

# Two-photon exchange processes in $e^+N$ scattering in resonance region: Analysis using $1/N_c$ expansion

C. Weiss (JLab), with J. Goity, JLab Positron WG Meeting 09 Aug 2023



Computed target normal single-spin asymmetry from two-photon exchange (TPE) in inclusive scattering  $eN \rightarrow e'X$  (and elastic scattering  $eN \rightarrow e'N$ ) in resonance region  $\sqrt{s} \lesssim 1.5$  GeV

Used systematic method based on  $1/N_c$  expansion of QCD: Parametric expansion, controlled accuracy,  $N$  and  $\Delta$  states related by spin-flavor symmetry

Planning several applications/extensions relevant to JLab positron program:  
Beam normal spin asymmetry,  $e^+$ - charge asymmetry, duality DIS — resonance region

J.L. Goity, C. Weiss, C.T. Willemyns, Phys. Lett. B 835, 137580 (2022) [\[INSPIRE\]](#)

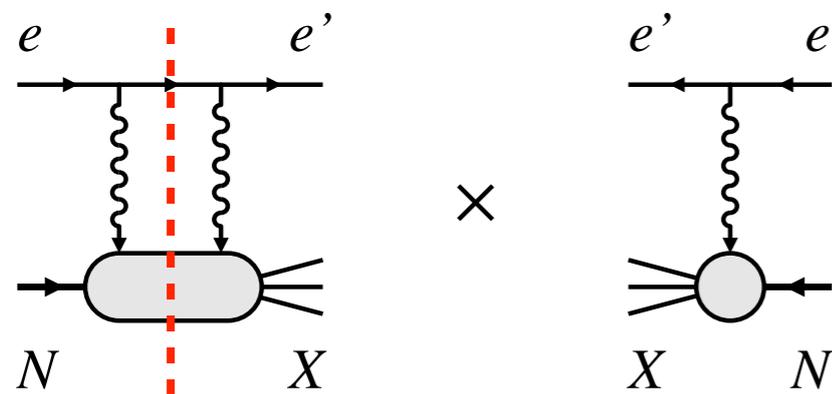
J.L. Goity, C. Weiss, C.T. Willemyns, Phys. Rev. D 107, 094026 (2023) [\[INSPIRE\]](#)

TPE has become field or research in its own right

Elastic  $ep$  cross section: TPE as radiative correction, involves  $\text{Re}(\text{TPE})$  and  $\text{Im}(\text{TPE})$   
 Much theoretical work, situation still inconclusive

Direct measurements:  $e^\pm N$  charge asymmetry,  $eN(\uparrow)$  target normal spin asymmetries

## Target normal single-spin asymmetry



Zero at  $O(\alpha^2)$ , pure  $O(\alpha^3)$  effect

Interference on one- and two-photon exchange  
 Also contribution from Bethe-Heitler - Virtual Compton interference

Involves only  $\text{Im}(\text{TPE})$ : Finite integral, on-shell amps

$eN \rightarrow e'X$  inclusive

$eN \rightarrow e'N$  elastic

Inclusive or elastic scattering

$$A_N = \frac{\sigma\uparrow - \sigma\downarrow}{\sigma\uparrow + \sigma\downarrow}$$

Can be measured in wide kinematic range:  
 Low-energy — resonance region — DIS

## Theoretical calculations

DIS region: QCD mechanism based on vacuum structure. Afanasev, Strikman, Weiss 2008

Various pQCD-based mechanisms. Metz, Schlegel, Goeke 2006; Metz et al. 2012; Schlegel 2013

Large variations  $A_N \sim 10^{-4} - 10^{-2}$

$A_N(\text{DIS}) \ll A_N(\text{low-energy elastic})$  because anomalous magnetic moment of quark  $\ll$  nucleon

*How does transition from low-energy to DIS regime happen? Need to explore resonance region!*

## Experiments

HERMES 2014: p target,  $W > 2$  GeV,  $A_N \sim 10^{-2}$

JLab Hall A Katich et al. 2014:  $^3\text{He}$  target,  $W = 1.7\text{-}2.9$  GeV,  $A_N \sim 10^{-2}$

Proposal JLab Hall A Grauvogel, Kutz, Schmidt 2021: p target,  $E_e = 2.2, 4.4, 6.6$  GeV

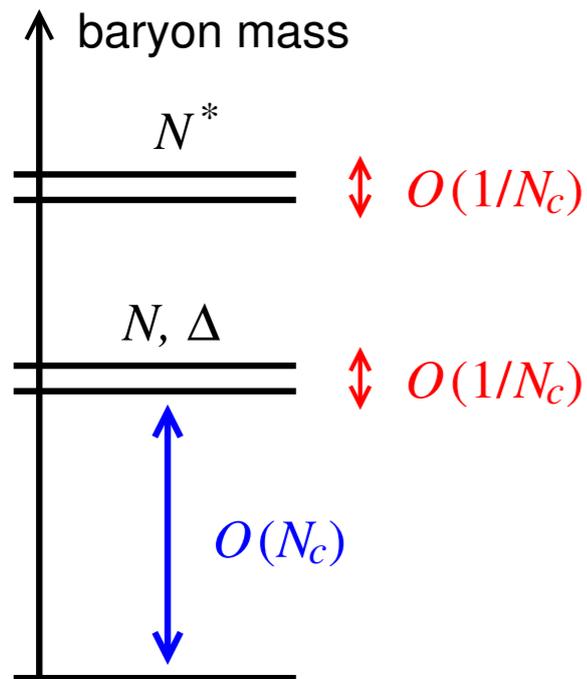
## Resonance region

$N, \Delta, N^*$  + nonresonant  $\pi N$  as final states and intermediate states in TPE

Need to combine contributions of channels at amplitude level – cancellations?

Need transition currents  $\langle \Delta | J | N \rangle, \langle \Delta | J | \Delta \rangle$  etc.

*Develop systematic approach based on  $1/N_c$  expansion!*



## Large- $N_c$ limit of QCD

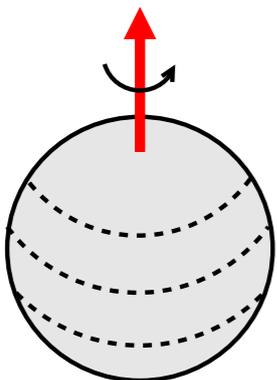
Semiclassical limit of QCD ‘tHooft 1974, Witten 1979

Hadron masses, couplings, matrix elements scale in  $N_c$   
 “Organization” of non-perturbative dynamics

Emerging dynamical spin-flavor symmetry  $SU(2N_f)$   
 Baryons in multiplets with masses  $O(N_c)$ , splittings  $O(1/N_c)$   
 Gervais, Sakita 1984; Dashen, Manohar, Jenkins 1993

$N \rightarrow N$  and  $N \rightarrow \Delta$  transitions related by symmetry:  
 $\langle \Delta | \mathcal{O} | N \rangle = [\text{symmetry factor}] \times \langle N | \mathcal{O} | N \rangle$

$S = I = 1/2, 3/2$



## $1/N_c$ expansion of hadronic matrix elements

Parametric expansion: Systematic, predictive, controlled accuracy

Applied to current matrix elements, hadronic amplitudes  
 Vector and axial currents: Fernando, Goity 2020

Generators of spin-flavor group algebra:  $\hat{S}^i, \hat{I}^a, \hat{G}^{ia}$

Matrix elements between ground-state baryons from symmetry:

$$\langle B(S', S'_3, I'_3) | \dots | B(S, S_3, I_3) \rangle = \text{fun}(N_c) \times \text{Clebsches} \quad S, S' = 1/2, 3/2 \quad B = N, \Delta$$

EM current operator expressed through generators:

$$J_S^\mu(q) = G_E^S(q^2) \frac{1}{2} g^{\mu 0} - i \frac{1}{2} \frac{G_M^S(q^2)}{\Lambda} \epsilon^{0\mu ij} q^i \hat{S}^j,$$

$$J_V^{\mu a}(q) = G_E^V(q^2) \hat{I}^a g^{\mu 0} - i \frac{6}{5} \frac{G_M^V(q^2)}{\Lambda} \epsilon^{0\mu ij} q^i \hat{G}^{ja},$$

$$J_{\text{EM}}^\mu(q) = J_S^\mu(q) + J_V^{\mu 3}(q),$$

$q^0 = \mathcal{O}(N_c^{-1}), q^i = \mathcal{O}(N_c^0)$   
momentum transfer

$G_{E,M}^{V,S}(q^2)$  form factors

Expresses parametric expansion in  $1/N_c$

Charges/form factors fixed from  $N \rightarrow N$  matrix elements

Predicts  $N \rightarrow \Delta$  and  $\Delta \rightarrow \Delta$  matrix elements

## Kinematic variables in inclusive electron scattering

$$e(k) + N(p) \rightarrow e(k') + X(p')$$

$$s = (k + p)^2$$

CM energy

$$q^2 = (k - k')^2$$

momentum transfer

$$M_X^2 = p'^2 = (p + q)^2$$

final-state mass

## Kinematic regimes in 1/N<sub>c</sub> expansion

	Energy regime	1/N <sub>c</sub> expansion regime	Channels open	Final states possible
I	$m_N < \sqrt{s} < m_\Delta$	$\sqrt{s} - m_N \sim N_c^{-1}, k \sim N_c^{-1}$	$N$	elastic
II	$m_\Delta < \sqrt{s} \ll m_{N^*}$	$\sqrt{s} - m_N \sim N_c^{-1}, k \sim N_c^{-1}$	$N, \Delta$	elastic or inelastic
III	$m_\Delta < \sqrt{s} \lesssim m_{N^*}$	$\sqrt{s} - m_N \sim N_c^0, k \sim N_c^0$	$N, \Delta, N^*$ (suppr)	elastic or inelastic

“low energies”

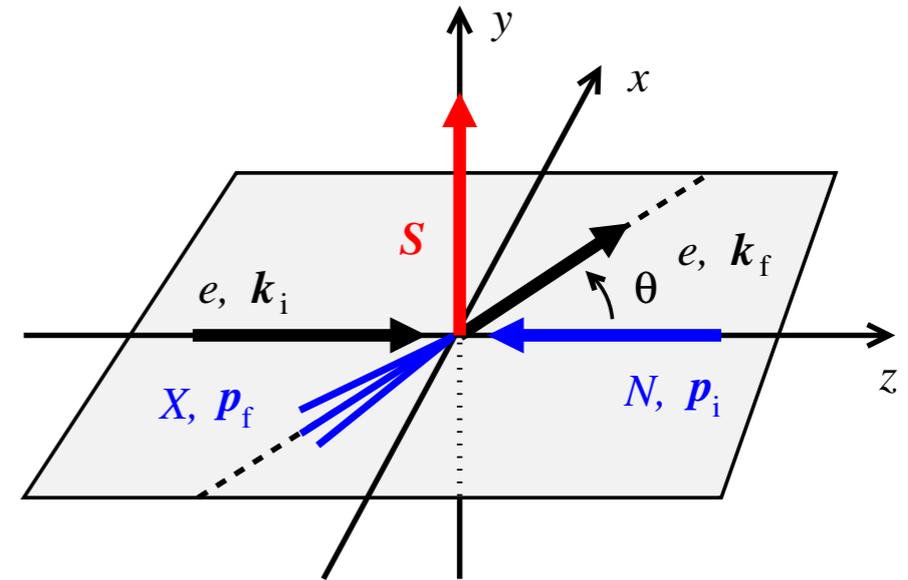
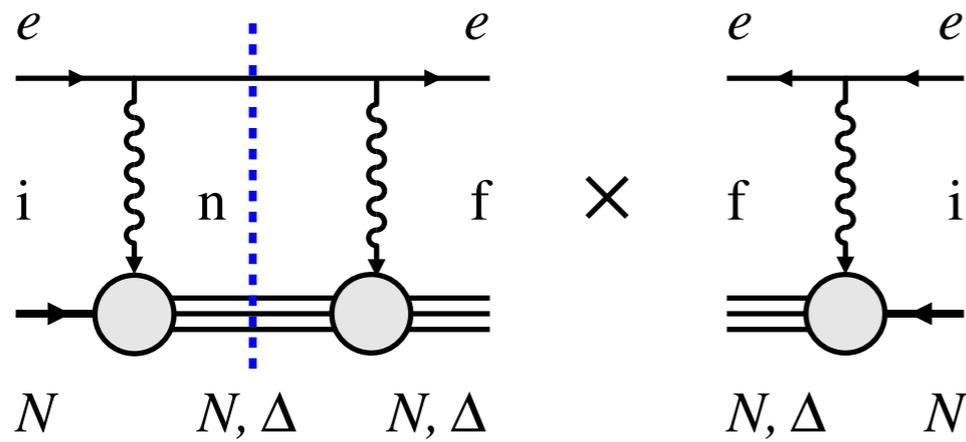
“intermediate energies”

$$k = (s - m^2)/2\sqrt{s} \quad \text{CM momentum}$$

Expansion can be applied in different kinematic regimes: Different “focus”, reach, accuracy

Systematic calculation, defined accuracy, could be improved by higher-order corrections

Non-resonant  $\pi N$  states suppressed in 1/N<sub>c</sub> relative to  $\Delta$

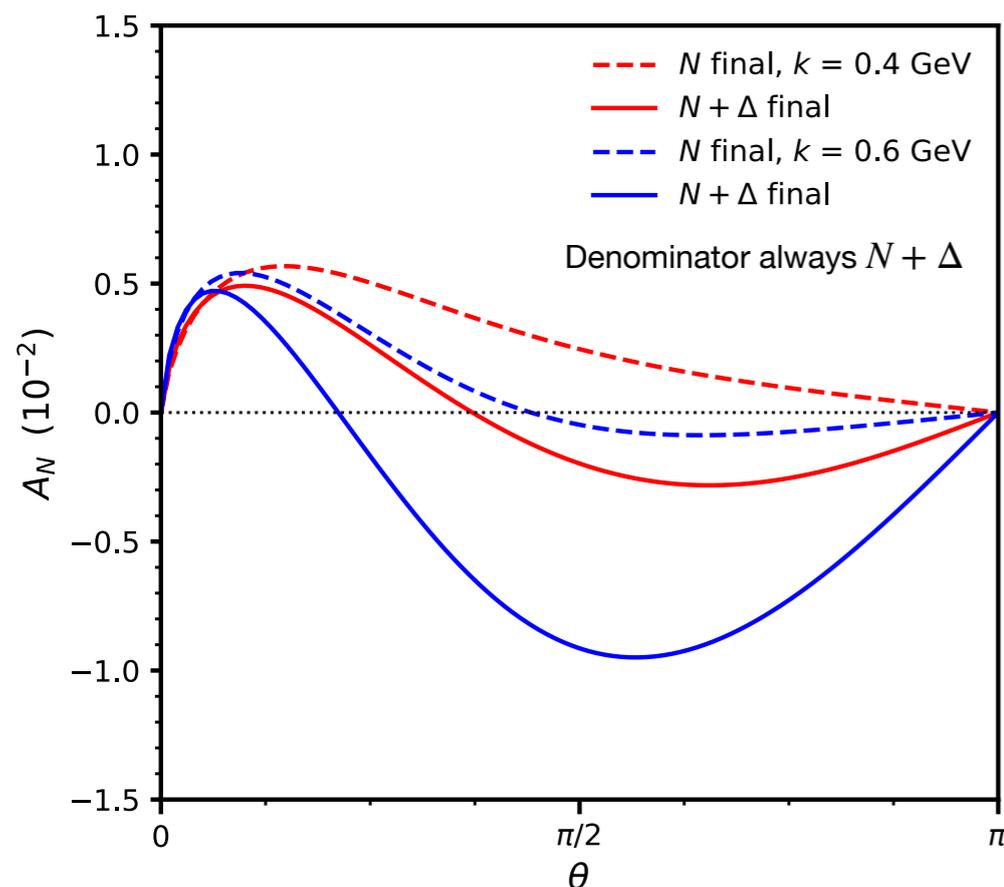


Calculate  $eB \rightarrow e'B'$  amplitudes for  $B, B' = N, \Delta$  with  $1/N_c$ -expanded currents

Integrate over phase space of intermediate state in TPE

Sum over intermediate and final states

Project out normal-spin dependent part of cross section



$A_N$  at intermediate energies (regime III)  
LO  $1/N_c$  expansion result

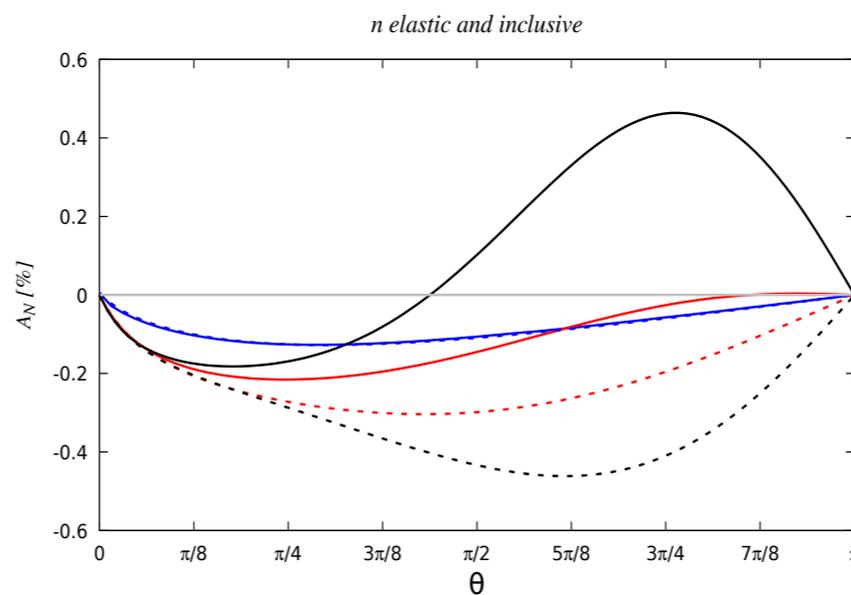
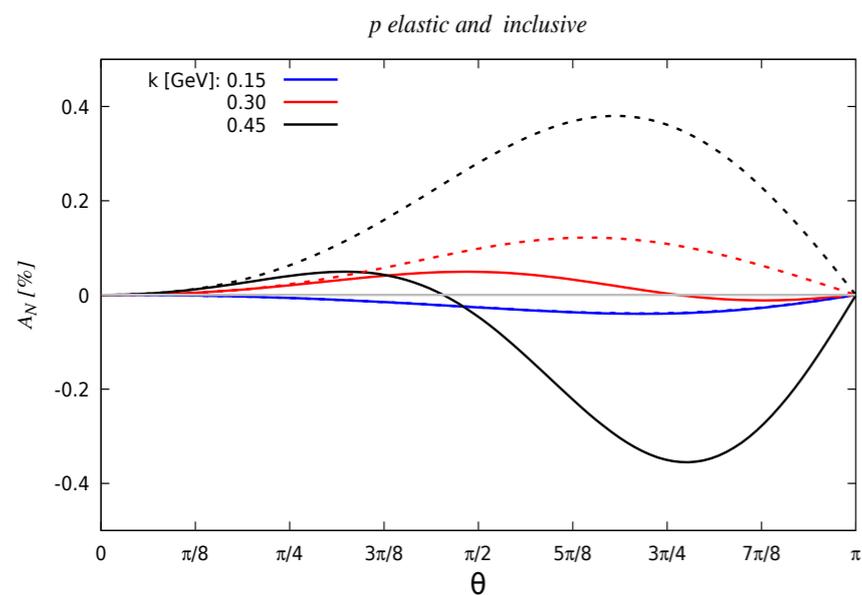
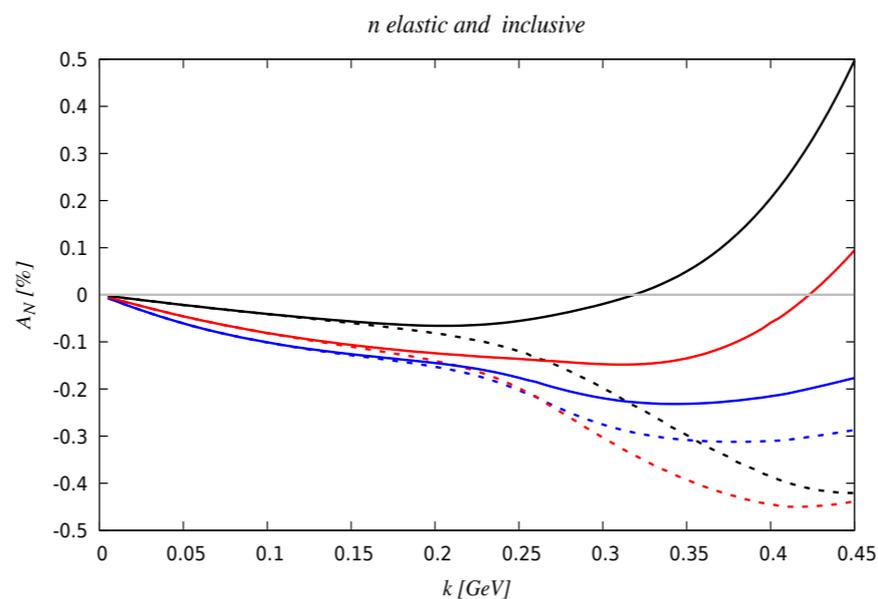
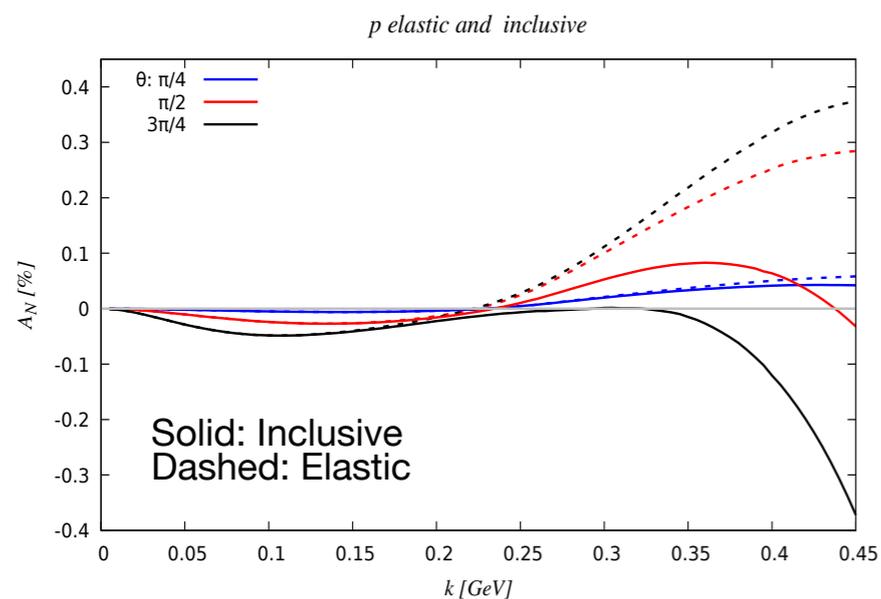
Valid for  $1.23 \text{ GeV} < \sqrt{s} \lesssim 1.5 \text{ GeV}$   
or  $0.3 \text{ GeV} < k \lesssim 0.6 \text{ GeV}$ ,  
and  $\theta \sim \pi/2$  “large angle”

$A_N \sim 10^{-2}$  predicted in intermediate-energy regime

Large contribution of  $\Delta$  final states at angles  $\theta \sim \pi/2$ , could be tested experimentally!

LO  $1/N_c$  expansion result: All transition currents magnetic isovector  $G^{ia}$ , simple structure. Electric currents come in at higher orders

$A_N$  is overall isovector:  $A_N(\text{proton}) = -A_N(\text{neutron})$



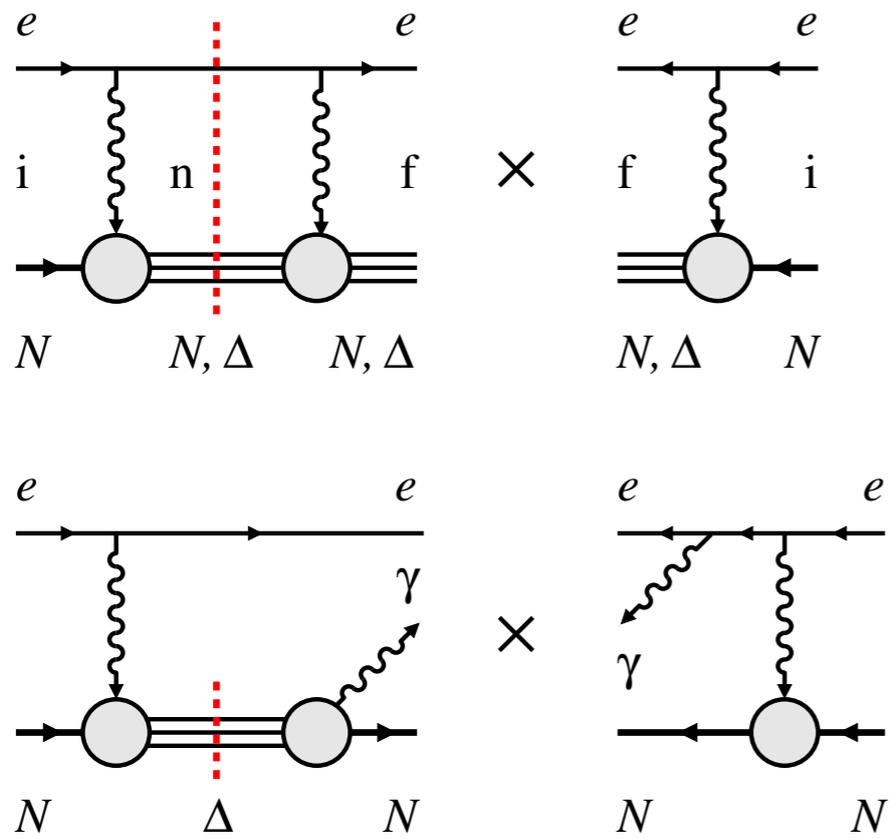
$A_N$  at low energies  
(regimes I and II)

LO + NLO  $1/N_c$   
expansion result

Includes finite  
 $\Delta$  width

$A_N$  rises steeply as function of energy above  $\Delta$  threshold (here: CM momentum  $k$ )

Large contribution of  $\Delta$  final states



$A_N$  in inclusive  $eN$  scattering also receives contribution from real photon emission channel

Interference of Virtual Compton Scattering and Bethe-Heitler amplitudes

$\text{Im}(\text{VCS}) \neq 0$  above  $\Delta$  threshold

$1/N_c$  expansion: Real photon emission process suppressed by  $1/N_c$  relative to TPE

$1/N_c$  expansion guides analysis and interpretation of TPE processes

Computed/analyzed  $A_N$  from TPE in  $eN$  scattering in resonance region  
in systematic approach based on  $1/N_c$  expansion

$A_N$  predicted to be  $\sim 10^{-2}$ , should be measurable

## Interesting features that could be studied experimentally

[→ Discussion](#)

Separate contributions of  $N$  and  $\Delta$  final states in spin-dependent cross section (= numerator)

Energy and angular dependence of spin-dependent cross section and  $A_N$

Isospin dependence proton-neutron of spin-dependent cross section and  $A_N$

Transition from resonance to DIS region - qualitative changes?

## Possible theoretical improvements

Higher-order  $1/N_c$  corrections in intermediate-energy regime  $\rightarrow N^*$  states, real  $\gamma$  emission

Combined chiral and  $1/N_c$  expansion in low-energy regime  $\rightarrow \pi N$  states

$1/N_c$  expansion enables systematic approach to  $eN$  scattering in resonance region:  
Organizes kinematics, channels  $\Delta \leftrightarrow \pi N$ , currents, calculation

## Applications to TPE and positron physics

Beam normal spin asymmetry: Pure TPE effect,  $\propto m_{\text{lepton}}$ , enhanced by collinear logarithm

Charge asymmetry of  $e^\pm N$  cross section: Involves also  $\text{Re}(\text{TPE})$ , obtained dispersion integral

Electroweak processes,  $\gamma Z$  exchange

## Applications to hadronic physics

Transition between resonance and DIS regions, quark-hadron duality

Spin effects in intermediate-energy  $eN$  scattering