Forward pion photoproduction with Regge phenomenology

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William & Mary, Williamburg, VA
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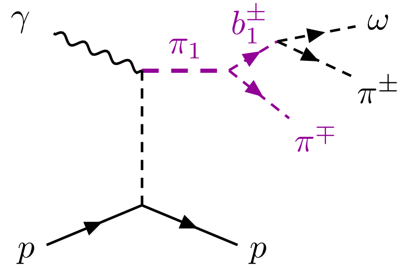
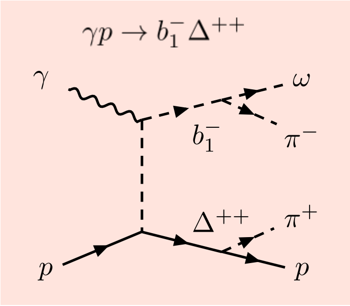
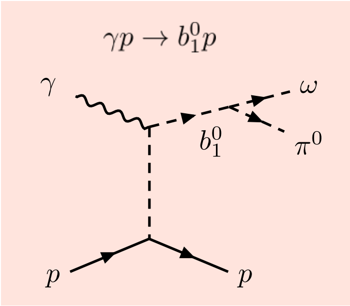
Search for exotic hybrid mesons in photoproduction with GlueX at JLab

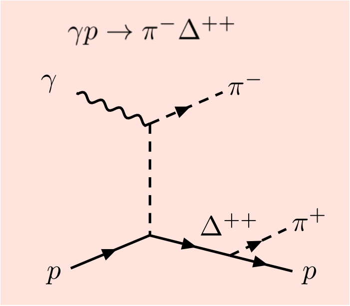
- Understanding the production mechanism in light meson photoproduction reactions is essential for the successful analysis of the data.
- Amplitude analyses of multi-meson final states require models for production amplitudes of several processes.



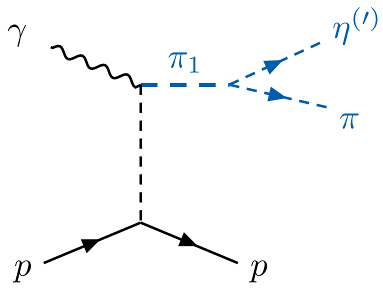


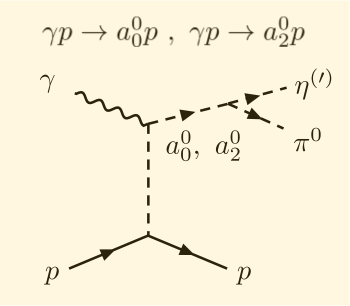
[Talk by A. Schertz, Fri PM]



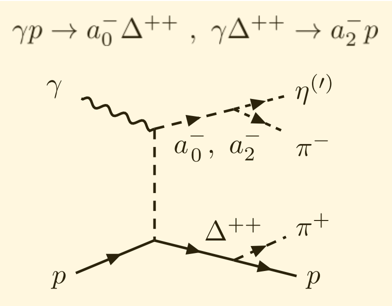


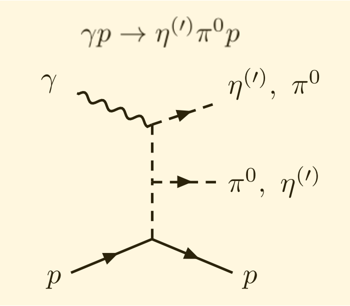
[Talk by V. Shastry, Sat AM]





[Talk by M. Albrecht, Tue AM]

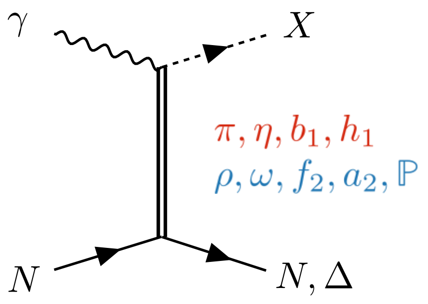




Begin by understanding the production mechanism of non-exotic mesons.

Polarized photoproduction at high energies

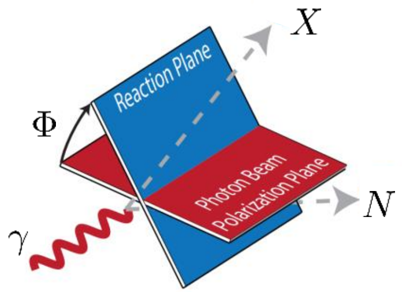
- At high energies, single meson photoproduction dominated by exchange of Regge trajectories in the t-channel.
- Linear photon beam polarization used to filter out the “naturality” of exchanged particle.



→ Unnatural ($P(-1)^J = -1$) parity: $0^-, 1^+, 2^-, 3^+, \ldots$

→ Natural ($P(-1)^J = 1$) parity: $0^+, 1^-, 2^+, 3^-, \ldots$

- Beam asymmetry



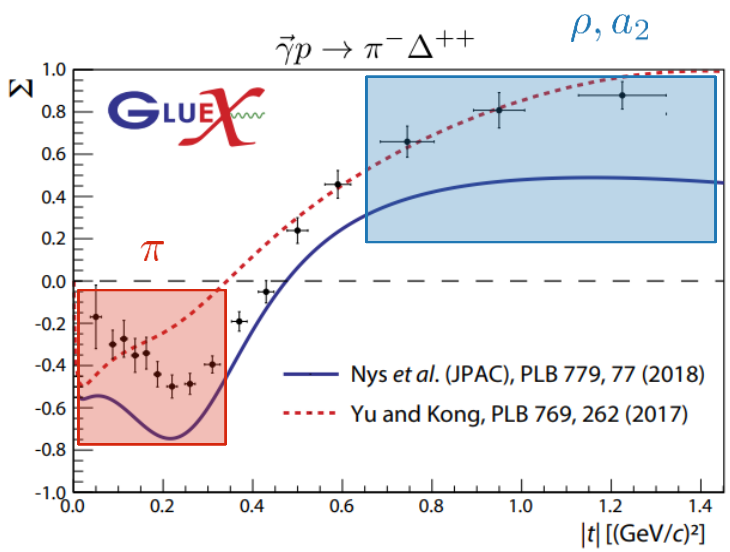
$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} \approx \frac{d\sigma_N - d\sigma_U}{d\sigma_N + d\sigma_U}$$

$\Phi = \frac{\pi}{2}$ $\quad\Phi = 0$

Natural exchange

Unnatural exchange

[V.Mathieu et al., *Phys.Rev.D* 92 (2015) 7]

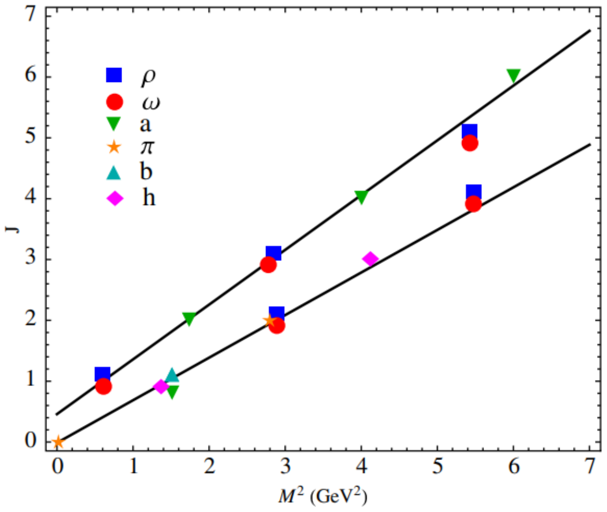


[GlueX Collaboration, *Phys.Rev.C* 103 (2021) 2, L022201]

Focus of this talk: pion exchange mechanism in pion photoproduction

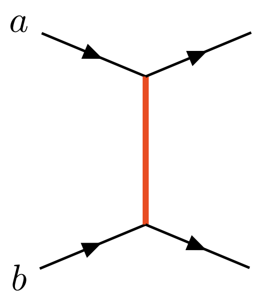
Reggeon trajectories

- Families with same quantum numbers but different spin J (even or odd).
- Almost straight lines (Chew-Frautschi plot)
- In standard Regge theory parameterized by: $\alpha(t) = \alpha' t + \alpha_0$

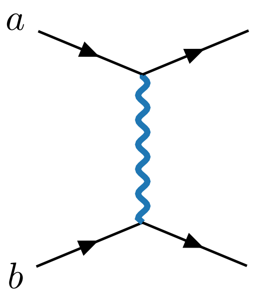


[V.Mathieu et al., *Phys.Rev.D* 98 (2018) 1, 014041]

$$\alpha_\pi(t) = 0.7(t - m_\pi^2) = 0.7t - 0.014$$
$$\alpha_\rho(t) = 0.9(t - m_\rho^2) + 1 = 0.9t + 0.466$$



Real particle with mass m and spin J $\qquad \dfrac{1}{t - m^2}$



Reggeon with mass $\sqrt{t}$ and spin $\alpha(t)$

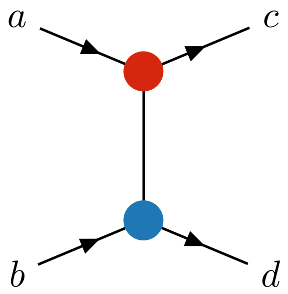
$$\mathcal{P}_{\text{Regge}} = \frac{\pi\alpha'}{\sin(\pi\alpha(t))} \frac{1 + \eta e^{-i\pi\alpha(t)}}{2} \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{1}{\Gamma(1 + \alpha(t))}$$

poles for integer $\alpha(t)$ — signature factor — asymptotic behavior — cancel non-physical poles

Implications of Regge pole amplitudes

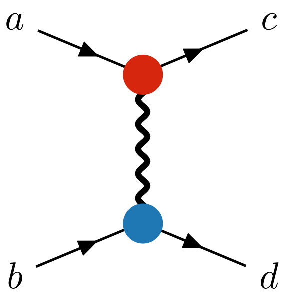
Factorization

Amplitude for particle exchange "factorizes" (follows from unitarity).



$$A(s,t) = g_{ac} \frac{1}{t-m^2} g_{bd}$$

The reggeon residue $\beta(t)$:



- Contains all information about incoming and outgoing particles.
- Related to the reggeon-hadron interaction vertices.
- Satisfies factorization: $\beta(t) = \beta_{ac}(t)\beta_{bd}(t)$

Power law energy dependence

$$A(s,t) \sim s^{\alpha(t)}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}|A(s,t)|^2 = s^{2\alpha(t)-2}$$

Leading Regge poles (biggest $\alpha(t)$) dominate asymptotically.

Phase

The phase comes from the signature factor: $\frac{1+\eta e^{-i\pi\alpha(t)}}{2}$

Exchange degeneracy (equal trajectories with opposite signatures) leads to rotating or constant phases.

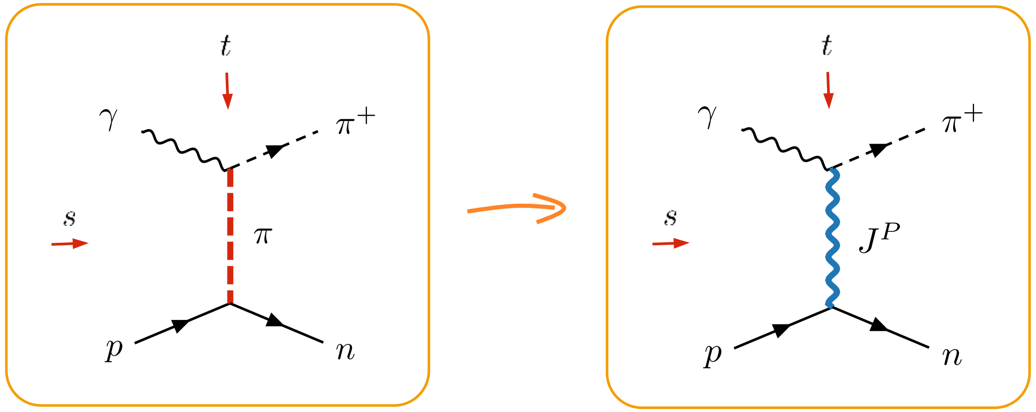
- Corrections to these hypothesis, usually ~10-20%. [J.Nys et al. (JPAC), *Phys.Rev.D* 98 (2018) 3, 034020]

Reggeization of pion exchange

- The exchanged pion is expected to reggeize.
- In the Regge-pole approximation:

$$\frac{1}{t-m_\pi^2} \longrightarrow \mathcal{P}_\pi^{\text{Regge}} = \frac{\pi\alpha'_\pi}{2}\frac{1+e^{-i\pi\alpha_\pi(t)}}{\sin\pi\alpha_\pi(t)}\left(\frac{s}{s_0}\right)^{\alpha_\pi(t)}$$

Pion trajectory: $\alpha_\pi(t) = \alpha'_\pi(t-m_\pi^2)$ with $\alpha'_\pi = 0.7$

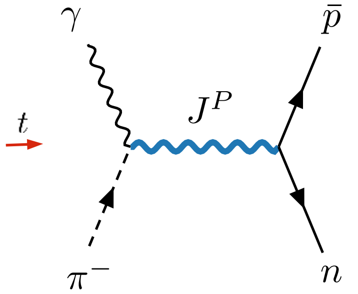


Rigorous reggeization: explicit exchange of Reggeons in the π trajectory, and sum the series of t-channel partial waves.

$$\gamma(k) + \bar{\pi}(-p_\pi) \to \bar{N}(-p_i) + N(p_f)$$

$$A_{\lambda_\gamma\lambda_1\lambda_2}(s,t) = \sum_J (2J+1)\, a^J_{\lambda_\gamma\lambda_i\lambda_f}(t)\, d^J_{\lambda_\gamma\lambda_i-\lambda_f}(\theta_t)$$

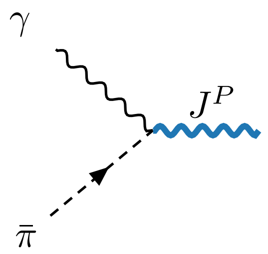
partial wave — Wigner function



$J = 0, 2, 4, \ldots$
$P = -1$

Exchange of arbitrary spin $J^P = (\text{even})^-$

- Build partial waves for the exchange of arbitrary spin:

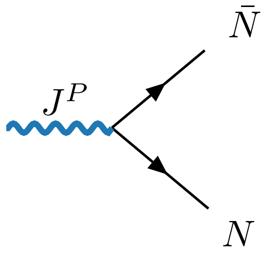


$$1^- \otimes 0^- = 1^+ \quad \left\{ \begin{array}{ll} L = 1 & J = 0 \\ L = \{J-1, J+1\} & J \geq 2 \end{array} \right\}$$

one L vs two L's

$$V_{\lambda_\gamma}(J) = 2\sqrt{2} e_\pi \left[k^{\nu_1} \cdots k^{\nu_J} \epsilon_\mu(k, \lambda_\gamma) p_\pi^\mu - (k \cdot p_\pi) k^{\nu_1} \cdots k^{\nu_{J-1}} \epsilon^{\nu_J}(k, \lambda_\gamma) \right] \epsilon^*_{\nu_1 \cdots \nu_J}(M)$$

→ Gauge invariant by construction



$$\tfrac{1}{2}^+ \otimes \tfrac{1}{2}^- = 0^- \oplus 1^- \rightarrow L = J$$

helicity non-flip (→ 0^-); helicity flip (→ 1^-)

$$V^{\text{NF}}_{\lambda_i \lambda_f}(J) = g P^{\nu_1} \cdots P^{\nu_J} \epsilon_{\nu_1 \cdots \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(-p_i) \qquad (P^\nu = p_i^\nu + p_f^\nu)$$

$$V^{\text{F}}_{\lambda_i \lambda_f}(J) = g P^{\nu_1} \cdots P^{\nu_{J-1}} \epsilon_{\nu_1 \cdots \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma^{\nu_J} \gamma_5 v_{\lambda_i}(-p_i)$$

$$V_{\lambda_\gamma}(J) \frac{1}{J - \alpha_\pi(t)} V^{\text{NF}}_{\lambda_i \lambda_f}(J) = a^J_{\lambda_\gamma \lambda_i \lambda_f}(t) d^J_{\lambda_\gamma \lambda_i - \lambda_f}(\theta_t)$$

Regge pole propagator

partial wave ($J > 0$): $a^J_{\lambda_\gamma \lambda_i \lambda_f}(t) \equiv \dfrac{2 e_\pi g}{J - \alpha_\pi(t)} (2\lambda_i \delta_{\lambda_i \lambda_f}) c_J^2 \sqrt{\dfrac{J+1}{J}} (-2 p_t k_t)^J t$

Analytical continuation to $J = 0$

- Reggeization of the pion trajectory requires the summation of partial waves with $J = 0, 2, 4, \ldots$

$$A_{\lambda_\gamma \lambda_1 \lambda_2}(s,t) = \sum_J (2J+1)\, a^J_{\lambda_\gamma \lambda_i \lambda_f}(t)\, d^J_{\lambda_\gamma \lambda_i - \lambda_f}(\theta_t)$$

... but we could only write gauge invariant partial waves for $J > 0$. The expression diverges for $J = 0$.

$$a^J_{\lambda_\gamma \lambda_i \lambda_f}(t) \equiv \frac{2e_\pi g}{J - \alpha_\pi(t)} (2\lambda_i \delta_{\lambda_i \lambda_f}) c_J^2 \sqrt{\frac{J+1}{J}} (-2p_t k_t)^J t$$

- Wigner d-functions can be expressed in terms of Jacobi polynomials, $d^J_{\lambda_\gamma \lambda'}(\theta_t) = \sqrt{\frac{J+1}{2J}} d^1_{\lambda_\gamma \lambda'}(\theta_t) P^{11}_{J-1}(z_t)$

  which satisfy the symmetry property $P^{ab}_n(-z) = (-1)^n P^{ba}_n(z)$

  and are defined via the regularized hypergeometric function: $\frac{1}{2J}(P^{11}_{J-1}(z_t) - P^{11}_{J-1}(-z_t))\Big|_{J=0} = \frac{2z_t}{z_t^2 - 1}$.

- The contribution to the amplitude from $J = 0$ is finite!

$$A^{J=0}_{\lambda_\gamma \lambda_i \lambda_f}(s,t) = a^0_{\lambda_\gamma \lambda_i \lambda_f}(t) d^0_{\lambda_\gamma \lambda_i - \lambda_f}(\theta_t) \approx -i2e_\pi g_{\pi NN} \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) \frac{t}{m_\pi^2 - t}$$

$$g|_{J=0} = \alpha' g_{\pi NN}$$

Spin summation

- For high spins, the coupling reflects the internal structure of the hadronic radii: $g = \alpha' g_{\pi NN} h_J (r_t r_b)^J$
- The kinematical factor has alternating poles and zeros for $J<0$. We approximate: $c_J^2 h_J (2 r_t r_b)^J \rightarrow \frac{j_p}{j_z}\frac{J+j_z}{J+j_p} R^{2J}$

effective zero

hadronic radius

effective pole

- We write the reggeized amplitude as:

$$A^{\text{Regge}\ \pi}_{\lambda_\gamma \lambda_i \lambda_f}(s,t) = -i 2 e_\pi g_{\pi NN} \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) t \times \mathcal{P}_\pi$$

$$\mathcal{P}_\pi^{j_p, j_z} = -i\alpha' \sin\theta_t \sum_{J=0,1,2,\ldots} \frac{(2J+1)(J+1)}{2J(J-\alpha_\pi(t))} \frac{j_p}{j_z} \frac{J+j_z}{J+j_p} (-\kappa)^J \frac{1}{2}\left[P_{J-1}^{|\lambda_\gamma - \lambda'||\lambda_\gamma+\lambda'|}(z_t) - P_{J-1}^{|\lambda_\gamma+\lambda'||\lambda_\gamma-\lambda'|}(-z_t)\right] \qquad (\kappa \equiv p_t k_t R^2)$$

… and use the generating function of the Jacobi polynomials to perform the spin summation:

$$\mathcal{P}_\pi^{j_p,j_z}(s,t) = \mathcal{P}_\pi^{J=0} - \frac{\alpha' \kappa \sin\theta_t}{2} \int_0^1 dy \left[\frac{1}{-\alpha} + y^{-\alpha} \frac{j_p(\alpha+j_z)(\alpha+1)(2\alpha+1)}{\alpha j_z(\alpha+j_p)} + y^{j_p} \frac{(j_z-j_p)(1-j_p)(1-2j_p)}{j_z(j_p+\alpha)} \right]$$
$$\times 2 \left[\frac{1}{G(z_t,y)} \frac{1}{(1+G(z_t,y))^2 - (\kappa y)^2} - (z_t \rightarrow -z_t) \right] \quad \text{with} \quad G(z_t,y) = \sqrt{1 + 2\kappa z_t y + (\kappa y)^2}$$

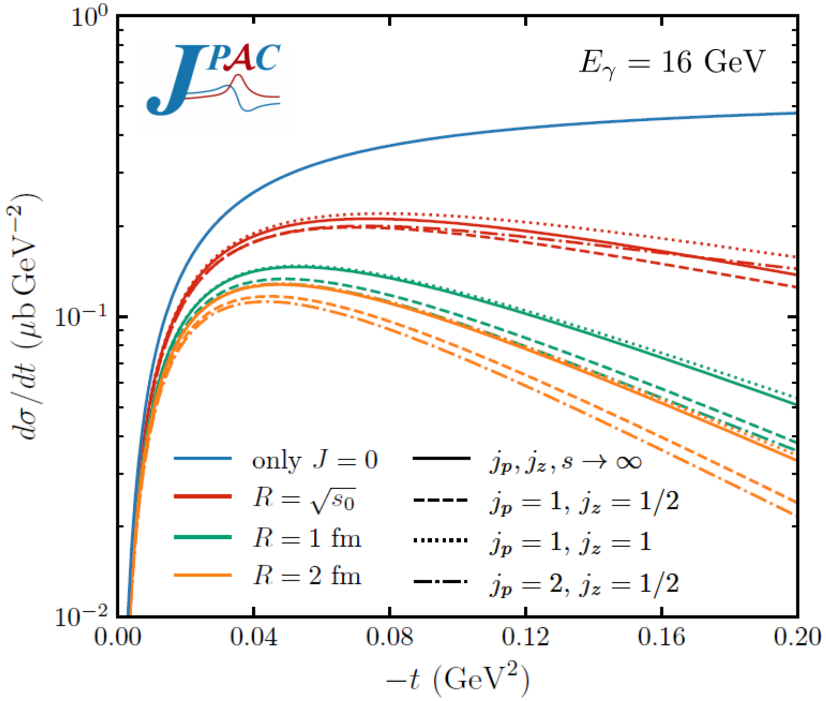
Option 1: solve numerically

Option 2: consider the dominant contribution of the Regge pole, in the limit of high *s*.

$$\mathcal{P}_\pi^{s\rightarrow\infty}(s,t) = -\frac{\alpha'}{\alpha}\left[1 - \left(1 - (sR^2)^\alpha \frac{\tau 2\sqrt{\pi}}{\sin \pi\alpha} \frac{\Gamma(\alpha+3/2)}{\Gamma(\alpha)}\right)\right] + O\left(\frac{1}{s^{j_p}}\right)$$
$$\approx -\frac{\alpha'}{\alpha} \frac{\tau 2\sqrt{\pi}}{\sin\pi\alpha} \frac{\Gamma(\alpha+3/2)}{\Gamma(\alpha)} (sR^2)^\alpha$$

Results

[G.Montana et al. (in preparation)]

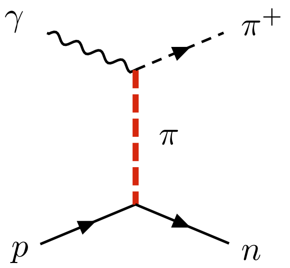


$$\mathcal{P}_\pi^{J=0} = -\frac{\alpha'}{\alpha(t)} = \frac{1}{m_\pi^2 - t}\ .$$

$$\mathcal{P}_\pi^{j_p,j_z}(s,t) = \mathcal{P}_\pi^{J=0} - \frac{\alpha'\kappa\sin\theta_t}{2}\int_0^1 dy\left[\frac{1}{-\alpha} + y^{-\alpha}\frac{j_p(\alpha+j_z)(\alpha+1)(2\alpha+1)}{\alpha j_z(\alpha+j_p)} + y^{j_p}\frac{(j_z-j_p)(1-j_p)(1-2j_p)}{j_z(j_p+\alpha)}\right]$$
$$\times 2\left[\frac{1}{G(z_t,y)}\frac{1}{(1+G(z_t,y))^2-(\kappa y)^2} - (z_t \to -z_t)\right]$$

$$\mathcal{P}_\pi^{s\to\infty}(s,t) = -\frac{\alpha' 2\sqrt{\pi}}{\sin\pi\alpha}\frac{\Gamma(\alpha+3/2)}{\Gamma(\alpha+1)}\frac{1+e^{-i\pi\alpha}}{2}(sR^2)^\alpha$$

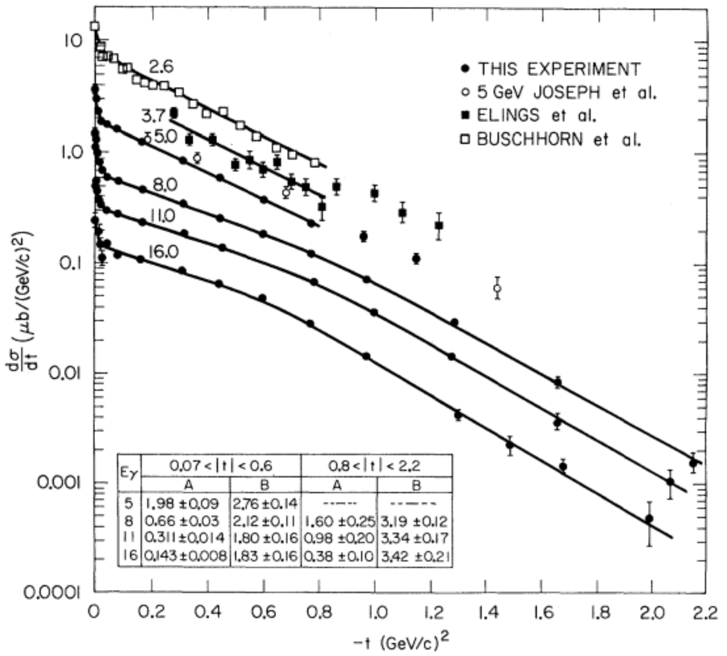
Charged pion photoproduction within Born models



- Description at low energies in terms of effective Lagrangians.
- High energies: reggeization of the pion Born diagram.

Known issues

- What is pion exchange and how does it reggeize?
- Cannot describe forward cross-section data in $\gamma p \to \pi^+ n$ (same for $np \to pn$).



E_γ	$0.07 < \lvert t \rvert < 0.6$		$0.8 < \lvert t \rvert < 2.2$	
	A	B	A	B
5	1.98 ±0.09	2.76 ±0.14	------	------
8	0.66 ±0.03	2.12 ±0.11	1.60 ±0.25	3.19 ±0.12
11	0.311 ±0.014	1.80 ±0.16	0.98 ±0.20	3.34 ±0.17
16	0.143 ±0.008	1.83 ±0.16	0.38 ±0.10	3.42 ±0.21

[A. Boyarski et al., *Phys.Rev.Lett.* 20 (1968) 300]

Proposed solutions

- Existence of parity-doublet conspirator of the pion.
  [J.S.Ball, W.R. Frazer and M. Jacob, *Phys.Rev.Lett.* 20 (1968) 518]
- Regge cuts and absorption (final state interactions).
  [F. Henyey, G.L.Kane, J.Pumplin, *Phys.Rev.* 182 (1969) 1579]
- Nucleon Born terms.
  [L.Jones, *Rev.Mod.Phys.* 52 (1980) 545]

Adding the nucleon Born diagrams

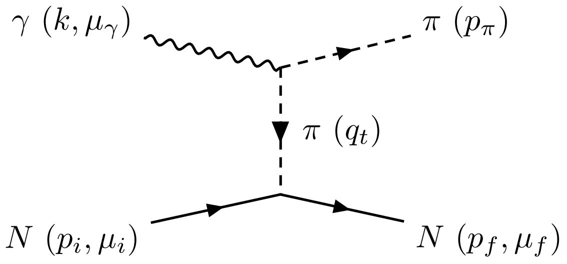
- *s*-channel reaction: $\gamma(k,\mu_\gamma) + N(p_i,\mu_i) \to \pi(p_\pi) + N(p_f,\mu_f)$
- Helicity amplitude: $A_{\mu_\gamma \mu_i \mu_f} = \bar{u}_{\mu_f}(p_f)\epsilon_{\mu_\gamma}(k) \cdot J_{\mu_i \mu_f} u_{\mu_i}(p_i)$,

The current is not conserved.

The amplitude is not gauge invariant.

The cross section is frame dependent.

t - channel Born diagram

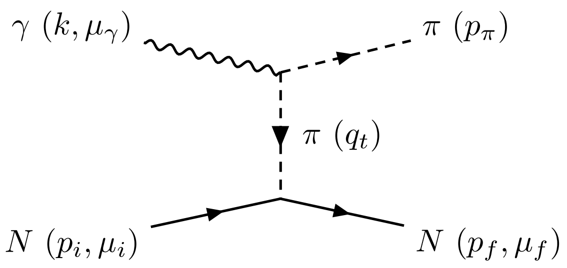


$$J^{\mu}_{\mu_i \mu_f, t} = -\sqrt{2} e_\pi g_{\pi NN} \frac{q_t^\mu - p_\pi^\mu}{t - m_\pi^2} \gamma_5$$

Adding the nucleon Born diagrams

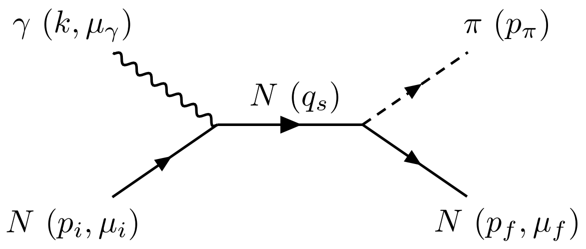
- *s*-channel reaction: $\gamma(k,\mu_\gamma) + N(p_i,\mu_i) \to \pi(p_\pi) + N(p_f,\mu_f)$
- Helicity amplitude: $A_{\mu_\gamma\mu_i\mu_f} = \bar{u}_{\mu_f}(p_f)\epsilon_{\mu_\gamma}(k)\cdot J_{\mu_i\mu_f} u_{\mu_i}(p_i)$, $J^\mu_{\mu_i\mu_f} = J^\mu_{\mu_i\mu_f,t} + J^\mu_{\mu_i\mu_f,s} + J^\mu_{\mu_i\mu_f,u}$ → The total current is conserved.

t - channel Born diagram



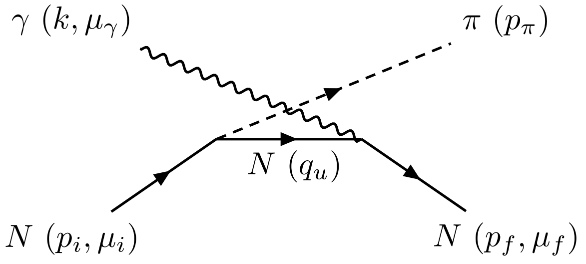
$$J^\mu_{\mu_i\mu_f,t} = -\sqrt{2}e_\pi g_{\pi NN}\frac{q_t^\mu - p_\pi^\mu}{t-m_\pi^2}\gamma_5$$

s - channel



$$J^\mu_{\mu_i\mu_f,s} = \sqrt{2}e_N g_{\pi NN}\bar{\gamma}_5\frac{\not{q}_s + m_p}{s-m_p^2}\gamma^\mu$$

u - channel



$$J^\mu_{\mu_i\mu_f,u} = \sqrt{2}e_N g_{\pi NN}\gamma^\mu\frac{\not{q}_u + m_p}{u-m_p^2}\gamma_5$$

- Separate electric and magnetic contributions: $A_{\mu_\gamma\mu_i\mu_f} = A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} + A^{\mathrm{m}}_{\mu_\gamma\mu_i\mu_f}$

$$A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2}g_{\pi NN}\left[e_\pi\frac{(\epsilon_{\mu_\gamma}\cdot p_\pi)}{t-m_\pi^2} + e_{N_i}\frac{(\epsilon_{\mu_\gamma}\cdot p_i)}{s-m_p^2} + e_{N_f}\frac{(\epsilon_{\mu_\gamma}\cdot p_f)}{u-m_p^2}\right]\bar{u}_{\mu_f}(p_f)\gamma_5 u_{\mu_i}(p_i)$$

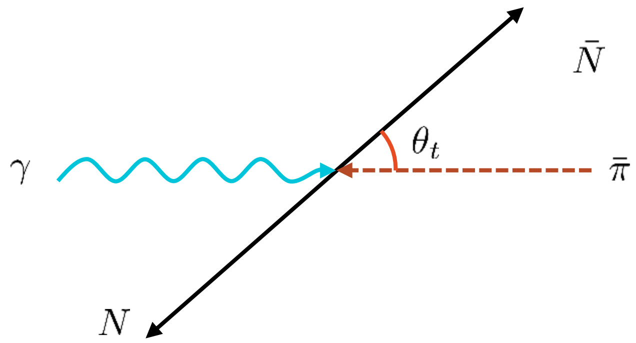
dominant contribution — vanish

$$A^{\mathrm{m}}_{\mu_\gamma\mu_i\mu_f} = \sqrt{2}g_{\pi NN}\left[\frac{e_{N_i}}{s-m_p^2} + \frac{e_{N_f}}{u-m_p^2}\right]\bar{u}_{\mu_f}(p_f)\gamma_5\not{k}\not{\epsilon}_{\mu_\gamma}u_{\mu_i}(p_i)$$

Electric term in the *t*-channel frame

- *t*-channel reaction: $\gamma(k,\lambda_\gamma)+\bar{\pi}(-p_\pi)\to\bar{N}(-p_i,\lambda_i)+N(p_f,\lambda_f)$

$$A^{\mathrm{e}}_{\lambda_\gamma\lambda_i\lambda_f}=2\sqrt{2}g_{\pi NN}\left[e_\pi\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{t-m_\pi^2}+e_{N_i}\frac{(\epsilon_{\lambda_\gamma}\cdot p_i)}{s-m_p^2}+e_{N_f}\frac{(\epsilon_{\lambda_\gamma}\cdot p_f)}{u-m_p^2}\right]\bar{u}_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)$$



- Using momentum conservation and electric charge conservation ($e_{N_i}=e_\pi-e_{N_f}$):

$$\begin{aligned}A^{\mathrm{e}}_{\lambda_\gamma\lambda_i\lambda_f}=\ &2\sqrt{2}g_{\pi NN}\Bigg[e_\pi\left(\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{t-m_\pi^2}+\frac{(\epsilon_{\lambda_\gamma}\cdot(p_i+p_f))}{s-u}\right)\\&+\frac{1}{2}e_{N_i}\left(\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{s-m_p^2}+\frac{(\epsilon_{\lambda_\gamma}\cdot(p_i+p_f))}{s-u}\frac{t-m_\pi^2}{s-m_p^2}\right)\\&-\frac{1}{2}e_{N_f}\left(\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{u-m_p^2}+\frac{(\epsilon_{\lambda_\gamma}\cdot(p_i+p_f))}{s-u}\frac{t-m_\pi^2}{u-m_p^2}\right)\Bigg]\bar{u}_{\lambda_f}(p_f)\gamma_5 v_{\lambda_i}(-p_i)\end{aligned}$$

→ Minimal gauge invariant (m.g.i.)

subleading at large *s*

$$A^{\mathrm{m.g.i}}_{\lambda_\gamma\lambda_i\lambda_f}(s,t)=2\sqrt{2}g_{\pi NN}e_\pi\left(\frac{(\epsilon_{\lambda_\gamma}\cdot p_\pi)}{t-m_\pi^2}+\frac{(\epsilon_{\lambda_\gamma}\cdot(p_i+p_f))}{s-u}\right)\approx -i2e_\pi g_{\pi NN}\lambda_\gamma(2\lambda_i\delta_{\lambda_i\lambda_f})\frac{t}{m_\pi^2-t}=A^{J=0}_{\lambda_\gamma\lambda_i\lambda_f}(s,t)$$

vanishes — dominant contribution

- Gauge invariance relates the pion and nucleon exchanges.
- The nucleon exchange diagram originates the pion pole in the t-channel frame.

Magnetic term

- The magnetic term is gauge invariant by itself.

$$A^{\mathrm{m}}_{\mu_\gamma \mu_i \mu_f} = \sqrt{2} g_{\pi NN} \left[\frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$
$$\approx \mu_\gamma 2 g_{\pi NN} (e_{N_i} - e_{N_f}) \delta_{\mu_\gamma \mu_i} \delta_{-\mu_i \mu_f}$$

- At $t \sim 0$ the electric term of the amplitude vanishes.
- The magnetic term has small dependence in t.

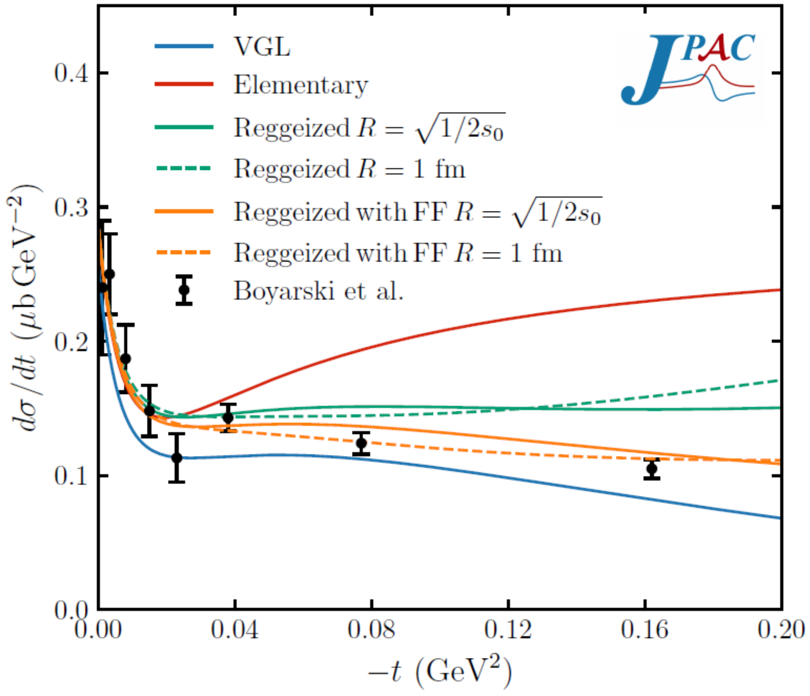
- Size of the cross section agrees with the data at $t \sim 0$.

- Corrections: add form factor to magnetic term $\beta(t) = \frac{\Lambda}{\Lambda^2 - t}$ with $\Lambda \sim 1 \text{ GeV}$

- Comparison with the VGL model → reggeizes by multiplying full Born amplitude by $(t - m_\pi^2) \times \mathcal{P}_{\text{Regge}}$

[M.Guidal, J.M.Laget and M. Vanderhaeghen, *Nucl.Phys.A* 627 (1997) 645-678]

[G.Montana et al. (in preparation)]



CONCLUSIONS

- A precise comprehension of the production mechanisms is crucial for the light hybrid meson searches.
- At high energies, meson photoproduction reactions are dominated by the exchange of Regge trajectories, in particular, the pion trajectory plays a major role at low momentum transfer.
- How do we reggeize the pion in an appropriate way?
    - Current conservation requires the nucleon Born terms (gauge invariance).
    - It was not clear how to add *t*- and *s*-channel consistently without double counting: *t*-channel and *s*-channel partial wave series should independently represent the full amplitude.
    - Examination of the analytical J dependence emerging from the contraction of the vertices coupling $\gamma\pi$ and NN to $J^P = (\text{even})^-$ reveals that it is analytical at $J = 0$ and physically contains part of the (*s*-channel, or *u*-channel depending on charge) nucleon exchange.

What's next?

- Revisit the pion exchange in $\gamma p \to \pi^- \Delta^{++}$ and understand Δ^{++} SDMEs.
- Extension of the formalism to natural parity exchanges.
- Amplitudes for photoproduction of $b_1,\ a_2,\ \pi_1$ with proton and Δ^{++} recoils.