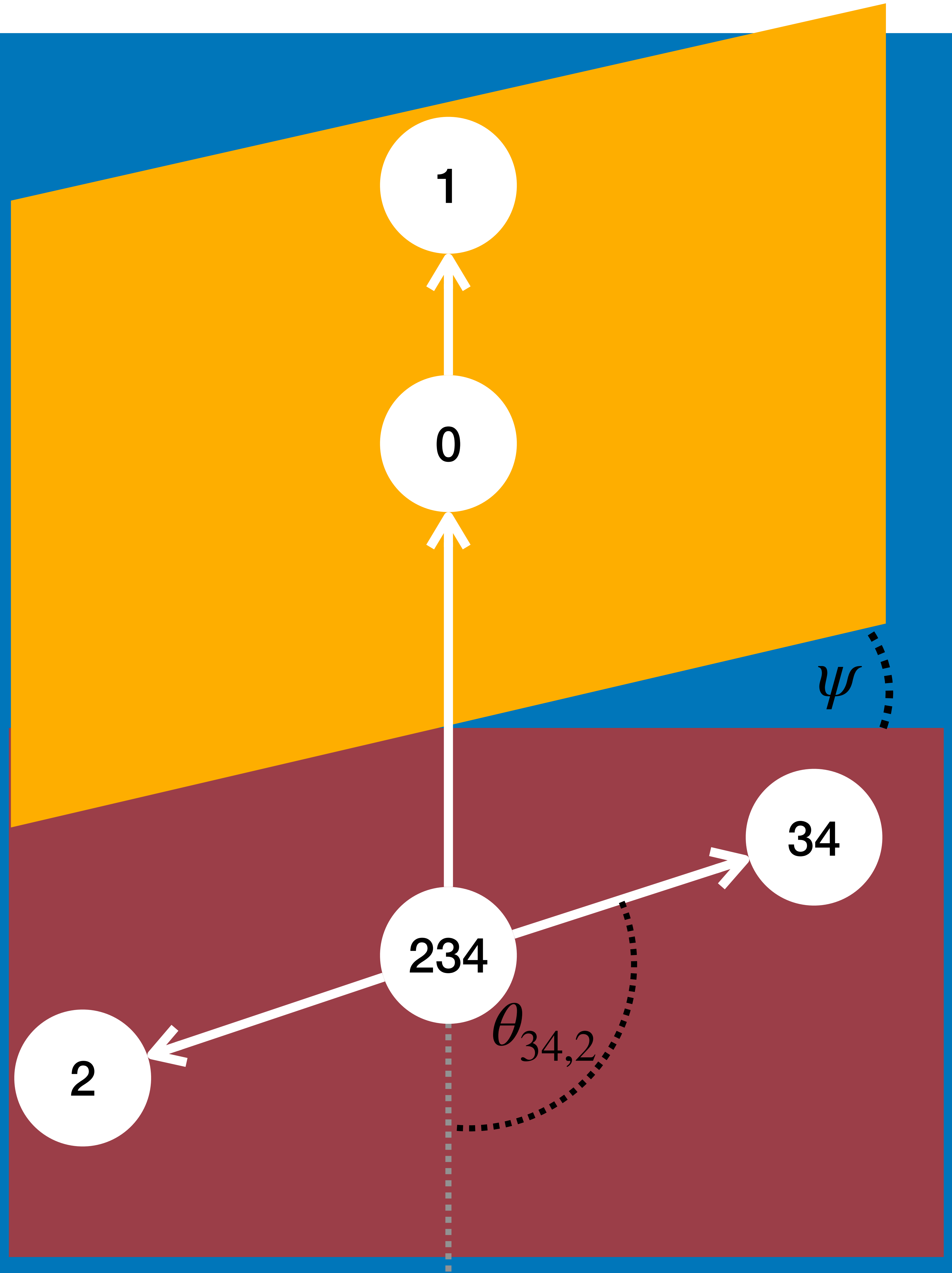

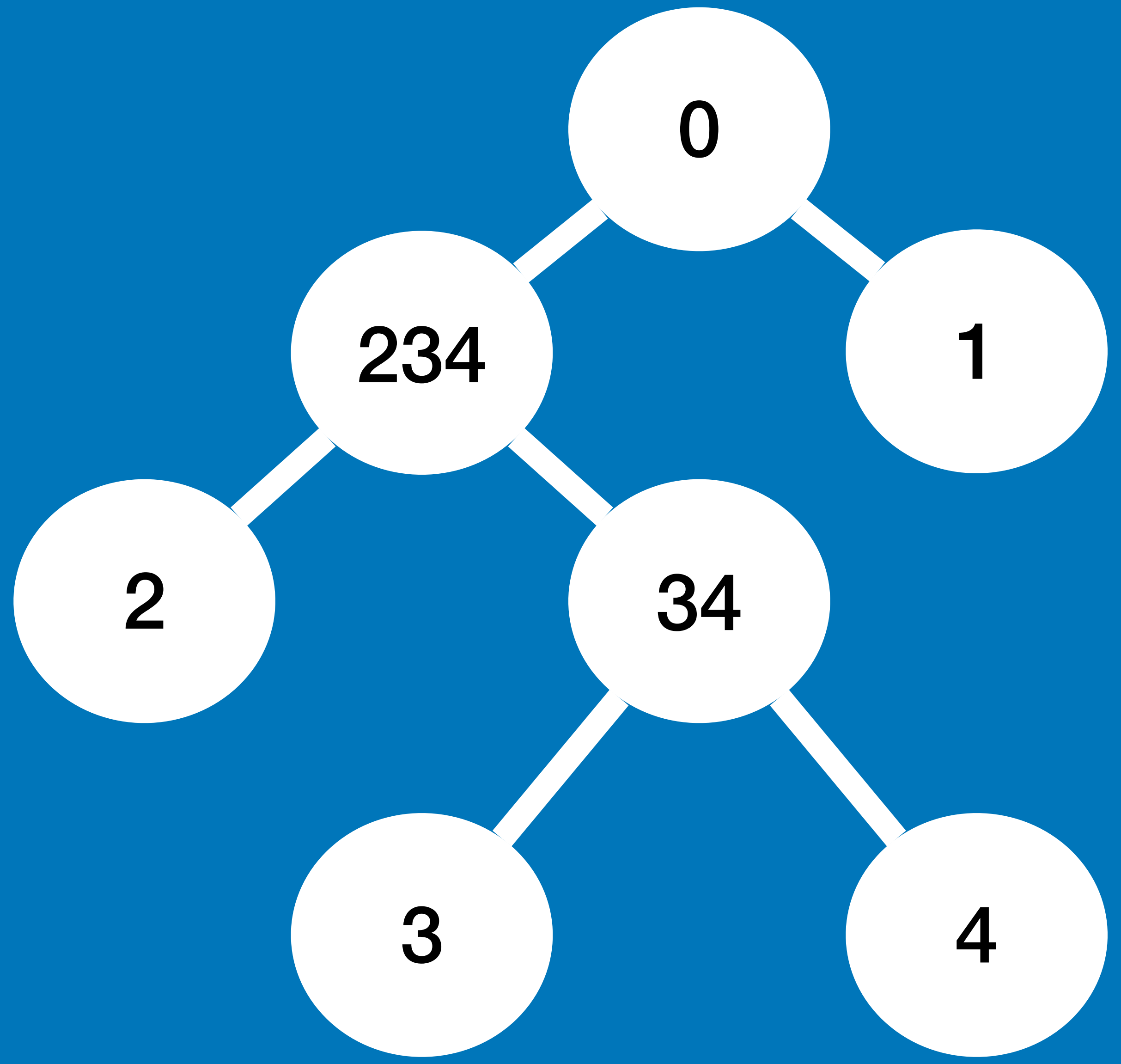

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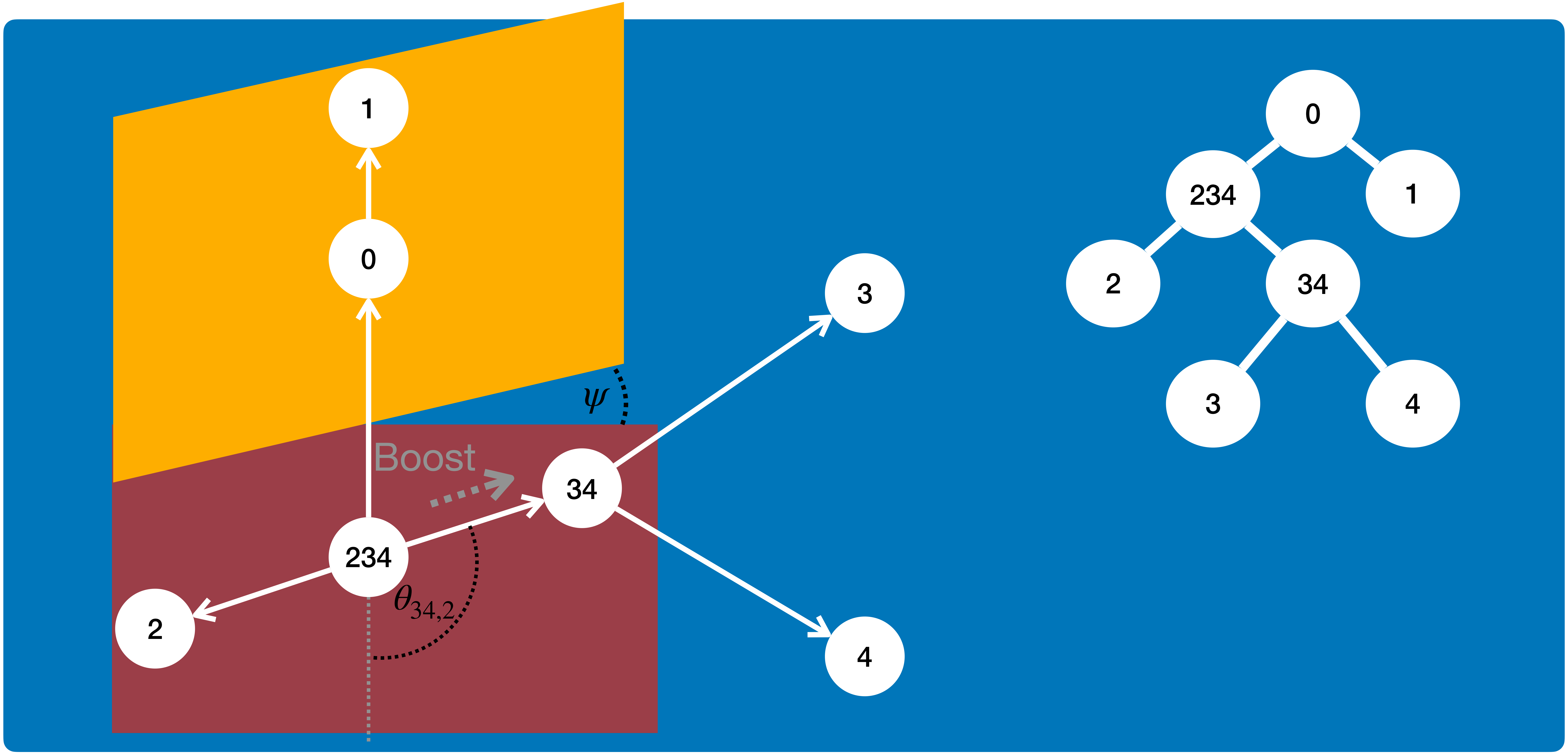

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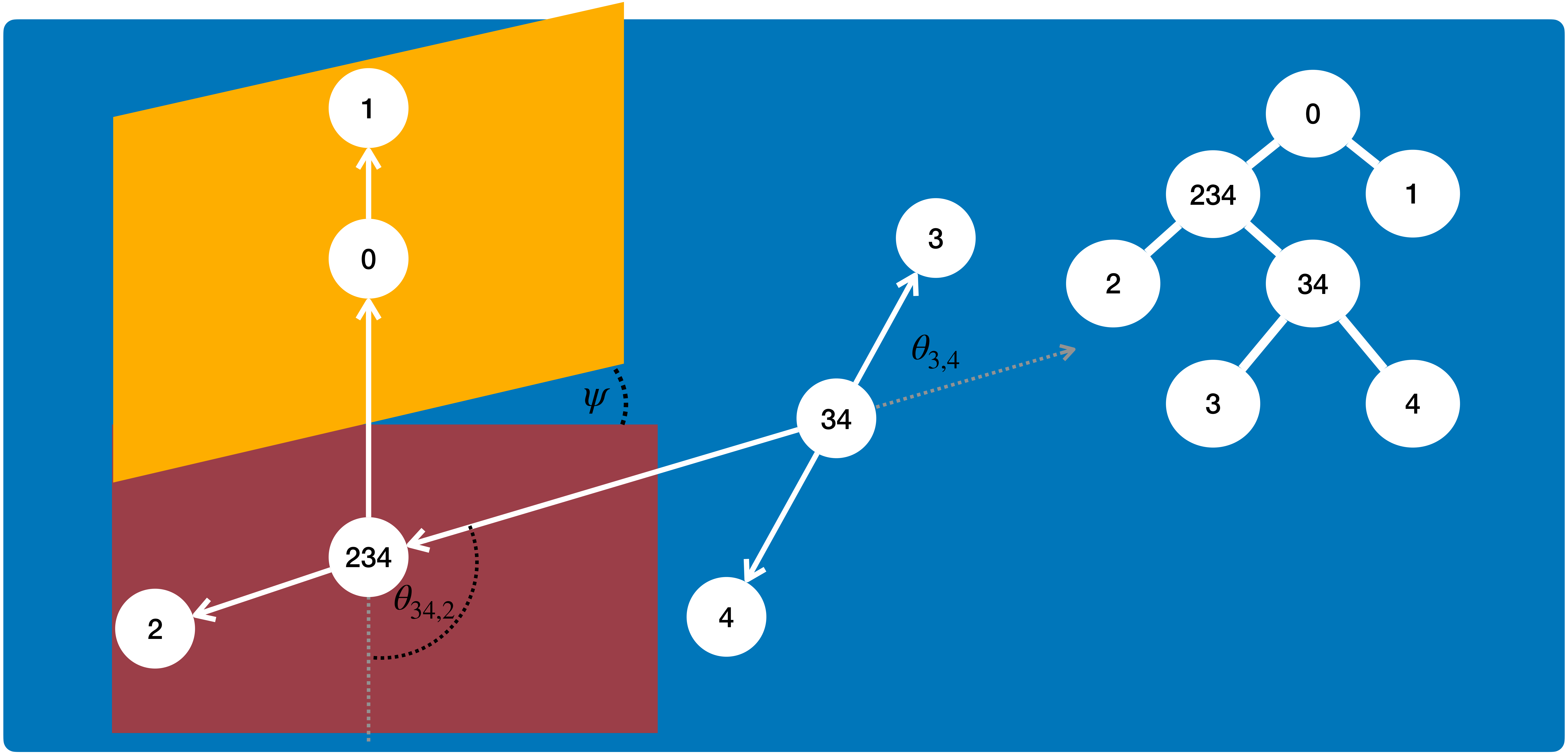

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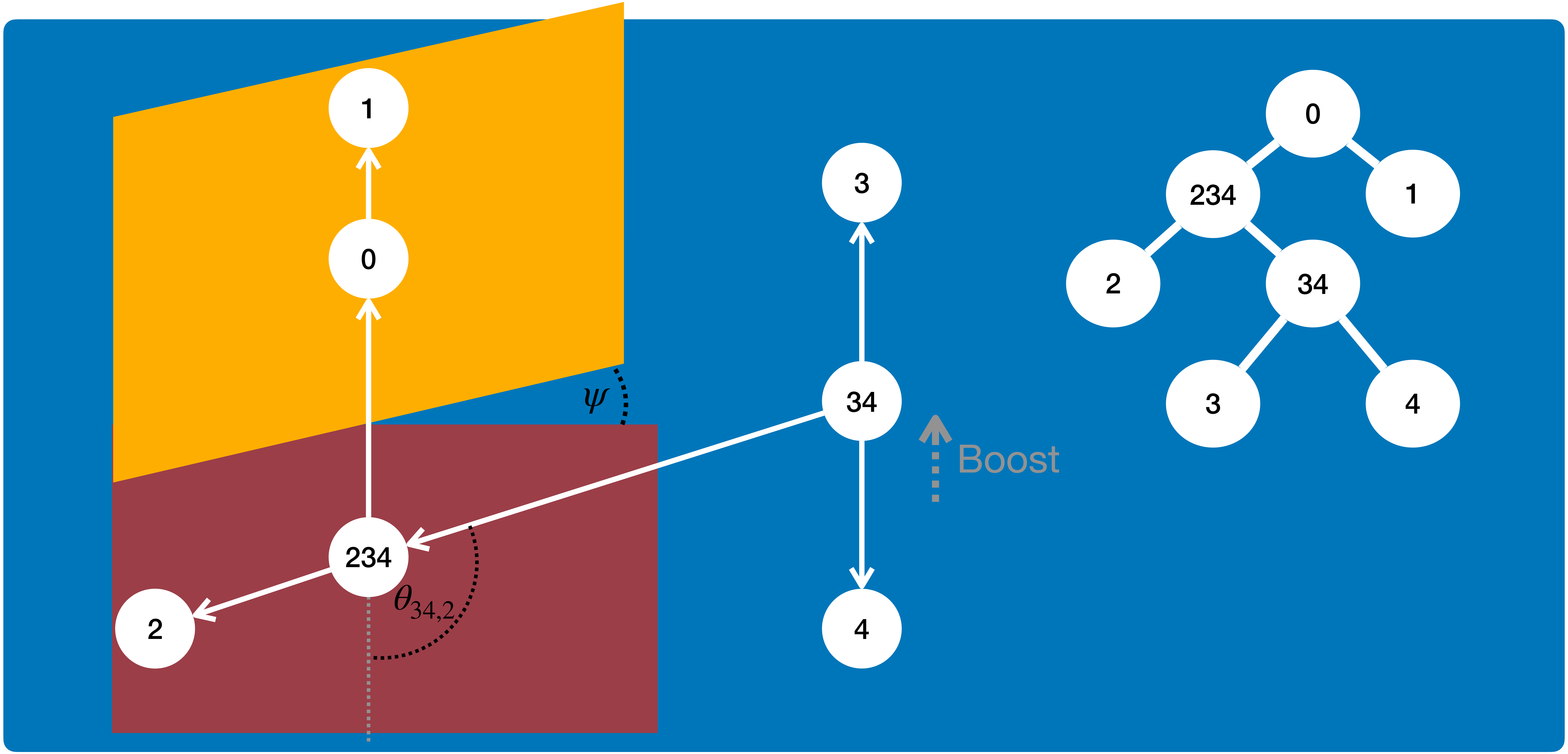
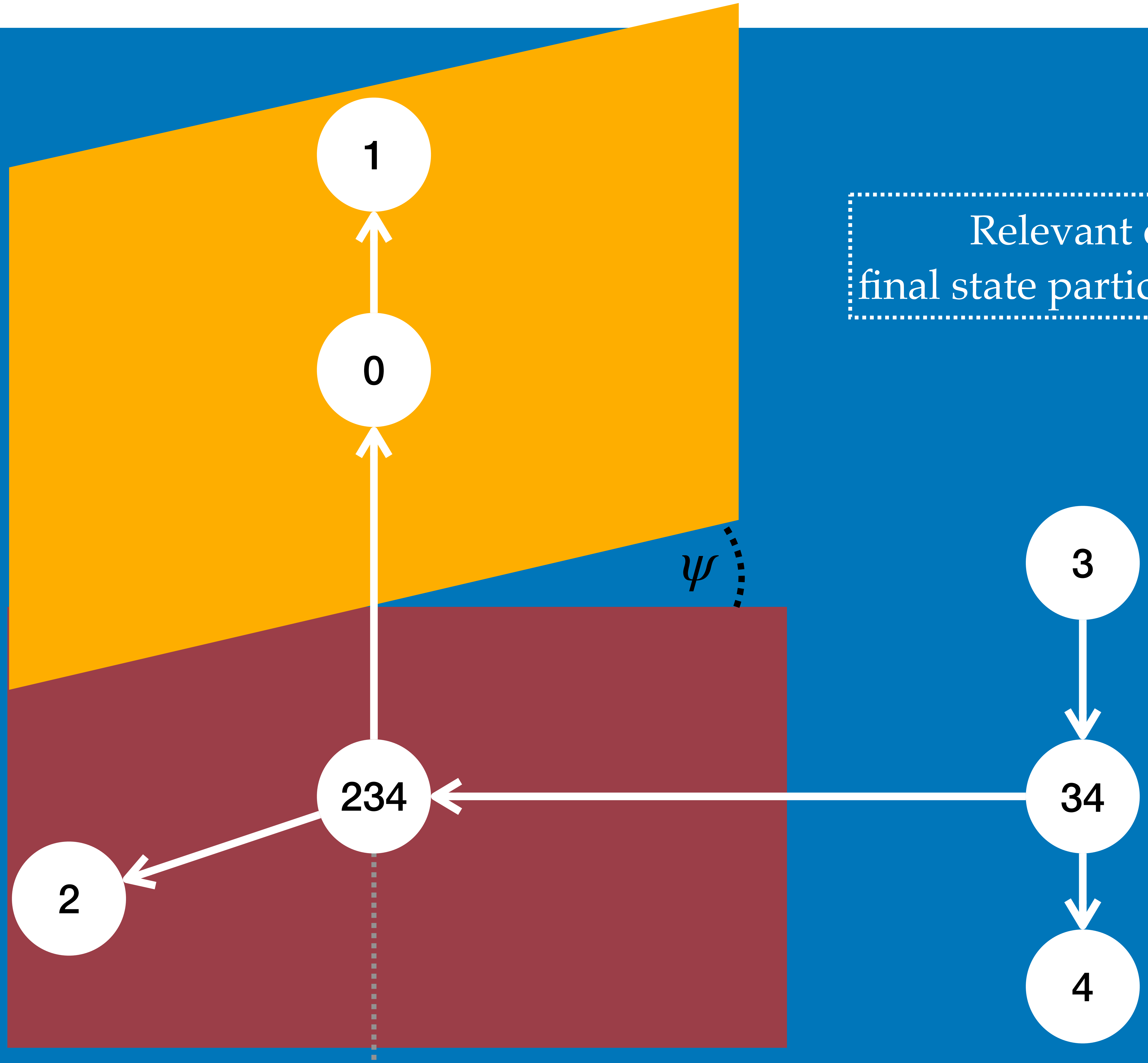
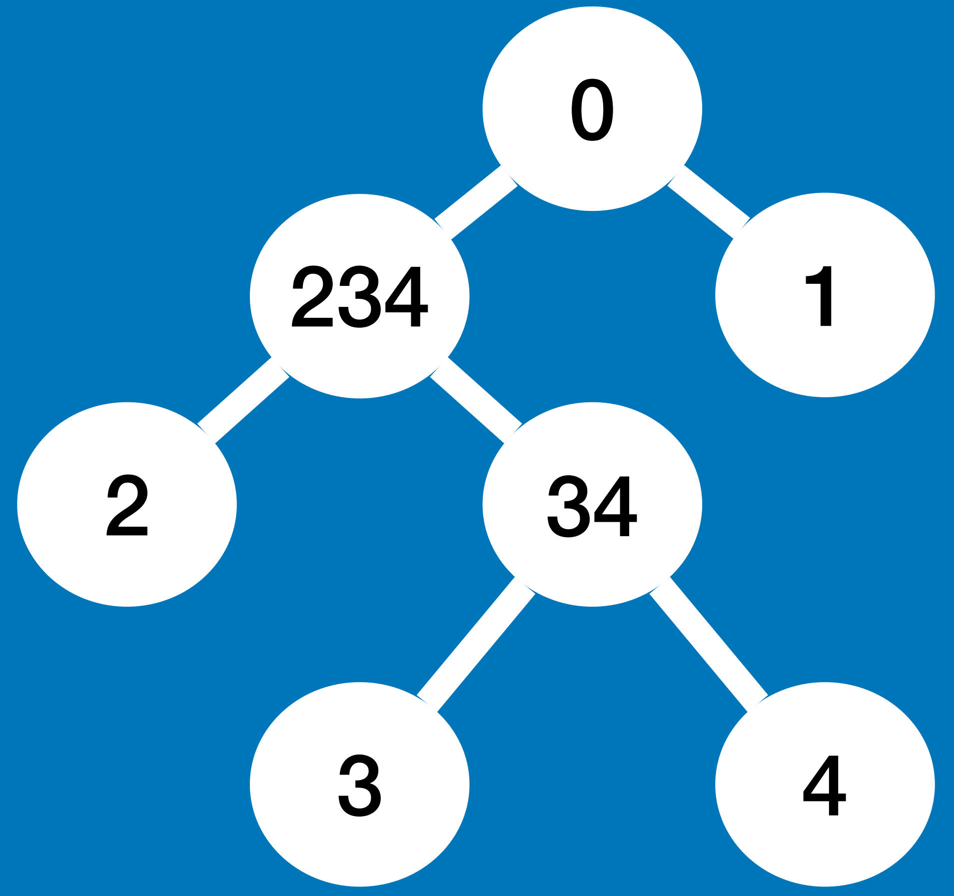


Multiple Topologies and Spin

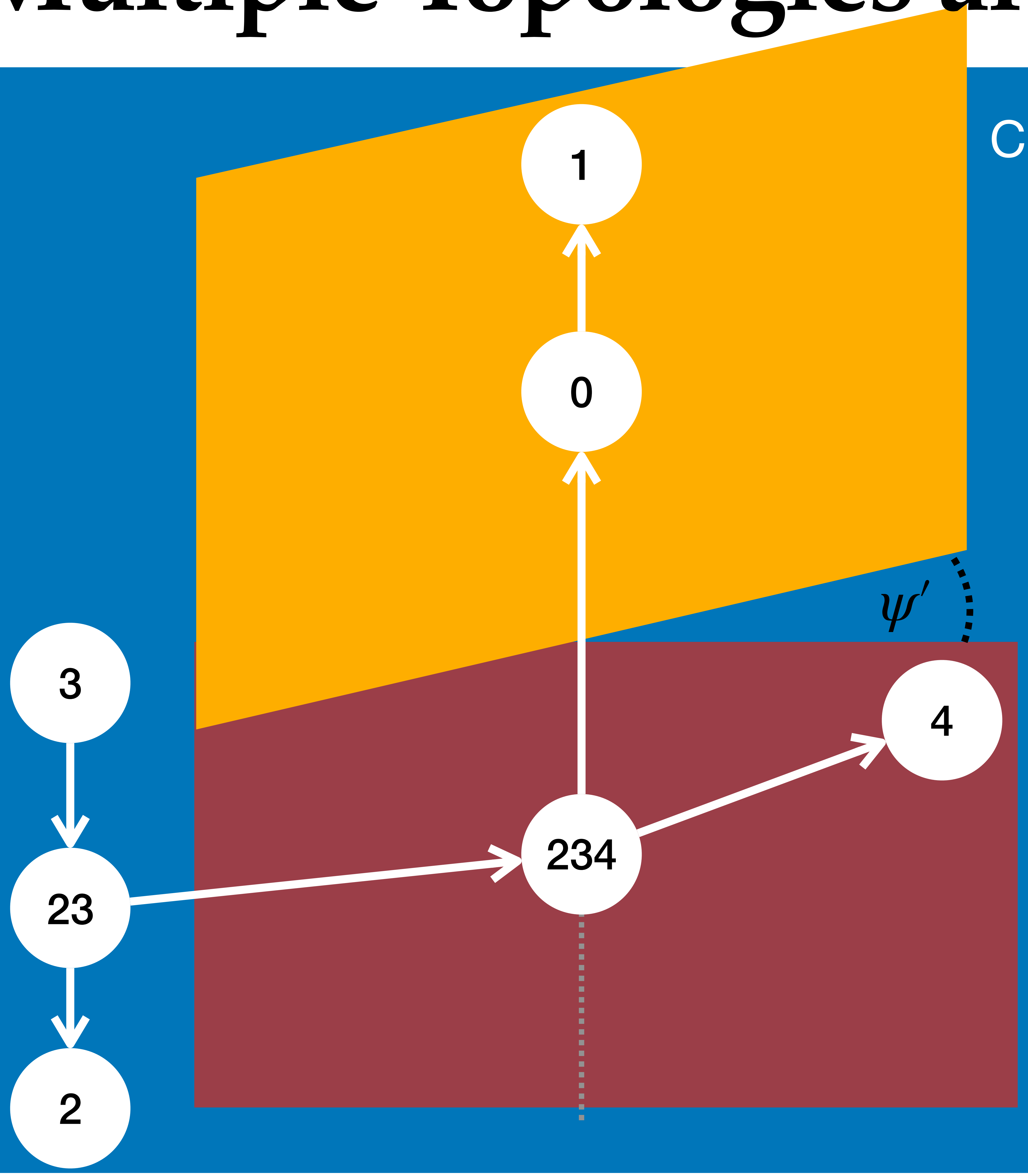


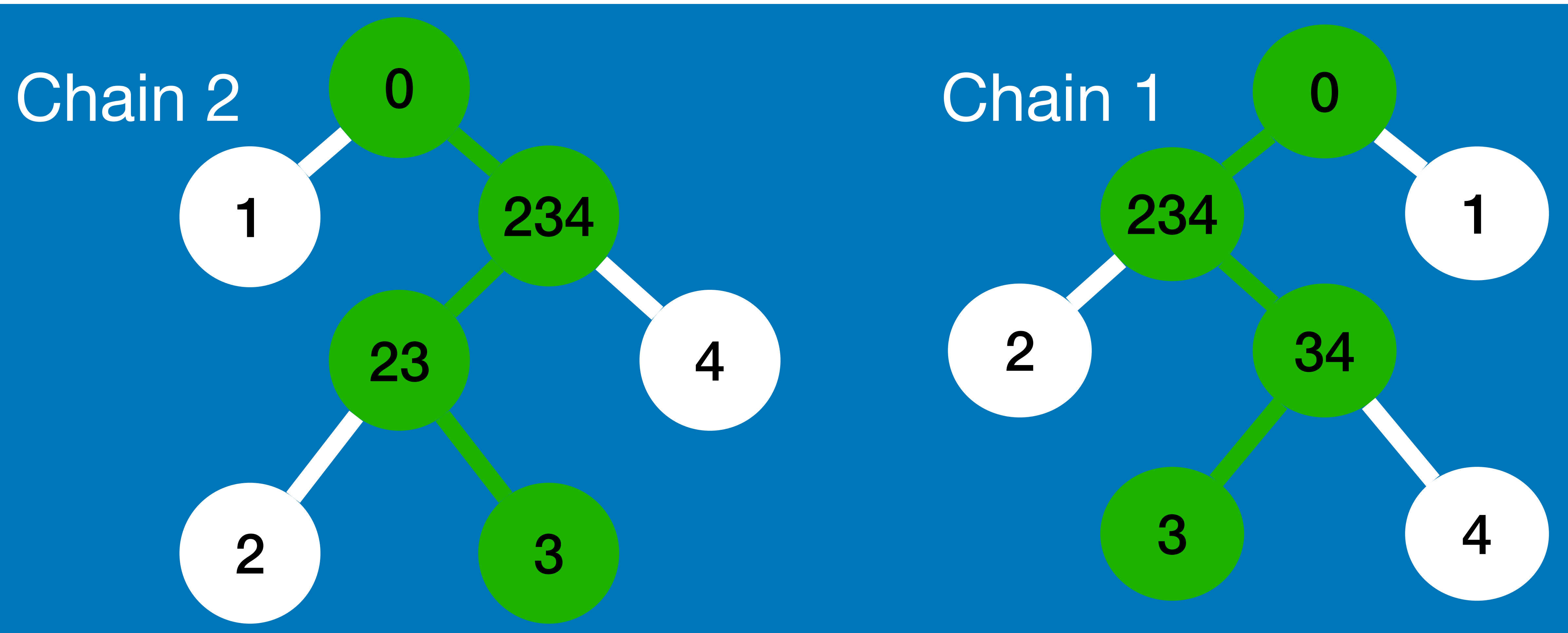
Relevant only for
final state particles with spin!



- Full transformation into rest frame of 3 for topology c:
  $\Lambda(0 \to 3)^c$
- Helicity of particle 3 is defined in particle 3 rest frame

Multiple Topologies and Spin





- Rotate final state frame 1 to final state frame 2 for particle 3:

  $$\Lambda_{\text{rot}} = \Lambda(0 \to 3)^2 \left(\Lambda(0 \to 3)^1\right)^{-1}$$

- Decode rotation angles $\theta_{\text{RF}}, \phi_{\text{RF}}, \psi_{\text{RF}}$ and change basis
- Issue: Spin 1/2 $\to$ 4π for full rotation

Decoding of Angles

Decode rotation angles and boost

$$\mathbf{P}_{\text{boosted}} = \Lambda \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \gamma \end{bmatrix} \Rightarrow \phi, \theta, \xi$$

$$\xi = \cosh^{-1}(\gamma/1) \qquad \theta = \cos^{-1}(p_z/|p|)$$

$$\phi = \tan^{-1}(p_y/p_x)$$

$$\Lambda_{\text{rf}} = \left[R_z(\phi)R_y(\theta)B_z(\xi)\right]^{-1}\Lambda$$

$$= B_z(-\xi)R_y(-\theta)R_z(-\phi)\Lambda$$

$$\Lambda = R_z(\phi)R_y(\theta)B_z(\xi)R_z(\phi_{\text{RF}})R_y(\theta_{\text{RF}})R_z(\psi_{\text{RF}})$$

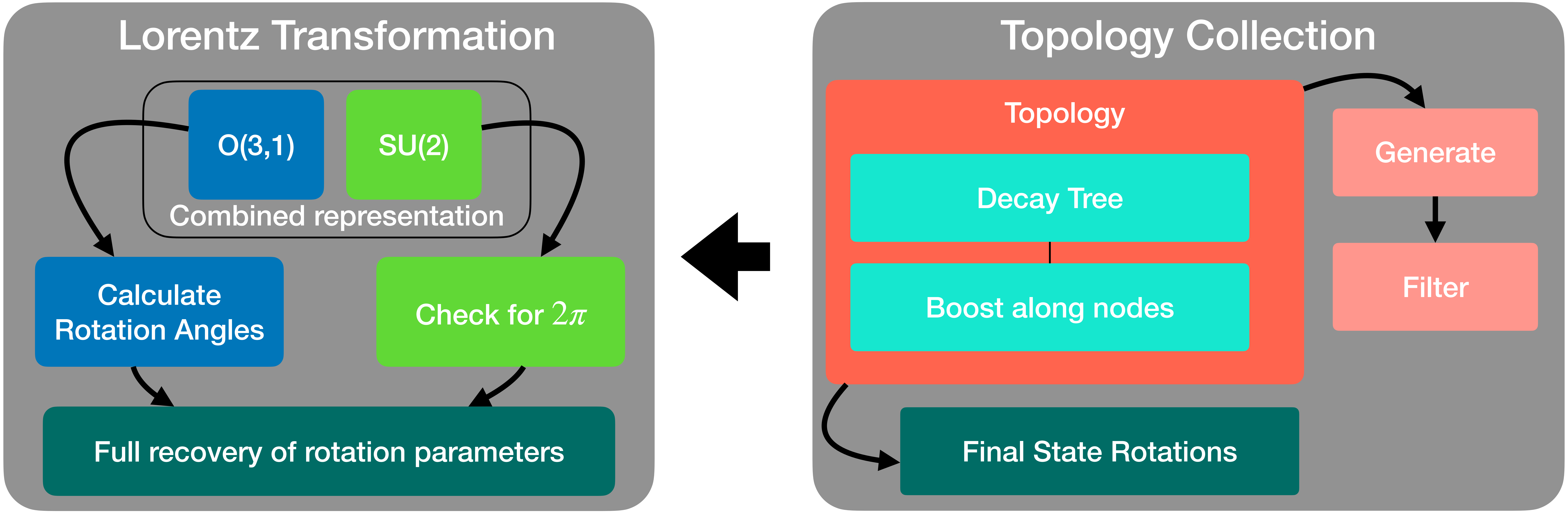
Λ_{rf} Is pure rotation

$\rightarrow$ decoding $\theta_{\text{RF}}, \phi_{\text{RF}}, \psi_{\text{RF}}$ via independent matrix entries

$$\Lambda^{2,2}(\theta, \phi, \xi, \phi_{\text{rf}}, \theta_{\text{rf}}, \psi_{\text{rf}}) = -\Lambda^{2,2}_{\text{orig}}$$

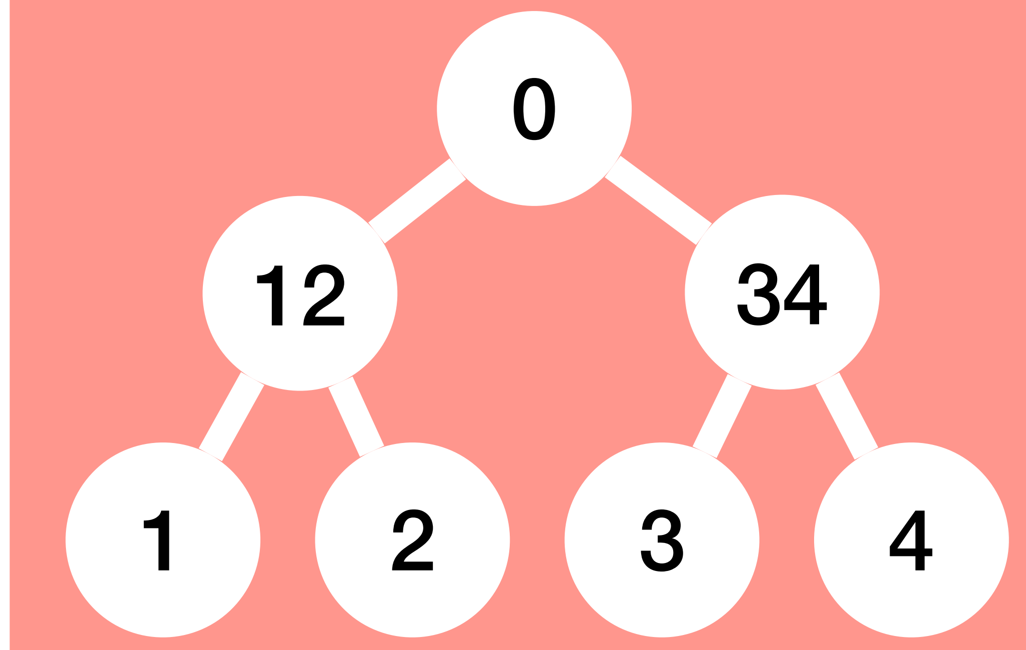
$$\Rightarrow \phi_{\text{rf}} \rightarrow \phi_{\text{rf}} + 2\pi$$

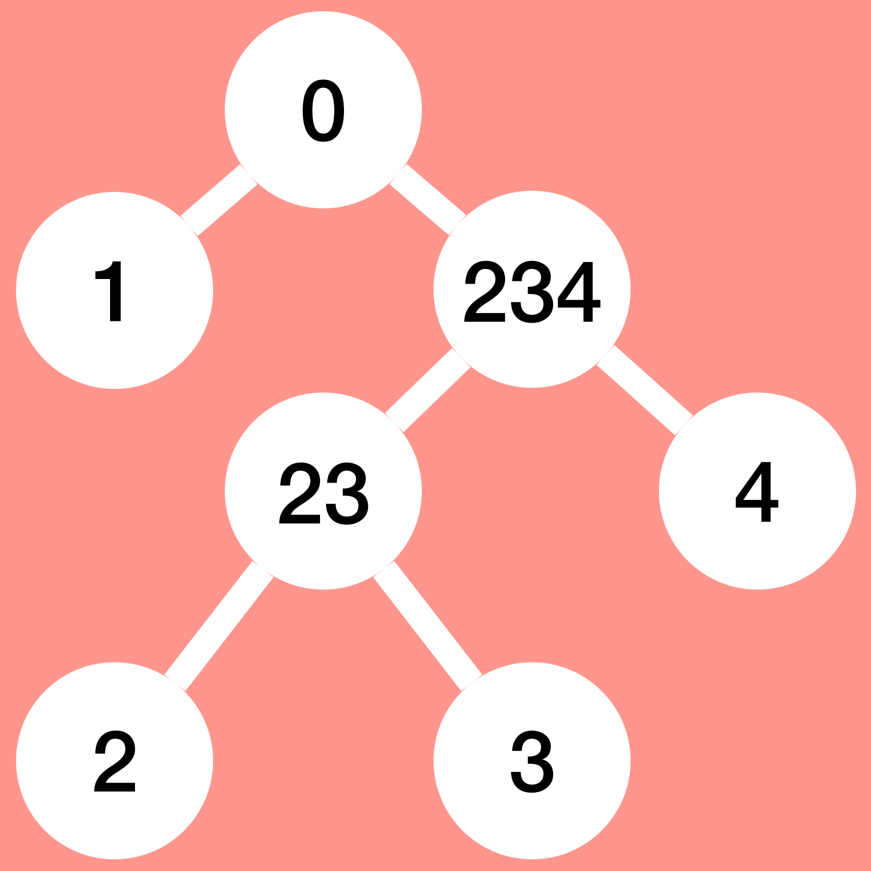
decayangle Software Framework



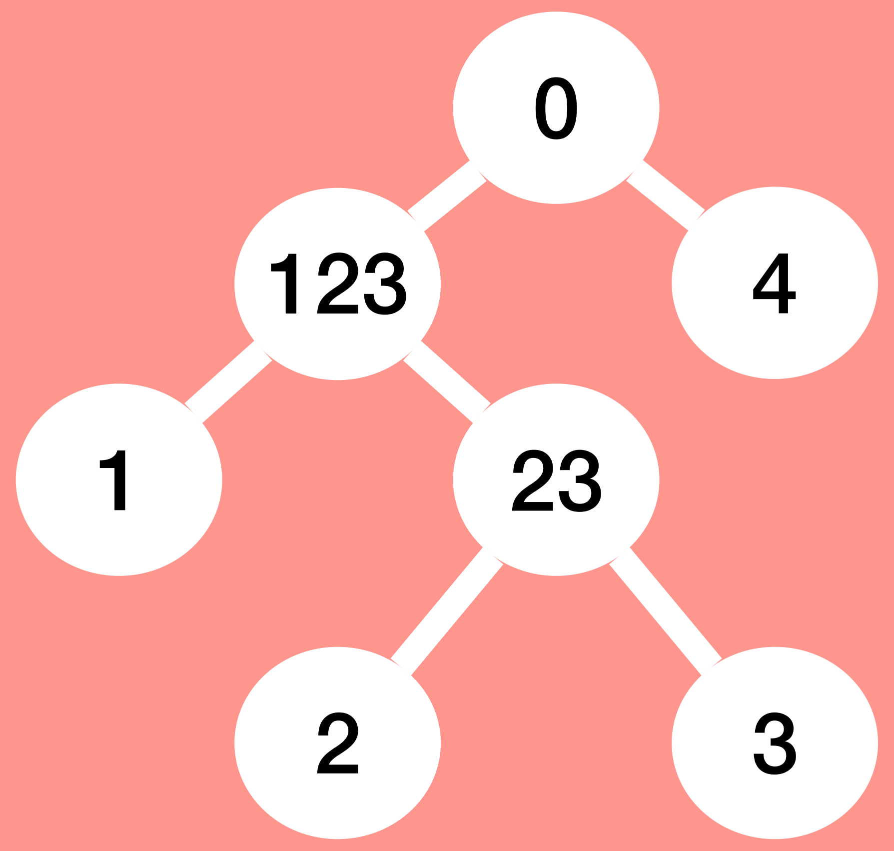
Full computation of helicity angles and relative final state rotations

Topologies

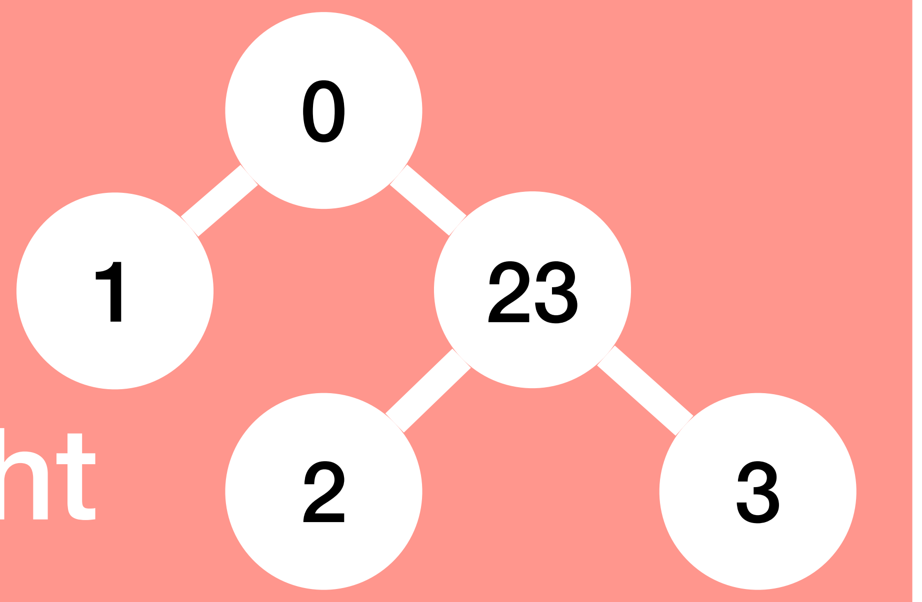




Generate via binary mask operation



- Represent next layer as number with one bit per constituent
- Binary representation i.e. (101) used as mask
  - 1 at position m
    -> constituent m goes left
  - 0 at position m
    -> constituent m goes left right
- Recurse till only one constituent



decayangle Topologies

The basic way to create and filter for the processes of interest

Generate

```
from decayangle.decay_topology import TopologyCollection

tg = TopologyCollection(0, [1, 2, 3]) # generate all decay topologies for 0 -> 1 2 3
tg.topologies # list of all decay topologies
```

Filter

```
tg = TopologyCollection(0, [1,2,3,4])
topologies = tg.filter((2, 1))
for topology in topologies:
    print(topology)
```

```
Topology: ( 0 -> ( (1, 2) -> 1, 2 ), ( (3, 4) -> 3, 4 ) )
Topology: ( 0 -> ( (1, 2, 4) -> ( (1, 2) -> 1, 2 ), 4 ), 3 )
Topology: ( 0 -> ( (1, 2, 3) -> ( (1, 2) -> 1, 2 ), 3 ), 4 )
```

decayangle Topologies

Create the topologies directly by defining the order of decays

Generate

```python
from decayangle.decay_topology import Topology

root = 0
topologies = [
    Topology(root, decay_topology=((1, 2), 3)),
    Topology(root, decay_topology=((1, 3), 2)),
    Topology(root, decay_topology=((2, 3), 1))
]
```

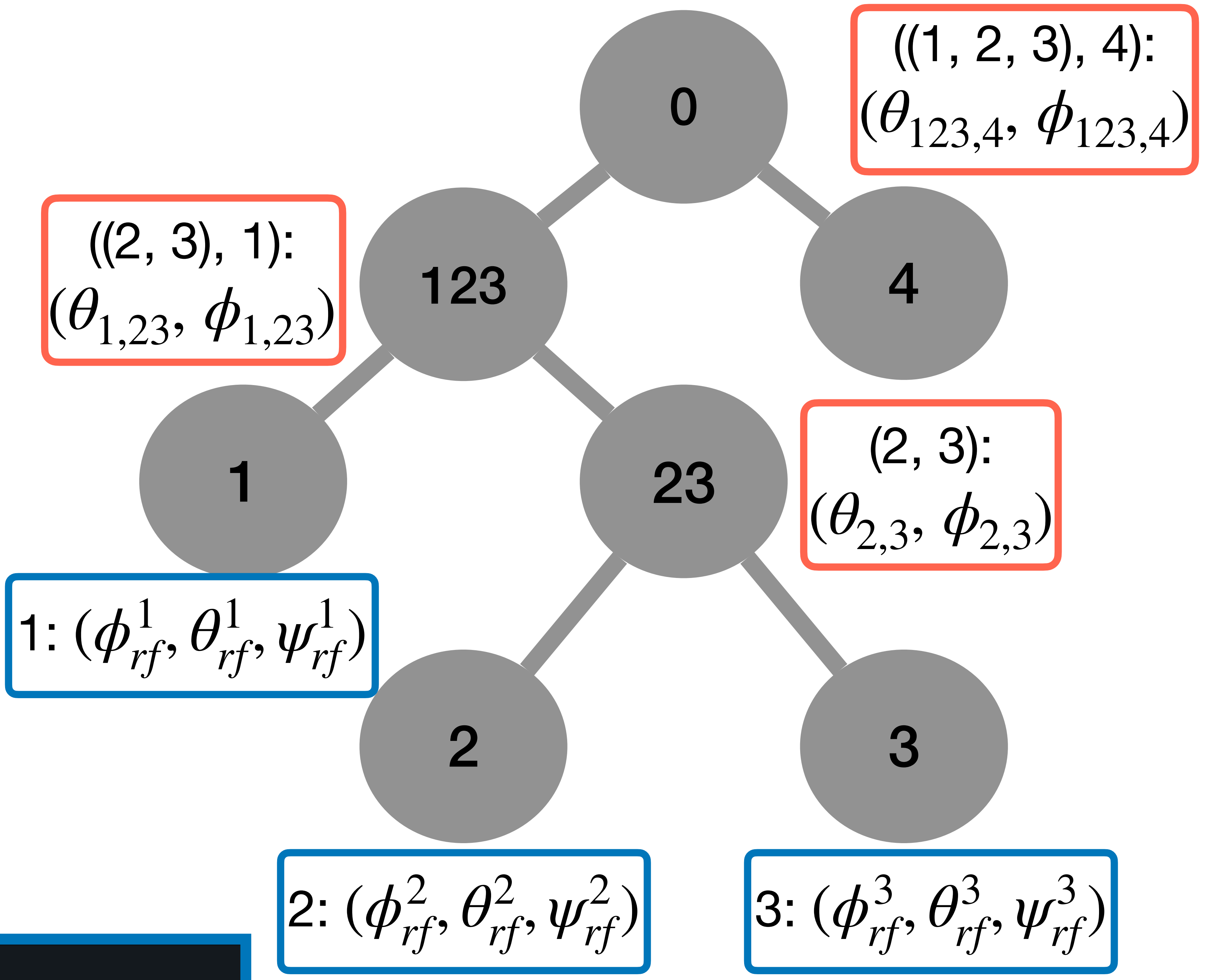
A lot of work and room for error in case of more complicated decays!

decayangle Angles

The computation of helicity angles and final state rotations

```
topology = topologies[0]

# `momenta` is a dict of particle momenta with
#  - key:  the final-state particle number
#  - value: np.ndarray or jax.numpy.ndarray with shape
# (..., 4)
angles = topology.helicity_angles(momenta)
```

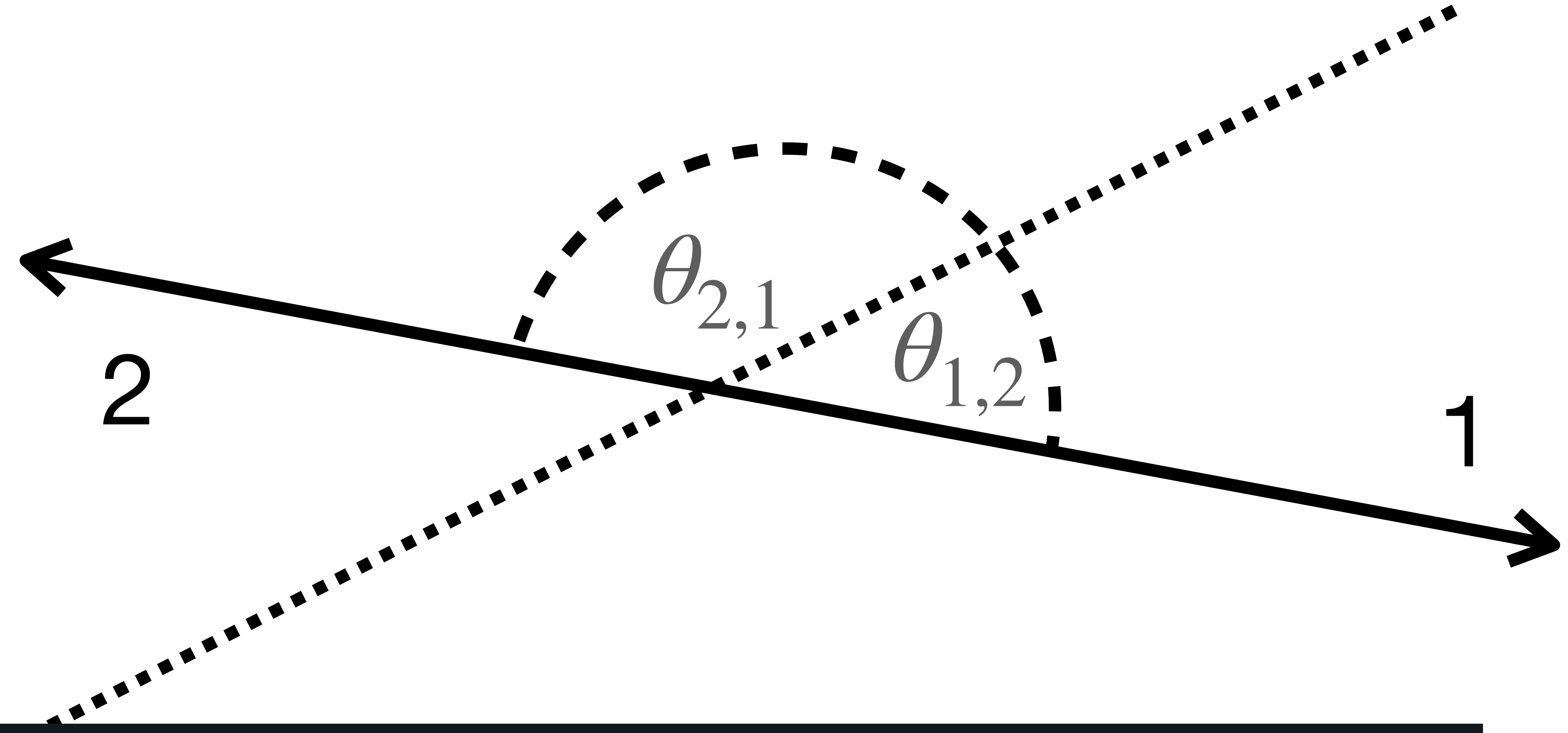



```
reference = topologies[0]
other = topologies[1]

relative_angles = reference.relative_wigner_angles(other, momenta)
```

Node Ordering

Effects of particle ordering




```python
from decayangle.decay_topology import Topology
from decayangle.config import config as cfg
cfg.sorting = "off"

root = 0
topologies = [
    Topology(root, decay_topology=((1, 2), 3)),
    Topology(root, decay_topology=((3, 1), 2)),
    Topology(root, decay_topology=((2, 3), 1))
]
```

- $\theta_{1,2} = \pi - \theta_{2,1}$
  - Ordering can have effect on angles
- decayangle orders by value of node as default
- Ordering can be turned off
  - Warning: generated topologies may have hard to predict ordering, without explicit scheme!

Ordering from Dalitz-plot decomposition
M. Mikhasenko et. Al.

Conclusion

- decayangle offers a easy-to-use solution to acquire all needed angles for an amplitude analysis
- Further support for selection of the desired decay chains is provided
- Approach of combined representations ensures correctness for all spin-carrying particles
- Extensive testing against analytic definition of angles
  - Dalitz Plot Decomposition
  - $\Lambda_c \to pK\pi$ aligned kinematics
  - General ability to reconstruct any set of angles and rapidities
- Available on PyPi:

Conclusion

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Thank you for listening!

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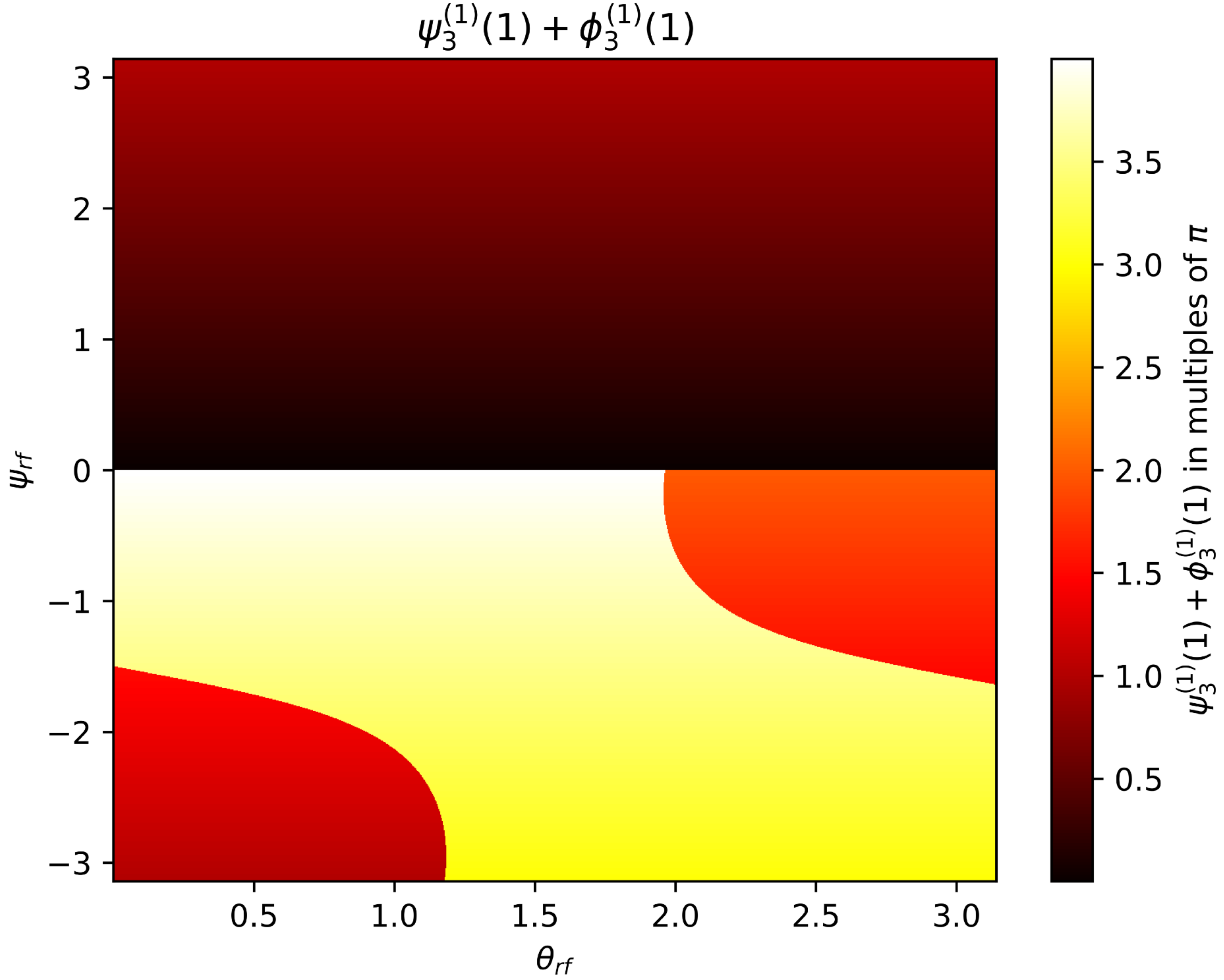
pypi v1.0.2 | python 3.8 | 3.9 | 3.10 | 3.11 | 3.12 | codecov 87%

Further Exploration

$$\Lambda_c \to p\, K\, \pi$$



- Aligned kinematics can be used to generate phase space points from angles ϕ_{rf} and ψ_{rf}
- Discontinuity can be seen nicely
- On the left:
  - $\psi_{rf} + \phi_{rf}$ from the relative Wigner rotation between the topology $0 \to (12 \to 1\ 2)\ 3$ labeled with index 3 and $0 \to (23 \to 2\ 3)\ 1$ labeled with index 1
  - ϕ_{rf} and ψ_{rf} are as found in topology $0 \to (23 \to 2\ 3)\ 1$

Further Exploration