The Jülich-Bonn dynamical coupled-channel approach

29th May, 2024 | Christian Schneider | Institute for Advanced Simulation, Forschungszentrum Jülich

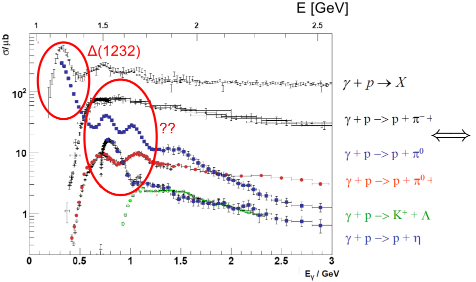
Supervisor: D. Rönchen, C. Hanhart

Supported by MKW NRW (Network NRW FAIR)
HPC support by Jülich Supercomputing Centre

The excited baryon spectrum:

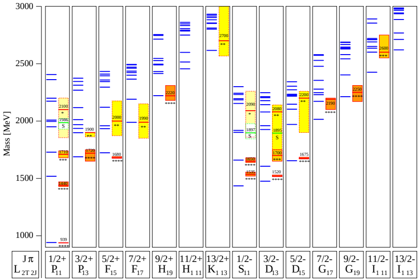
Connection between experiment and QCD in the non-perturbative regime

Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Major source of information:

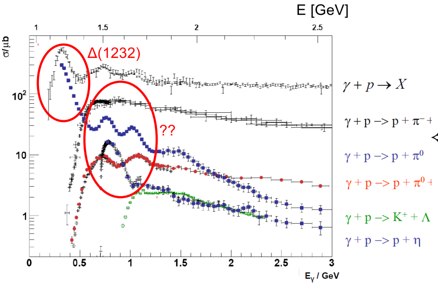
- In the past: πN**-scattering** $\to$ "missing resonance problem"
- In recent years: **photoproduction reactions** $\to$ enlarged data base with high quality (double) polarization observables
- In the future: **electroproduction reactions**

JÜLICH Forschungszentrum

The excited baryon spectrum:

Connection between experiment and QCD in the non-perturbative regime

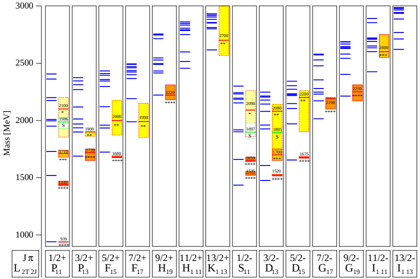
Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

⟺

Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

⇒ Partial wave decomposition:
decompose data with respect to a conserved quantum number:

total angular momentum and parity J^P

⇒ search for resonances/excited states in those partial waves:
poles on the unphysical Riemann sheet

The Jülich-Bonn DCC approach for N^* and Δ^*

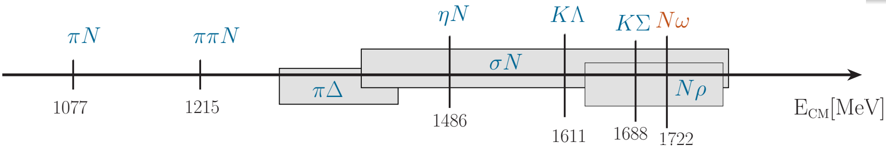
pion-induced reactions

EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- $\pi\pi N$ through effective channels $(\pi\Delta, \sigma N, \rho N)$
  $\Rightarrow$ 2 body unitarity and analyticity respected

JÜLICH Forschungszentrum

The Jülich-Bonn DCC approach for N^* and Δ^*

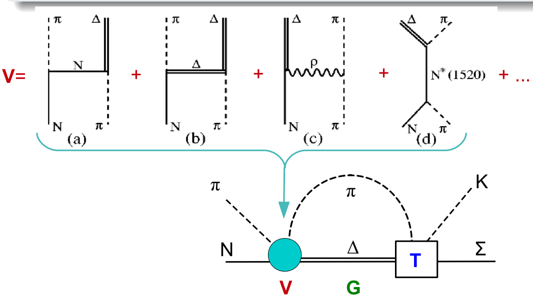
pion-induced reactions

EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P

  genuine resonance states
- t- and u-channel: T^{NP}

  dynamical generation of poles
- contact terms

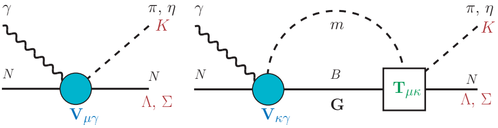
Photoproduction in a semi-phenomenological approach

EPJ A 50, 101 (2015)

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

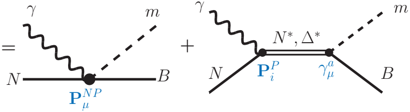


$m = \pi, \eta, K$, $B = N, \Delta, \Lambda$

$T_{\mu\kappa}$: full hadronic T-matrix as in pion-induced reactions

Photoproduction potential: approximated by energy-dependent polynomials (field-theoretical description numerically too expensive)

$\mathbf{V}_{\mu\gamma}(E,q) =$

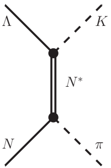


$$= \frac{\tilde{\gamma}^{a}_{\mu}(q)}{m_N} P^{\mathrm{NP}}_{\mu}(E) + \sum_i \frac{\gamma^{a}_{\mu;i}(q) P^{\mathrm{P}}_{i}(E)}{E - m^{b}_{i}}$$

Simultaneous fit of pion- & photon-induced reactions

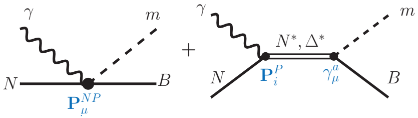
Free parameters

- $\pi N \to \pi N, \eta N, KY$:
  s-channel: resonances (T^P)



$m_{bare} + f_{mBN^*}$

- $\gamma p \to \pi N, \eta N, KY$: couplings of the polynomials and s-channel parameters



- couplings in contact terms: one per PW, couplings to πN, ηN, $\pi\Delta$, $K\Lambda$, $K\Sigma$
- t- & u-channel parameters: cut-offs, mostly fixed to values of previous JüBo studies (couplings fixed from SU(3))

$\Rightarrow$ $>$ 900 fit parameters in total

- large number of fit parameters, many from polynomials
- can be regarded as advantage: prevents the inclusion of superfluous s-channel states to improve fit
- χ^2-minimization using Minuit on a supercomputer [JURECA, JSC, Journal of large-scale research facilities, 2, A**62** (2016)]

Two potential formalism

$$T_{\mu\nu} = T^{P}_{\mu\nu} + T^{NP}_{\mu\nu} \qquad V_{\mu\nu} = V^{P}_{\mu\nu} + V^{NP}_{\mu\nu}$$

- Non pole part (”background”): $V^{NP}_{\mu\nu}$ $\sim$ t- and u-channels

$$T^{NP}_{\mu\nu} = V^{NP}_{\mu\nu} + \sum_{\kappa} V^{NP}_{\mu\kappa} G_{\kappa} T^{NP}_{\kappa\nu}$$ (numerically demanding)

- Pole part (resonances): s-channels

$$V^{P}_{\mu\nu} = \frac{\gamma^{a}_{\mu}\gamma^{c}_{\nu}}{z - m_b}$$ $\gamma^{a,c}_{\mu}$ $\sim$ bare annihilation/creation vertex, m_b $\sim$ bare mass

$$T^{P}_{\mu\nu} = \frac{\Gamma^{a}_{\mu}\Gamma^{c}_{\nu}}{z - m_b - \Sigma}$$ $\Rightarrow$ T^P evaluated from T^{NP}

with dressed vertices $\Gamma^{c}_{\mu} = \gamma^{c}_{\mu} + \sum_{\nu} \gamma^{c}_{\nu} G_{\nu} T^{NP}_{\nu\mu}$, $\Gamma^{a}_{\mu} = \gamma^{a}_{\mu} + \sum_{\nu} T^{NP}_{\mu\nu} G_{\nu} \gamma^{a}_{\nu}$
and self-energy $\Sigma = \sum_{\mu} \gamma^{c}_{\mu} G_{\mu} \Gamma^{a}_{\mu}$,

- Set of ”fast” parameters in T^P optimized for each step in ”slow” parameters
$\rightarrow$ ”Nested” fit strategy

JüBo2024: Data base

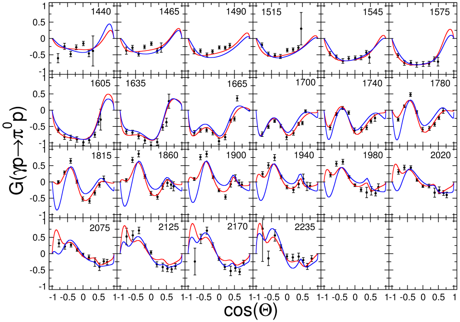
Reaction	Observables (# data points)	p./channel
$\pi N \to \pi N$	PWA GW-SAID WI08 (ED solution)	8,396
$\pi^- p \to \eta n$	$d\sigma/d\Omega$ (676), P (79)	755
$\pi^- p \to K^0\Lambda$	$d\sigma/d\Omega$ (814), P (472), β (72)	1,358
$\pi^- p \to K^0\Sigma^0$	$d\sigma/d\Omega$ (470), P (120)	590
$\pi^- p \to K^+\Sigma^-$	$d\sigma/d\Omega$ (150)	150
$\pi^+ p \to K^+\Sigma^+$	$d\sigma/d\Omega$ (1124), P (551) , β (7)	1,682
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (3287), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393+198), H (225), E (1227+495), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	27,355
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5670), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86+217), H (128), E (903)	9,674
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112+320), Σ (535+80), P (63), T (291), F (144),	
	E (306), G (47), H (56)	10,954
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2563), P (1663), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	6,072
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4381), P (402), Σ (280)	
	T (127) , $C_{x'}$ (94), $C_{z'}$ (94), O_x (127), O_z (127)	5,632
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (281), P (167)	448
	in total	73,066

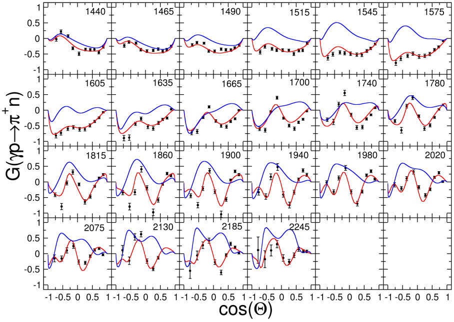
JÜLICH Forschungszentrum

Recent updates to JüBo I

preliminary

- Double polarization observable $\mathbb{G}$ for $\vec{\gamma}\vec{p} \to \pi^0 p$ and $\vec{\gamma}\vec{p} \to \pi^+ n$ [CLAS, Phys. Lett. B 817 (2021) 136304]
  (blue: 2022, red: 2024)

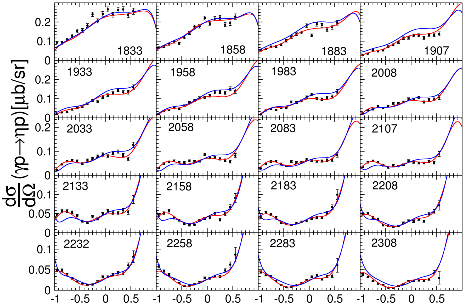


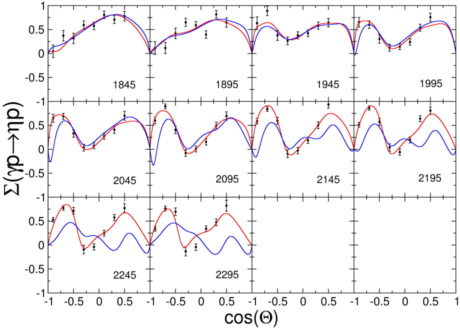


Recent updates to JüBo II

preliminary

- $\frac{d\sigma}{d\Omega}$ and Σ for $\vec{\gamma} p \to \eta p$ [LEPS, Phys. Rev. C 106 (2022) 3, 035201]
  (blue: 2022, red: 2024)

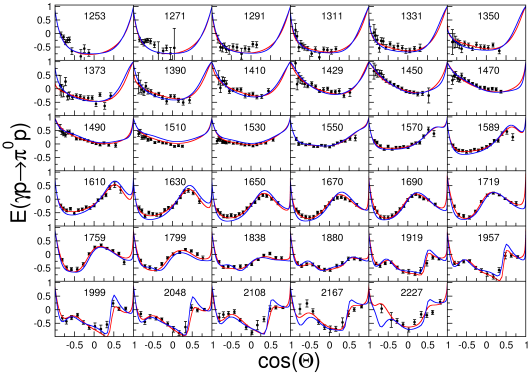




Recent updates to JüBo III

preliminary

- Double-spin-polarization observable $\mathbb{E}$ for $\vec{\gamma}\vec{p} \to \pi^0 p$ [CLAS, Eur.Phys.J.A 59 (2023) 9, 217]
  (blue: 2022, red: 2024)



Change of Pole Positions

preliminary

$N(1710)$ 1/2$^+$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1586.8	107.6	2.8	−108.0
2022	1605 ± 14	115 ± 9	5.5 ± 4.7	-114 ± 57
PDG 2024	1700 ± 50	120 ± 40	7 ± 3	190 ± 70

$N(1520)$ 3/2$^-$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1496.3	100.4	24.4	−18.2
2022	1482 ± 6	126 ± 18	27 ± 21	-36 ± 48
PDG 2024	1510 ± 5	112.5 ± 7.5	35 ± 3	-10 ± 5

$\Delta(1600)$ 3/2$^+$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1592.8	84.2	9.7	−114.4
2022	1590 ± 1	136 ± 1	11 ± 1	-106 ± 2
PDG 2024	1520 ± 50	280 ± 40	25 ± 15	210 ± 30

$\Delta(1700)$ 3/2$^-$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1680.3	360.2	38.0	−6.2
2022	1637 ± 64	295 ± 58	15 ± 23	-13 ± 147
PDG 2024	1665 ± 25	250 ± 50	25 ± 15	-20 ± 20

$N(1900)$ 3/2$^+$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1903.5	141	1.07	−95.9
2022	1905 ± 3	93 ± 4	1.6 ± 0.3	44 ± 21
PDG 2024	1920 ± 20	130 ± 40	4 ± 2	-10 ± 30

$N(1720)$ 3/2$^+$ ∗∗∗∗	Re E_0 [MeV]	−2Im E_0 [MeV]	$\|r\|_{\pi N\to\pi N}$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]
2024	1698.5	132.7	9.7	−8.2
2022	1726 ± 8	185 ± 12	15 ± 2	-60 ± 5
PDG 2024	1680 ± 20	200 ± 50	15 ± 5	-110 ± 50

GDH sum rule

- Photoproduction process $\gamma N \to X$ can be characterized in terms of integrals of cross-sections
- For circularly polarized photons on longitudinally polarized nucleons either $\Delta\sigma = \sigma_{3/2} - \sigma_{1/2}$ or $\sigma_{tot} = \sigma_{3/2} + \sigma_{1/2}$

Gerasimov Drell Hearn (GDH) sum rule

$$I_{GDH} = \int_{E_\gamma^{thr}}^{\infty} \frac{\Delta\sigma}{E_\gamma}\,\mathrm{d}E_\gamma = \frac{2\pi^2\alpha}{M^2}\kappa^2$$

Ann. Rev. Nucl. Part. Sci. 54, 69 (2004).

$\alpha = \frac{e_0^2}{4\pi}$

E_γ^{thr} $\sim$ pion photoproduction threshold,

M $\sim$ nucleon mass,

$\kappa_p = \mu_p - 1,\ \kappa_n = \mu_n$

- Values for photoproduction on p or n targets: [I. Strakovsky et al. Phys.Rev.C 105 (2022) 4, 045202]

$$I_{GDH,p} = 204.784482(35)\mu\mathrm{b}$$
$$I_{GDH,n} = 232.25159(13)\mu\mathrm{b}$$

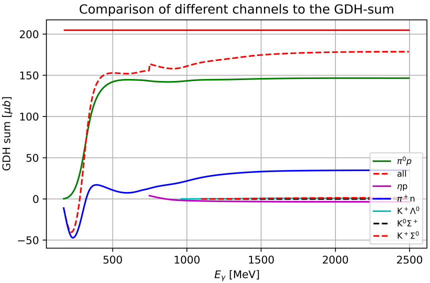
GDH sum rule - JüBo

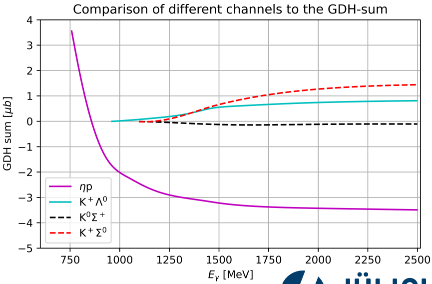
preliminary

- From JüBo 2024 fits for $\gamma p \to X$

$$I_{GDH} = \int_{E_\gamma^{thr}}^{E_\gamma} \frac{\Delta\sigma}{E'_\gamma}\,\mathrm{d}E'_\gamma, \qquad E_\gamma \text{ : Upper integration limit}$$

- $\pi^0 p$ main contribution, followed by $\pi^+ n$
- ηp mainly negative
- missing contribution expected from 2π-channels





Summary

Jülich-Bonn DCC model:

- Extraction of the N^* and Δ^* spectrum in a simultaneous analysis of pion- and photon-induced reactions:

  - $\pi N \to \pi N, \eta N, K\Lambda$ and $K\Sigma$
  lagrangian based description, unitarity & analyticity respected

  - $\gamma N \to \pi N, \eta N, K\Lambda$ and $K\Sigma$ in a semi-phenomenological approach
  hadronic final state interaction: JüBo DCC analysis

  $\to$ analysis of $\sim$73,000 data points

- New data sets and preliminary fit updates 2024
- GDH sum rule contribution of different channels

Outlook:

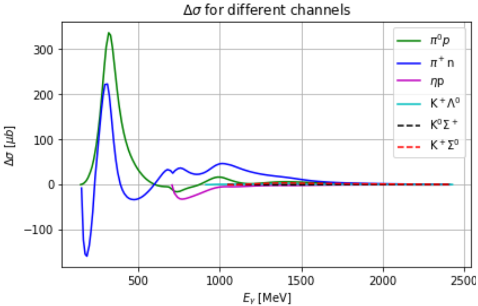
- Include $\gamma n \to X$ and calculate GDH sum rule for these processes
- Simultaneous fit of pion-, photon-induced and electroproduction data.

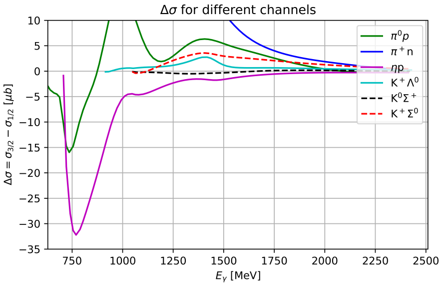
Thank you for your attention!

Appendix

GDH sum rule - JuBo Δ_{31}

preliminary

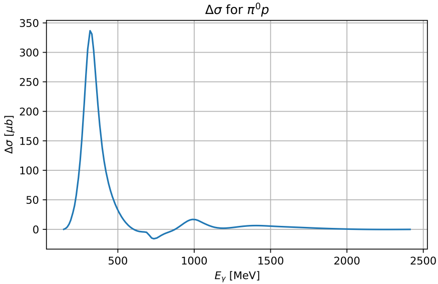


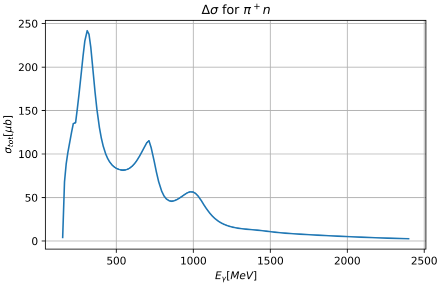


JÜLICH Forschungszentrum

GDH sum rule - JuBo Δ_{31}

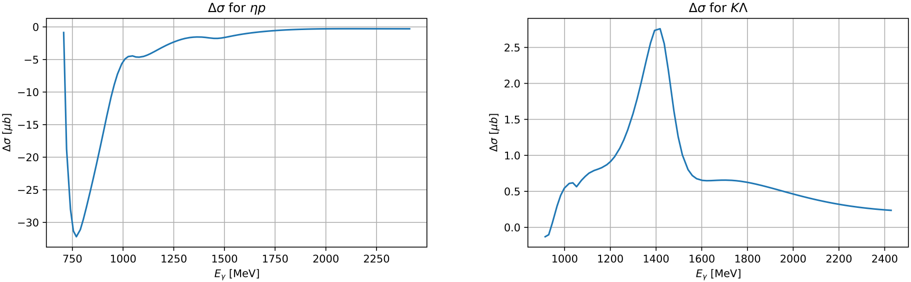
preliminary





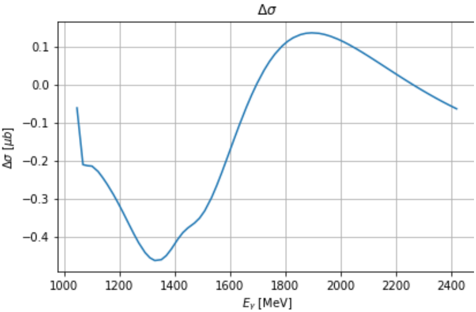
GDH sum rule - JuBo Δ_{31}

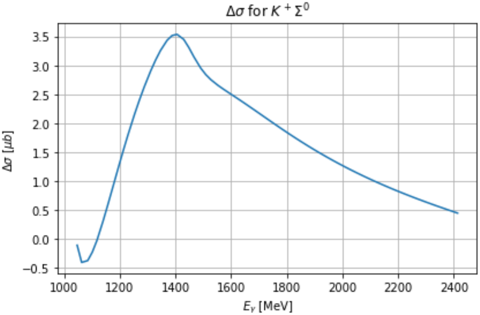
preliminary



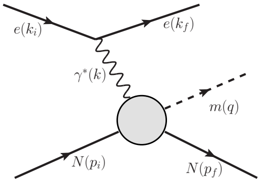
GDH sum rule - JuBo Δ_{31}

preliminary





Electroproduction of pseudoscalar mesons



Construction of the multipole amplitude $M^{IJ}_{\mu\gamma}$

Different approaches

- **Field theoretical** approaches : DMT, ANL-Osaka, **Jülich-Athens-Washington**, ...

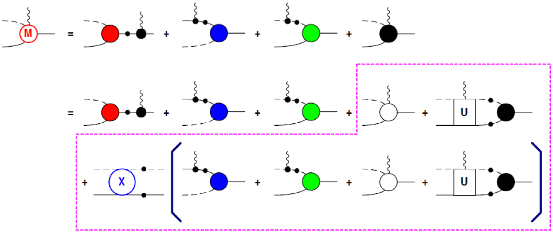
  Example: Gauge invariant formulation by Haberzettl, Huang and Nakayama

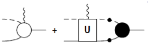
  Phys. Rev. C56 (1997), Phys. Rrev. C74 (2006), Phys. Rev. C85 (2012)

  - satisfies the generalized off-shell Ward-Takahashi identity
  - earlier version of the Jülich-Bonn model as FSI

Photoproduction amplitude:

$$M^\mu = \underbrace{M^\mu_s + M^\mu_u + M^\mu_t}_{\text{coupling to external legs}} + \underbrace{M^\mu_{int}}_{\text{coupling inside hadronic vertex}}$$



Strategy: Replace 

by phenomenological contact term such that the generalized WTI is satisfied

- Alternative gauge invariant chiral unitary method: Borasoy *et al.*, Eur. Phys. J. A **34** (2007) 161

Details of the formalism

Polynomials:

$$P_i^{\mathsf{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathsf{P}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathsf{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathsf{NP}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{\mu,n+1}^{\mathsf{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathsf{P}}$, $g_{\mu,j}^{\mathsf{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

back

Construction of the potential V: phenomenological vs fieldtheoretical

Phenomenological

- implementation easier (e.g. polynomials)
- numerically advantageous

Fieldtheoretical

- development based on $\mathcal{L}$ complicated, numerically demanding
- information on the dynamical content
- in case of incomplete data base: model constrained by well-established physics
  $\rightarrow$ minimize uncertainties due to lack of complete data / high-quality data
- 3-body unitarity requires discontinuities from t-channel ex. simultaneously with discontinuities from s-channels
  $\rightarrow$ meson ex. arises naturally from requirements of the S-matrix
- make predictions

... depends on your goal and your resources (data, computing power)

The scattering potential: s-channel resonances

$$V^{\mathrm{P}} = \sum_{i=0}^{n} \frac{\gamma^a_{\mu;i}\,\gamma^c_{\nu;i}}{z - m_i^b}$$

- i: resonance number per PW
- $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$): creation (annihilation) vertex function
  with **bare coupling** f (free parameter)
- z: center-of-mass energy
- m_i^b: **bare mass** (free parameter)

Vertex	$\mathcal{L}_{int}$
$N^*(S_{11})N\pi$	$\frac{f}{m_\pi}\,\bar\Psi_{N^*}\,\gamma^\mu\,\vec\tau\,\partial_\mu\vec\pi\,\Psi \;+$ h.c.
$N^*(S_{11})N\eta$	$\frac{f}{m_\pi}\,\bar\Psi_{N^*}\,\gamma^\mu\,\partial_\mu\eta\,\Psi \;+$ h.c.
$N^*(S_{11})N\rho$	$f\,\bar\Psi_{N^*}\,\gamma^5\gamma^\mu\,\vec\tau\,\vec\rho_\mu\,\Psi \;+$ h.c.
$N^*(S_{11})\Delta\pi$	$\frac{f}{m_\pi}\,\bar\Psi_{N^*}\,\gamma^5\,\vec S\partial_\mu\vec\pi\,\Delta^\mu \;+$ h.c.

- $J \leq 3/2$:

  $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$) from effective $\mathcal{L}$

- $5/2 \leq J \leq 9/2$:
  correct dependence on L (centrifugal barrier)

$$(\gamma^{a,c})_{\frac{5}{2}-} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{5}{2}+} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

$$(\gamma^{a,c})_{\frac{7}{2}-} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}-} \qquad (\gamma^{a,c})_{\frac{7}{2}+} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}+}$$

$$(\gamma^{a,c})_{\frac{9}{2}-} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{9}{2}+} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

The scattering potential: t- and u-channel exchanges

	πN	ρN	ηN	πΔ	σN	KΛ	KΣ
πN	N,Δ,$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$	N, Δ, Ct., π, ω, a_1	N, a_0	N, Δ, ρ	N, π	Σ, Σ*, K*	Λ, Σ, Σ*, K*
ρN		N, Δ, Ct., ρ	-	N, π	-	-	-
ηN			N, f_0	-	-	K*, Λ	Σ, Σ*, K*
πΔ				N, Δ, ρ	π	-	-
σN					N, σ	-	-
KΛ						Ξ, Ξ*, f_0, ω, ϕ	Ξ, Ξ*, ρ
KΣ							Ξ, Ξ*, f_0, ω, ϕ, ρ

Free parameters: cutoffs Λ in the form factors: $F(q) = \left(\frac{\Lambda^2 - m_x^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^n, n = 1,\ 2$

Interaction potential from effective Lagrangian

J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); U.-G. Meißner, Phys. Rept. **161**, 213 (1988); B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A **11**, 5183 (1996).

- consistent with the approximate (broken) chiral $SU(2) \times SU(2)$ symmetry of QCD

Vertex	$\mathcal{L}_{int}$	Vertex	$\mathcal{L}_{int}$
$NN\pi$	$-\frac{g_{NN\pi}}{m_\pi}\Psi\gamma^5\gamma^\mu\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi$	$NN\omega$	$-g_{NN\omega}\bar{\Psi}[\gamma^\mu - \frac{\kappa_\omega}{2m_N}\sigma^{\mu\nu}\partial_\nu]\omega_\mu\Psi$
$N\Delta\pi$	$\frac{g_{N\Delta\pi}}{m_\pi}\bar{\Delta}^\mu\vec{S}^\dagger\cdot\partial_\mu\vec{\pi}\Psi$ + h.c.	$\omega\pi\rho$	$\frac{g_{\omega\pi\rho}}{m_\omega}\epsilon_{\alpha\beta\mu\nu}\partial^\alpha\vec{\rho}^\beta\cdot\partial^\mu\vec{\pi}\omega^\nu$
$\rho\pi\pi$	$-g_{\rho\pi\pi}(\vec{\pi}\times\partial_\mu\vec{\pi})\cdot\vec{\rho}^\mu$	$N\Delta\rho$	$-i\frac{g_{N\Delta\rho}}{m_\rho}\bar{\Delta}^\mu\gamma^5\gamma^\mu\vec{S}^\dagger\cdot\vec{\rho}_{\mu\nu}\Psi$ + h.c.
$NN\rho$	$-g_{NN\rho}\Psi[\gamma^\mu - \frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\partial_\nu]\vec{\tau}\cdot\vec{\rho}_\mu\Psi$	$\rho\rho\rho$	$g_{NN\rho}(\vec{\rho}_\mu\times\vec{\rho}_\nu)\cdot\vec{\rho}^{\mu\nu}$
$NN\sigma$	$-g_{NN\sigma}\bar{\Psi}\Psi\sigma$	$NN\rho\rho$	$\frac{\kappa_\rho g^2_{NN\rho}}{2m_N}\bar{\Psi}\sigma^{\mu\nu}\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\rho}_\nu)$
$\sigma\pi\pi$	$\frac{g_{\sigma\pi\pi}}{2m_\pi}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\sigma$	$\Delta\Delta\pi$	$\frac{g_{\Delta\Delta\pi}}{m_\pi}\bar{\Delta}_\mu\gamma^5\gamma^\nu\vec{T}\Delta^\mu\partial_\nu\vec{\pi}$
$\sigma\sigma\sigma$	$-g_{\sigma\sigma\sigma}m_\sigma\sigma\sigma\sigma$	$\Delta\Delta\rho$	$-g_{\Delta\Delta\rho}\bar{\Delta}_\tau(\gamma^\mu - i\frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta}\sigma^{\mu\nu}\partial_\nu)$
			$\cdot\vec{\rho}_\mu\cdot\vec{T}\Delta^\tau$
$NN\rho\pi$	$\frac{g_{NN\pi}}{m_\pi}2g_{NN\rho}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\pi})$	$NN\eta$	$-\frac{g_{NN\eta}}{m_\pi}\bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\eta\Psi$
NNa_1	$-\frac{g_{NN\pi}}{m_\pi}m_{a_1}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi\vec{a}_\mu$	NNa_0	$g_{NNa_0}m_\pi\bar{\Psi}\vec{\tau}\Psi\vec{a_0}$
$a_1\pi\rho$	$-\frac{2g_{\pi a_1\rho}}{m_{a_1}}[\partial_\mu\vec{\pi}\times\vec{a}_\nu - \partial_\nu\vec{\pi}\times\vec{a}_\mu]\cdot[\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu]$	$\pi\eta a_0$	$g_{\pi\eta a_0}m_\pi\eta\vec{\pi}\cdot\vec{a}_0$
	$+\frac{2g_{\pi a_1\rho}}{2m_{a_1}}[\vec{\pi}\times(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)]\cdot[\partial^\mu\vec{a}^\nu - \partial^\nu\vec{a}^\mu]$		

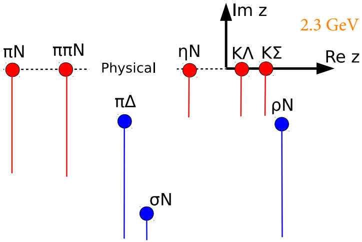
Thresholds of inelastic channels

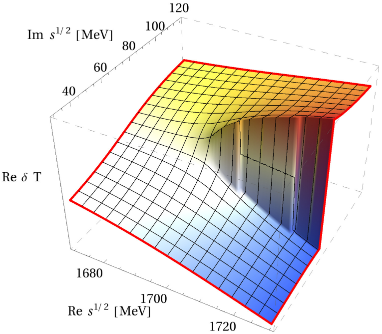
- (2 body) unitarity and analyticity respected (no on-shell factorization, dispersive parts included)
- opening of **inelastic channels** $\Rightarrow$ **branch point** and new **Riemann sheet**

3-body $\pi\pi N$ channel:

- parameterized effectively as $\pi\Delta$, σN, ρN
- $\pi N/\pi\pi$ subsystems fit the respective phase shifts

↳ branch points move into complex plane





Example: ρN branch point at

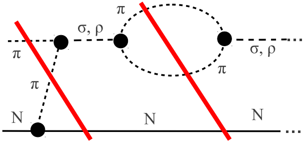
$$M_N + m_{rho} = 1700 \pm i75 \text{ MeV}$$

Inclusion of branch points important to avoid false resonance signal!

Theoretical constraints of the S-matrix

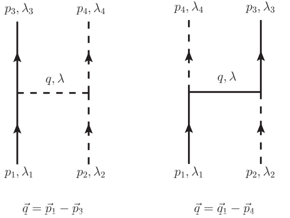
Unitarity: probability conservation

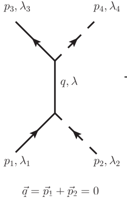
- 2-body unitarity
- 3-body unitarity:
  discontinuities from t-channel exchanges
  → Meson exchange from requirements of the S-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]



Analyticity: from unitarity and causality

- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon u-channel exchange





→ Resonances

Inclusion of the $\pi N \to \omega N$ channel

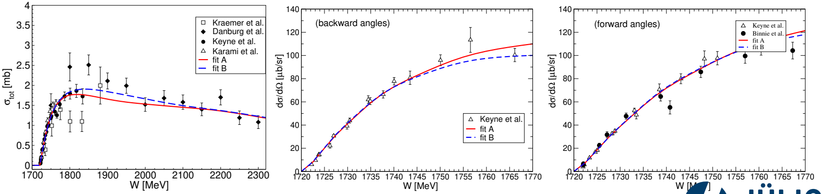
Yu-Fei Wang *et al.*

Motivation

- Completion of the Jülich model (not far above the previously highest threshold $K\Sigma$) $\to$ refined analyses of the hadron spectra
- Preparation of the study of $\gamma N \to \omega N$ (abundant high quality data)
- importance of ω in nuclear matter [H. Shen et al. 1998 NPA]
- Scattering length $a_{\omega N} \to$ whether or not there are in-medium bound states

Numerical fit

- 304 parameters (38 new), over 10000 data points (178 new)
- Two fit scenarios:
  - fit A – non-pole parameters close to the previous solution
  - fit B – non-pole parameters are changed more
- Selected fit results: Total cross section, backward/forward differential cross section



Data: Kraemer et al. 1964 PR, Danburg et al. 1970 PRD, Binnie et al. 1973 PRD, Keyne et al. 1976 PRD, Karami et al. 1979 NPB

Further Improvements

- **elastic πN channel**: not data but GWU SAID PWA

Correlated χ^2 fit

$$\chi^2(A) = \chi^2(\hat{A}) + (A - \hat{A})^T \hat{\Sigma}^{-1} (A - \hat{A})$$

PRC 93, 065205 (2016)

$\hat{\Sigma}$ ~ covariance matrix
A ~ vector of fitted PWs,
$\hat{A}$ ~ vector of SAID SE PWs

$\rightarrow$ same χ^2 as fitting to data up to nonlinear order (included)
$\rightarrow$ needed for error analysis

- "Missing resonance problem" $\rightarrow$ What resonances are relevant?

Least Absolute Shrinkage and Selection Operator

$$\chi_T^2 = \chi^2 + \lambda \sum_{i=1}^{i_{max}} |a_i|$$

[PRC 95, 015203 (2017); J. R. Stat. Soc. B 58, 267 (1996)]

λ ~ penalty factor,
a_i ~ fit parameter

$\rightarrow$ LASSO to find minimal model
$\rightarrow$ Numerically very demanding!

JÜLICH Forschungszentrum

JüBo2022: Data base

Eur.Phys.J.A 58 (2022) 229

Reaction	Observables (# data points)	p./channel
$\pi N \to \pi N$	PWA GW-SAID WI08 (ED solution)	8,396
$\pi^- p \to \eta n$	$d\sigma/d\Omega$ (676), P (79)	755
$\pi^- p \to K^0 \Lambda$	$d\sigma/d\Omega$ (814), P (472), β (72)	1,358
$\pi^- p \to K^0 \Sigma^0$	$d\sigma/d\Omega$ (470), P (120)	590
$\pi^- p \to K^+ \Sigma^-$	$d\sigma/d\Omega$ (150)	150
$\pi^+ p \to K^+ \Sigma^+$	$d\sigma/d\Omega$ (1124), P (551) , β (7)	1,682
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (3287), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393), H (225), E (1227), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	26,662
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5670), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86), H (128), E (903)	9,457
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (535), P (63), T (291), F (144),	
	E (306), G (47), H (56)	10,554
$\gamma p \to K^+ \Lambda$	$d\sigma/d\Omega$ (2563), P (1663), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	6,072
$\gamma p \to K^+ \Sigma^0$	$d\sigma/d\Omega$ (4381), P (402), Σ (280)	
	T (127) , $C_{x'}$ (94), $C_{z'}$ (94), O_x (127), O_z (127)	5,632
$\gamma p \to K^0 \Sigma^+$	$d\sigma/d\Omega$ (281), P (167)	448
	in total	71,756