Gen—T

GENERALITAT VALENCIANA

Decays of heavy mesons: a theoretical perspective using effective field theories

David Alejandro Barón Ospina

PWA13/ATHOS8

2024

Special acknowledge

- Patricia Camargo Magalhães
- Diego Alejandro Milanes Carreño

Document available at:

https://repositorio.unal.edu.co/handle/unal/85546

Outline

1) Motivation

2) The $D_S^+ \to K^+K^-K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Outline

1) Motivation

2) The $D_S^+ \to K^+K^-K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Motivation

Motivation

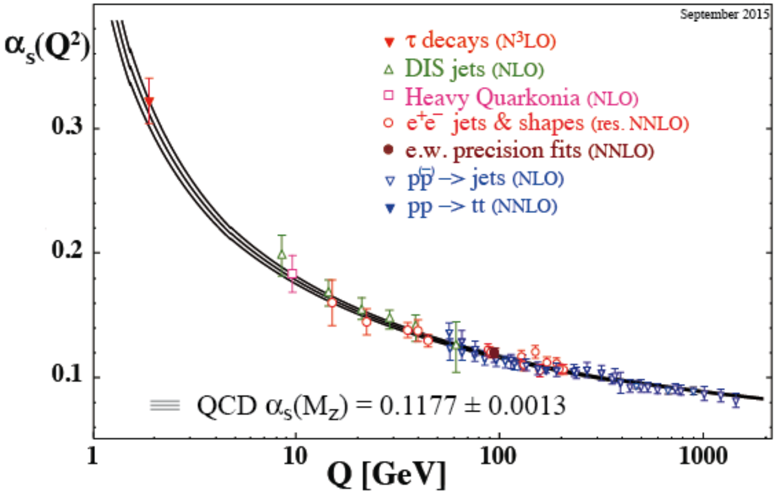


Image taken from :
https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Motivation

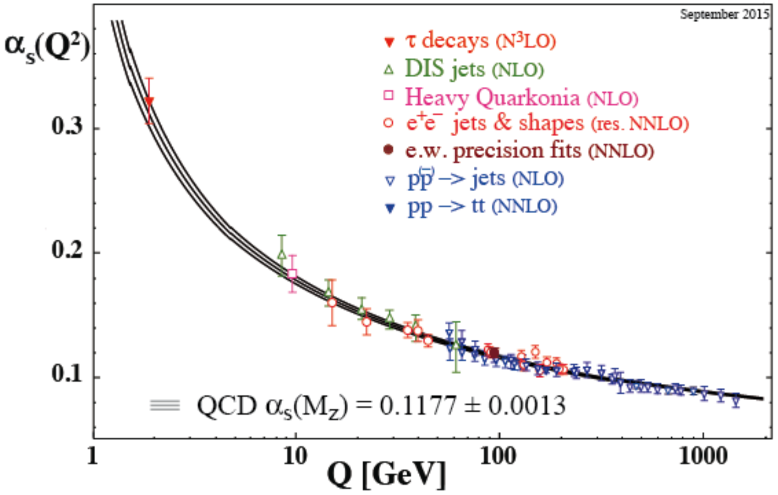


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Motivation

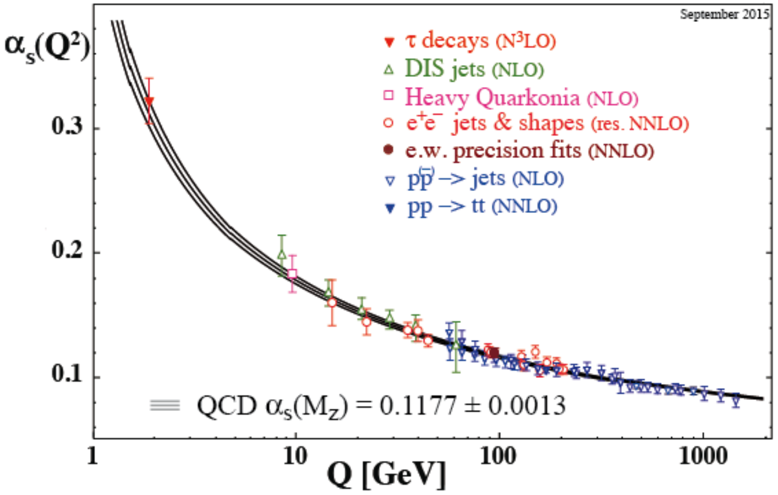


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t$$

Gen–T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Motivation

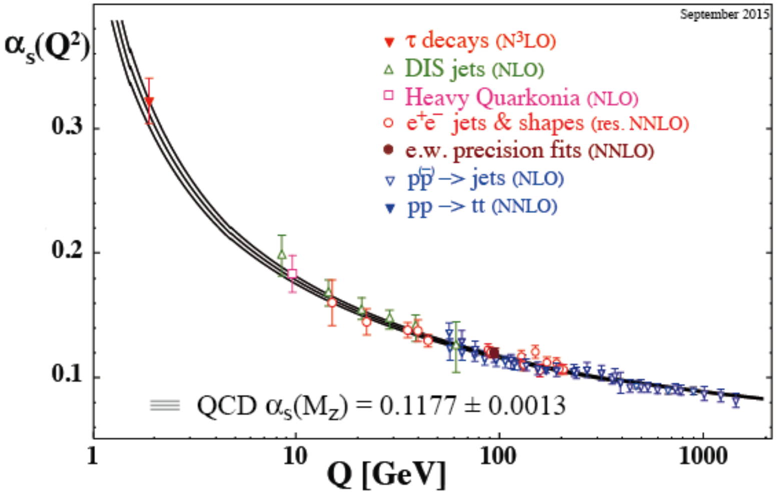


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t \quad !$$

Motivation

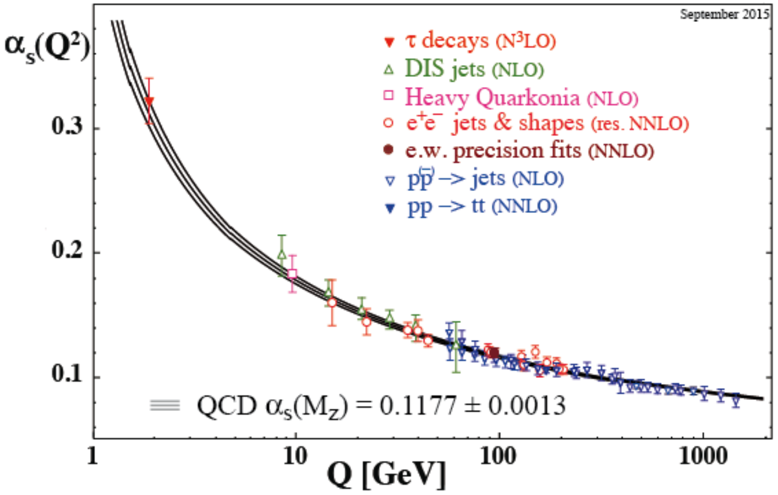


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t \quad !$$

New degrees of freedom

Motivation

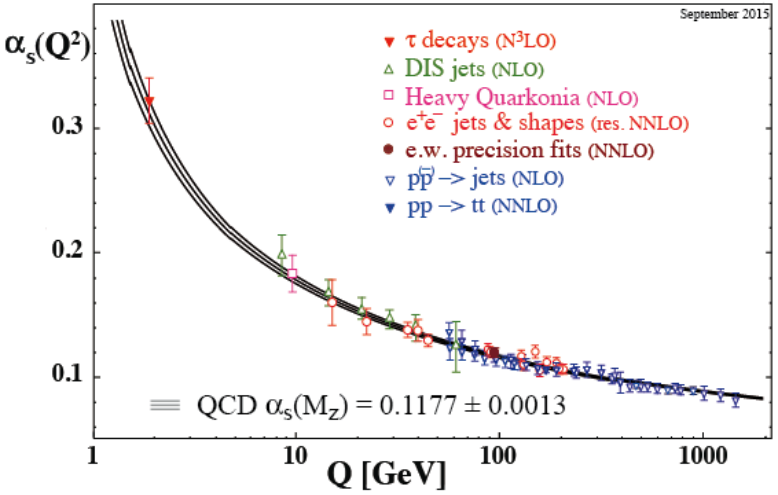


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t \ \ !$$

New degrees of freedom

$$\pi, K, \eta, D, B, \ldots..$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Motivation

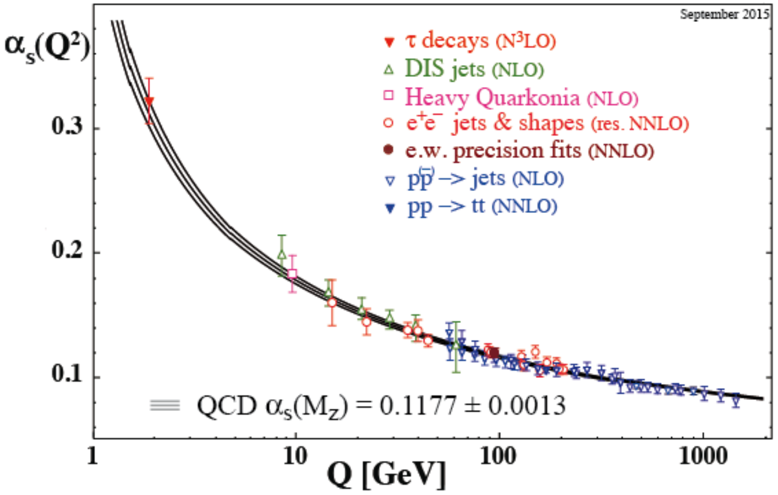


Image taken from :
https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t \quad !$$

New degrees of freedom

$$\underbrace{\pi, K, \eta}_{\chi pt}, D, B, \ldots..$$

Gen–T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Motivation

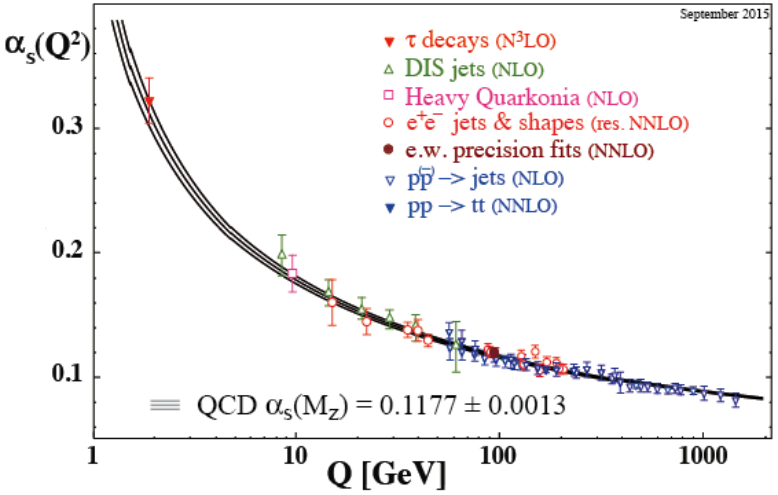


Image taken from : https://www.researchgate.net/figure/Summary-of-measurements-of-a-s-as-a-function-of-the-energy-scale-Q-The-respective-degree_fig2_287249926

Degrees of freedom

$$u, d, s, c, b, t \;\; !$$

New degrees of freedom

$$\underbrace{\pi, K, \eta}_{\chi pt}, D, B, \ldots..$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

1) Motivation

2) The $D_S^+ \to K^+ K^- K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

1) Motivation

2) The $D_S^+ \to K^+K^-K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

The $D_s^+ \to K^+K^-K^+$ decay

The $D_s^+ \to K^+K^-K^+$ decay

Topologies

The $D_s^+ \to K^+K^-K^+$ decay

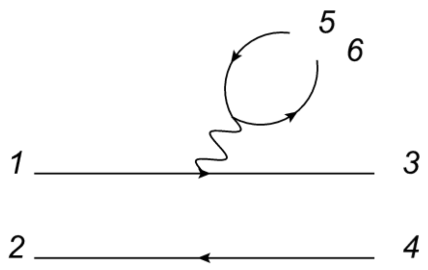
Topologies

L. Chau. Quark Mixing in Weak Interactions, 1983

The $D_s^+ \to K^+K^-K^+$ decay

Topologies

L. Chau. Quark Mixing in Weak Interactions, 1983

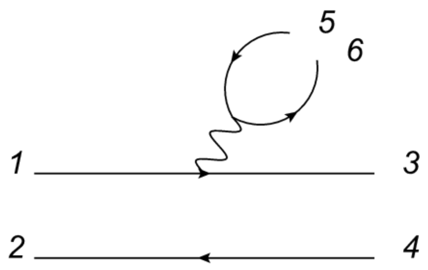


External W-emission

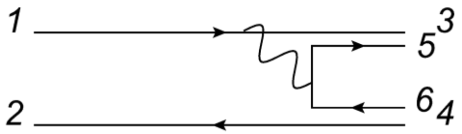
The $D_s^+ \to K^+K^-K^+$ decay

Topologies

L. Chau. Quark Mixing in Weak Interactions, 1983



External W-emission

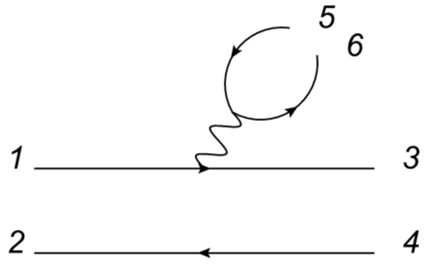


Internal W-emission

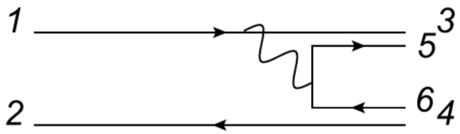
The $D_s^+ \to K^+ K^- K^+$ decay

Topologies

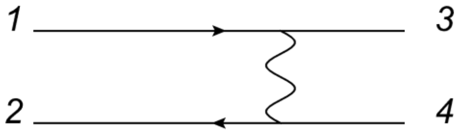
L. Chau. Quark Mixing in Weak Interactions, 1983



External W-emission



Internal W-emission

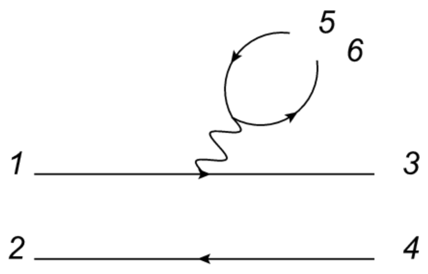


W-exchange

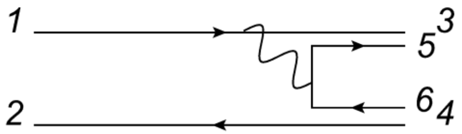
The $D_s^+ \to K^+K^-K^+$ decay

Topologies

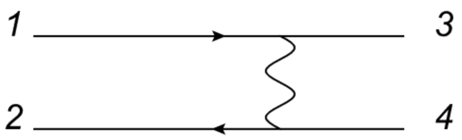
L. Chau. Quark Mixing in Weak Interactions, 1983



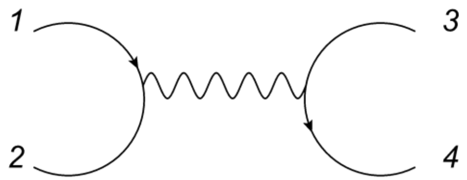
External W-emission



Internal W-emission



W-exchange

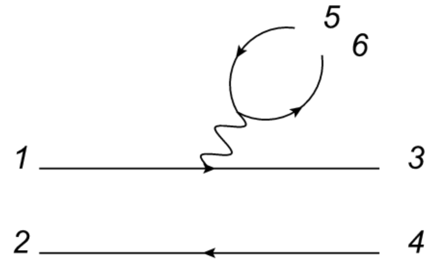


W-annihilation

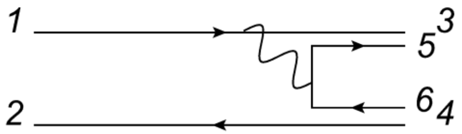
The $D_s^+ \to K^+ K^- K^+$ decay

Topologies

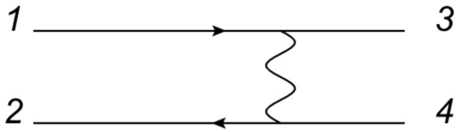
L. Chau. Quark Mixing in Weak Interactions, 1983



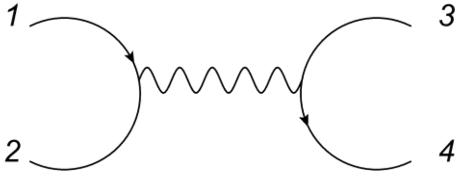
External W-emission



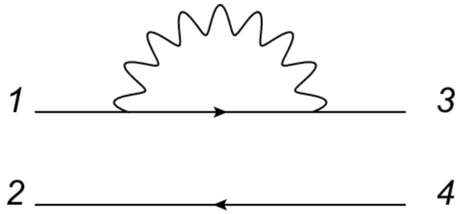
Internal W-emission



W-exchange



W-annihilation



Penguin

Gen—T

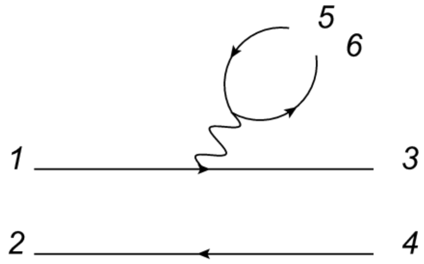
IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

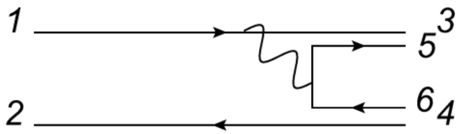
The $D_s^+ \to K^+ K^- K^+$ decay

Topologies

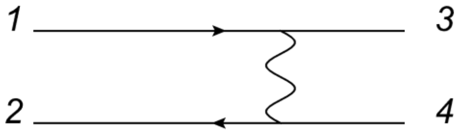
L. Chau. Quark Mixing in Weak Interactions, 1983



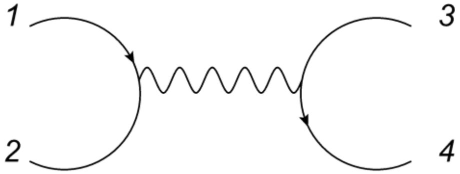
External W-emission



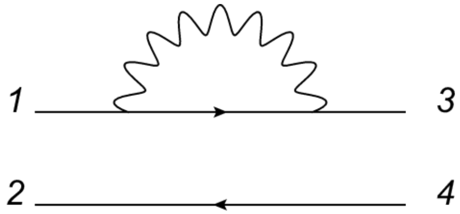
Internal W-emission



W-exchange



W-annihilation



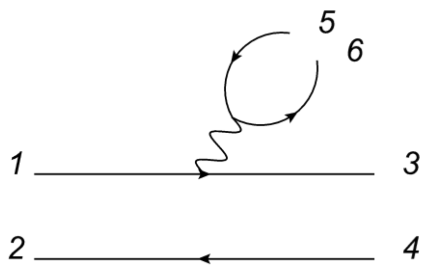
Penguin

Sideways Penguin

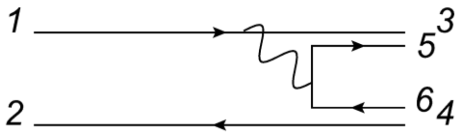
The $D_s^+ \to K^+K^-K^+$ decay

Topologies

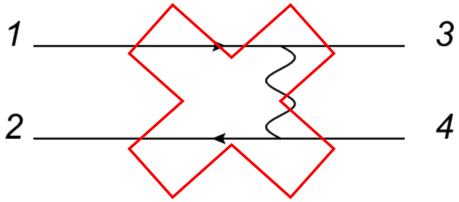
L. Chau. Quark Mixing in Weak Interactions, 1983



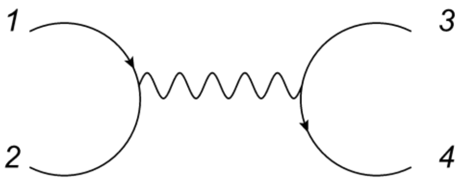
External W-emission



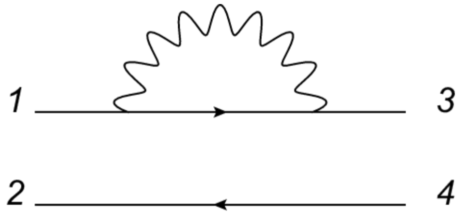
Internal W-emission



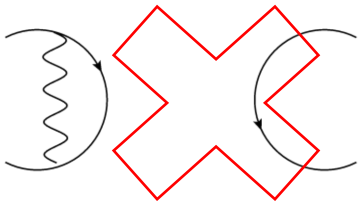
W-exchange



W-annihilation



Penguin

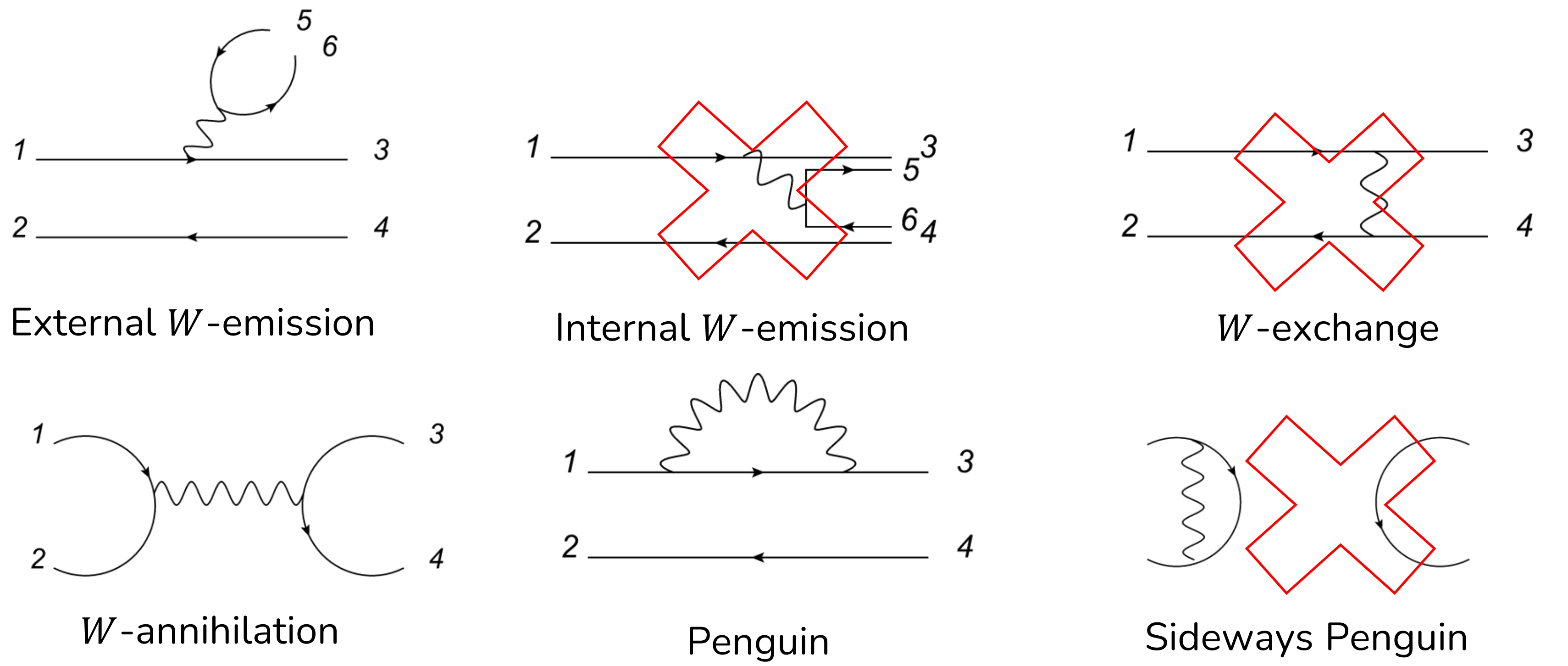


Sideways Penguin

The $D_s^+ \to K^+ K^- K^+$ decay

Topologies

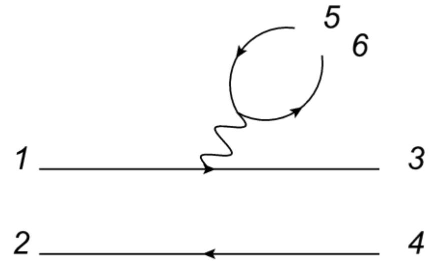
L. Chau. Quark Mixing in Weak Interactions, 1983



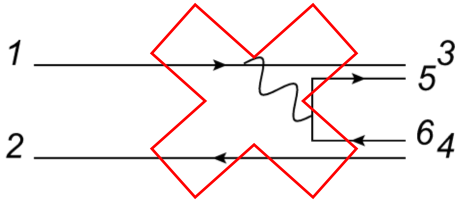
The $D_s^+ \to K^+K^-K^+$decay

Topologies

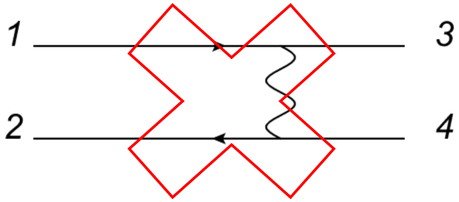
L. Chau. Quark Mixing in Weak Interactions, 1983



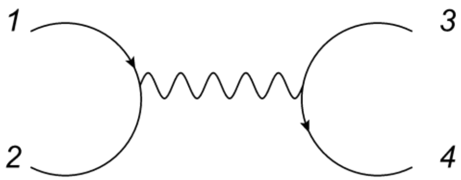
External W-emission



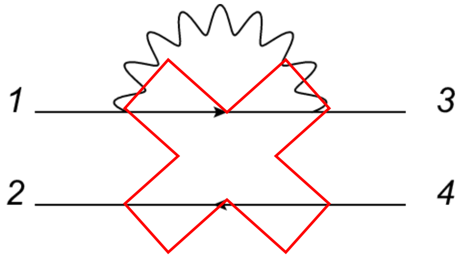
Internal W-emission



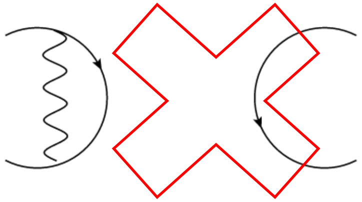
W-exchange



W-annihilation



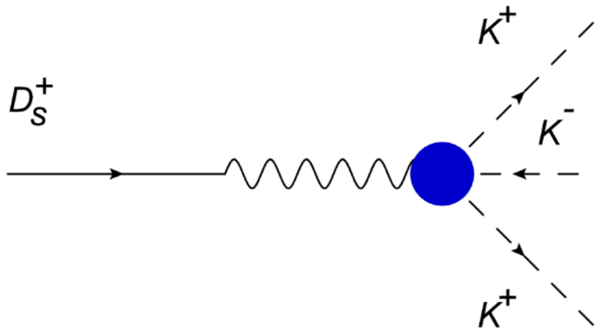
Penguin

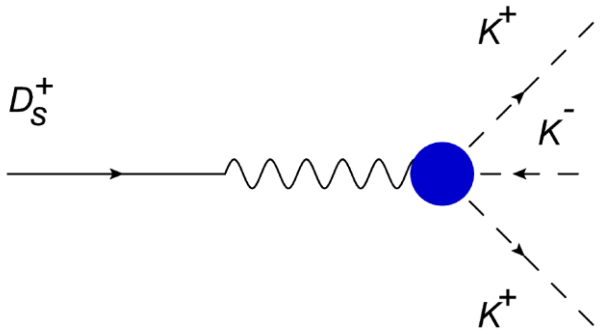

Sideways Penguin

Gen-T

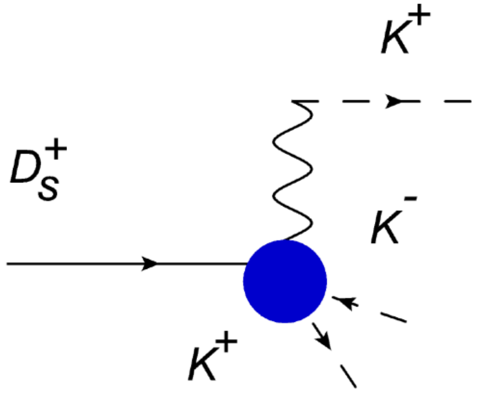
IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

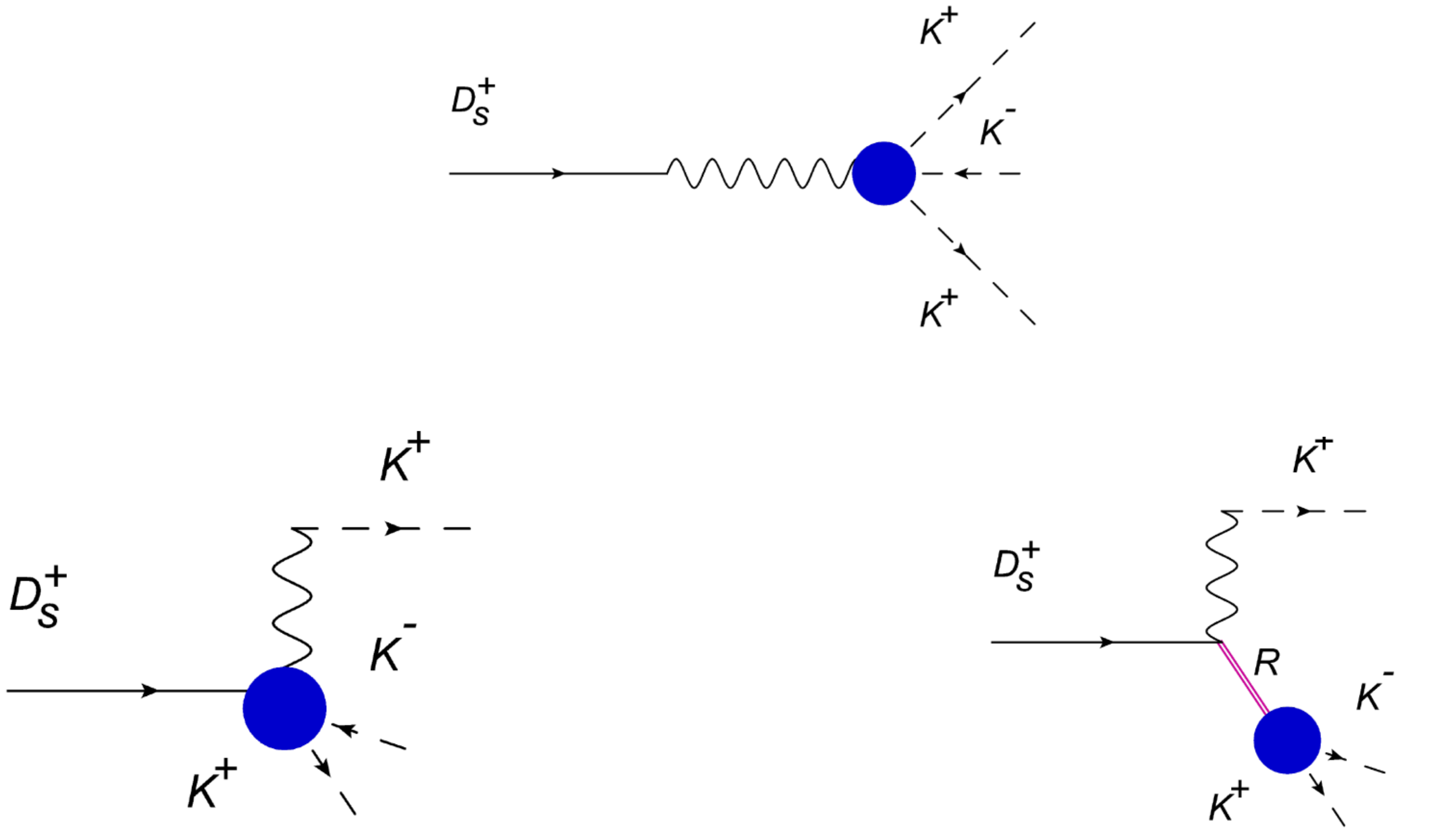

D_s^+
K^+
K^-
K^+



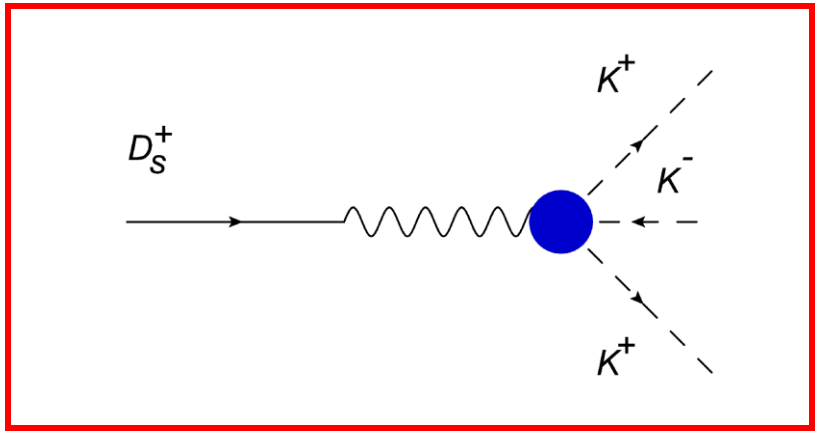
D_s^+
K^+
K^-
K^+



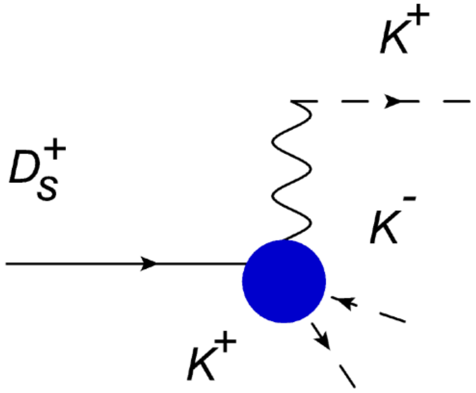
D_s^+
K^+
K^-
K^+

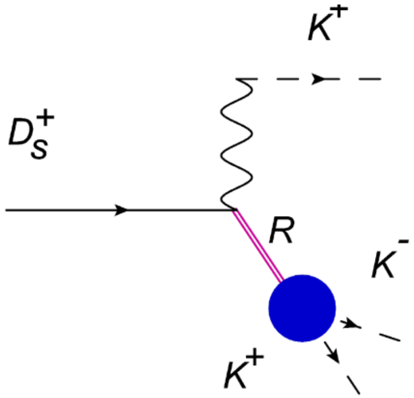


D_s^+
K^+
K^-
K^+
K^+
D_s^+
K^-
K^+
K^+
D_s^+
R
K^-
K^+



R. T. Aude et al. Multimeson model for the $D^+ \rightarrow K^+K^-K^+$decay amplitude, 2018

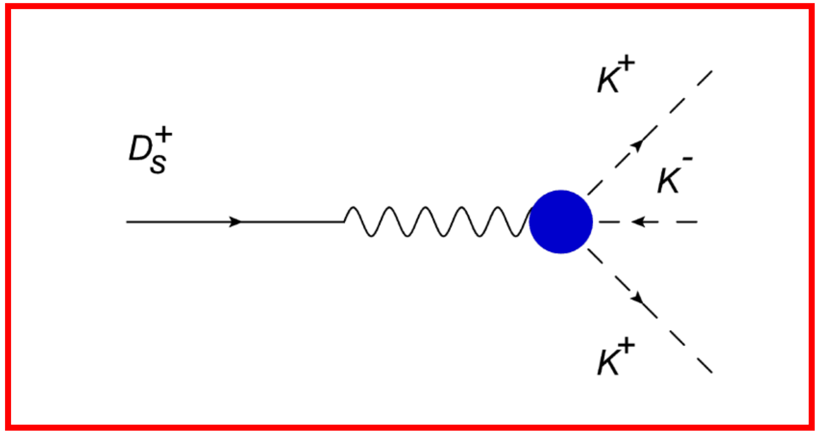




Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

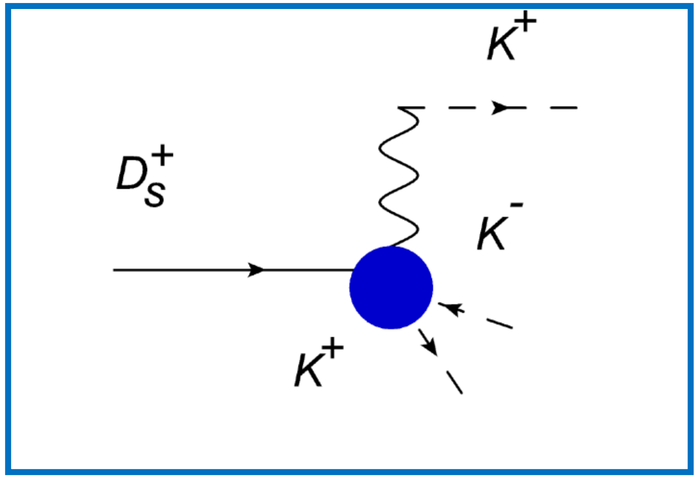


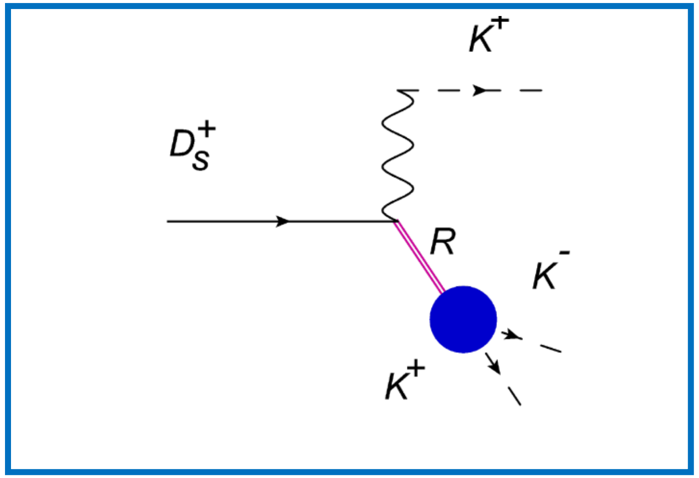
R. T. Aude et al. Multimeson

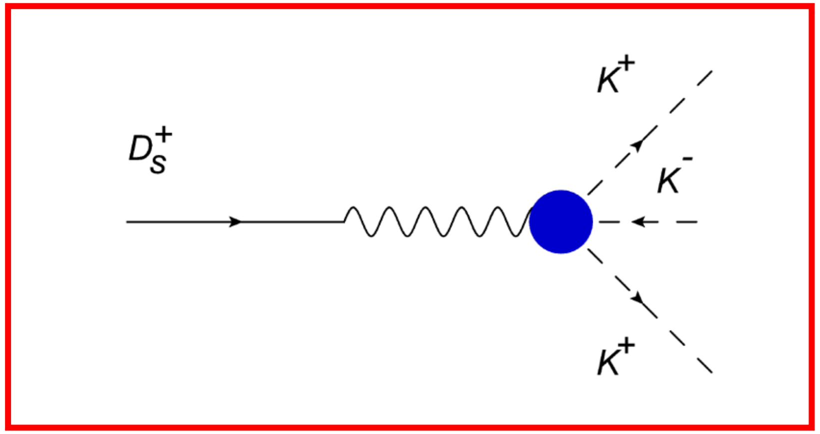
model for the

$D^+ \rightarrow K^+K^-K^+$decay amplitude

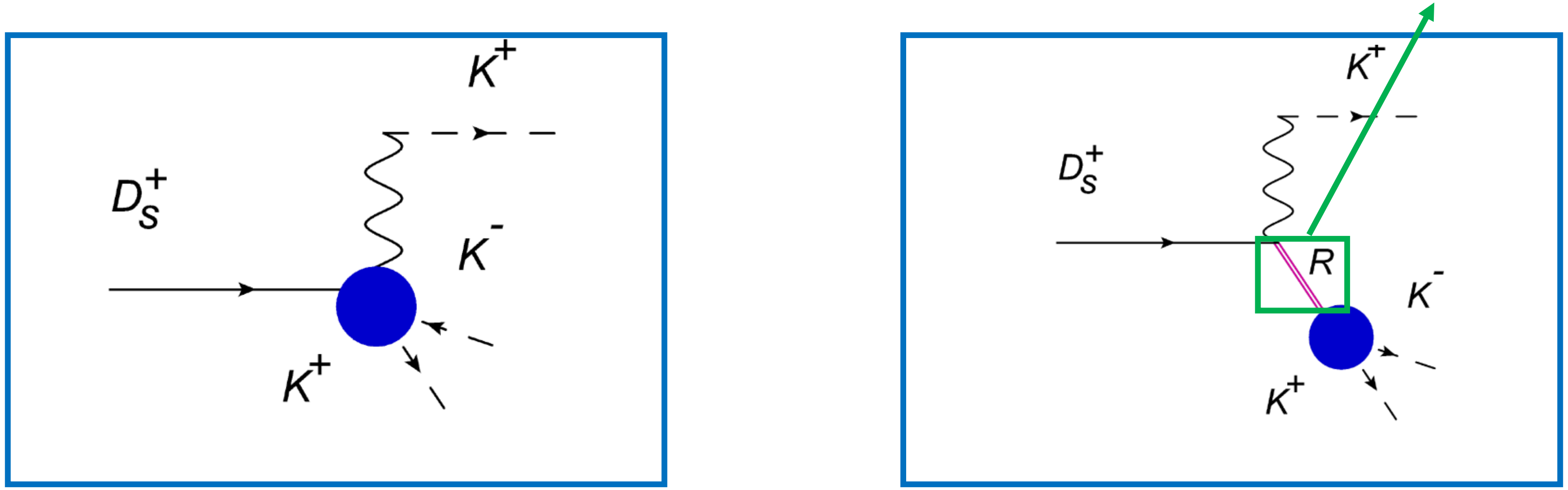
, 2018



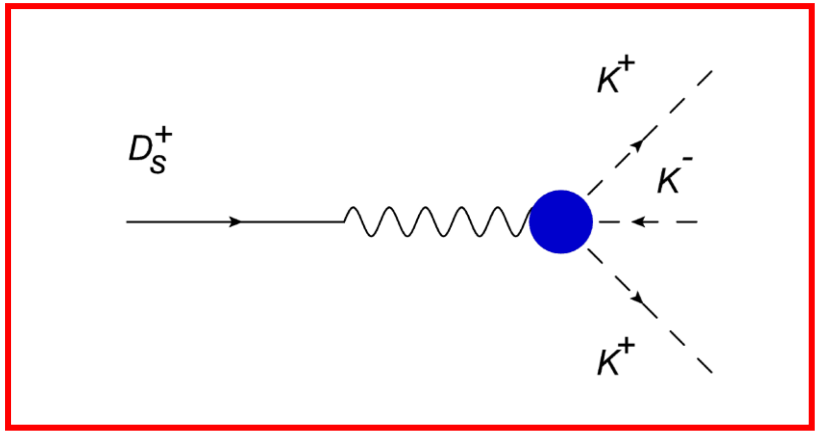




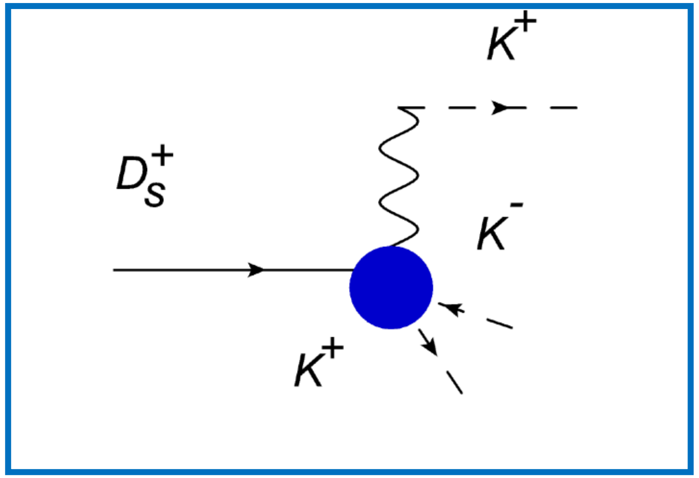
R. T. Aude et al. Multimeson model for the $D^+ \rightarrow K^+K^-K^+$decay amplitude, 2018

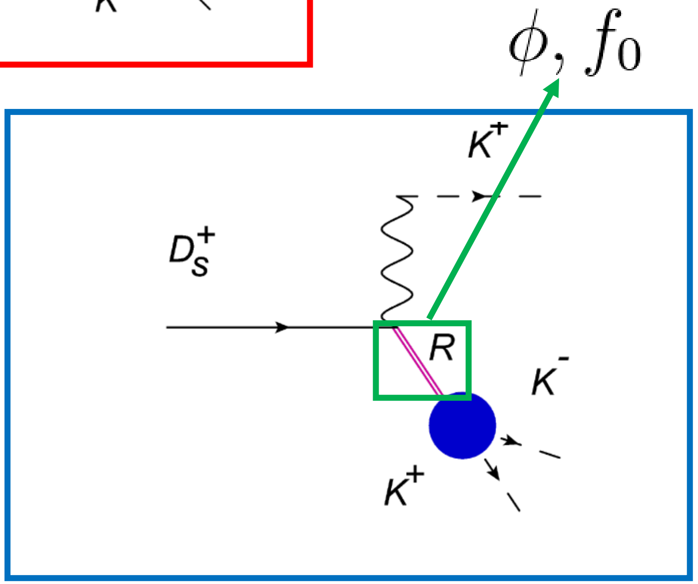


Gen-T
IFIC INSTITUT DE FÍSICA CORPUSCULAR
VNIVERSITAT DE VALÈNCIA

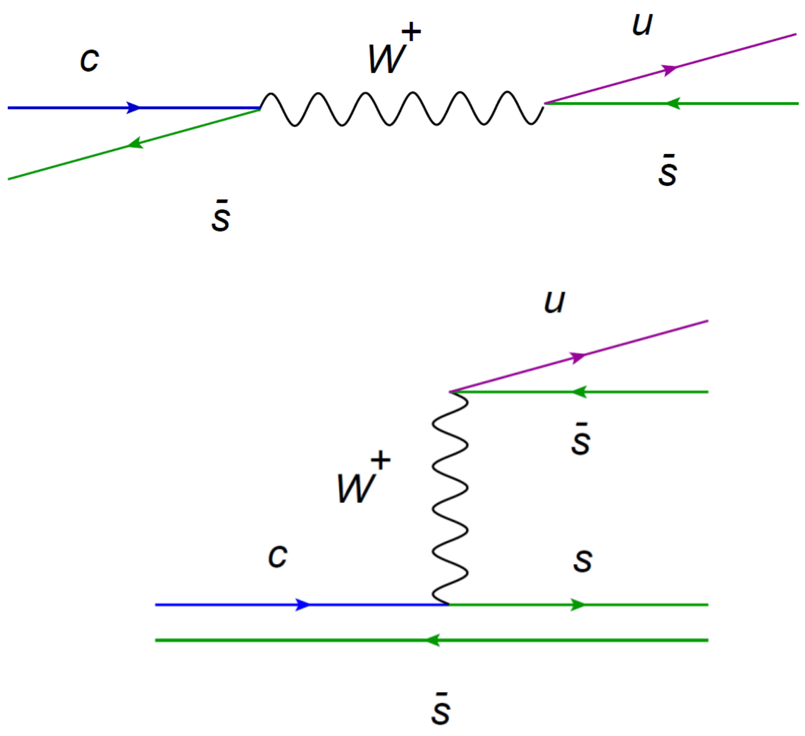


R. T. Aude et al. Multimeson model for the $D^+ \to K^+K^-K^+$ decay amplitude, 2018

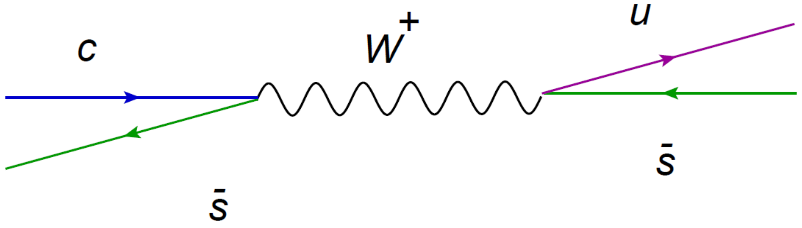


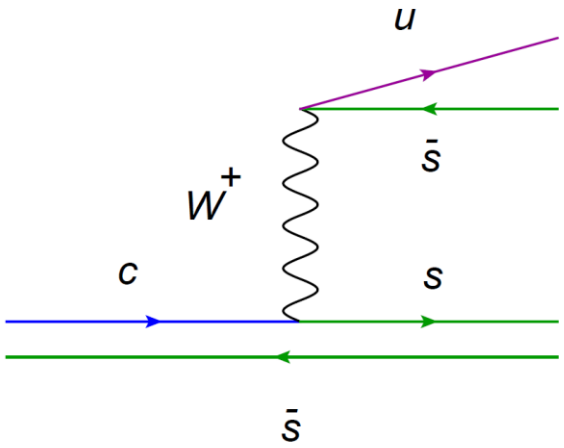


$$D_S^+ \to K^+ K^- K^+$$

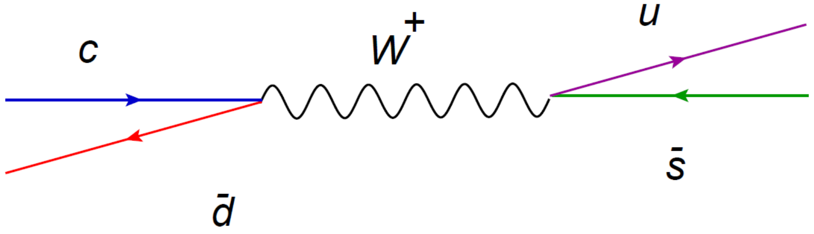


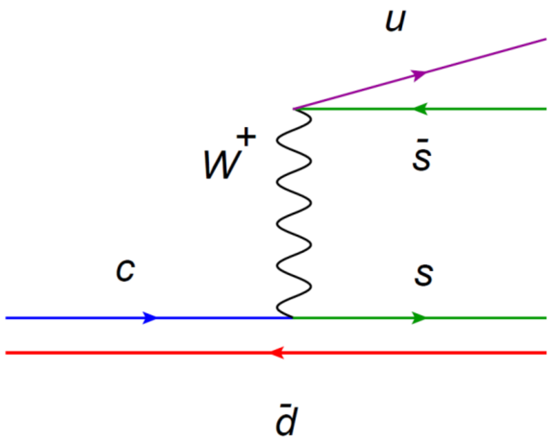
$$D_S^+ \rightarrow K^+K^-K^+$$





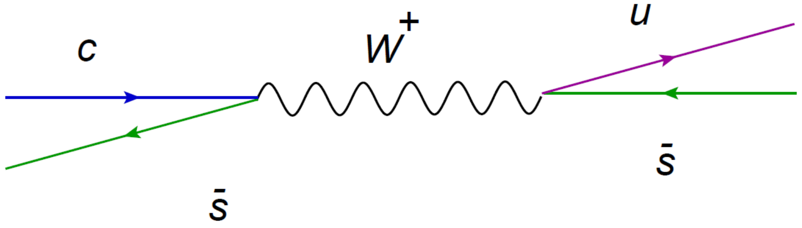
$$D^+ \rightarrow K^+K^-K^+$$

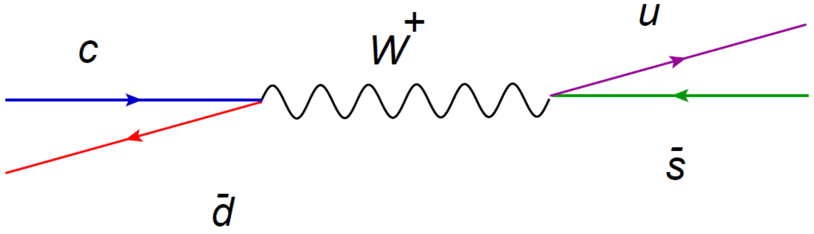


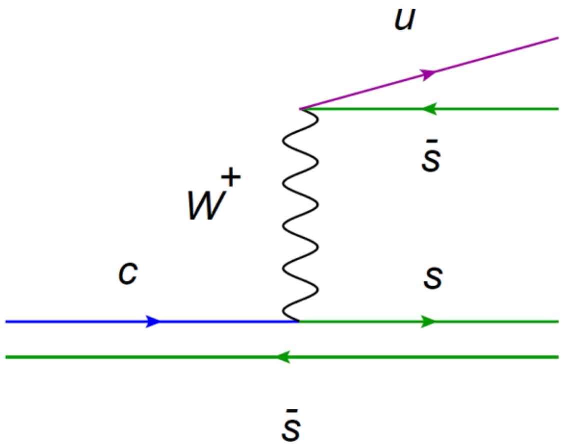


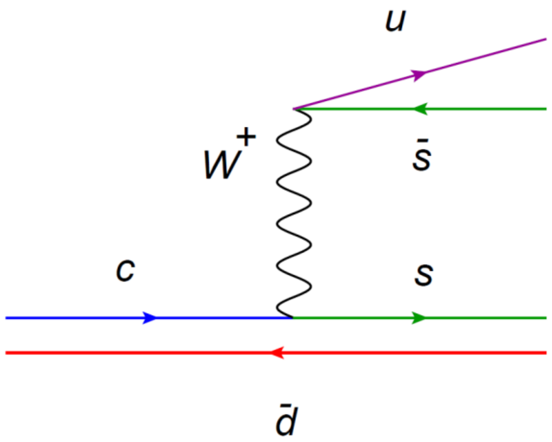
$$D_S^+ \to K^+ K^- K^+$$

$$D^+ \to K^+ K^- K^+$$





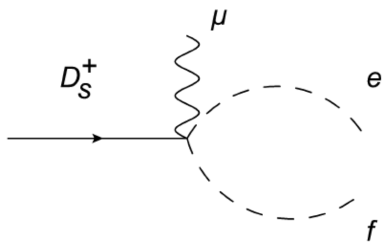




Additional

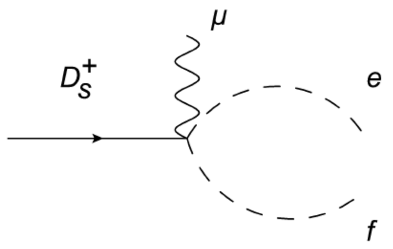
interactions needed!

Model: weak vertex



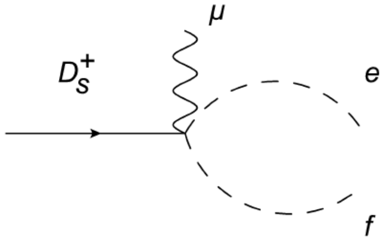
Model: weak vertex

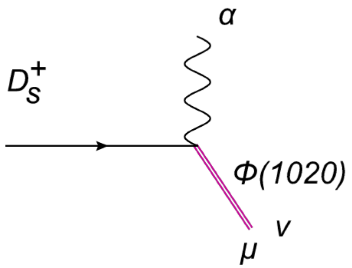
HQSS



Model: weak vertex

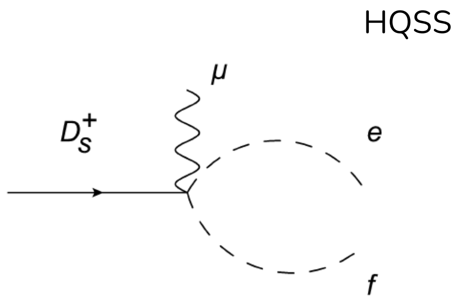
HQSS

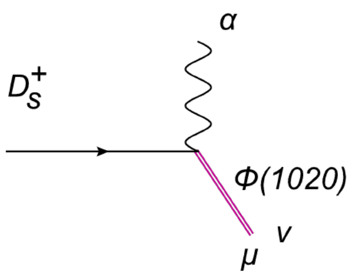




$$\approx V_{cs}^{*} g \sin\theta \left(f_{(V)} \delta^{\alpha\nu} P_D^{\mu} - (\mu \leftrightarrow \nu) \right)$$

Model: weak vertex





$$\approx V_{cs}^{*} g \sin\theta \left(f_{(V)} \delta^{\alpha\nu} P_D^{\mu} - (\mu \leftrightarrow \nu)\right)$$

$$\begin{aligned} |f_0(980)\rangle &= \cos\varepsilon\,|S_1\rangle + \sin\varepsilon\,|S_8\rangle\,, \\ |f_0(1370)\rangle &= -\sin\varepsilon\,|S_1\rangle + \cos\varepsilon\,|S_8\rangle \end{aligned}$$

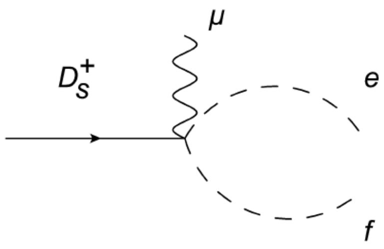
Gen-T

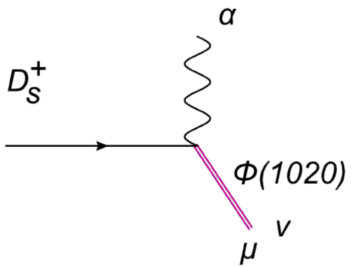
IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Model: weak vertex

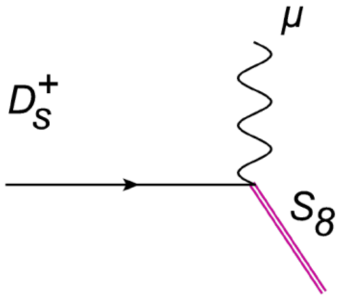
HQSS



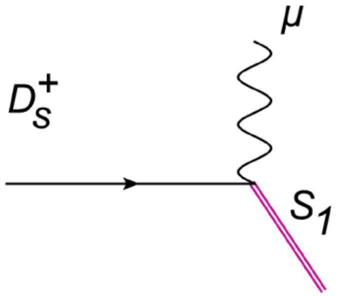


$$\approx V_{cs}^{*} g \sin\theta \left(f_{(V)} \delta^{\alpha\nu} P_D^{\mu} - (\mu \leftrightarrow \nu) \right)$$

$$\begin{aligned} |f_0(980)\rangle &= \cos\varepsilon\,|S_1\rangle + \sin\varepsilon\,|S_8\rangle\,, \\ |f_0(1370)\rangle &= -\sin\varepsilon\,|S_1\rangle + \cos\varepsilon\,|S_8\rangle \end{aligned}$$



$$\approx \quad V_{cs}^{*} g f_{(so)} P_D^{\mu}$$

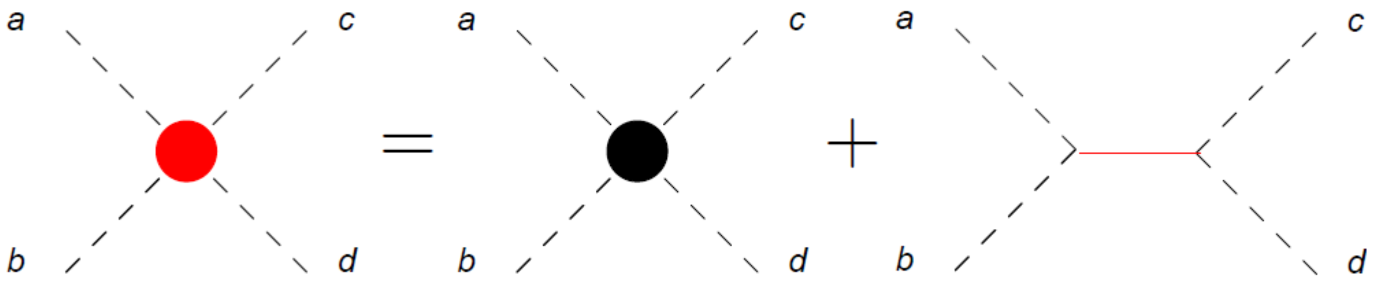


$$\approx \quad V_{cs}^{*} g f_{(ss)} P_D^{\mu}$$

Gen-T
IFIC INSTITUT DE FÍSICA CORPUSCULAR
VNIVERSITAT DE VALÈNCIA

Model: Final states interactions (FSI)

Model: Final states interactions (FSI)

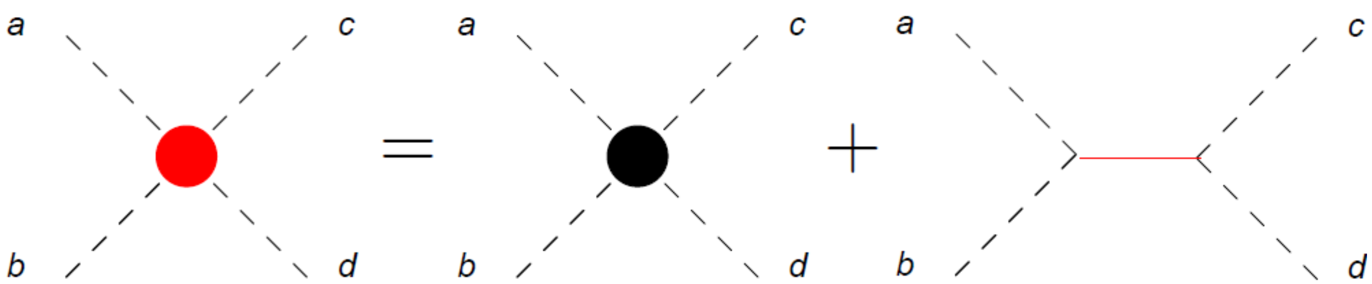


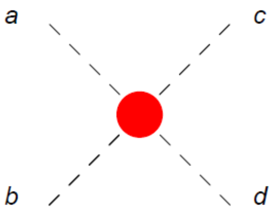
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

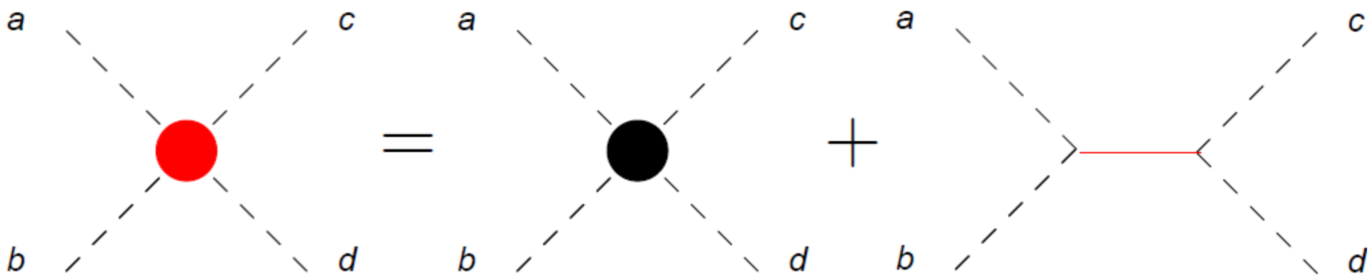
Model: Final states interactions (FSI)

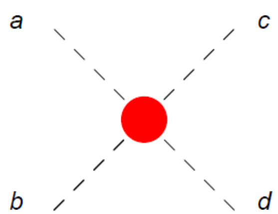




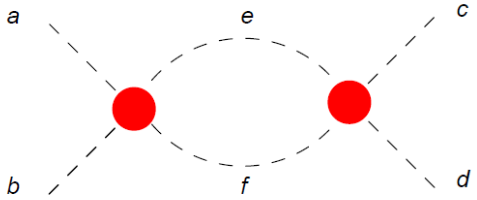
$$= i\kappa_{ab|cd}$$

Model: Final states interactions (FSI)





$$= i\kappa_{ab|cd}$$



$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

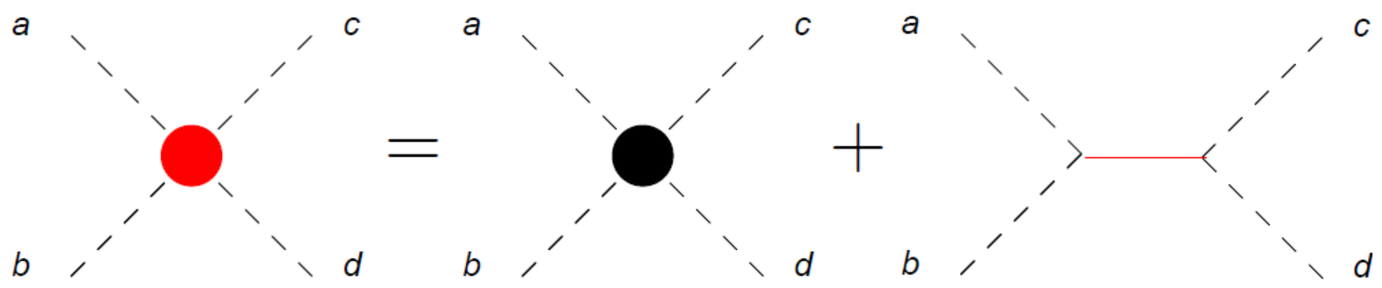
$$= \sum_{ef} i\kappa_{ab|ef} M_{ef|cd},$$

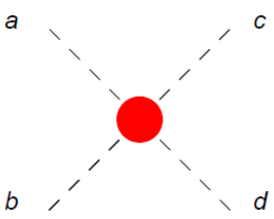
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

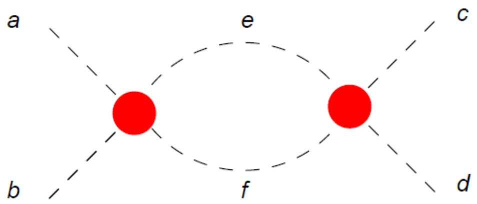
VNIVERSITAT DE VALÈNCIA

Model: Final states interactions (FSI)





$$= i\kappa_{ab|cd}$$



$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

$$= \sum_{ef} i\kappa_{ab|ef}M_{ef|cd},$$

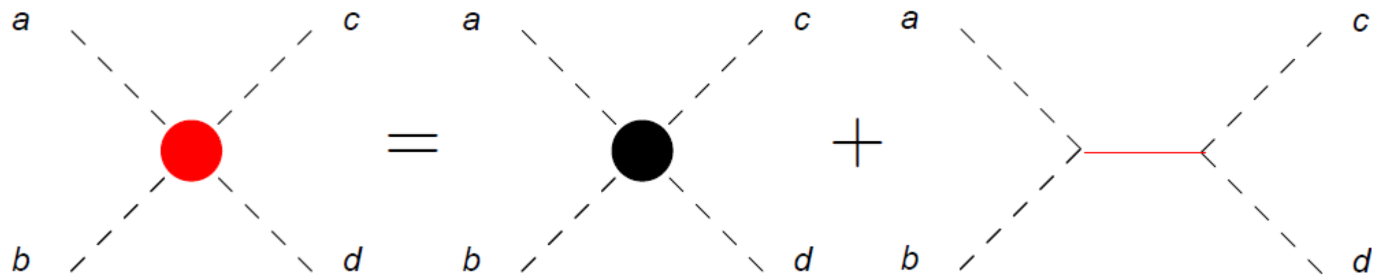
$$\sum_{ef} i\kappa_{ab|ef}(\delta_{ef|cd} + M_{ef|cd} + M^2_{ef|cd} + \ldots..) = \sum_{ef} i\kappa_{ab|ef}\sum_{n=0}^{\infty} M^n_{ef|cd}$$

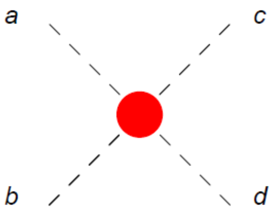
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

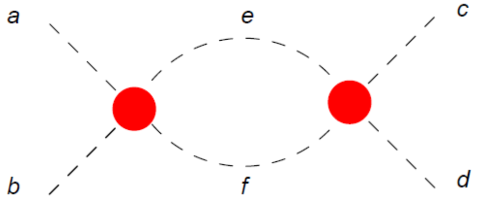
VNIVERSITAT DE VALÈNCIA

Model: Final states interactions (FSI)





$$= i\kappa_{ab|cd}$$



$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

$$= \sum_{ef} i\kappa_{ab|ef}M_{ef|cd},$$

$$\sum_{ef} i\kappa_{ab|ef}(\delta_{ef|cd} + M_{ef|cd} + M^2_{ef|cd} + \ldots..) = \sum_{ef} i\kappa_{ab|ef}\sum_{n=0}^{\infty} M^n_{ef|cd}$$

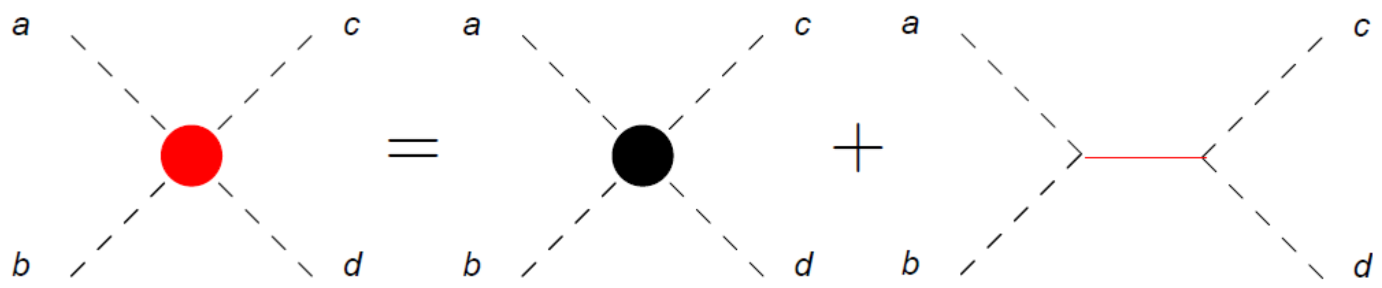
$$\langle ab|cd\rangle = \sum_{ef} i\kappa_{ab|ef}\left(\frac{1}{1-M}\right)_{ef|cd}$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

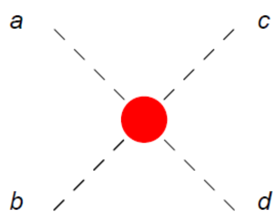
VNIVERSITAT DE VALÈNCIA

Model: Final states interactions (FSI)

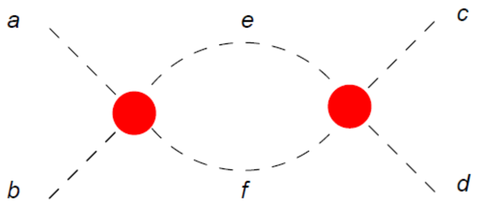


P.C Magalhães et al.

Multibody decay analyses -- a new phenomenological model for meson-meson subamplitudes, 2020



$= i\kappa_{ab|cd}$



$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

$$= \sum_{ef} i\kappa_{ab|ef} M_{ef|cd},$$

$$\sum_{ef} i\kappa_{ab|ef}(\delta_{ef|cd} + M_{ef|cd} + M^2_{ef|cd} + \ldots..) = \sum_{ef} i\kappa_{ab|ef} \sum_{n=0}^{\infty} M^n_{ef|cd}$$

$$\langle ab|cd\rangle = \sum_{ef} i\kappa_{ab|ef}\left(\frac{1}{1-M}\right)_{ef|cd}$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

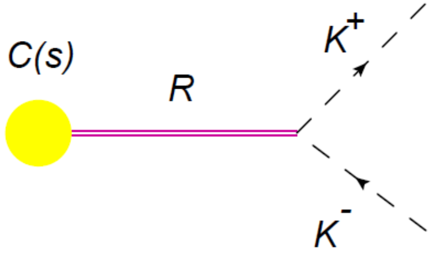
Model: Why FSI?

Gen—T

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

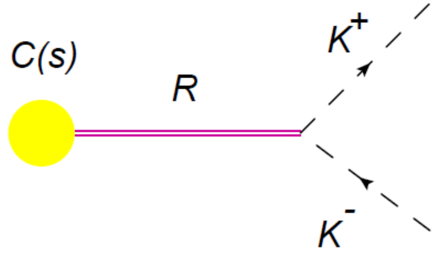
VNIVERSITAT DE VALÈNCIA

Model: Why FSI?

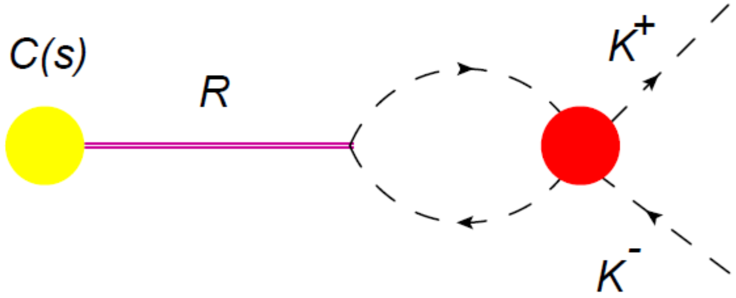


$$= \frac{c(s)}{s - m_R^2}$$

Model: Why FSI?

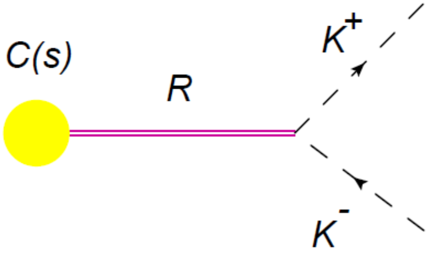


$$= \frac{c(s)}{s - m_R^2}$$

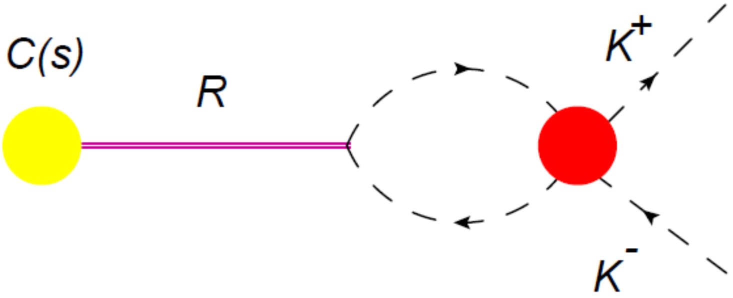


$$= \frac{c(s)}{(s - m_R^2)(1 + \Omega\kappa)}$$

Model: Why FSI?



$$= \frac{c(s)}{s - m_R^2}$$



$$= \frac{c(s)}{(s - m_R^2)(1 + \Omega\kappa)}$$

I. The interaction kernel times the pole is not zero at the mass of the resonance

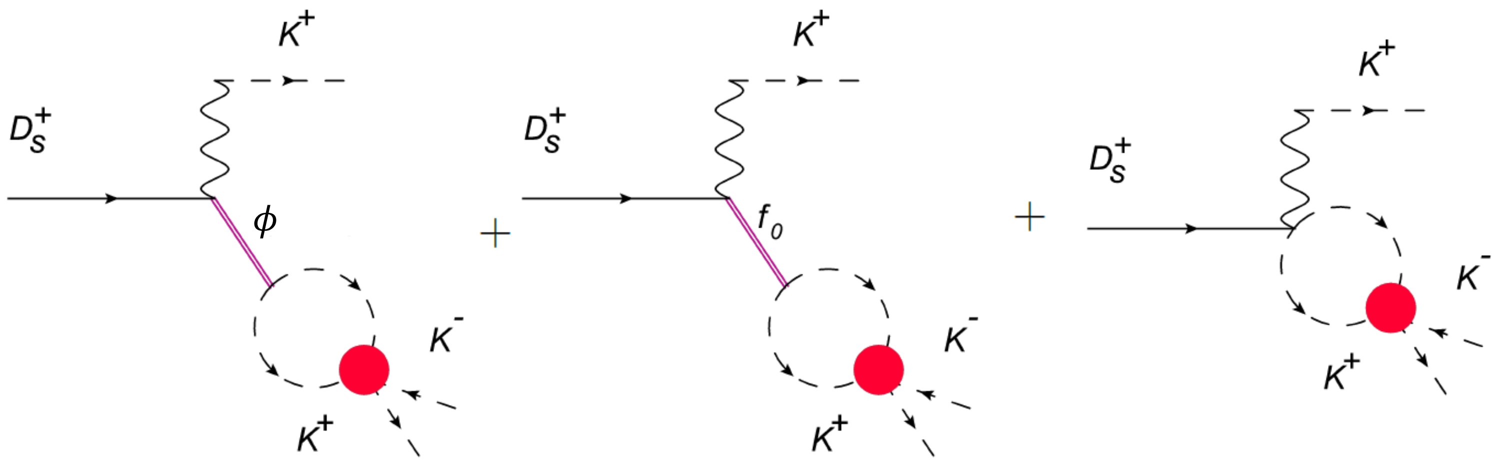
II. Considering only the imaginary part of the loop (K matrix approximation) a width to the resonance propagator is given

Gen-T
IFIC INSTITUT DE FÍSICA CORPUSCULAR
VNIVERSITAT DE VALÈNCIA

Decay amplitude

Decay amplitude

$$\langle K^- K^+ K^+ | D_s^+ \rangle \ =$$



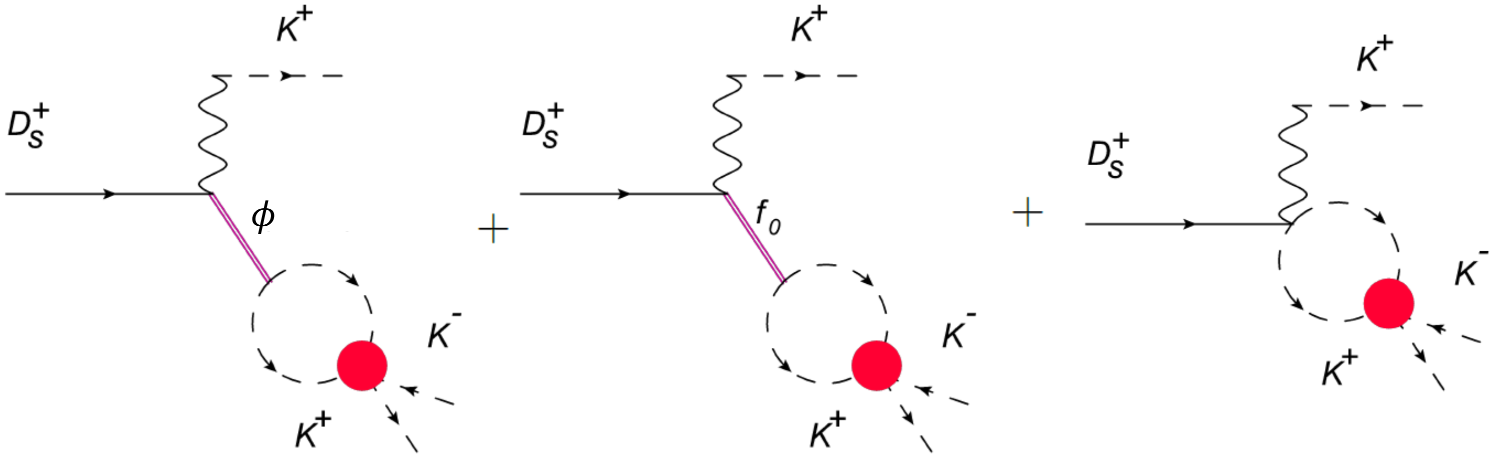
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Decay amplitude

$$\langle K^- K^+ K^+ | D_s^+ \rangle =$$



Parameter	Value		
m_π	0.1395GeV	m_K	0.4937GeV
m_D	1.9685GeV	m_ϕ	1.0194GeV
m_{f_a}	0.980GeV	m_{f_b}	1.370GeV
f	0.093GeV	f_D	0.216MeV
c_d	0.032GeV	c_m	0.042GeV
$\tilde{c}_d$	0.018GeV	$\tilde{c}_m$	0.025GeV
G_V	0.066GeV	f_{ss}	1.7
f_{so}	1.7	$\sin\theta$	0.605
V	1	ε	$\frac{\pi}{3}$

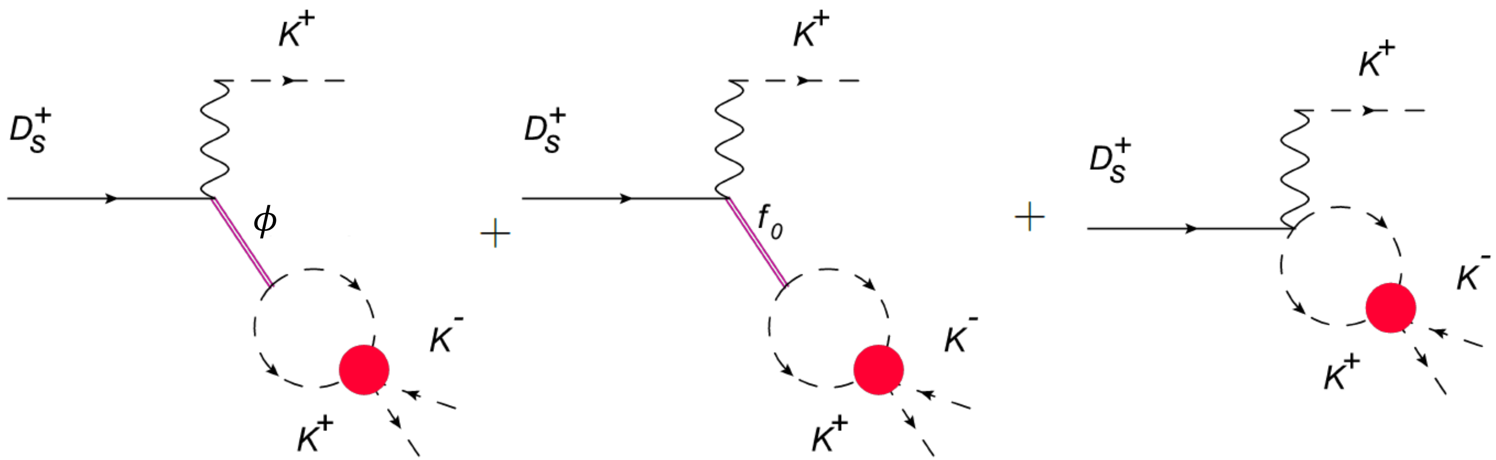
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Decay amplitude

$$\langle K^- K^+ K^+ | D_s^+ \rangle =$$



J. Gasser et al. Chiral perturbation theory: expansions in the mass of the strange quark, 1984

Parameter	Value		
m_π	0.1395GeV	m_K	0.4937GeV
m_D	1.9685GeV	m_ϕ	1.0194GeV
m_{f_a}	0.980GeV	m_{f_b}	1.370GeV
f	0.093GeV	f_D	0.216MeV
c_d	0.032GeV	c_m	0.042GeV
$\tilde{c}_d$	0.018GeV	$\tilde{c}_m$	0.025GeV
G_V	0.066GeV	f_{ss}	1.7
f_{so}	1.7	$\sin\theta$	0.605
V	1	ε	$\frac{\pi}{3}$

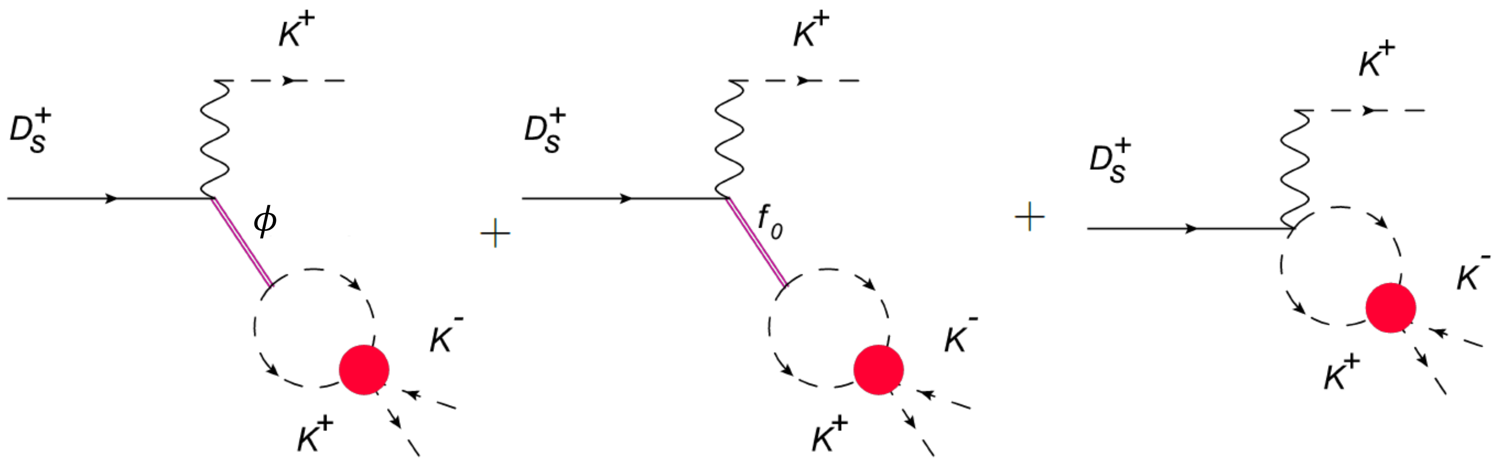
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Decay amplitude

$$\langle K^- K^+ K^+ | D_s^+ \rangle =$$



J. Gasser et al. Chiral perturbation theory: expansions in the mass of the strange quark, 1984

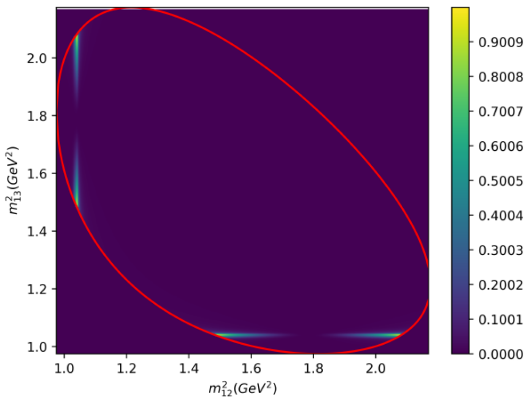
Parameter	Value		
m_π	0.1395GeV	m_K	0.4937GeV
m_D	1.9685GeV	m_ϕ	1.0194GeV
m_{f_a}	0.980GeV	m_{f_b}	1.370GeV
f	0.093GeV	f_D	0.216MeV
c_d	0.032GeV	c_m	0.042GeV
$\tilde{c}_d$	0.018GeV	$\tilde{c}_m$	0.025GeV
G_V	0.066GeV	f_{ss}	1.7
f_{so}	1.7	$\sin\theta$	0.605
V	1	ε	$\frac{\pi}{3}$

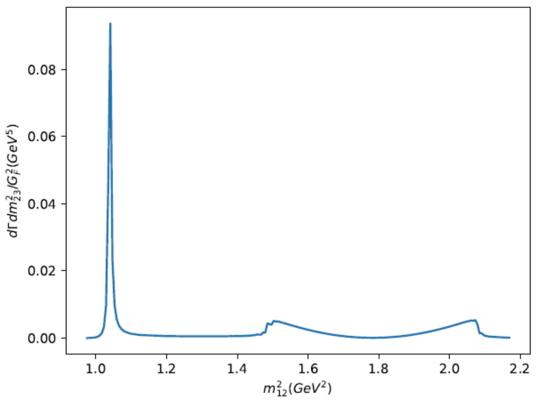
G. Ecker et al. The role of resonances in chiral perturbation theory, 1988

Main results

Main results

$[\phi]$





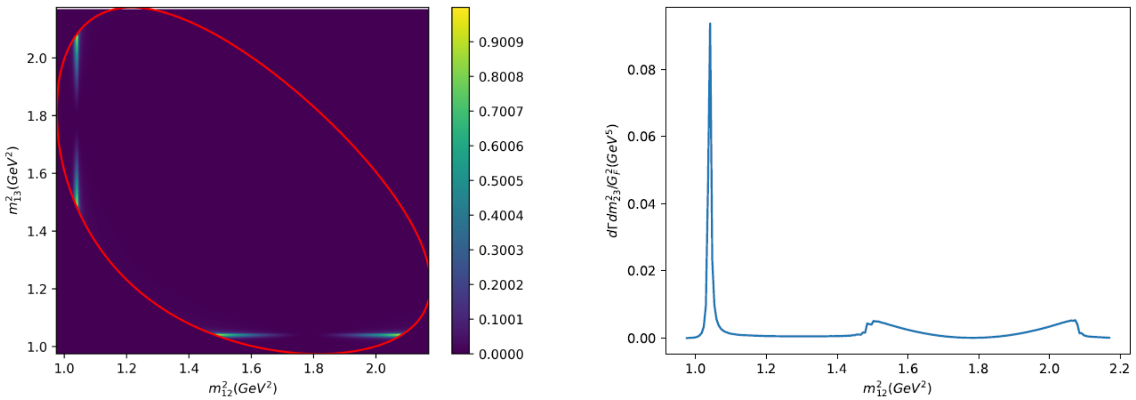
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

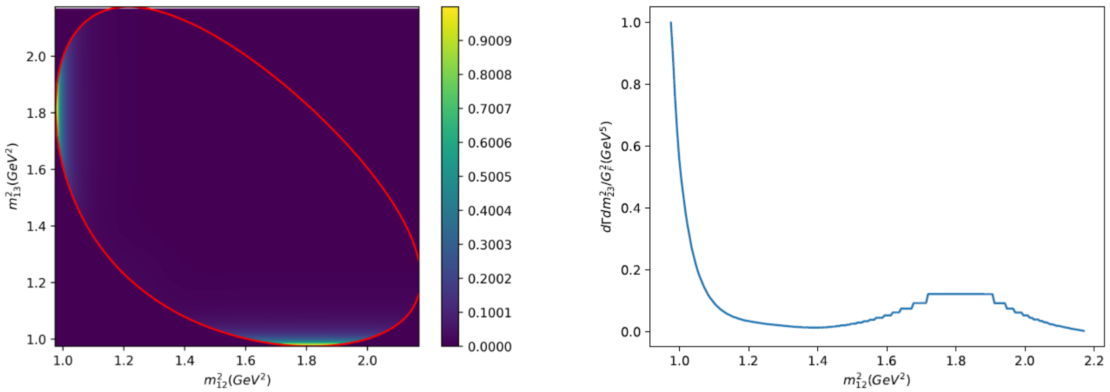
VNIVERSITAT DE VALÈNCIA

Main results

$[\phi]$



$[f_0]$



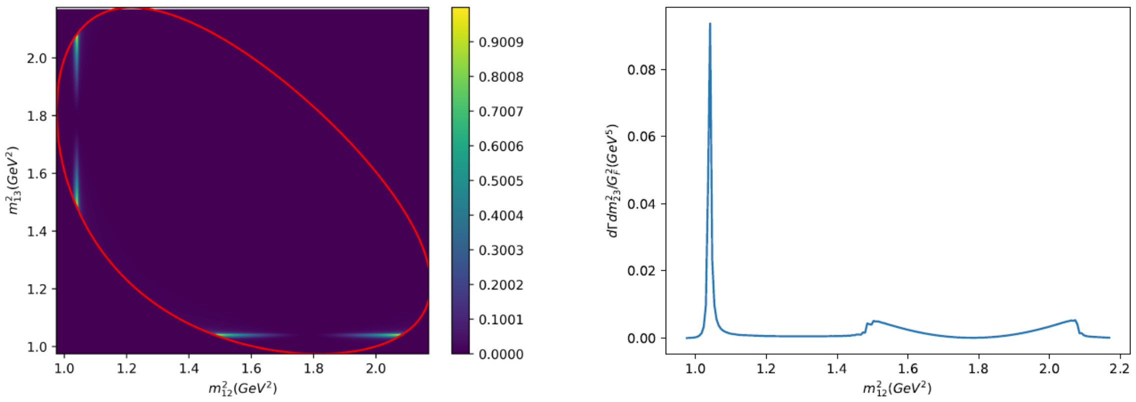
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

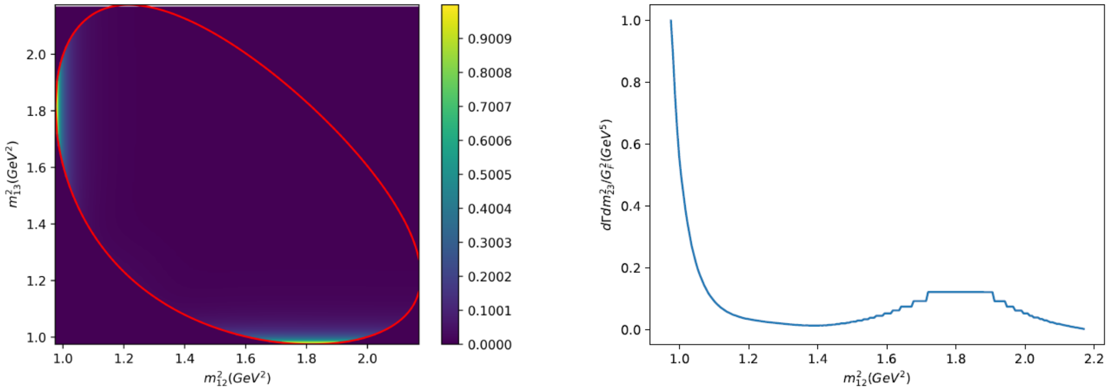
VNIVERSITAT DE VALÈNCIA

Main results

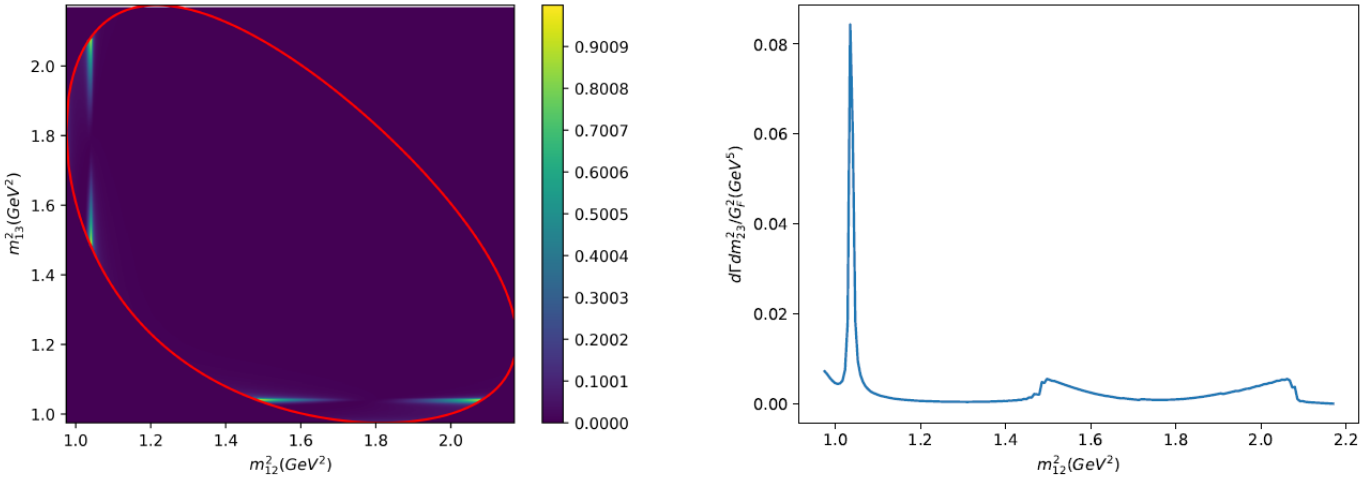
$[\phi]$



$[f_0]$

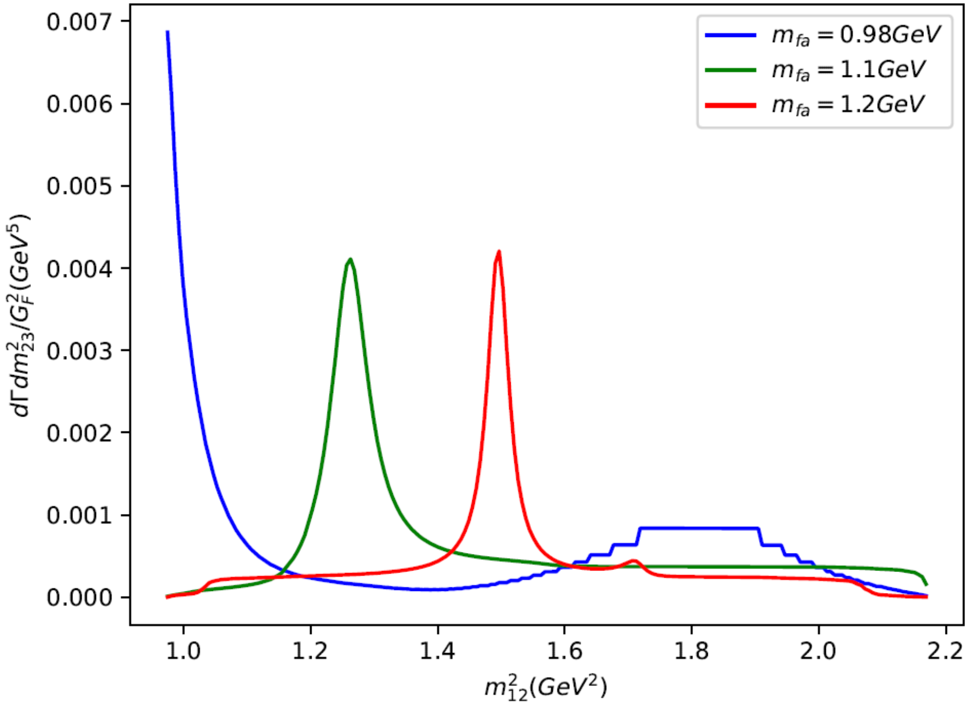


Combined



Parameters of the model

Parameters of the model

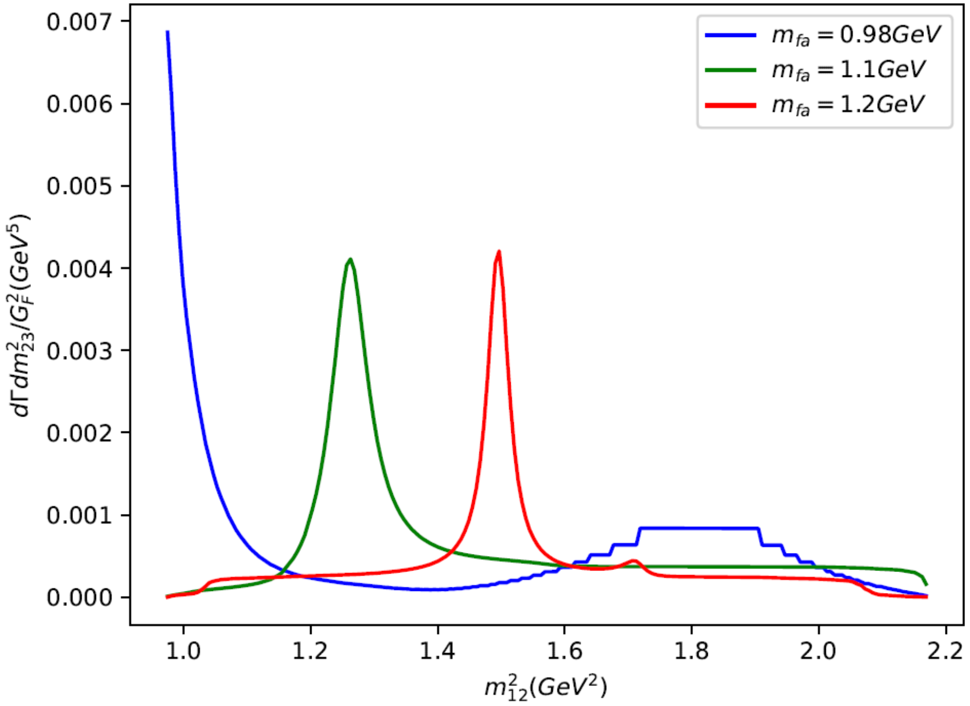


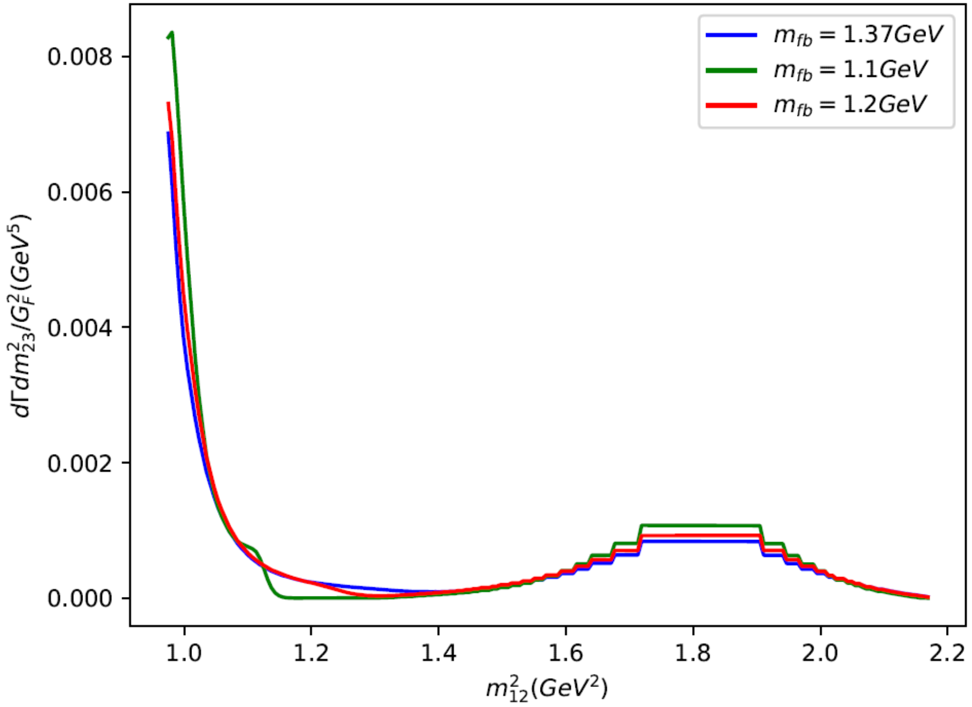
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Parameters of the model

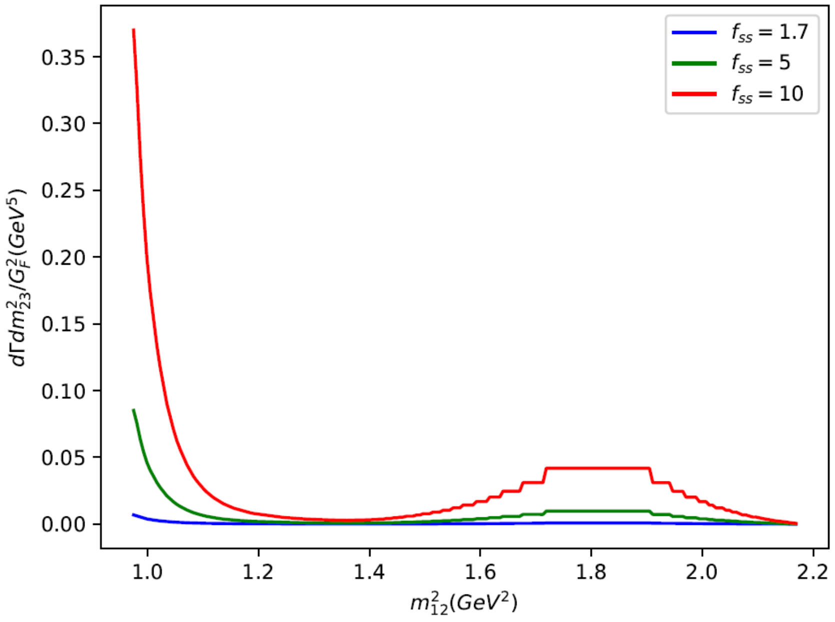




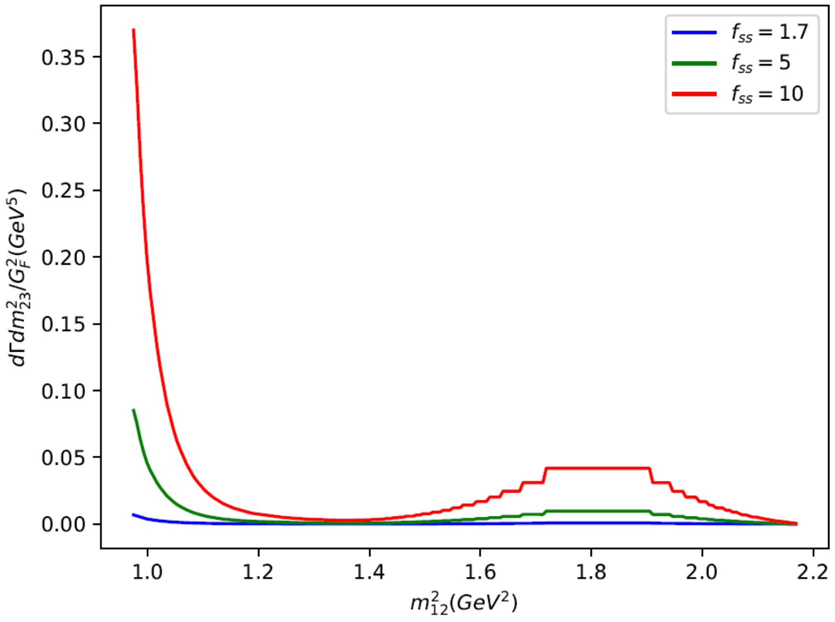
Gen—T

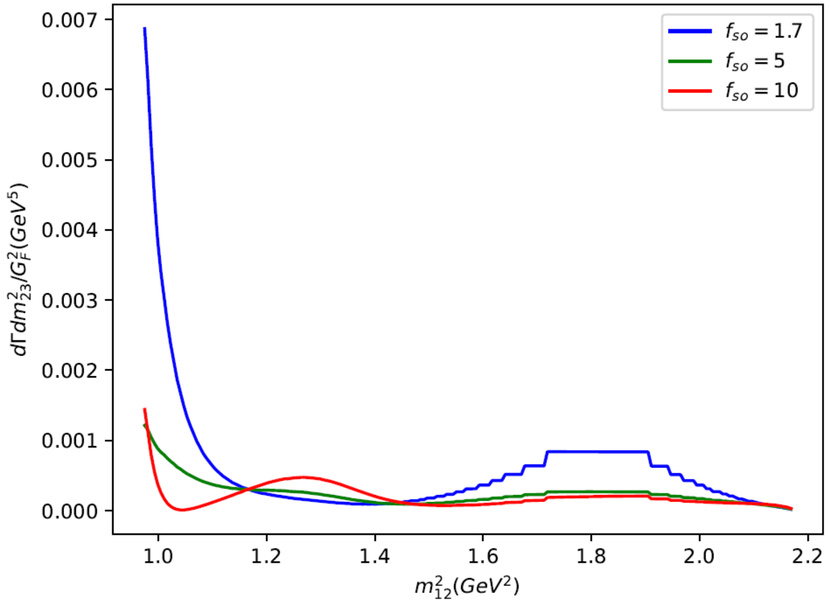
IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA



$f_{ss} = 1.7$
$f_{ss} = 5$
$f_{ss} = 10$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$d\Gamma dm_{23}^2/G_F^2(GeV^5)$
1.0
1.2
1.4
1.6
1.8
2.0
2.2
$m_{12}^2(GeV^2)$

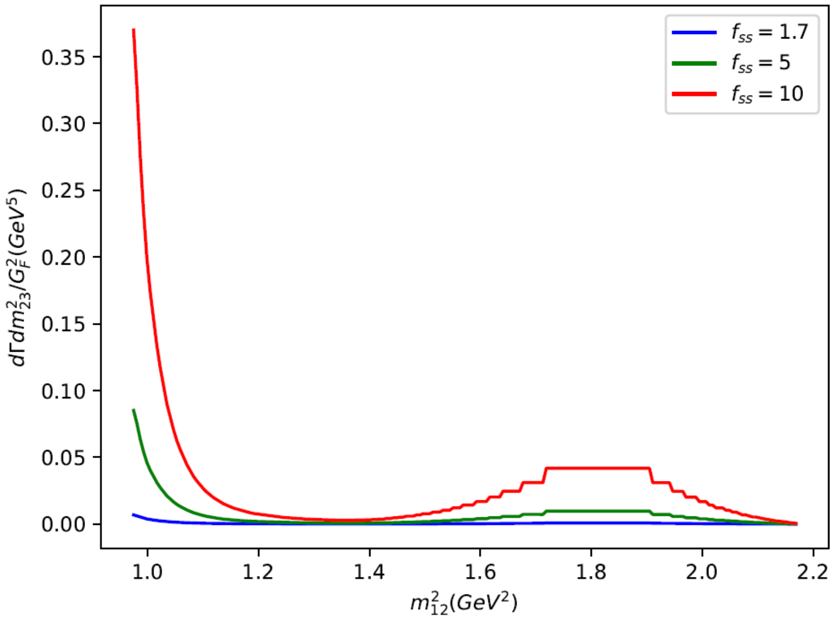


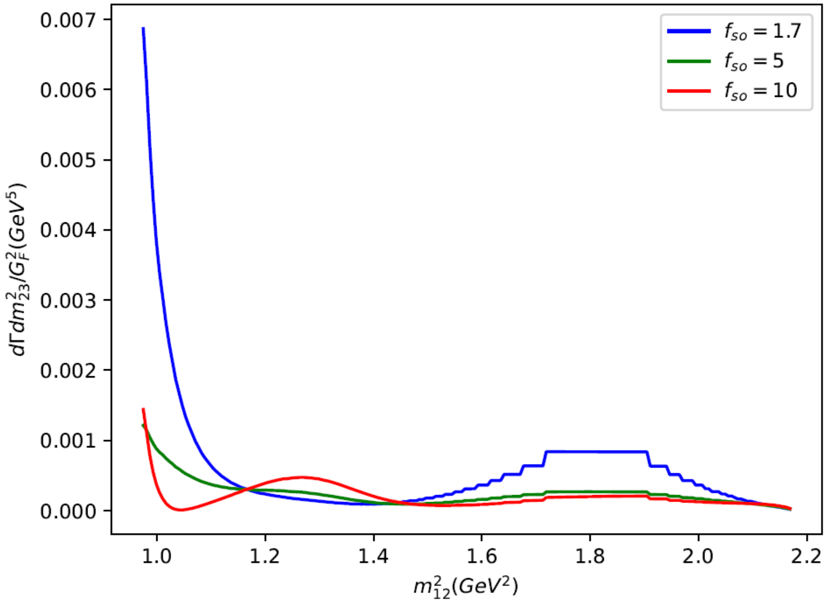


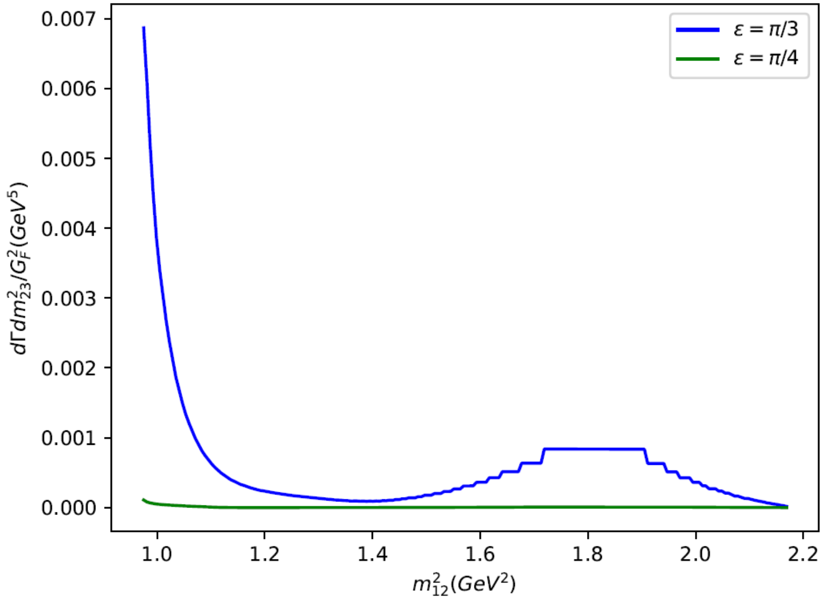
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA







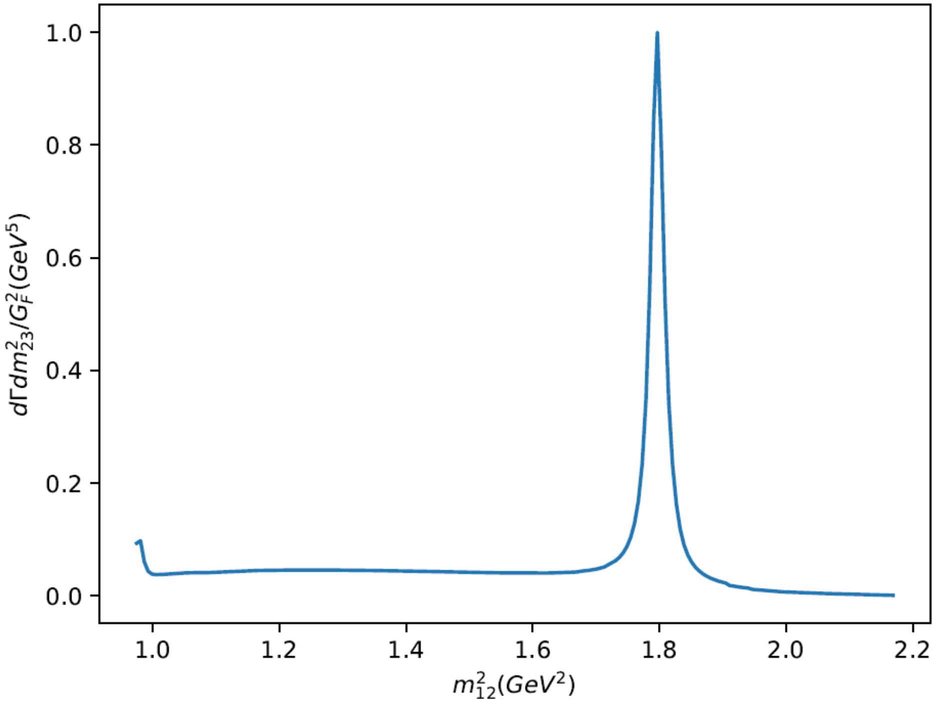
Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Model implemented with parameters taken from recent data analysis

Model implemented with parameters taken from recent data analysis



Parameters taken from: R. Aaij et al. Dalitz plot analysis of the $D^+ \rightarrow K^+K^-K^+$ decay, 2019.

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

1) Motivation

2) The $D_S^+ \to K^+ K^- K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

1) Motivation

2) The $D_S^+ \to K^+K^-K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Inclusive charmonium production from B meson decays

Inclusive charmonium production from B meson decays

Theory: NRQCD factorization

Inclusive charmonium production from B meson decays

Theory: NRQCD factorization

$$\Gamma(Y \to H(c\bar{c})X) = \sum_n \Gamma[n] < O_n^H >$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Inclusive charmonium production from B meson decays

Theory: NRQCD factorization

Long-distance Dynamics

$$\Gamma(Y \to H(c\bar{c})X) = \sum_n \Gamma[n] < \overline{O_n^H} >$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Inclusive charmonium production from B meson decays

Theory: NRQCD factorization

Long-distance Dynamics

$$\Gamma(Y \to H(c\bar{c})X) = \sum_n \Gamma[n] < O_n^H >$$

Short-distance dynamics

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Full theory: Electroweak and strong interactions

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X$$

Gen—T

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X \qquad\qquad b \to c\bar{c}\ \ q$$

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c}) \;\; X$$

$$b \to c\bar{c} \;\; q$$

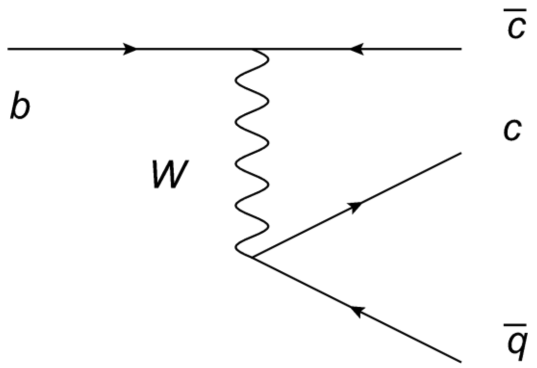
Same short-distance dynamics

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X$$

$$b \to c\bar{c}\ \ q$$

Same short-distance dynamics

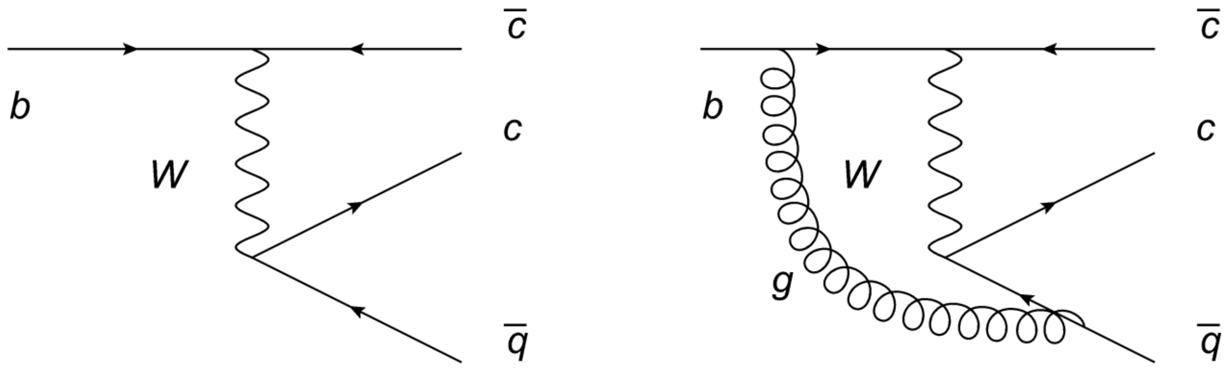


Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X$$

$$b \to c\bar{c}\ \ q$$

Same short-distance dynamics

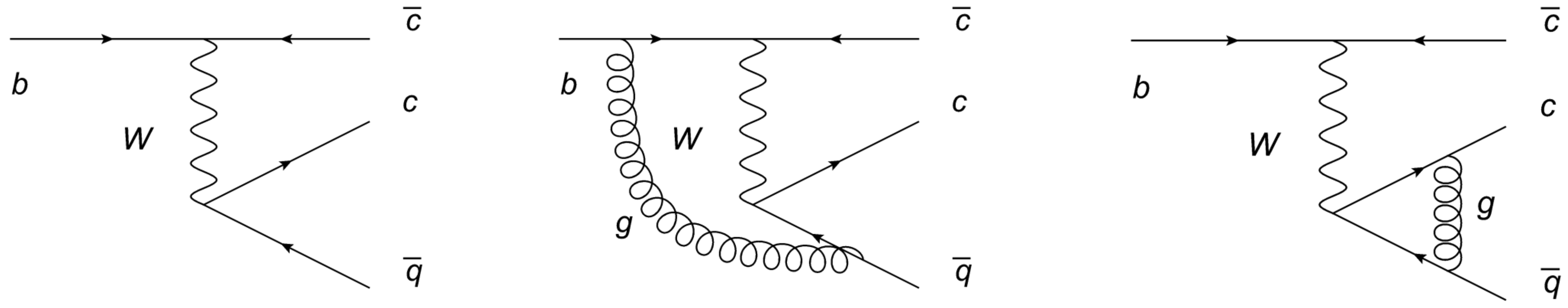


Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X$$

$$b \to c\bar{c}\ \ q$$

Same short-distance dynamics

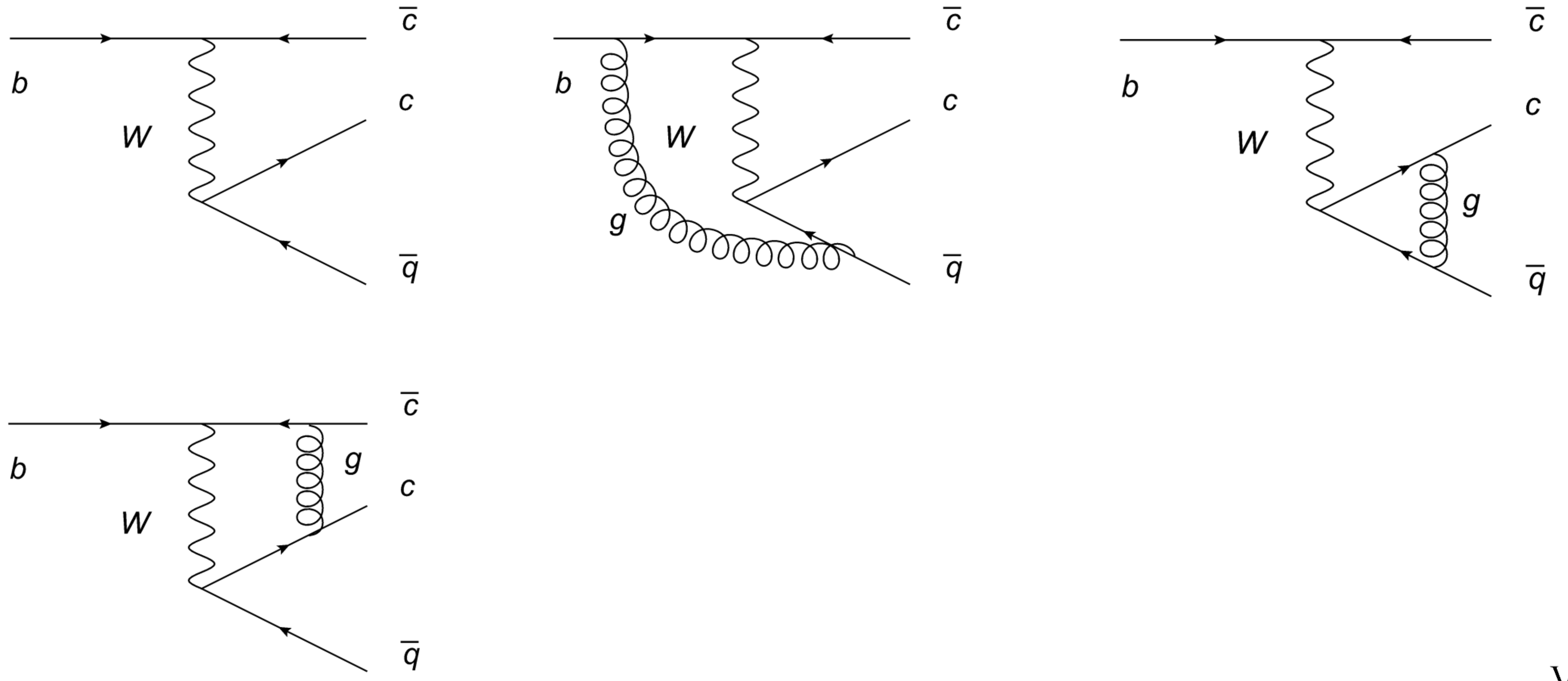


Gen-T
IFIC INSTITUT DE FÍSICA CORPUSCULAR
VNIVERSITAT DE VALÈNCIA

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X \qquad\qquad b \to c\bar{c}\ \ q$$

Same short-distance dynamics

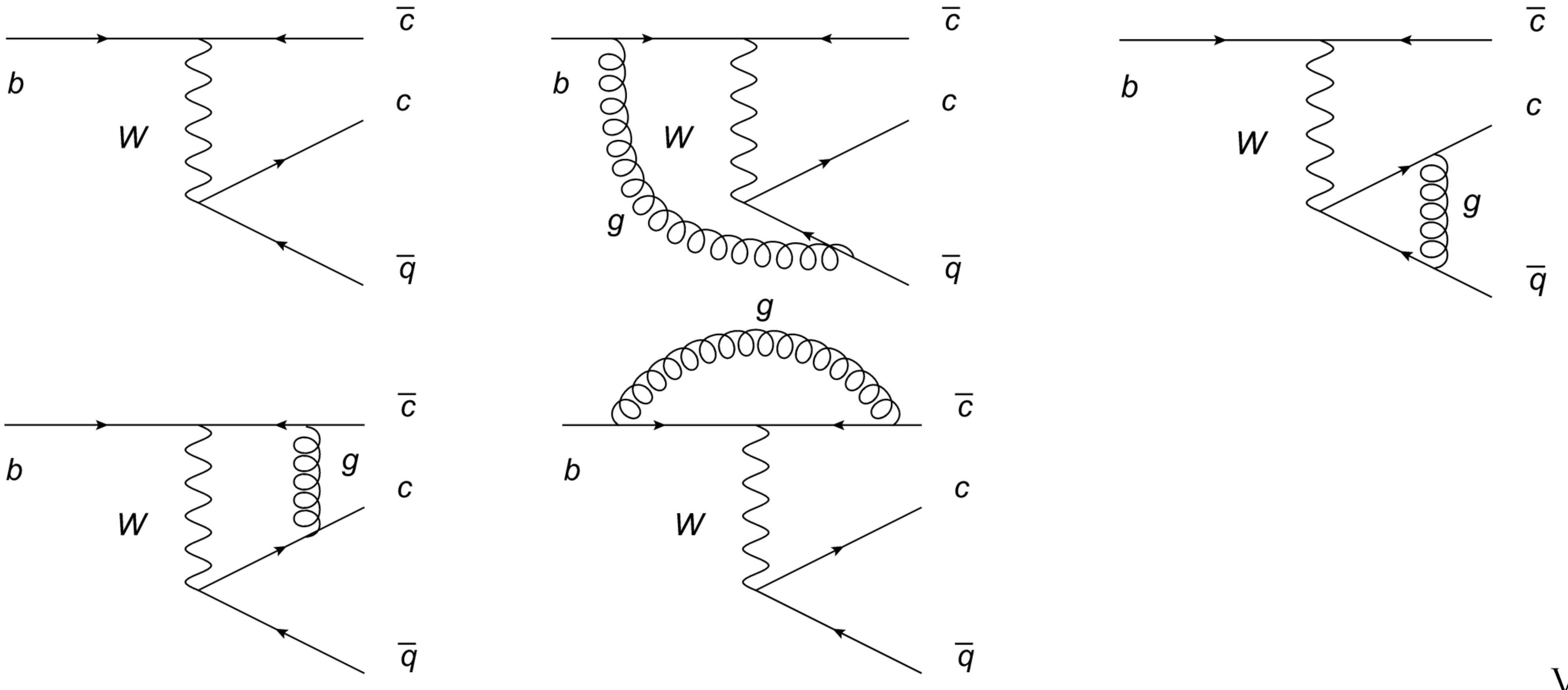


Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ \ X$$

$$b \to c\bar{c}\ \ q$$

Same short-distance dynamics



Gen-T

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

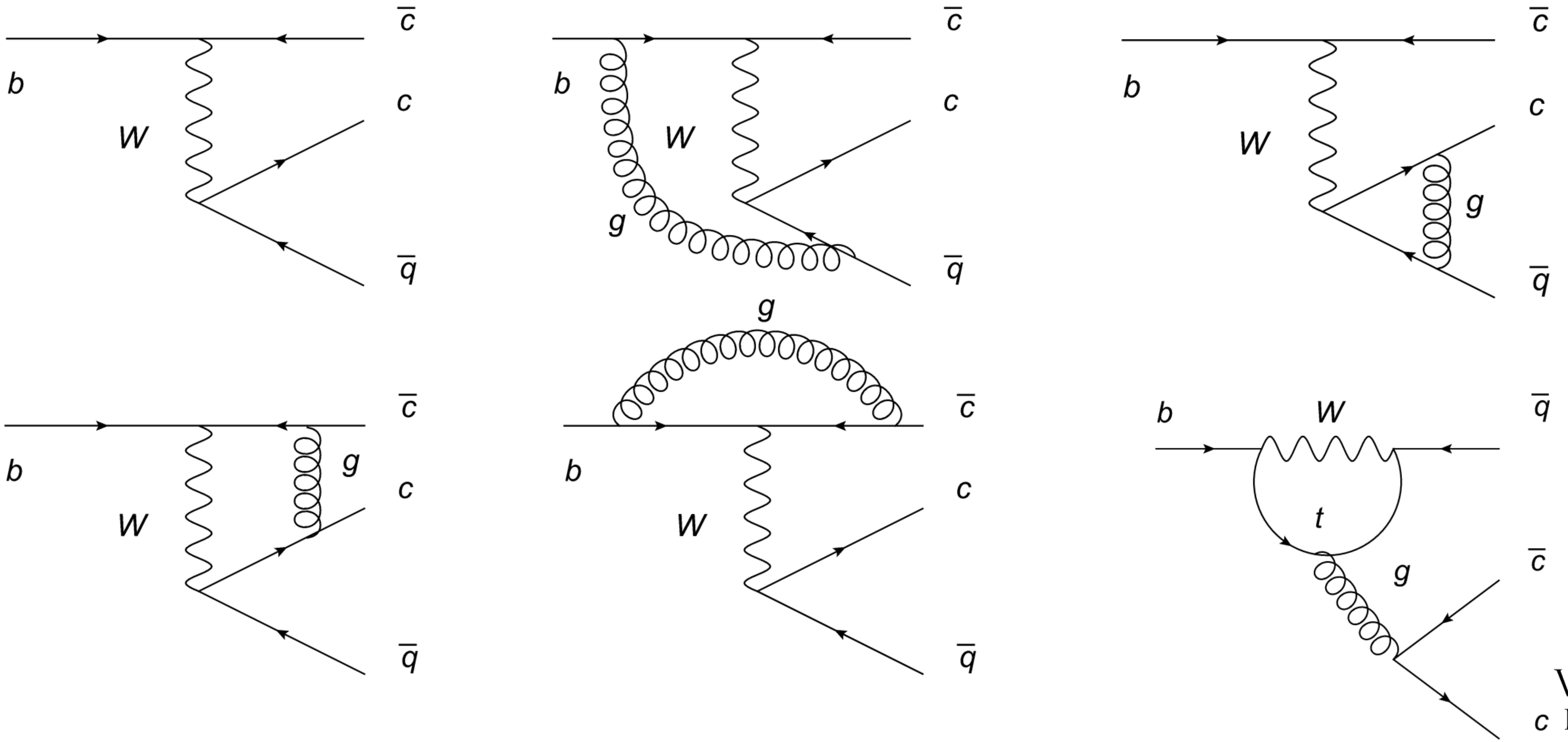
VNIVERSITAT DE VALÈNCIA

Full theory: Electroweak and strong interactions

$$B \to H(c\bar{c})\ X$$

$$b \to c\bar{c}\ q$$

Same short-distance dynamics



$$\mathscr{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i]\}$$

$$\mathscr{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i]\}$$

Current-current operators

$$\mathscr{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i]\}$$

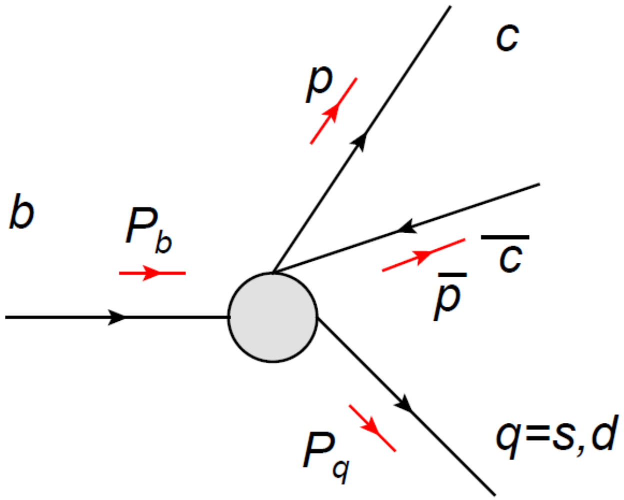
Current-current operators

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i]\}$$

Current-current operators

Virtual top penguin operators

$$\mathscr{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{ V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i] \}$$



Current-current operators

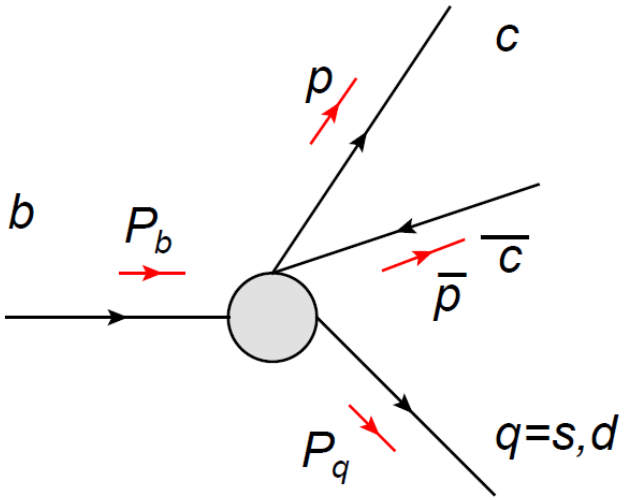
Virtual top penguin operators

G. Buchallla et al. Weak decays beyond leading logarithms, 1996

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_{s,d} \{ V_{cb}^* V_{cq} [\frac{1}{3} C_1 O_1 + C_8 O_8] - V_{tb}^* V_{tq} \sum_{i=3}^{6} [c_i O_i] \}$$

Current-current operators

Virtual top penguin operators



LO result

LO result

$$\Gamma[n] = \Gamma_0 C^2_{[1,8]} f[n](\eta)(1+\delta_P[n])$$

LO result

Function	State $[n]$	
$3(1-\eta)^2$	${}^1S_0^{(1)}$	$f[n](\eta)$
$(1-\eta)^2(1+2\eta)$	${}^3S_1^{(1)}$	
$2(1-\eta)^2(1+2\eta)$	${}^3P_1^{(1)}$	
$\frac{9}{2}(1-\eta)^2$	${}^1S_0^{(8)}$	
$\frac{3}{2}(1-\eta)^2(1+2\eta)$	${}^3S_1^{(8)}$	
$3(1-\eta)^2(1+2\eta)$	${}^3P_1^{(8)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^1S_0^{(1)}$	$\delta_p[n]$
$\frac{2(3(C_3+C_5)+C_4+C_6)}{C_1}$	${}^3S_1^{(1)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^3P_1^{(1)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^1S_0^{(8)}$	
$\frac{4(C_4+C_6)}{C_8}$	${}^3S_1^{(8)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^3P_1^{(8)}$	

$$\Gamma[n] = \Gamma_0 C_{[1,8]}^2 f[n](\eta)(1+\delta_P[n])$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

LO result

Function	State $[n]$	
$3(1-\eta)^2$	${}^1S_0^{(1)}$	$f[n](\eta)$
$(1-\eta)^2(1+2\eta)$	${}^3S_1^{(1)}$	
$2(1-\eta)^2(1+2\eta)$	${}^3P_1^{(1)}$	
$\frac{9}{2}(1-\eta)^2$	${}^1S_0^{(8)}$	
$\frac{3}{2}(1-\eta)^2(1+2\eta)$	${}^3S_1^{(8)}$	
$3(1-\eta)^2(1+2\eta)$	${}^3P_1^{(8)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^1S_0^{(1)}$	$\delta_p[n]$
$\frac{2(3(C_3+C_5)+C_4+C_6)}{C_1}$	${}^3S_1^{(1)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^3P_1^{(1)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^1S_0^{(8)}$	
$\frac{4(C_4+C_6)}{C_8}$	${}^3S_1^{(8)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^3P_1^{(8)}$	

$$\Gamma[n] = \Gamma_0 C^2_{[1,8]} f[n](\eta)(1+\delta_P[n])$$

$$\eta = \frac{4m_c^2}{m_b^2}$$

LO result

Function	State $[n]$	
$3(1-\eta)^2$	${}^1S_0^{(1)}$	$f[n](\eta)$
$(1-\eta)^2(1+2\eta)$	${}^3S_1^{(1)}$	
$2(1-\eta)^2(1+2\eta)$	${}^3P_1^{(1)}$	
$\frac{9}{2}(1-\eta)^2$	${}^1S_0^{(8)}$	
$\frac{3}{2}(1-\eta)^2(1+2\eta)$	${}^3S_1^{(8)}$	
$3(1-\eta)^2(1+2\eta)$	${}^3P_1^{(8)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^1S_0^{(1)}$	$\delta_p[n]$
$\frac{2(3(C_3+C_5)+C_4+C_6)}{C_1}$	${}^3S_1^{(1)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^3P_1^{(1)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^1S_0^{(8)}$	
$\frac{4(C_4+C_6)}{C_8}$	${}^3S_1^{(8)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^3P_1^{(8)}$	

$$\Gamma[n] = \Gamma_0 C_{[1,8]}^2 f[n](\eta)(1+\delta_P[n])$$

$$\eta = \frac{4m_c^2}{m_b^2}$$

E. Braaten et al. Helicity decomposition

for inclusive J /Ψ production, 1996.

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

LO result

Function	State $[n]$	
$3(1-\eta)^2$	${}^1S_0^{(1)}$	$f[n](\eta)$
$(1-\eta)^2(1+2\eta)$	${}^3S_1^{(1)}$	
$2(1-\eta)^2(1+2\eta)$	${}^3P_1^{(1)}$	
$\frac{9}{2}(1-\eta)^2$	${}^1S_0^{(8)}$	
$\frac{3}{2}(1-\eta)^2(1+2\eta)$	${}^3S_1^{(8)}$	
$3(1-\eta)^2(1+2\eta)$	${}^3P_1^{(8)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^1S_0^{(1)}$	$\delta_p[n]$
$\frac{2(3(C_3+C_5)+C_4+C_6)}{C_1}$	${}^3S_1^{(1)}$	
$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^3P_1^{(1)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^1S_0^{(8)}$	
$\frac{4(C_4+C_6)}{C_8}$	${}^3S_1^{(8)}$	
$\frac{4(C_4-C_6)}{C_8}$	${}^3P_1^{(8)}$	

$$\Gamma[n] = \Gamma_0 C_{[1,8]}^2 f[n](\eta)(1+\delta_P[n])$$

$$\eta = \frac{4m_c^2}{m_b^2}$$

E. Braaten et al. Helicity decomposition for inclusive J /Ψ production, 1996.

A. Petrelli et al. NLO production and decay of quarkonium, 1998.

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

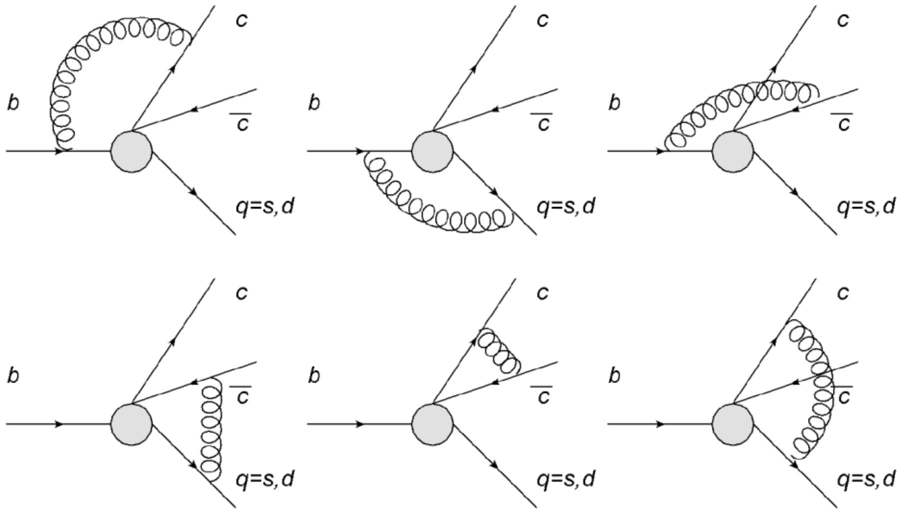
NLO result

NLO result

Virtual gluon corrections

NLO result

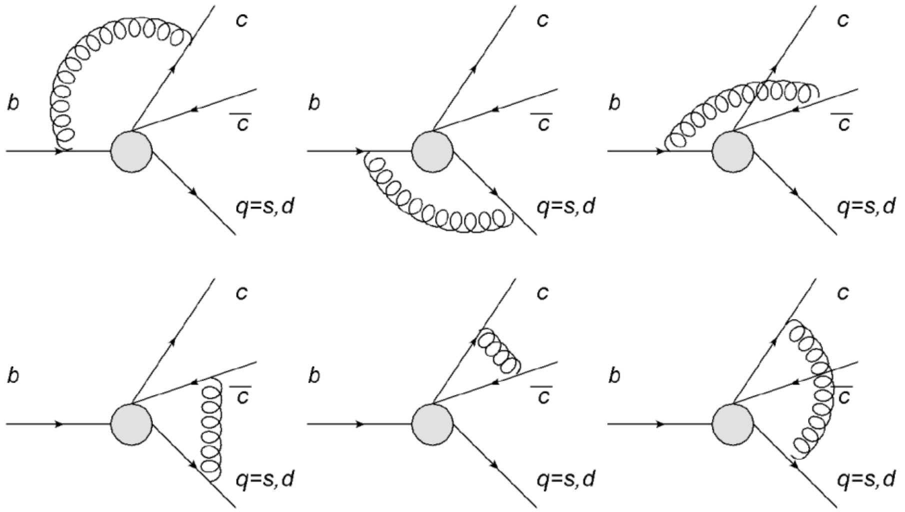
Virtual gluon corrections



NLO result

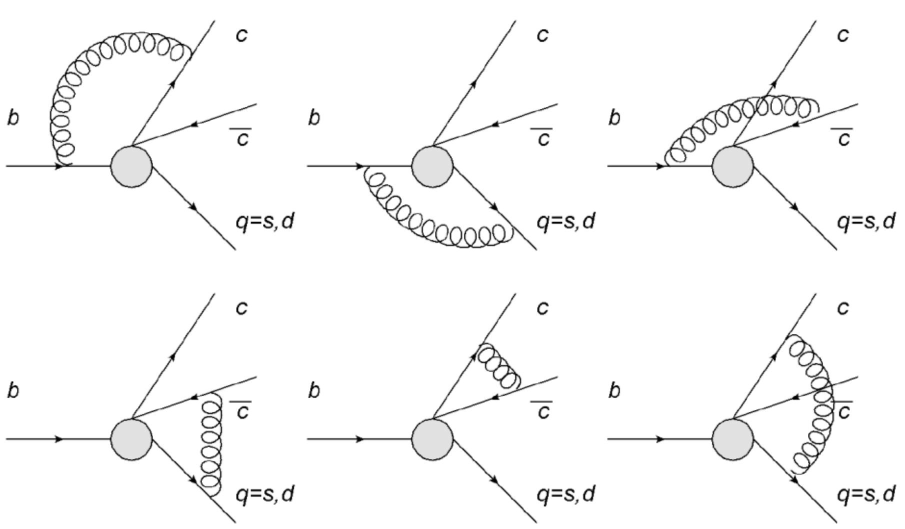
Virtual gluon corrections

Real gluon corrections

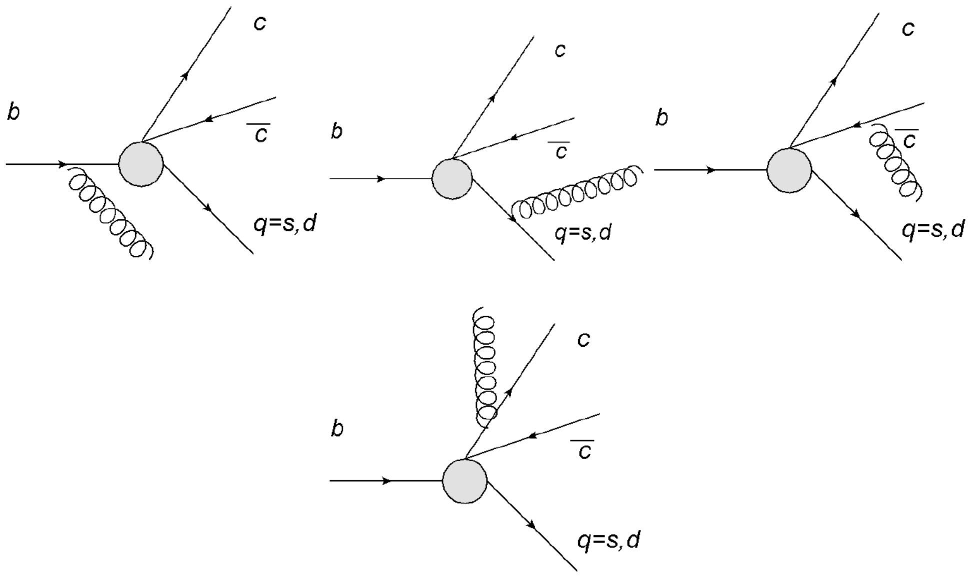


NLO result

Virtual gluon corrections

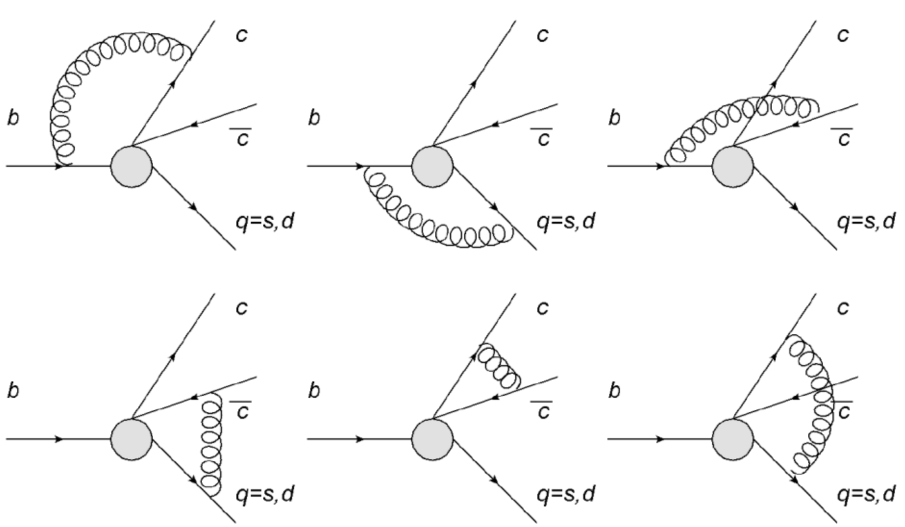


Real gluon corrections

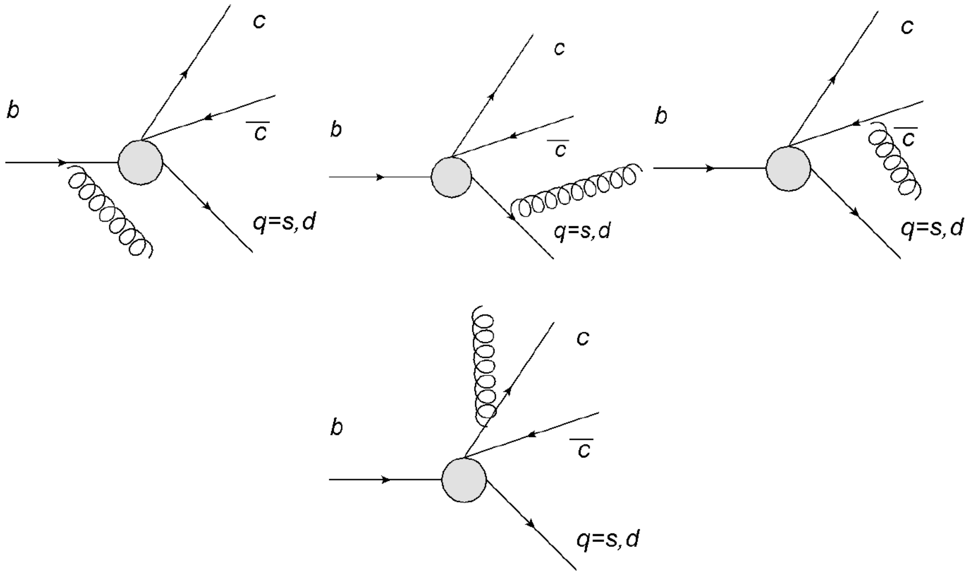


NLO result

Virtual gluon corrections



Real gluon corrections



$$\Gamma[n] = \Gamma_0 \left[C_{[1,8]}^2 f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\left(C_{[1]}^2 g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]}g_2[n](\eta,\mu,\tilde{\mu}) + C_{[8]}^2 g_3[n](\eta,\mu,\tilde{\mu}) \right) \right]$$

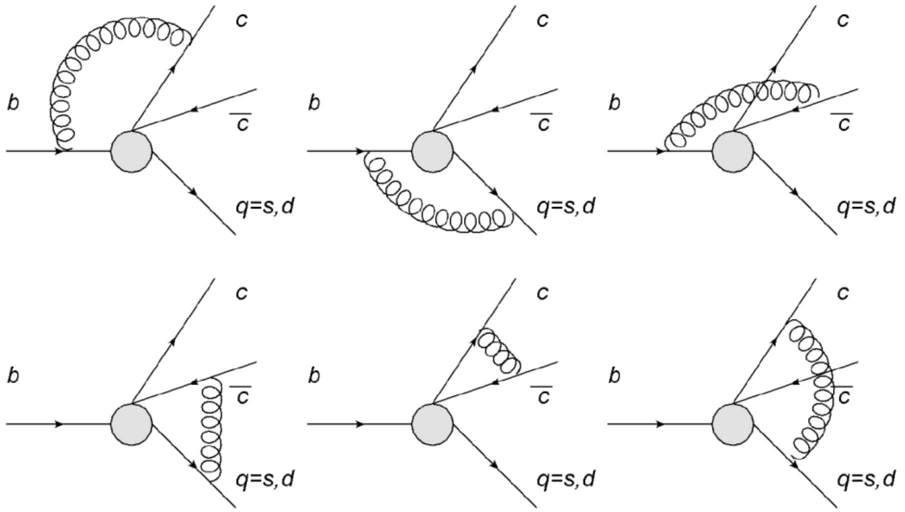
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

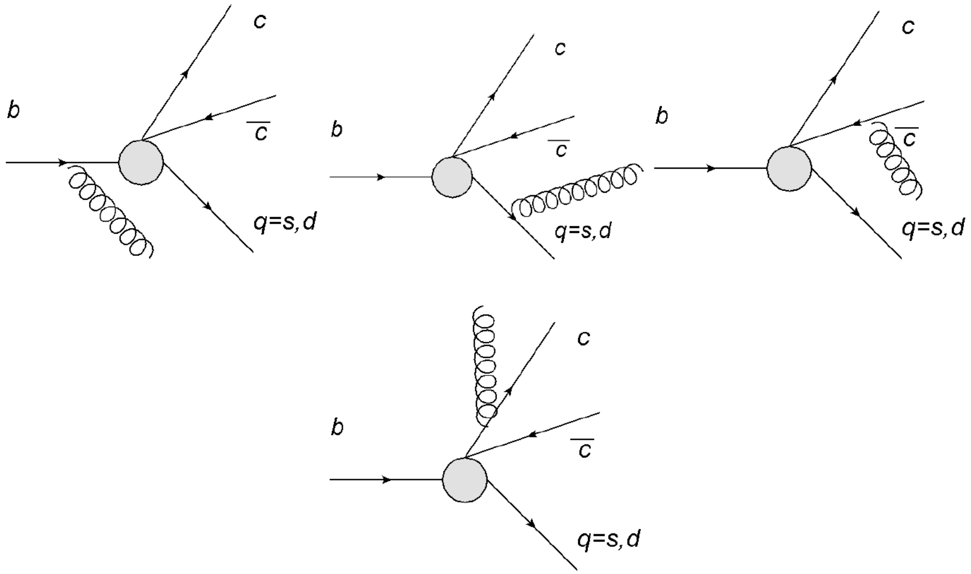
VNIVERSITAT DE VALÈNCIA

NLO result

Virtual gluon corrections



Real gluon corrections



$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu}) \Big) \Big]$$

QCD renornalization scale

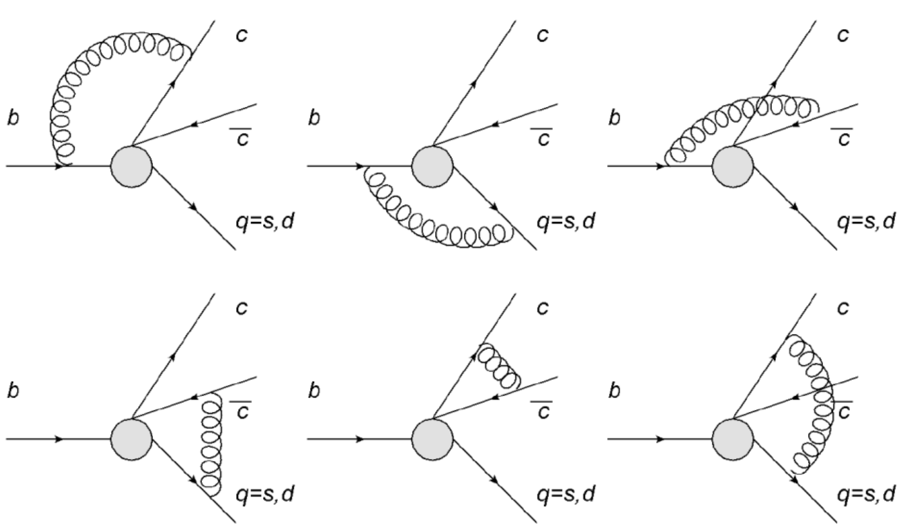
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

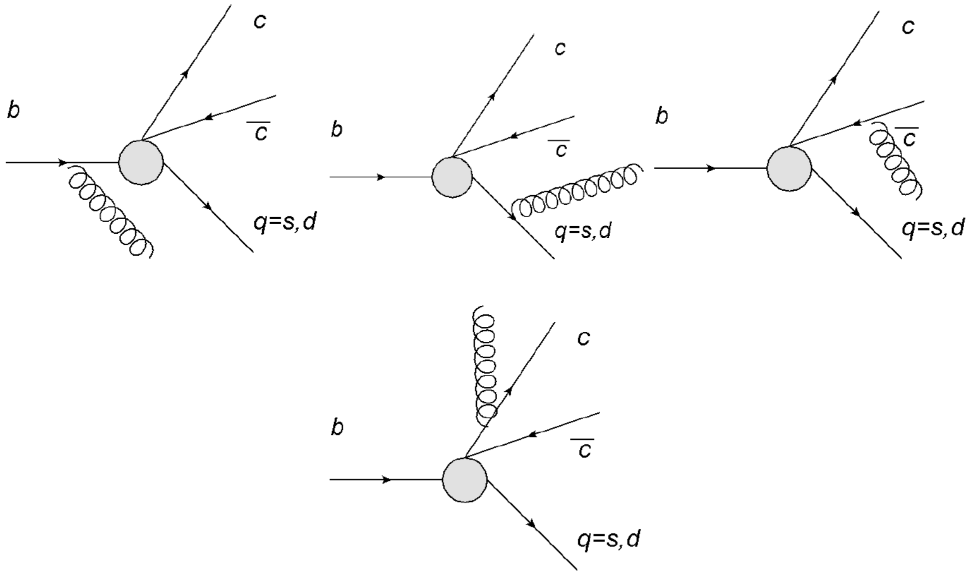
VNIVERSITAT DE VALÈNCIA

NLO result

Virtual gluon corrections



Real gluon corrections



$$\Gamma[n] = \Gamma_0 \left[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi} \left(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu}) \right) \right]$$

NRQCD renornalization scale

QCD renornalization scale

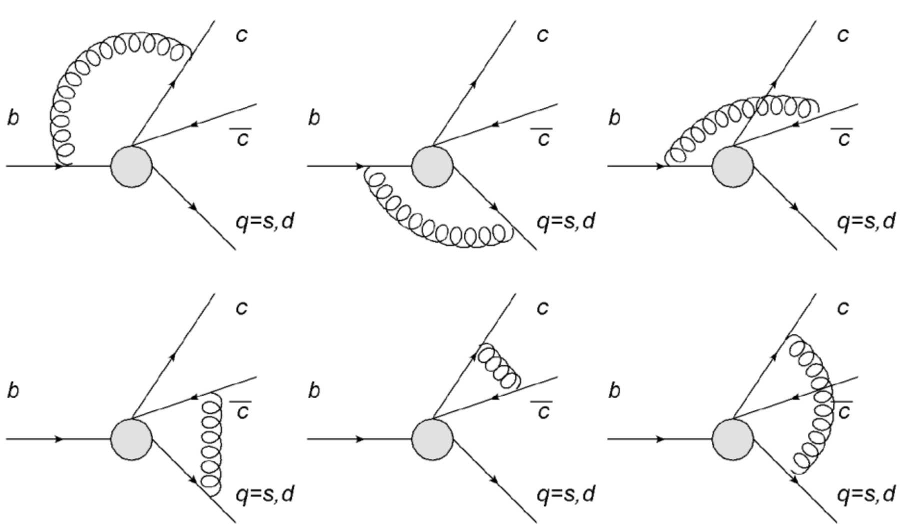
Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

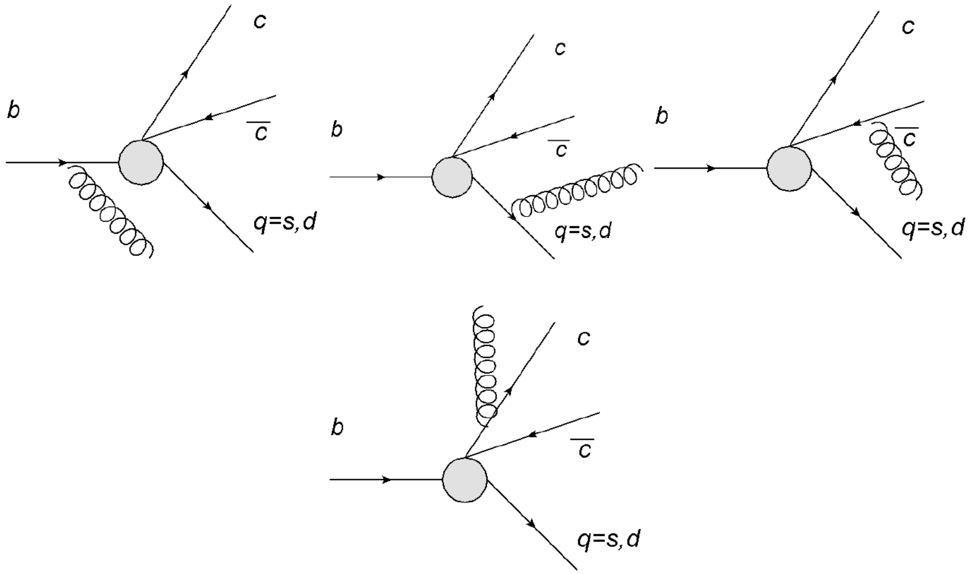
VNIVERSITAT DE VALÈNCIA

NLO result

Virtual gluon corrections



Real gluon corrections



$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu}) \Big) \Big]$$

NRQCD renornalization scale

QCD renornalization scale

M. Beneke et al. QCD analysis of inclusive B decay into charmonium, 1999.

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu}) \Big) \Big]$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} \underline{g_3[n](\eta,\mu,\tilde{\mu}}) \Big) \Big]$$

NLO

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}\underline{C_{[8]} g_2[n](\eta,\mu},\tilde{\mu}) + C^2_{[8]}\underline{g_3[n](\eta,\mu},\tilde{\mu}) \Big) \Big]$$

NNLO

NLO

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu}) + 2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu})\Big)\Big]$$

NNNLO NNLO NLO

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi} \Big(C^2_{[1]} \underbrace{g_1[n](\eta,\mu,\tilde{\mu})}_{\text{NNNLO}} + 2C_{[1]} \underbrace{C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})}_{\text{NNLO}} + C^2_{[8]} \underbrace{g_3[n](\eta,\mu,\tilde{\mu})}_{\text{NLO}} \Big) \Big]$$

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi} \Big(2C_{[1]} C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu}) \Big) + \Big(\frac{\alpha_s(\mu)}{4\pi} \Big)^2 C^2_{[8]} \frac{(g_2[n](\eta,\mu,\tilde{\mu}))^2}{f[n]} \Big]$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} \underline{g_1[n](\eta,\mu,\tilde{\mu})} + 2C_{[1]} \underline{C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})} + C^2_{[8]} \underline{g_3[n](\eta,\mu,\tilde{\mu})} \Big) \Big]$$

NNNLO NNLO NLO

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(2C_{[1]} C_{[8]} g_2[n](\eta,\mu,\tilde{\mu}) + \underline{C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu})} \Big) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C^2_{[8]} \frac{(g_2[n](\eta,\mu,\tilde{\mu}))^2}{f[n]} \Big]$$

NLO

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(C^2_{[1]} \underline{g_1[n](\eta,\mu,\tilde{\mu})} + 2C_{[1]} \underline{C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})} + C^2_{[8]} \underline{g_3[n](\eta,\mu,\tilde{\mu})} \Big) \Big]$$

NNNLO — NNLO — NLO

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi}\Big(\underline{2C_{[1]} C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})} + \underline{C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu})} \Big) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C^2_{[8]} \frac{(g_2[n](\eta,\mu,\tilde{\mu}))^2}{f[n]} \Big]$$

NNLO — NLO

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Color singlet channels, a problem to be faced

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi} \Big(\underbrace{C^2_{[1]} g_1[n](\eta,\mu,\tilde{\mu})}_{\text{NNNLO}} + 2C_{[1]}\underbrace{C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})}_{\text{NNLO}} + C^2_{[8]}\underbrace{g_3[n](\eta,\mu,\tilde{\mu})}_{\text{NLO}} \Big) \Big]$$

$$\Gamma[n] = \Gamma_0 \Big[C^2_{[1,8]} f[n](\eta)\,(1+\delta_P[n]) + \frac{\alpha_s(\mu)}{4\pi} \Big(\underbrace{2C_{[1]}C_{[8]} g_2[n](\eta,\mu,\tilde{\mu})}_{\text{NNLO}} + \underbrace{C^2_{[8]} g_3[n](\eta,\mu,\tilde{\mu})}_{\text{NLO}} \Big) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \underbrace{C^2_{[8]} \frac{(g_2[n](\eta,\mu,\tilde{\mu}))^2}{f[n]}}_{\text{NNLO}} \Big]$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Experimental fitting for χ_c states production

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{B}(B\to\chi_{c0}+X) &= A\frac{\langle O_1^{\chi_{c0}}({}^3P_0)\rangle}{m_c^2}+B\langle O_8^{\chi_{c0}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c1}+X) &= C\frac{\langle O_1^{\chi_{c1}}({}^3P_1)\rangle}{m_c^2}+B\langle O_8^{\chi_{c1}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c2}+X) &= D\frac{\langle O_1^{\chi_{c2}}({}^3P_2)\rangle}{m_c^2}+B\langle O_8^{\chi_{c2}}({}^3S_1)\rangle
\end{aligned}
$$

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{B}(B\to\chi_{c0}+X) &= A\frac{\langle O_1^{\chi_{c0}}({}^3P_0)\rangle}{m_c^2}+B\langle O_8^{\chi_{c0}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c1}+X) &= C\frac{\langle O_1^{\chi_{c1}}({}^3P_1)\rangle}{m_c^2}+B\langle O_8^{\chi_{c1}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c2}+X) &= D\frac{\langle O_1^{\chi_{c2}}({}^3P_2)\rangle}{m_c^2}+B\langle O_8^{\chi_{c2}}({}^3S_1)\rangle
\end{aligned}
$$

Parameter	Value	Corresponding quantum number ${}^{2s+1}L_J$ and color channel.
A	$-0.0252\,\mathrm{GeV}^{-3}$	${}^3P_0$ Singlet
B	$0.3279\,\mathrm{GeV}^{-3}$	${}^3S_1$ Octet
C	$-0.0147\,\mathrm{GeV}^{-3}$	${}^3P_1$ Singlet
D	$-0.0200\,\mathrm{GeV}^{-3}$	${}^3P_2$ Singlet

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{B}(B\to\chi_{c0}+X) &= A\frac{\langle O_1^{\chi_{c0}}({}^3P_0)\rangle}{m_c^2}+B\langle O_8^{\chi_{c0}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c1}+X) &= C\frac{\langle O_1^{\chi_{c1}}({}^3P_1)\rangle}{m_c^2}+B\langle O_8^{\chi_{c1}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c2}+X) &= D\frac{\langle O_1^{\chi_{c2}}({}^3P_2)\rangle}{m_c^2}+B\langle O_8^{\chi_{c2}}({}^3S_1)\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle O_1^{\chi_{cJ}}({}^3P_J)\rangle &= (2J+1)\langle O_1^{\chi_{c0}}({}^3P_0)\rangle \\
\langle O_8^{\chi_{cJ}}({}^3S_1)\rangle &= (2J+1)\langle O_8^{\chi_{c0}}({}^3S_1)\rangle
\end{aligned}
$$

Parameter	Value	Corresponding quantum number ${}^{2s+1}L_J$ and color channel.
A	$-0.0252\,\mathrm{GeV}^{-3}$	${}^3P_0$ Singlet
B	$0.3279\,\mathrm{GeV}^{-3}$	${}^3S_1$ Octet
C	$-0.0147\,\mathrm{GeV}^{-3}$	${}^3P_1$ Singlet
D	$-0.0200\,\mathrm{GeV}^{-3}$	${}^3P_2$ Singlet

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{B}(B\to\chi_{c0}+X) &= A\frac{\langle O_1^{\chi_{c0}}({}^3P_0)\rangle}{m_c^2} + B\langle O_8^{\chi_{c0}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c1}+X) &= C\frac{\langle O_1^{\chi_{c1}}({}^3P_1)\rangle}{m_c^2} + B\langle O_8^{\chi_{c1}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c2}+X) &= D\frac{\langle O_1^{\chi_{c2}}({}^3P_2)\rangle}{m_c^2} + B\langle O_8^{\chi_{c2}}({}^3S_1)\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle O_1^{\chi_{cJ}}({}^3P_J)\rangle &= (2J+1)\langle O_1^{\chi_{c0}}({}^3P_0)\rangle \\
\langle O_8^{\chi_{cJ}}({}^3S_1)\rangle &= (2J+1)\langle O_8^{\chi_{c0}}({}^3S_1)\rangle
\end{aligned}
$$

Parameter	Value	Corresponding quantum number ${}^{2s+1}L_J$ and color channel.
A	$-0.0252\,\mathrm{GeV}^{-3}$	${}^3P_0$ Singlet
B	$0.3279\,\mathrm{GeV}^{-3}$	${}^3S_1$ Octet
C	$-0.0147\,\mathrm{GeV}^{-3}$	${}^3P_1$ Singlet
D	$-0.0200\,\mathrm{GeV}^{-3}$	${}^3P_2$ Singlet

$$
\begin{aligned}
O_1 &\equiv \langle O_1^{\chi_{c0}}({}^3P_0)\rangle / m_c^2 \\
O_8 &\equiv \langle O_8^{\chi_{c0}}({}^3S_1)\rangle
\end{aligned}
$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Experimental fitting for χ_c states production

$$
\begin{aligned}
\left|H\left({}^{2S+1}L_J\right)\right\rangle &= O(1)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1]}\right)\right\rangle \\
&+ O(v)\left|Q\bar{Q}\left({}^{2S+1}(L\pm 1)_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S'+1}L_{J'}^{[8]}\right)g\right\rangle \\
&+ O\left(v^2\right)\left|Q\bar{Q}\left({}^{2S+1}L_J^{[1,8]}\right)gg\right\rangle \\
&+ \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{B}(B\to\chi_{c0}+X) &= A\frac{\langle O_1^{\chi_{c0}}({}^3P_0)\rangle}{m_c^2}+B\langle O_8^{\chi_{c0}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c1}+X) &= C\frac{\langle O_1^{\chi_{c1}}({}^3P_1)\rangle}{m_c^2}+B\langle O_8^{\chi_{c1}}({}^3S_1)\rangle \\
\mathscr{B}(B\to\chi_{c2}+X) &= D\frac{\langle O_1^{\chi_{c2}}({}^3P_2)\rangle}{m_c^2}+B\langle O_8^{\chi_{c2}}({}^3S_1)\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle O_1^{\chi_{cJ}}({}^3P_J)\rangle &= (2J+1)\langle O_1^{\chi_{c0}}({}^3P_0)\rangle \\
\langle O_8^{\chi_{cJ}}({}^3S_1)\rangle &= (2J+1)\langle O_8^{\chi_{c0}}({}^3S_1)\rangle
\end{aligned}
$$

Parameter	Value	Corresponding quantum number ${}^{2s+1}L_J$ and color channel.
A	$-0.0252\,\mathrm{GeV}^{-3}$	${}^3P_0$ Singlet
B	$0.3279\,\mathrm{GeV}^{-3}$	${}^3S_1$ Octet
C	$-0.0147\,\mathrm{GeV}^{-3}$	${}^3P_1$ Singlet
D	$-0.0200\,\mathrm{GeV}^{-3}$	${}^3P_2$ Singlet

$$
\begin{aligned}
O_1 &\equiv \langle O_1^{\chi_{c0}}({}^3P_0)\rangle/m_c^2 \\
O_8 &\equiv \langle O_8^{\chi_{c0}}({}^3S_1)\rangle
\end{aligned}
$$

Numerical results

Numerical results

Measurement	Experimental value
$\mathscr{B}(b \to \chi_{c0}\ X)$	$(3.02 \pm 0.47 \pm 0.23 \pm 0.94_{\mathscr{B}}) \times 10^{-3}$
$\mathscr{B}(b \to \chi_{c1}\ X)$	$(2.76 \pm 0.59 \pm 0.23 \pm 0.89_{\mathscr{B}}) \times 10^{-3}$
$\mathscr{B}(b \to \chi_{c2}\ X)$	$(1.15 \pm 0.20 \pm 0.07 \pm 0.36_{\mathscr{B}}) \times 10^{-3}$
$\frac{\mathscr{B}(b\to\chi_{c1}\ X)}{\mathscr{B}(b\to\chi_{c0}\ X)}$	$0.92 \pm 0.20 \pm 0.02 \pm 0.14_{\mathscr{B}}$
$\frac{\mathscr{B}(b\to\chi_{c2}\ X)}{\mathscr{B}(b\to\chi_{c0}\ X)}$	$0.38 \pm 0.07 \pm 0.01 \pm 0.05_{\mathscr{B}}$

R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \to \phi\phi\phi$, 2017

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Numerical results

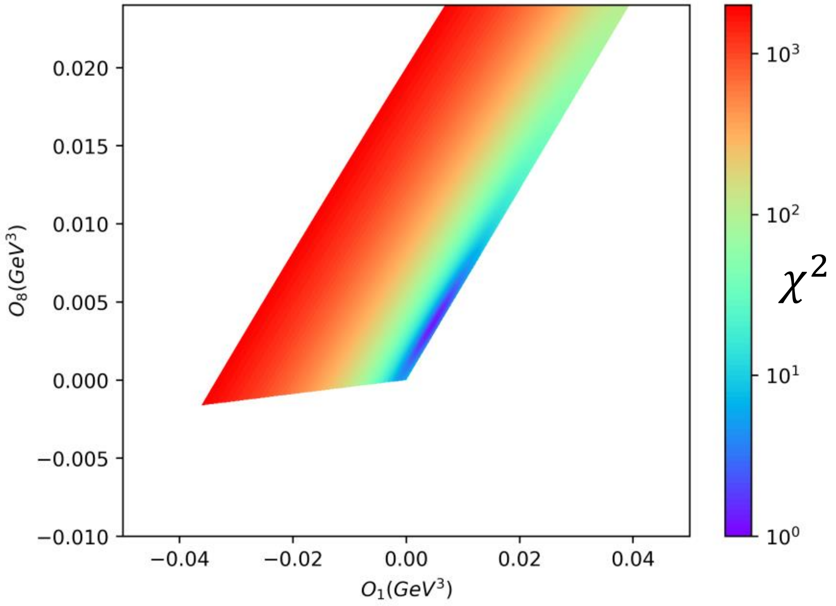
Measurement	Experimental value
$\mathcal{B}(b \to \chi_{c0}\ X)$	$(3.02 \pm 0.47 \pm 0.23 \pm 0.94_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c1}\ X)$	$(2.76 \pm 0.59 \pm 0.23 \pm 0.89_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c2}\ X)$	$(1.15 \pm 0.20 \pm 0.07 \pm 0.36_{\mathcal{B}}) \times 10^{-3}$
$\frac{\mathcal{B}(b\to\chi_{c1}\ X)}{\mathcal{B}(b\to\chi_{c0}\ X)}$	$0.92 \pm 0.20 \pm 0.02 \pm 0.14_{\mathcal{B}}$
$\frac{\mathcal{B}(b\to\chi_{c2}\ X)}{\mathcal{B}(b\to\chi_{c0}\ X)}$	$0.38 \pm 0.07 \pm 0.01 \pm 0.05_{\mathcal{B}}$

R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \to \phi\phi\phi$, 2017

Numerical results

Measurement	Experimental value
$\mathcal{B}(b \to \chi_{c0}\ X)$	$(3.02 \pm 0.47 \pm 0.23 \pm 0.94_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c1}\ X)$	$(2.76 \pm 0.59 \pm 0.23 \pm 0.89_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c2}\ X)$	$(1.15 \pm 0.20 \pm 0.07 \pm 0.36_{\mathcal{B}}) \times 10^{-3}$
$\frac{\mathcal{B}(b \to \chi_{c1}\ X)}{\mathcal{B}(b \to \chi_{c0}\ X)}$	$0.92 \pm 0.20 \pm 0.02 \pm 0.14_{\mathcal{B}}$
$\frac{\mathcal{B}(b \to \chi_{c2}\ X)}{\mathcal{B}(b \to \chi_{c0}\ X)}$	$0.38 \pm 0.07 \pm 0.01 \pm 0.05_{\mathcal{B}}$

R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \to \phi\phi\phi$, 2017



Gen—T

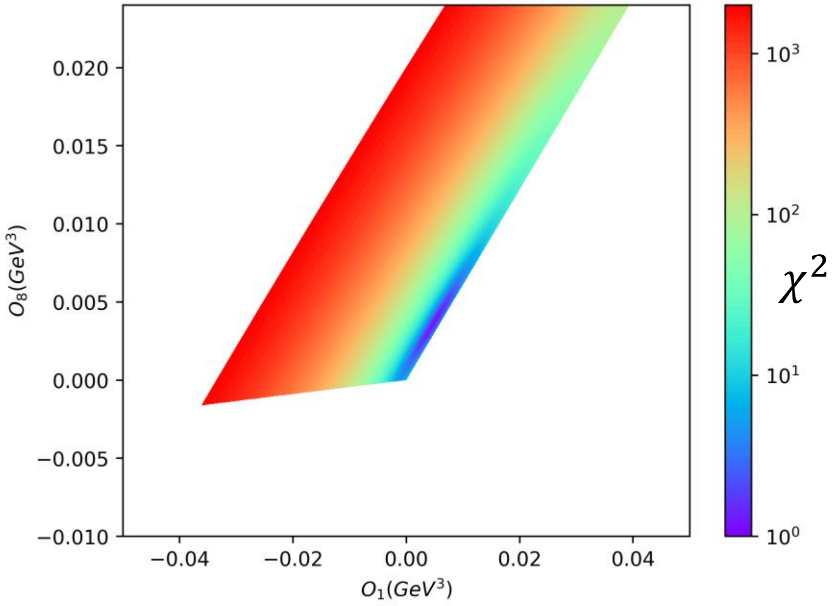
IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Numerical results

Measurement	Experimental value
$\mathcal{B}(b \to \chi_{c0}\ X)$	$(3.02 \pm 0.47 \pm 0.23 \pm 0.94_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c1}\ X)$	$(2.76 \pm 0.59 \pm 0.23 \pm 0.89_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c2}\ X)$	$(1.15 \pm 0.20 \pm 0.07 \pm 0.36_{\mathcal{B}}) \times 10^{-3}$
$\frac{\mathcal{B}(b \to \chi_{c1}\ X)}{\mathcal{B}(b \to \chi_{c0}\ X)}$	$0.92 \pm 0.20 \pm 0.02 \pm 0.14_{\mathcal{B}}$
$\frac{\mathcal{B}(b \to \chi_{c2}\ X)}{\mathcal{B}(b \to \chi_{c0}\ X)}$	$0.38 \pm 0.07 \pm 0.01 \pm 0.05_{\mathcal{B}}$

R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \to \phi\phi\phi$, 2017



Gen-T

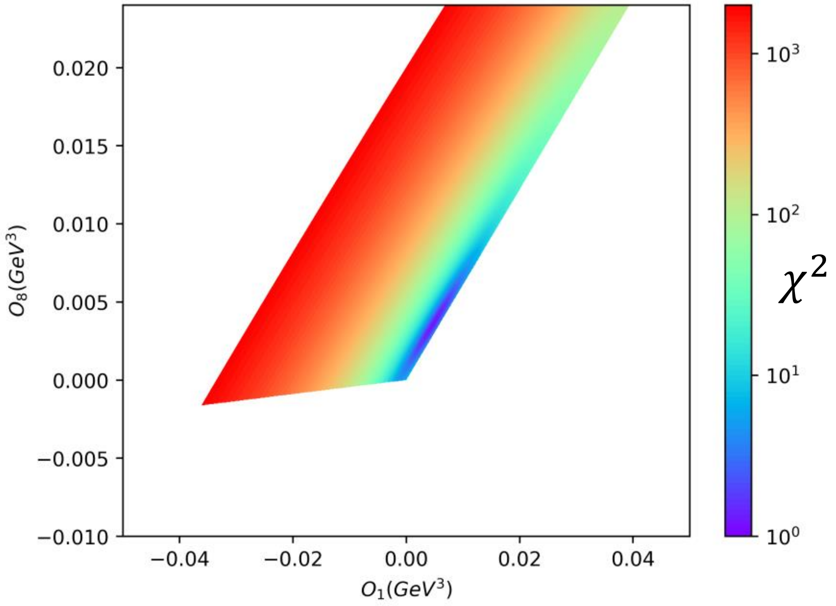
IFIC INSTITUT DE FÍSICA CORPUSCULAR

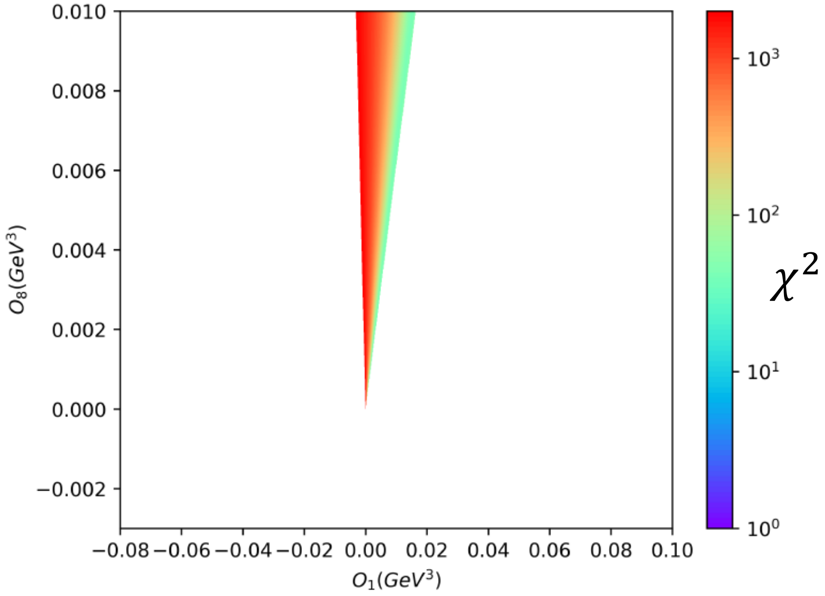
VNIVERSITAT DE VALÈNCIA

Numerical results

Measurement	Experimental value
$\mathcal{B}(b \to \chi_{c0}\ X)$	$(3.02 \pm 0.47 \pm 0.23 \pm 0.94_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c1}\ X)$	$(2.76 \pm 0.59 \pm 0.23 \pm 0.89_{\mathcal{B}}) \times 10^{-3}$
$\mathcal{B}(b \to \chi_{c2}\ X)$	$(1.15 \pm 0.20 \pm 0.07 \pm 0.36_{\mathcal{B}}) \times 10^{-3}$
$\frac{\mathcal{B}(b\to\chi_{c1}\ X)}{\mathcal{B}(b\to\chi_{c0}\ X)}$	$0.92 \pm 0.20 \pm 0.02 \pm 0.14_{\mathcal{B}}$
$\frac{\mathcal{B}(b\to\chi_{c2}\ X)}{\mathcal{B}(b\to\chi_{c0}\ X)}$	$0.38 \pm 0.07 \pm 0.01 \pm 0.05_{\mathcal{B}}$

R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \to \phi\phi\phi$, 2017





Gen-T
IFIC INSTITUT DE FÍSICA CORPUSCULAR
VNIVERSITAT DE VALÈNCIA

Theoretical uncertainties management

Theoretical uncertainties management

$$m_b = (4.8 \pm 0.3)\text{GeV}$$

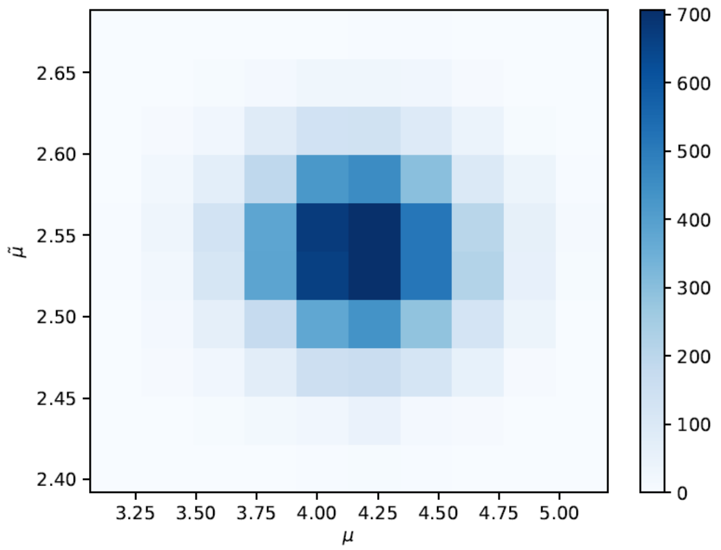
$$2m_c = (2.54 \pm 0.04)\text{GeV}$$

Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

Theoretical uncertainties management



$$m_b = (4.8 \pm 0.3)\mathrm{GeV}$$

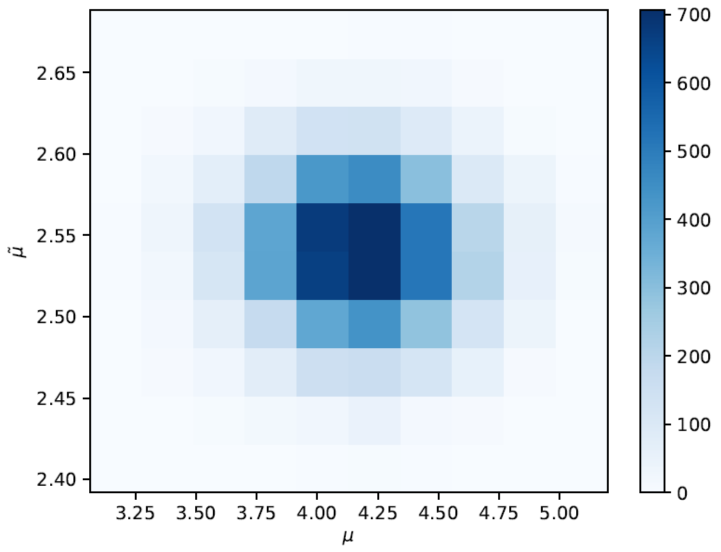
$$2m_c = (2.54 \pm 0.04)\mathrm{GeV}$$

Gen-T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

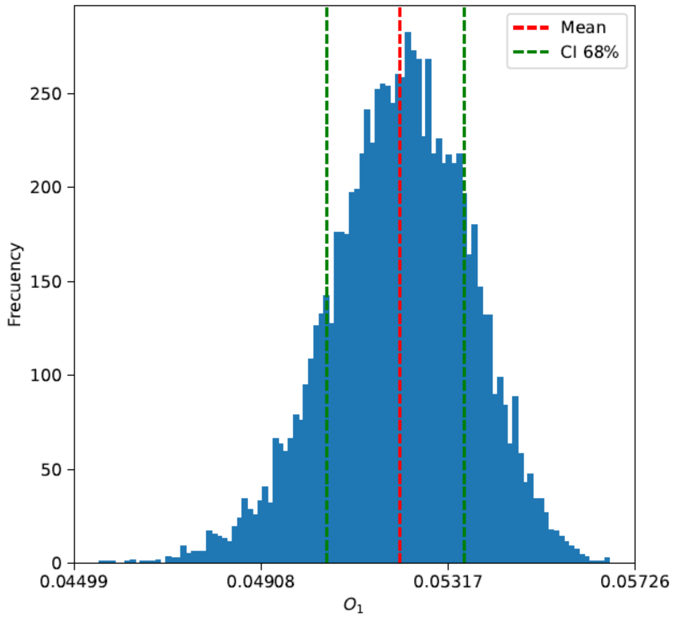
VNIVERSITAT DE VALÈNCIA

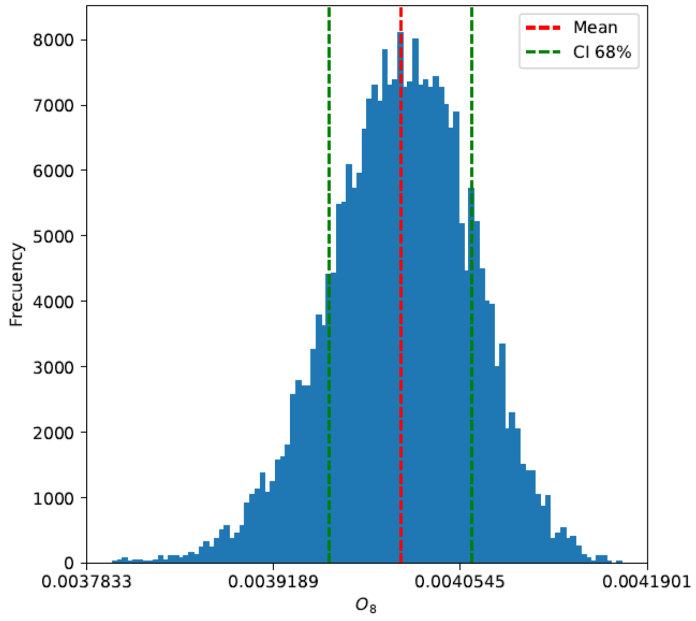
Theoretical uncertainties management



$$m_b = (4.8 \pm 0.3)\text{GeV}$$

$$2m_c = (2.54 \pm 0.04)\text{GeV}$$



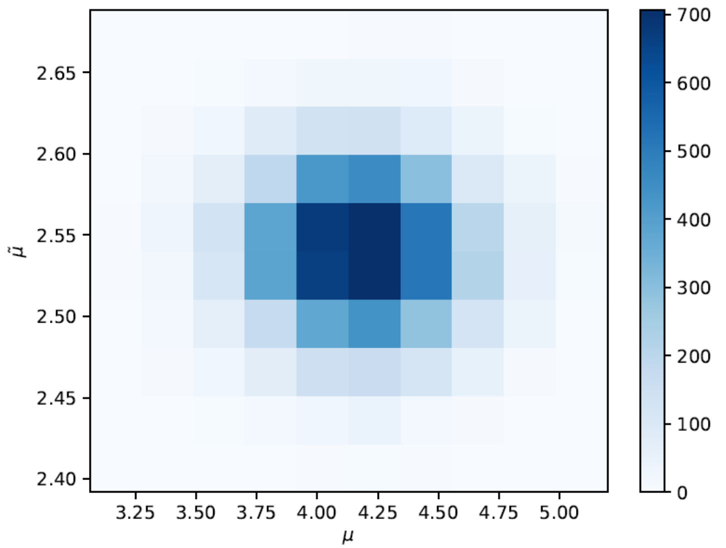


Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

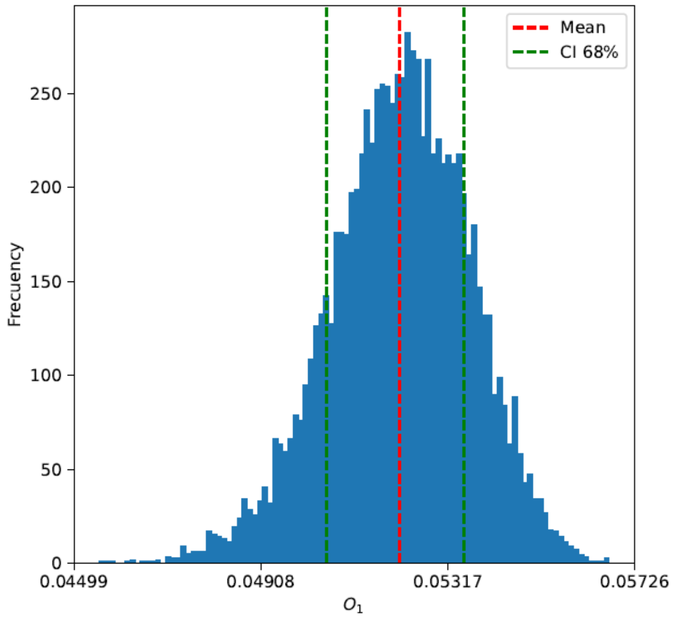
Theoretical uncertainties management

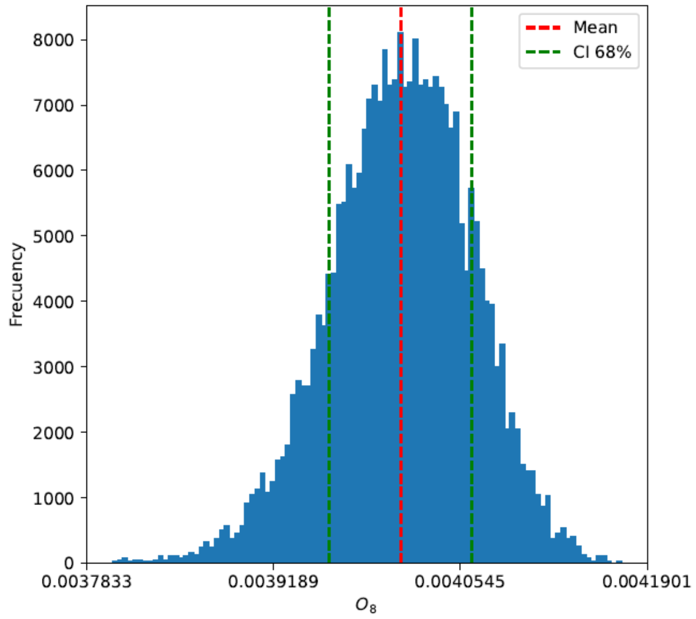


$$m_b = (4.8 \pm 0.3)\text{GeV}$$

$$2m_c = (2.54 \pm 0.04)\text{GeV}$$

	Value
O_1	$\left(0.05205 \pm 0.046488^{+0.00153}_{-0.00147}\right)\ \text{GeV}^3$
O_8	$\left(0.00400 \pm 0.002864^{+0.00006}_{-0.00004}\right)\ \text{GeV}^3$





Gen—T

IFIC INSTITUT DE FÍSICA CORPUSCULAR

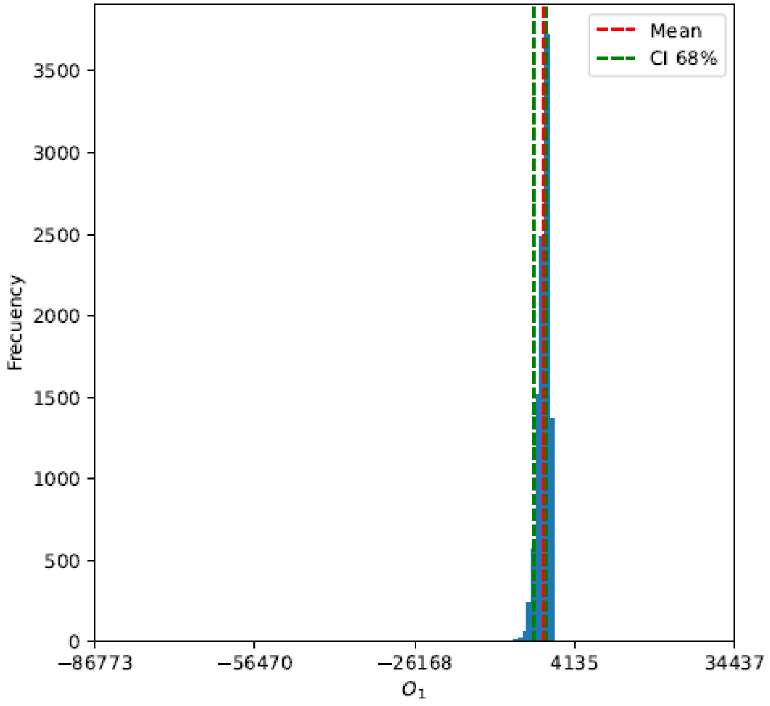
VNIVERSITAT DE VALÈNCIA

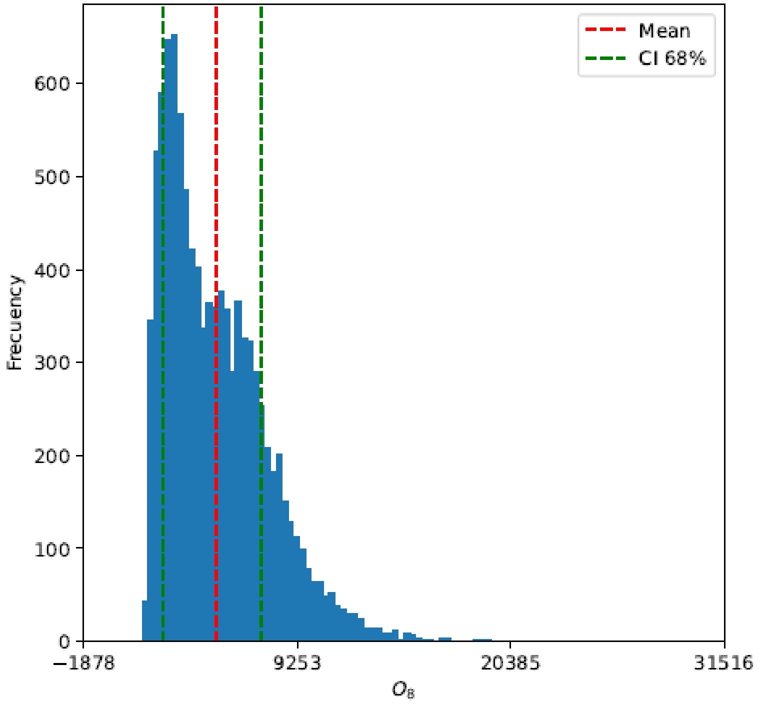
There is one problem....

For the ratio of the branching fractions

There is one problem....

For the ratio of the branching fractions





1) Motivation

2) The $D_S^+ \to K^+K^-K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

1) Motivation

2) The $D_S^+ \to K^+ K^- K^+$ decay

3) Inclusive charmonium production from B meson decays

4) Conclusions and perspectives

Conclusions and perspectives

- For the $D_s^+ \to 3K$ decay a model to the external W-emission topology has been achieved.
- It is necessary to fit the $D_s^+ \to 3K$ decay model parameters not only using the external W-emission topology but also constraining them even more by the inclusion of the W-annihilation topology
- Inclusive charmonium production from B meson decays can be modeled using NRQCD. However, the fitting of the LDMEs for χ_c states has proven that either the model or the minimization process within the fitting is no accurate enough for the ratio of the branching fractions.
- The next step would be to compute the SDCs at NNLO in QCD to estimate in a proper way the SDCs in the double parameter power counting.

References

- L. Chau. Quark Mixing in Weak Interactions, Phys. Rept., 95:1–94, 1983.
- R. T. Aude et al. Multimeson model for the D^+ →K^+ K^− K^+decay amplitude Phys. Rev. D, 98(5):056021, 2018.
- P. C. Magalhães et al. Multibody decay analyses: A new phenomenological model for meson-meson subamplitudes. Phys. Rev. D, 102(7):076012, 2020.
- J. Gasser and H. Leutwyler. Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark. Nucl. Phys. B, 250:465–516, 1985.
- G. Ecker et al. The Role of Resonances in Chiral Perturbation Theory. Nucl. Phys. B, 321:311–342, 1989.
- R. Aaij et al. Dalitz plot analysis of the $D^+ \rightarrow K^+K^-K^+$ decay, JHEP, 04:063, 2019.
- G. Buchallla et al. Weak decays beyond leading logarithms, Reviews of Modern Physics, 68(4):1125–1244, Oct 1996.
- E. Braaten et al. Helicity decomposition for inclusive J/Ψ production, Physical Review D, 54(5):3216–3227, Sep 1996.
- A. Petrelli et al. NLO production and decay of quarkonium, Nuclear Physics B, 514(1-2):245–309, Mar 1998.
- M. Beneke et al. QCD analysis of inclusive B decay into charmonium, Phys. Rev. D, 59:054003, 1999.
- R. Aaij et al. Study of charmonium production in b-hadron decays and first evidence for the decay $B_s^0 \rightarrow \phi\phi\phi$, Eur. Phys. J. C, 77(9):609, 2017.
- S. Barsuk et al. Test of nrqcd with charmonium production in inclusive b-hadron decays. LAL 17-051, jul 2017.