E45 PWA/Data Analysis Framework

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1. Develop programs to calculate differential cross sections using given partial wave amplitudes for π N -> π N elastic and π N -> π π N channels.
   https://www.phy.anl.gov/theory/research/anl-osaka-pwa/

2. Develop a partial wave program (PWA) to extract resonance parameters for given measured differential cross sections for π N -> π N elastic and π N -> π π N channels .

3. Develop event generators using the differential cross sections from (1) for data analysis.

Differential Cross Section for Event Generator

- Develop differential cross section programs
  - Interpolation using resonance amplitude grid tables from different models (ANL-Osaka model, SAID model, etc.)
  - Differential cross section calculation is model-independent
  - π N -> π N elastic and π N -> π π N channels
    - π N -> π N elastic channel.

Partial wave grid tables

$$\frac{d\sigma_{MB\to M'B'}}{d\Omega_{k'}} = \frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M}m_{j_B}}\sum_{m'_{j_M}m'_{j_B}}|<M'B'|t(W)|MB>|^2$$

    - π N -> π π N channel
      - π N -> π Δ ->π π N
      - π N -> σ N ->π π N
      - π N -> ρ N ->π π N

π N -> π N elastic differential cross sections

- Partial wave resonance amplitude tables from ANL-Osaka/ SAID model websites

  https://www.phy.anl.gov/theory/research/anl-osaka-pwa/

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  ANL-osaka amplitudes : < MB (nf)|T| MB (ni) >
  Relation to S-matrix : S = 1 + 2 i T
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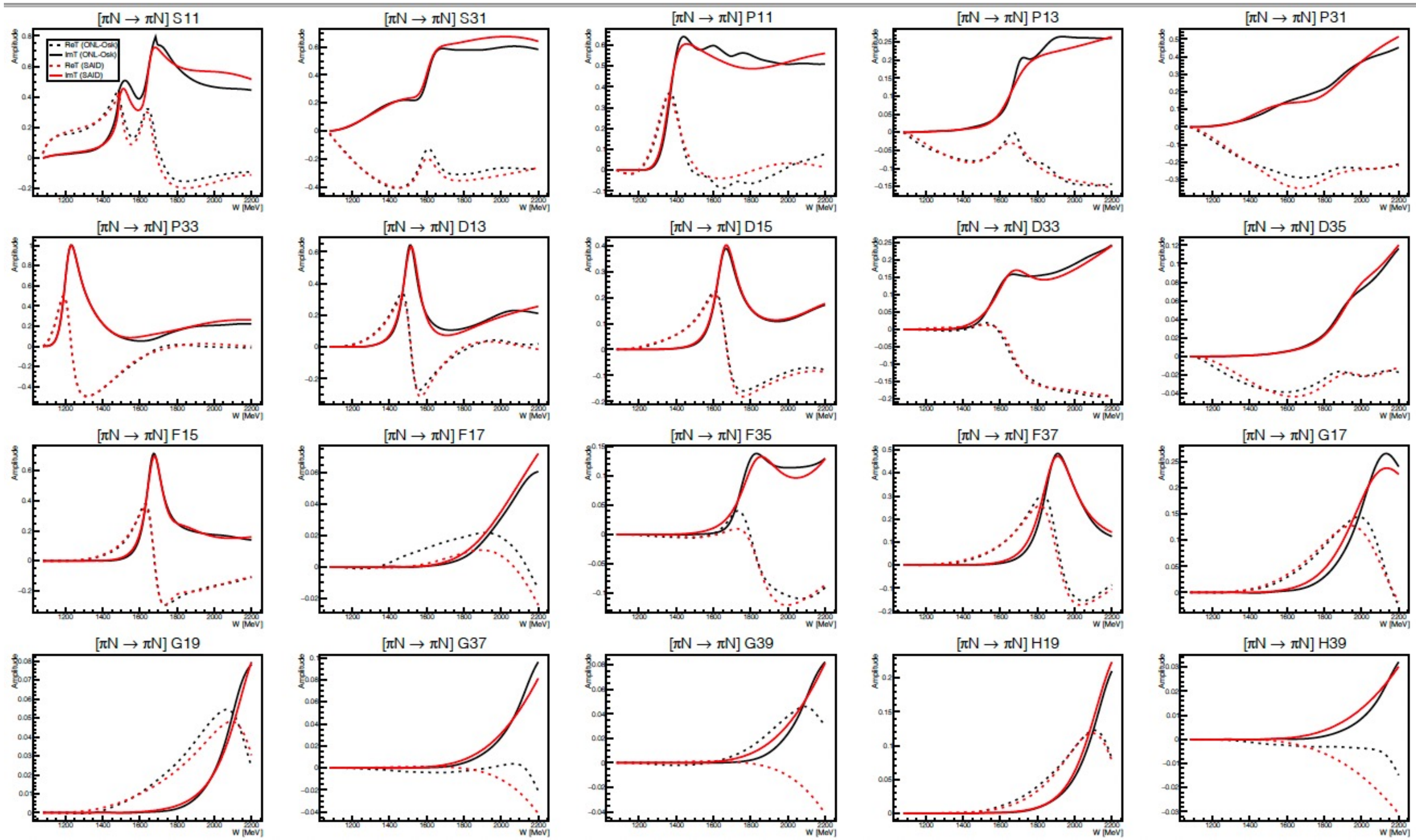
 Requested Amplitudes
 MB(nf)=  pi-N    ,   MB(ni)=  pi-N


  Partial-wave = S11
  W (MeV)    < MB (nf)|T| MB (ni) >
 0.1080E+04  0.3166E-01  0.9945E-03
 0.1085E+04  0.5210E-01  0.2561E-02
 0.1090E+04  0.6643E-01  0.4459E-02
 0.1095E+04  0.7784E-01  0.6283E-02
 0.1100E+04  0.8749E-01  0.7943E-02
 0.1105E+04  0.9593E-01  0.9486E-02
```

Comparison between ANL-Osaka and SAID

Partial wave grid tables

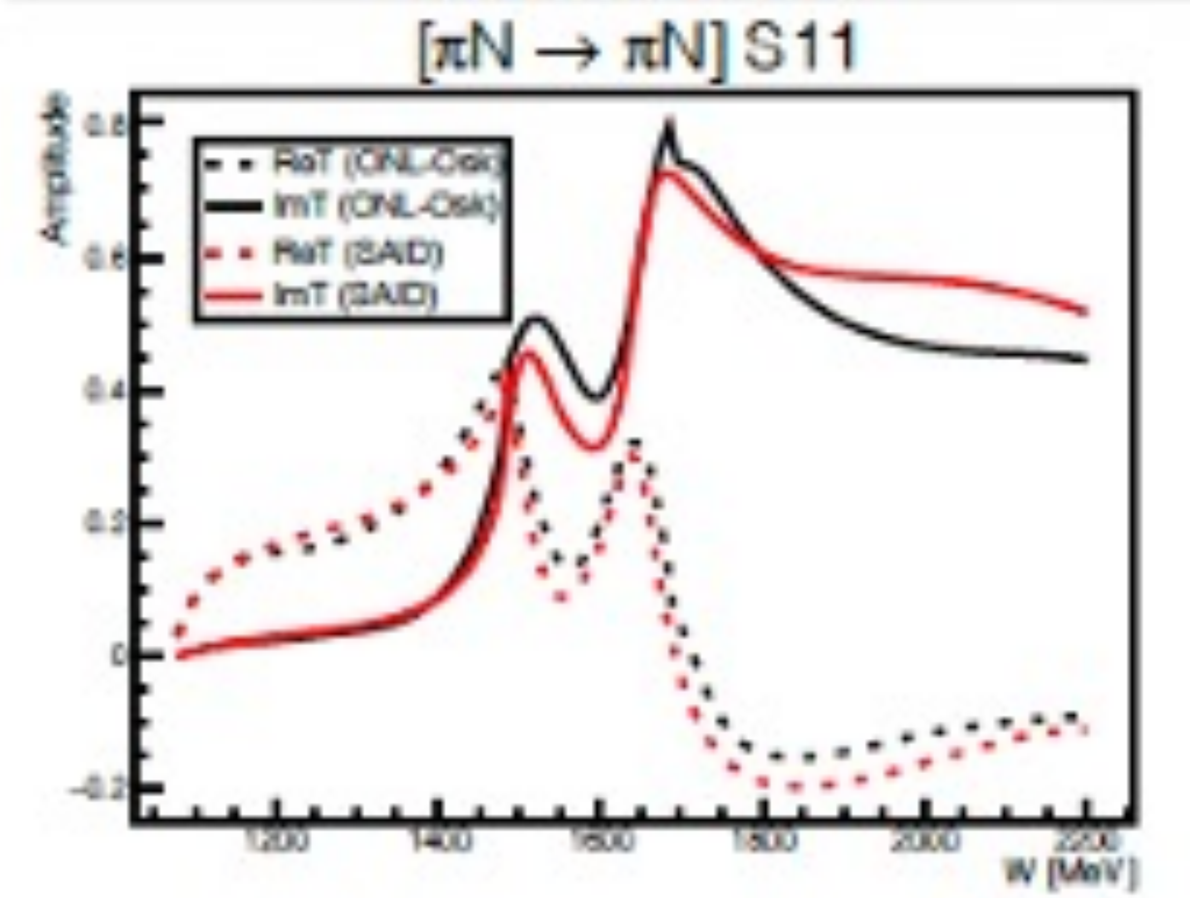
$$\frac{d\sigma_{MB\to M'B'}}{d\Omega_{k'}} = \frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M} m_{j_B}}\sum_{m'_{j_M} m'_{j_B}} |<M'B'|t(W)|MB>|^2$$



Comparison between ANL-Osaka and SAID

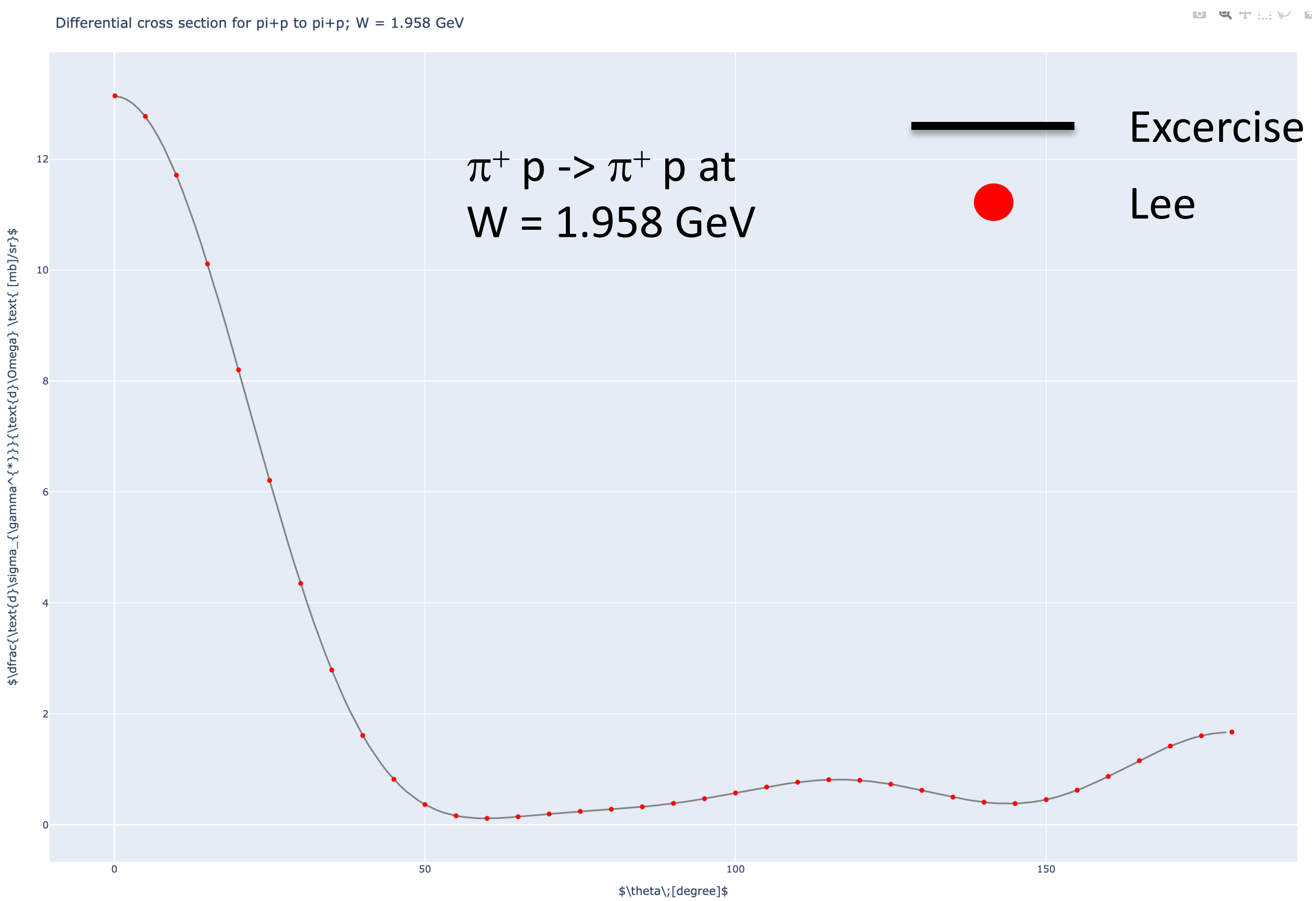
Partial wave grid tables

$$\frac{d\sigma_{MB\to M'B'}}{d\Omega_{k'}} = \frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M}m_{j_B}}\sum_{m'_{j_M}m'_{j_B}}|<M'B'|t(W)|MB>|^2$$



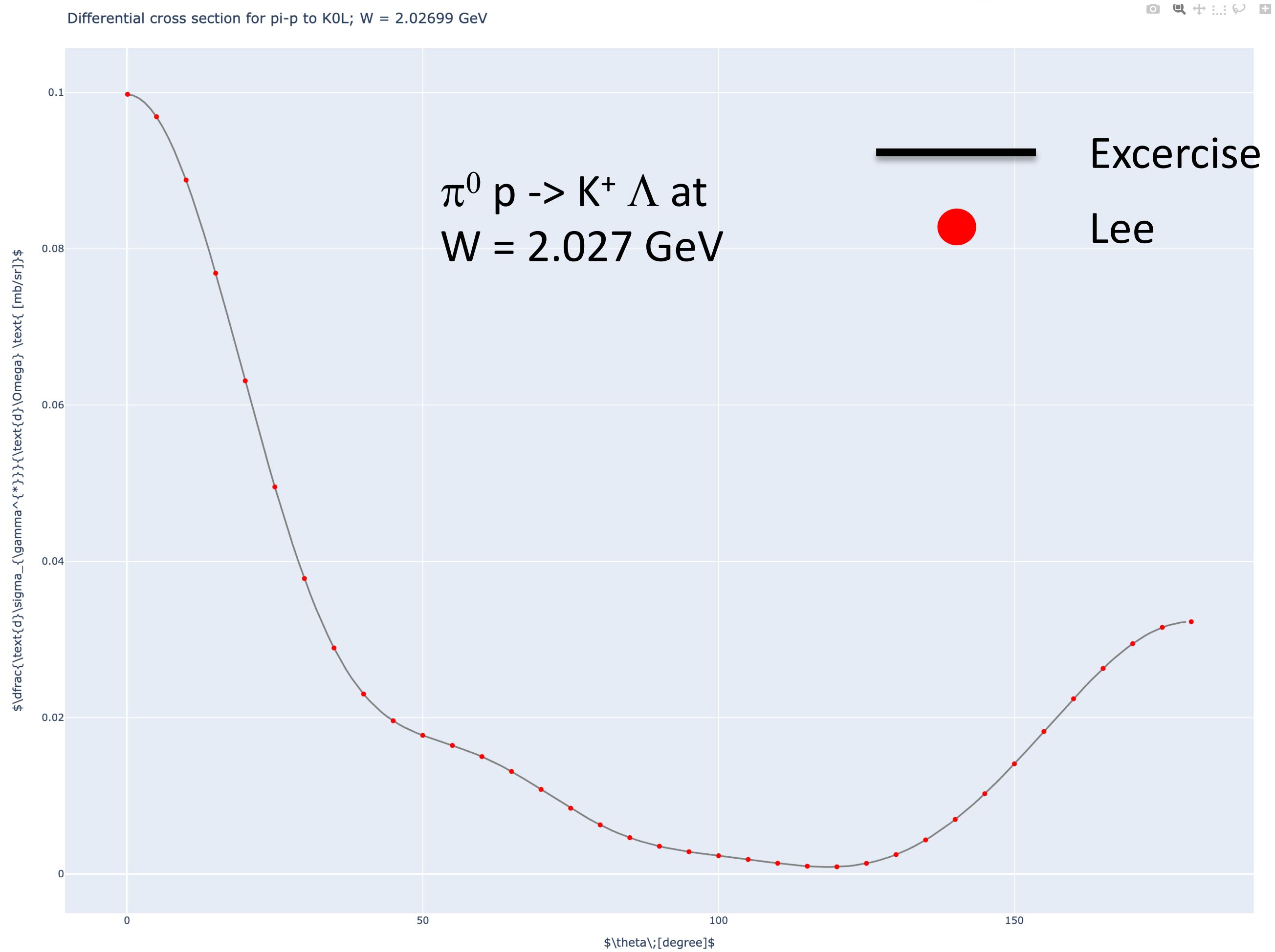
Validation of π N -> π N elastic differential cross sections

$$\frac{d\sigma_{MB\to M'B'}}{d\Omega_{k'}} = \frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M}m_{j_B}}\sum_{m'_{j_M}m'_{j_B}}|<M'B'|t(W)|MB>|^2$$



Validation of π N -> K Λ differential cross sections

$$\frac{d\sigma_{MB\to M'B'}}{d\Omega_{k'}} = \frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M}m_{j_B}}\sum_{m'_{j_M}m'_{j_B}}|<M'B'|t(W)|MB>|^2$$



π N -> π π N Differential Cross Section

- π N -> π Δ -> π π N
- π N -> σ N -> π π N
- π N -> ρ N -> π π N

2D table (W, p) from ANL-Osaka website

$$< \pi\Delta|T(W)|\pi N >= -\rho^{1/2}_{\pi\Delta}(p_\Delta)t^{JT}_{L'S'\pi\Delta,LS\pi N}(p_\Delta,k,W)\rho^{1/2}_{\pi N}(k) \ ,$$
$$< \rho N|T(W)|\pi N >= -\rho^{1/2}_{\rho N}(p_\rho)t^{JT}_{L'S'\rho N,LS\pi N}(p_\rho,k,W)\rho^{1/2}_{\pi N}(k) \ ,$$
$$< \sigma N|T(W)|\pi N >= -\rho^{1/2}_{\sigma N}(p_\sigma)t^{JT}_{L'S'\sigma N,LS\pi N}(p_\sigma,k,W)\rho^{1/2}_{\pi N}(k) \ ,$$

○ π N -> π π N Differential Cross Section

- π N -> π Δ -> π π N
- π N -> σ N -> π π N
- π N -> ρ N -> π π N

2D resonance amplitude table (W, p)

```
Partial-wave = S11 (pi-N);  L= 1, S= 1/2 (sigma-N )
 W=  1220  MeV
p (MeV)    < MB (nf)|T| MB (ni) >
     0.0  -0.0000E+00  -0.0000E+00
     5.0   0.1445E-03  -0.8761E-03
    10.0   0.4084E-03  -0.2477E-02
    15.0   0.7496E-03  -0.4546E-02
    20.0   0.1153E-02  -0.6990E-02
    25.0   0.1608E-02  -0.9753E-02
    30.0   0.2110E-02  -0.1279E-01
    35.0   0.2652E-02  -0.1608E-01
    40.0   0.3231E-02  -0.1960E-01
    44.0   0.3717E-02  -0.2254E-01
```

π N -> π π N Total Cross Section

- π N -> π Δ -> π π N
- π N -> σ N -> π π N
- π N -> ρ N -> π π N

$$\sigma_{\pi N \to aN} = \frac{4\pi}{k_0^2} \sum_{JT,L'S',LS} \frac{2J+1}{(2S_N+1)(2S_\pi+1)} |\rho_{aN}^{1/2}(k) t^{JT}_{L'S'aN,LS\pi N}(k,k_0;W) \rho_{\pi N}^{1/2}(k_0)|^2 \times \langle t_\pi, t_N, t_\pi^z, t_N^z | T, T^z \rangle^2 \ . \quad (135)$$

2D table (W, p) from ANL-Osaka website

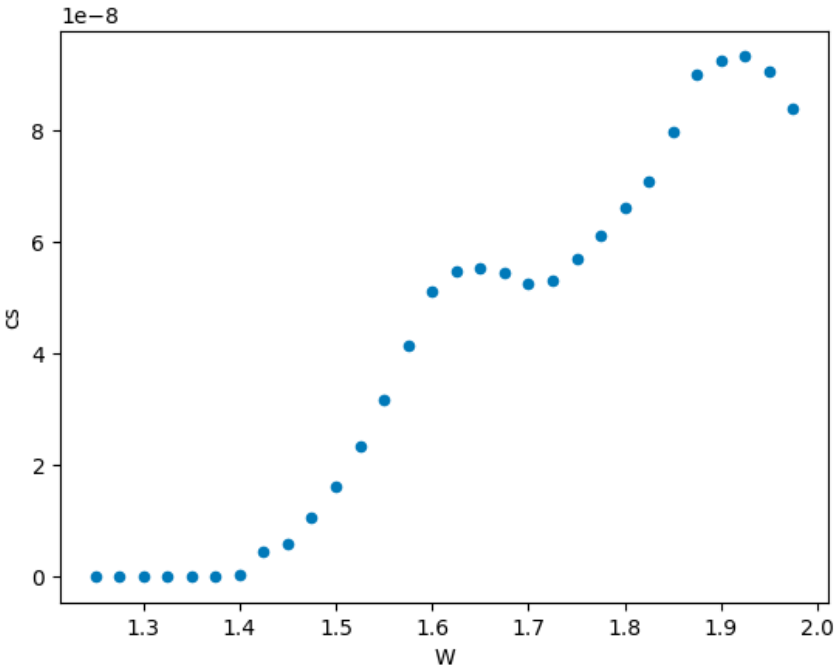
$$< \pi\Delta | T(W) | \pi N > = -\rho_{\pi\Delta}^{1/2}(p_\Delta) t^{JT}_{L'S'\pi\Delta,LS\pi N}(p_\Delta, k, W) \rho_{\pi N}^{1/2}(k) \ ,$$

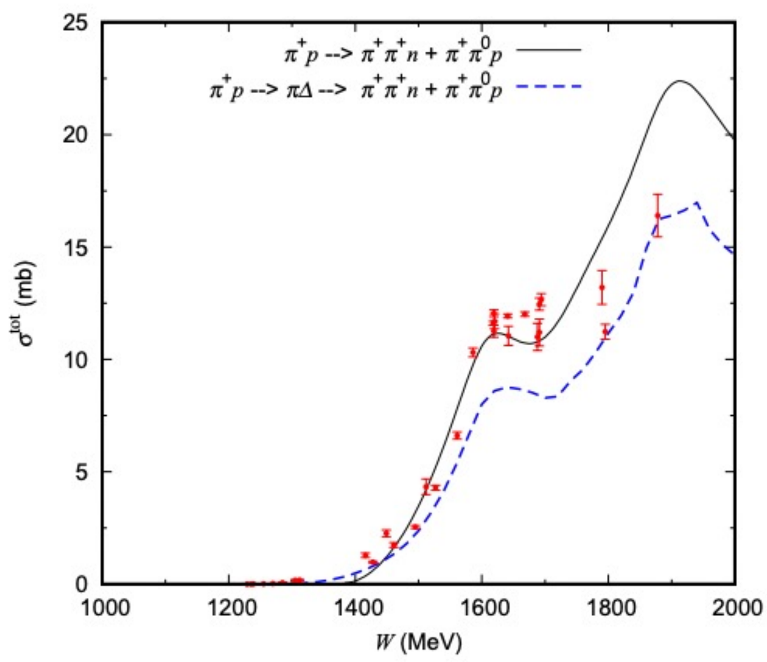
$$< \rho N | T(W) | \pi N > = -\rho_{\rho N}^{1/2}(p_\rho) t^{JT}_{L'S'\rho N,LS\pi N}(p_\rho, k, W) \rho_{\pi N}^{1/2}(k) \ ,$$

$$< \sigma N | T(W) | \pi N > = -\rho_{\sigma N}^{1/2}(p_\sigma) t^{JT}_{L'S'\sigma N,LS\pi N}(p_\sigma, k, W) \rho_{\pi N}^{1/2}(k) \ ,$$

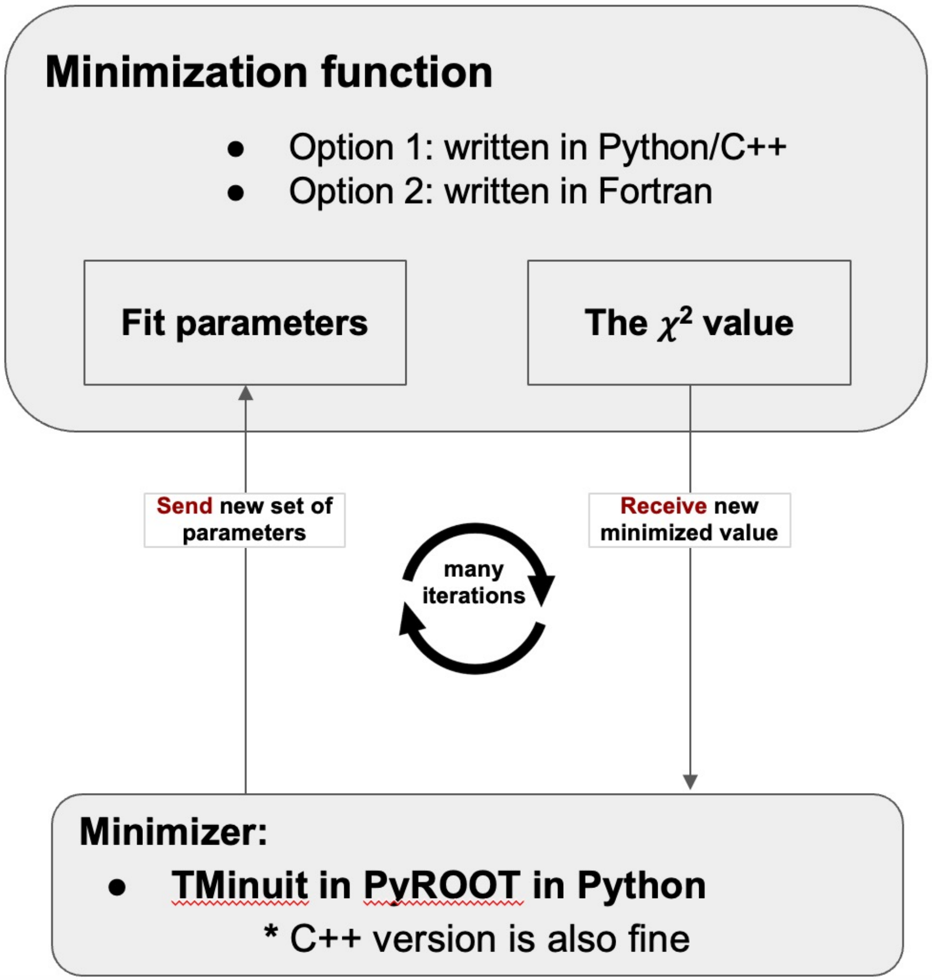
○ π p -> π Δ Total Cross Section

$$\sigma_{\pi N\to aN} = \frac{4\pi}{k_0^2}\sum_{JT,L'S',LS}\frac{2J+1}{(2S_N+1)(2S_\pi+1)}|\rho_{aN}^{1/2}(k)t^{JT}_{L'S'aN,LS\pi N}(k,k_0;W)\rho_{\pi N}^{1/2}(k_0)|^2 \times\langle t_\pi,t_N,t_\pi^z,t_N^z|T,T^z\rangle^2\ . \quad (135)$$





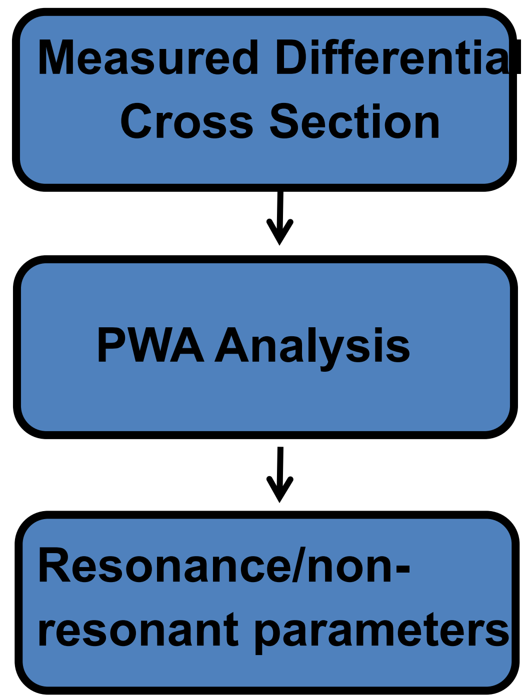
Partial Wave Analysis



$$\chi^2 = \sum_i^N \frac{\left(y_{experimental} - y_{theoretical}\right)^2}{\Delta y^2}$$

where N is the number of experimental data points

Develop partial analysis program by combining all the pieces A+B

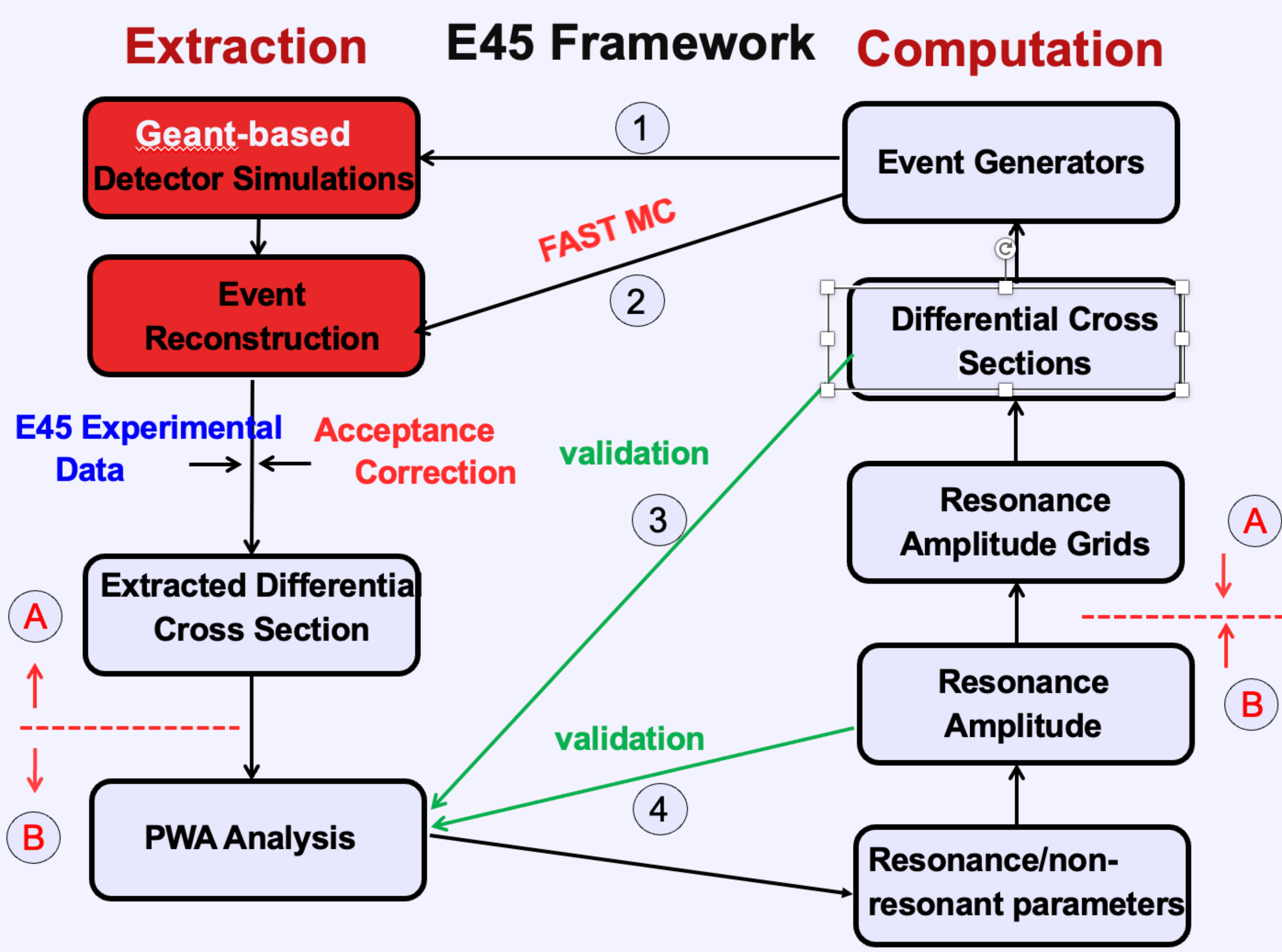


Challenges

1. Fit the JPARC data by varying ANL-Osaka PWA such as polynomials, dipole form or gaussian form.

2. There will have many parameters like 200 or more (M resonance x N parameters x r channels), and it is a challenge to fit the data with so many parameters.

3. It is the biggest unknown to me and it needs to develop strategies.


Extraction
E45 Framework
Computation
Geant-based
Detector Simulations
1
Event Generators
FAST MC
2
Event
Reconstruction
Differential Cross
Sections
E45 Experimental
Data
Acceptance
Correction
validation
3
Resonance
Amplitude Grids
A
Extracted Differential
Cross Section
A
B
Resonance
Amplitude
validation
4
B
PWA Analysis
Resonance/non-
resonant parameters