From pole parameters to line shapes and branching ratios

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What is the plan?

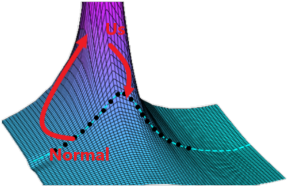
What is the usual procedure?

→ Analysis of data to determine resonance parameters

What do we want to do?

→ Change the direction: Recreate from resonance parameters

- Partial wave amplitude t_ℓ
- Form factors F_ℓ
- Branching ratios



One full channel ($\pi\pi$) one resonance ($f_0(500)\,/\,\rho(770)$)

Naive approach: one–channel resonance exchange

Naive model: Resonance exchange (on the $(+/-)$ sheet)

$$\mathrm{i}\,T^{(+/-)} = \underbrace{\mathrm{i}g}_{\text{coupling}} \cdot \underbrace{\frac{\mathrm{i}}{s - m^2 + g^2\Pi(s)^{(+/-)}}}_{\text{propagator}} \cdot \underbrace{\mathrm{i}g}_{\text{coupling}}$$

→ g, m needed to fix pole (2 real parameters)

→ residue then automatically determined by pole postion and Π

→ Π is scalar 2-particle loop: self-energy contribution

→ We found: need to decouple pole and residue

$$\boxed{\frac{\sigma}{16\pi^2} \log \frac{\sigma+1}{\sigma-1}}$$

⇒ Background interaction needed

Two–potential–formalism

- Conceived to solve nucleon scattering (Nakano et al. 1982, PRC 26)
- Divides interaction potential into two distinct parts: $V_1 + V_2$
- Was used before to describe meson interaction (Hanhart 2012, PLB 715)
- Divide resonance and background physics:

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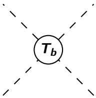

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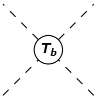
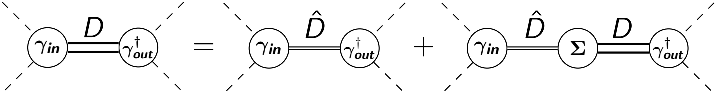
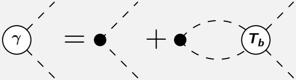


$T_R \Rightarrow$

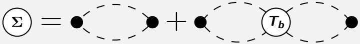


Two-potential formalism



$$\Leftrightarrow \gamma = 1 + GT_b$$

$$\mathrm{Disc}(\gamma) = 2i\gamma^{I}(s)T_b^{II}(s)$$



$$\Leftrightarrow \Sigma = G + GT_bG = \gamma G$$

$$\mathrm{Disc}(\Sigma) = 2i\rho\gamma^{I}\gamma^{II}(s)$$

Background model

$$\boxed{T_b = \frac{f_0}{f(s) - f_0 \Pi(s)} = \frac{1}{\rho(s)\cot(\delta_b) - i\rho(s)}}$$

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Possible energy dependencies:

- Polynomials (resonances)
- Left-hand cuts (Danilkin et al. 2023, PRD 107)

$$\omega(s) = \frac{\sqrt{s - s_L} - \sqrt{s_E - s_L}}{\sqrt{s - s_L} + \sqrt{s_E - s_L}}$$

T–matrix construction

$$f(s) = 1 + \sum_{k=1} f_k \omega(s)^k + \sum_{n=1} g_n s^n$$

T–matrix construction

For correct onset of LHC (from ChPT)

$$f(s) = 1 + f_1\left(\omega(s) + 2\omega(s)^2\right) + f_{\mathrm{R}} s \qquad f_{\mathrm{R}} = \frac{1}{(2\mathrm{GeV})^2}$$

For T_b vanishing faster than a $\log$

T–matrix construction

$$f(s) = 1 + f_1\left(\omega(s) + 2\omega(s)^2\right) + f_R s \qquad f_R = \frac{1}{(2\text{GeV})^2}$$

Function	Formula
Vertex function $\gamma(s)$	$\exp\left\{\frac{s}{\pi}\int_{s_{thr}}^{\infty}\frac{\delta_b(s')ds'}{(s'-s)s'}\right\}$
Self-energy $\Sigma(s)$	$\frac{s}{\pi}\int_{s_{thr}}^{\infty}\frac{\rho(s')\lvert\gamma(s'^2)\rvert ds'}{(s'-s)s'}$

$$T = \frac{f_0}{f(s) - f_0\Pi(s)} - \frac{g^2\gamma(s)^2}{s - m^2 + g^2\Sigma(s)}$$

T–matrix construction $\ell \neq 0$

$$f(s) = 1 + f_1\left(\omega(s) + 2\omega(s)^2\right) + f_R s \qquad f_R = \frac{1}{(2\mathrm{GeV})^2}$$

Function	Formula
$\gamma_\ell(s) = \xi^{2\ell}\hat{\gamma}_\ell$	$\xi^{2\ell} \exp\left\{ \frac{s}{\pi} \int_{s_{thr}}^{\infty} \frac{\delta_b^\ell(s')ds'}{(s'-s)s'} \right\}$
$\Sigma_\ell(s)$	$\frac{s}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')\xi^{2\ell}\lvert\hat{\gamma}_\ell(s')\rvert^2 ds'}{(s'-s)s'}$

$$\boxed{\xi^{2\ell} = \frac{p^{2\ell}}{2\ell+1} B_\ell^2 \left(\frac{s-s_{thr}}{s_0-s_{thr}}\right)}$$

$$T = \frac{\xi^{2\ell} f_0}{f(s) - f_0 \Pi_\ell(s)} - \frac{g^2\gamma_\ell(s)^2}{s - m^2 + g^2\Sigma_\ell(s)}$$

Parameter determination

Pole adjustment

$$s_p - m^2 + g^2\Sigma^{II}(s_p) \stackrel{!}{=} 0$$

$\rightarrow$ m and g fixed by pole position

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Residue adjustment

$$\frac{1}{2\pi \mathrm{i}} \oint_C T^{II}(s', f_0, \dots, f_i) ds' = \frac{1}{2\pi \mathrm{i}} \oint_C \frac{-Zg^2 {\gamma^{II}}^2}{s - s_p} ds' = -\tilde{g}^2(f_0, \dots, f_i)$$

- Determine background parameters $(f_0, \dots, f_i)$ from residue numerically

Parameter determination

Pole adjustment

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Residue adjustment

$$\frac{1}{2\pi \mathrm{i}} \oint_C T^{ll}(s', f_0, ..., f_i) ds' = \frac{1}{2\pi \mathrm{i}} \oint_C \frac{-Zg^2 {\gamma^{ll}}^2}{s - s_p} ds' = -\tilde{g}^2(f_0, ..., f_i)$$

- Determine background parameters $(f_0, ..., f_i)$ from residue numerically

$\Rightarrow$ one pole and one residue (=4 inputs) $\rightarrow$ m, g, f_0, f_1

Different versions of the model

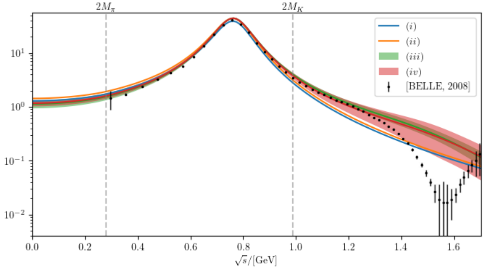
(i) $f_0 = f_1 = 0 \quad \Rightarrow$ no background at all

(ii) $f_0 \neq 0, \quad f_1 = 0 \Rightarrow$ only an additional constant

(iii) $f_0,\ f_1 \neq 0 \quad \Rightarrow$ constant and lhc

(iv) $f_0,\ f_1 \neq 0$ and $K\bar{K}$ included fully coupling to resonance

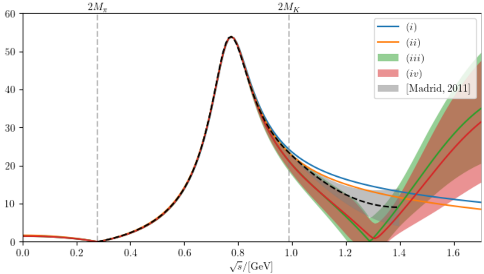
Results: the $\rho(770)$

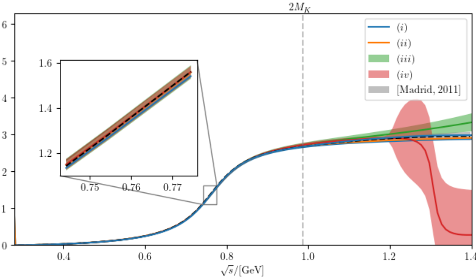


(i) Already residue within 2%

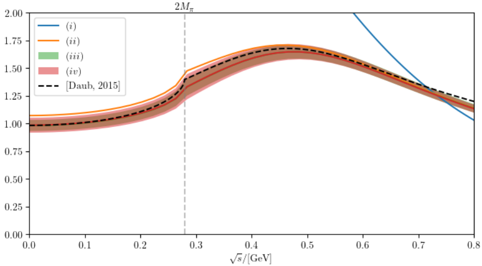
(ii) Residue within 1%

(iii) Exact residue
additional resonance
structure in vicinity of ρ'

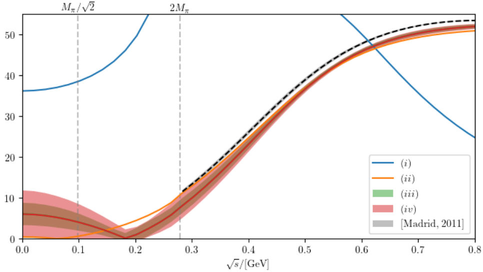


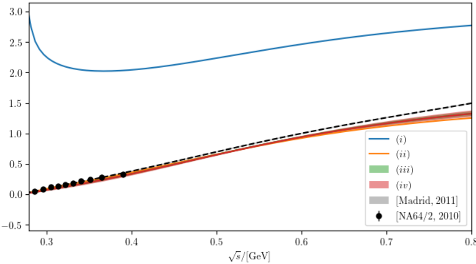


Results: The $f_0(500)$



- (i) Phase of residue off by 100%
- (ii) Residue within 10%
  $\rightarrow$ zero below threshold
- (iii) Perfect residue
  $\rightarrow$ zero below threshold





How to include more channels

For the $\rho(\mathbf{770})$:

1. $\bar{K}K$ included fully via VMD approximation $g^2_{\bar{K}K} = \frac{g^2_{\pi\pi}}{2}$
2. $\pi\gamma$ defined absolute value of coupling $|\tilde{g}_{\pi\gamma}|^2 = |Zg^2_{\pi\gamma}|$ contribution in propagtor neglected

For the $\boldsymbol{f_0}(\mathbf{500})$:

1. $\bar{K}K$ included fully via third fitting parameter
2. $\gamma\gamma$ defined absolute value of coupling $|\tilde{g}_{\gamma\gamma}|^2 = |Zg^2_{\gamma\gamma}|$ contribution in propagtor neglected

$$Z = \frac{\tilde{g}_i^2}{g_i^2 \gamma_i^{II}(s_p)^2}$$

Branching ratios

Narrow-width formula connects pole $\sqrt{s_p} = (M_R - \mathrm{i}\Gamma_R/2)$, residue $\tilde{g}_i$ and branching ratios:

$$\Gamma_i = \frac{|\tilde{g}_i|^2}{M_R} \rho_i(M_R^2)$$

Branching ratios can be calculated

$$\mathrm{BR}(R \to i) = \frac{\Gamma_i}{\Gamma_R}$$

- Why is there an absolute?
- What happens with closed channels?
- How well does this work for non-trivial lineshapes?

Narrow-width limit in our formalism

$$D^{(n)}(s_p)^{-1} = s_p - m^2 + \sum_i g_i^2 \Sigma_i^{(n)}(s_p) = 0$$

$$\Gamma_R = \frac{1}{M_R}\mathrm{Im}\left(\sum_i g_i^2 \Sigma_i^{(n)}(s_p)\right) = \sum_i \underbrace{(-1)^{(n)} \frac{g_i^2}{M_R} |\mathrm{Im}\left(\Sigma_i^{(n)}(s_p)\right)|}_{=\Gamma_i^{(p)} \Rightarrow \Gamma_i^{(p,m)} = |\Gamma_i^{(p)}| / \sum_i |\Gamma_i^{(p)}|}$$

$$\approx \sum_k \underbrace{\frac{g_k^2}{M_R}\rho_i(M_R^2)}_{\approx \Gamma_i^{(\mathrm{nw})}}$$

Can be negative!

Limit of non-interacting mesons ($\Sigma \approx \Pi$), narrow resonances ($s_p \approx M_R^2$) and all channels open ($(-1)^{(n)} = 1$)

A better way to determine branching ratios

$\rightarrow$ Count events

$$r_i = \int_{s_{thr,i}}^{\infty} ds |D(s)|^2 g_i^2 \gamma_i^2 \rho_i(s)$$

$$\mathrm{BR}^{(\mathrm{cr})}(R \rightarrow i) = \frac{r_i}{\sum_j r_j}$$

$\sum_j r_j = \pi$, Källén-Lehmann representation

$$\sum_j r_j = \int_0^{\infty} \frac{\sum_j g_j^2 \rho_j |\gamma_j|^2 \Theta(s - s_{thr,j})}{|s - m^2 + \sum_k g_k^2 \Sigma_k(s)|^2} ds = -\int_0^{\infty} \mathrm{Im}(D(s)) ds$$

Branching ratio results

resonance	channel (a)	narrow width $\mathrm{Br}_a^{(\mathrm{nw})}$	pole location $\mathrm{Br}_a^{(\mathrm{p})}$	$\mathrm{Br}_a^{(\mathrm{p,m})}$	this work $\mathrm{Br}_a^{(\mathrm{cr})}$
$f_0(500)$	$\pi\pi$	0.8(1)	1.03(1)(0)	0.97(0)(0)	0.970(5)(12)
	$\gamma\gamma \quad \times 10^6$	3.0(7)	5.0(1.6)(0.3)	4.8(1.5)(0.3)	1.4(4)(3)
	$\bar{K}K$	0	−0.03(1)(0)	0.03(1)(0)	0.030(5)(12)
	sum	0.8	1.0	1.0	1.0
$\rho(770)$	$\pi\pi$	1.007(14)	1.04(1)(4)	0.96(1)(3)	0.95(4)(3)
	$\pi\gamma \quad \times 10^4$	5.1(1.1)	3(1)(6)	3(1)(3)	12(1)(4)
	$\bar{K}K$	0	−0.05(1)(3)	0.04(1)(3)	0.05(4)(3)
	sum	1.0	1	1	1

Conclusion and outlook

- Flexible parametrization for resonances
- Adjusted parameters to recreate residue and pole
- Used lineshapes to determine branching ratio unambiguously
- Improve: more fully implemented channels, more resonances
- Outlook: model used in project for analysis of $\pi\pi$ and $\pi\omega$ data

Thank you!

Numerics – challenges that we overcame (mostly)

$\Rightarrow$ Stacked integrals en masse

- $\Pi(s)$ needs dispersion integral
- $\Omega(s)$ depends on $\Pi(s)$ and needs dispersion integral
- $\Sigma(s)$ depends on $\Omega(s)$ and needs dispersion integral
- $\tilde{g}$ depends on all of them and needs circular integral

Pythons vectorization and precalculated Πs to ran code reasonably fast

Calculation on a lattice of parameter pairs (f_0, f_1)

The barrier discussion -positivity what?

Positivity

$$\Rightarrow \int_0^\infty \mathrm{d}\mu^2 \rho(\mu^2) \stackrel{!}{=} 1 \Rightarrow \rho(\mu^2) \propto \left(\frac{1}{\mu^2}\right)^{1+\frac{n}{2}} \quad n \stackrel{!}{>} 0$$

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Propagator can be expressed as $\Delta'(p^2) = \int_0^\infty \mathrm{d}\mu^2 \frac{\rho(\mu^2)}{\mu^2+p^2-i\epsilon}$

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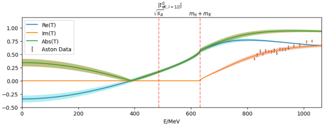
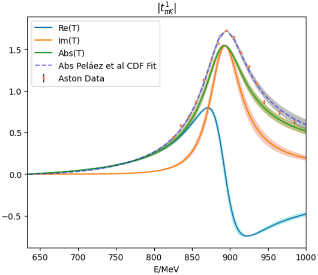
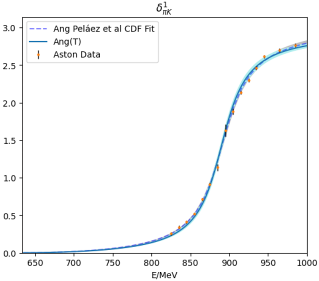
$\forall n > 0 \Rightarrow \lim_{p^2\to\infty} \Delta'(p^2) \propto \frac{1}{p^2}$

If propagator falls faster, spectral density $\rho = -\frac{1}{\pi}\mathrm{Im}(D)$ not normalized

Some πK results







Adler zeros

Chiral symmetry causes zero in scalar QCD amplitudes
Dependent on reaction and isospin configuration
Chiral limit $s_A = 0$
Non-zero quark masses $\to$ s_A gets corrections (with every O in chiral expansion)

$$s_A^{(2)} = \frac{M_\pi^2}{2}\,, \qquad \frac{s_A^{(4)}}{s_A^{(2)}} = -0.12(3)\,, \qquad \frac{s_A^{(6)}}{s_A^{(2)}} = -0.03(1)$$