Strange-Meson Spectroscopy with COMPASS

Stefan Wallner
(swallner@mpp.mpg.de)

Max Planck Institute for Physics

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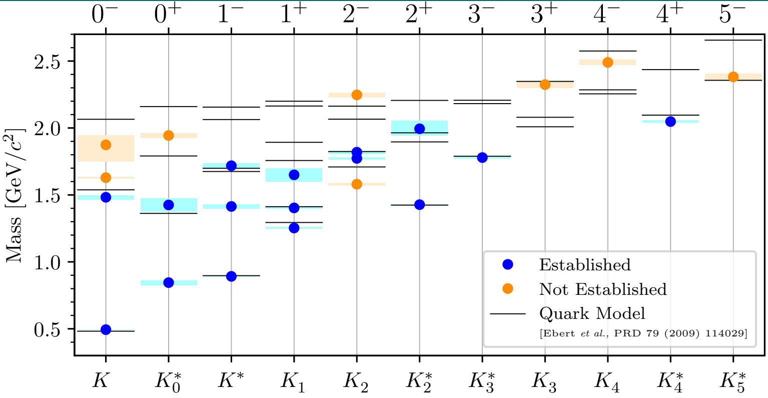
The Strange-Meson Spectrum

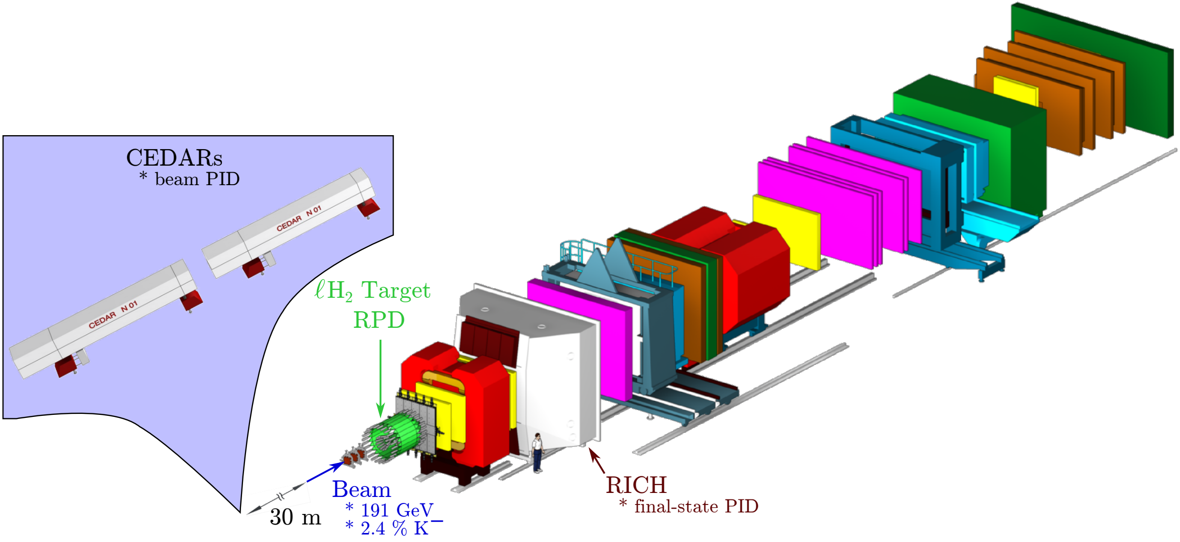


PDG lists 25 strange mesons (2022)

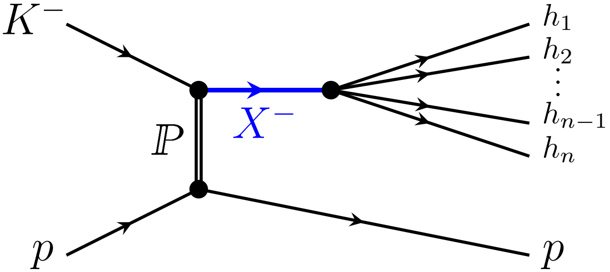
- 16 established states, 9 need further confirmation
- Missing states with respect to quark-model predictions
- Many measurements performed more than 30 years ago

COMPASS Setup for Hadron Beams

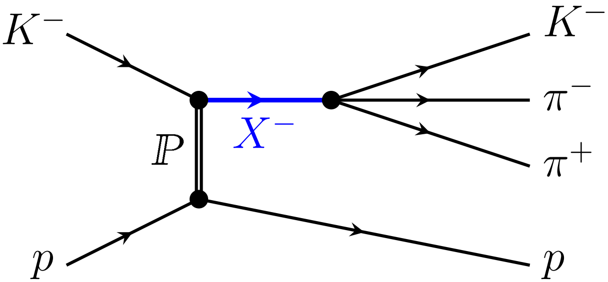
[COMPASS, Nucl. Instrum. Methods **779** (2015) 69]



Strange-Meson Spectroscopy with COMPASS
Production of Strange Mesons



- Diffractive scattering of high-energy kaon beam
- Strange mesons appear as intermediate resonances X^-
- Decay to multi-body hadronic final states
- $K^-\pi^-\pi^+$ final state
  - Study in principle all strange mesons
  - Study a wide mass range
  - COMPASS measured world's largest data set of about 720 k events



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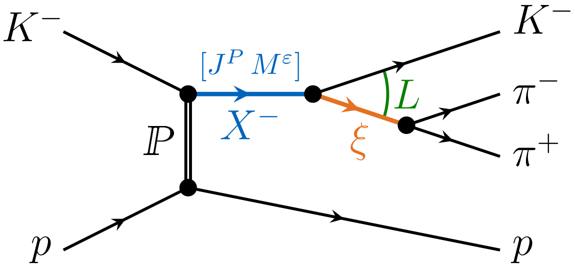
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Partial wave: $J^P\ M^\varepsilon\ \xi\ b^-\ L$

- J^P spin and parity
- M^ε spin projection
- ξ isobar resonance
- b^- bachelor particle
- L orbital angular momentum



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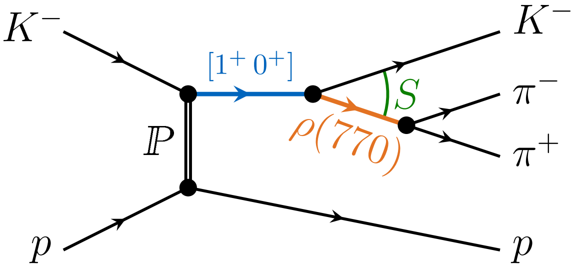

Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Data: 720 k diffractively produced $K^-\pi^-\pi^+$ candidates

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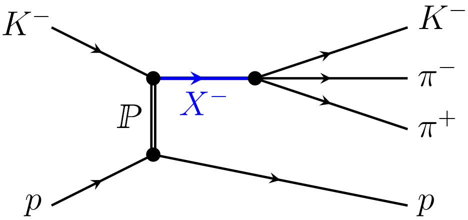
(I) Partial-Wave Decomposition

Performed independently in narrow $(m_{K\pi\pi}, t')$ cells

No assumption about $K\pi\pi$ resonances

Partial waves: Intensities and relative phases as a function of $(m_{K\pi\pi}, t')$



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[SW PhD]

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(II) Resonance-Model Fit

Model $m_{K\pi\pi}$ dependence of partial waves

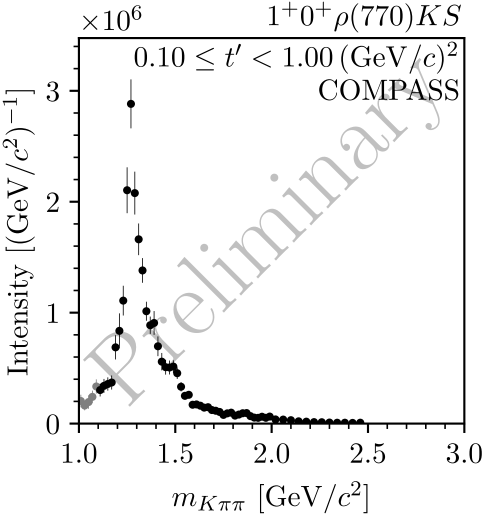
$K\pi\pi$ resonances and background

Resonance parameters: Masses and widths of the strange-meson resonances

Partial-Wave Decomposition

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_{a,b\in\mathbb{W}_z(m_{K\pi\pi},t')} \Psi_a(\tau)\rho_{ab}(m_{K\pi\pi}, t') \left[\Psi_b(\tau)\right]^*$$

- Measure spin-density matrix $\rho_{ab}(m_{K\pi\pi}, t')$ in independently $(m_{K\pi\pi}, t')$ cells
  - No assumption about $K^-\pi^-\pi^+$ resonances
- Wave set $\mathbb{W}_z(m_{K\pi\pi}, t')$ inferred from data using regularization-based model-selection techniques
- Bootstrap resampling to improve uncertainty estimates
  - Performed about $20\,\mathrm{M}$ fits
- Detailed Monte Carlo input-output studies

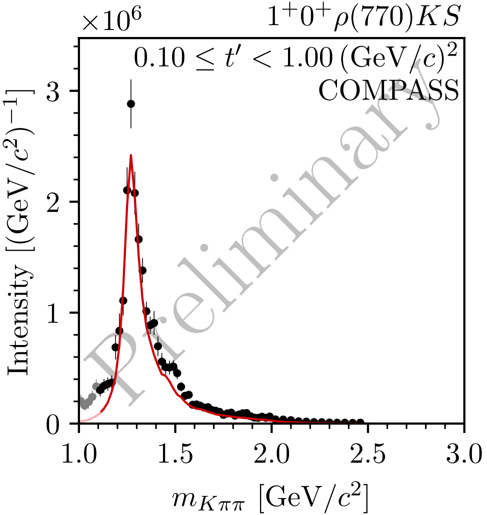


Resonance-Model Fit

$$\hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') = \hat{\mathcal{T}}_a(m_{K\pi\pi}, t') \left[\hat{\mathcal{T}}_b(m_{K\pi\pi}, t')\right]^*$$

$$\hat{\mathcal{T}}_a(m_{K\pi\pi}, t') = \sum_{k \in \mathbb{S}_a} \mathcal{K}(m_{K\pi\pi}, t')^k \mathcal{C}_a(t')\, \mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$$

- Model $m_{K\pi\pi}$ dependence of partial-wave amplitudes
- Breit-Wigner amplitudes for $K^-\pi^-\pi^+$ resonance components
- Coherent non-resonant component parameterizing other $K^-\pi^-\pi^+$ production mechanisms



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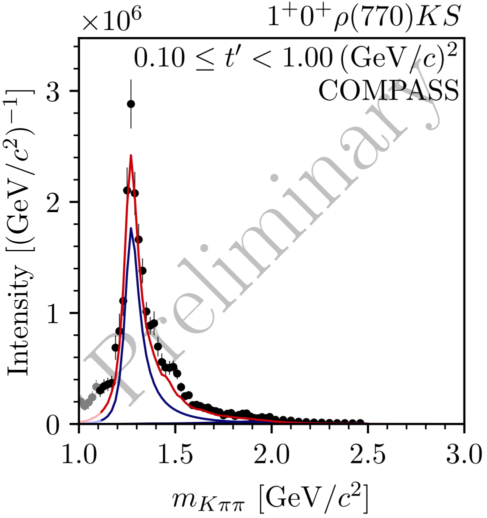

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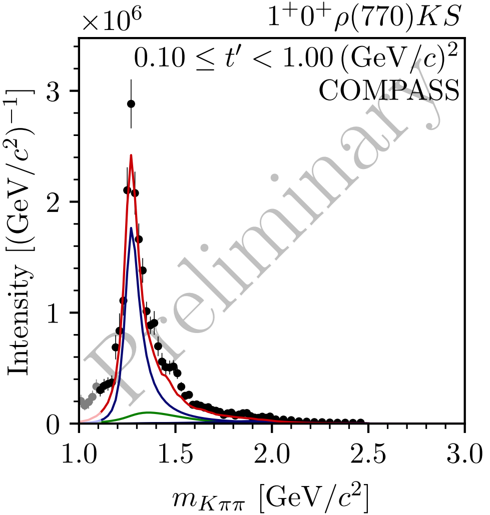

Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

[SW PhD]

Incoherent Backgrounds

- Incoherent background from π^- diffraction to $\pi^-\pi^-\pi^+$ and other reactions (in total about 10 %)
- Very good model for dominant $\pi^-\pi^-\pi^+$ background from COMPASS $\pi^-\pi^-\pi^+$ analysis
  - Study background in partial waves by
    - Generate pseudodata from $\pi^-\pi^-\pi^+$ model
    - Apply $K^-\pi^-\pi^+$ reconstruction event selection
    - Project into $K^-\pi^-\pi^+$ partial waves
  - Large in some waves, e.g. with $\rho(770)$ isobar
  - Small in other waves, e.g. with $K^*(892)$ isobar

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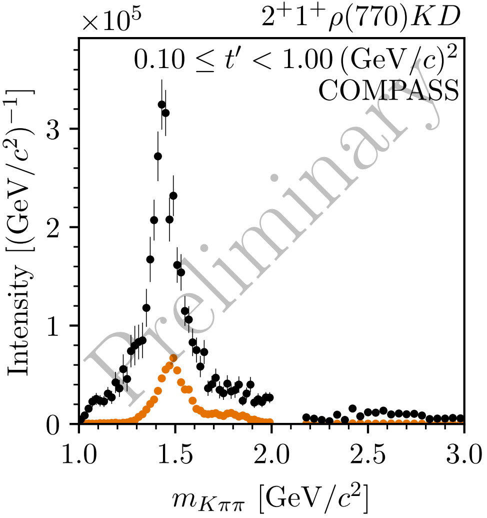

$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data

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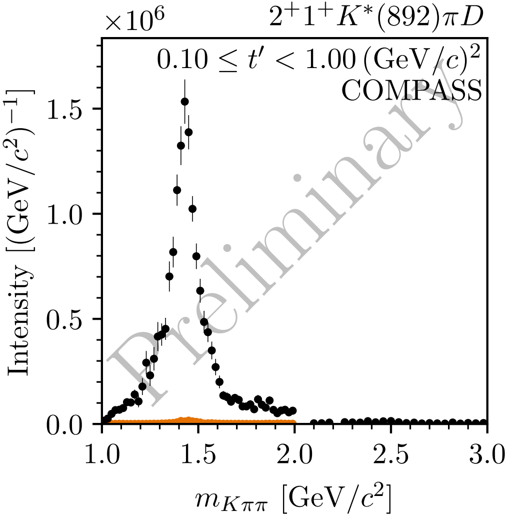

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Handling of Incoherent Backgrounds

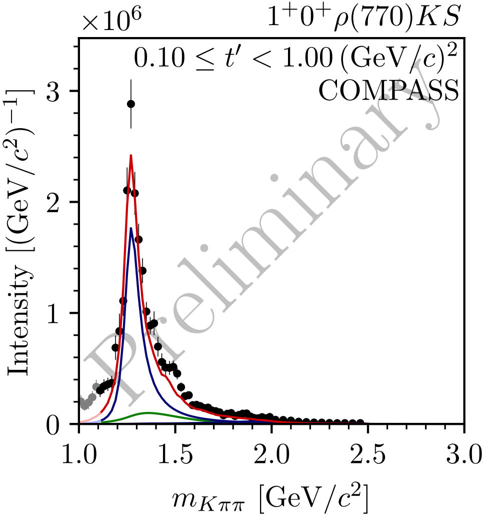
- Challanging to explicitly treat in partial-wave decomposition
  - ➥ Effectively taken into account

$$\rho_{ab} = \sum_z \mathcal{T}_a^z \, [\mathcal{T}_b^z]^*$$

  - ➥ Measured ρ_{ab} include background
- Explicitly model them in resonance-model fit

$$\hat{\rho}_{ab}(m_{K\pi\pi}, t') = \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t')$$

  - $\pi^-\pi^-\pi^+$ background modeled by partial-wave projection of $\pi^-\pi^-\pi^+$ pseudodata
    - Yield is only free parameter
  - Incoherent effective background component for other background processes



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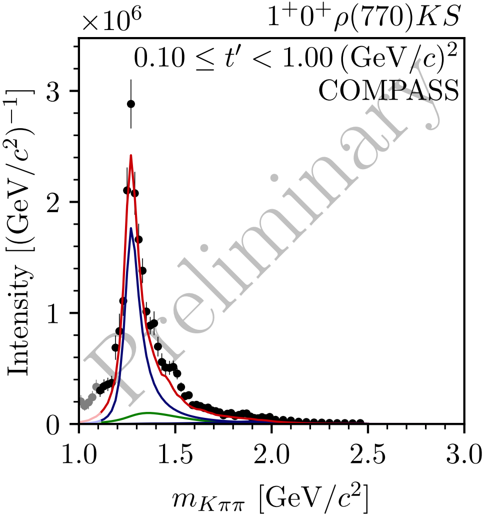

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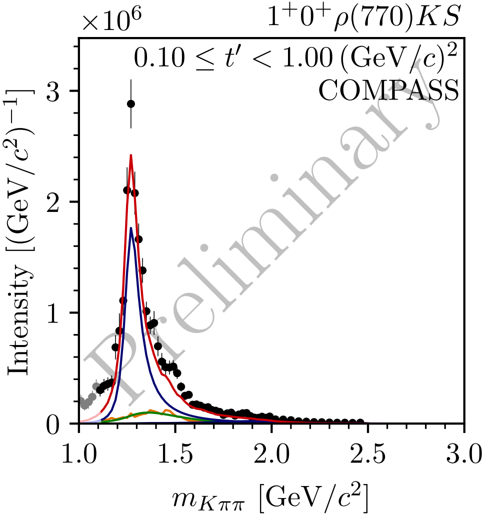

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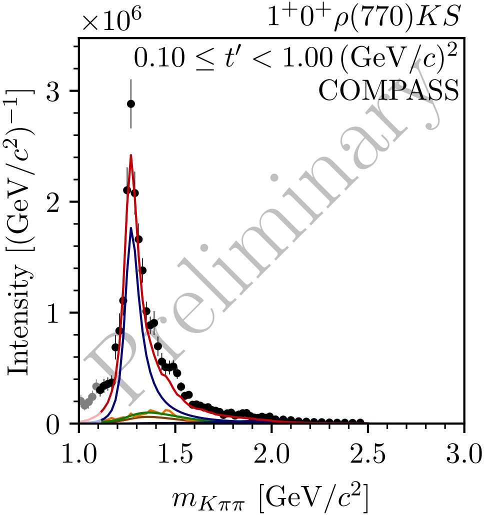

Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

- Simultaneously included 14 partial waves in resonance-model fit
- Modeled by 13 strange-meson resonance components
- Using measured intensities and interference terms (relative phases)

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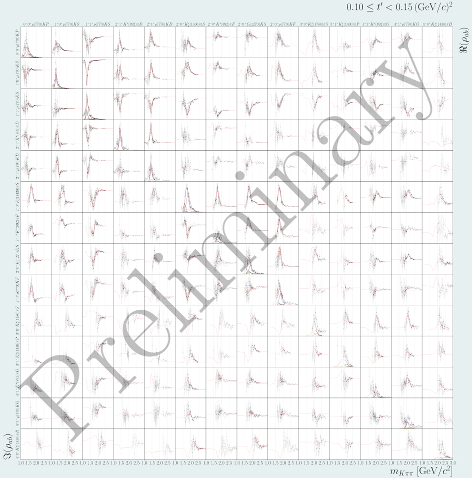

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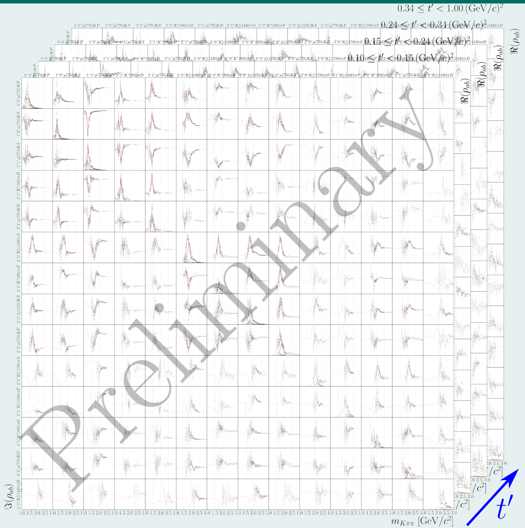
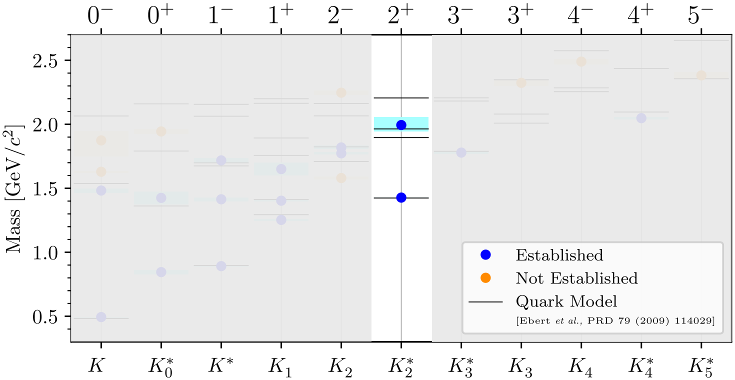


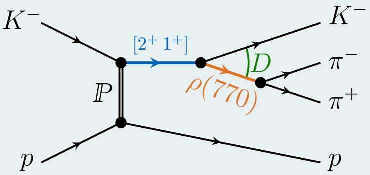


PDG

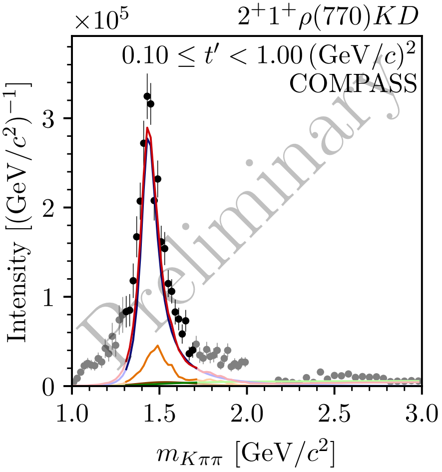
(2022)

- $K_2^*(1430)$ well known resonance

Partial Waves with $J^P = 2^+$



- $K_2^*(1430)$ signal
  - $m_0 = (1430.9 \pm 1.4\,^{+3.1}_{-1.5})\ \mathrm{MeV}/c^2$
  - $\Gamma_0 = (111 \pm 3\,^{+4}_{-16})\ \mathrm{MeV}/c^2$
- In different decays
  - $\rho(770)\,K\,D$
  - $K^*(892)\,\pi\,D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data
- Fitted yield of $\pi^-\pi^-\pi^+$ background consistent with expectation



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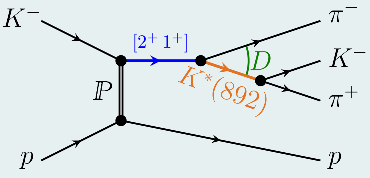

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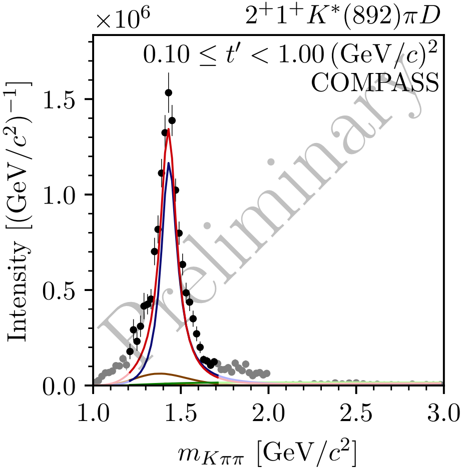

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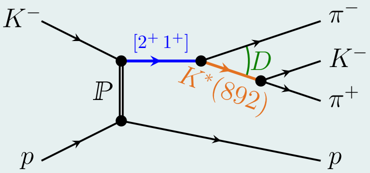

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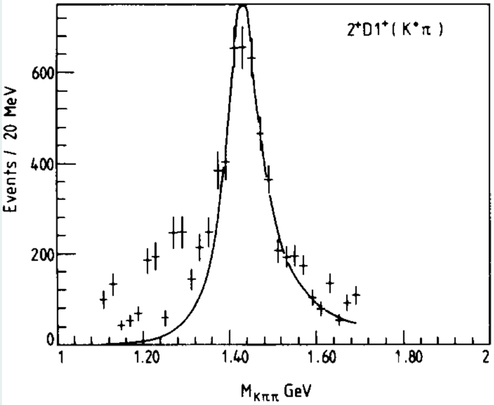
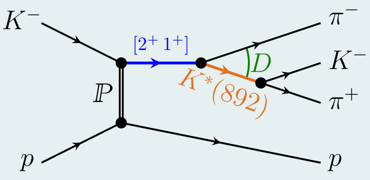


WA03 (CERN), 200 000 events, ACCMOR, Nucl. Phys. B **187** (1981)



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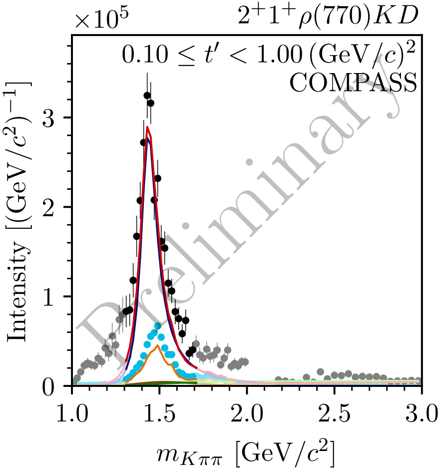

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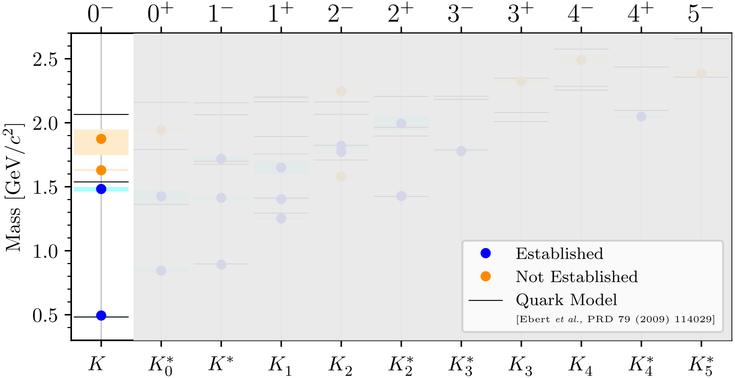
Searching for Exotic Strange Mesons with $J^P = 0^-$



PDG (2022)

- $K(1460)$ and $K(1830)$
- $K(1630)$
  - Unexpectedly small width of only $16\,\text{MeV}/c^2$
  - J^P of $K(1630)$ unclear

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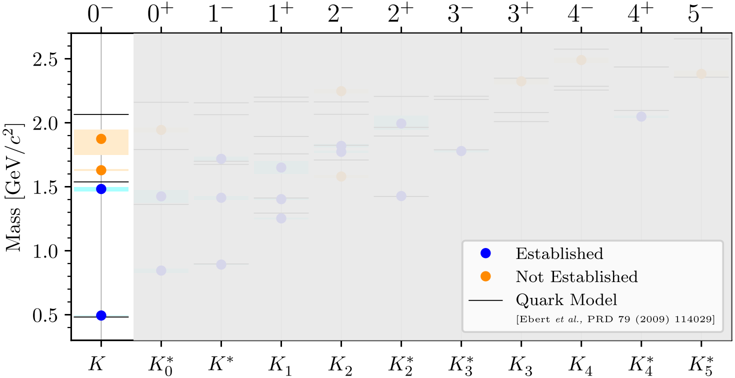

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  - Established $K(1460)$
  - But, $m_{K\pi\pi} \lesssim 1.5$ GeV$/c^2$ region weakly affected by known analysis artifacts
- Second peak at about 1.7 GeV$/c^2$
  - $K(1630)$ signal with $8.3\,\sigma$ statistical significance
  - Accompanied by rising phase
- Weak signal at about 2.0 GeV$/c^2$
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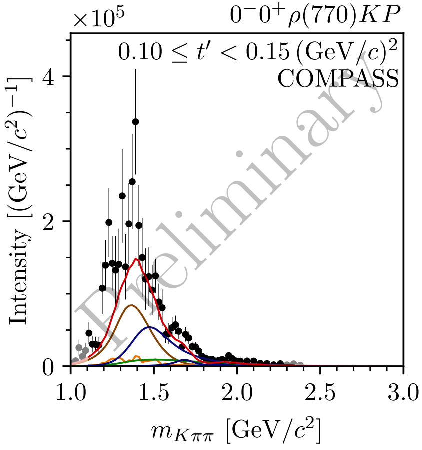

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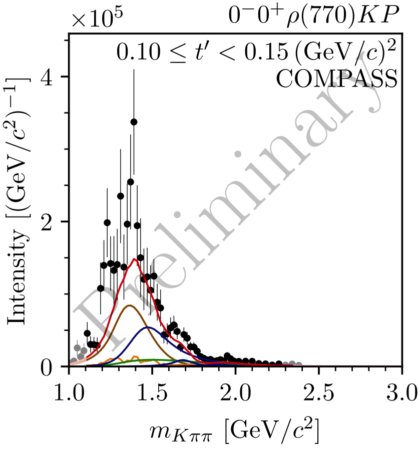

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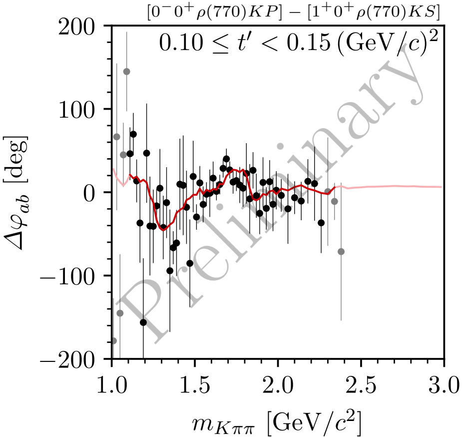

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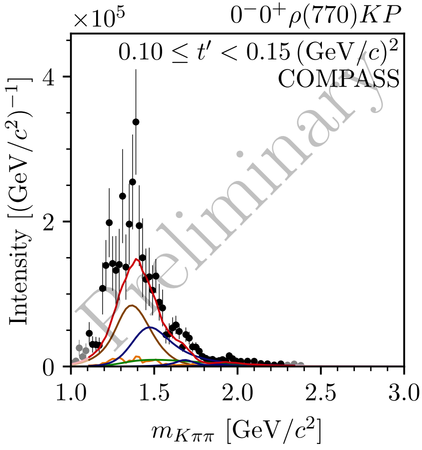
Searching for Exotic Strange Mesons with $J^P = 0^-$

COMPASS $K^-\pi^-\pi^+$ data

- Peak at about 1.4 GeV$/c^2$
  - Established $K(1460)$
  - But, $m_{K\pi\pi} \lesssim 1.5$ GeV$/c^2$ region weakly affected by known analysis artifacts
- Second peak at about 1.7 GeV$/c^2$
  - $K(1630)$ signal with $8.3\,\sigma$ statistical significance
  - Accompanied by rising phase
- Weak signal at about 2.0 GeV$/c^2$
  - $K(1830)$ signal with $5.4\,\sigma$ statistical significance



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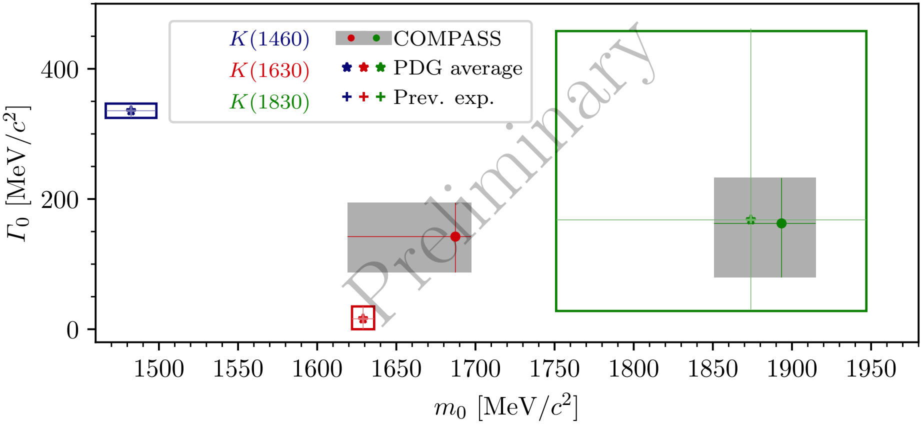
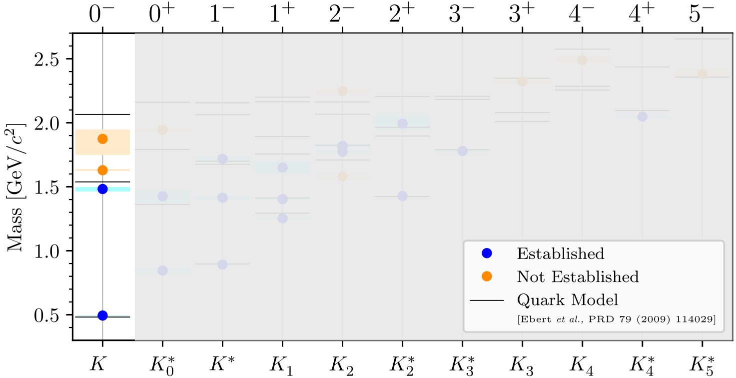


- $K(1830)$ parameters in good agreement with LCHb measurement [PRL 118 (2017) 022003]
- Expected $K(1630)$ width of about 140 MeV/c^2

Searching for Exotic Strange Mesons with $J^P = 0^-$



- Indications for 3 excited K from a single analysis
- Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$
  - ➥ $K(1630)$ supernumerary signal
  - ➥ Candidate for exotic non-$q\bar{q}$ state; other explanations possible ($K^*(892)\,\omega$ threshold nearby)

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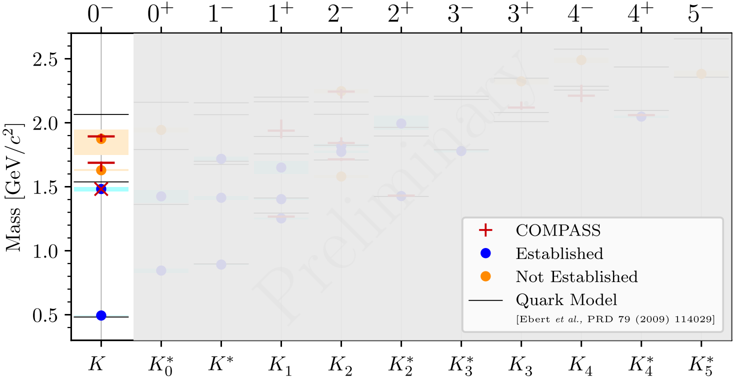

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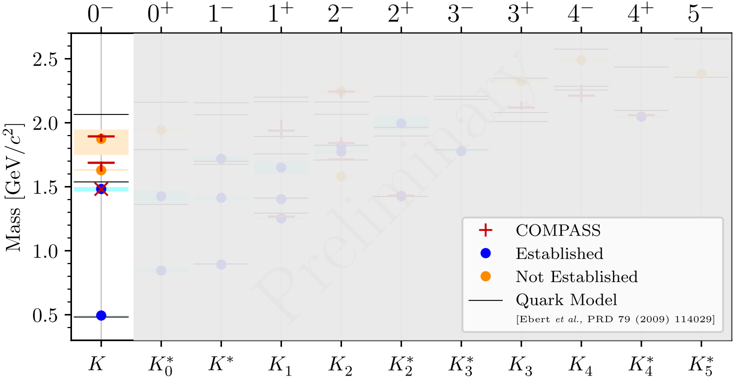

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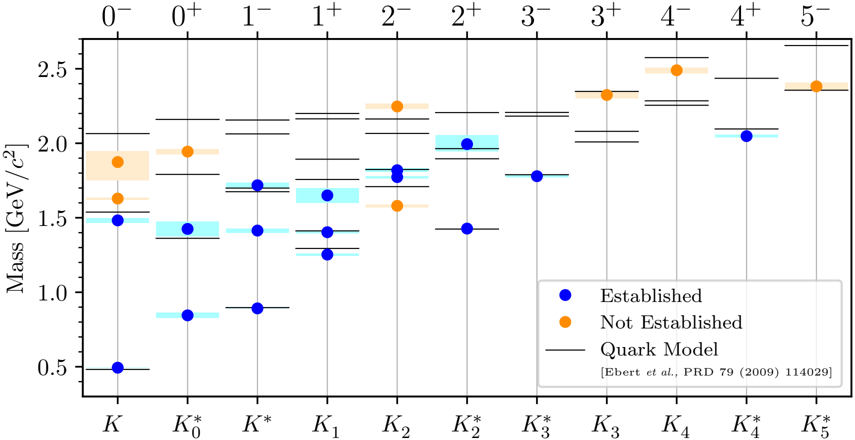
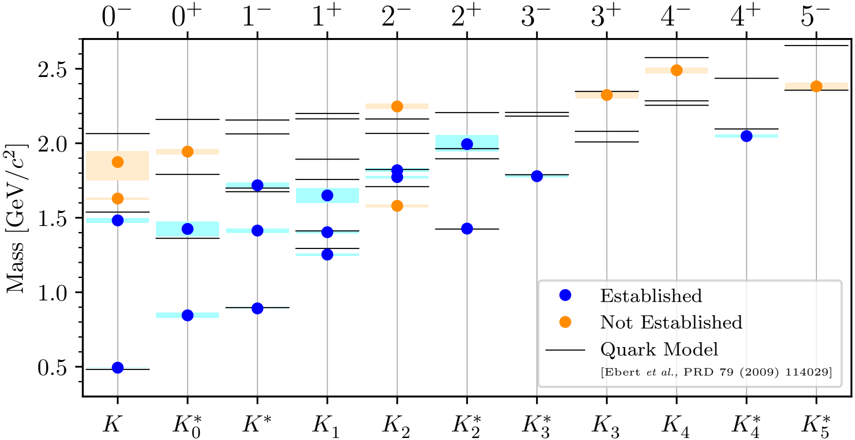


The Strange-Meson Spectrum

- Many strange mesons require further confirmation
- Search for strange partners of exotic non-strange light mesons

Summary



COMPASS

- World's largest data sample on $K^-\pi^-\pi^+$ $\Rightarrow$ Most detailed and comprehensive analysis
- Candidate for exotic strange-meson signal with $J^P = 0^-$

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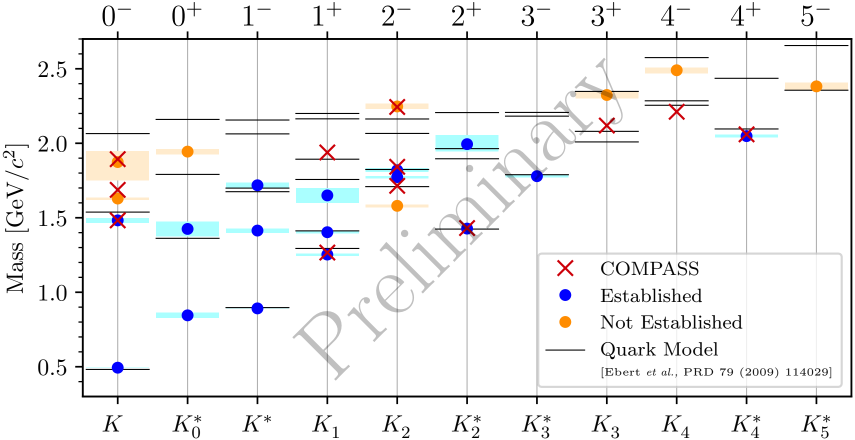

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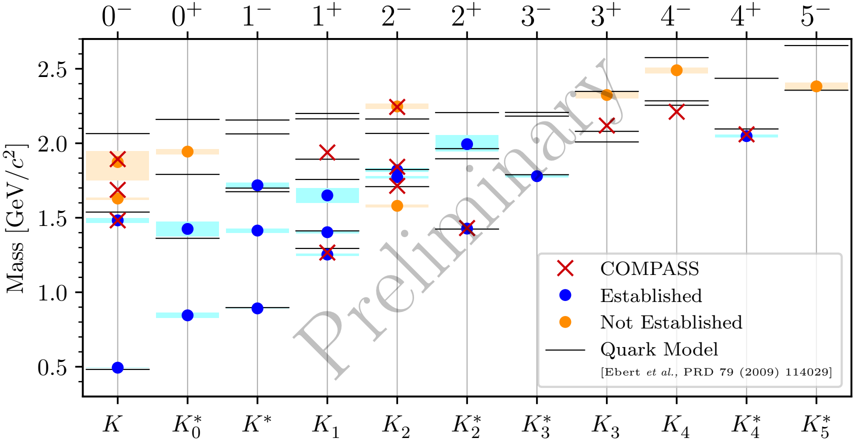
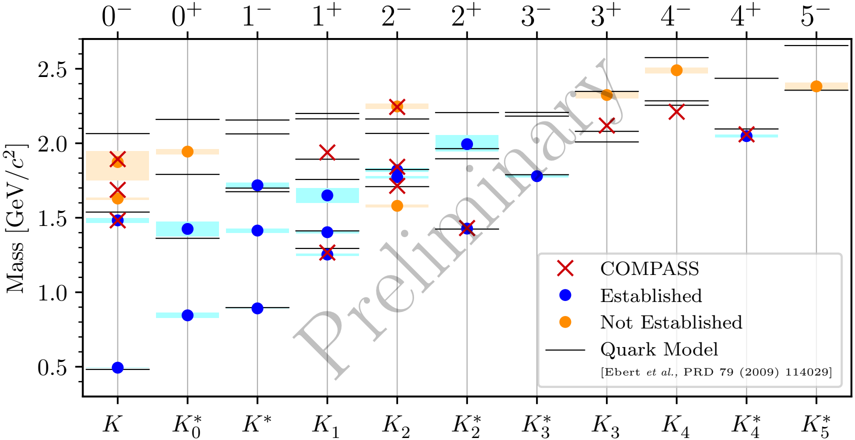


AMBER: Proposal for High-Precision Strange-Meson Spectroscopy

- Goal: Collect 10 – 20 $\times 10^6$ $K^-\pi^-\pi^+$ events using high-energy kaon beam
- AMBER is open for interested collaborators to join

Summary



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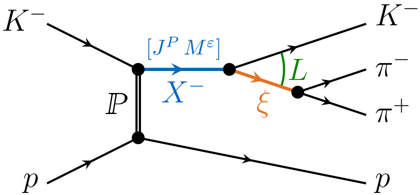
Backup

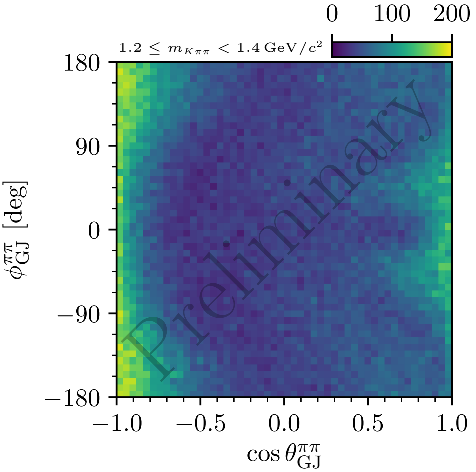
Outline



Partial wave

$$J^P M^\varepsilon \xi b L$$

- $J^P M^\varepsilon$: Spin, parity, and spin projection of X^-
- ξ: Isobar
- b: Bachelor particle. Here: Spectator K^-
- L: Angular momentum between bachelor and isobar

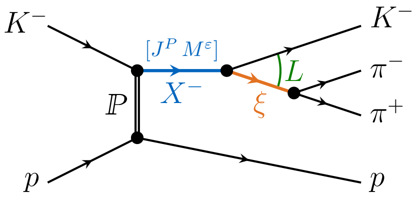


Partial-Wave Decomposition

Model intensity

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_{z} \left| \sum_{a \in \mathbb{W}_z(m_{K\pi\pi}, t')} \mathcal{T}_a^z(m_{K\pi\pi}, t') \Psi_a^z(\tau; m_{K\pi\pi}) \right|^2$$

- Model intensity distribution
  - in 5D $K^-\pi^-\pi^+$ phase-space
  - for a given $(m_{K\pi\pi}, t')$ cell
  - as incoherent sum over coherent sectors z
    - "Rank" of the partial-wave model = number of coherent sectors
- Ψ_a^z known, assuming the isobar model
- Wave set $\mathbb{W}_z(m_{K\pi\pi}, t')$ inferred from data using regularization-based model-selection techniques
- $\mathcal{T}_a^z$ extracted in maximum-likelihood fit, independently for each $(m_{K\pi\pi}, t')$ cell



Spin-Density Matrix

$$\rho_{ab} = \sum_z \mathcal{T}_a^z \, [\mathcal{T}_b^z]^*$$

Partial-Wave Decomposition

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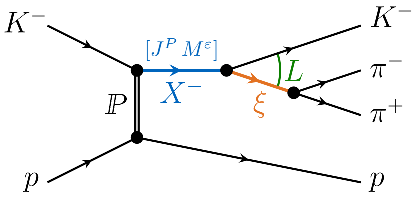

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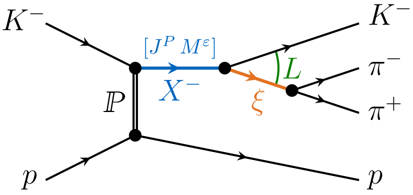

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Approach

- Effectively take into account in partial-wave decomposition by incoherently adding additional coherent sectors z
  (Model background by $K^-\pi^-\pi^+$ partial waves)
  - ➥ Increasing the rank of the spin-density matrix ρ_{ab}
  - ➥ Signal not separated from background in partial-wave decomposition
  - ➥ Partial-wave amplitudes include background
- Model signal and background contributions in resonance-model fit using more constrained signal model
  - ➥ Separate signal from background

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_{z} \left| \sum_{a \in \mathbb{W}_z(m_{K\pi\pi}, t')} \mathcal{T}_a^z(m_{K\pi\pi}, t') \Psi_a^z(\tau; m_{K\pi\pi}) \right|^2$$

$$\rho_{ab} = \sum_{z} \mathcal{T}_a^z \, [\mathcal{T}_b^z]^*$$

Partial-Wave Decomposition

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

True physics intensity distribution

$$\mathcal{I}(\tau) = \left| \sum_{a}^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau) = \eta(\tau)\,\mathcal{I}(\tau)$$

- Take into account different processes $\mathfrak{p}$
  - Different model intensities $\mathcal{I}^{\mathfrak{p}}$
  - Different experimental acceptance $\eta^{\mathfrak{p}}$
  - Formulated in terms of different phase-space variables $\tau^{\mathfrak{p}}$
    - Jacobian terms $J(\tau^{K\pi\pi} \to \tau^{\mathfrak{p}})$ from variable transformation

True physics intensity distribution for process $\mathfrak{p}$

$$\mathcal{I}^{\mathfrak{p}}(\tau\ ) = \left|\sum_{a}^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}}\,\Psi_a^{\mathfrak{p}}(\tau\ )\right|^2$$

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- $\mathcal{I}^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation

- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $\mathcal{I}^{\mathfrak{p}}$ unknown
  - Unknown background channels

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Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \, \Psi_a^{\mathfrak{p}}(\tau^{\mathfrak{p}}) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a^{\text{waves}} \tilde{\mathcal{T}}_a^{\mathfrak{p}} \, \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

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Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \eta^{K\pi\pi}(\tau^{K\pi\pi}) \, \mathcal{I}(\tau^{K\pi\pi})$$

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $\mathfrak{p}$
  - Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
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$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b}^{\text{waves}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \, \rho_{a,b} \, [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_\mathrm{r} > 1$

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Partial-Wave Decomposition

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

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$$\rho_{a,b} = \sum_{\mathfrak{r}}^{N_\mathrm{r}} \mathcal{T}_a^{\mathfrak{r}} [\mathcal{T}_b^{\mathfrak{r}}]^*$$

- Experimentally measurable quantities are spin-density matrix elements
  - ➥ Transition amplitudes $\mathcal{T}_a^{\mathfrak{p}}$ are only effective parameters
  - ➥ Cannot determine $\mathcal{T}_a^{\mathfrak{p}}$ of individual processes
  - ➥ Cannot separate different processes

Partial-Wave Decomposition

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

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$$\rho_{a,b} = \sum_{\mathfrak{r}}^{N_\mathrm{r}} \mathcal{T}_a^{\mathfrak{r}} [\mathcal{T}_b^{\mathfrak{r}}]^*$$

- Large number of fit parameters: $N_\mathrm{para} = N_\mathrm{r}(2N_\mathrm{waves} - N_\mathrm{r})$
- Sufficient rank of spin-density matrix must be determined
  - Rank two needed to describe pure $\pi^-\pi^-\pi^+$ Monte Carlo sample using $K^-\pi^-\pi^+$ partial waves
  - Used rank three to model $K^-\pi^-\pi^+$ sample

Resonance-Model Fit

Data

720 k diffractively produced $K^-\pi^-\pi^+$ candidates

(I) Partial-Wave Decomposition

Partial Waves

Intensities and relative phases of the partial waves

(II) Resonance-Model Fit

Resonance Parameters

Masses and widths of the meson resonances

Resonance-Model Fit

- Spin-density matrix $\rho_{ab}(m_{K\pi\pi}, t')$ measured in partial-wave decomposition
- Model spin-density matrix in resonance-model fit

$$\hat{\rho}_{ab}(m_{K\pi\pi}, t') = \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{3\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{\mathrm{Bkg}}(m_{K\pi\pi}, t')$$

Model transition amplitudes as coherent sum over various components

$$\hat{\mathcal{T}}_a^z(m_{K\pi\pi}, t') = \sum_{k \in \mathbb{S}_a} K(m_{K\pi\pi}, t')\, {}^k\mathcal{C}_a^{K\pi\pi}(t')\, \mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$$

- Dynamic functions $\mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$
  - For resonances: rel. Breit-Wigner
  - For non-resonant terms: $\mathcal{D}_k^{\mathrm{NR}}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{\mathrm{thr}})^{a_k} e^{-b(c_k)\, \tilde{q}_k^2(m_{K\pi\pi})}$
- "Coupling amplitudes": ${}^k\mathcal{C}_a^z(t')$
  - Independent coupling amplitude for each t' bin
- Kinematic factor $K(m_{K\pi\pi}, t')$
- Coherently summed over all assumed model components

Model transition amplitudes as coherent sum over various components

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  - For non-resonant terms: $\mathcal{D}_k^{\mathrm{NR}}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{\mathrm{thr}})^{a_k}\, e^{-b(c_k)\, \tilde{q}_k^2(m_{K\pi\pi})}$
- "Coupling amplitudes": ${}^k\mathcal{C}_a^z(t')$
  - Independent coupling amplitude for each t' bin
- Kinematic factor $K(m_{K\pi\pi}, t')$
- Coherently summed over all assumed model components

Model transition amplitudes as coherent sum over various components

$$\hat{\mathcal{T}}_a^z(m_{K\pi\pi}, t') = \sum_{k\in\mathbb{S}_a} K(m_{K\pi\pi}, t')\, {}^k\mathcal{C}_a^{K\pi\pi}(t')\, \mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$$

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3π spin-density matrix

$$\hat{\rho}_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t') = \left|\mathcal{C}^{\pi\pi\pi}\right|^2 \rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$$

- $\rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$ obtained from PWD of $\pi^-\pi^-\pi^+$ pseudodata sample
  - $m_{K\pi\pi}$ dependence fixed
  - t' dependence fixed
  - Rel. strength between partial waves fixed (freed in a study)
- One global real-valued yield parameter $\left|\mathcal{C}^{\pi\pi\pi}\right|^2$

Background spin-density matrix

- Additional incoherent contribution form other processes: $K^- K^- K^+$, ...
- Transition amplitudes modeled by non-resonant parameterizations for each partial wave

$$\hat{\mathcal{T}}_a^{\mathrm{eBKG}}(m_{K\pi\pi}, t') = \mathcal{K}(m_{K\pi\pi}, t')\ \mathcal{C}_a^{\mathrm{eBKG}}(t')\,\mathcal{D}_{k_a}^{\mathrm{eBKG}}(m_{K\pi\pi}; a_{k_a}, c_{k_a})$$

Resonance-Model Fit

χ^2 Fit Procedure

- χ^2 fit of the real and imaginary parts of the spin-density matrix
  - Taking into account correlations between spin-density matrix elements
  - Shape parameters (m_0, Γ_0, ...) and coupling amplitudes are free parameters
- For the main fit, we performed 2000 fit attempts with random start-parameter values for the shape parameters, e.g. mass and width parameters, and the coupling and branching amplitudes.
- Start-parameter ranges for the shape parameters are chosen according to previous measurements (see note)
- The best result is the one which yielded the smallest χ^2 value

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Wave-Set Selection

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \left| \sum_{a \in \mathbb{W}(m_{K\pi\pi}, t')} \mathcal{T}_a(m_{K\pi\pi}, t') \Psi_a(\tau; m_{K\pi\pi}) \right|^2$$

Challenge: Find the "best" set of waves that describes the data

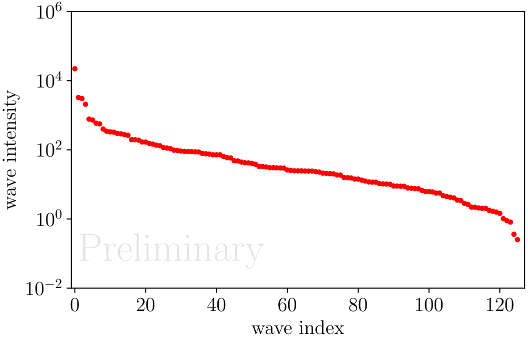
- If the wave set is too large
  - ➥ Starting to describe statistical fluctuations
- If waves that contribute to the data are missing
  - ➥ Intensity can be wrongly attributed to other waves
  - ➥ Model leakage

Wave-Set Selection

Infer wave set from data

- Systematically construct large set of allowed partial waves
  - ➥ "Wave pool"
- Fit wave pool to data
  - Impose penalty on $|\mathcal{T}_a|^2 \Rightarrow$ regularization
  - Suppress insignificant waves
- Select waves that significantly contribute to data
  - ➥ "Best" subset of waves that describe the data

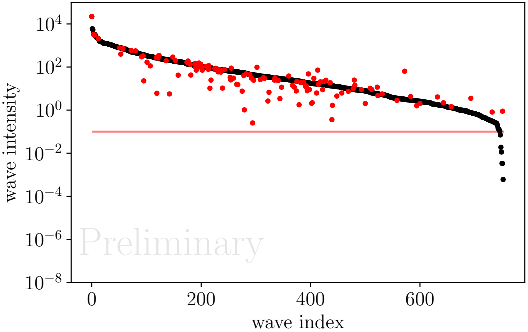
Wave-Set Selection



- $\pi^-\pi^-\pi^+$ Monte Carlo mock data set with 126 partial waves
- Fitting wave pool of 753 waves
  - ➥ Massive overfitting
  - ➥ Almost all waves pick up intensity

Courtesy F. Kaspar, TUM

Wave-Set Selection



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$$\ln \mathcal{L}_{\text{fit}} = \ln \mathcal{L}_{\text{extended}} + \sum_{a}^{\text{waves}} \ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \{c_{\text{para}}\})$$

LASSO/L1 regularization[1]

$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \lambda) = -\lambda |\mathcal{T}_a|$$

- Maximum at $|\mathcal{T}_a| = 0$
- Well established[2]
- "Smoothing" at $|\mathcal{T}_a| = 0$

$$|\mathcal{T}_a| \to \sqrt{|\mathcal{T}_a|^2 + \varepsilon}$$

[1] Robert Tibshirani. "Regression Shrinkage and Selection via the Lasso". In: Journal of the Royal Statistical Society. Series B 58.1 (1996)
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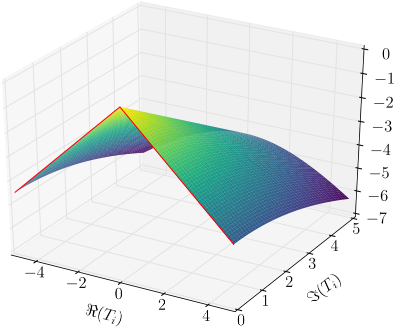

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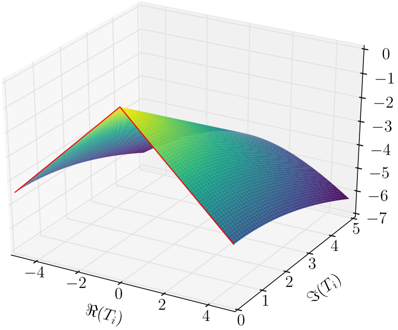

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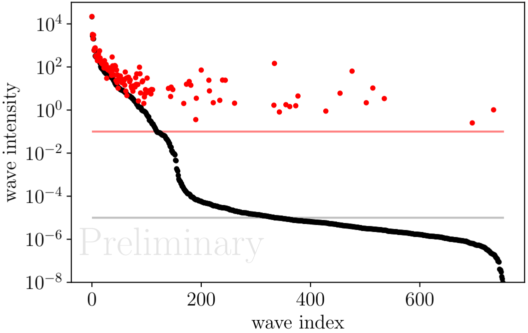


$\lambda = 0.3$

$\varepsilon = 10^{-5}$

- Bias also on large transition amplitudes
- Some additional waves
- Some waves missing

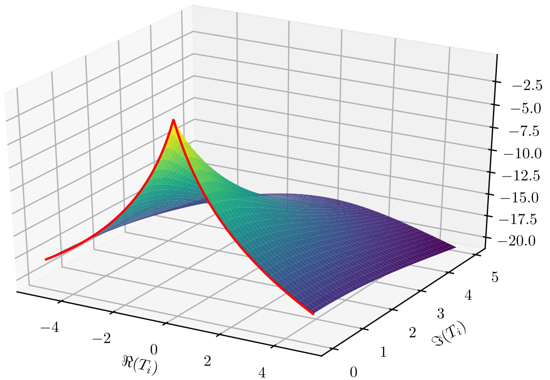
Courtesy F. Kaspar, TUM

Generalized Pareto[1]

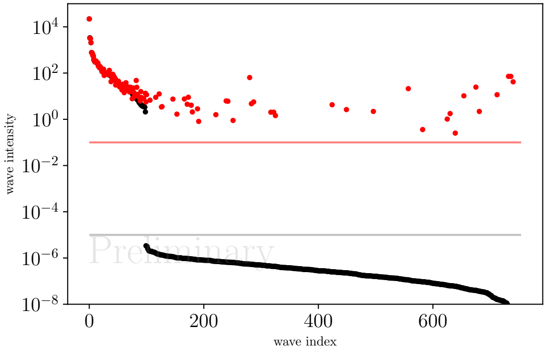
$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \Gamma, \zeta) = -\frac{1}{\zeta} \ln \left[1 + \zeta \frac{|\mathcal{T}_a|}{\Gamma} \right]$$

- Wave intensities spread over orders of magnitudes
- Use logarithmic prior
  - ➥ Heavy-tailed
  - ➥ Less bias on large waves
- LASSO-like for $|\mathcal{T}_a| \to 0$
- "Smoothing" at $|\mathcal{T}_a| = 0$

$$|\mathcal{T}_a| \to \sqrt{|\mathcal{T}_a|^2 + \varepsilon}$$



[1] Artin Armagan, David B. Dunson, and Jaeyong Lee. "Generalized double Pareto shrinkage". In: Statistica Sinica (2013). doi: 10.5705/ss.2011.048.



$\zeta = 0.5$

$\Gamma = 0.1$

$\varepsilon = 10^{-5}$

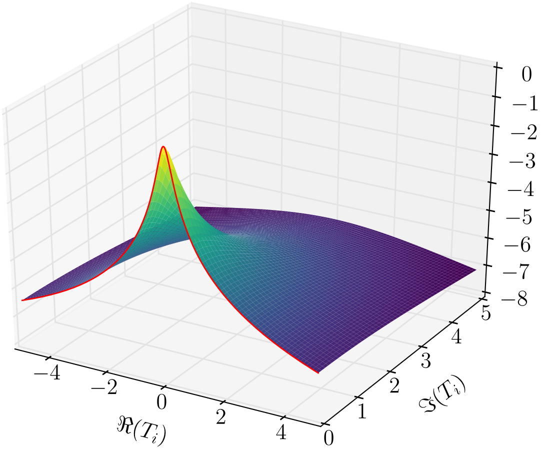
- Less bias on large transition amplitudes
- Clear kink in intensity distribution to smoothing scale $\Rightarrow$ Selection
- Less additional waves
- Some small waves missing

Courtesy F. Kaspar, TUM

"Cauchy"

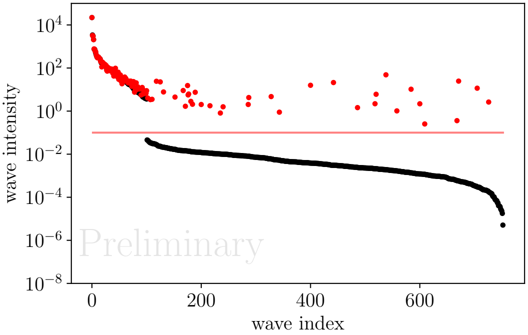
$$\ln \mathcal{L}_{\mathrm{reg}}(|\mathcal{T}_a|;\Gamma) = -\ln\left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2}\right]$$

- Logarithmic prior
- L2-like for $|\mathcal{T}_a| \to 0$



Wave-Set Selection
Regularization: Cauchy

$\Gamma = 0.2$



- Less bias on large transition amplitudes
- Clear kink in intensity distribution
- Few additional waves
- Few small waves missing

Courtesy F. Kaspar, TUM

Wave pool

- Spin $J \leq 7$
- Angular momentum $L \leq 7$
- Positive naturality of exchange particle
- 12 isobars
  - $[K\pi]_S^{K\pi}$, $[K\pi]_S^{K\eta}$, $K^*(892)$, $K^*(1680)$, $K_2^*(1430)$, $K_3^*(1780)$
  - $[\pi\pi]_S$, $f_0(980)$, $f_0(1500)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$

$\Rightarrow$ "Wave pool" of 596 waves

"only" 720 k events

Wave-Set Selection

For the $K^-\pi^-\pi^+$ Final State

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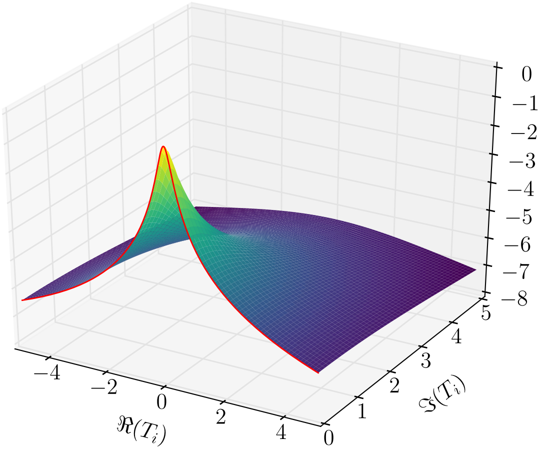
Regularization

$$\ln \mathcal{L}_{\mathrm{reg}}(|\mathcal{T}_a|; \Gamma) = -\ln\left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2}\right]$$

- Use Cauchy regularization
- Scale of $|\mathcal{T}_a|$ depends on experimental acceptance
  - Apply penalty on expected number $\bar{N}_a$ of observed events

$$\Gamma_a = \frac{\Gamma}{\sqrt{\bar{\eta}_a}} \quad \Rightarrow \quad \frac{|\mathcal{T}_a|^2}{\Gamma_a^2} = \frac{\bar{N}_a}{\Gamma^2}$$

  - Γ is a universal parameter



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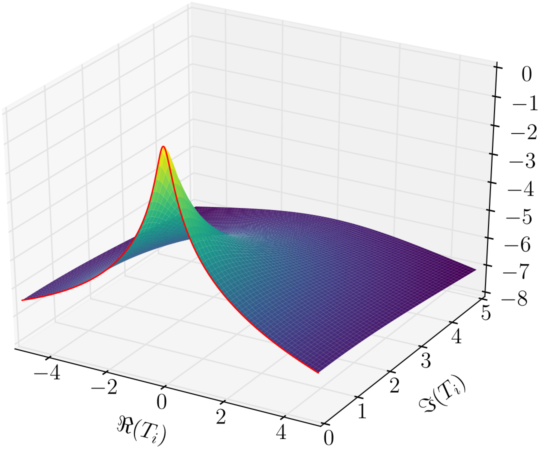

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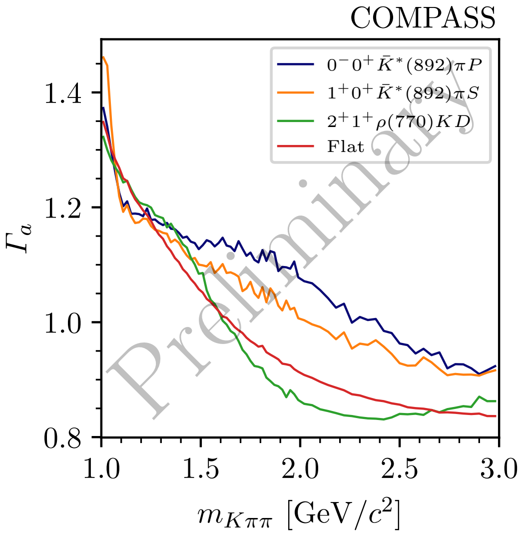

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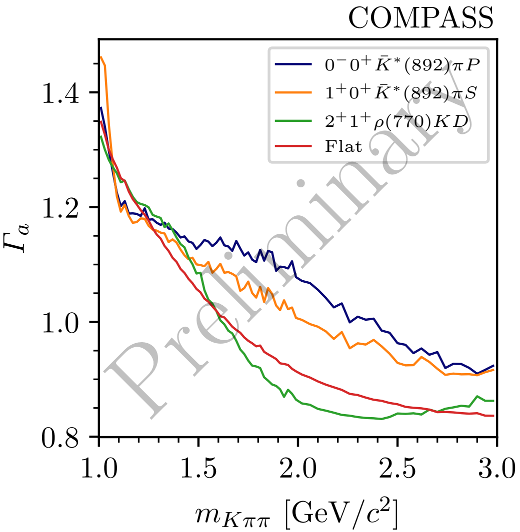


Imposing continuity of the wave set

- Wave-set inferred independently for each $(m_{K\pi\pi}, t')$ cell
- Impose continuity of the wave set in $m_{K\pi\pi}$ by adding additional regularization term

$$\ln \mathcal{L}_{\mathrm{cont}}(\{\mathcal{T}_a(m_{K\pi\pi}, t')\}; \lambda) = \sum_{j=i-3}^{j=i+3} \lambda \Big| \mathcal{T}_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^{j+1}) - \mathcal{T}_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^{j}) \Big|^2 ,$$

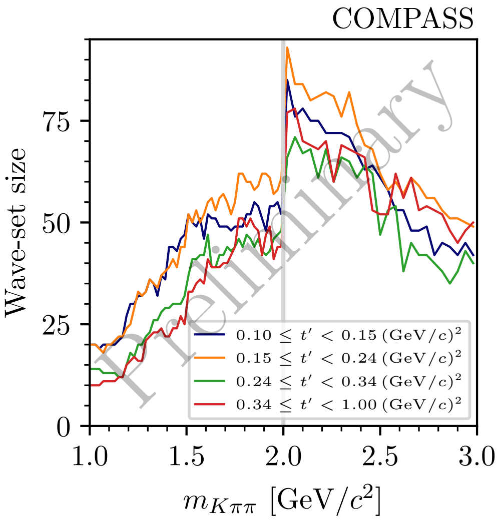
which suppresses fluctuations among neighboring $m_{K\pi\pi}$ bins

Wave-Set Selection

For the $K^-\pi^-\pi^+$ Final State

Wave-set size

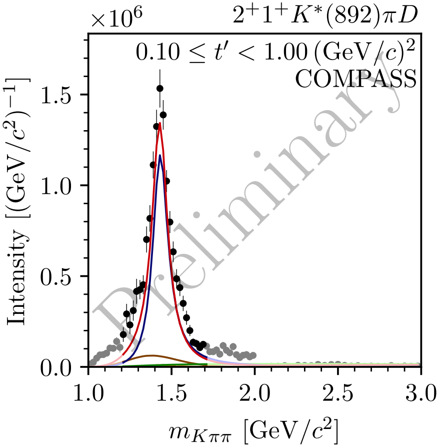
- 5 to 90 waves per $(m_{K\pi\pi}, t')$ cell
- Larger wave set for larger binning in $m_{K\pi\pi}$
- Larger wave set in t' bins with more events



Wave-Set Selection

For the $K^-\pi^-\pi^+$ Final State

- Selection of large signals
- as well as of signals at per-mil level



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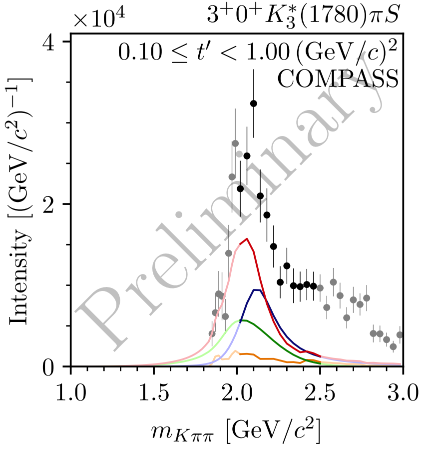
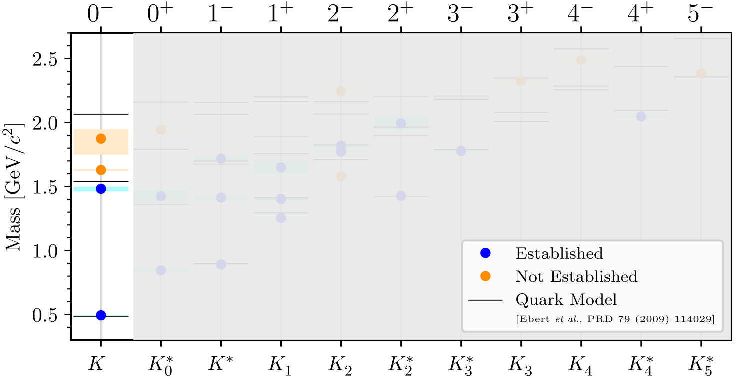
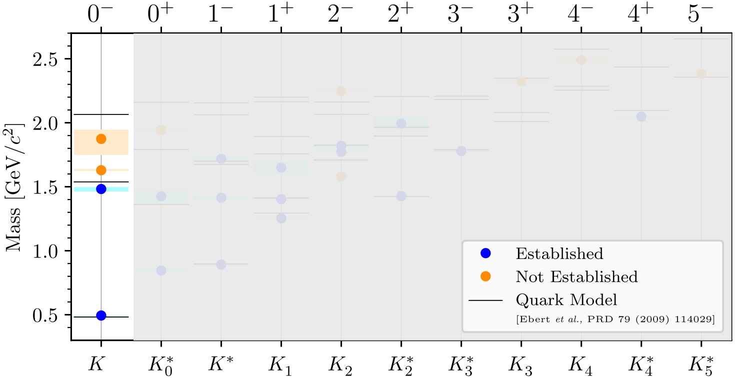




PDG
(2022)

- $K(1460)$ and $K(1830)$
- $K(1630)$
  - Unexpectedly small width of only 16 MeV/c^2
  - J^P of $K(1630)$ unclear



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COMPASS $K^-\pi^-\pi^+$ data

- Peak at about 1.4 GeV/c^2
  - Potentially from established $K(1460)$
  - But, $m_{K\pi\pi} \lesssim 1.5$ GeV/c^2 region affected by analysis artifacts
- Second peak at about 1.7 GeV/c^2
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  - Accompanied by rising phase
- Weak signal at about 2.0 GeV/c^2
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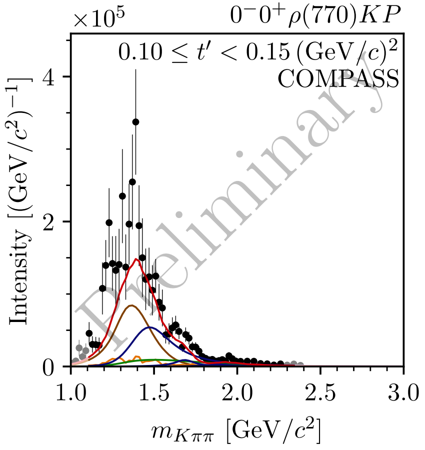

total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background

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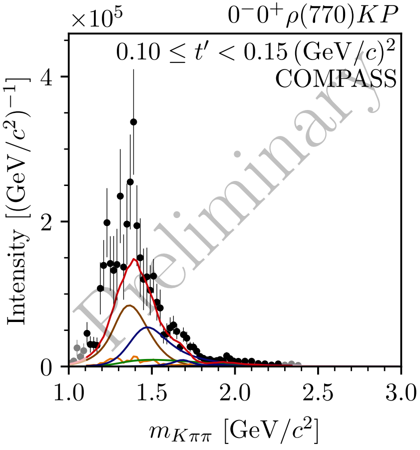

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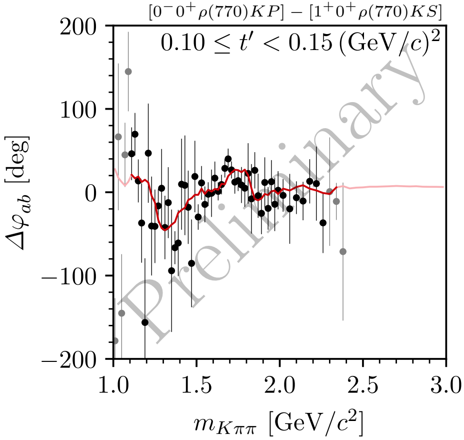

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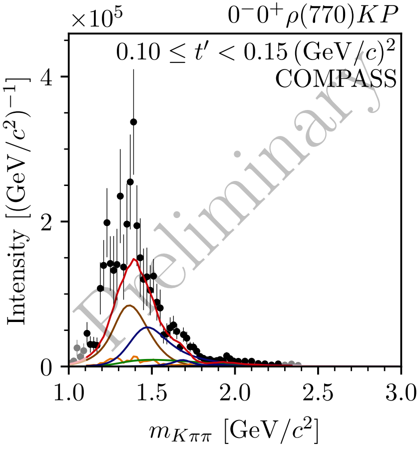

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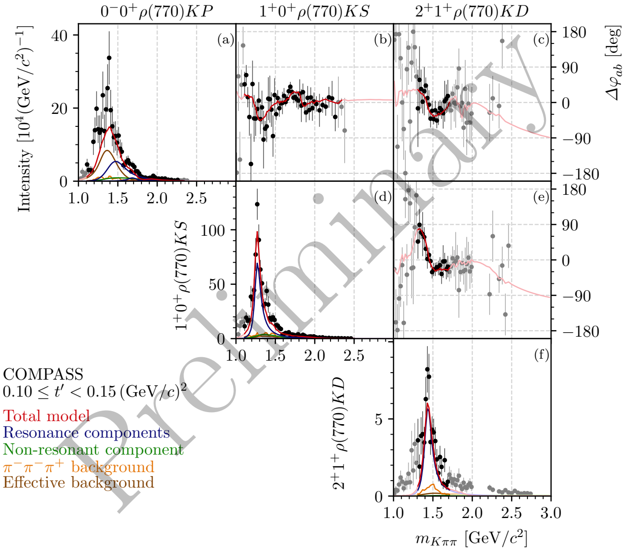
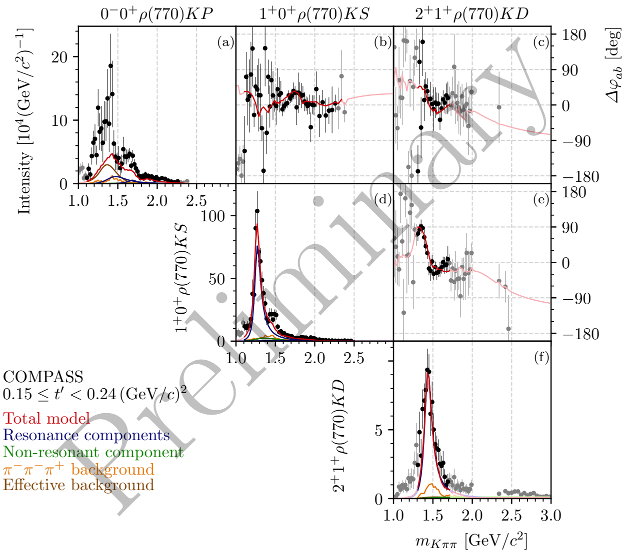
14-Wave Resonance-Model Fit

Searching for Exotic Strange Mesons with $J^P = 0^-$

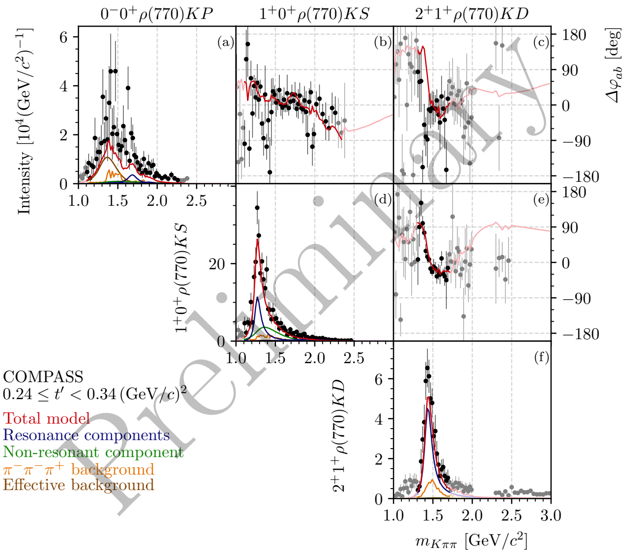




$0^-0^+\rho(770)KP$
$1^+0^+\rho(770)KS$
$2^+1^+\rho(770)KD$
(a)
(b)
(c)
(d)
(e)
(f)
Intensity $[10^4(\mathrm{GeV}/c^2)^{-1}]$
$1^+0^+\rho(770)KS$
$2^+1^+\rho(770)KD$
$\Delta\varphi_{ab}$ [deg]
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
COMPASS
$0.15 \leq t' < 0.24\,(\mathrm{GeV}/c)^2$
Total model
Resonance components
Non-resonant component
$\pi^-\pi^-\pi^+$ background
Effective background
Preliminary

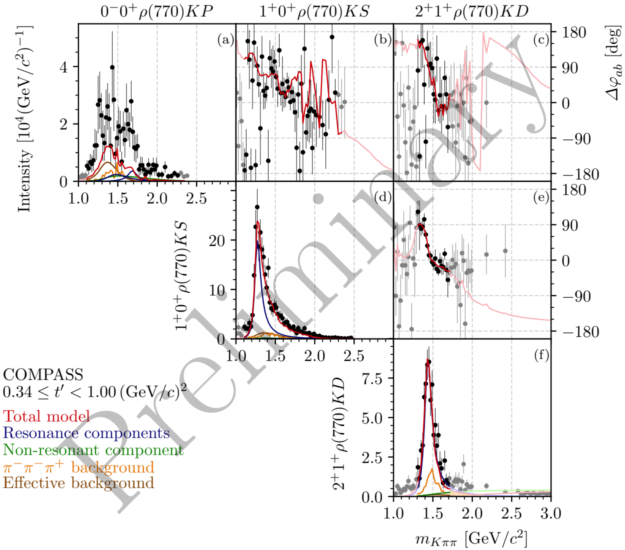
14-Wave Resonance-Model Fit

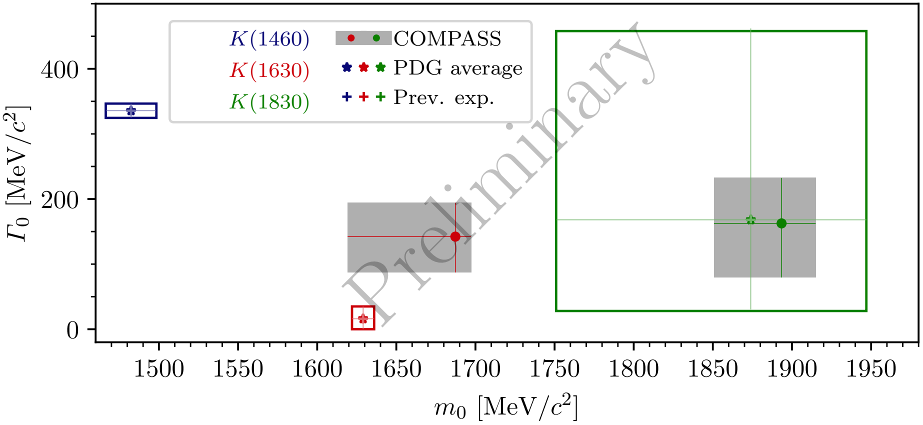
Searching for Exotic Strange Mesons with $J^P = 0^-$



14-Wave Resonance-Model Fit

Searching for Exotic Strange Mesons with $J^P = 0^-$





- $K(1830)$ parameters in good agreement with LCHb measurement [PRL 118 (2017) 022003]
- Realistic $K(1630)$ width of about 140 MeV/c^2

14-Wave Resonance-Model Fit

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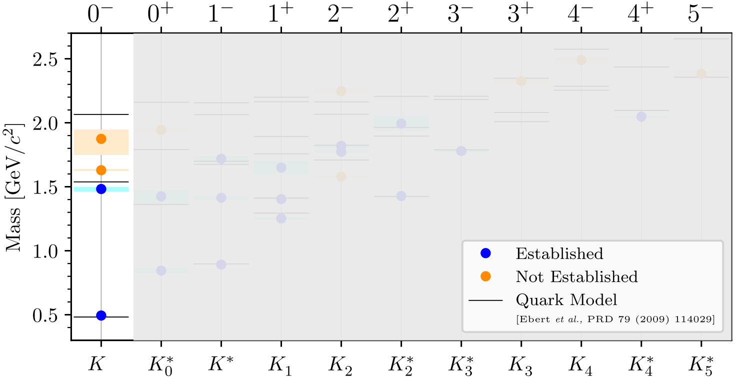

- Indications for 3 excited K from a single analysis
- Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$
  - ➥ $K(1630)$ supernumerary signal
  - ➥ Candidate for exotic non-$q\bar{q}$ state; other explanations possible ($K^*(892)\,\omega$ threshold nearby)

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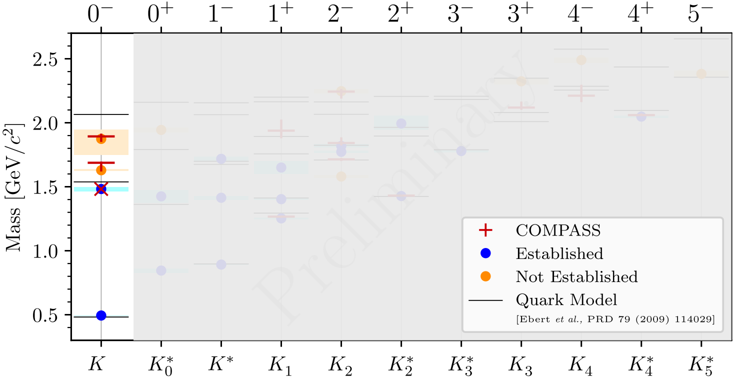

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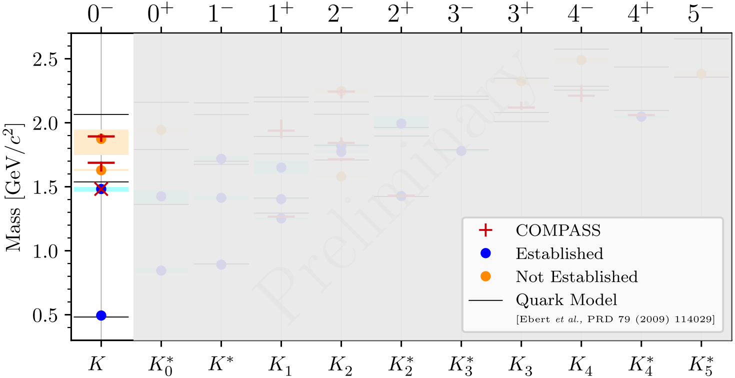

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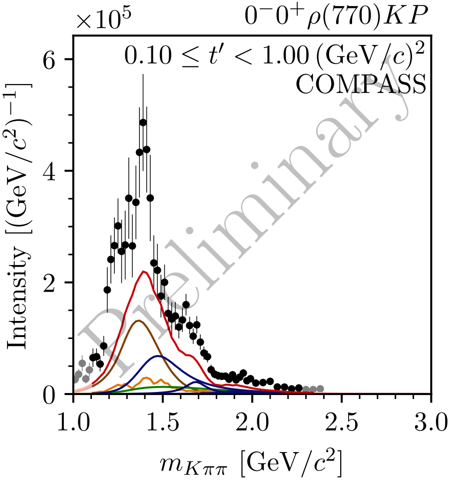
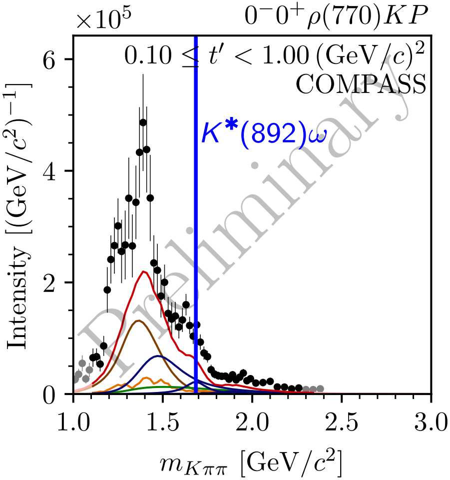


$\times 10^5$
$0^-0^+\rho(770)KP$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
6
4
2
0
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$



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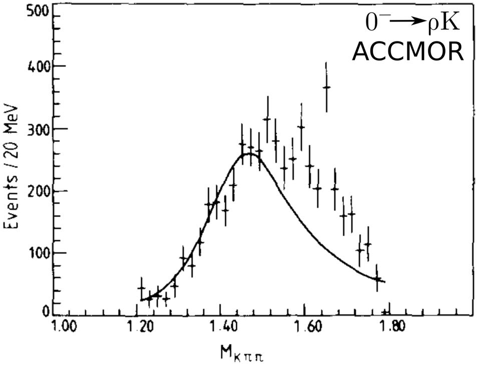
$K^-\pi^-\pi^+$ from ACCMOR

- Potential $K(1630)$ signal already in ACCMOR analysis

$K^-\pi^-\pi^+$ from LHCb

- Measurement of $D^0 \to K^\mp\pi^\pm\pi^\pm\pi^\mp$ at LHCb
  - Study strange mesons in $K\pi\pi$ subsystem
  - MIPWA of $J^P = 0^-$ amplitude
  - Potential signal above 1.6 GeV/c^2
  - Limited by kinematic range



WA03 (CERN), 200 000 events, ACCMOR, Nucl. Phys. B **187** (1981)

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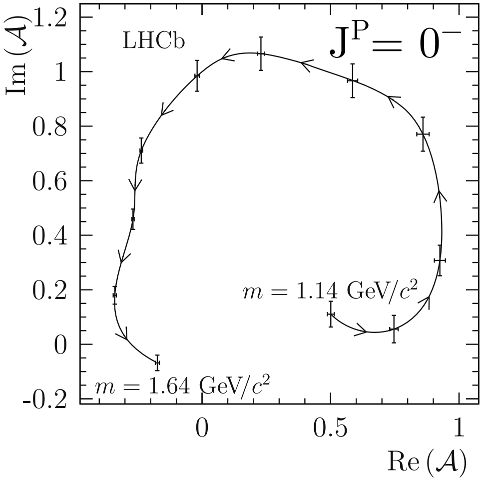

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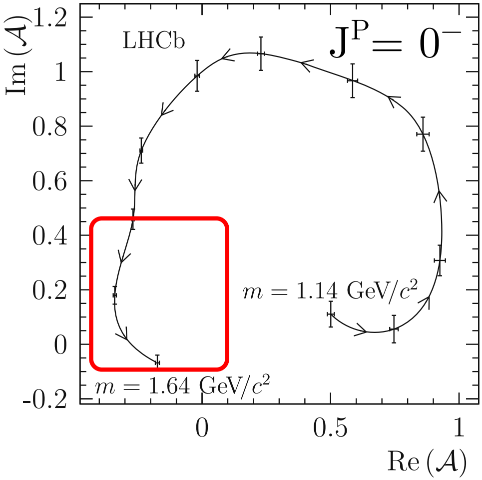

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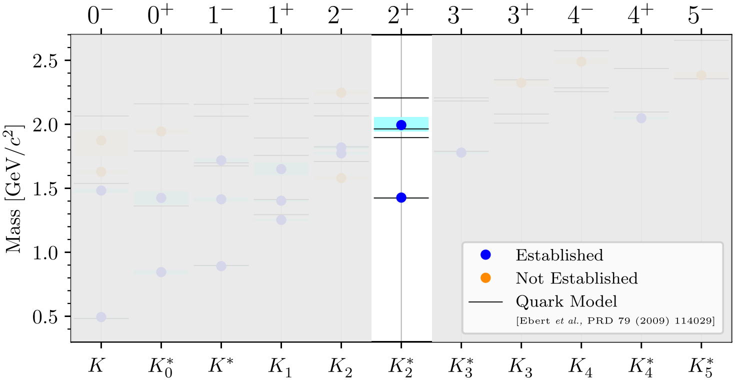
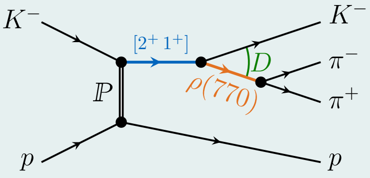
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^+$

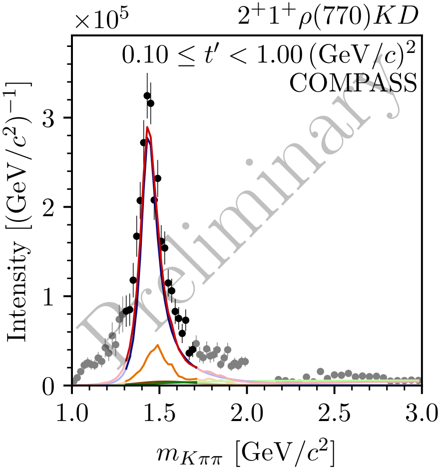


PDG (2022)

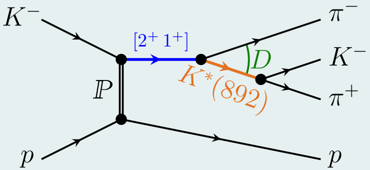
- $K_2^*(1430)$ well known resonance



- Signal in $K_2^*(1430)$ mass region
- In different decays
  - $\rho(770)\, K\, D$
  - $K^*(892)\, \pi\, D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data



total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background



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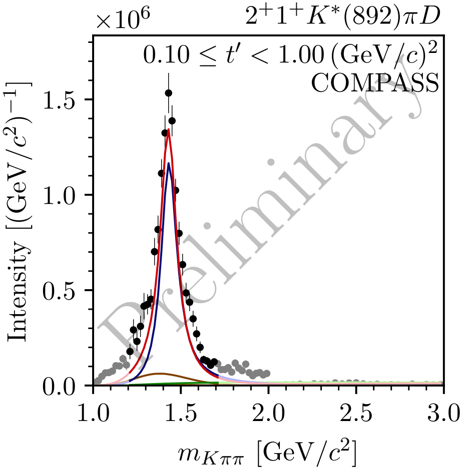

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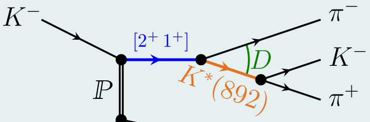

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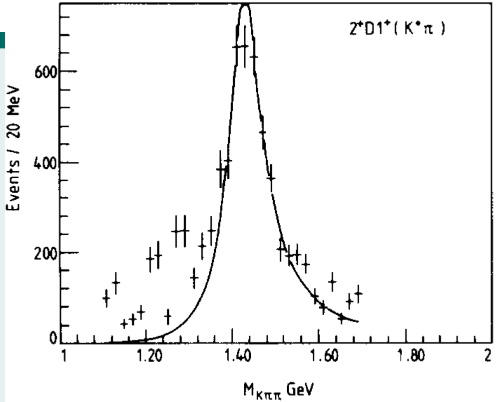
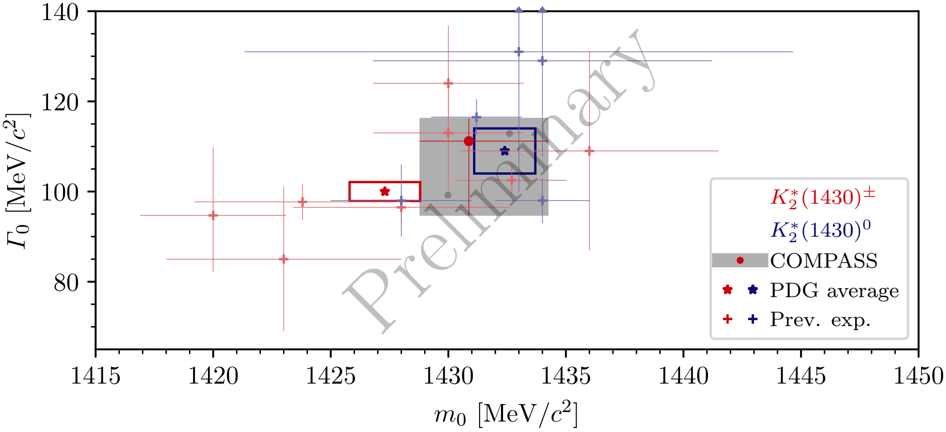


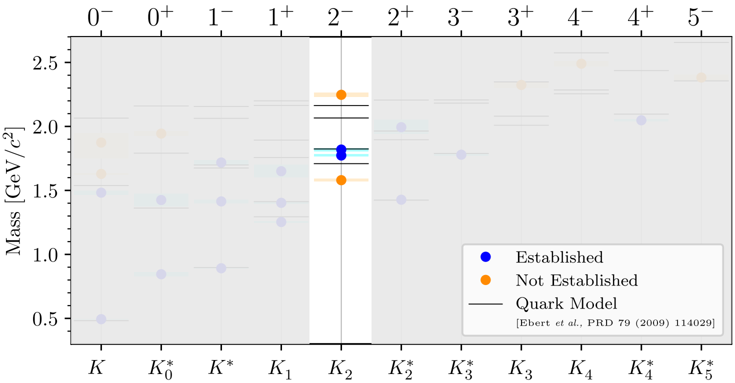
WA03 (CERN), 200 000 events, ACCMOR, Nucl. Phys. B **187** (1981)

14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^+$



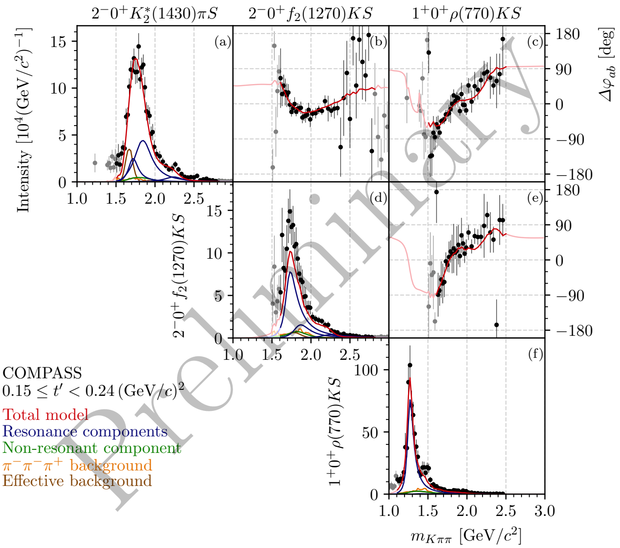
- $K_2^*(1430)$ parameters consistent with previous observations
- Better agreement with PDG average values for neutral $K_2^*(1430)$



PDG (2022)

- Established $K_2(1770)$ and $K_2(1820)$
- $K_2(2250)$ need further confirmation

- Simultaneously fit 4 waves with $J^P = 2^-$
- $1.8\,\mathrm{GeV}/c^2$ peak modeled by $K_2(1770)$, $K_2(1820)$
- High-mass shoulder modeled by $K_2(2250)$
- Different intensity spectra and large phase motions among 2^- waves



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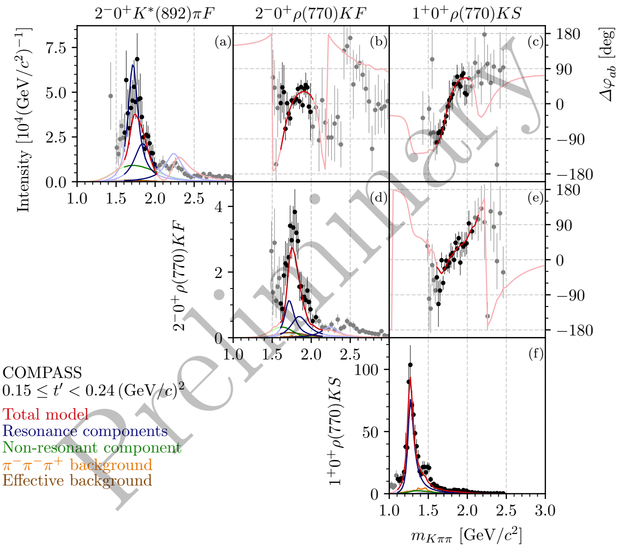
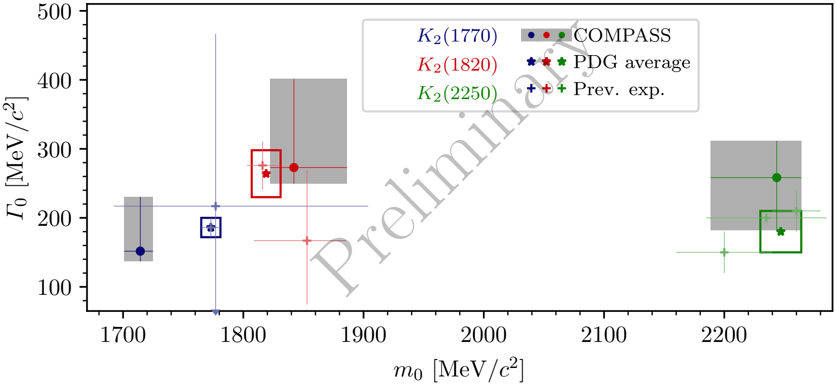


14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$

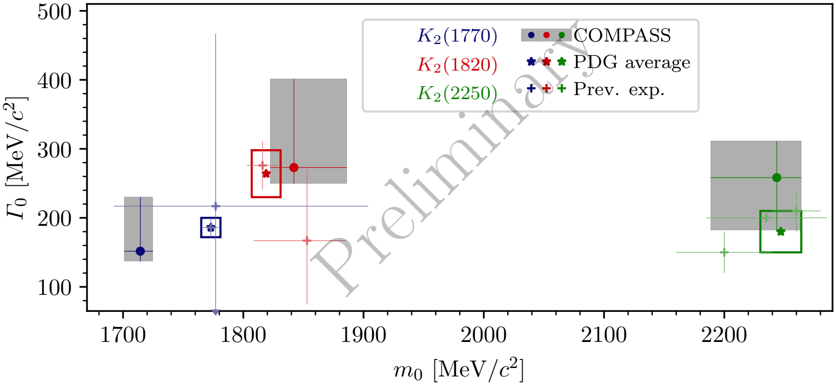


$K_2(1770)$ and $K_2(1820)$

- Two states were considered by only three measurements ACCMOR, LASS, LHCb
- Only LHCb measurement could confirm two states ($3\,\sigma$ statistical significance)
- We observe two sates with $11\,\sigma$ statistical significance

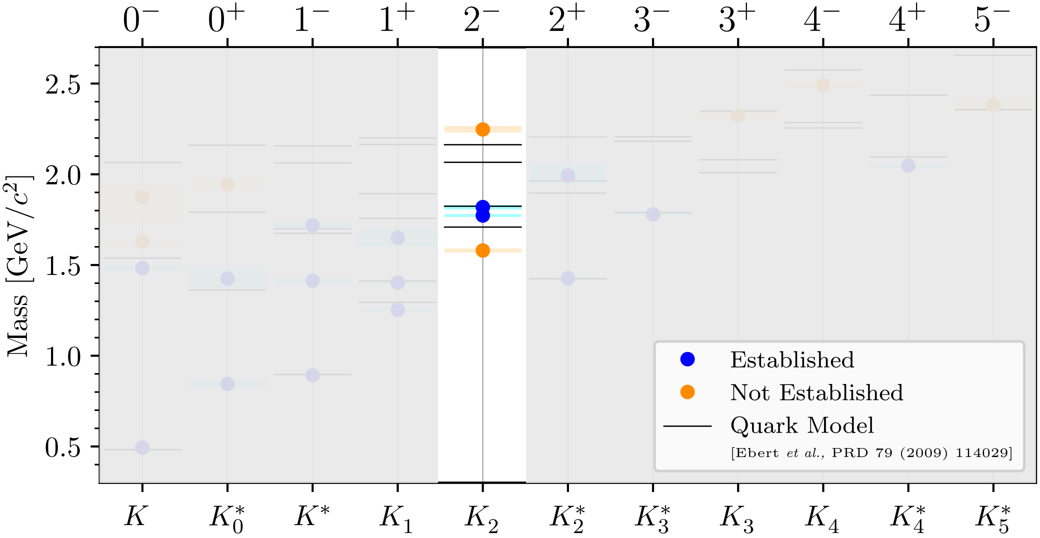
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$



$K_2(2250)$

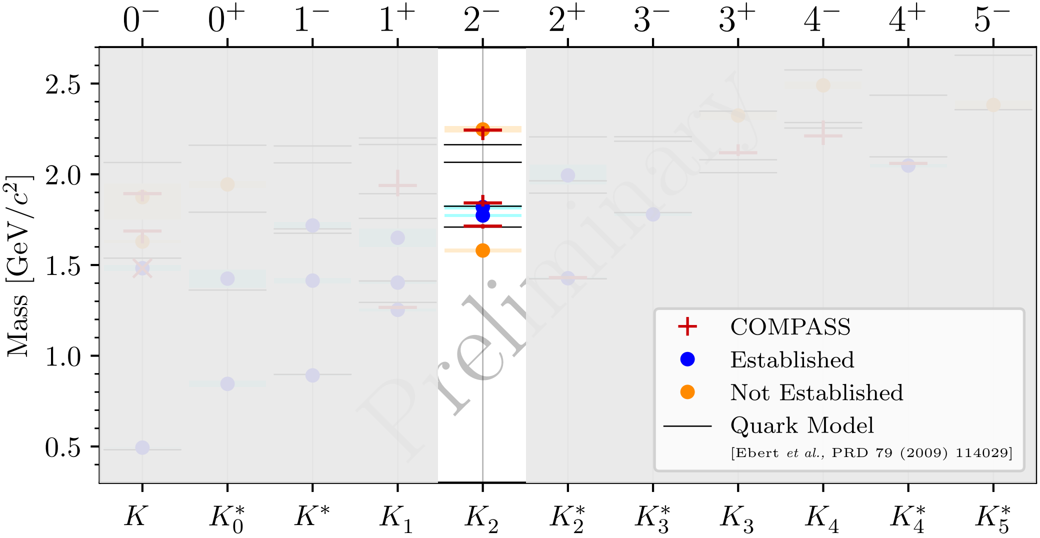
- Studied so far mainly in $\overset{(-)}{\Lambda}\overset{(-)}{p}$ final states
- First simultaneous measurement of $K_2(1770)$, $K_2(1820)$, and $K_2(2250)$
- Resonance parameters consistent with previous observations



0^- 0^+ 1^- 1^+ 2^- 2^+ 3^- 3^+ 4^- 4^+ 5^-
Mass [GeV/c^2]
2.5
2.0
1.5
1.0
0.5
K K_0^* K^* K_1 K_2 K_2^* K_3^* K_3 K_4 K_4^* K_5^*
Established
Not Established
Quark Model
[Ebert *et al.*, PRD 79 (2009) 114029]

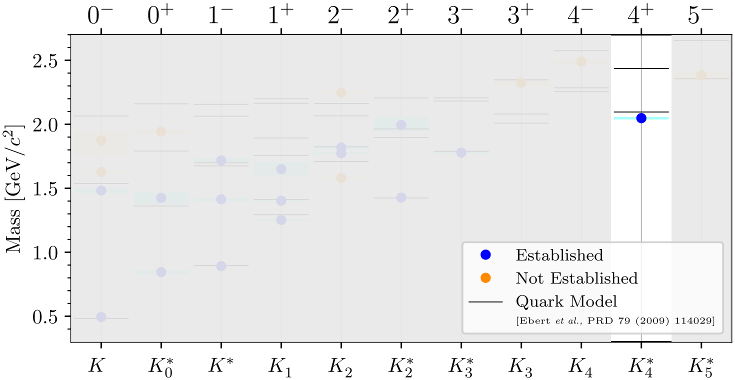
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$



14-Wave Resonance-Model Fit

Partial Waves with $J^P = 4^+$

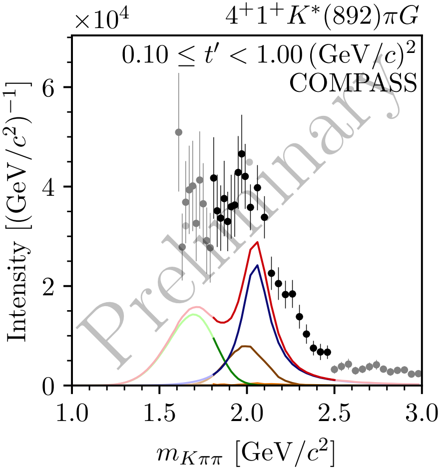


PDG

(2022)

- $K_4^*(2045)$ known resonance

- Signal $K_4^*(2045)$ signal in $K^*(892)\ \pi$ and $\rho(770)\ K$ decays



total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background

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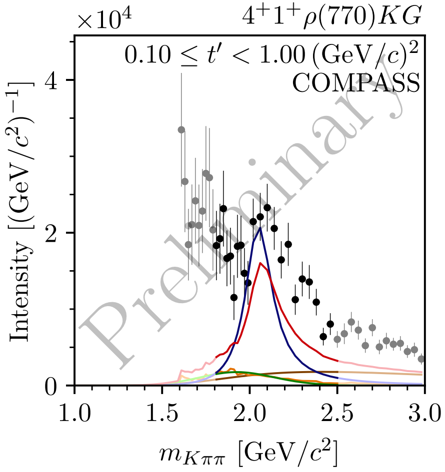

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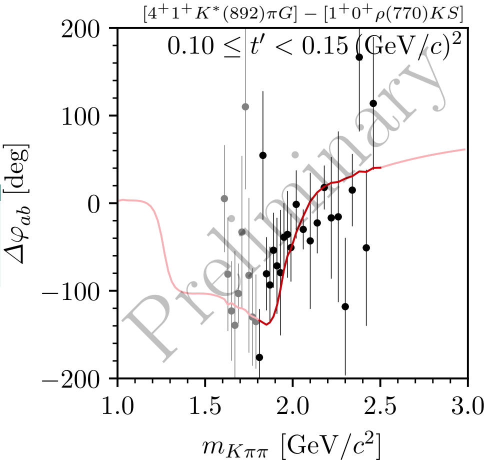

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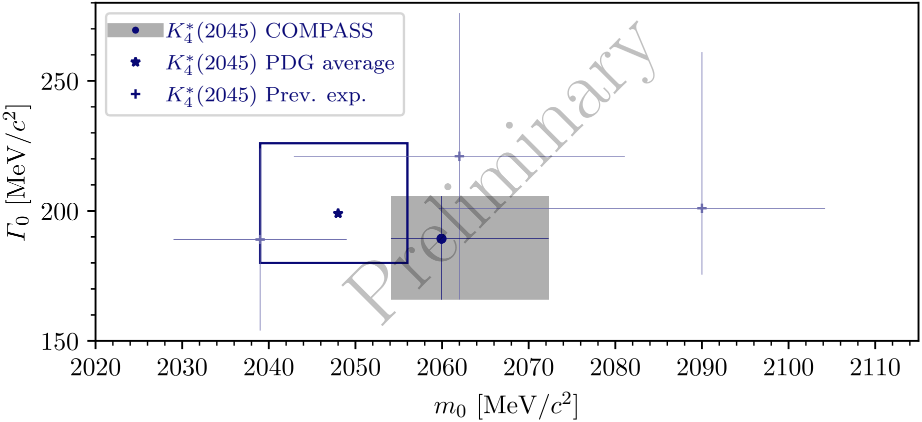
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 4^+$

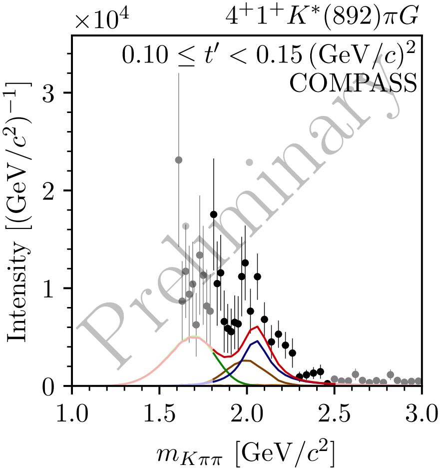


14-Wave Resonance-Model Fit

Partial Waves with $J^P = 4^+$

- Imperfect description of magnitude of intensity,
- Also, real and imaginary parts of interference terms described well, including their magnitude
- Intensities and real and imaginary parts of interference terms not directly related as $\mathrm{Rank}[\rho_{ab}] > 1$ $|\rho_{ab}| \neq \sqrt{|\rho_{aa}|\,|\rho_{bb}|}$
  - ➥ Analysis artifacts in intensities of small waves, which are the least constrained by data

- Results validated by Monte Carlo input-output and systematic studies
- Imperfections considered in systematic uncertainties
- Results in agreement with previous experiments



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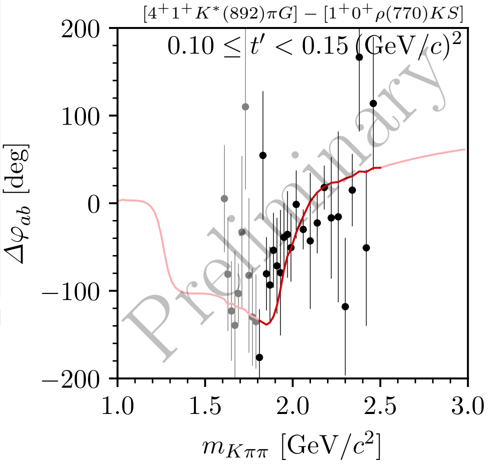

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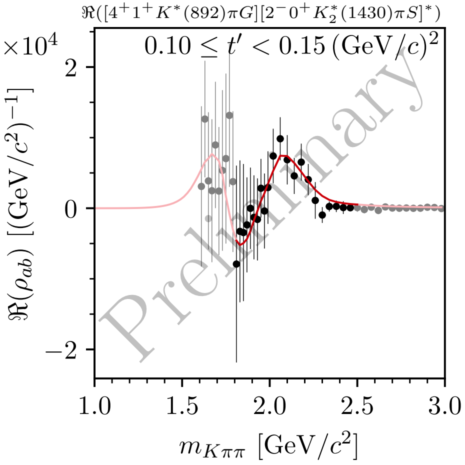

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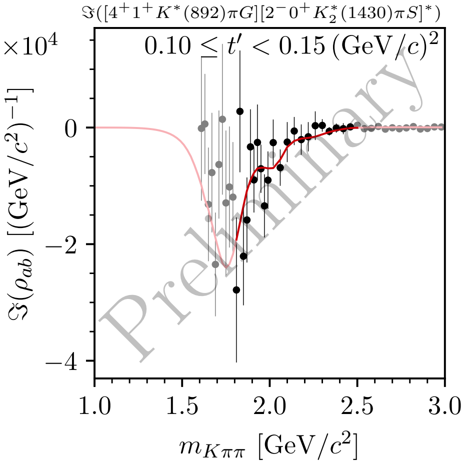

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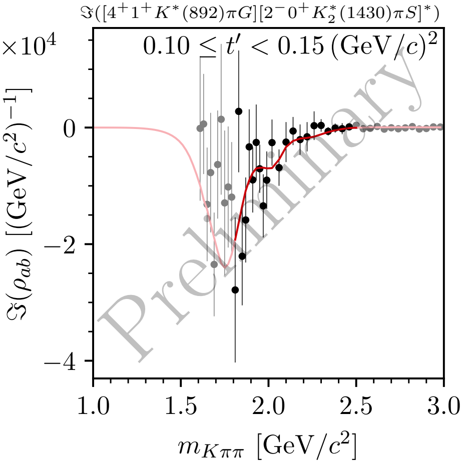

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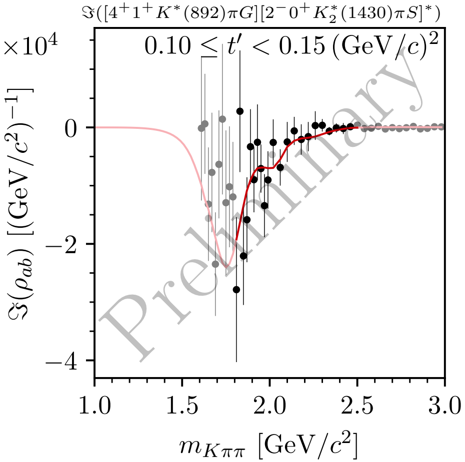

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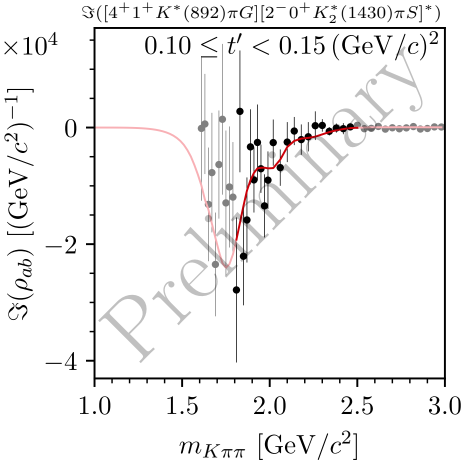

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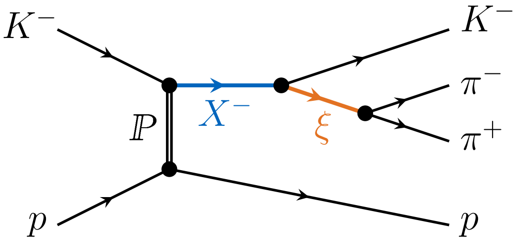
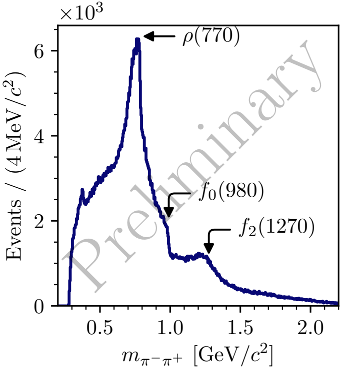
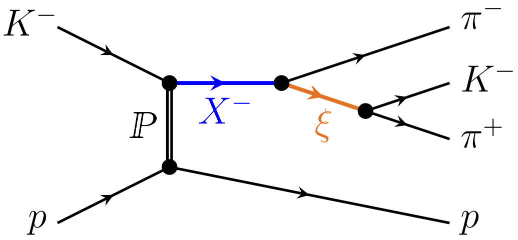
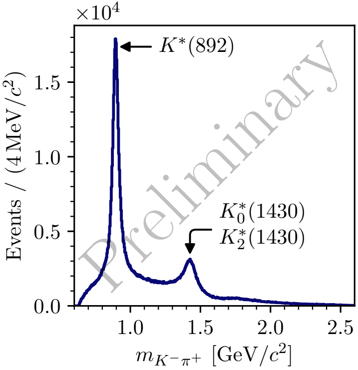




- Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems
  - ➥ Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar
- Also structure in angular distributions





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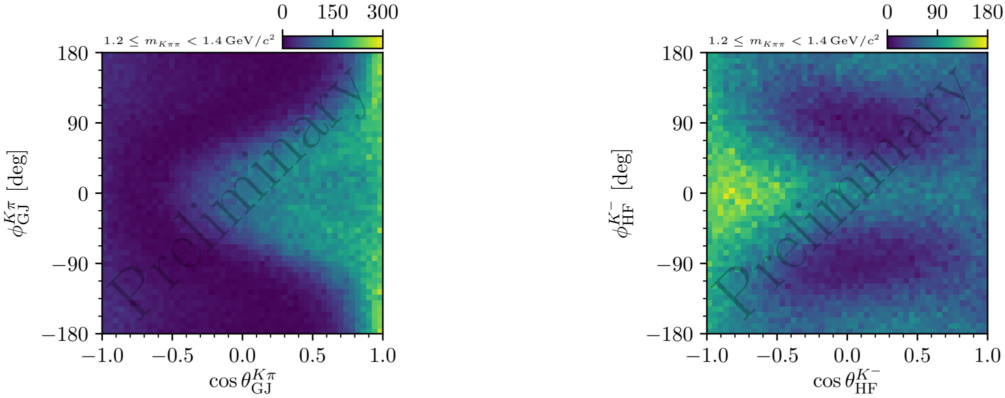
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem



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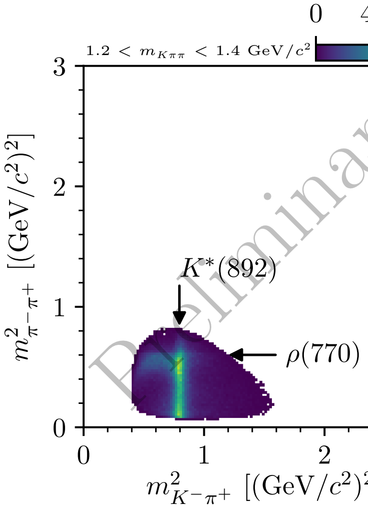
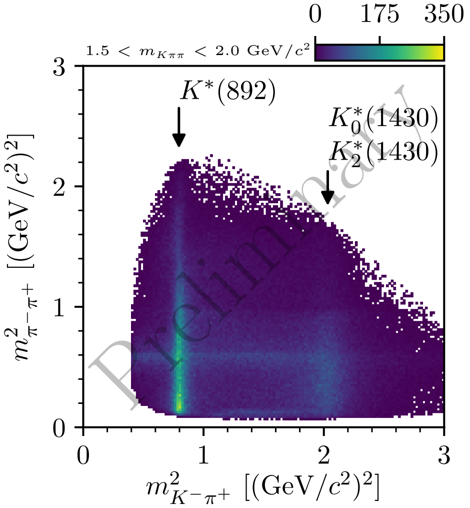
  - ➥ Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar
- Also structure in angular distributions

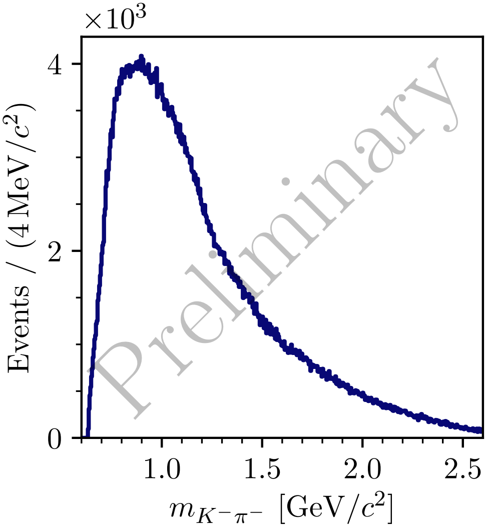


0 400 800
$1.2 < m_{K\pi\pi} < 1.4\ \mathrm{GeV}/c^2$
$K^*(892)$
$\rho(770)$
Preliminary
$m^2_{\pi^-\pi^+}\ [(\mathrm{GeV}/c^2)^2]$
$m^2_{K^-\pi^+}\ [(\mathrm{GeV}/c^2)^2]$

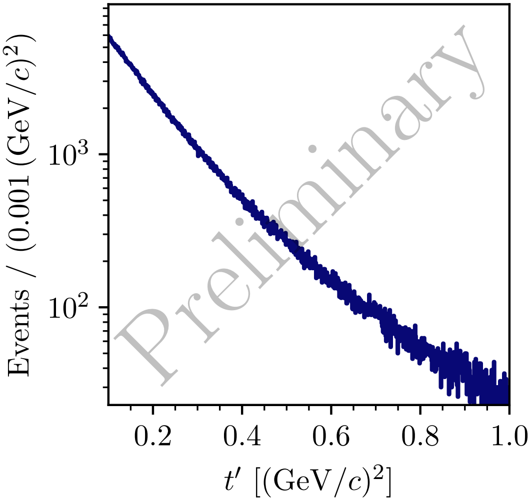


0 175 350
$1.5 < m_{K\pi\pi} < 2.0\ \mathrm{GeV}/c^2$
$K^*(892)$
$K_0^*(1430)$
$K_2^*(1430)$
Preliminary
$m^2_{\pi^-\pi^+}\ [(\mathrm{GeV}/c^2)^2]$
$m^2_{K^-\pi^+}\ [(\mathrm{GeV}/c^2)^2]$

- No dominant resonant structures

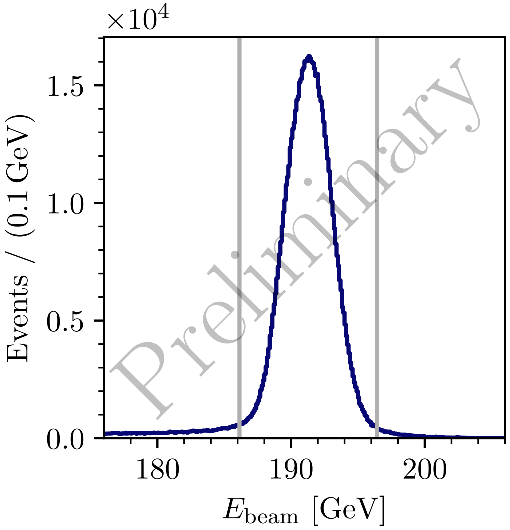


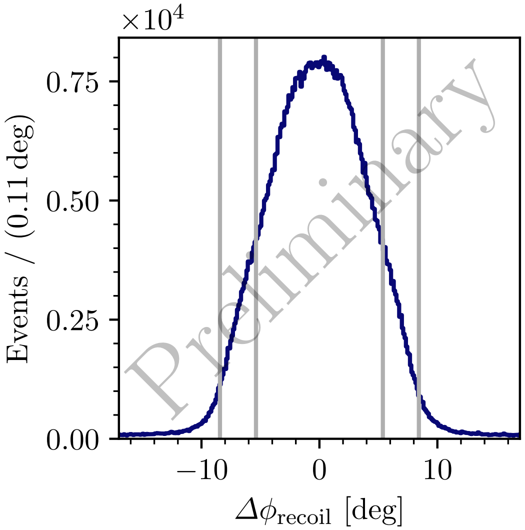
- Exponential shape
- Shallower for larger t'



Kinematic Distribution of $K^-\pi^-\pi^+$ Events

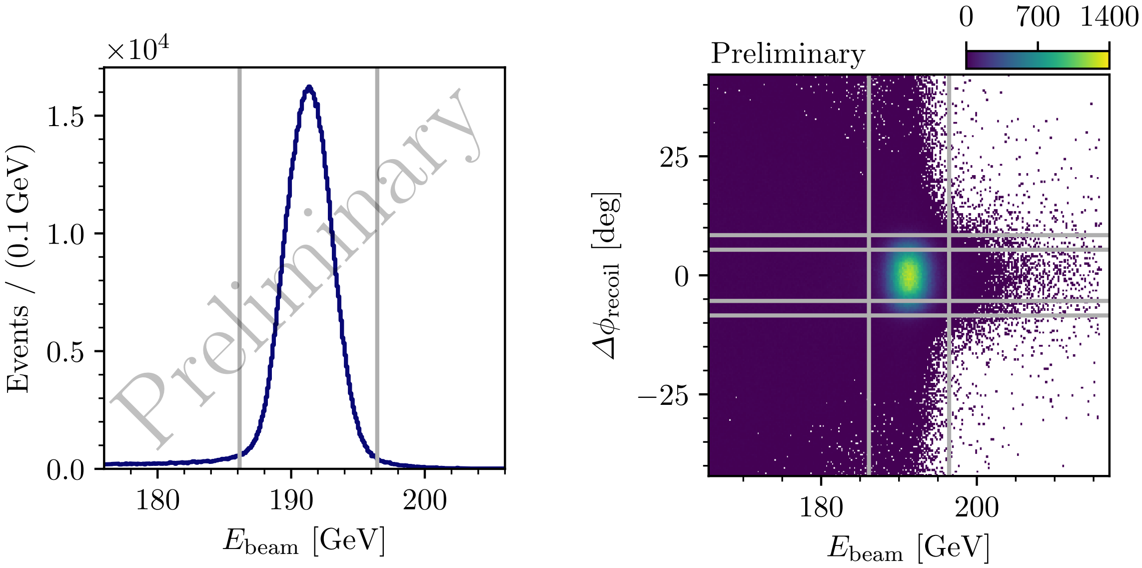
Exclusivity

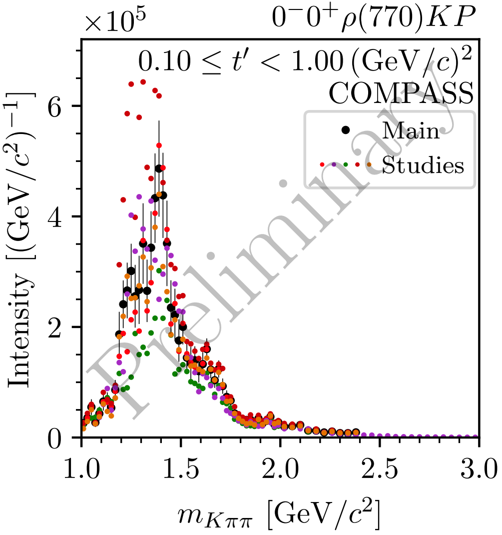




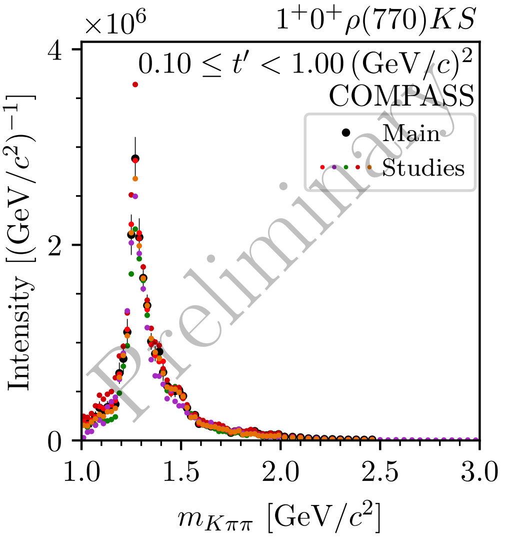
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Exclusivity

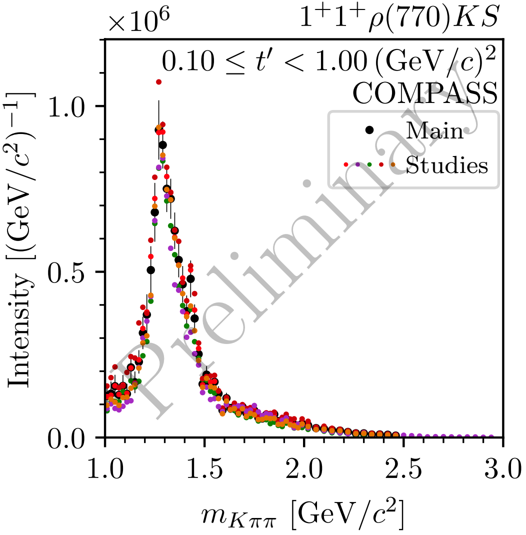


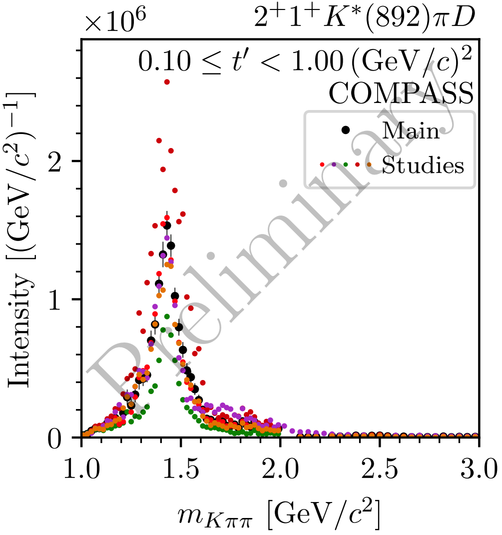


$\times 10^5$
$0^-0^+\rho(770)KP$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
2
4
6
1.0
1.5
2.0
2.5
3.0

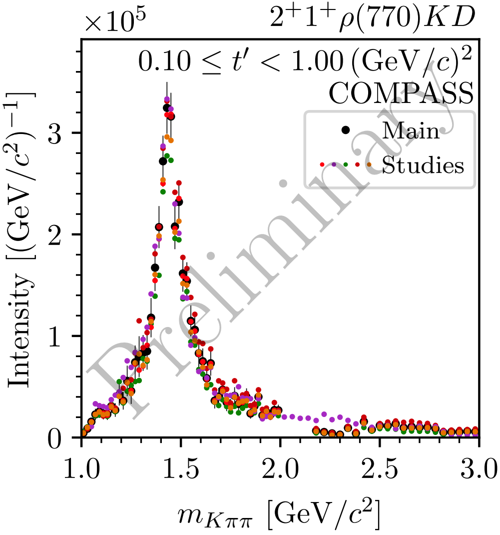


$1^+0^+\rho(770)KS$
$\times 10^6$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
2
4
1.0
1.5
2.0
2.5
3.0

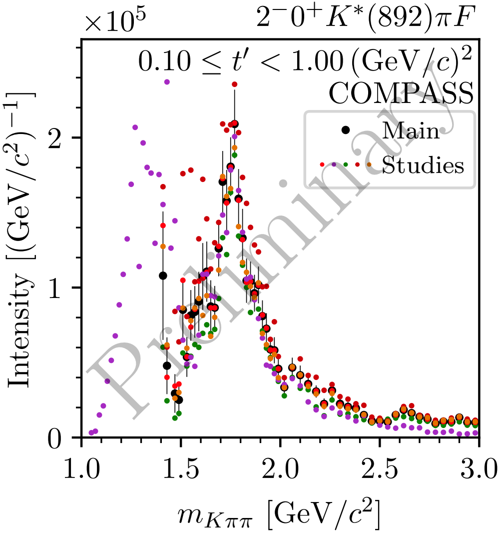

$\times 10^6$
$1^+1^+\rho(770)KS$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
1.0
0.5
0.0
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$

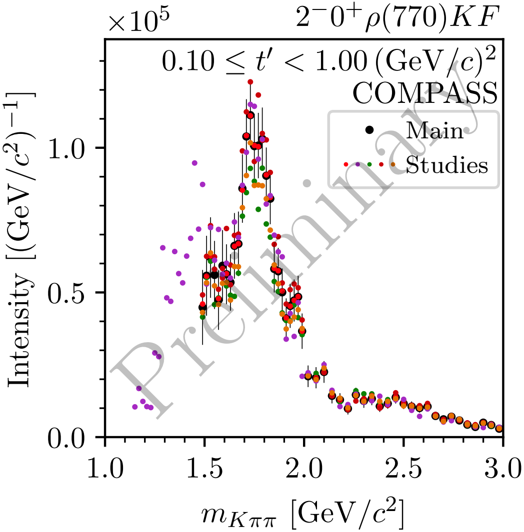


$\times 10^6$
$2^+1^+K^*(892)\pi D$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
1
2
1.0
1.5
2.0
2.5
3.0

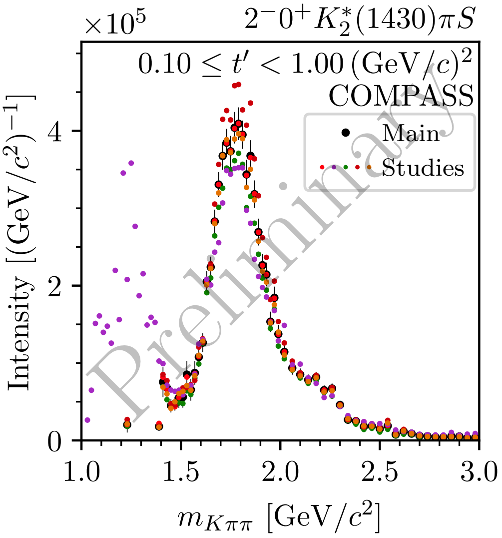


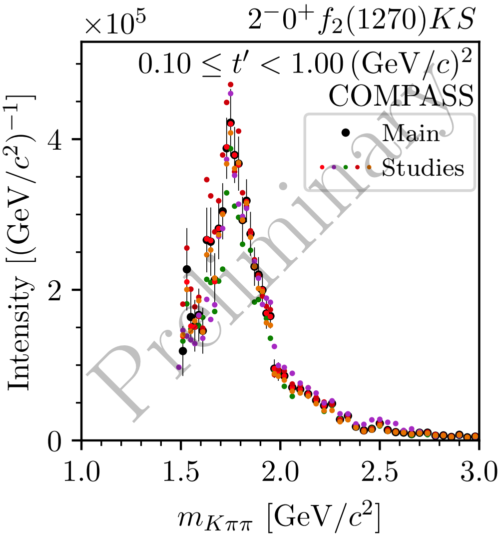
$\times 10^5$
$2^+1^+\rho(770)KD$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
0
1
2
3
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$

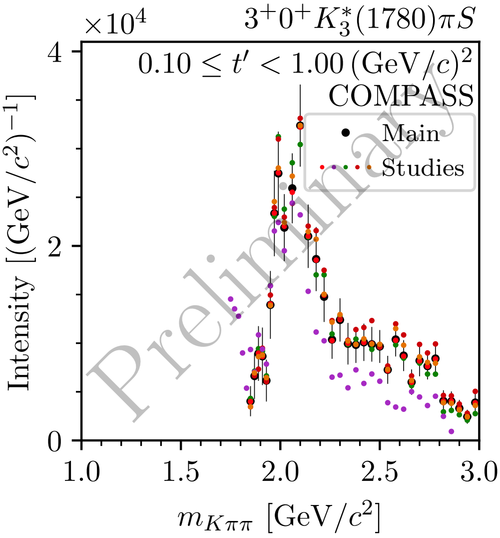

$\times 10^5$
$2^-0^+K^*(892)\pi F$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
1
2
1.0
1.5
2.0
2.5
3.0



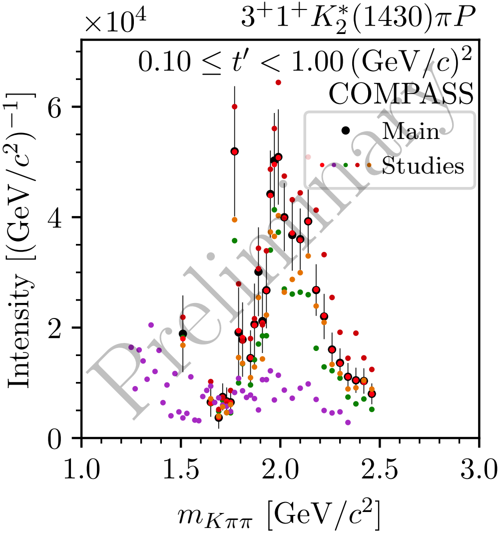
$\times 10^5$
$2^-0^+\rho(770)KF$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
0.0
0.5
1.0
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$

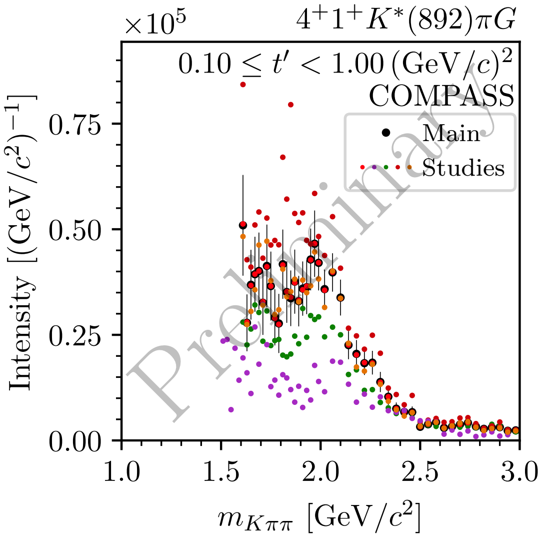

$\times 10^5$
$2^-0^+K_2^*(1430)\pi S$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
0
2
4
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$


$\times 10^5$
$2^-0^+ f_2(1270) K S$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
0
2
4
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$

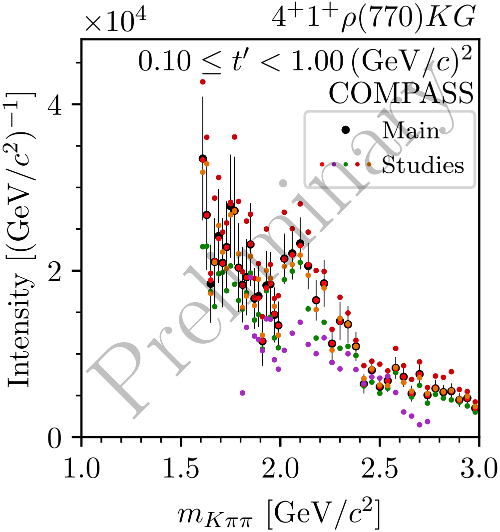

$\times 10^4$
$3^+0^+K_3^*(1780)\pi S$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
2
4
1.0
1.5
2.0
2.5
3.0

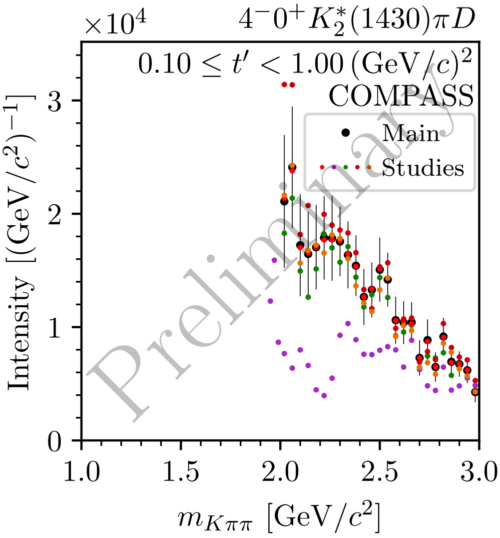
Systematic Studies of the Partial-Wave Decomposition
14 Waves

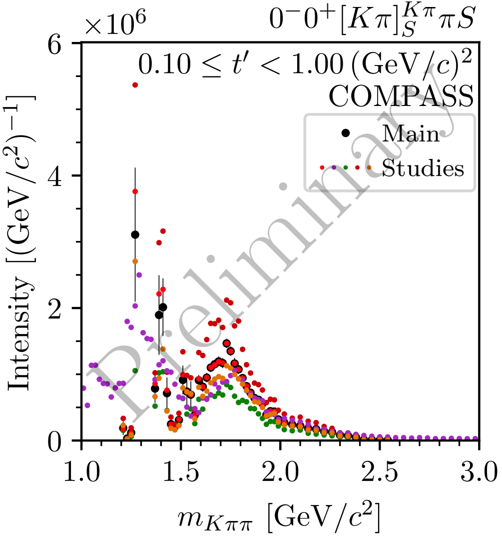


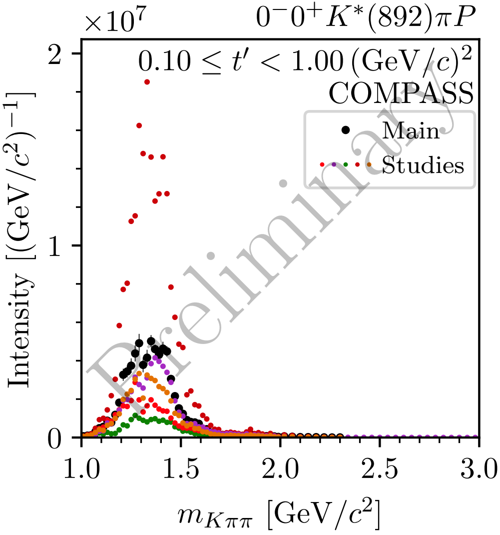

$\times 10^5$
$4^+1^+K^*(892)\pi G$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
0.75
0.50
0.25
0.00
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$

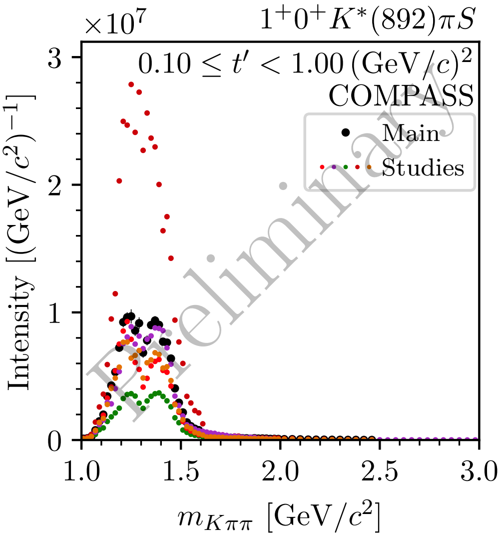
Systematic Studies of the Partial-Wave Decomposition
14 Waves

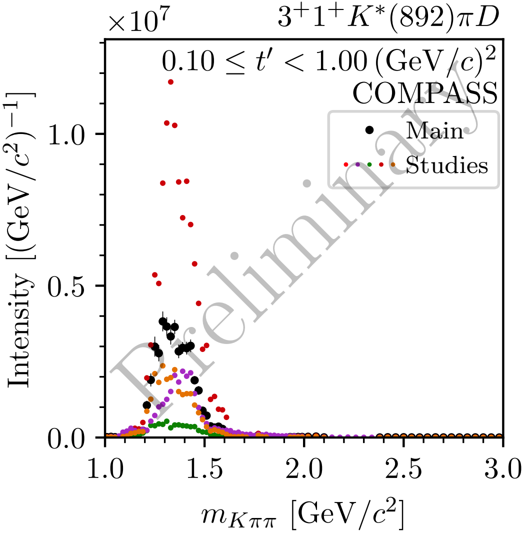



$\times 10^4$
$4^- 0^+ K_2^*(1430)\pi D$
$0.10 \le t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
1
2
3
1.0
1.5
2.0
2.5
3.0


$0^-0^+[K\pi]_S^{K\pi}\pi S$
$\times 10^6$
$0.10 \le t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
2
4
6
1.0
1.5
2.0
2.5
3.0

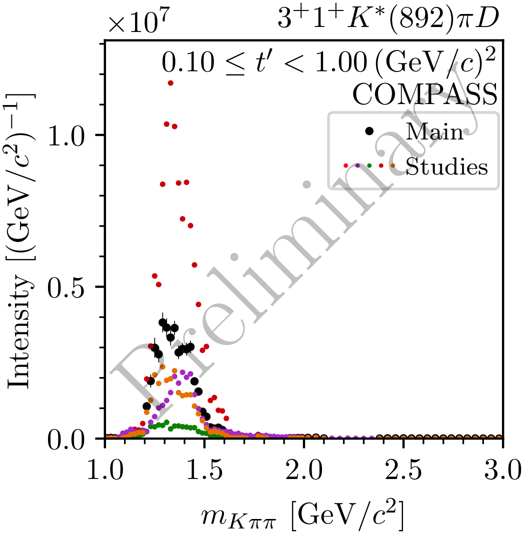

$\times 10^7$
$0^-0^+K^*(892)\pi P$
$0.10 \leq t' < 1.00\,(\mathrm{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\mathrm{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\mathrm{GeV}/c^2]$
0
1
2
1.0
1.5
2.0
2.5
3.0


$\times 10^7$
$1^+0^+K^*(892)\pi S$
$0.10 \leq t' < 1.00\,(\text{GeV}/c)^2$
COMPASS
Preliminary
Main
Studies
Intensity $[(\text{GeV}/c^2)^{-1}]$
$m_{K\pi\pi}$ $[\text{GeV}/c^2]$
0
1
2
3
1.0
1.5
2.0
2.5
3.0


$\times 10^7$
$3^+1^+K^*(892)\pi D$
$0.10 \le t' < 1.00\,(\text{GeV}/c)^2$
COMPASS
Main
Studies
Preliminary
Intensity $[(\text{GeV}/c^2)^{-1}]$
1.0
0.5
0.0
1.0
1.5
2.0
2.5
3.0
$m_{K\pi\pi}$ $[\text{GeV}/c^2]$

Leakage Effect

- Unexpected low-mass enhancement in $3^+ \, 1^+$ $K^*(892)\,\pi\,D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - ➡ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected



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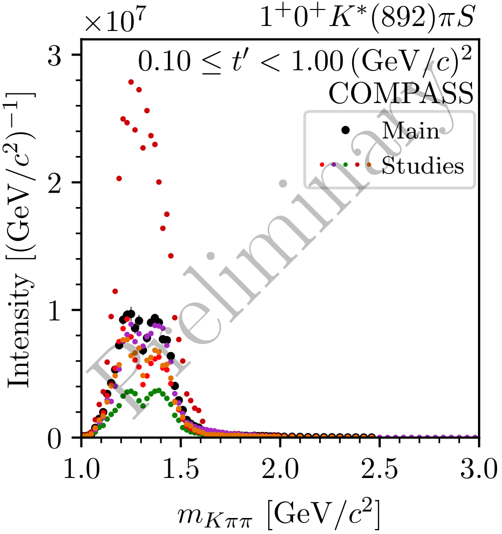

Leakage Effect

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- Similar to dominant 1^+ wave
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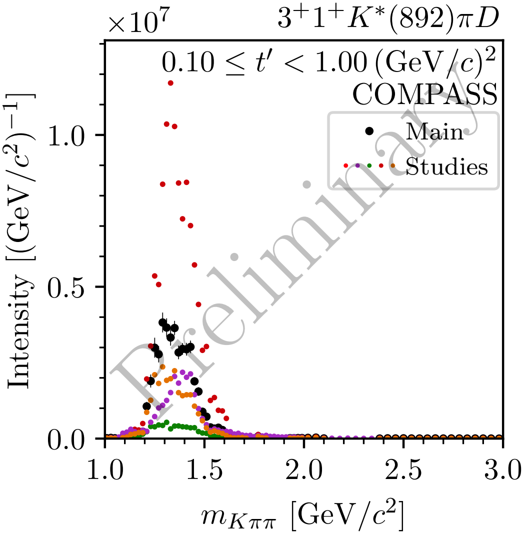
  - ➡ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected



Leakage Effect

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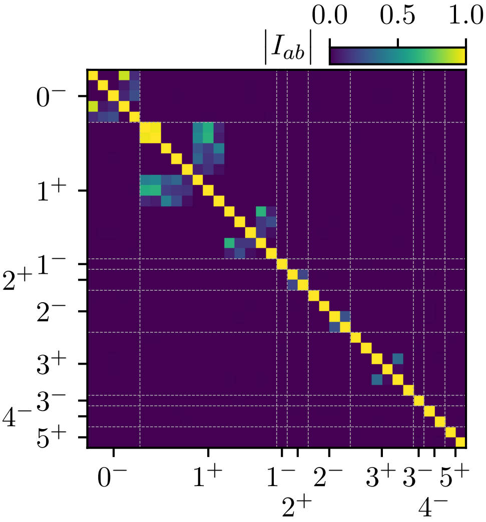
- Loss of orthogonality taking acceptance into account
  - ➡ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected

$$I_{a,b} = \int \mathrm{d}\varphi_3(\tau)\ \Psi_a(\tau)\Psi_b^*(\tau)$$



Leakage Effect

- Unexpected low-mass enhancement in $3^+ \, 1^+ \, K^*(892)\, \pi \, D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
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- Loss of orthogonality taking acceptance into account
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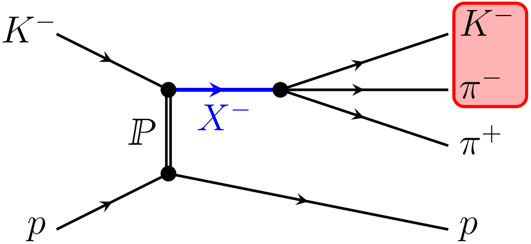
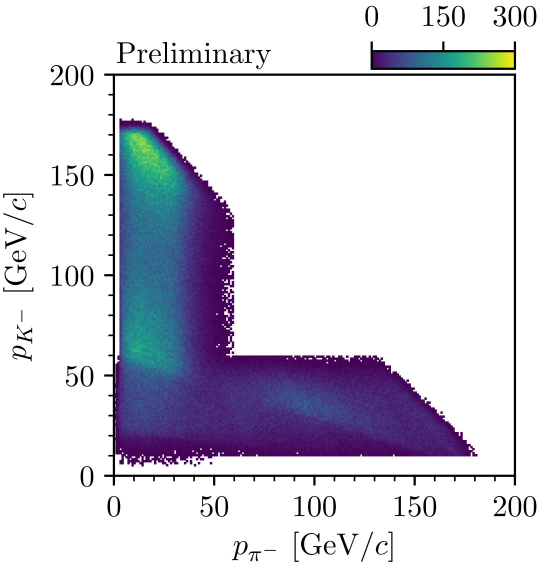
- Only a sub-set of partial waves affected



Leakage Effect

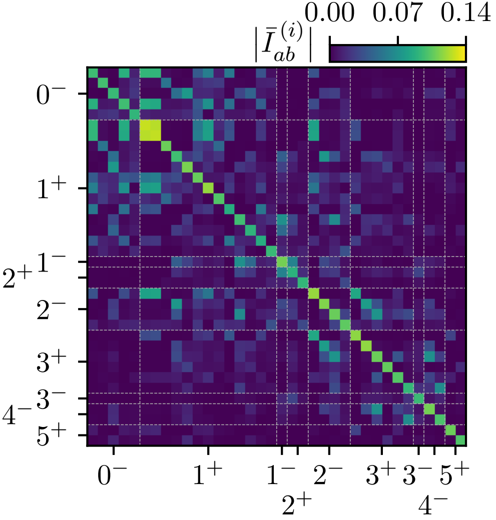
- Unexpected low-mass enhancement in $3^+ \, 1^+$ $K^*(892) \, \pi \, D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - ➡ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected



Leakage Effect

- Unexpected low-mass enhancement in $3^+ 1^+$ $K^*(892)\,\pi\; D$ wave
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - ➥ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected

$$\bar{I}_{a,b} = \int \mathrm{d}\varphi_3(\tau)\; \eta(\tau)\, \Psi_a(\tau) \Psi_b^*(\tau)$$



Leakage Effect

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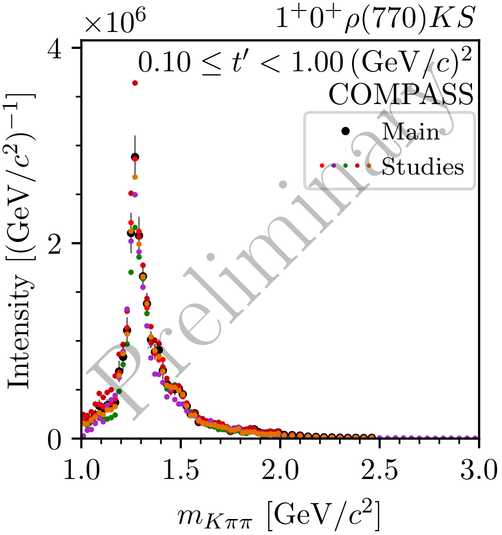
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - ➥ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected



Leakage Effect

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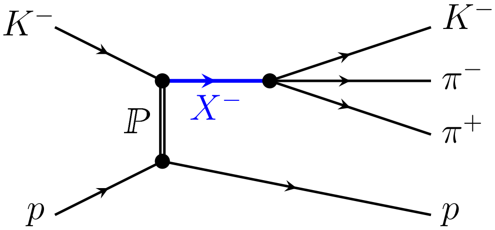
- Similar to dominant 1^+ wave
- Sensitive to systematic effects
- Decay amplitudes of different J^P are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - ➥ Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID:
  - Expect small contamination from beam π^-
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
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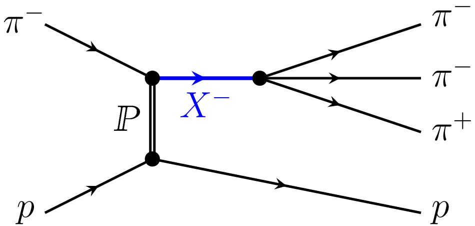
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
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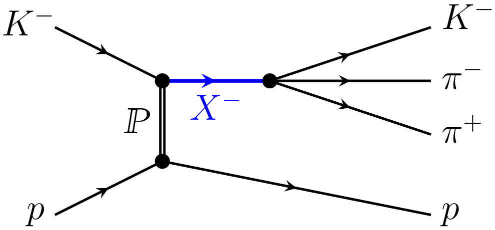
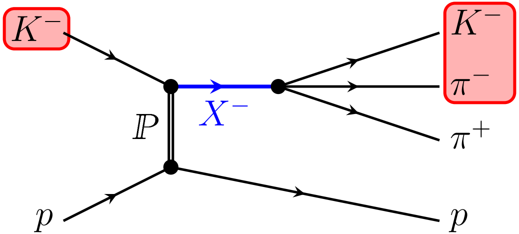
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➡ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➡ Dominant background in $K^-\pi^-\pi^+$ sample



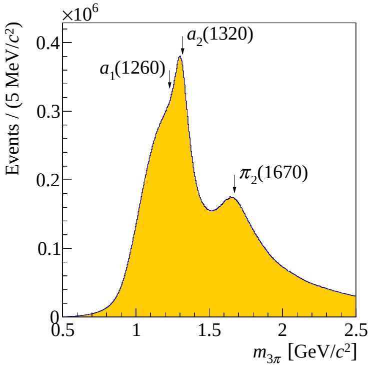
Incoherent $\pi^-\pi^-\pi^+$ Background

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID:
  - Expect small contamination from beam π^-
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - ➥ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
  - ➥ Dominant background in $K^-\pi^-\pi^+$ sample



Incoherent $\pi^-\pi^-\pi^+$ Background

- Well established model for $\pi^- + p \to \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction and selection as for $K^-\pi^-\pi^+$
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample



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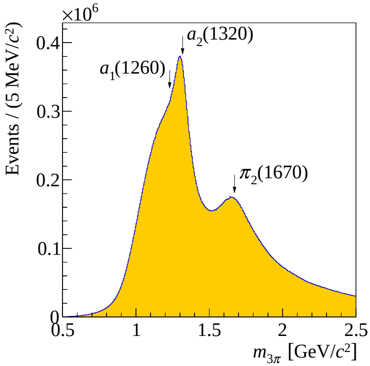

Incoherent $\pi^-\pi^-\pi^+$ Background

[Phys. Rev. D 95 (2017) 032004]

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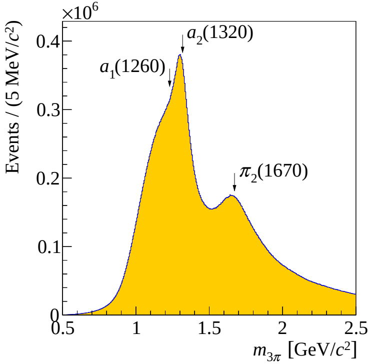

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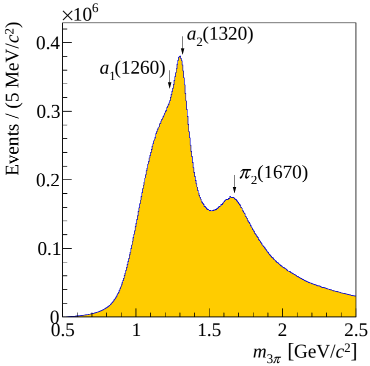
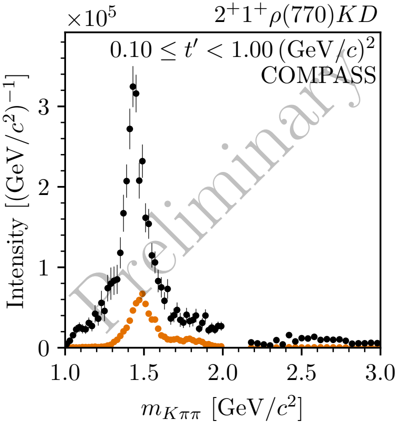
➥ Study $\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves



Incoherent $\pi^-\pi^-\pi^+$ Background

- Significant contribution to waves with $\rho(770)$ isobar
- $\pi^-\pi^-\pi^+$ produces peaking structures
- Largest relative contribution to $2^+ 1^+ \rho(770)\, K\, D$ wave
- Small contribution to waves with $K^*(892)$ isobar
- Also significant contribution to waves with $f_2(1270)$ and $K_2^*(1430)$ isobars
- No contribution to flat wave



$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data

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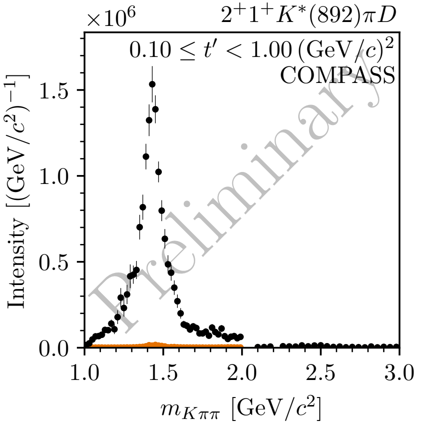

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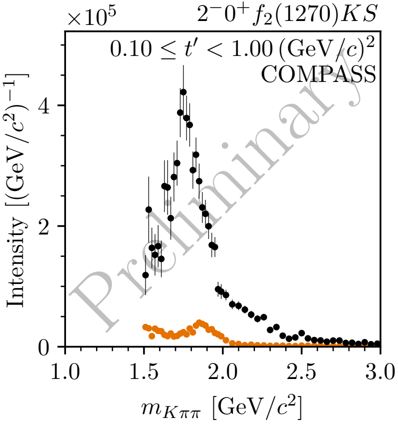

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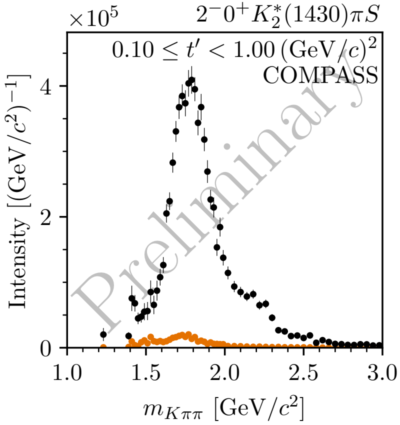

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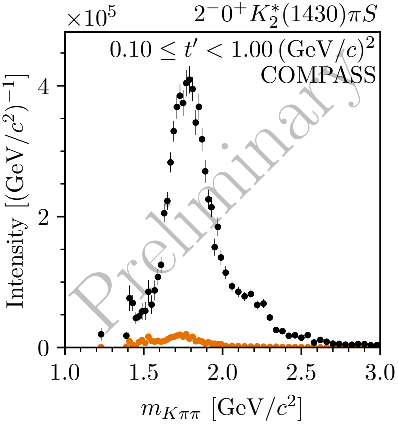
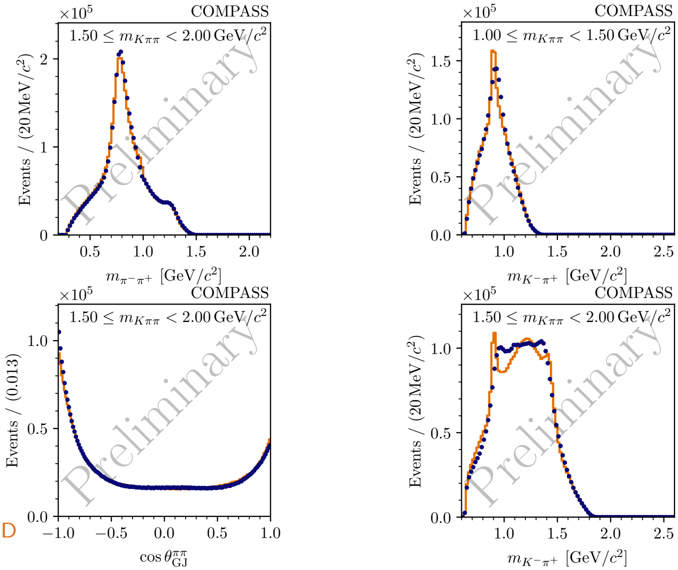


$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data

Incoherent $\pi^-\pi^-\pi^+$ Background

- 238-wave set can describe main features of $\pi^-\pi^-\pi^+$ pseudodata sufficiently well
- Largest deviation for $K^-\pi^+$ isobar system at thigh $m_{K\pi\pi}$



$\pi^-\pi^-\pi^+$ pseudo data,
prediction (weighted-MC) of $K^-\pi^-\pi^+$ PWD to $\pi^-\pi^-\pi^+$ pseudo data